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PREFACE

American Civil Engineering Practice presents the fundamental principles, procedures, and data of modern civil engineering in concise form for reference purposes with illustrations from current practice. It replaces the *American Civil Engineers' Handbook*, which was first published in 1910 with Professor Mansfield Merriman as editor in chief. This handbook was revised through five editions, all based upon the original plan devised by Professor Merriman. It was recognized from the beginning as one of the most useful reference books available to the civil engineering profession. By 1950, however, the handbook had become obsolete because of the extraordinary development of new knowledge and new methods in civil engineering. It was quite apparent that an additional revision to bring it up to date would not adequately serve the needs of the modern civil engineer. For this reason *American Civil Engineering Practice* was written and is now offered as an entirely new reference work with changed scope and format.

The original handbook was intended as "a source of facts, formulas, tables, and methods which could be found more readily there than by referring to textbooks." The new work goes beyond that objective to present the fundamentals and techniques of the entire field of civil engineering. It is intended to be a comprehensive reference source and a time-saving expedient for professional engineers and engineering students. Derivations generally are omitted, but methods not adaptable to mathematical expression are clearly explained and illustrated by examples. Where practicable, emphasis has been placed upon the planning aspects of civil engineering as well as technical design and construction.

In scope *American Civil Engineering Practice* covers the recognized technical and professional specialties. The fundamental engineering sciences, such as mathematics, mechanics, electricity, thermodynamics, and fluid mechanics are fully treated in a companion volume, *Handbook of Engineering Fundamentals*, edited by Ovid W. Eshbach. Likewise, data on materials of construction are presented in *Handbook of Engineering Materials*, edited by Douglas F. Miner and John B. Seastone. Together these three books, all published by John Wiley & Sons, provide a complete reference library covering all of the usual problems likely to arise in civil engineering.

American Civil Engineering Practice is subdivided into thirty-four sections and published in three volumes. Each volume covers a broad field of civil engineering and is a complete entity. Volume I contains metropolitan and community planning, surveying, traffic, highways, airports, railroads, soil mechanics and site planning, foundations, earthwork, tunnels, and mathematical tables. Volume II is devoted to the general fields of hydraulic, sanitary, and harbor engineering, and Volume III to masonry, reinforced concrete, steel, and timber structures. Each volume contains its own index.

In the planning of the book it was decided that each section should be written by a recognized authority in that specialty. Fortunately this was possible, and it will be noted that the list of contributors contains the names of some of America's most distinguished consulting engineers, engineers in civil service, and professors of civil engineering.

The editor takes this opportunity to thank the contributors for their cooperation. Appreciation is also expressed for the cooperation of many who permitted the use of material published elsewhere. Such contributions are recognized where they appear in the text, and it is hoped that none have been overlooked.

The preparation of a reference book of this magnitude involves many problems of planning and organization of material and consistency in style and typography. It is practically impossible to eliminate all errors in a first edition, but earnest efforts have been exerted by the contributors and by the editor to keep them to a minimum.

ROBERT W. ABBETT

New York, N. Y.
August, 1956

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SECTION 1

METROPOLITAN AND COMMUNITY PLANNING

BY

THEODORE T. McCROSKY AND VINCENT DE PASCIUTO

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METROPOLITAN AND COMMUNITY PLANNING

INTRODUCTION

The field with which this section is concerned is planning for the unified development of cities and their metropolitan environs, as expressed through determination of the comprehensive arrangement of land uses and occupancy and the regulation thereof.[1] As thus defined, planning provides the basic physical framework for all urban public works and private developments that the civil engineer is called upon to design. However, the planning process involves much more than a meshing together of construction projects. Behind the physical plan, there must be a considered background of economic and social objectives which are desirable and feasible of realization.

A planning program may customarily be divided into four logical steps:

1. Surveys—collection of the facts.
2. Analysis and interpretation to determine future objectives and requirements.
3. Formulation of plans and of procedures for their effectuation.
4. Actual construction and implementation of the plan.

In practical operation, these steps will overlap to a considerable extent. The work must be divided into appropriate subject headings, each of which will involve the four steps or processes listed above. Local priorities must be taken into account, but the various component features of the plan should be carried forward simultaneously and progressively. For example, it would evidently not be good practice to complete a school building program one year and to start a street and highway plan the next, or to evaluate the future economy of the community without reference to the determination of required residential lot sizes.

Civil engineers who are experienced in planning techniques may be placed in charge of any or all of the four basic steps in the program, as they relate to one or more of the component features of the plan. These engineers will work closely with professionals trained in economics, public administration, finance, sociology, urban geography, architecture, landscape architecture, and law. City and regional planning embraces all these fields, and the professional planner must have great breadth of knowledge and perception to enable him to correlate the contributions of each.

The civil engineer who is engaged in planning will logically become coordinator of the work done by the engineers who design individual projects included in his plan. The basic capacities of structures, highways, sewers, water mains, and electric power lines are determined by the concepts, objectives, and framework of the plan. This fundamental interrelationship between general planning and detailed engineering design has made the planning field very attractive to civil engineers. They are particularly qualified to assume the responsibility for the many decisions that together determine the technical soundness and adequacy of the plan. More and more, the general public is recognizing the special fitness of the civil engineer for making the over-all plan as well as the detailed designs that derive from it. Many early city plans of great imagination, daring, and foresight failed of realization for lack of practicality and from ignorance of sound design principles.

It is because of this key position of the planning function in the entire field of civil engineering that the decision was reached to place the treatment of its principles and techniques at the beginning of this handbook. No attempt is made to summarize the history of past accomplishments and progress in urban planning. Nor will the reader find any impressive array of statistics. Up-to-date figures are readily available in publications that are issued currently. With each year they change. The statistics in this section have been included, not for factual reference, but rather to illustrate basic principles, standards of good practice, methods of analysis, and comparisons between related factors.

Certain subject headings that are properly a part of the urban planning field have been omitted from this section because they are covered fully in other sections of the handbook. In this category are railroads and railroad terminals, airport location and design, water supply, and sewerage systems. Certain other subjects, of which traffic and transit are examples, are here treated only as to their impact on metropolitan planning. Traffic

[1] This definition is based upon the statement of purposes of the American Institute of Planners, modified to omit those phases of planning that are at state, regional, and national levels.

engineering is the subject of a separate section, and transit engineering is properly a combination of the railroad, automotive, and electrical fields.

The basic planning problems of urbanized areas may be said to follow the sequence of a cycle characterized by four recognizable stages: (*a*) growth, (*b*) congestion, (*c*) decay, (*d*) decentralization.

It is against the evil effects of this cycle that the planner is pitted in all his professional work. Growth needs to be guided in the public interest. Congestion can be prevented when foreseen in time, and otherwise must be relieved. The danger signs of decay should be recognized by the planner before others see them. Decentralization to the environs of a metropolitan region provides the opportunity for guided redevelopment of the center. The old area is thus enabled to start afresh and, aided by sound plans, to avoid the pitfalls of its past.

1. ECONOMIC BASE

In order to plan for the future of a region, a city, or a town, it is necessary first to evaluate its economic potential. The likelihood of growth, both in productivity and in number of people, should be predicted on the basis of several alternate and reasonable sets of assumptions. Industrial, service, and retail opportunities all require analysis, and there are evidently a multiplicity of factors that must be taken into consideration.

Basic vs. Service Employment. The U. S. Census of the Labor Force provides a first measure of the economic type of the community. As a general rule, for each worker in basic manufacturing employment, there is at least one person employed in trade, service, and professional occupations. Any figure over 50% for manufacturing workers is indicative of an exceptionally high degree of industrialization, of which Gary, Ind., is an example. New York City, with its great emphasis on merchandising, banking, general business, and amusements, reveals only 28.0% manufacturing employment (1950). Washington, D. C., has but 7.3% of its resident employees working in industries. Figures for these and other cities with a wide range of characteristics are given in Table 1.

Table 1. Manufacturing Employment for Representative Cities, as a Percentage of Total Employment, 1950

City	Per Cent
Gary, Ind.	54.3
Elizabeth, N. J.	51.6
Bridgeport, Conn.	51.3
Dayton, Ohio	43.2
Camden, N. J.	43.1
Toledo, Ohio	38.2
Chicago, Ill.	36.7
New York, N. Y.	28.0
Evanston, Ill.	18.3
Atlanta, Ga.	16.8
San Francisco, Calif.	16.6
Sacramento, Calif.	9.4
Miami, Fla.	8.0
Washington, D. C.	7.3

Source: 17th U. S. Census: Census of Population, Vol. II, Characteristics of Population, Table 35

Barring exceptional cases, such as a predominantly resort town, any marked imbalance between manufacturing and other employment points to the desirability of seeking to broaden the local economic base by encouraging activities that are deficient but hold promise of being strengthened.

Economic Position Relative to a Larger Territory. Figure 1 illustrates a simple method for comparing the economic position of a community with that of the county or larger area of which it is a part, or with neighboring communities. At the same time, the trends of growth, or retrogression, between business censuses are presented. For any given economic activity such as retail sales receipts, the relation of a town to its county is derived by the following formula:

$$I_t = \frac{R_{t/c}}{P_{t/c}} \times 100$$

where I_t = index number for the retail-trade position of the town relative to the county.
$R_{t/c}$ = per cent of the county's retail trade done in the town
$P_{t/c}$ = per cent of the county's population living in the town.

By using this method, the strong points and weaknesses of the community can be quickly ascertained. If, for example, the community were the principal trading center of the county, the value of I_t would be 150–200, or even higher. In Fig. 1, the reverse is true. Retail trade is weak in the town, and $I_t = 61.3$ for sales receipts, 1948. At the right side of Fig. 1, the percentage changes are shown for the interval 1939 to 1948. After careful analysis and appropriate study of contributory local conditions, it becomes possible to advise the community as to which economic activities offer the best promise for continuing strength, which activities require reinforcement in the interest of balancing the economy, and which are so seriously deficient that they appear to be unsuited to the locality and unrewarding as subjects for attention.

Figure 1 shows the application of the relative index number method only to retail sales and establishments as a whole. It should be separately applied to the major census classifications of retail trade (food, apparel, furniture, etc.) and is equally adaptable to census

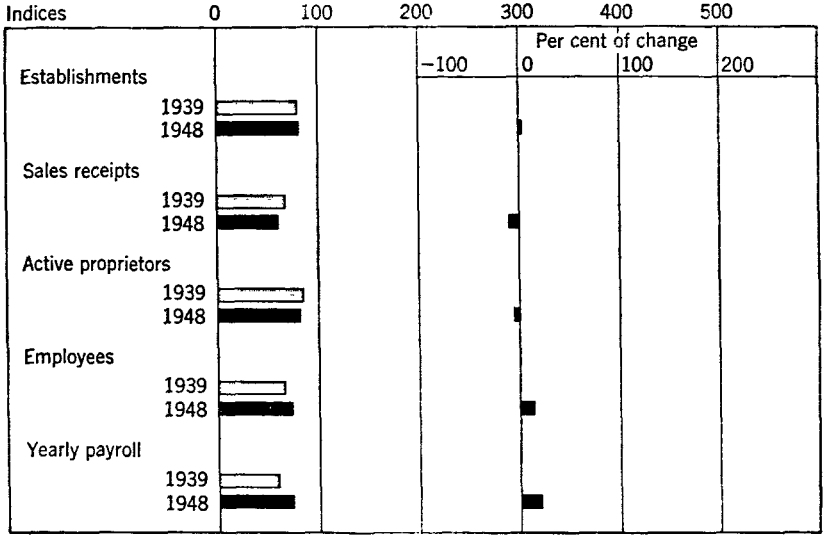

FIG 1 Retail trade indices.

data for manufacturing employment, number of manufacturing establishments, and value added by manufacture. It can also be used to advantage for evaluating wholesale trade and service occupations, and for such economic yardsticks as postal receipts, bank clearings, savings bank deposits, and electric power sales.

Factors in Industrial Location. From the standpoint of industry, centrality in relation to a large distribution market is important. However, in view of the strong trend toward dispersion of industry to surburban locations, coupled with the construction of through highways, a sufficient degree of centrality can often be achieved on the outskirts of a metropolis The factor that is usually second in importance is an adequate labor force, either locally available or residing within easy commuting distance. Third, and most obvious, there must be available industrial sites on good land. Other factors that influence industrial location and therefore affect the industrial potential of a community are transportation network, including interchange terminals, availability of raw materials; cheap power and fuel; nearness to related industries; local living conditions; cheap land or cheap factory space for rent; and local assessment and tax levels.

Causes of Economic Growth and Retrogression. It is necessary to ascertain *why* the economic position of a community is changing, and not merely to record the amount of the changes. In some cases, growth or emerging weakness may result from broad national or regional trends over which no single community can exercise control. More frequently, and especially in retrogressions, the fault is local and can be ascertained and corrected. For example, lack of adequate and convenient automobile parking space in the heart of a business district will cause sales to fall off and is an open invitation to the development of competing retail centers. Public opinion surveys have been found valuable as a means

of determining what the local residents themselves consider to be the major deficiencies of their town. Table 2 represents an 85% canvass of all families living in a community

Table 2. Opinion Survey of Local Conditions

Services and Conditions Voted as Needing Improvement	Per Cent of votes
Street paving	48
Sidewalks	58
Street lighting	45
Refuse removal	8
Parks and street trees	40
Fire Department	8
Police Department	20
Water supply	21
Sewerage system	10
Grade crossings	84
Swamp conditions	44
Playgrounds	40

of 5000 population. As a matter of practice, a representative 15–25% sample would be sufficiently indicative. The listing of items should be varied to cover local problems and requirements.

2. POPULATION

The economic base of a community and its attractiveness as a place to live are the principal determinants of its future population expectancy. Growth may be faster or slower than normal, depending upon national economic trends that affect the birth rate. However, if the local economic base is unstable, out-migration will counterbalance even birth-rate peaks such as have been experienced during and after World War II.

The fact that the sole purpose of all facilities and services in a community is to meet the physical, economic, social, cultural and governmental needs of the population makes it evident that a study and understanding of the growth, distribution, composition and other characteristics of the population, and the trends in such characteristics, are basic in the preparation of a plan for the future development of the community.[1]

Growth. Analysis of past growth helps to evaluate the strength of the fundamental economic and social forces that have controlled the development of a community, and to estimate the extent to which these forces may continue their control in the future. The slope of a growth curve plotted on normal cross-section paper indicates the arithmetic or absolute rate of population increase. If plotted on semilogarithmic paper, the slope becomes the percentage rate of increase. Both methods should be used in a thorough analysis. In Fig. 2, the left-hand diagram is on arithmetic scale, and the right-hand on semilogarithmic. The two communities A and B are growing at the same *percentage* rate, and their curves therefore appear as parallel lines on the semilogarithmic scale; but as A is larger than B, its arithmetic growth curve, at the left, rises much faster than that of B. By contrast, communities C and D have the same *arithmetic* rate of growth; i.e., each is increasing by 200 persons per year. Hence, curves C and D are parallel on the arithmetic scale at the left; but D—the smaller community—has a faster percentage growth, as shown by its steeper curve in the right-hand diagram.

Separate growth curves for a group of cities of comparable size and characteristics are helpful in the analysis of past trends, and also as a rough guide to the future of the city being studied.

Future population can be estimated by many methods. Those that are purely mathematical generally prove to be fallacious, even when applied to very large areas. At one time, population curves were extrapolated by straight line projection, on arithmetic or semilogarithmic paper, whichever seemed better to fit the records of the past. When growth was observed to occur at a faster and faster rate, exponential curves came into favor. Later, because of the slowing down of decennial population increments, equations producing reversed curves that approach a horizontal asymptote were found to give reasonable results. All these methods consist of fitting an empirical equation to past data, and then calculating its values for future years. They therefore ignore future basic changes in the economic potential and the incidence of unforeseen inventions and social forces

[1] *Local Planning Administration*, by Ladislas Segoe, International City Managers' Association, 2nd Ed., 1948, p. 67.

"The result has a dangerous fascination because the prediction seems to be growing out of the past and to be an inevitable statement of what the past implies for the future." [2]

The shortcomings of purely mathematical methods based on past statistics are best illustrated by the sudden "flattening out" of population curves during the depression of

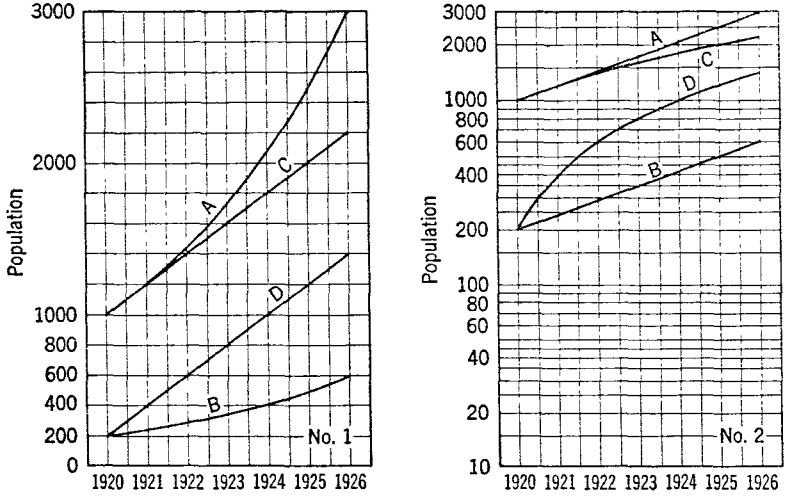

FIG 2 Absolute and percentage rates of population growth

the 1930's, followed by the swift upturns resulting from the war migration and high birth rates that characterized the 1940's. Neither phenomenon was related to any statistical evidence available in the censuses of 1930 and prior decades

An acceptable method of prediction, which is less rigid mathematically, is to plot the percentage that the population of area A has borne to that of one or several larger areas of which A is a part (Fig 3). These percentage curves can then be extended 10–30 years into the future by the exercise of considered judgment. The fact that several curves are used requires an adjusted interplay of their projections. The merit of this method is that

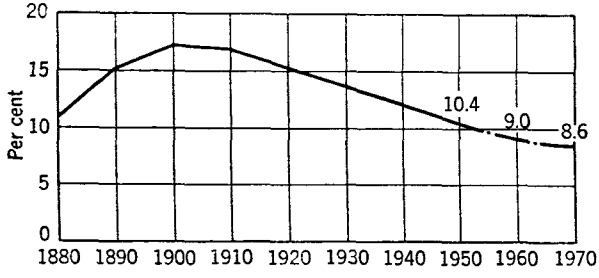

FIG. 3 Morristown, N. J , population as percentage of Morris County population, 1880–1970

only the largest area has to be estimated in absolute terms. The populations of all smaller areas are derived from it by application of the percentages. Unusual forces will have smaller effect upon very large areas, such as a group of states or the entire United States. Another advantage of this method is that the effects of the anticipated local economic potential can be brought into the process of estimating.

An entirely different method of estimating population growth, illustrated in Figs. 4 and 5, derives from projecting the trend of differences between birth rates and death rates, and the trend of differences between in-migration and out-migration. The extrapolations

[2] *Regional Survey of New York and Its Environs*, Vol. II, 1929, p. 110. See pp. 107 ff. for detailed discussion.

may be done by personal judgment or by the use of empirical equations. This general procedure is useful in predicting future school population over a short term of years, but presents pitfalls when applied to total population growth. Accurate migration data are difficult to obtain for small areas, and their extrapolated trend derives not from past experience but from the incidence of the future economic potential. Crude birth rates and crude death rates (1000 times total births or deaths divided by total population) are inadequate and actually misleading as an index of population trends. "This apparent paradox arises from the fact that the birth rate is the resultant of two influences, namely, first,

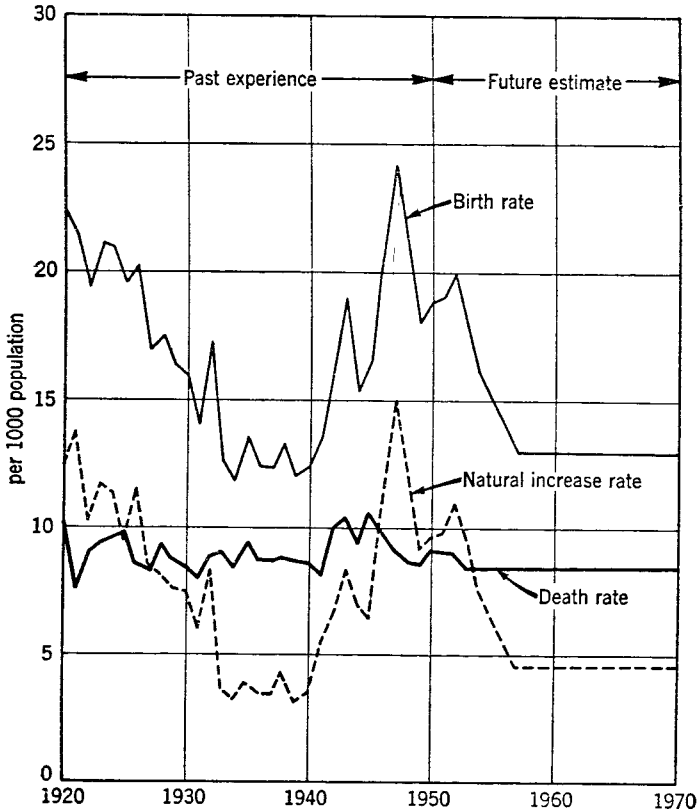

FIG. 4 Vital statistics, West Orange, N. J., 1920–1970

the fertility of that part of the population which is in the reproductive period of life; and second, the prevailing age distribution (population pyramid) which determines what proportion of the total population thus contributes to the annual progeny." [3] For good results, therefore, it is necessary to utilize specific birth rates derived from the number of women of child-bearing age, and to compute separate death rates for each of many age groups.

In order to estimate population for intercensal years, or to project a curve for a few years beyond the last census, an accepted method is to obtain the record of building permits (or preferably certificates of occupancy, if available) issued in each year. These figures, after adjustment for vacancies, are then multiplied by the estimated size of family taken from the trend curve. Other methods involve use of public utility meter records, telephone installations, or birth and death statistics. The most appropriate technique will generally depend upon the form and availability of local data.

[3] Modern Trends in the Birth Rate, by Alfred J. Lotka, *Annals Am. Acad. Political and Social Sci.*, Vol. 188 (November 1936).

Sources of Demographic Data. For most planning purposes, the published reports of the Bureau of the Census, U. S. Department of Commerce, furnish all necessary basic population data. Where small areas are involved, it may be necessary to arrange for special tabulations by the Bureau.

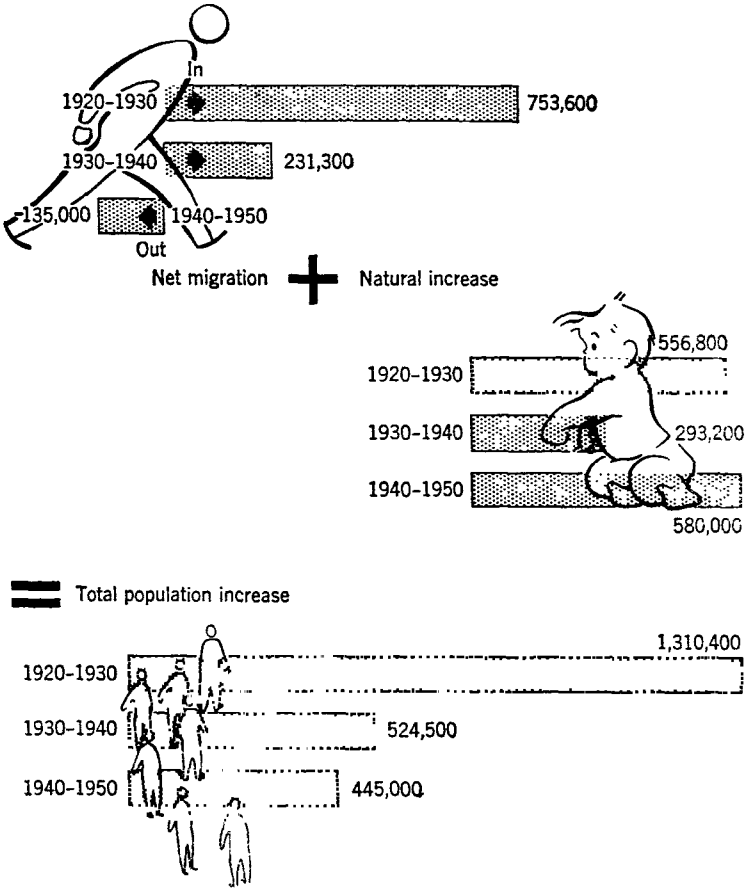

FIG 5. Net migration and natural increase in New York City, 1920 to 1950. City Planning Commission, New York, N. Y.

Records of births, deaths, and marriages by years are obtained from municipal, county, and state offices of vital statistics. Birth and death rates must be computed from acceptable intercensal estimates of population.

Authoritative predictions for the growth of the United States are contained in:

Forecasts of the Population of the United States, by Age and Sex, 1945–2000, Bureau of the Census, 1946.
Forecasts of the Population of the United States, 1945–1975, by P. K. Whelpton, Bureau of the Census, 1947.

Periodic revised estimates and forecasts are issued by the Bureau of the Census for the United States and for major political subdivisions.

For states and smaller areas, forecasts are customarily prepared by state development agencies and by county and local planning boards or bureaus of vital statistics. All such estimates should be checked and evaluated for their degree of reliability.

Distribution. Studies of the geographic distribution of population are often of greater significance to the planning engineer than knowledge and prediction of total growth. Dis-

tribution gives an accurate picture of density as related to housing types and zoning and is essential in calculating future requirements for schools and local recreation areas. It also is a vital factor in evaluating highway and public transportation facilities.

Population distribution is customarily shown on "dot maps," where each dot represents a definite number of persons or families (Fig. 6). By utilizing dots, circles, and other sym-

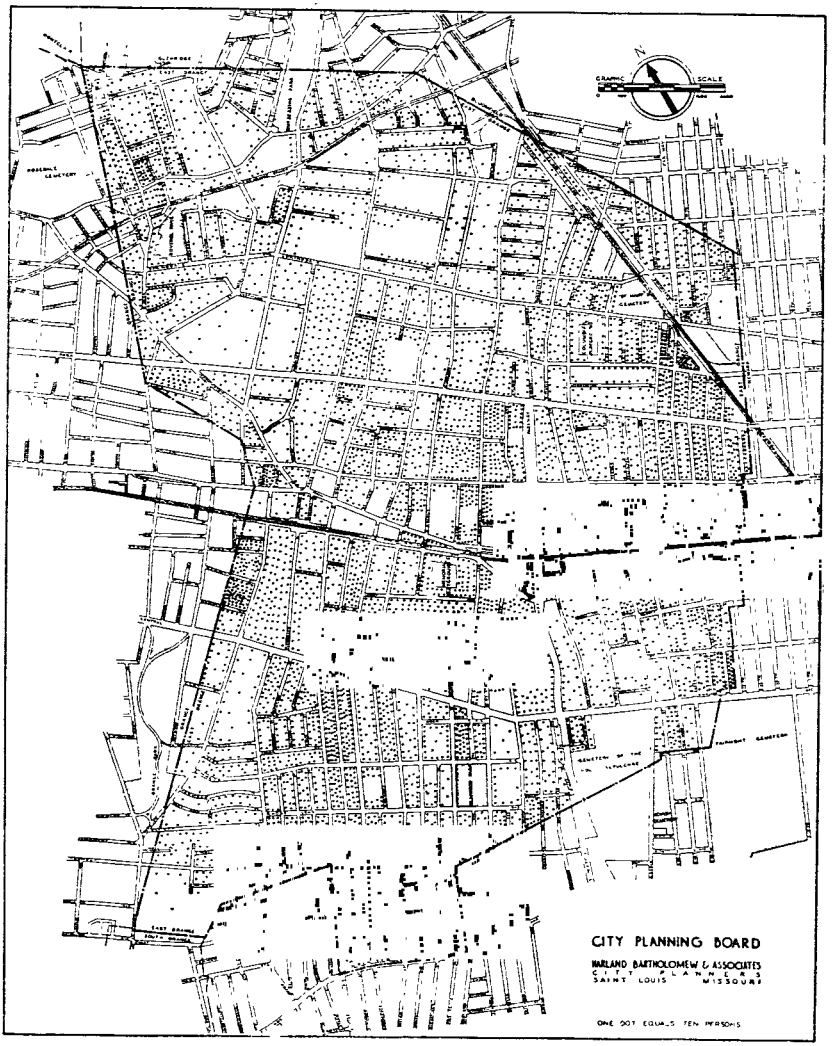

FIG. 6 Distribution of population, 1940, City of East Orange, N J

bols, the distribution pattern for several consecutive census years can be shown on one map. Population density is best mapped by means of graduated shadings (Fig. 7). Dot maps and density maps are useful as visual aids and for general public information. For detailed office calculations, they should be supplemented by census tract and block data statistics, in both tabular and map form.

Composition. Characteristic groupings which make up the population structure, and which indicate changes in structure through the years, have a vital bearing on planning for the future.

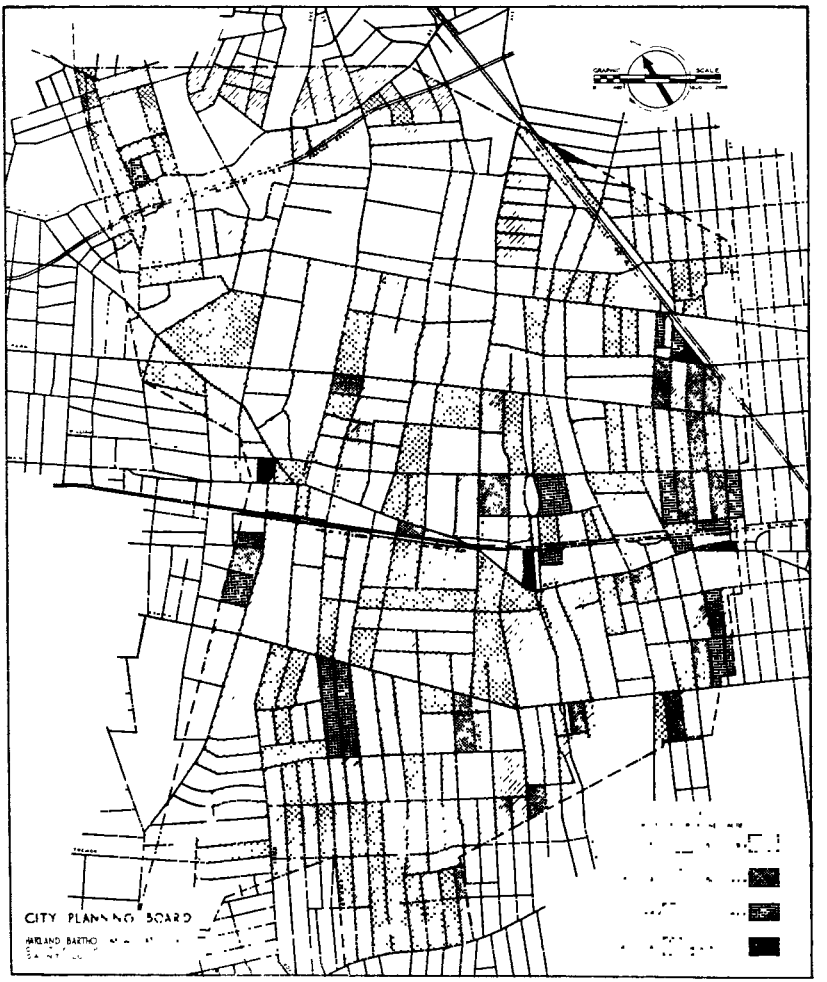

FIG 7 Density of population, 1940, City of East Orange, N J

Age composition is best presented in terms of numbers and percentage of persons in each age group, as recorded by the U. S censuses. Charts usually take the form of "population pyramids," with separate figures for males and females (see Fig 8). The trend of changes in age composition must be taken into account in estimating future school classroom requirements, and in determining space needs for playgrounds, small parks, and other public improvements appropriate to the various age groupings. The present trend toward an older population means an increased demand for facilities to care for the aged In Boston, the number of people in the age group of 60 years and older increased 63.5% between 1930 and 1950, despite the fact that the total city population grew by only 2 6%.

Race and nationality of population, when large numbers of a particular ethnic or foreign-origin group are involved, have resulted in the formation of well-defined colonies that have a significant bearing on neighborhood structure and social needs. The present trend is toward the breaking up of these colonies, but it will be many years before the process is complete. Because of restrictions on immigration from abroad, and the rapid assimilation of second-generation families, many of the "foreign" colonies have already lost their former specialized characteristics, and rarely present any unusual problems in the field of physical

planning. Established Negro districts, because of the lower average income level of their inhabitants, call for thoughtful analysis and appreciative understanding of their social and economic needs. In most cities these districts are characterized by serious overcrowding of housing, which was not designed for the present occupants, with resultant excessive population density and high rates of tuberculosis and infant mortality.

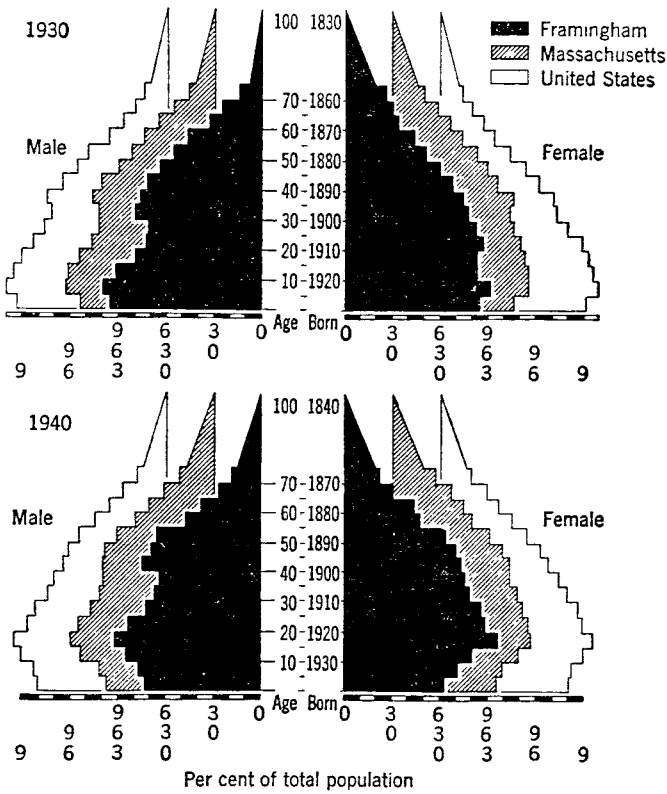

FIG. 8. Age distribution, the city of Framingham, the state of Massachusetts, and the United States, 1930 and 1940.

Size of family is the determinant of housing demand for a given municipal population. This is illustrated by the figures in Table 3, which are for an eastern town which has had only a moderate population growth. In connection with Table 3, it may be observed that

Table 3. Effect of Size of Family on Housing Demand

Year	Population	Size of Family	Required Dwelling Units *
1900	11,267	5.20	2170
1910	12,507	4.73	2646
1920	12,548	4.32	2905
1930	15,197	4.07	3731
1940	15,270	3.80	4018
1950	17,078	3.48	4900

* Excluding allowances for vacancies and doubling up.

in 50 years the total population increased roughly 50%, while the required number of dwelling units is 125% greater.

Size of family has also exercised a marked influence on the type of housing and number of rooms desired. The current preponderant demand for four-room and five-room houses,

and for three-room apartments, would not have fitted the normal family size of a generation ago. For purposes of analyzing housing demand, average family size is not sufficient, and must be supplemented by breakdown figures giving the number of families of each size in the potential market.

The increasing longevity of the American population may be expected to counteract the long-term contraction in average size of family. And the presence of more old people will produce a renewed demand for houses with three and four bedrooms.

It is significant to note that the phenomenal rise in the birth rate from 1941 through 1953 has not reversed the trend toward smaller family size. The "war babies" were born to young couples whose marriages had been deferred by service in the armed forces. On present evidence these families do not appear to be raising more children, on average, than the families that were formed 10 years earlier.

3. LAND-USE PROPORTIONS

Existing Land-Use Map. As an essential preliminary to a master plan, it is necessary to map and analyze the existing uses of the land. The land-use map should show recognized classifications, usually those listed in Tables 4 and 5, as a minimum. Local circumstances will often require further refinement of these classifications, such as single-family on plots over 3 acres, single-family on 1–3 acres, etc.; and automobile filling and service stations, automobile garages and parking lots; storage uses.

Selection of precise land-use classifications for a particular community should accord with the existing zoning ordinance use district classifications or with proposed new zoning classifications—and preferably with a workable combination of both, when feasible. Otherwise, it is difficult if not impossible to correlate the land uses with their corresponding zones. This correlation is highly desirable as a measure of the degree of conformity of zoning provisions with the pattern of existing uses.

The land-use map can best be prepared by means of waxed-paper patterns, so that black-and-white reproductions can be made in quantity at full or reduced scale. Individual prints may be colored by hand for display purposes.

Land-Use Tabulations. After the existing land-use map is completed, the areas of land in each category should be measured and tabulated. Percentages of land area by category, and acreage in use per 100 persons, can then be derived.

Table 4 gives percentages of "net developed area" by principal classifications, for five municipalities. Communities vary so greatly in the extent of vacant land and land in streets that these items are shown separately as surcharges, after first being deducted from total usable area.

Table 4. Proportions of Net Developed Area of Various Municipalities, by Major Urban Land Uses

("Net developed area" is the total city area less streets, vacant land, and land under water.)

	Maplewood, Mo.	Cape Girardeau, Mo.	Montclair, N. J.	Morristown, N. J.	Yonkers, N. Y.
Single-family	63.4	52.4	68 0	42.9	29.7
Two-family	4.7	1.6	4.8	11.4	4.6
Multifamily	2.2	0.7	2.4	7.9	7.2
Total residence	70.3	54.7	75.2	62.2	31.5
Business	4.0	3.0	3.0	7.4	5 8
Industry and railroads	20.3	28.0	3.5	6.9	4 7
Parks and playgrounds	1 3	3 8	7.9	11.0	8.0
Other public and semi-public	4.1	10.5	10.4	12.5	40 0
Total net developed area	100.0	100.0	100 0	100 0	100.0
Surcharge for streets	29.6	64 6	22 2	19.1	28.5
Surcharge for vacant	43.1	264.0	17 7	41.7	49 3
Total per cent of net developed area	172.7	428.6	139.9	160.8	177.8

Sources: Figures for Maplewood and Cape Girardeau are derived from data in *Urban Land Uses*, by Harland Bartholomew, Harvard University, 1932. Montclair figures are derived from a city plan report prepared by Harold M. Lewis. Morristown and Yonkers data are from the files of McHugh and McCrosky (1949–50).

Table 5 presents the same data as Table 4, but in terms of acreage per 100 persons. Even after the above-noted adjustments for vacant land and streets, it will be observed that there are wide variations in both tables within each category. Obviously, no rigid set of "standards" can or should be established as a guide for future planning. There is

Table 5. Acres per 100 Persons for Major Urban Land Uses in Various Municipalities

	Maplewood, Mo.	Cape Girardeau, Mo.	Montclair, N. J.	Morristown, N. J.	Yonkers, N. Y.
Single-family	2.90	3.96	4.79	2.92	1.28
Two-family	0.22	0.01	0.33	0.78	0.20
Multifamily	0.10	0.06	0.17	0.54	0.31
Total residence	3.22	4.03	5.29	4.24	1.79
Business	0.18	0.23	0.21	0.50	0.25
Industry and railroads	0.94	2.13	0.25	0.47	0.20
Parks and playgrounds	0.06	0 29	0.55	0.75	0.34
Other public and semi-public	0.19	0.80	0.73	0.85	1.72
Total net developed area	4.59	7.48	7.03	6.81	4.30
Streets	1.55	4.91	1.57	1.30	1.22
Vacant	1.98	20.09	1.24	2 84	2.12
Total usable area	8.12	32.48	9.84	10.95	7.64

Sources: Maplewood and Cape Girardeau data are from *Urban Land Uses*, by Harland Bartholomew, Harvard University, 1932. Montclair data are derived from planning report by Harold M. Lewis. Morristown and Yonkers data are from files of McHugh and McCrosky (1949–50).

no substitute for detailed knowledge of the community and understanding of its aspirations and realizable economic potential. In comparing one community with another, the special characteristics and peculiarities of each must be known more fully than they can be presented in tables such as these.

The wide degree of variation in land-use proportions is exemplified by the data in Table 6, which gives the low, average, and high figures for 48 self-contained cities.

Table 6. Percentage of Developed Area Occupied by Major Urban Uses in 48 Self-Contained Cities

Type of Use	Less than 50,000 17 Cities	50,000–150,000 20 Cities	150,000 and Over 11 Cities	Total All 48 Cities
Single-family residence	32.86	35.42	32.49	33.53
Two-family dwelling	3.29	3.23	4.39	3.88
Multiple dwelling	1.75	1.52	3.26	2.51
Commercial use	2.60	2.54	3.24	2.93
Public and semipublic use	11.25	7.61	9.87	9.24
Parks and playgrounds	5.07	6.40	8.10	7.21
Light-industrial use	4.19	2.29	3.17	2 97
Heavy industrial use	2.63	3.38	3.75	3.51
Railroad use	4.09	4.75	4.69	4.65
Streets and alleys	32.27	32.86	27.04	29.57
Total developed area	100 0	100.0	100.0	100.0

Source: Urban Land Use—1949, Eldridge Lovelace, *J. Am. Inst. Planners*, summer, 1949.

Notwithstanding the fact that rigid standards would be undesirable, it is evident that marked divergence from normal proportions is indicative of special local conditions. They can then be properly taken into account in future land-use planning.

Municipal versus "Contributory Area" Land-Use Map Boundaries. When possible, the land-use map and resulting quantitative analysis should not be confined to the usually arbitrary political boundaries of the municipality. For communities of small to medium size, the statistical area should embrace all developed territory that is directly contributory to the business center. For large cities with semi-independent satellites, separate mapping and tabulation will usually be indicated for the city itself and for the city plus satellites. Problems of jurisdiction frequently impede the analysis of land uses over as broad an area as would be desirable. Judgment of the relative importance in a particular case has to be balanced against the complications involved.

Proportionate Land-Use Trends in New York Region. The Regional Plan Association, Inc., publishes basic data on land-use acreage and proportions for all counties within the New York Region.[1] As evidence of long-term trends, it is significant that from 1925 to 1940 the population of the Region as a whole increased 26%, while the area of closely developed residential land expanded 56%. This change was accompanied by a decrease in population density from 41 to 33 persons per gross acre. Another interesting trend was revealed: 90% of the areas closely developed between 1925 and 1940 were within 1 mile of railroad stations or rapid-transit lines, as contrasted with 98% before 1925. As of 1940, 59% of the total developed area within the commuter zone was used for urban residence and local business, 5% for principal industries, and 36% for public and semipublic uses.

Land Uses and Future Planned Proportions in New York City. Mapping and tabulating existing land uses are principal tools for shaping a long-term land-use master plan and for considered revision of the zoning ordinance and map that are to effectuate the plan. Table 7 is derived from the 1948 land-use survey of New York City and gives predicted acreage and proportionate requirements for the year 1970. All figures have been calculated on the basis of "net developed area," with streets and vacant land shown as surcharges, as in Table 4.

Table 7 is an illustration of long-range quantitative calculations. In determining required areas for 1970, allowance has been made for reduction of population density and

Table 7. Land-Use Proportions in New York City, 1948, and Planned Estimates for 1970

	1948			1970		
Classification	Acres	Per Cent of Net Developed Area	Acres per 100 Persons	Acres	Per Cent of Net Developed Area	Acres per 100 Persons
Residence *	41,020	42.1	5.23	59,550	48.6	6.95
Commerce	1,930	2.0	0.25	2,160	1.8	0.25
Auto storage and services	1,450	1.5	0.18	1,880	1.5	0.22
Manufacturing and industry	6,660	6.8	0.85	10,160	8.3	1.18
Transport airports	5,360	5.5	0.68	5,360	4.4	0.62
Other transportation uses	3,570	3.7	0.46	3,570	2.9	0.42
Parks, playgrounds, public and private institutions, and cemeteries	37,250	38.4	4.76	39,720	32.5	4.62
Net developed area	97,240	100.0	12.41	122,400	100.0	14.26
Surcharge for streets	57,790	59.4	7.39	57,790 †	47.1	6.73
Surcharge for vacant	36,540	37.6	4.66	11,380 ‡	9.3	1.32
Total usable land	191,570	197.0	24.46	191,570	156.4	22.31
Population	7,835,099			8,585,000		
Population per residential acre	191			144		
Population per net developed acre	81			70		
Population per usable acre	41			45		

* Includes residential buildings with stores on the ground floor.
† Sufficient vacant land is mapped with streets to accommodate the population increment.
‡ Principally in the Borough of Richmond (Staten Island).

Source: This table is derived from data in *Plan for Rezoning the City of New York*, by Harrison, Ballard, and Allen, 1950, and from supplemental official sources.

diminishing size of family in residential districts, and for parking and recreation space in new developments. For industry, the 1970 figures take into consideration the trend toward one-story construction and the need for accessory automobile parking space. This statistical plan for 1970 was the basis for the proposed new zoning ordinance and map, which were published in 1950.

[1] Regional Plan Association, Inc., New York, *Bulls.* 63 (1944) and 73 (1947).

4. RESIDENTIAL AREAS

Types of Area. Both in analyzing existing residential areas and in planning for their future, it is good practice to classify them in terms of quality and potential, as well as by type of housing development. The Chicago Plan Commission set up eight categories, with proportions of total residential area in the city as given below:[1]

	Per Cent
1. Blighted	6.0
2. Near blighted	8.6
3. Conservation	36.3
4. Stable	23 3
5. Arrested development	7.4
6. Progressive development	0.9
7. New growth	4.7
8. Vacant—suitable for residence	12.8
	100 0

Each category was statistically defined in terms of percentage brackets for age of structure, condition, rent, and extent of residential use, as determined from land-use survey data The incidence of each combination of factors corresponding to one of the eight categories was determinable by superimposition of block characteristics maps For this purpose, acetate overlays afford a simple technique.

In New York City, the Mayor's Committee on City Planning utilized a series of twelve categories, which included the indicated future type of housing development, as well as factors of economic and physical condition. The New York list and the proportions of total residential area were as follows.[2]

	Per Cent
1. Single-family high rent	8.0
2. Single-family medium rent	29 5
3. Two-family	8.8
4. Unripe and impractical	19 7
5. Prematurely developed	5 9
6. Declining residential (in transition to non-residential)	1 4
7. Rehabilitation	4.2
8. Demolition and low-rent housing	2 7
9. High-rent apartments	1.6
10. Medium-rent apartments	12.4
11. High-rent garden apartments	2.0
12. Medium-rent garden apartments	3 8
	100.0

Assignment of blocks and areas to the above twelve categories involved quantitative correlation of over thirty factors derived from the land-use survey and other planning studies. This detail was fully justified by the complexities of the New York pattern and the great amount of basic data that had been compiled.

For small cities, the following residential categories, or a similar listing, is generally accepted to be sufficient:

1. Blighted.
2. Conservation.
3. Stable.
4. New growth.

These can later be further subclassified by housing type, for purposes of the future land-use plan and zoning revision.

Establishment of Neighborhood Boundaries for Planning Purposes. By the above-described process of residential land-use analysis, the boundaries of many clearly defined and reasonably homogeneous communities logically emerge upon the maps. Such communities should be broken down into neighborhoods, with areas and population sizes appropriate to be served by an elementary school, public library branch, recreation spaces, and other usual facilities which people share. The neighborhood thus becomes the unit or cell for replanning the residential districts of the city.

[1] *Master Plan of Residential Land Use of Chicago*, Chicago Plan Commission, 1943, pp 67 ff

[2] *City Wide Studies*, Part I, *Basic Factors in the Planning of the City of New York*, Mayor's Committee on City Planning, 1940, p. 74.

ABRIDGED BLOCK SCHEDULE A — ENVIRONMENTAL SURVEY — CITY _____

Identification and Reference Data

Identification A: Ward or Tract _____ Block _____ Identification B: District _____ Block _____

Boundaries:
Frontage 1 _____ 2 _____ 3 _____ 4 _____ (5) _____

Net Block Dimensions _____ x _____ Net Block Area: sq ft _____ acr _____

Entries by: Item 1 _____ 4 _____ 6 _____ 7 _____ 10 _____

Date: Item 1 _____ 4 _____ 6 _____ 7 _____ 10 _____

1. COVERAGE BY STRUCTURES

a. Calculations

1. Area covered: sq ft _____ % of block area _____

2. Average height of structures: stories _____

b. Rating

Per cent from line 1 above	Average height of structures: pen. pts. 1–2.9 sty.	3–5.9 sty.	6 + sty.
0–19.9	☐ 0 pts.	☐ 0 pts.	☐ 0 pts.
20–39.9	☐ 1 "	☐ 3 "	☐ 5 "
40–59.9	☐ 6 "	☐ 8 "	☐ 14 "
60–79.9	☐ 15 "	☐ 18 "	☐ 24 "
80 or more	☐ 25 "	☐ 28 "	☐ 30 "

PENALTY SCORE _____

4. RESIDENTIAL YARD AREAS

a. Calculations

1. Premises: Non-res. _____; res. (incl. mixed) _____

2. Res. premises lacking minimum unbuilt area: No. _____; % _____

3. Res. struct. _____; units _____; avg. Un/Str _____

b. Rating

Per cent from line 2 above	Average units/structure: pen. pts. 1–2.9 u./s.	3–5.9 u./s.	6+ u./s.
0–9.9	☐ 0 pts.	☐ 0 pts.	☐ 0 pts.
10–39.9	☐ 2 "	☐ 4 "	☐ 6 "
40–69.9	☐ 7 "	☐ 9 "	☐ 11 "
70 or more	☐ 12 "	☐ 16 "	☐ 20 "

PENALTY SCORE _____

6. NON-RESIDENTIAL LAND USE: LINEAR INCIDENCE

Per cent frontage in non-res. use	Frontage 1	2	3	4	(5)	Pen. Pts.
0.0	☐	☐	☐	☐	___	0
0.1–9.9	☐	☐	☐	☐	___	2
10–29.9	☐	☐	☐	☐	___	5
30–49.9	☐	☐	☐	☐	___	8
50–69.9	☐	☐	☐	☐	___	12
70–89.9	☐	☐	☐	☐	___	16
90 or more	☐	☐	☐	☐	___	20
PENALTY SCORE	___	___	___	___	___	

10. STREET TRAFFIC

Type of street	Frontage 1	2	3	4	(5)	Pen. Pts.
a. Residential	☐	☐	☐	☐	___	0
b. Comm'l service	☐	☐	☐	☐	___	3
c. Minor traffic	☐	☐	☐	☐	___	7
d. Major traffic	☐	☐	☐	☐	___	11
Trolley cars: on *a*	☐	☐	☐	☐	___	2
on *b*	☐	☐	☐	☐	___	3
on *c*	☐	☐	☐	☐	___	4
PENALTY SCORE	___	___	___	___	___	

7. SPECIFIC NON-RESIDENTIAL NUISANCES AND HAZARDS (Scored from Rating Table 7A)

Sources (enter number)	This Block M*	C*	E*	Pen. Pts.	Opp. Front. 1 M*	C*	E*	Pen. Pts.	Opp. Front. 2 M*	C*	E*	Pen. Pts.	Opp. Front. 3 M*	C*	E*	Pen. Pts.	Opp. Front. 4 M*	C*	E*	Pen. Pts.
a. Noise or vibration	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___
b. Objectionable odors	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___
c. Fire or explosion	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___
d. Rodents or vermin	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___
e. Localized smoke, dust	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___	___
f. Glare at night	___	___			___	___			___	___			___	___			___	___		
g Dilapidated structure, insanitary vacant lot	___			___	___			___	___			___	___			___	___			___
PENALTY SCORE				___				___				___				___				___
COMBINED PENALTY SCORE (block plus opposite frontage)								___				___				___				___

* Code: M = Moderate, C = Considerable, E = Extreme

Remarks_____

FIG 9. Environmental survey form, Committee on the Hygiene of Housing, American Public Health Association, 1946

A caution should be expressed against carrying the process too determinedly into the field of administrative government. Often times, clear neighborhood boundaries do not exist and cannot be realistically drawn. Also, the area of service for an elementary school is not necessarily the appropriate area for a police precinct, a fire precinct, or a garbage-collection district. Experience shows that the planning neighborhood is best limited in its application to fundamental residential problems of housing, education, and recreation. Furthermore, it is a unit created for convenience in statistical and physical planning, and by no means an area set apart with a theoretical wall around it. Nothing could be further from the democratic tradition or from the purposes for which its boundaries are established.

Delineation of Substandard and Redevelopment Areas. Techniques of analysis for blighted residential areas have been developed more highly than those for residential neighborhoods presenting less critical problems. The Committee on Hygiene of Housing, American Public Health Association, has been a leader in this field. Detailed schedules and a system of "penalty points" have been evolved. The method has been applied in many cities. The results justify the amount of work required, by giving comparative data that are beyond challenge in their completeness and reliability. An APHA environmental survey form is reproduced in Fig. 9. For application of the method, reference should be made to the published manual of the committee.[3]

When an APHA environmental survey cannot be undertaken, satisfactorily detailed data can be abstracted from the U. S. Census of Housing reports giving block statistics. Combined incidence of such factors as high percentage of structures needing major repairs or lacking private baths, low rent, and old age makes it possible to demarcate the badly blighted residential areas in a preliminary way. Field checks should then be conducted to refine the boundaries.

In a simplified statistical method used in St. Louis, residential buildings were analyzed by application of penalty scores as follows.[4]

	Penalty Points
1. No running water	25
2. Age of building	20
3. No private bath	20
4. Needing major repair	20
5. Tenant-occupied	10
6. Overcrowding	5
Maximum total	100

The above technique was supplemented by various studies of social conditions, including juvenile delinquency, adult crime, incidence of various diseases, and other available indices.

Statistics must be reinforced by mature judgment. They cannot be used blindly or arbitrarily without reference to public opinion. In practice, the detailed survey data do not themselves *determine* the areas most appropriate for clearance and redevelopment; rather they furnish the basis for selection of those areas that professional and political judgment agree to be most advisable.

Rebuilding Old Areas. Rebuilding obsolete and substandard urban areas for middle- and low-income families is one of the greatest challenges to the professional planner, to government, and to private capital. The most authoritative treatment of this complex problem is contained in two volumes edited by Coleman Woodbury: *Urban Redevelopment: Problems and Practices*, and *The Future of Cities and Urban Redevelopment*, University of Chicago Press, 1953. In this article, it is not feasible to do more than present the basic factors, the present framework for solutions, and an analysis of a typical redevelopment project.

With some notable exceptions of semicharity and fiduciary developments, private initiative has not been able to make outstanding progress in the rebuilding of blighted neighborhoods. The costs of land acquisition and new construction have been too great, and the prospective economic return too small. Efforts have had to be confined principally to rehabilitation of existing groups of buildings, which does not generally create needed enlargement of open space or make these buildings fit the living requirements of tenant families of an income group for which they were not originally designed. In many ways, the converted town mansion is potentially a worse slum than the "old law" tenement.

In the late 1930's, the United States Government, acting through local housing authorities, embarked on a program of low-income housing construction, with federal subsidies

[3] An Appraisal Method for Measuring the Quality of Housing, *Operations Manual* (3 Vols.), Committee on the Hygiene of Housing, American Public Health Association.

[4] Criteria Used in Delimiting Redevelopment Areas, *J. Am. Inst. Planners*, Summer 1950.

to achieve the desired level of rents. The original program has been extended and amplified by further legislation.

In recognition of the fact that the difficulty encountered by private enterprise in rebuilding blighted areas stems largely from the economic problem of high land costs, the National Housing Act of 1949 (Title I) established a new program of slum clearance and urban redevelopment. This program provided for a write-down of land-acquisition and site-preparation costs to the economic reuse value, with the difference financed by a combination of federal and municipal capital subsidies Under this program, predominantly residential land may be redeveloped for either new residential or non-residential use Existing non-residential land may be redeveloped only for residences, and vacant land is also eligible for residential development under certain conditions As of December 1953, federal capital grant reservations totaling $246,468,000 had been approved for 211 localities.

This new approach to urban rebuilding holds great promise. Its success will depend upon the volume of private capital that can be attracted. A principal limitation is that the areas must usually be redeveloped for a higher-income group than their present occupants, and this higher group is smaller numerically than the families now living under substandard conditions.

The Housing Act of 1954 continued the general policies of the 1949 Act, and placed added stress on local municipal cooperation in the whole field of urban renewal, with improved coordination between public housing and the newer program of redevelopment projects financed by private funds under federal mortgage guarantee.

When rebuilding slum areas for new residence use, it is generally desirable that the new population density should not exceed the old. Otherwise, problems of school adequacy at once arise. Reductions in density should be the goal in badly crowded neighborhoods. For economic reasons, this is not always possible of achievement. With good land planning, plus provision of schools and play spaces outside the project site, satisfactory conditions can result even with considerably higher densities than prevailed before demolition. The most notable examples are furnished by the work of the New York City Housing Authority. Its projects rely on low site coverage, to compensate for high buildings and high population density. Coverages of 13–22% are typical practice, and heights of apartment projects range from 6 to 14 stories.

Among major private-investment housing developments in New York, Peter Cooper Village has a net coverage of 24% and rises 15 stories, and Stuyvesant Town covers 25% of its site, with a height of 12 stories Both are owned by the Metropolitan Life Insurance Company.

In developing framework plans for a city, the professional planner needs to consider the level of densities for residential rebuilding in terms of resulting land requirements for accessory uses, service uses, and places where the people will work. A New York City analysis revealed that increase in net density in residential areas beyond a certain point will result in a decrease in theoretical gross density for the entire section of the city to which the residential districts are contributory.[5] A density point of diminishing economic returns is therefore reached It cannot be exceeded without (*a*) sacrificing proper space standards for facilities accessory to residence use, or (*b*) diminishing the economic base of the community.

Derivation of the maximum effective density can be achieved for known or reasonably assumed values of the many variables involved. It is not readily susceptible to presentation in the form of an equation that can be applied to any city. This field of inquiry merits further research, as it may prove to be the determinant of the future economic stability of our cities, as well as the best guide to planning their amenities.

Table 8 shows a hypothetical example evaluating the future residential utilization of a slum site selected for redevelopment.

In the example given in Table 8, if the land had been calculated at its full cost of $3.75 per square foot, instead of at $1 00, the average rent per apartment per month would have had to be increased from $117.50 to $143.00. Calculations such as these are greatly expedited by the use of specially prepared nomograms.

For simplicity, the example in Table 8 was set up with rent as the unknown factor to be ascertained. In practice, a certain rent level must be achieved to meet the predetermined housing market. In such cases, the problem must be "worked from both ends," so as to find the upset maximum capital cost of land and buildings per dwelling unit. Then, by deducting the construction cost, the required reuse value of the land is obtained, and the necessary write-down of land cost per dwelling unit can readily be derived This may

[5] Broad Specifications for Urban Redevelopment, by F. Dodd McHugh, *J. Am. Inst. Planners*, Winter 1946.

Table 8. Analysis of a Residential Redevelopment Project

Present conditions		
Site area	20 acres	
Present assessed value, total	$2,315,000	
Present assessed value, per square foo	2.65	
Acquisition cost, per square foot	3.00	
Site preparation	0.75	
Total site cost	3.75	
Reuse value	1.00	
Capital grant	2.75	
Dwelling units now on site	1215	
Vacant dwelling units	35	
Families eligible for public housing	480	Determined by survey
Families able to relocate on site	200	Determined by survey
Families which must be privately rehoused elsewhere, and which possibly require the assistance of a relocation bureau	500	
Proposed project		
Net coverage	13%	
Number of stories	7	
Floor area per dwelling unit	1000 sq ft (gross)	
Density	39.6 families per acre	
Total dwelling units	792	
Land cost per family, at reuse value	$1,100	
Building cost per family	9,900	
Total project cost per family	11,000	
Interest rate on 70% mortgage	5 1/4%	
Amortization period of mortgage	20 years	
Assumed percentage of vacancy	3%	
(*a*) Rent required to pay carrying charges on mortgage	$54.25 per d.u. per month	
Per cent return on 30% equity	7%	
Amortization period of equity	40 years	
(*b*) Rent required to pay carrying charges on equity	$21.00 per d.u. per month	
Ratio of assessed value to project cost	50%	
Tax rate	3 5%	
(*c*) Rent required to pay taxes	$16.50 per d.u. per month	
Maintenance cost	$80.00 per room per year	
Avg. number of rooms per d.u.	3.75	
(*d*) Rent required to pay maintenance	$25.75 per d.u. per month	
Total required average rent ($a + b + c + d$)	$117.50 per d.u. per month	
or	$33.50 per room per month	

then be expressed in terms of dollars per square foot, and also as total dollars of required capital grant.

Development of New Areas.[6] Opening up new areas for residential growth generally involves the laying out of new streets and lots. This process of land subdivision is subject to detailed review and approval by official local planning boards, pursuant to the enabling acts of a great many states. Legal and administrative procedures are discussed in Art. 13. Here, the matter of standards of design and layout requires brief treatment.

Since the mid-1930's, the trend has been increasingly toward integrated development, with the subdivision of land and the construction of houses both carried out by one company. The evils of premature subdivision, auction sales of vacant lots, petty land speculation, and sporadic home building are thus largely avoided. The Federal Housing Administration's function of guaranteeing mortgage loans has been a great stimulus to orderly development. It has established standards for street layout, street improvements, lot sizes and arrangement, and building design and construction, which are higher than formerly prevailed. Builders not operating under FHA mortgage guarantee must evidently compete for purchasers with those who do.

[6] Much of this material has been abstracted from the author's subdivision development review in Present Status of City Planning, Zoning, and Subdivision Development in the United States, a paper presented at the Pan-American Engineering Congress, Rio de Janeiro, 1949, by Harry W Alexander, Harold M Lewis, and Theodore T McCrosky, Members ASCE

Good modern subdivision design provides combinations of curvilinear streets, loop streets, and cul-de-sacs, rather than the old-fashioned gridiron pattern, which failed to discourage through vehicles from traversing residential neighborhoods and causing traffic hazards. Figure 10 illustrates an effective rearrangement of a gridiron plat, in conformity with up-to-date principles of design.[7] The developer obtained an increase in number of lots and a decrease in street area and improvement costs. The city secured dedication of a school site. Note also that the redesign provides off-street parking at one of the groups of shops.

Modern practice usually requires a minimum lot width of 60 ft, with widths of 75–100 ft and over by no means uncommon.[8] Lot depth should be twice the width, as a general rule, but shallower depth may be dictated by local custom.

Subdivisions that are intended to be served by septic tanks require larger lot sizes than those that have sanitary sewers. Minimum size will depend on character of the soil and slope of the land. A stipulation of 10,000 sq ft is reasonable for average conditions. When, in addition, the water supply is to be furnished by wells, still larger lot sizes are necessary to protect the public health. Depending again on soil conditions, minimum areas of 20–25,000 sq ft are good practice.[9]

Street area can be held close to 20% of the total, as against 25–35% a generation ago. The difference may now be dedicated or reserved for parks, playgrounds, and school sites.

In developments that are only moderately curvilinear, blocks are best laid out with their long dimensions roughly north and south. In temperate latitudes, this has been proved to give the best sunlight penetration. Blocks may have a maximum length up to 2000 ft, but, when longer than 800–1000 ft, public cross-block walks should generally be provided. These are a great convenience to pedestrians but may be considered a nuisance to abutting owners. Cul-de-sac streets should not be longer than 300–500 ft, depending on lot sizes and number of families to be served. At their dead-ends, turnaround circles at least 80 ft in diameter between curbs should be provided.

Minor residential streets are now laid out 50 or 60 ft wide between property lines, and roadways are generally 26–34 ft between curbs. Sidewalks can be omitted, on one or both sides, in developments featuring large lots, provided that the street pattern is designed so that little traffic will be generated by the frontage between intersections.

Maximum and minimum street grades are customarily fixed by local planning boards. Their values necessarily depend in part on local topographic conditions. For minor streets, grades should be held to a maximum of 8%, if feasible; a minimum grade of 0.5% is good practice to assure adequate surface drainage.

There is an interesting trend toward inclusion within the same building development of houses having a considerable range of price. This is recognized as socially and economically sound, as it gives a healthy balance to the life of the community. In a single project, prices may range from $10,000 to $18,000 or from $14,000 to $25,000. Great extremes are not usually advisable.

Increasing thought is being given to the integrated layout and construction of developments of neighborhood unit scale—sufficient in total population to support a grade school, shopping district, library, churches, and other community facilities.

An entire new town, Park Forest, has been built 30 miles south of Chicago. This will eventually house 8000 families in five contiguous neighborhoods, each with its own elementary school. Land has also been set aside for a planned industrial district, so that the town can be a reasonably self-contained economic unit.

Figure 11 provides a means of determining the land cost per lot for varying widths and depths of lot and various land prices per acre.[10] This chart is premised on rectangular blocks of 600-ft or 1200-ft length. It may be used, by interpolation, for other block lengths, and with discretionary adjustments it will give approximate land costs for lots in street layouts that are moderately curvilinear.

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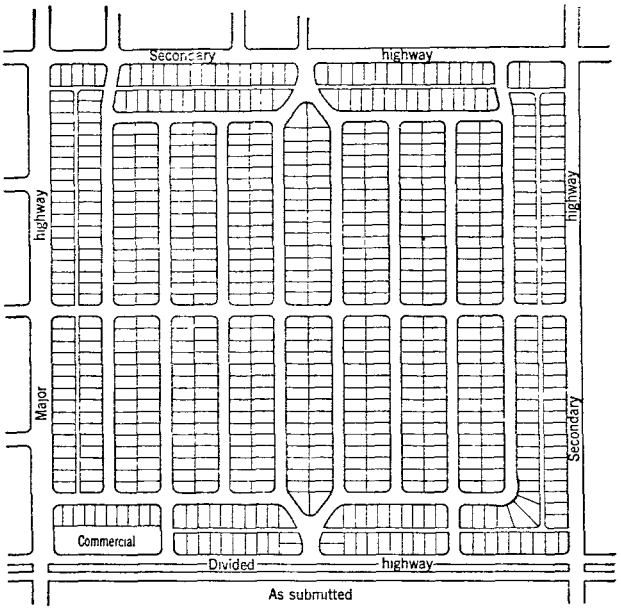
[7] *Accomplishments 1950*, City Planning Commission, Los Angeles, Calif., p. 31.

[8] *The Community Builders Handbook*, Urban Land Institute, 1954, p. 68. See also *Home Builders Manual for Land Development*, National Association of Home Builders, 1951.

[9] For detailed treatment, see Land Planning for Sanitary Drainage and Water Supply in Surburban and Open Country Subdivisions, by Robert C. Hoover, *J. Am. Inst. Planners*, Winter 1951.

[10] This diagram was prepared by the author and previously published in Land Subdivision, *ASCE Manual of Engineering Practice*, Nov. 16, 1939.



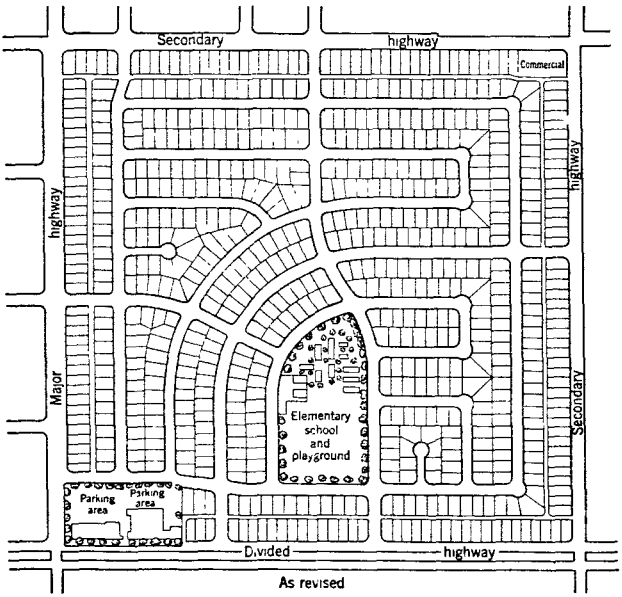


FIG 10 Effective rearrangement of a gridiron plat The revised plat below discourages through traffic on local residential streets, thereby reducing the traffic hazard School facilities are located in proper relation to the area to be served Advantages to the subdivider are lower street improvement costs resulting from decreased street area, and an increase in the number of lots for sale Department of City Planning, Los Angeles, Calif.

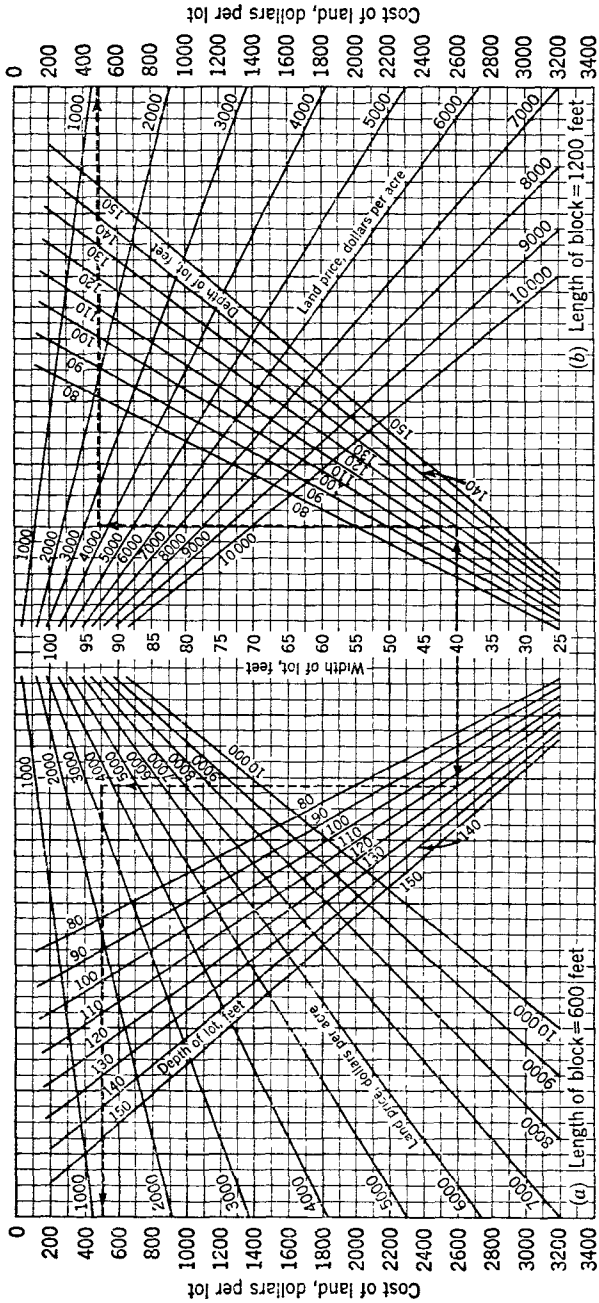


FIG. 11. Variations in land area and cost of land per lot for varying widths and depths of lots and various land prices per acre. American Society of Civil Engineers

5. BUSINESS DISTRICTS

Central Districts. *Growth of Area of Influence.* The size of metropolitan area that is capable of being contributory to a central business district has grown with the progress of the transportation industry. Before the railroads, the center could be reached only on foot or by horse or horse-drawn vehicle. The railroads greatly increased the radius of accessibility, but only within narrow sectors along their lines. With the advent of the electric streetcar and rapid-transit trains, the existing sectors widened out and new sectors became suitable for residential development. The automobile, used as a commuting vehicle, has virtually eliminated sector boundaries and at the same time has again extended the radius of convenient access.

In the mid-nineteenth century, a city rarely extended more than 2 miles from its center, giving an effective area of influence of 12 sq mi. Today, the radius of the metropolis is 15–30 miles, and its area may therefore encompass 700 to over 2500 sq mi. In the 1930's, it was anticipated that the radius would be still further increased by airplane commutation service and the use of private planes. This development has not materialized to any significant extent

The Urge for Centrality. As transportation developments widened the area of influence of the urban hub, it was inevitable that business would concentrate more and more around the center. Skyscrapers are evidence of the demand for centrality. As need for accessible space increased, property values at the center rose rapidly, more skyscrapers were built, and the business focus began to spread out. This, in turn, caused site values along surrounding streets to rise in anticipation of potential higher profit. As the price of land rose, the opportunity for a profitable return engendered the construction of more skyscrapers, thus straining the capacity of streets and transit systems and producing a demand for more transportation facilities to improve access to the center.[1] Accessibility created concentration. Concentration created congestion. Congestion was relieved, for a while at least, by increasing the means of accessibility. This is a vicious circle. "We have pursued the ideal of location to the point of destroying its advantages." [2] With the limits of concentration and of accessibility now nearly reached, the circle has begun to break. Congestion has become the force that is impelling decentralization.

Solution by Creation of Multiple Foci. If all transit lines lead to one center, that is the sole focus. But, if the transit lines are laid out to provide four major junction points within the central city, then there are four foci. This simple truism was realized too late to prevent overconcentration in our large cities, but may aid those that are not yet choked by congestion. The principle applies equally to the layout of urban expressways. Further single-focus concentration can be discouraged, and new centers encouraged, by a system of radial highways stretching outward, not from the old center, but from a ring highway surrounding it at a reasonable distance.[3]

Solution by Rebuilding the Center. The age-old centripetal forces are being weakened by congestion and counteracted by the centrifugal urge toward decentralization. In these circumstances, two schools of thought have developed. One would rebuild our urban centers, advocating high concentration of both business and residence, but with sufficient open space to prevent a repetition of congestion. The other school emphasizes the construction of new satellite towns in the environs of the urban region, which would absorb the future increment of metropolitan business and residential growth and also siphon out excess concentration from the present center.[4]

Le Corbusier, a leading exponent of concentration, envisages "green cities" composed of great skyscrapers surrounded by wide open spaces. The Empire State Building has a density of nearly 10,000 workers per acre. Such a building would stand alone in a park-like setting. The largest-scale approach to Le Corbusier's doctrine is probably the Radio City development in New York, which by conscious civic design combines massive buildings and high density with a considerable area of open space The working population is roughly 3000 per acre. Another example is the United Nations group in New York.

The satellite-town school is originally indebted to the writings and constructive work of Ebenezer Howard in England, who was the moving spirit in developing Letchworth and Welwyn Garden Cities. His thesis has been carried forward by F. J Osborne and

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[1] For the full exposition of this thesis, see Effect of Urban Decentralization upon Transit Operation and Policies, by Harland Bartholomew, *Proc. Annual Convention Am. Transit Assoc.*, 1940.

[2] *Surging Cities*, by T. T. McCrosky, Charles A. Blessing, and J. Ross McKeever, Greater Boston Development Committee, Inc., 1948, p. 101.

[3] This principle is illustrated by the expressway plan for the Boston region. Long-range plans for Chicago, San Francisco, and other major cities also furnish examples in point.

[4] This treatment is developed in part from *The Urban Pattern*, by Gallion and Eisner, 1950.

others in England, where new towns are being built under government auspices, and by many able professionals in the United States.

The American movement for satellite towns has been given impetus by recognition that decentralization of industry provides, among its other advantages, a means of reducing the potential damage from atomic attack. If industry be deliberately decentralized, as a matter of policy, it is manifest logic to provide for the accompanying decentralization of the homes of employees

The difference of opinion between the advocates of well-designed concentration and the exponents of satellite towns cannot be resolved on the philosophical plane. The central cores of our cities contain many obsolete areas, which are far more likely to be replanned and rebuilt than to be abandoned. Industry and residence *are* decentralizing of their own accord; and satellite towns, both old and new, are growing much faster than central cities. The answer is a combination of planned rebuilding at the center and planned decentralized development on the fringes The latter should absorb at least the increment of metropolitan business and residential growth. The problem was ably summed up by Thomas Adams in these words. [5]

> The evils of congestion in centers are a consequence of badly organized and ill-planned centralization but not of centralization in itself. The remedy is to be found neither by stemming the tide of outward movement from the centers nor by artificially promoting the outward flow without any improvement of organization and planning.
>
> For ourselves we prefer the term "Recentralization" to define the kind of movement that is wanted. . . .

And at a later point in his discussion:

> What is needed is properly to regulate the movement so as to increase its usefulness in defeating the evils of over-centralization on the one hand, and to counteract the bad effects of ribbon development and the spoliation of country districts, on the other hand.

Principles for Central District Rebuilding. When opportunities can be created for large-scale rebuilding of central business districts, reference to the following check list of principles and design features will assist in avoiding the causes of congestion:

1. *Assembly of entire block—or preferably superblock—sites for redevelopment*, with redesign of the local circulatory system and elimination of minor streets when feasible.
2. *Restriction, by zoning provisions, of permitted building bulk.* The floor-area ratio of the Equitable Building in New York is 35. It was about the last of the sheer wall skyscrapers. The New York City Zoning Resolution of 1916 prevented its repetition. The floor-area ratio of the Empire State Building is 32, but the street façades rise only five stories, all the rest of the floor space being in the tower. The floor-area ratio of the Radio City group is only 12.
3. *Mandatory provision of off-street loading bays for trucks*, in appropriate sliding-scale ratio to floor area.[6]
4. *Mandatory provision of off-street parking space for passenger automobiles*, proportioned to floor area and type of use.[7]
5. *Street widening by arcading of sidewalks under the buildings.*
6. *Consideration of two-level streets*, as typified by Wacker Drive in Chicago.
7. *Construction of expressways*, preferably depressed below surface grade in congested areas, and leading from the periphery of the business district to the principal satellite towns. Where the terrain permits, these radial expressways should stem from a ring expressway having a diameter of $^1/_2$–2 miles and surrounding the center.
8. *Concentration of new office space in towers*, separated from each other by open space or by low buildings. (Space on the lower floors of office buildings is less desirable, and its quantity should therefore be minimized.)
9. *Underground access to rapid-transit stations from all principal buildings*, by construction of subsurface arcades.

Shopping Subcenters and Neighborhood Centers. With the outward spread of urban areas, local shopping centers of varying size and importance logically developed to serve their contributory communities and immediate residential neighborhoods. Stores were located at street corners and along main arteries in ribbon fashion. Little or no provision was made for automobile parking. These out-dated business developments have created insoluble parking and traffic problems and constitute a danger to vehicles and pedestrians alike.

New local shopping centers should be integrated into the neighborhood they serve. Parking area, in the ratio of 2–3 times the retail floor area, is essential. It may be situated

[5] *Recent Advances in Town Planning*, by Thomas Adams, 1932, pp. 5 and 10.
[6] See Table 15.
[7] See Table 14.

in front of or behind the stores, or on near-by lots permanently reserved for this essential accessory use. If abutting upon a main artery, the shopping center should be provided with a service road on which no parking is allowed for any purpose.

Surburban Regional Shopping Centers. As a combined result of residential decentralization plus traffic congestion in central retail districts, a new pattern of merchandising has come into being. The large-scale suburban regional shopping center is a post-World War II development. It is designed to serve virtually all the customer desires of an entire sector of a metropolitan region.

Statistics on several of the principal centers are given in Table 9.

Table 9. Suburban Regional Shopping Centers

Name and Location	Site Area, acres	Selling Area, sq ft	Auto Parking
Westchester Shopping Center, Los Angeles	73	262,000	3300
National Suburban Center, Framingham, Mass.	70	502 000	6000
Northgate, Seattle	50	650 000 *	3000 *
Eastland Plaza, Detroit	103	1,250,000	6000

* Plus area for planned expansion.

If the average area required for parking one car is taken at 300 sq ft (including aisles), the parking ratio for these shopping centers ranges from 1.4 to 3.8 sq ft of parking space per square foot of selling area. Local conditions, including extent of public-transit patronage, must be taken into account in determining parking needs. Best practice, for average conditions, indicates a minimum ratio of 2.5–3.5.

A suburban regional shopping center should preferably be located at the intersection of two arterial highways or expressways, but with all access provided by its own service roads. Location within or adjoining an established retail district is usually not desirable, as the center should not be subjected to local competition and must not be hampered by street traffic.

The following is a well-considered statement of the objectives that these large centers should achieve:[8]

1. To be completely, conveniently, and flexibly geared to the private automobile. . . .
2. To provide an even greater density of productive pedestrian traffic per square foot of displayed merchandise than exists in present cities.
3. To reduce costs of occupancy, selling, publicity, and delivery . . . in structures designed to obtain a higher ratio of net-selling space to gross area, greater sales per square foot, and economical operation at either normal or peak.
4. To make available in the area the style and luxury merchandise not now available there, and keep within the area some of the purchasing power being dispersed elsewhere. . . .
5. To provide a calculated balance between shopping and recreational facilities, each classification contributing increased volume to the other.

Suburban regional shopping centers involve a great investment and must therefore be most carefully analyzed to determine their potential sales volume and earning power. The primary trade area is customarily delimited by including all communities within 30-min average driving time by preferential routes. The number of families and average income by municipality or section should be determined from census data. Appropriate percentage discounts from the total families in each community are next applied, depending on driving time involved. The greater the time, the smaller the proportion of families that can be expected to patronize the center.

As a further step, the residue of families not eliminated by the time-distance factor must be reduced by a judgment percentage to allow for the competitive attraction of local stores in or near each home community. Separate competition factors are needed for (*a*) general merchandise, apparel, furniture, furnishings and appliances, and (*b*) food. The above categories are census classifications for which detailed sales figures are published for all but very minor political subdivisions.

By application of this method, with extensions and refinements to suit the particular case, a sound estimate of the trade volume of a projected shopping center can be prepared. However, no procedure of this kind can be purely statistical, and experienced judgment must be used.

Analysis of the tributary area will determine the number and type of stores that should be included within a suburban regional shopping center. It is recognized standard practice to build the pulling power of the center around a branch of a well-known city depart-

[8] The North Shore Shopping Center, *The Am. City*, July 1947, p 124.

ment store and to include one or more branches of high-grade "specialty shops." One or two super markets are usually included. Competition within the center is a favorable factor as it attracts a larger total patronage.

With these as the nucleus, there can be 15–50 or more small shops each offering a particular line of goods or services. A commercial bank is practically essential, and a branch savings bank and a post office are desirable. Restaurants of various price ranges are a necessity, and at least one gasoline filling station is required. Some major centers have found it advantageous to include an office building for professional and general business tenants. The question of whether to allot space for a moving picture theater and other commercial recreation uses must also be considered. Lastly, and of great importance to the success of these enterprises, is the provision of free nurseries and playgrounds with professional supervision.

Suburban shopping centers may be expected to play an increasingly important role in the economy of metropolitan regions. They are a major manifestation of the decentralization trend. If well planned and operated, they should help to reduce congestion in the old central retail districts and to achieve a better balance between the forces of centripetal and centrifugal growth.

6. INDUSTRIAL DEVELOPMENT

The professional planner is concerned with the problem of providing for the location and space needs of industries that are deemed suitable to the community or region, and with the changes in transportation requirements that accompany any major industrial growth or shift. He also has occasion to make plans for redevelopment of existing industrial districts. Actual design of industrial sites—placement of buildings and location of spur tracks, handling facilities, and employee parking areas—does not generally come within his field of work. It is important to stress, however, that the modern trend of industrial requirements dictates a pattern of one- or sometimes two-story buildings laid out for belt-line production and surrounded by a large acreage of land for plant expansion, parking, and landscaping

Old Heavy-Manufacturing Districts. Established heavy industrial plants and districts must usually be accepted by public authority as they stand. Plant investment is so great that they cannot be moved, except by decision of management. Public provision can be made for amelioration of the traffic conditions they create, and smoke-control measures can minimize their adverse effects upon adjacent development.

However, when such industrial plants face the necessity for expansion, it will frequently be found that their increment of floor space can more advantageously be built at a decentralized location. This will usually have a beneficial effect on peak-hour transit loading, as many workers will commute outward from the city to the plant, rather than inward to the old center. Sometimes, even very large plants are abandoned, as the result for example, of moving to a more efficient satellite site. In such cases, the municipality, through its planning board, gains the opportunity to study the optimum reuse classification of the land and to make appropriate zoning revisions, if indicated

Old Light-Manufacturing Districts. Light manufacturing presents different problems, and usually offers greater scope for gradual improvement of the districts where it is established. Many such districts are in central locations and are characterized by lack of parking space and by old buildings that are inefficient for modern production, because of inadequate floor area, poor column spacing, narrow staircases, small elevators, lack of handling space on the ground floor, etc. Such buildings are often intermingled with retail stores, tenements, and office buildings, so that efficiency is further reduced by street traffic congestion [1] Unfortunately, there are many marginal enterprises that can make a profit only by leasing cheap space in outmoded structures. However, with increasing emphasis on efficiency, as a result of high labor costs for lost time and motion, the demand for good layout is increasing.

The opportunity exists to demolish whole areas of these substandard buildings, replan their sites, and redevelop with modern loft buildings that will provide efficient space for tenant manufacturers, whether their need be for a mere 2000 sq ft or for several whole floors. If the obsolescent district is predominantly blighted residential, as measured by land area, its redevelopment for light manufacturing qualifies for federal capital grants-in-aid, under the terms of Title I of the National Housing Act of 1949 (see also Art. 4, p. 1–19.)

[1] *Surging Cities*, by T. T. McCrosky, Charles A. Blessing, and J. Ross McKeever, Greater Boston Development Committee, Inc , 1948, p 106.

The principal desideratum in redevelopment for light industry is to achieve extensive plottage. The new buildings can then offer large rentable area on a single floor, and still have open space for light and air and for off-street loading and parking facilities. Sites 1 acre in size are by no means excessive for a single loft building. Sites smaller than 100 x 100 ft are no longer considered good practice, and even this size is inadequate for handling generated traffic.

Planned Industrial Districts. A large industrial corporation, when faced by the problem of costly congestion in its present location, coupled often with the need for expansion, can move out and build itself a new plant in open country. A small corporation, with limitations on its financial capacity, may not have this choice. It is dependent upon rented space and the availability of streets and spur tracks used jointly with other companies. To meet the requirements of small industries, without sacrifice of quality of facilities provided, many variations of cooperative enterprise have been evolved. The basic pattern has been summarized as follows:[2]

A private development corporation acquires a large acreage of industrial land, lays out and builds necessary streets and railroad spur tracks, installs water mains and utilities, provides storage warehouses, constructs docks, if the location is on a waterfront, and then sells or leases parcels of land or buildings to individual industrial companies. The corporation undertakes to finance both land purchases and construction costs. It usually erects the buildings, relieving individual industries of this problem Location and style of all structures are made to conform with general plans for the development of the entire tract. The corporation usually maintains appropriate cafeterias, club rooms, and medical clinics. It centralizes the management of all rail and truck operations. The cost of these many features is included in the lease or purchase agreements of the individual companies, which thus obtain an extent and quality of service that they could not otherwise afford.

Outstanding examples of this type of enterprise are the Central Manufacturing District in Chicago, which owns an aggregate area of 850 acres of land, with more than 300 industrial companies; and the Clearing Industrial District, which has over 2000 acres on the outskirts of Chicago. Other examples are Bush Terminal in New York, the Fairfax and North Kansas City Industrial Districts in Kansas City, Northwest Terminal Company in Minneapolis, and the Boston Wharf Company. The latter controls nearly 100 buildings with 4,500,000 sq ft of floor space.[3]

7. TRANSIT

City Planning Objectives. The construction of expressways induces new volumes of automobile drivers and passengers, part of whom are diverted from the public transit lines. Because of this, it is sometimes considered that the expressway and the mass-transportation route are competing facilities. From the planning standpoint, this is not the case.

New express highways have not cured transit jams, and new subways have not solved highway traffic congestion. Better transit and better highways are both essential parts of the modern transportation network. The problem is not *which* to build, but how much more of each Probably no region can afford a completely adequate system of express highways and subways.[1]

The following statement summarizes the objectives of good transit planning as related to preparation of a comprehensive master plan.[2]

(1) Coordination of routes and services in order to bring about the most desirable distribution of urban population;
(2) Direct routes from the central business district to all residential areas having a population density of 10 persons per acre or more;
(3) Integration of streetcar and bus lines into a well-coordinated system, and elimination of any unnecessary duplication of lines and service;
(4) Adequate cross-town routes, including direct service to large centers of industrial employment, large recreational areas, and the like, and
(5) Good routing plan and expeditious flow in the central business district.

In the achievement of any or all of these, the transit system is dependent upon the proper execution of the other parts of the comprehensive plan, particularly the street plan and the zoning plan.

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[2] *Ibid.*, p. 108.

[3] This subject is treated in detail in Organized Industrial Districts, by Robert L. Wrigley, Jr., *J. Land and Public Utility Econ.*, May 1947.

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[1] *Surging Cities*, by T. T. McCrosky, Charles A. Blessing, and J. Ross McKeever, Greater Boston Development Committee, Inc., 1948, p 96

[2] Modern Transit—Key to Community Planning, by Harland Bartholomew, *Tomorrow's Cities*, American Transit Association, 1944, p. 8.

Types of Transit Facility and Their Capacities. Table 10 lists the principal types of transit vehicles and compares their hourly passenger-carrying capacity with that of the private automobile.

Table 10. Passenger-Carrying Capacities per Lane per Hour

Mode of Transport	Number of Passengers per Lane per Hour
Automobiles on city street	600–1,270 *
Automobiles on expressway	2,250–2,550 †
Buses on city street	3,870–9,100 ‡
Buses on expressway	6,300–15,000 §
Streetcars on city street	6,300–13,500 ‖
Streetcars on own right-of-way	10,800–16,200 ¶
Local subway trains	32,000 **
Express subway trains	50,000 ††

* Range is computed from 1.5 persons per car (rush hour) to 1.7 persons per car, and from 400 to 750 cars per lane per hour.

† Range is computed from 1.5 to 1.7 persons per car, at 1500 cars per lane per hour.

‡ Range is computed for buses with 32 seats plus 11 standees, and buses with 55 seats plus 15 standees, for schedules of 90–130 buses per hour.

§ Range is computed for buses with 55 seats plus 15 standees, for schedules of 90–215 buses per hour.

‖ Range is computed for streetcars carrying 90 passengers, for schedules of 70–150 streetcars per hour.

¶ Range is computed for streetcars carrying 90 passengers, for schedules of 120–180 streetcars per hour.

** Trains carrying 800 passengers, scheduled at 40 trains per hour

†† Trains carrying 1000 passengers, scheduled at 50 trains per hour.

Theoretical and actual capacities are seen to vary widely with size of vehicle and scheduling. Table 10 gives a considerable range of values, because the schedule that can be maintained is usually dependent upon the size and efficiency of terminal or other loading facilities. With expressway operation, it is theoretically possible to schedule a solid lane of buses on 6-sec headway, giving a discharge capacity of 600 buses or 42,000 passengers per hour. The Port of New York Authority's bus terminal operates on 5-sec outbound headway in rush hours (720 buses per hour), but its discharge makes use of both westbound lanes in the Lincoln Tunnel. It is desirable to point out that the loading of buses in stalls at a terminal gives them a decided advantage over the loading of streetcars at a platform alongside the main track. The streetcar lane capacities given in Table 10 presuppose (*a*) multiple loading positions on several platforms in a terminal, or (*b*) a confluence of routes, with passengers boarding the streetcars before the routes come together.

It can be computed from Table 10 that a lane of large buses on a city street can carry 8.1 times as many passengers per hour as a lane of automobiles averaging 1.5 persons per car under the favorable condition of a 750-car-per-hour discharge rate. It is evident that public transit, if well patronized, is the most efficient way to obtain maximum utilization of available roadway area in congested downtown districts.

Streetcars vs. Motor Buses. It used to be considered that the motor bus should be used only for outlying feeder service and for light-load downtown lines, and that the streetcar was far superior for heavy-traffic city routes. This doctrine has been negated by the experience of a great many large cities where streetcars have been replaced by buses even on the most heavily traveled lines. Transit capacity has been increased, because the streetcars used to be caught behind slow trucks, which the bus can avoid. Furthermore, traffic flow has been improved, because the streetcars impeded the progress of private automobiles. Public acceptance of the bus appears to be complete, and there are few if any surface streetcar lines that need be considered permanent fixtures in the transportation plan. Exception should be made of operations involving streetcars that run a part of their route in subways and the balance on the surface, as in Boston and Philadelphia.

Trolley-buses are not included in Table 10. Their passenger-carrying capacity per lane is recognized to be somewhat less than that of streetcars, but in excess of buses. Their special application is in situations where the electric railway company has a large unamortized investment in power houses, and therefore finds that converting to trolley-buses is financially more feasible than to gasoline buses. The trolley-bus also has the operating advantage of a faster acceleration than the gasoline bus. It suffers from some of the shortcomings of the streetcar: rigidity of route and maintenance cost of overhead wires.

Subways. Because of their exceedingly high construction cost, subways are financially possible only in very large cities, and even then are restricted to special and unusual conditions of routing and population distribution. There must be a dense residential concentration along the route, or the certainty that high density will be built up as a result of the subway construction. Also, there must be a highly concentrated business district for the subway to serve. Major cities that have grown to their present size without sub-

ways can rarely meet these two conditions. The downtown center may be highly concentrated but the residential population is usually spread thinly over the land, so that only a multiplicity of surface transit lines can give service close to the people's homes. The Chicago subway system is not an exception, as it is intended as the eventual replacement for the elevated lines which shaped the earlier pattern of growth. San Francisco, however, should be mentioned as a possible exception. Its exceedingly hilly terrain and the fact that Market Street is the backbone of the street system combine to indicate that a subway might be logical. In seeking the solution to transit problems, the planning engineer should not count too heavily on unstudied proposals to build subways. Thorough technical and economic analysis is essential. Also, it should be remembered that subway lines have sometimes proved the cause of congestion rather than its cure.

Railroad Commuting Service. In the larger metropolitan regions, railroad commutation service long preceded the development of transit lines. During recent years, the railroads have been losing commuting riders to public transit and private automobiles, despite great increases in total suburban population. On lightly traveled lines, the railroads are faced with the alternatives of continuing an unprofitable operation or abandoning service entirely. On heavy-traffic lines, the demand for service sometimes exceeds trunk-line track and terminal capacity. A compensating factor, of potential significance, is the reverse-flow rush-hour traffic induced by industrial decentralization.

From the planning standpoint, the indicated long-term solution may take one of the following forms

1. Consolidation and interconnection of railroad commuter service and existing mass-transit service, using the railroad tracks but operating the transit equipment under transit management.

2. Abandonment of railroad commuter trackage and reuse of right-of-way for automobiles and express buses.

Public Owership of Transit Systems. Detailed evaluation of the trend toward municipal or public authority ownership and operation of transit lines is beyond the scope of this article. Mass transportation of passengers is essential to every metropolitan region. When rising costs make its continuance no longer financially possible under private corporate auspices, there is no other alternative than public ownership. The corporation's service should, in the long run, pay its own way. In most cases, in actual practice, the deficit from commuter-line operation is made up from trunk-line freight revenues. Public transit need not pay its own way, as deficits can be defrayed from real estate taxes. At present cost levels, if a privately or publicly owned transit system were to fix its fares on a self-supporting basis, they would usually be at or above the level of diminishing returns. A substantial proportion of the system's passengers would desert the mass-transit lines and drive their own cars, or in some cases would move back into the central city.

Railroad Electrification. Unique planning opportunities arise when a railroad electrifies its passenger and terminal operations. The tracks can then be covered over and used as sites for new buildings that will literally change the face of the city. In the days of steam-train operation in New York, Park Avenue above 42 Street was an open railroad yard. The luxury hotels and apartment houses that now line this avenue stand on columns erected between the tracks. Part of the Park Square office building district in Boston was also a former railroad yard.

Routing of Transit Lines. The principle of multiple foci of transit routes has been discussed in Art. 5 as being the most effective means for creating a decentralized pattern of central business district growth. The outstanding example is the "inner circle" of the underground railway system in London. Trains are operated in both directions around this route and intercept every radial line. The junction stations are the focal points for business centers that would not otherwise have come into being. The most nearly equivalent American example is the elevated loop in Chicago, which made it possible for the central district to spread out over an area of more than $^1/_2$ sq mi.

By contrast, New York has no inner circle or loop. The early elevated and subway lines were laid out roughly parallel to the axis of Manhattan. They focused all their traffic in the downtown financial district, and extended across the East River to make this district accessible to the principal population concentrations of Brooklyn. Only in comparatively recent years, with the construction of the line across Manhattan at 59 Street into Queens, and with the construction of other cross-town routes further south, has there been created a system of junction points, where decentralized growth can logically develop. However, with one exception, all the New York subway lines are radials rather than intercepting circumferentials.

With modern emphasis on construction of expressways, the planning engineer has the opportunity to correct many major deficiencies of past practice in the layout of transit routes. As a general rule, every expressway and most express parkways should be designed for use by transit facilities. Transit may take the form of rail service on a central

right-of-way, or may be provided by express bus service on the highway itself. In the latter case, special design of bus turnouts at loading stations is required. These stations should be of double-deck construction to provide for transfer to transit lines situated on arteries that cross the expressway above or below its grade.

Integration of transit with expressway design is the most significant planning development in the field of urban transportation. Failure to integrate will result in congestion of the expressways by commuting private cars, and in continuance of inadequate and unpopular transit service on crowded surface streets.

Riding-Habit and New-Route Analysis. Table 11 gives consolidated riding-habit figures for representative cities. The "riding habit" is the quotient of the total revenue transit rides per annum divided by the population in the area served. Riding habit is a useful measure of over-all transit demand and has its greatest value in analyses of the financial

Table 11. Riding Habit in Representative Cities

City	Riding Habit,* 1950
Population over 1,000,000	
Chicago	218
Detroit	154
Los Angeles	96
Philadelphia	211
Population 500,000–1,000,000	
Baltimore	168
Buffalo	170
Cincinnati	162
Milwaukee	260
New Orleans	229
San Francisco	241
Population 250,000–500,000	
Dallas	154
Fort Worth	115
Indianapolis	136
Louisville	195
Rochester, N. Y.	190
San Antonio	136
San Diego	109
Population 100,000–250,000	
Corpus Christi	78
Des Moines	130
Evansville	177
Fort Wayne	145
Gary	54
Omaha	154
Savannah	119
Shreveport	155
Tacoma	131
Trenton	123
Utica	102
Population 50,000–100,000	
Binghamton	92
Evanston	177
Harrisburg	193
Springfield, Ill.	139
York, Pa.	58

* Total revenue passengers, divided by population in area served (including suburbs).

Source: Transit Operating Reports, 1950, Part 1, System-Wide Totals, *Bull.* 72, American Transit Association, July 1951.

feasibility of general service improvements. For study of individual routes, and especially of projected new routes, other data are required, as the riding habit gives only the consolidated experience of all operating lines, whether heavily or lightly patronized.

To evaluate the prospective earnings of a new route, it is best to seek comparison with the most closely equivalent line already in local operation. Items that should be ascertained for the old line include:

1. Population served per route mile.
2. Rides per capita of contributory population per annum.
3. Effects of feeder and junction lines.
4. Extent of profit or loss.

In applying these comparative data to the proposed line, with necessary adjustments, a factor must be introduced for new building development. This should include housing known to be projected, plus additional residential construction that the new line would be expected to induce.

In many cases, the importance of developing general good will by rendering a desired service to the public will be the deciding factor. However, the transit operator must have an advance estimate of the probable extent of losses, before deciding whether it is prudent to assume a new liability.

Terminals. Modern practice requires the provision of off-street terminals for radial bus and streetcar lines. The continuing use of street space is inefficient and constitutes a cause of congestion that should assuredly be removed whenever feasible. Progress so far has been largely confined to construction of terminals for suburban lines, the local city services usually continue to load on the public streets. This is often inevitable, as these local lines are through-haul operations and may not have any dominant loading point where an off-street station would be feasible

In December 1950, the Port of New York Authority opened its terminal for bus lines using the Lincoln Tunnel to New Jersey. This terminal occupies the entire Manhattan

block between 8 and 9 Avenues from 40 to 41 Streets (200 by 800 ft). Eighty-five per cent of the buses enter and leave on grade-separated roadways directly connected with the tunnel portals, and therefore are entirely removed from the congested surface streets. There are 53 loading berths or platform positions for long-haul buses, and 87 for short-haul suburban service. The building is 4 stories high, with provision for parking 450 private automobiles on its roof. Over 50% of costs are recouped from rentals of stores and concessions.

Today, a bus terminal is needed even in the small town of 10,000–20,000 population. Its retail-trade economy is dependent in part upon bus-riding patronage. These potential customers should not be expected to wait on street corners in the rain or to huddle in the entrances of stores. Provision should be made in the master plan for an attractive bus station and waiting room, and a sufficient number of covered bus stalls off the street. Local requirements of layover and loading will determine the number of stalls to provide. The original design should be capable of expansion to handle anticipated future increases in service.

Operating Practices. Determination of schedules, locations of stops, and practices regarding fare collections and transfers are usually outside the province of the planning engineer. However, his special knowledge of the integration of transit with the city plan may give him opportunities to suggest improved operating practices For example, inauguration of skip-stops and a "pay-as-you-leave" policy in the downtown area may contribute materially to the relief of traffic congestion, and thus obviate the need for restrictive traffic regulations and expensive street improvements.

8. MOVEMENT OF GOODS

This article is mainly restricted to those aspects of freight and commodity handling that relate directly to the work of the planning engineer. Section 6 of this volume includes treatment of the problems of railroad terminal design, and Sect. 10, Vol. II, is devoted to port development.

Port Facilities. All seventeen of the cities in the United States with over 500,000 population are ports. Most owe their origin and, in varying degree, their subsequent growth to water-borne commerce. Although the freight car and the motor truck have largely superseded the cargo vessel in handling domestic freight shipments, the role of a port in an urban economy will continue to be of great significance. A major ocean port generally is and will remain the most vital single factor in the future economy of its metropolitan region.

A ship may be said to earn money only when under way. Time at dock is lost time. A principal requirement for an efficient port is therefore the provision of facilities for rapid handling of cargo. In St. Louis, it has been estimated that 60% of the cost of river freight is in the handling at terminals.[1]

Few planning engineers will have occasion to design docks or to determine their equipment. However, they should take steps to interest the proper authorities in assuring their adequacy to meet the needs of shipping and to prevent loss of trade to competing ports.

The particular role of the planning engineer is to integrate the physical development plans for the port with the over-all master plan for the city and region. In basic terms, the port must have good accessibility by rail and by highway. No matter how good the facilities of the port itself, it cannot function efficiently if its traffic is bottlenecked in congested city streets.

Rail Freight. The land-use pattern of many cities and regions has been crystallized by the location of the railroad network, yards, and terminals. In the interests of efficient planning, freight terminals should be close to the primary origins and destinations of the commodities they handle. Freight classification yards, by contrast, should be on the periphery of the urban area, so that they will not constitute barriers to anticipated growth or damaging influences upon adjacent development. Also, peripheral location reduces train and car movements in the congested area, with resulting savings in time and cost. Chicago furnishes an example of railroad yards that were once at the edge of the built-up area but have long since been swallowed up by the spread of the city. Planning opportunities exist in Chicago and many other cities to relocate railroad yards on the outskirts, and thus to recapture their present sites for higher and more profitable economic uses.

The larger urban regions present the problem of multiple service by many separately owned freight railroads, each having its own tracks and each handling shipments to and

[1] *Guide Plan, Missouri-Illinois Metropolitan Area Development*, prepared for the Metropolitan Plan Association, Inc., by Harland Bartholomew and Associates, 1948.

from all parts of the metropolitan area. The recognized solution is to provide a belt-line railroad, under joint ownership. Inbound freight can then be routed from the trunk railroad via the belt line to the terminal that is most convenient for ultimate delivery to consignee. Outbound freight can be similarly handled. By this means, length of truck haul through city streets will be reduced to a minimum.

Chicago has both an outer and an inner belt line. The New York Connecting Railroad performs the same function, but has recourse to carfloat ferries for transfer between the Brooklyn waterfront and the trunk-line terminals on the west side of the Hudson River in New Jersey. Equivalent belt-line facilities, in various forms and under differing types of ownership, exist in most of the large metropolitan regions. St. Louis and Boston are other outstanding examples. As a general rule, belt lines have not been provided in the smaller cities. Their absence presents a planning problem of increasing importance.

The situation has been summed up as follows: [2]

> If a merchant or manufacturer receives goods from New York or New Orleans or Milwaukee, or if he wishes to make shipments to Pittsburgh or Indianapolis or Denver, he should not be compelled to go to a different part of the city to receive or deliver the goods arriving or leaving by a particular line. He should be able to use the nearest and most conveniently located freight station, whatever may be the destination of his goods or by whatever line they may be carried. This is a fundamental economic principle which holds good whether the town be large or small, whether the transportation lines serving it be few or many.

A major urban planning requirement, not always recognized, is the provision of rail-truck interchange terminals, for handling pick-up and delivery of l.c.l. (less-than-carload-lot) freight. Such terminals, when served by a belt line, can concentrate a substantial proportion of these movements at a centrally located facility where consignments can be assembled, combined, and handled with maximum efficiency. The outstanding example is Inland Freight Terminal Number One, completed in 1932 by the Port of New York Authority. It occupies an entire block, 800 by 200 ft, on the lower west side of Manhattan, and provides joint service to nine trunk-line railroads. This great terminal has 15 floors of warehouse, manufacturing, office, and loading space, and is the third largest building in the world, by cubical content. It handles more than 300,000 tons of l.c.l. freight plus 225,000 tons of railway express per year. It has resulted in most substantial savings in trucking, handling, and rail-freight costs, and by consolidation of truck loads has reduced congestion of pick-up and delivery service on the streets.

Trucking. Movement of freight by truck, other than in pick-up and delivery service, is a recent development and presents new planning problems that have grown to critical proportions.

The high incidence of over-the-road trucking is emphasized by the following representative figures: [3]

Commodity	Per Cent Received by Truck
Milk, 75 metropolitan areas with 69,000,000 population	100
Live poultry, 11 principal metropolitan areas	99.7
Shell eggs, 14 principal metropolitan areas	97.7
Livestock, total receipts at stockyards	75.6
Butter, 11 principal metropolitan areas	55.4
Fruits and vegetables, 20 principal metropolitan areas	46.5
Cheese, 11 principal metropolitan areas	43.0
Sand and gravel, total products of mines	70.4

The Automobile Manufacturers Association has reported a study of all freight and express traffic originating in the New York Central System territory, conducted by the "Ship Coal by Rail Committee" of East Syracuse, N. Y. The proportions of total tonnage shipped by truck, classified by length of haul, were found to be as follows: [4]

	Per Cent
Intra-terminal	96
25–100 miles	82
100–300 miles	68
300–500 miles	44
500–1000 miles	28
Over 1000 miles	3

[2] *Planning the Modern City*, by Harold McLean Lewis, John Wiley & Sons, 1949, Vol. 1, pp. 67–68.

[3] All data, except for sand and gravel, are from the U. S. Department of Agriculture, as abstracted from tables published by Automobile Manufacturers Association. Sand and gravel data are from U. S. Bureau of Mines. Figures are for 1952, except as follows: milk, 1948; sand and gravel, 1951.

[4] *Motor Truck Facts*, Automobile Manufacturers Association, 1949, p. 26.

A questionnaire survey by the Association of American Railroads in 1949 [5] revealed that the preponderant reason why shippers use trucks was "shorter transit time." This answer constituted 75% of all replies. Other significant primary reasons were "lower costs" and "less handling." Such explanations as "less loss and damage" and "less marking and packing" were cited as primary reasons by negligible percentages of the shippers.

The above facts and figures emphasize the vital role that the motor truck plays in the American metropolitan economy. There are nearly 10,000,000 trucks in the United States (1954), and approximately 30 per cent are registered in the 115 most important urban counties. If they are to function with efficiency and without disruption of street-transit and passenger-automobile traffic, the following special types of facilities must be provided as intrinsic components or recommendations of the master plan:

1. Mixed-traffic express highways leading close to the center of the metropolitan region (as contrasted with parkways reserved exclusively for private passenger cars).
2. Truck terminals for interchange between over-the-road and local pick-up and delivery trucks.
3. Off-street loading berths for all buildings that are major origins or destinations of truck freight.

In addition, under the heading of traffic regulation, it has been found desirable to limit the length of trucks permitted in central business areas during congested hours of the day,[6] and in extreme cases to restrict pick-up and delivery operations to off-peak hours.

The Port of New York Authority has constructed two large Union Motor Truck Terminals. Their statistical descriptions are given in Table 12. The Authority published the following statement, in explanation of the need for its New York terminal: [7]

The current situation, which this union terminal seeks to eliminate, is characterized by:

1. Wasteful use of over-the-road equipment to effect deliveries and pick-ups in congested areas.
2. Inadequate and scattered individual terminals which result in:
   (a) Wasteful utilization of over-the-road units to transport interchange freight between terminals in contrast to transferring freight across a platform in a union terminal.
   (b) Wasteful utilization of local pick-up and delivery equipment because trucks operate only partially loaded if restricted to small zones, or if fully loaded, have to cover large zones, thus wasting equipment and manpower.
   (c) Waste of manpower occasioned by lower productivity per man, both in local delivery and in individual platform operations, than would be secured by union terminal operations.

Table 12. Port of New York Authority Union Motor Truck Terminals

	New York	Newark *
Location	Lower Manhattan, near Holland Tunnel	Route U. S. 1, north of Newark Airport
Site area	3.67 acres	29 acres
Dimensions of building	1000 x 160 ft	1174 x 202 ft
Dimensions of freight platform	800 x 80 ft	1000 x 100 ft
Number of truck berths	144	160
Rated capacity	2000 tons/day	2500 tons/day
Capacity per truck berth	13 9 tons/day	15.6 tons/day
Approximate cost	$10,000,000	$8,200,000
Opened for operation	November 1949	July 1950
Estimated savings in truck-miles	1,830,000 per yr	2,000,000 per yr

* This terminal was leased in 1951 to the U. S. Army Air Force.

Source: The Port of New York Authority Marine and Inland Terminals, 1949.

In 2 1/2 sq mi of St. Louis, there are reported to be 114 separate truck terminals. "Seventy-eight buildings serving 170 truck lines were located in compliance with the proposed St Louis Master Plan, while 36 buildings serving 115 truck lines are located in conflict with the zoning provisions of the proposed plan." [8] Construction of one or more consolidated freight interchange terminals for rail-truck-water transport is recognized by the Bi-State Development Agency of the Missouri-Illinois Metropolitan District as requiring a high priority in its planning and construction program [9]

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[5] Abstracted from summary report in *Motor Truck Facts*, Automobile Manufacturers Association, 1949, p. 26.

[6] Example: Regulations of New York City Department of Traffic, 1951.

[7] *New York Union Motor Truck Terminal*, The Port of New York Authority, May 1949.

[8] *Guide Plan, Missouri-Illinois Metropolitan Area Development*, prepared for the Metropolitan Plan Association, Inc., by Harland Bartholomew and Associates, 1948, p. 31.

[9] A Bi-State Compact for Metropolitan St Louis Area Development, by W. Philip Shatts, *State Governments*, magazine published by the Council of State Governments, April 1949.

In 1947, the Boston City Planning Board issued a detailed report recommending construction of a union motor truck terminal.[10] The statistical data therein presented are indicative of good practice in the analysis of metropolitan requirements.

Many of the findings of the report were derived from a questionnaire sent to a 20% sample taken from the membership list of the Massachusetts Motor Truck Association (242 letters sent) plus a large number (188 letters) to other common carrier trucking firms. The following major conclusions were derived from the questionnaire and other sources of information:

1. Approximately 7200 tons of freight in small consignments are picked up or delivered daily in the downtown area by roughly 2000 trucks traveling 30,000 truck-miles.
2. About 2300 tons of freight are interchanged daily between common carriers in the Boston area.
3. The present loading factor for the average of all trucks is only 44% of capacity.
4. A truck interchange terminal could attract 2400 tons per day of pick-up and delivery business, plus 400 tons of interchange freight.
5. Average loading factor could be doubled.
6. A 12.5% reduction in all pick-up and delivery truck traffic through the streets would be realizable.
7. The terminal would save 27 cents per ton of freight handled at its platforms, plus 51 cents on each ton of freight picked up or delivered.
8. Gross annual saving to truckmen and shippers (and eventually to the public) would be in the order of magnitude of $800,000.

The Boston City Planning Board recommended that a single terminal designed to handle 2800 tons daily should have the following features:

Site area of at least 9 acres.

Administration and storage floor area of 90,000 sq ft

Platform or platforms totaling 1250 ft in length, with back-up spaces for 200 trucks, and equipped with a mechanical conveyor system.

Access roadways and parking areas.

Further analysis of Boston's problems of freight origination, routing, and downtown congestion led the Planning Board to recommend that the above capacity be divided between two terminals, one to the north and the other to the south of the central district

Examination of the St. Louis and Boston cases indicates clearly that major union truck interchange terminals, whether financed by public or private corporate funds, will not and should not be viewed as replacements for all existing private facilities. Their role in the metropolitan plan is to consolidate the centers of operation of large numbers of the smaller companies at one or several strategic locations, and to attract thereto a significant portion of less-than-truckload business of the major over-the-road common carriers This function of *partial* unification will reduce street congestion and costs of operation. *Total* unification in any large city, on the other hand, would tend to increase haulage distances, overconcentrate the streams of truck traffic, and thereby defeat its own prime purposes.

These considerations point to the great importance in the regional plan of reaching an authoritative determination of the number and location of union truck terminals to be provided. The following check list of factors should be considered:

1. Location and relative tonnage of existing truck terminals.
2. Location, number, and relative tonnage of truck freight generation centers (light-manufacturing districts, etc.).
3. Number of main radial highways extensively utilized for intercity truck shipments, and relative tonnage by route.
4. Number of trucking companies and other truck operators by type of operation and size of fleet: local pick-up and delivery common carriers, industry-owned pick-up and delivery fleets, metropolitan area common carriers, general over-the-road common carriers, interterminal operators, etc. Representative loading factors for each.
5. Influence lines for origin and destination of heavy truck traffic volumes
6. Determination as to which existing terminals should best be superseded by the proposed union terminals.
7. Estimate of tonnage, by classifications, to be diverted to the proposed union terminals.
8. Check of alternate terminal-location plans, to secure near-minimum truck travel on downtown streets. Calculation of potential savings on haulage costs.
9. Congested streets and intersections to be relieved by provision of new terminals.
10. Optimum routing between proposed terminals, for interchange traffic.

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[10] *Report on a Union Motor Truck Terminal for Boston*, City Planning Board, Boston, October 1947.

11. Traffic flow on local streets induced by proposed terminal sites, adequacy of street widths and driveway accesses, and effect upon surrounding properties.
12. Determination as to whether truck-trailers are to be restricted in pick-up and delivery operations.
13. Land value of alternate sites, foundation conditions, etc.
14. Determination of number of berths required, site and building sizes, capital cost, potential savings in handling costs, etc.

A theoretical principle is implicit in the planning of union truck terminal locations: The terminals, in an ideal plan, would be spaced circumferentially at or near the intersections of the inner belt highway [11] with the most important radial arteries, and their relative sizes would be determined by combined weighted judgment of the magnitude of truck traffic on the radials plus the nearness of the terminals to the major truck tonnage generators.

9. TRAFFIC GENERATORS

Section 3 of this volume is devoted to traffic engineering. This article is restricted to those aspects of traffic generation that relate directly to the work of the planning engineer, i.e., the determination of the quantitative relation of land use to traffic generation as a basis for restrictions on building use and bulk and for the requirements of off-street loading and parking.

Traffic Generation by Size of City. The following tabulation shows the percentages of all persons bound for the central business district who travel by private automobile, for a series of cities and groups of cities in which recent check counts have been made:

	Per Cent by Automobile
Average of 8 cities under 100,000 population *	81.2
Average of 6 cities of 100,000–500,000 population *	69.1
Washington, D. C.	65
Kansas City	51
Boston	25
San Francisco	25
Chicago	20
New York (Manhattan)	10

* *Parking*, Eno Foundation for Highway Traffic Control, 1946, p. 26.

These percentages are principally influenced by the availability, speed, convenience, and popular acceptance of mass-transportation lines. With some exceptions, public carrier service has been most highly developed in the largest cities, which at the same time present the worst conditions of street congestion and traffic delay to the private driver.

Traffic Generation by Express Highways. The inference may be drawn that any marked improvement in public transportation would diminish the proportion of travelers using private automobiles. However, when the improvement is accompanied by a material increase in fares, this result may fail to be achieved. On the opposite hand, any notable improvement in highway facilities, such as construction of a new radial expressway, will invariably induce a new volume of traffic that did not previously exist. It will thus increase the percentage of automobile riders to the central district. This induced traffic may amount to a surcharge of 30% or more.

Expressways relieve the traffic load on existing parallel streets, even though they may themselves become congested. However, except for a small proportion of through intercity vehicles, all traffic on a radial expressway must eventually become local traffic on local streets, in order to reach ultimate destinations. The only exception is the traffic that terminates at the exit from the expressway, either in a convenient parking lot or in a garage located for direct access. Recognition of the importance of discouraging expressway vehicles from congesting local streets was a prime reason for construction of the Battery Parking Garage in lower Manhattan, at the junction of the West Side Express Highway, the Brooklyn Battery Tunnel, and the Battery Park Underpass connecting with East River Drive. This garage has a capacity of 1050 cars and is the property of the New York City Bridge and Tunnel Authority.

Traffic Generation by Specific Land Uses. The majority of the parking surveys conducted in recent years have determined parking demand in terms of quantity of space, duration of use, and purpose of trip. It has been less usual to relate the quantitative demand to the floor area or other appropriate measure of size of the various types of

[11] See Art. 5, Business Districts, Solution by Creation of Multiple Foci.

building that the automobile parkers have as their destinations. Several pilot studies are reported in *Urban Traffic, a Function of Land Use*, by Robert B. Mitchell and Chester Rapkin (Columbia University Press, 1954). This significant book is primarily concerned with the analysis of basic concepts related to the interaction of land use and vehicular traffic. There is need for more field research, and a broad study has been initiated by the Eno Foundation for Highway Traffic Control (July 1954). An actual survey was conducted in Oakland, Calif., in August 1949. It covered 1161 off-street parking stall spaces provided by the Downtown Merchants Association on a sales ticket validation basis (free parking for first hour, and 15 cents per hour thereafter) Table 13 gives an abbreviated summary of the findings.[1]

Table 13. Summary of Parking-Space Demand

(Created on 24-hr week day by various traffic generators, and relationships with floor area or other basic units.)

Generator	Gross Floor Area or Other Basic Unit per Parking Space Required
Department store A	283 sq ft (selling area)
Department store B	518 sq ft (selling area)
Office building A	1628 sq ft (net rentable area)
Office building B	818 sq ft
Retail and mail-order store	1816 sq ft
General market	199 sq ft
Neighborhood shopping community	813 sq ft
Hotel	2.7 guest rooms
Theater	19.1 seats *
Public high school	1263 sq ft or 7.5 students

* An unusually low experience figure. See Table 14. One parking space for each 4–12 seats is the general range of design requirements

The parking requirements given in Table 13 are representative of the type of data needed for technical solution of the parking problem. These data furnish the basis for private off-street parking regulations to be included in the zoning ordinance, and also measure the demand for public parking areas to be provided by the municipality, so as to overcome existing deficiencies.

Actual figures for the demand generated by different land uses will vary widely from city to city. Extents of the public transit trade and walk-to-shop trade are the principal variables. Free parking versus parking for a fee is also an important factor.

Traffic Generation by an Office Building. The following example is illustrative of the type of analysis required to evaluate the effects upon the city plan of major traffic generators. The figures are entirely hypothetical, but derive from reasonable assumptions and representative experience.

A skyscraper office building is proposed for a central site in a city of 133,000 people. The plot measures 200 x 100 ft, fronting on three streets. The zoning ordinance permits a 10-story structure on 60% average lot coverage The skyscraper will therefore contain 120,000 sq ft of gross floor area and may be assumed to house a working population of 1000.

In a city of this size, at least 60% of the workers would normally arrive by automobile. Allowing 1.5 employees per car in the rush hour, the building will generate an all-day parking demand for 400 car spaces Utilization of the open area on the lot (40%) would accommodate only about 30 cars. If the balance of 370 spaces were provided on a separate open parking lot, its area would need to be 2.1–2.5 acres (250–300 gross sq ft per car stall). If 370 spaces were built into the building itself, they would occupy 5 or 6 floors—an obvious impracticability.

In addition to employee parking, there must be provision for visitors Assume 50 visitors' automobiles to be the maximum at any one time. Curb space across the 200-ft building front plus 100 ft on each of two side frontages would accommodate only about 15 cars, after allowance for corner clearance, fire hydrants, etc. Hence, 35 visitors' cars must preferably be parked off the street, as other curb locations may be expected to be needed for the customers of other buildings. The end result is that our 10-story office building should have a separate accessory parking area of 2.25–2.75 acres This roughly corresponds to an entire city block measuring 200 x 500–600 ft, and is approximately equal to the total gross floor area of the building.

[1] *Off-Street Parking Requirements*, by John G. Marr, *Planning*, American Society of Planning Officials 1949, p. 55

What happens when the 400 employees' cars converge upon this office building in the morning? Assume that they all arrive within a 30-min period, and that they approach in equal volume from four directions, on four access streets. Each street will then have a generated traffic load, for this building alone, of 100 cars in 30 mins. For typical traffic-light-controlled streets, this is the equivalent of roughly one-half of one vehicular lane, running at comfortable capacity.

Now let it be assumed that 30% of the 1000 total workers arrive by bus. These 300 transit riders will require only five or six modern buses in all, divided among the four directions of the compass and spaced out over 30 min. It is evident that this does not result in any abnormal demand for bus terminal facilities. Even if *all* the workers arrived in buses, not more than 20 loads would be needed; spaced out over half an hour, and allowing 4 min per bus in the terminal, 3 or 4 bus stalls would suffice.

The remaining 10% of the workers arrive on foot. They present no problem. If the entire 1000 workers enter the building in 30 min through doorways on three of its sides, and if the sidewalks are 15 ft wide and each person spends an average of 1 min on the sidewalk outside the building, there would be, at any one time, 180 sq ft of sidewalk area per pedestrian bound to the building. Studies indicate that comfortable walking area is only 15 sq ft per person, and averages as low as 6 sq ft prevail on crowded sidewalks.

Traffic Generation of a Business District. It is worth while to carry this hypothetical analysis several steps further. Our city of 133,000 population can be assumed to have roughly 50,000 total workers. If representative proportions are used, the following breakdown of employment results:[2]

Manufacturing	40%	or	20,000 workers
Retail and wholesale	25		12,500
Business and professional	8		4,000
Transportation, Government, Education, Other	27		13,500
Total workers	100%		50,000

It may next be assumed that the following will require central business district space:

Manufacturing	15%	or	3 000 workers
Retail and wholesale	60		7,500
Business and professional	80		3,200
Other classifications	10		1,300
Total central district workers			15,000

From the above tabulation, it is seen that the rush-hour traffic load, before shoppers arrive, will be 15 times the traffic generation of our 10-story skyscraper. Required aggregate off-street parking area, assuming that all curb space is needed for transients, will be roughly 40 acres, including space for visitors other than shoppers. If there is a 60-min spread in morning arrival times, the 6000 automobiles will require 15 full lanes on the access streets. The 4500 bus passengers can be handled in 90 buses, or roughly one every 3 3/4 min from each of the four directions. A minimum of 6–8 bus stalls would be indicated. As in the single building example, the people who walk to work would present no problem of sidewalk congestion.

Zoning Requirements for Parking. In order to help prevent future building construction from intensifying the already critical parking problem, an increasing number of cities now require the provision of off-street parking area as an accessory to new buildings of certain types. Over 300 cities now have regulations of this kind.

Table 14 briefly reviews the requirements found in the ordinances of 155 cities,[3] with additional data drawn from other sources as noted. This table is actually a composite of present practice, rather than an authoritative guide. When the planner is drafting zoning provisions for a particular city, he should first analyze the local quantitative demands for parking space

Examination of Table 14 leads to the conclusion that the range of zoning requirements for a particular land use is too broad to be justified by variations in local conditions. There is also a great diversity of methods for computing these requirements, the extent of which is only partially revealed by the tabulated data. John Marr has pointed out in

[2] Based on Fort Wayne, Ind., with some modifications.

[3] Abstracted from Zoning for Parking Facilities, Highway Research Board, *Bull.* 24.

Table 14. Zoning Requirements for Off-Street Parking Space

Land Use	Square Feet of Parking Space Required: Minimum	Average	Maximum	Other Basis for Computation of Requirements *
1- and 2-fam. dwellings	100 sq ft per d.u.	181.6 sq ft per d.u.	300 sq ft per d u	1 space per d.u.
Multiple-fam d.u.'s	33 sq ft per d u	166 8 sq ft per d u	420 sq ft per d u	1 space per d.u.
Hotels	20 sq ft per room	81.2 sq ft per room	300 sq ft per room	1 space per 3–6 rooms
Hospitals		26 7 sq ft per bed 0.72 space per sq ft of sleeping area	0.6 sq ft per sq ft of gross floor area	1 space per 6–10 beds
Mortuaries		480 sq ft per room	0 6 sq ft per sq ft of gross floor area, plus 0 3 sq ft for each sq ft of gross floor area over 3000	1 space per 50 sq ft of floor area
Restaurants		1 space per 50 sq ft of patron use * 1 space per 100 sq ft of gross floor area *		1 space per 5 seats
Theaters	13 sq ft per seat 0.08 space per seat	30 sq ft per seat 0.23 space per seat	50 sq ft per seat 0 5 space per seat	1 space per 4–12 seats
Auditoriums and churches	12.6 sq ft per seat	29 7 sq ft per seat	50 sq ft per seat	
Schools		40 sq ft per seat (college) 12 6 sq ft per seat (high school)		
General business office buildings	15 sq ft per ft of lot frontage	32 sq ft per ft of lot frontage	40 sq ft per ft of lot frontage	0.92 sq ft per sq ft of floor area 1 space per 400 sq ft of net floor area
Industry	1 space per 4 employees	1 space per 3 employees * 1 space per 750 sq ft of floor area *	1 space per 2 employees *	Total parking space equaling 1–3 times floor area
Retail stores				1 space per 200 sq ft of floor area 1 space per 300 sq ft. of floor area exceeding 4000 sq ft

* The figures in this column and other figures marked * are drawn from miscellaneous sources, including *Parking Manual*, American Automobile Association, and various zoning ordinances not covered in the Highway Research Board's bulletin cited below.

Source. Data are from Zoning for Parking Facilities, Highway Research Board, *Bull* 24, except as noted above

his analysis of these same data, for example, that there are twenty-one different units of measurement for general business requirements.[4]

It may be anticipated that further survey knowledge, supported where necessary by empirical computations, will lead to greater uniformity of space requirements Variations would then reflect only the differences in local conditions. Also, when this type of zoning provision becomes less experimental, a higher degree of standardization in units of measurement will logically follow.

Another problem, integrally related to the provision of private off-street parking space, is the location and adequacy of ingress and egress driveways. This can be handled satisfactorily by requiring that the plan of driveway facilities be submitted for approval by the Zoning Board of Appeals, and subject to advisory technical report by the Planning Board. In this connection, it should be pointed out that certain concentrated generators of traffic, such as outdoor theaters, require an ample car reservoir outside their pay gates. Otherwise, traffic will back up on the running lanes of the tributary highways.

Loading Space. Many zoning ordinances require in new buildings of certain types the provision of off-street loading and unloading bays for trucks. New York City requirements are as given in Table 15. There are certain exceptions, for which reference should be made to the full text of the zoning amendment. The New York City Planning Com-

[4] Off-Street Parking Requirements, by John G Marr, *Planning*, American Society of Planning Officials, 1949, p. 61

Table 15. Required Off-Street Loading Berths for New Construction, City of New York *

For the Uses Set Forth in the Subsections Below	In the Following Area Districts	Off-Street Loading Berths Shall Be Provided as Set Forth Below: Square Feet of Floor Area	Required Berths
For all hospitals, in addition to any spaces provided for ambulances	In all area districts	From 10,000 to 300,000	1
		For each additional 300,000 or fraction thereof	1 additional
For all undertaking establishments and funeral parlors	In all area districts	From 2,500 to 5,000	1
		For each additional 10,000 or fraction thereof	1 additional
For all hotels and offices	1. In B area districts	From 75,000 to 300,000	1
		For each additional 300,000 or fraction thereof	1 additional
	2. In all other area districts	From 25,000 to 100,000	1
		From 100,000 to 300,000	2
		For each additional 300,000 or fraction thereof	1 additional
For all retail sales and service uses	1. In B area districts	From 25,000 to 40,000	1
		From 40,000 to 100,000	2
		For each additional 150,000 or fraction thereof	1 additional
	2. In all other area districts	From 8,000 to 25,000	1
		From 25,000 to 40,000	2
		From 40,000 to 60,000	3
		From 60,000 to 100,000	4
		For each additional 150,000 or fraction thereof	1 additional
For all wholesale, manufacturing and storage uses, or any combination of such uses	1. In B area districts, except where such districts coincide with manufacturing and unrestricted use districts	From 15,000 to 40,000	1
		From 40,000 to 80,000	2
		For each additional 80,000 or fraction thereof	1 additional
	2. In B area districts, where such districts coincide with manufacturing and unrestricted use districts, and in all other area districts	From 8,000 to 25,000	1
		From 25,000 to 40,000	2
		From 40,000 to 60,000	3
		From 60,000 to 80,000	4
		For each additional 80,000 or fraction thereof	1 additional

* As amended to April 29, 1954.

mission also drafted a set of less restrictive retroactive truck-loading requirements for existing buildings devoted to retail sales, service, wholesale, manufacturing, and storage uses. However, these regulations for existing buildings were disapproved by the Board of Estimate and are therefore not in effect.

10. RECREATION AREAS

Park and Playground Standards. Requirements for outdoor recreation areas differ from city to city and depend upon such variable factors as economic and social backgrounds, population density and age distribution, local scenic resources, topography, and climate. Precise standards therefore cannot be established, but as the basic needs of all urban populations are similar, it is possible to indicate appropriate ranges of requirements. By exercise of judgment, these can be used both for evaluating the adequacy of existing facilities and as a basis for planning additions to park systems.

Table 16 is a synthesis of recommended standards compiled from studies made by the National Recreation Association and other agencies

As a very rough measure of adequacy, it is sometimes stated that combined park and playground area should equal 10% of city area and provide 1 acre of recreation space per 100 population. These two standards are mutually consistent only when the population density is equal to 10 persons per gross acre of the city. For higher densities, 10% of city area gives insufficient recreation space. New York City has 13.0% of its total area in parks, but still provides only 1 acre per 293 people.

It is apparent that the 10% standard can be very misleading. For example, a town might have one large park, but practically no small neighborhood parks and playgrounds.

Calculating recreation needs in terms of 1 acre for each 100 population is open to similar criticism. However, if recognized purely as a combined standard which must later be divided into its components, it will yield reasonably good results for communities of medium and high population density.

Table 16. Park and Playground Standards

Type of Facility	Minimum Size	Maximum Travel Distance	Size Related to Population	Age Group Served
Playlot	2400–5000 sq ft	1/4-mile walk	1 playlot per 30–60 families	2–6 yr
Playground	3.25 acres for 2000 pop. 4.00 acres for 3000 pop. 5.00 acres for 4000 pop. 6.00 acres for 5000 pop.	1/4–1/2-mile walk	1 acre per 800 pop 2.3 acres for 200 children 3.05 acres for 450 children 4.45 acres for 800 children 6.25 acres for 1200 children	6–14 yr
Playfield	20–30 acres	1/2–1-mile walk	1 acre per 800 pop 1 playfield per 20,000 pop.	High school students and adults
Neighborhood park	1 1/2–2 acres	1/4–1/2-mile walk	1 acre per 500–1500 pop	Primarily adults
Large recreation park	50–100 acres	1–3 miles	1 park per 40,000 pop	All ages
Park reservation	1000 acres	Ready access by auto or public transportation	Serves a city or urban region	

Sources: Most of the data in this table are taken from Standards for Municipal Recreation Areas, by George D Butler, *Recreation*, July–August 1948. Mr. Butler has drawn on the Standards of the National Recreation Association, of which he is Director of Research, and also on recommendations of the American Public Health Association, National Park Service, and other authorities.

Table 17 gives percentages of park area, and acres of park per 100 persons, for a wide range of American cities over 25,000 population. It will be observed that many of the listed cities have better than 10% of park area, whereas only a few show more than 1 acre of park per 100 persons.

Table 17. Park Acreage as a Function of City Area and Population, for Representative Cities, 1950

City	Population *	Total Land Area,† acres	Park Acreage ‡	Park Acreage per 100 Population §	Per Cent of City Area in Parks §
New York, N. Y.	7,835,099	204,982	26,644	0.341	13.0
Chicago, Ill.	3,606,436	132,200	6,024	0.168	4.6
Philadelphia, Pa.	2,064,794	81,408	8,393	0.406	10.3
Cleveland, Ohio	905,636	46,800	2,360	0.261	5.0
St. Louis, Mo.	852,623	39,000	3,048	0.358	7.8
Boston, Mass.	790,863	28,100	3,019	0.382	10.7
Seattle, Wash.	462,440	43,800	2,953	0 639	6.7
Toledo, Ohio	301,358	23,700	2,154	0 715	9.1
Pasadena, Calif.	184,087	12,400	1,223	0 663	9.9
Salt Lake City, Utah	181,718	33,600	1,555	0 856	4.6
Wichita, Kan.	166,306	13,500	2,650	1.591	19.6
Utica, N. Y.	101,429	10,100	686	0.677	6.8
Huntington, W. Va.	78,836	8,060	800	1.017	9.9
Springfield, Mo.	66,302	8,700	453	0 684	5.2
Ogden, Utah	56,910	10,570	1,200	2 110	11.4
Asheville, N. C.	52,208	9,280	400	0 766	4.3
Fargo, N. D.	37,981	3,200	470	1 239	14.7
Beloit, Wis.	29,541	3,070	323	1 093	10 5
Santa Fe, N. M.	27,547	17,320	31	0.113	0.2
Attleboro, Mass.	23,665	17,480	37	0 156	0.2

* U. S. Census of Population, 1950.

† *Areas of the United States*, 1940, U. S. Census.

‡ *Recreation and Park Yearbook*, National Recreation Association, 1951. "Park acreage" includes playgrounds and playfields other than those on school sites.

§ Computed by the author.

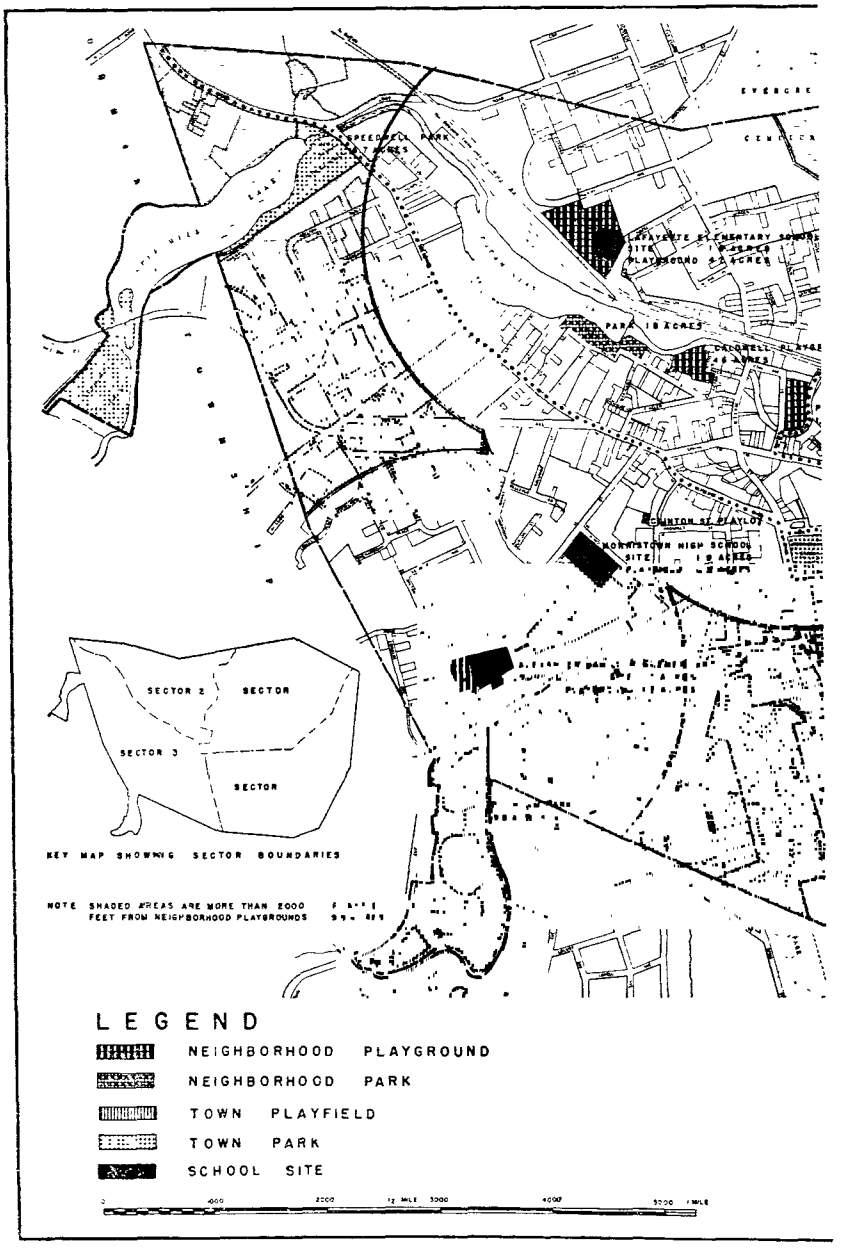
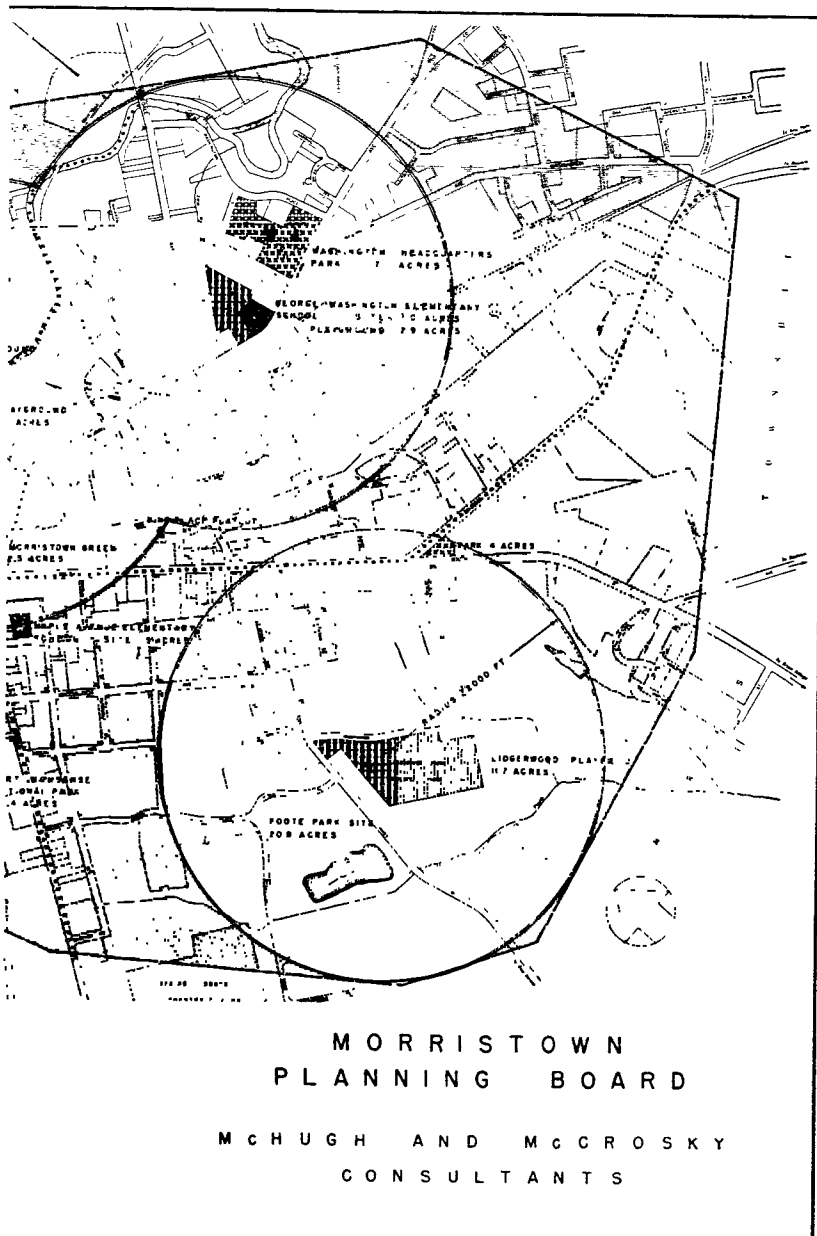


FIG. 12. Parks and playgrounds,



Morristown, N. J , 1949.

The National Recreation Association has published suggested standards for a large number of special athletic facilities that should be provided within the public parks and play areas. The following tabulation gives the more important of these standards:[1]

Baseball	1 diamond per 6000 population
Softball	1 diamond per 3000 population
Tennis	1 court per 2000 population
Golf	1 hole per 3000 population (18-hole course sufficient for typical city of 50,000)
Swimming pool	Adequate to serve 3% of population at one time, on basis of 15 sq ft of water area per person in the pool[2] (100 x 60-ft pool thus adequate for a community of 13,000)

Recreation Surveys. In order to measure the sufficiency of a recreation system and to plan for its future, two basic surveys are customarily indicated. First, there must be an inventory of the existing facilities: *what* and *where* and *how large* are the questions to be answered. Separate maps should generally be prepared for playlots and playgrounds, for playfields, and for parks. The available areas and types of recreational service should be tabulated. Second, it is necessary to know the intensity of local use of each facility, so as to gage the standards that would be appropriate to apply. These will generally fall within the ranges given in Table 16. Sufficient indication of intensity of use may be obtainable from Park Department and Athletic Commission records of attendance, supplemented by personal observation. If statistical data are inadequate, an attendance survey should be made.

Determination of Adequacy of Recreation Areas. The recreation standards in Table 16 involve minimum sizes, radius of influence, and ratio of size to population. There should be plotted on the survey maps the effective area of influence of each separate existing recreation space. If population density is low, this area will be circumscribed by the maximum walking distance determined as the suitable standard for the community. In communities of irregular street systems, or restricting topographic barriers, the area of influence will evidently not be a true circle. Corrective adjustments must be made. When population density is medium to high, more people will live within the allowable walking distance than can be served by the available recreation area. In such cases, the effective area of influence can best be determined from the population dot map or from block statistics of dwelling units. Any unusual age composition characteristics of the contributory population should be taken into account, such as a concentration of small apartments containing a low proportion of children of playground age. A simple example of a map showing existing parks and tributary areas is given in Fig. 12.

Some existing neighborhood parks and playgrounds will be found to be smaller than the minimum sizes indicated in Table 16. These must be analyzed on the basis of the limited scope of facilities they can provide. Other recreation areas may be of adequate size, but far from fully developed in terms of baseball diamonds, tennis courts, swimming pools, and other facilities and equipment they could accommodate. These present an administrative rather than a physical planning problem. The land is there, and the planning engineer should recommend its proper development as funds become available.

At this stage, the series of survey maps will show all areas of the community that are adequately served by one or several different types of recreation area, and all those areas that are deficient in the provision of playlots, playgrounds, playfields, and neighborhood parks. As the problem is generally an immediate one, these maps should be based on present population, so as to reveal existing space deficiencies that require current remedial action.

Correction of Existing Recreation Space Deficiencies. The unserved or insufficiently served areas of the community must next be investigated for potential park and play sites. Required sizes should be determined by the same procedure described for evaluating existing recreation spaces, combined computations involving radius, population to be served, and minimum area, for each separate type of facility.

General preferential locations will be revealed by the maps. Sometimes public land is available in other administrative jurisdictions, and transfers to the Park Department can be arranged. It is also profitable to search the records of real estate tax delinquency. Many cities and towns have found that they can obtain needed sites by foreclosure of tax-defaulted lands. Emphasis should also be placed on opportunities to enlarge existing parks and playgrounds that are below acceptable minimum size.

By this process, an immediate recreation plan is built up from the survey data. This

[1] Abstracted from *Schedule for the Appraisal of Community Recreation*, National Recreation Association, 1940.

[2] A revised standard. In 1940, the National Recreation Association suggested 20 sq ft per person in the pool.

plan, however, is limited to the correction of *existing* recreation space deficiencies for the present population. It is only the first step toward the long-term plan.

The Recreation Master Plan. To provide for the future, the immediate recreation plan must be restudied in the light of anticipated population growth and distribution. Economic base studies and the general future land-use plan give the foundation for population expectancy in each section of the community. The analysis procedure described in the previous sections should be repeated, substituting future population statistics for 10 or 20 years hence. Different areas of influence will result for the existing recreation spaces, and different area requirements for additions to the system will emerge. Vacant lands that are ripe for residential growth must be brought into consideration for the first time.

The resulting recreation master plan will indicate the desirability of modifications in the immediate plan. For example, if the immediate plan shows need for a 4-acre playground in an unserved neighborhood, and the master plan reveals that 6 acres will be required 20 years from now, it will usually be wise to obtain the entire 6-acre site at one time, and then to develop its full potential gradually as the population increases. By similar reasoning, in districts of expected high-density residential growth, it will be better to provide one 2-acre neighborhood park immediately, and a second of the same size within a few years, than to acquire a 3-acre site halfway between, to satisfy only the immediate need.

Just as the long-range master plan is an extension of the immediate plan in order to meet future expansion, so the first-stage acquisition program derives from splitting up the master-plan proposals into a schedule of priorities.

The initial step of preparing a tentative immediate plan is sometimes omitted. This can have a great disadvantage from the standpoint of public relations. The people know their present problem, at least in terms of the general deficiencies they want corrected at once. The planner should not, therefore, ask them to comprehend a plan for 20 years hence, while he himself deliberately ignores technical analysis of the problem facing the public today. The people will readily grasp the immediate plan, and can then appreciate the reasons for minor modifications that will make it fit into the plan for the future.

Regional Parks. Suitable sites for large parks will often be found to lie well beyond the local municipal boundary lines. Provision of these facilities is generally a matter for the jurisdiction of the state or county, or an intermunicipal park authority. The location and extent of these regional recreation areas will depend upon the available land, its special attractiveness and scenic features, and on the relative needs for particular types of use such as beaches, mountain resorts, and yachting, as evaluated in terms of financing considerations.

11. SITES FOR PUBLIC BUILDINGS

Civic Centers. In the early years of the twentieth century, as a result of the influence of the Chicago World's Fair of 1893, many American cities embarked on ambitious plans for civic centers. They had outgrown their old city halls and courthouses and aimed to create harmonious groupings of new buildings of monumental design. These great centers were not intended to be built in a day. They were planned on ample sites, and new buildings have been added through the years. Among the outstanding civic-center developments of this tradition are those of San Francisco, St. Louis, Cleveland, and Denver. A good many other cities made similar plans, which for various reasons—usually financial—could not be carried into execution. A common error was that these plans were too grandiose, and often unrelated to the needs of the community and the practicalities of land replanning.

The second half of the twentieth century offers new opportunities to achieve good civic-center planning and construction, scaled to immediate and clearly emerging needs. Many large cities are now nearing their probable ultimate population expectancy. Future government floor-area requirements can therefore be estimated more closely than was possible 40 years ago. The old public buildings are clearly outdated and inadequate. In fact, it has become customary for cities to rent private office space to supplement their public quarters. There is also an awakened popular understanding of the enlarged responsibilities and needs of government. This combination of crystallized need and favorable reception by the people fits into the pattern of the urban redevelopment program established by the National Housing Act of 1949.[1] It is quite usual to find several blocks of antiquated tenements practically in the shadow of the central business district. Replacement of the old city hall can become part of the plan for rebuilding the slum areas of the city.

[1] See Art. 4, under the heading Rebuilding Old Areas.

Local requirements and desires must always determine the particular governmental functions to be given place within a civic center. The following is a considered check list, but should not be interpreted as representative of any accepted standard of good practice:

Aquarium	High school
Art gallery	Hospital
Auditorium	Museum
Board of education	Opera house
Bus terminal	Planetarium
City hall	Police headquarters
College	Post office
County offices	Public library
Courts of law	Public market
Federal offices	Radio and television station
Fire headquarters	Railroad passenger terminal
Garage for city-owned vehicles	Sports arena
Garage for public parking	State offices
Hall of records	Tourist information bureau

Many functions on the above list could evidently be combined into one building, for example, county offices and county courts of law, federal offices and post office, art gallery and museum. Certain of the tabulated uses are usually considered mutually incompatible: public markets, hospitals, and high schools furnish instances in point. None of these are normal parts of a civic center. They are included in the list for the sake of completeness, because under special local site conditions a plan might embrace a very great range of facilities without loss of integrity and harmony. One effective site composition is to provide a grouping of civic buildings around a plaza, which can be used for public assembly. The market places of medieval towns are obvious examples of this concept of design.

Civic Subcenters. In large municipalities, the planning board needs to prepare separate master plans for the location of police precinct stations, fire stations, branch libraries, health centers, and other types of public services that require sites within separate neighborhoods rather than a single structure for city-wide coverage. Such specialized parts of the master plan are customarily worked out in close collaboration with the cognizant administrative departments. The planner's position of strength in these studies is that he can integrate and coordinate a wide range of separate and seemingly unrelated requirements. Through this process, he can evolve plans for combinations of services and groupings of public buildings in effective civic subcenters, often focusing around a community high school. The planned development can be laid out *in toto*, and the entire needed site acquired. Construction of individual facilities should then proceed gradually as capital funds become available. There can be no set rules. The functions of the planning engineer are to synthesize space requirements, to select suitable sites for adoption as parts of the master plan, to prepare schematic site plans, and to include cost allowances in the long-term capital program. Subsequent detailed development of such projects is the province of the architect. The indicated procedure obviates random site selections by individual departments without reference to other municipal requirements, and without control by an over-all master plan.

Individual and Isolated Buildings. By no means all municipal buildings can or should be grouped in civic centers and subcenters. Centralization of municipal functions must be examined by the common rules of need and reason. It can be greatly overdone.

The seat of government is customarily kept separate, for example, from public markets and docks. Water filtration and pumping stations, sewage-treatment plants, and garbage incinerators have their locations fixed primarily by geographic factors. A municipal garage for public parking might be badly needed in the business district, whereas it might be too far off the beaten track if placed in the civic center.

In every city and town there will be special or local site requirements for individually placed municipal services. Under the more advanced state planning statutes, *all* public building locations are subject to review by the planning board in relation to the master plan. In some instances, the plan will have already made provision for the particular need. In others, review by the planning board will serve to assure that the proposed facility will not be incompatible with other long-term improvements planned for the immediate neighborhood.

12. PUBLIC SCHOOLS

Capacity and Space Standards. Size of city, population density, traditions of past practice, and financial limitations all exercise great influence on local policy as to standards and planning for future school requirements. The problems are very complex. Partly for this reason, even large school systems with competent staffs of analysts are prone to

call in specialized consultants or seek the assistance of professional planners. As a result, there is a trend toward greater uniformity of standards, and an increasing administrative desire to overcome the recognized inadequacies of criteria used in the past.

Table 18 gives the basic school standards used by the Los Angeles City Planning Commission, in preparation of its master plan. Los Angeles has been selected for purposes of illustration because it has the acute problems that are characteristic of fast-growing cities

Table 18. School Types and Space Standards in Los Angeles *

Type of School	Grades	Age Group, years	Service Radius, miles	Students per School	Site Area,† acres
Elementary	Kindergarten–6	5–11	3/8–5/8	250–750 ‡	10
Junior high	7–9	12–14	1–1 1/2	1000–1500	20
Senior high	10–12	15–17	2–2 1/2	1800–2200	40
Junior college	13–14	18–20	5–6	2500–6000	100

* As used by the City Planning Commission for its Master Plan of School Requirements.

† These site standards are more generous than ordinary practice (see text)

‡ From the standpoint of administrative economy and efficiency, some authorities consider that 400 elementary pupils is the minimum effective number, and that schools over 700 do not yield any economic or educational advantages (see *Planning the Modern City* by H. M. Lewis, John Wiley & Sons, Vol. I, p. 224.

of great size, and at the same time has set its standards at a quality level rarely found except in small and progressive municipalities. This has been possible because Los Angeles still has large open areas within its corporate boundaries, and provision for sites conforming to these standards can generally be made at the time of land subdivision. It should also be pointed out that Los Angeles builds single-story school structures, as is now usual in modern practice. The site areas shown in Table 18 are planned for utilization in the following approximate proportions:

	Per Cent
Building coverage	30
Playground	50
Campus and automobile parking	20
Total	100

It will be noted at once in Table 18 that Los Angeles has a K6-3-3 educational system,[1] plus provision for 2 years of junior college. This accords with the general trend of educational thinking. The old and traditional K8-4 system is being progressively changed over, but there are many jurisdictions where it still prevails. In some school districts, a K4-4-4 system has been found to be the most desirable. Decisions as to type of organization and the best time to change from one to another vest in the local board of education and not in the planning board. However, considerations related to scheduling of major bonded improvements in the planning board's capital program may provide an opportunity to work with the board of education in coping with this problem

Tributary Areas for School Plant Planning. The total area under the jurisdiction of a board of education is referred to as a "school district." It may be of greater or less extent than the territory within the corporate limits of a single municipality. One school district will usually include a considerable number of school buildings, each of which will have its own "attendance district," for purposes of efficient and convenient apportioning of the total school district enrollment among the several schools.

Except in very large school districts, the determination of the capacity requirements of the district as a whole, plus the approximate siting of new buildings, will usually complete the contribution of the professional planner. The board of education and school superintendent can then proceed to work out attendance district boundaries, which must frequently be adjusted from year to year. However, in cases where the school district extends over a wide and sparsely settled area, embraces several communities, or takes in an entire city of substantial size, the planner will need to make separate statistical studies of the attendance districts tributary to each existing school or geographic grouping of related schools.

Data Needed for Planning Future School Requirements. To the extent that proves feasible, it is desirable to develop the following basic and derived data and estimates for each geographic area for which a separate calculation of school plant requirements is to be prepared:

[1] Elementary school. Kindergarten through 6th grade
Junior high 7th, 8th, and 9th grades
Senior high 10th, 11th, and 12th grades

1. Total population and average size of family, for 2 or 3 U. S. census years.
2. Intercensal estimates of population and number of families, for each year.
3. Number of births, by years.
4. Birth rate, by years.
5. Estimated future birth rate, by years.
6. Change in total number of dwelling units, for each year since the last U. S. census.
7. Predicted future changes in total number of dwelling units, by years.
8. Predicted future number of families, by years (allowing for appropriate vacancy ratios).
9. Past school enrollment, by years and grades, extending back at least 20 years if comparable statistics are available.
10 Calculated total births for each future year.
11. Past-experience factor: kindergarten enrollment, expressed as percentage of total births 5 years prior.[2] (This percentage is conditioned by enrollments in parochial and private schools and by the mobility of the population.)
12 Past-experience table for recent classes graduating from high school, showing the size of class at completion of each grade, expressed as a percentage of the size of each class when it completed kindergarten.[2]
13. Past-experience coefficients:[3]
   (*a*) Number of grade school pupils per family.
   (*b*) Number of junior high school pupils per family.
   (*c*) Number of high school pupils per family.
14. Recent experience as to number and per cent of enrollment by grades represented by children *not* born in the statistical district (in-migration).
15. Recent experience as to number and per cent of lost enrollment by grades, represented by children born in the statistical district who moved away before completion of the next higher grade.
16. Preschool census of all children by year of birth and probable year of entry into kindergarten.[2]
17. Separate determination of anticipated experience coefficients for number of children per family due to enter school, by years, from:
   (*a*) New single-family-house districts.[4]
   (*b*) New multiple-dwelling districts.[4]
18. Experience factors for most recent U. S. census year:
   (*a*) Per cent of 5- to 14-year age group represented by K-8th grade enrollment.
   (*b*) Per cent of 15- to 19-year age group represented by 9-12th grade enrollment.
19. Estimate of the numerical size of the 5- to 14-year and 15- to 19-year age groups for future intercensal and census years.

Factors Affecting Choice of Procedure. The list given under the previous heading has been stated to be *desirable*. It will be recognized that many of the items provide alternate or overlapping methods of prediction, which are useful for the purpose of checking and adjusting separate estimates for mutual consistency. It will frequently be found that some of these items cannot be ascertained without extensive surveys and research, which may not be feasible to undertake. The planner must therefore use his judgment in developing a sound procedure consistent with existing or readily obtainable data.

The combined incidence of items 14 and 15 is reflected in the experience coefficients of item 12, unless very recent building construction is initiating a markedly different trend.

In large jurisdictions, such as whole cities, item 13 is often used as a substitute for items 10–12 and 14–19. The application of these coefficients, with appropriate adjustments for the future, will give total estimated annual school enrollment, by groups of grades, without the refinement of separate predictions for each grade. In Los Angeles, this method was applied to the probable ultimate total population and number of families in each well-established residential neighborhood. For new areas not yet developed, Los Angeles used the following coefficients, which were derived from recent local experience:

	Pupils per Family
K–6th grade	0.5
7–9th grade	0.214
10–12th grade	0.214
13–14th grade (junior college)	0.214

[2] In many school districts it will be preferable to use the first grade as the basis, because a substantial number of children do not enter school in the kindergarten grade.

[3] The grades should be grouped to accord with local policy: K8-4, or K6-3-3, etc.

[4] Because the age composition of families moving into newly constructed developments will generally be younger than average, these coefficients may diverge widely from past school-enrollment experience in old-established neighborhoods.

Items 16 and 17 provide a very accurate method for determination of coefficients for estimating enrollment in future kindergarten and primary grades. They should be resorted to in cases where internal statistical evidence indicates that other, more generalized procedures result in manifest disparities.

By contrast, items 18 and 19 represent a short-cut method of estimating, which has application when more refined procedures cannot be applied, for lack of basic data or available staff time. If the future numerical size of the basic age groups is carefully calculated from birth-rate trends and considered estimates of new residential construction, it has been found that satisfactory predictions will result from this technique.

From the previous discussion, it will be clear that no single procedure has general application. Availability of data and special conditions in the local jurisdiction will largely determine the estimating method to be adopted. In any particular case, this procedure should be tested against the 2 or 3 years of the immediate past, to determine whether it yields figures that are in accord with the actual enrollments for these years. For example, if school records for 1953 and prior years, plus new building construction data for 1954, are utilized, the estimated enrollments for 1954 should agree closely with the actual enrollments.

Calculation of Future School Enrollments. When the most practicable procedure has been determined, it should be applied to the years of the future, so as to prepare a detailed tabulation for each year of the anticipated enrollment by grade or groups of grades.

Illustrative examples of the methods of prediction described herein would serve no valid purpose, as all the indicated computations can easily be made for any given set of data. The local experience of individual school districts varies widely, and none may be considered typical. Estimates made in the years following the close of World War II have had to take account of abnormal fluctuations in the birth rate. The postwar boom in residential construction, to correct the housing shortage, has also produced an atypical situation, which in many cases requires detailed analysis.

Because of the many variables and "unknowns" that are involved in the prediction of school capacity requirements, it is not sound policy to extrapolate very far into the future. Quite apart from unpredictable birth-rate fluctuations, any major divergence from the assumed rate of new residential construction will introduce a substantial cumulative error. In view of the special conditions after World War II, to which reference has been made, current good practice indicates that estimates should be projected forward until the peak enrollment has been passed. In terms of combined enrollment from kindergarten through twelfth grade, this wave will usually be found to reach its anticipated crest between 1961 and 1965.

Again because of the imponderables involved, every prediction of school requirements should be re-examined frequently as new data on actual residential construction, actual birth rates, and actual enrollments become available. Best practice calls for recalculating all estimates once a year. This process will also lead to refinements in procedure and in the selection of coefficients to fit emerging local conditions. The economic base of the community and its school districts is fundamental to all such processes of prediction. If the economy shows signs of change, or of rate of change, it is essential to sound planning that all school enrollment estimates be revised.

Provision of Required Increments in School Plant Capacity. The practical or optimum capacity and also the maximum realizable capacity of the present school plant can be determined in consultation with the school authorities.[5] The differences between anticipated enrollment and present capacity measure the additional seating needs for future years. These figures are then converted into required number of new rooms and number of new schools, on the basis of local standards for room capacity and school size.[6]

The determination of where and how to provide the indicated future additions to school plant capacity is an intrinsic part of the comprehensive master plan. Some existing schools may have ample sites to support physical additions to the present buildings. In other cases, existing school sites can be enlarged.

Where the development of new residential areas is involved, or where existing schools cannot be enlarged or are too far apart, it is evident that new sites must be found. Land

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[5] Planning for the long term should be premised on optimum capacities, but temporary enrollment peaks can be presumed to be handled by utilizing maximum-capacity figures.

[6] Room-capacity standards vary widely. For grade schools, 30 pupils per room is a usual figure, but school districts generally prefer to reduce classes to 25 or less when financially feasible. Design capacities of 35 and even 40 are not uncommon in large cities where the cost of providing small classes is beyond the bonding and operating budget limitations of the board of education. In junior high schools and high schools, the platoon system is usually in operation, with rooms permanently assigned to particular subjects, rather than as the "home rooms" of individual classes of predetermined enrollment. The capacity of a platoon-system school is therefore a function of the sizes of the various student groups taking different subjects during the hour when the maximum number of classes is scheduled.

acquisition should usually be undertaken as soon as the requirements have been clearly ascertained, lest the most suitable sites be developed for private purposes before they can be obtained. Figure 13 illustrates a composite plan for existing and future school sites and recreation areas.

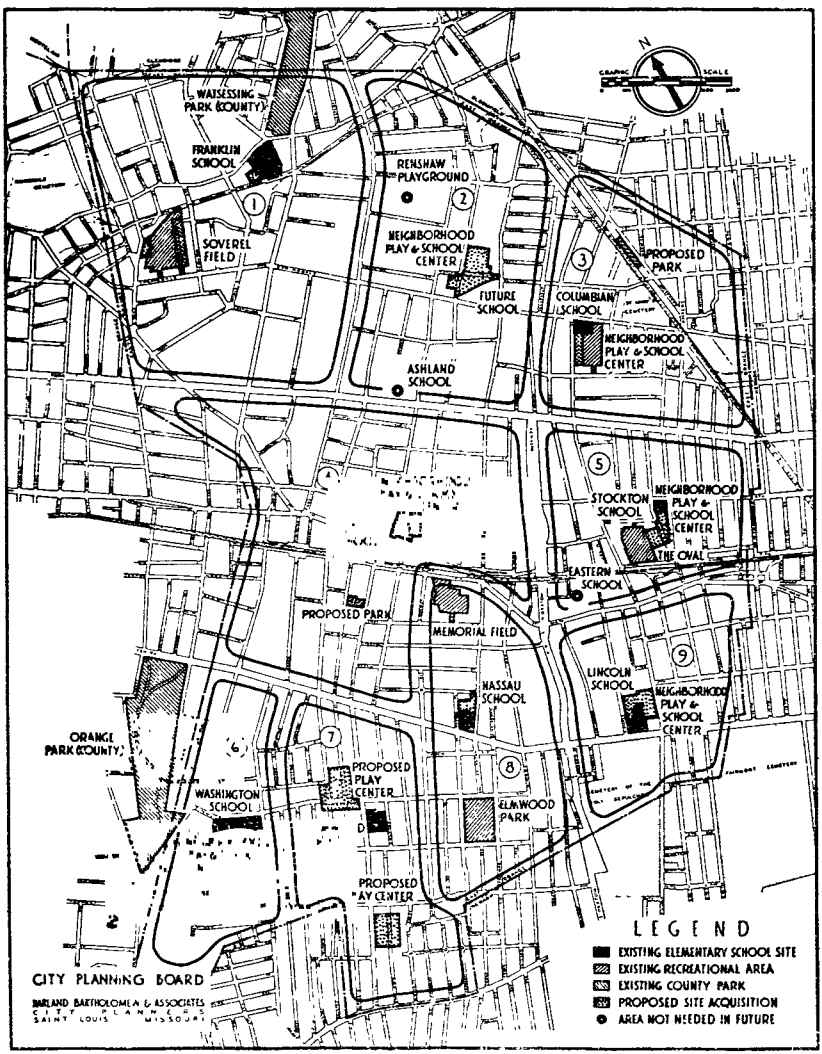

FIG 13 Proposed system of elementary-school and recreational areas, City of East Orange, N. J

Size and spacing of sites should accord with good modern usage tempered by local policies and conditions. Where possible, close spacing of moderate-sized grade schools is preferable to siting larger schools further apart. All sites should be of ample area to support a reasonable extent of unforeseen enrollment growth.

It is axiomatic that grade-school children ought to be able to walk to school without crossing heavy traffic arteries. This aim cannot always be achieved in the near-term future, but should be realized in maximum possible degree when the entire school construction program is completed. In open areas, the provision of school buses will be perma-

nently necessary, and efficient routing of these buses will be a ruling factor in choosing new school locations. Present and future public-transit lines will frequently be a principal determinant for high school sites.

As a general rule, sites fronting directly on main streets and highways should be avoided for elementary schools. Such sites may have special advantages for high schools, provided that ample building setbacks are possible. It is not considered good practice to place schools in established business districts, or adjacent to land that is zoned for business expansion. The types of retail stores that are prone to grow up around a school are usually considered incompatible with the best interests of the pupils.

13. PLANNING LAW AND PROCEDURES

State Enabling Acts for Municipal Planning. Local official planning is performed as an exercise of the police power—the protection of the public health, safety, morals, and general welfare. This is among the powers implicitly reserved to the several states by Article X of the Constitution of the United States. As the police power vests in the *state*, it can be exercised by political subdivisions of the state only when officially delegated to them. In the case of planning powers, this delegation takes the form of enabling acts adopted by the state legislature, or charter provisions established pursuant to state authority.

The great majority of state planning enabling acts are patterned on "A Standard City Planning Enabling Act," which was prepared by the Advisory Committee on City Planning and Zoning of the U. S. Department of Commerce in 1925–1927. The first printed edition was issued in 1928, with a foreword by Herbert Hoover, then Secretary of Commerce. He stated:

> The report recommends, first, a clearly defined permanent planning branch in the local government, in the form of a commission which formulates a comprehensive plan and keeps it up to date. The commission then advises the legislative and executive branches of the municipality, and the public, as to the importance of the plan and promotes conformance to it in the laying out of new streets, the construction of public works and utilities, and the private development of land. Close attention was given to every detail here, as elsewhere in the act, that would help make good planning popular and effective.

In subsequent years, the separate state enabling acts have been extensively amended, with the result that they now differ considerably from each other in matters of detail.

The discussion in this article is principally confined to the New York State Planning Enabling Act, which is reasonably typical and representative of good practice.

When a municipality creates a planning board pursuant to the New York State Planning Enabling Act,[1] the board, *ipso facto*, has certain powers stipulated in the act. These include the authority to prepare a master plan, to review public improvements in relation thereto, to report on proposed changes in the official map, and to prepare general reports on any matter relating to the planning and development of the municipality.

Certain other powers recited in the state enabling act are permissive only. The local governing body may grant them to its planning board or may withhold them, as it sees fit. If not specifically granted by ordinance, they are deemed to be withheld. The authority to review and approve subdivision plats is the most important of these permissive powers. Another, which has rarely been granted, is the power to modify zoning regulations with respect to a submitted plat, simultaneously with its approval. A third, of considerable importance, is the provision that the local governing body may "provide for the reference of any matter or class of matters to the planning board before final action thereon. . . ."

Municipal Planning Ordinances. The local ordinance establishing a municipal planning board should customarily repeat the wording of the state enabling act as to the delegation of specific powers, or alternatively may state that the board shall exercise such and such powers "pursuant to the provisions of the state enabling act," which should then be cited by its official designation. It is considered unwise to paraphrase or otherwise tamper with the verbiage of the state law, as to do so will invite the danger of adverse court decision on grounds of an unauthorized delegation of authority. Unless there are compelling local reasons to the contrary, the local legislative body will be well advised to grant to its planning board all the permissive powers recited in the state law. The maximum benefit will thus be derived from the planning process.

The Master Plan. The master plan, in New York State, is required to show:

> . . . existing and proposed streets, bridges and tunnels and the approaches thereto, viaducts, parks, public reservations, roadways in parks, sites for public buildings and structures, zoning districts, pier-

[1] There are separate acts for cities, villages, and towns, but their basic provisions differ very little.

head and bulkhead lines, waterways and routes of public utilities and such other features existing and proposed as will provide for the improvement of the city and its future growth, protection, and development, and will afford adequate facilities for the public housing, transportation, distribution, comfort, convenience, public health, safety and general welfare of its population.

The above list was later simplified and grouped under seven headings by Edward M. Bassett. His statement reads as follows [2]

It is a mistake for legislation to require and for planning commissions to include in a master plan subjects which are no part of a community plan that affects land areas. The following subjects should be shown in the master plan and no others

1. Streets which include pedestrian and vehicular bridges and tunnels,
2. Parks which include parkways;
3. Sites for public buildings,
4. Public reservations,
5. Zoning districts, which words refer to any governmental unit where the regulations differ in different districts [3]
6. Routes for public utilities;
7. Pierhead and bulkhead lines.

It is universal good practice for the master plan, which is in effect a map or series of maps each showing certain features of the plan, to be accompanied by extensive recommendations in text or tabular form. These will generally include standards for population density, subdivision regulations, traffic regulations, suggested housing and building code provisions, a capital budget and long-term capital program, and proposals for improvement of the economic base. These items, although often considered to be parts of the master plan, should more properly be classified as means for effectuation of the plan, or as basic components of the planning process. In a strict construing of the law, the master plan is limited to those community uses and markings of land areas that comprise the elements of planning and can be shown on a map.[4]

The master plan is an informal document prepared and adopted by the planning board, after public hearing, and designed to guide the future development of the municipality by showing the approximate locations of all component improvements, both existing and proposed.

The Official Map. The official map is a separate and distinct document from the master plan, under the laws of New York and other states that follow the standard enabling act The official map requires formal adoption by the local legislative body. In contrast with the master plan, it shows the *exact* location of improvements, existing or authorized. For example, a new expressway would be indicated diagrammatically on the master plan, whereas it would not be placed on the official map until its precise location, alignment, and grades had been surveyed and determined Many cities adopt an "as is" official map, which is in fact no more than a street and park map, drawn approximately to scale. This action is taken partly as a means of preventing the construction of buildings in the bed of paper streets shown on the "official map," and partly as a first step toward a more exact series of documents. Adoption of these approximate official maps should not be prevented, but it is important to recognize them for what they are. In law, the dimensions recorded on a true official map are final and conclusive with respect to the precise location of lines or boundaries of streets, parks, and other public places. The official map of the City of New York consists of a great many thousand separate surveys, each showing the alignment and grade or change of alignment and grade of a street or group of related streets, etc. The ordinary city map, prepared by a city engineer's office, is merely an approximate representation, at small scale, of the full detail contained on a true official map.

Integrity of Master Plan and Official Map. Once adopted, a master plan becomes a frame of reference for guiding the location of all principal municipal improvements, streets, parks, public building sites, housing projects, redevelopment areas, public school sites, etc In some states, all such projected improvements are required by state law to be referred to the planning board for report, and a report of disapproval can be overridden only by a two-thirds vote of the local legislative body. In other states, the board

[2] *The Master Plan*, by Edward M. Bassett, 1938, p. 142.

[3] In administrative practice, zoning districts are customarily shown on a separate map, which accompanies and forms part of the zoning ordinance. This is essential, as the zoning ordinance and map require local legislative enactment, whereas the master plan usually does not Also, the zoning map is customarily prepared under the jurisdiction of a zoning commission, which may or may not be the same body as the planning board It is appropriate for the master plan to show a *future* pattern for the distribution of land uses This is more nearly ideal, and more generalized as to boundaries, than the precise map of zoning districts.

[4] Paraphrased from statement by Edward M Bassett, *op. cit.*, p. 11.

renders purely advisory reports. Under either procedure, the planning board becomes the protector of the integrity of the master plan.

Similar processes apply to the official map. After the map has been adopted by the governing body, no street or park can be laid out or opened unless it conforms. If it does not conform, the official map may be amended by the local legislature, after report by the planning board. Since the official map is an integral step in effectuating the master plan, the planning board's reports on official-map amendments are again highly significant in protecting the recommendations of the plan.

Subdivision Regulations and Approval of Plats. It has previously been noted that the power to approve plats, under the terms of the standard enabling act, is in the permissive category. When granted to the planning board by the local governing body—and such

Table 19. Outline of Subdivision Regulations

Introductory statement	Importance of adequate regulation, advantages to the municipality, avoidance of ill-considered street layouts, equal treatment for all land developers, etc.
Authority of the planning board	Excerpts from the state enabling act and the local ordinance empowering the board to approve plats Formal statement that all subdivisions, as defined by law, must be submitted to the board and approved before being filed for record.
Definitions	Copy of the state-law definition of a "subdivision." (In some states, the division of a tract of land into two or more parcels is a subdivision. In other states, a subdivision must also include one or more new streets not shown on the official map.) Other definitions as desirable.
General requirements and principles for the subdivision of land	Conformance with master plan and official map. Standards for minimum and maximum block lengths and widths, arrangement of street and lot patterns, cul-de-sacs, visibility on horizontal and vertical curves, maximum grades, easements for utilities, minimum lot sizes, building lines, off-street parking area, buffer strips adjacent to industrial tracts, avoidance of frontaging lots on arterial highways, location of water lines and sewers, required park and playground areas, setting aside of school sites, preservation of natural features of the terrain, etc.
Preliminary plat	Explanation that the subdivider shall submit a "preliminary plat," which need not be an exact survey, to facilitate review by the board and furnish the basis for requested modifications of layout and stipulating required site improvements. This procedure saves the subdivider the expense of detailed surveys and other requirements for the final plat, until his contemplated scheme has received general approval.
	Listing of information to be shown on the preliminary plat or submitted with it, including contours, profiles of streets, tentative plans for drainage and sewage lines, relation to adjoining street system, etc.
	Procedure for approval of preliminary plat as a condition precedent to submittal of final plat.
Final plat	Procedure for submission, and statutory fee, if any. Number of required copies, size, scale, etc.
	Listing of information to be shown on the final plat or submitted with it, including surveyed lines of streets, certificate of surveyor, notation of lands offered for dedication or specifically withheld, location of monuments, street profiles and cross sections, certificate of title to the property, etc. The final plat usually shows less information than the preliminary plat, but all recorded lines must be dimensioned in accordance with an actual certified survey of the land.
	Certificate of owner that all required improvements have been installed pursuant to the final plat, or in lieu thereof the filing of a performance bond or cash deposit, in amount fixed by the board or the municipal engineer, sufficient to cover the full cost of all such improvements. It is now accepted practice to require the subdivider to install street paving, curbs, gutters, sidewalks, sewers and water lines, house connections, and other improvements necessary to the utilization of the lots shown on his plat Provision of certain improvements can be waived in special cases, and the list of requirements will vary with local policy and practice
	Procedure for public advertisement and hearing, and for official approval of the final plat Requirement that the plat, when approved, be filed for record in the office charged with the recording of deeds.
Specifications and standards for improvements	Detailed official requirements of the municipality, governing type of pavement, method of construction, installation of utilities, etc These may be covered by local law or separate regulation, in which case the text should be reprinted as an appendix to the subdivision regulations
Procedure for resubdivisions	In general practice, a resubdivision is subject to the same regulations as an original subdivision
Administration	Provision for written waiver and variation of subdivision regulation requirements. Procedure for amendment of regulations

grant is customary—it is important that the planning board prepare and adopt a set of subdivision regulations, for the guidance of developers and the information of the general public. Failure to adopt subdivision regulations can be the source of great uncertainty and ill-feeling as to the requirements that may be imposed as prerequisites to approval of specific plats. The regulations establish uniform standards of layout and design. They should be written in great detail and should contain a saving clause providing that the planning board may elect to waive parts of its requirements in a particular case.

The headings and descriptions in Table 19 conform to good practice for the content of subdivision regulations.

State laws customarily include a saving clause providing that, if a planning board fails to act upon a subdivision plat within a stipulated number of days, such plat shall be deemed to have been approved, and a certificate to that effect shall be issued by the appropriate officer

Street Lines, and Buildings Projected in the Bed of Mapped Streets. The laws of many states, following the provisions of the standard enabling act, provide that the master plan and official map may show street lines for the widening of existing streets or for the establishment of new streets, in advance of actual public acquisition of the land included within such lines (These lines, in some jurisdictions, are called "building setback lines ") Court decisions supporting the establishment of such street lines in advance of land takings have been handed down in Pennsylvania, New York, Ohio, Virginia, Michigan, and other states. There have also been adverse decisions. Planning engineers should proceed cautiously in the application of this procedure They will usually be on safe ground if their new street lines are reasonably related to the public safety and general welfare; and if the municipality has the definite intention to acquire the land, by purchase or condemnation, within a reasonable number of years after fixing these street lines on its official map.[5]

It is important to make clear that the establishment of a street line on the official map, without concurrent payment of compensation, is an exercise of the police power. Subsequent acquisition of fee to the land may be by dedication, purchase, or eminent domain ("condemnation") proceedings. Official-map street lines are a method for laying out and opening, or widening, streets. These lines must be clearly distinguished from front-yard requirements written into the text of a zoning ordinance. Fixing the depth of a front yard is also an exercise of the police power, but in sound legal practice it must not be utilized as a subterfuge to facilitate subsequent widening of a street. For new subdivisions, a planning board may require the dedication of land for the widening of bounding streets, provided that such action be reasonable

Once a street line or lines have been established on the official map, no owner may thereafter be granted a permit to erect a structure within the lines defining the bed of the street He must set back to the new street line; and if the zoning ordinance requires a front yard, its depth is measured from this new street line. As a safety valve to take care of exceptional hardships, the zoning board of appeals "shall have power . . . to grant a permit for a building in such street . . . which will as little as practicable increase the cost of opening such street . . . , or tend to cause a change in the official map. . . . , and such board may impose reasonable requirements as a condition of granting such permit which requirements shall inure to the benefit of the city." [6]

Legal Authority for Planning beyond Municipal Boundaries. It is not uncommon for state enabling acts to authorize the geographic extension of planning-board powers into territory beyond the corporate limits of the municipality. Subdivisions within 3 or 5 miles of the municipal line may thus be made subject to review, and the master plan may embrace contiguous areas deemed by the board to be affected. In usual practice, such extraterritorial powers are limited to lands not situated in any political jurisdiction which has a separate planning board.

County and Regional Planning Boards. It is axiomatic that neither a master plan nor the control of future land development can, in effective practice, stop at the arbitrary boundaries of a city, town, or village. In order to achieve geographic continuity, the laws of an increasing number of states provide for the official establishment of county planning boards and of regional boards, to exercise their functions in cooperation with existing local boards, or to act in lieu of local boards where none exist

County or regional boards have been created in the Los Angeles Region, in Lane County, Oregon, in Alleghany and Bucks Counties, Pennsylvania, and in more than ten of the 22 counties in the New York Region. There are also a large number of joint city-county

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[5] For detailed treatment of this subject, reference should be made to *Building Lines and Reservations or Future Streets*, by Russell Van Nest Black, 1935, and *Information Rept.* 1, American Society of Planning Officials, 1949.

[6] Quoted from New York State Enabling Act for Cities, General City Law, Art. 3, Sect. 35.

boards, in many states, where the principal city dominates the economy of its county. At the present stage of administrative thinking, there is an open question as to whether a regional planning board should be granted other than advisory power, unless the many political jurisdictions it embraces are bound together in some structure of integrated regional government.

Where no statutory provision exists for creation of a county or regional planning board, and where extraterritorial power is not delegated, it devolves upon each local board to achieve coordination of planning and uniformity of regulations by means of exchanges of views between cognizant agencies.

Public "Authorities." Although somewhat apart from the field of municipal planning as such, it is desirable, for purposes of completeness, to refer briefly to the public corporate authorities that have been created and empowered by law both to plan and to construct and operate public improvements. These authorities finance their work by the sale of revenue bonds, to the payment of which the faith and credit of the state or municipality are usually not pledged. They are extramunicipal corporations rather than planning agencies *per se*. Notable examples in point are the Metropolitan District Commission (Boston), the Port of New York Authority (New York-New Jersey), the St. Louis Bi-State Agency (Missouri-Illinois), the Mystic River Bridge Authority (Boston), the California Toll Bridge Authority (San Francisco Bay area), and the New Jersey Turnpike Authority.

The following check list gives the principal types of functions that have been delegated to planning and development authorities. Most such authorities have a specialized character, and in no instance have all these activities been given into the jurisdiction of any single agency.

Functions of Regional Authorities

1. Port development, including ownership and operation of piers and terminals.
2. Airport development—planning, construction, and operation.
3. Toll bridges and toll tunnels, together with their connections to state highways and city streets.
4. Regional park and recreational facilities, beyond the capabilities of local municipalities.
5. Rail-truck-water interchange terminals.
6. Metropolitan transit systems and interurban bus terminals.
7. Grain elevators and storage warehouses.
8. Water supply and conservation.
9. River and harbor pollution and sanitation control; sewerage systems.
10. Belt-line freight railroads.
11. Produce markets.
12. Industrial development.

14. ZONING

Definition. "Zoning is the regulation by districts under the police power of the height, bulk, and use of buildings, the use of land, and the density of population. Most state enabling acts for zoning require that the regulations shall be uniform for buildings of the same class throughout the district." [1] For example, an apartment house in one district would be governed by a different set of provisions than an apartment house in another district, but the regulations for all apartment houses in a given district would be the same. Zoning also is recognized to include requirements for minimum spaces around buildings.

Extent of Zoning in the United States. As of 1953 there were between 1800 and 2000 municipal zoning ordinances in effect.[2] Only one city over 250,000 did not have a zoning ordinance, and all but about five cities over 100,000 were zoned.

Ruling Legal Decisions. The legality of zoning as a proper exercise of the police power has been firmly established. For legal instruction on the many intricacies of zoning, the reader should refer to standard textbooks on the law.[3]

State Zoning Enabling Acts. In the majority of states, the zoning enabling acts, like planning laws, now closely follow the provisions originally recommended by a committee

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[1] *Zoning*, by Edward M. Bassett, 1936, p. 45.

[2] *The Municipal Year Book*, 1954, p. 293.

[3] The following are among the standard works: *The Law of City Planning and Zoning*, by Frank B. Williams, 1922; *Legal Aspects of Zoning*, by N. F. Baker, 1927; *Zoning*, by Edward M. Bassett, 1936; *The Law of Zoning and Planning*, by Charles A. and Arden H. Rathkopf, 1949, with Supplement and Digest, 1951.

of the Department of Commerce.[4] This Standard State Zoning Enabling Act included a grant of power to regulate population density as such, which under previous laws had been subject only to indirect controls.

The Pennsylvania City Zoning and Building Regulations Enabling Act is typical and representative of good practice.[5] The following excerpts give the basic grants of power:

Section 4110. Power to Regulate and Restrict Building Operations; Districts. Cities may, by ordinance regulate and restrict the height, number of stories, bulk, and size of buildings and other structures, the percentage of lots that may be occupied, the size, depth, and width of yards, courts, and other open spaces, the density of population, and the location and use of buildings, structures and land for trade, industry, residence, or other purposes, and may make different regulations for different districts thereof, and may alter the same . .

Section 4113 Board of Appeals. Council shall appoint a board of appeals. . . . The board of appeals shall have the following powers

1 To hear and decide appeals where it is alleged there is error in any order, requirement, decision, or determination made by an administrative official in the enforcement of the provisions of this subdivision or of any ordinance adopted pursuant thereto.

2. To hear and decide special exceptions to the terms of the ordinance upon which such board is required to pass under such ordinance

3. To authorize upon appeal in specific cases such variance from the terms of the ordinance as will not be contrary to the public interest where, owing to special conditions, a literal enforcement of the provisions of the ordinance will result in unnecessary hardships, so that the spirit of the ordinance shall be observed and substantial justice done.

Zoning Ordinances—Municipal Procedure. Pursuant to the state enabling act, the governing body of a municipality [6] appoints a zoning commission or committee to prepare its original zoning ordinance. In some states, the planning board may be designated as the zoning commission. The ordinance is required to be based on a comprehensive plan, and to show reasonable consideration to the character of the districts and their peculiar suitability for particular uses. It is therefore essential that the planning staff prepare a detailed map of land uses as they now exist, and a more generalized map representing the ultimate desiderata of a future land-use plan. The latter can be adopted as a part of the master plan. The zoning map will generally depict a pattern of permitted uses that is less ideal and less restrictive than the plan, but more logical in arrangement and more restrictive than the existing conditions revealed by the "as-is" land-use map.

The detailed provisions in the text of the ordinance and the location of district boundaries on the zoning map must be *reasonable*. This is the fundamental criterion for judging every zoning question in the courts of law. The decision as to what is reasonable may hinge on legal precedent or may be reached on the basis of the inherent merit of the instant case. The law is a living thing, and the courts have usually taken a constructive attitude toward new techniques and procedures if they are shown to be clearly related to the general welfare.

There is an increasing tendency to write zoning ordinances, and to construe them, as positive measures to *protect* property owners, rather than as negative measures that merely *restrict* improper utilization of land. In modern practice, therefore, a good ordinance states, in so far as possible, what *may* be done, rather than what may *not*. The distinction is in part psychological, but is of considerable importance in achieving good legal draftsmanship.

Form of Zoning Ordinance. The form and arrangement of zoning ordinances vary greatly. They are determined to some extent by the requirements of content appropriate to the particular municipality. The original New York City Zoning Resolution (1916) provided three separate maps to define use, height, and area districts, respectively. Single-map ordinances are now practically the universal rule. Most of the older ordinances, following the New York model in their textual form, contained individual sections for each use district, stating permitted and prohibited uses only The regulations for height, area of lot, coverage, yard dimensions, density of population, etc., were then placed in entirely separate sections, one for each such subject. In modern usage, the text is generally arranged so that each use-district section is self-inclusive, with all the principal height and area clauses, etc., contained therein. Certain subject matter, common with minor modifications to all districts, is still usually reserved for separate sections with headings such as Supplemental Use Regulations and Supplemental Height and Area Regulations. The first would include the special provisions for garages and off-street

[4] *A Standard State Zoning Enabling Act*, Advisory Committee on Zoning, appointed by Secretary Herbert Hoover, Rev. Ed., 1926.

[5] This act applies to third-class cities. Separate but equivalent acts are in effect for municipalities of other classes.

[6] In certain states (Wisconsin and California, among others) there is legislative authority for county zoning ordinances.

parking, truck-loading berths, clubs and institutions, filling stations, advertising signs, etc. The second would group together all the special clauses relating to height (steeples, water towers, etc.), and the permitted variations in standard lot and yard requirements.

Some of the most recent ordinances set up all their basic use, height, and area provisions in the form of tables, which are legally constituted as parts of the ordinance. For a simple situation, all fundamental information can be assembled in a single large table. If the requirements are more complex it is easier to understand the provisions if they are separated into one or several use tables, plus a height and area table. In any case, there will still be necessity for text sections giving supplemental regulations, as previously described. The basic provisions in the tables should be referenced by footnotes to these supplemental regulations, in cases where significant exceptions are involved. The tabular form of ordinance is not recommended for very large cities, except as noted below. The complexities of big-city zoning require too extensive supplemental text, with the result that the tabulated basic requirements become subject to a great multiplicity of special provisos, and partake of the nature of an index of references. An exception should be made of the classification of permitted uses. The proposed new zoning resolution for the City of New York arranges *all* uses into eighteen groups. A simple table then shows which combinations of groups are permitted in each of the fifteen basic zoning districts.[7]

Outline of Zoning Ordinance Content. Table 20 gives the table of contents of a zoning ordinance which is considered representative good practice for a medium-sized city (150,000), with a normal incidence of complexities. In this example, the ordinance is in modi-

Table 20. Section Headings and Subheadings, for a Representative Zoning Ordnance *

(Minor paragraph subject headings omitted)

SECTION 1	Title and Short Title
SECTION 2	Declaration of Purpose
SECTION 3	Application of Regulations
SECTION 4	Definitions
4-A	Meaning and Scope of Certain Terms
4-B	Definitions of Terms
SECTION 5	Districts and Zoning Map
SECTION 6	Use District Regulations
6-A	Table: Residential and Commercial-Residential Uses
6-B	Table: Public, Semipublic, and Institutional Uses
6-C	Table: Business, Commercial, and Industrial Uses, excluding Manufacturing Operations, Storage, and Moto -Vehicle Uses
6-D	Table: Motor-Vehicle Uses
6-E	Table: Storage Uses
6-F	Table. Manufacturing Uses
6-G	Prohibited Uses
6-H	Supplemental Use Regulations
6-H-10	Residential and Commercial-Residential Uses
6-H-20	Clubs, and Philanthropic or Similar Private Institutions
6-H-30	Motor Vehicles, Private Garages for
	Motor Vehicles, Private Open-Air Parking Areas for
6-H-40	Motor Vehicles, Semipublic Garages for
	Motor Vehicles, Semipublic Open-Air Parking Areas for
6-H-50	Motor Vehicles, Public Garages for
	Motor Vehicles, Public Open-Air Parking Areas for
6-H-60	Truck Loading or Unloading Berths
6-H-70	Motor-Fuel Filling Stations and Service Stations for Motor Vehicles
6-H-80	Advertising Signs in B or BA Districts
6-H-90	Manufacturing Operations Not Otherwise Classified
SECTION 7	Non-conforming Uses and Non-conforming Structures
7-A	Unlawful Uses Not to be Construed as Non-conforming
7-B	Continuance of Non-conforming Use
7-C	Extension of Non-conforming Use
7-D	Reconstruction of Structure Used by a Non-conforming Use
7-E	Change of Non-conforming Use
7-F	Abandonment of Non-conforming Use
7-G	Discontinuance of Non-conforming Advertising Signs
SECTION 8	Building Regulations
8-A	Building Regulations by Districts (large table)
8-B	Supplemental Building Regulations
8-B-10	Residential Buildings in More Intensive Use Districts
8-B-20	Floor-Area Ratios
8-B-30	Height

[7] *Plan for Rezoning the City of New York*, by Harrison, Ballard, and Allen, 1950, pp. 172 ff

Table 20. Section Headings and Subheadings, for a Representative Zoning Ordinance *
—Continued

(Minor paragraph subject headings omitted)

8-B-40	Lot Area, Lot Width, and Lot Area per Family
8-B-50	Lot Coverage
8-B-60	Side Yards and Rear Yards
8-B-70	Front Yards
8-B-80	Courts
SECTION 9	Completion of Existing Buildings
SECTION 10	Board of Appeals
10-A	Organization
10-B	Appeals on Interpretation of Zoning Ordinance and Map
10-C	Special-Exception Uses
10-C-10	Conditions and Guiding Principles
10-C-20	Procedure
10-C-21	Expiration of Permits for Special-Exception Uses
10-C-30	Special-Exception Uses Construed as Conforming Uses
10-C-31	Special Uses Previously Authorized
10-C-40	Residential and Commercial-Residential Special-Exception Uses
10-C-50	Public-Semipublic and Institutional Special-Exception Uses
10-C-60	Business, Commercial, and Industrial Special-Exception Uses, excluding Manufacturing-Operations, Storage, and Motor-Vehicle Uses
10-C-70	Motor-Vehicle Special-Exception Uses
10-C-80	Storage Special-Exception Uses
10-C-90	Manufacturing Special-Exception Uses
10-D	Variances
10-D-10	Guiding Principles, Pursuant to General City Law
10-D-20	Procedure
10-D-21	Expiration of Permits for Variances
10-D-30	Specific Types of Variances
10-D-40	Variances Construed as Non-conforming Uses or Structures
SECTION 11	Interpretation
11-A	Zoning Ordinance Prescribes Minimum Requirements
11-B	Relation of Zoning Ordinance to Other Provisions of Law
SECTION 12	Enforcement
12-A	General
12-B	Building Permits
12-C	Certificate of Occupancy and Use Permit
12-C-10	Certificate of Occupancy for a Building
12-C-20	Use Permit for a Structure Other than a Building, and for the Use of a Lot or Land
12-C-30	Fee for Certificate of Occupancy or Use Permit
12-D	Inspection of Premises
12-E	Violations, Penalties, and Injunctions
SECTION 13	Validity of Ordinance
SECTION 14	Amendments, Supplements, and Changes in Regulations and District Boundaries
SECTION 15	When Effective

* Ordinance prepared by T. T. McCrosky.

fied tabular form, with quite extensive supplemental regulations. The headings should not be followed too closely but are useful as a check list. In larger cities, many other intricacies and special problems must be met Each such city requires individual analysis. It would be unwise to apply a standard form, however detailed, as the basis for writing a zoning ordinance for a city of great size.

The Zoning Board of Appeals.[8] Nearly all states now provide for municipal appointment of a zoning board of appeals. This is a quasi-judicial body whose basic function is to rule upon exceptional matters that cannot be detailed in the ordinance. Its main purposes are well stated in the excerpt from Pennsylvania law, previously quoted under the heading State Zoning Enabling Acts.

Variances should be confined to cases involving unique circumstances of hardship, peculiar to the particular lot in question, and must not adversely affect the neighborhood. It has been clearly established by the courts that the board of appeals may not usurp the municipal legislative prerogative. In other words, if a requested variance is of such a radical nature that it should more properly be made the subject of a legislative change in the zoning map, the board should deny the variance. Permission to build a retail store in a residential zone would be an example in point. The granting of improper use variances will seriously undermine the validity of the zoning map. In New Jersey and some

[8] In some states, known as "zoning board of adjustment."

other states, to guard against the abuse of the power to grant variances for use, the board may only recommend to the local legislature that a use variance be granted.

In all states, variances are construed as an appellate jurisdiction of the board. Adjudication of the location of zoning district boundaries and of the meaning of clauses in the text of the ordinance is also appellate jurisdiction. By contrast, the power to permit special-exception uses is a matter of original jurisdiction. A special-exception use is one which is specifically listed in the ordinance text as permissible in one or more zoning districts "on special approval of the board of appeals and subject to appropriate conditions and safeguards." In good practice, the board of appeals section of the ordinance contains a detailed statement of general standards and principles, enunciated by the local legislative body for the instruction of the board in passing upon special-exception uses. Each such use should further be enumerated, with an appropriate statement of the type of specific conditions and safeguards that the board will be expected to impose.[9] As an example, in permitting a gasoline filling station in a local retail district, the ordinance might list the following subject matter of conditions and safeguards:

1. Location of pumps on private property and set back from street line.
2. Location and adequacy of entrances and exits.
3. Planting and screening of adjacent properties.
4. Prevention of glare from flood lights.
5. Provision of off-street parking for cars awaiting service.
6. Location and type of signs.

As a condition precedent to board of appeals' jurisdiction over a filling station as a special-exception use, the ordinance might also provide that no filling station may be located within 200 ft of the entrance to a church or school or place of public assembly.

Some special-exception uses are customarily made subject to advisory report by the planning board, as a supplement to their review by the board of appeals. This applies to types of uses that directly affect the master plan, such as the location of public schools, playgrounds, airports, large-scale residential developments—often with accessory retail stores, designed regional shopping centers, etc. The original jurisdiction should, however, vest in the board of appeals, which body would then be required to refer the matter to the planning board for report. A few post-World War II ordinances have delegated original jurisdiction over special-exception uses to the planning board. This raises a doubt as to legal validity, inasmuch as state zoning enabling acts uniformly provide that the zoning board shall be the body having jurisdiction over special-exception uses, and do not recognize planning boards in this connection.

Special Zoning Problems. *Overzoning.* Most municipalities whose ordinances date back many years are overzoned for business and also overzoned for apartments. This defect will be determined by quantitative tabulation of existing land uses as contrasted with the areas of corresponding zoning districts. In the long run, overzoning usually depreciates property values, although it may encourage profitable short-term speculation in unripe vacant land. The problem is a difficult one. When engaged in comprehensive revision of a zoning ordinance and map, the planning engineer should seek to reduce overzoning to a percentage surcharge that is consonant with prospective demand and sufficient to avoid creating scarcity values. Whether or not this is locally practicable, some part of the excess general business frontage can at least be rezoned for local retail business, and some part of the excess apartment zoning can be reclassified for two-story garden apartments or group houses.

Retroactive Elimination of Non-conforming Uses. Most zoning ordinances provide for the indefinite continuance of lawful non-conforming uses (i.e., uses not permitted by the zoning ordinance in the district in which they are situated, but legally established therein before its enactment). In the early days of zoning, it was expected that these non-conforming uses would gradually be abandoned. This hope has not been realized.

Many non-conforming uses are seriously prejudicial to the general welfare. Factories, stores, filling stations, and billboards in residential districts are examples in point. It is possible in some states to zone them out retroactively,[10] but in many jurisdictions it will be found inexpedient to do so. When writing such provisions, it is wise to stipulate a reasonable period of continuance, as a matter of right. At the close of this period, the owners of non-conforming uses should further be accorded the privilege of appeal to the zoning board for a further continuance for a limited additional period, which would be granted only if they can produce evidence that they have not yet been able to amortize their investment. The length of the first period should depend on whether the non-conformity is merely a

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[9] For detailed treatment see *The Necessity for Adequate Standards for Boards of Zoning Appeals in Special Exception Cases*, by Ralph W. Crolly, 1949.

[10] Hadacheck v. Sebastian, 36 S. Ct. 143 (1915).

use of land, such as a junk yard, or a substantial building. The length of the second period should be discretionary with the zoning board, with indication of minima and maxima stated in the ordinance for its guidance

Off-Street Parking and Loading Space. This subject is treated in Art. 9, Traffic Generators. Because automobile registration is still increasing relative to number of families, it is wise to require for multifamily residence districts accessory parking space in the ratio of one car space per family. However, it must be recognized that this requirement will often not be feasible in the most densely built areas of large cities.

Zoning Protection of Transport Airports. Airplane take-off and approach zones beyond the ends of runways at commercial airports require limitation of building heights in order to protect the public safety. In preparing appropriate regulations, reference should be made to the latest technical standards of the Civil Aeronautics Administration.

New Departures in Zoning. *Statement of Legislative Purposes.* It is now considered good practice to start a zoning ordinance with a detailed listing of local legislative purposes and intents. These derive from the police power, but should be stated in more specific terms than the customary phraseology of the state enabling act. For example, instead of a declaration that the ordinance is adopted simply "to protect the public health, safety, and general welfare," the following objectives can be spelled out. A listing such as this is important in any court case where the intent of the local legislative body may be called into question.

Declaration of Purposes

This zoning ordinance is adopted pursuant to the State Zoning Enabling Act of the State of . . . , to promote and protect the public health, safety, and general welfare, and in furtherance of the following related and more specific purposes

1. To guide and regulate the orderly growth, development, and redevelopment of the City of . . . in accordance with a well-considered plan and with long-term objectives, principles, and standards deemed beneficial to the interests and welfare of the people.
2. To protect the established character and the social and economic well-being of both private and public property
3. To promote, in the public interest, the utilization of land for the purposes for which it is most desirable and best adapted.
4. To secure safety from fire and other dangers, and to provide adequate light, air, and convenience of access.
5. To prevent overcrowding of the land and excessive density of population.
6. To minimize and, where possible, to prevent traffic congestion on public streets and highways.
7. To conserve the value of buildings and to enhance the value of land throughout the city.

Minimum and "Look-Alike" Houses. Rising building costs since World War II have led to the widespread construction of houses of very limited size, so as to avoid pricing new homes out of the market. These houses are much smaller than those built in the past. Many have been built with less than 700 sq ft of gross floor area. Also, to achieve maximum economy, groups of them are customarily built alike, or almost alike. Municipalities have found that minimum houses do not pay enough taxes to defray their share of the expense of local government and public education. Property owners' associations consider that developments of minimum and "look-alike" houses depreciate the character and value of established neighborhoods. Can they be restricted by zoning provisions, as an exercise of the police power? There is disagreement among zoning authorities. More court decisions of a clarifying character are needed.

In New Jersey, and also in Texas, a minimum floor area per family, applied uniformly throughout the municipality, has been upheld by the courts as being reasonably related to the public health and welfare. But to require 1800 sq ft in a "good" district and only 1000 sq ft in a modest district is not supported by these cases, and has been the subject of an adverse decision in Pennsylvania.[11] It may be added that the courts have decided, time and again, that zoning cannot stipulate a minimum price or value as a criterion for permissible house construction. Pursuant to this line of reasoning, minimum houses can continue to invade "good" neighborhoods.

Is the "public welfare" substantially involved? If several score large houses were shown to have depreciated in value as the direct result of the intrusion of minimum houses into their neighborhood, and if these large houses obtained reductions in their assessed valuations on this ground, then the tax rate of all other properties in the municipality would be proportionately increased. This might furnish the foundation for a court case, on the grounds that an increase in the tax rate brought about by construction of mini-

[11] Lionshead Lake, Inc., v. Wayne Township, New Jersey, 89 A. 2d. 693, Supreme Court (1952) Thompson v. City of Carrollton, Texas, 211 S. W. 2d. 970. Medinger et ux., v. Springfield Township, Pennsylvania, 118 Atlantic Reporter 2d. 118, Supreme Court (1954).

mum houses would be against the public welfare. No clear-cut case of this kind has as yet been decided (July, 1954).

In order to prevent "look-alike" houses, some zoning ordinances have been amended to require that new houses on the same street, or within a stated distance of each other, shall have stipulated features of distinctive difference. In some cases, such provisions have been made the subject of separate special ordinances. The legality of these clauses has not been established and must be viewed with reasonable doubt. As in the case of minimum houses, it would need to be proved that the regulations bear a demonstrable relationship to the public welfare. Esthetic considerations have been upheld when secondary to some other legally recognized purpose, but not as an end unto themselves.

Requirement of Acreage Lots for Residences. It has been mentioned that the early enabling acts did not provide for regulation of population density. Consequently, many ordinances were drawn without minimum-lot-size clauses. Neighborhoods of large homes and estates thus obtained no zoning protection. They had to rely entirely upon private deed restrictions to prevent the cutting up of adjoining land into small lots.

Within recent years, it has become customary to establish zoning districts requiring 1/2-acre, 1-acre, and 2-acre lots. There are instances of 5 acres and even larger required area. Numerous court cases have approved a 1-acre classification, and 2-acre lots were upheld in a ruling decision of the New York Court of Appeals (highest state court).[12] The test of validity is the reasonableness of the regulation for the lands to which it applies.

The right of a municipality to zone for 1-acre and 2-acre lots, in neighborhoods already established in this pattern, can be extended to apply to vacant areas where provision of municipal services for more intensive development would be impractical and unreasonable.

The purchasers of minimum and "look-alike" houses can ill afford, and usually do not want, an acre of ground. And the builder of small homes cannot generally provide a lot of this size in any suburban location suitable for development. As a result, a zoning requirement of 1 acre or 2 acres per house is an effective method for preventing the infiltration of minimum houses into good neighborhoods, and can also be used to discourage inappropriate colonies of "look-alike" houses in new subdivisions. This procedure, under present court decisions, offers the most practical solution for achieving these objectives.

Exclusion of New Residential Construction from Industrial Zones. Just as residential neighborhoods are entitled to protection from the encroachment of industry, so it has come to be recognized that established industrial districts should be protected from residential intrusion. In each instance, there is mutual incompatibility. Modern ordinances therefore prohibit all future residential construction in heavy industrial zones. Exception must be made for living quarters of caretakers and watchmen employed upon industrial premises. The retroactive elimination of existing residential buildings has not been attempted.

Floor-Area Ratio. Recent zoning ordinances for large cities employ the ratio:

$$\frac{\text{Total gross area on all floors of building}}{\text{ground area of lot}}$$ [13]

as the basic control over building bulk. Other controls (height restrictions, yard and court sizes, land coverage, and building setbacks) become secondary. These latter serve to assure reasonable placing and proportioning of the building, but its over-all size is governed by the floor-area ratio. (See also Art. 5.) Appropriate floor-area ratios are determined for each residential and non-residential district by considering current trends in desirable building construction practice and also the long-term objectives of the master plan.

Usable Open Space. In residential districts, a few recent ordinances require that every lot shall provide a certain number of square feet per family of "usable open space." This is to assure at least a modicum of recreation area, in addition to required off-street parking spaces and small landscaped or paved yards. Minimum size standards must be stipulated. The proposed New York City zoning resolution permits the inclusion of roof and balcony areas as usable open space in the higher-density districts.

Unobstructed Area for Light Access to Windows. The proposed New York City zoning resolution incorporates an ingenious procedure for assuring light and air to legally required windows. This new method, developed by Harrison, Ballard, and Allen, requires that for a given district a certain number of "units of light access" be unobstructed from each window. Measurements and calculations are made with the aid of devices resembling protractors.

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[12] Dilliard v. Village of North Hills (N. Y.) 276 App. Div. 369.

[13] Some zoning practitioners prefer to calculate the floor-area ratio from the total netrentable floor area, exclusive of halls, stairs, elevators, and garage and storage space.

Large-Scale Developments The theory and practice of zoning grew up in the years when every building was erected on a distinct and separate lot of its own. With the modern trend toward large-scale campus-type developments, there may be a dozen or more buildings on one superlot. Demarkation of individual rear, side, and front yards for each building would be meaningless, even if possible of achievement. Good zoning practice now therefore provides that such developments be considered as a whole, preferably as special-exception uses under the original jurisdiction of the board of appeals. They may also be made subject to an advisory report by the planning board (addressed to the zoning board) on features of their site plan. It is usual for the ordinance to stipulate the spacing between buildings, which may be proportioned to their height, with a stated irreducible minimum. Ordinances frequently specify also that the *average* density of the entire development shall not exceed that permitted by the zoning ordinance for the district in which the project is situated. This allows for combinations of elevator apartments, walk-up garden apartments, and group houses in a zone which would not otherwise permit the elevator-apartment density.

Shopping centers on sites of 2 up to 100 or more acres can similarly be regulated as special-exception uses in business zones. There is an alternate procedure, somewhat less desirable, which derives from the fact that a zoning change from a residence to business classification is usually required. This probability can therefore be foreseen by establishing a "designed shopping center district" in the ordinance text, but not upon the zoning map. Later, when a desirable project is put forward, the legislative body amends the map by placing the site in this designed shopping center district.

"Mixed Residential" Theory As a corollary to the modern trend toward large-scale residential developments, some planners advocate that residential use zoning be done solely on the basis of density, without regard to type of building (single-family, group house, multifamily, etc.). This principle would permit low-density elevator apartments in established single-family detached-home neighborhoods. It is considered by other authorities to be a reversion to the limited zoning techniques of the early ordinances, which were adopted before the establishment of single-family districts was clearly upheld by the courts. Home ownership has grown up under the protection of these single-family zones. There will be instances where "mixed residential" zoning will find valid utilization, but the application of this theory should be approached with extreme caution in any situation involving neighborhoods of owner-occupied homes.

Non-Nuisance Business and Industry in Residential Neighborhoods. Certain types of non-residential uses, such as research laboratories, insurance offices, and book-publishing plants, are not particularly objectionable in residential neighborhoods, provided they have low land coverage on large lots and are screened from adjoining properties. Performance standards for degrees of noise and odor have been established in order to determine whether a given industry should be permitted in a residential zone. On this basis, it is argued by some that qualifying industries should be permitted in residential zones as a matter of right, or as special exceptions subject to approval by the zoning board of appeals or by the planning board or both. Conservative authorities fear that this doctrine may be construed as putting aside the dicta of the Euclid Village case, in which the U. S. Supreme Court ruled that industry and business can properly be excluded from residential areas.[14] They adhere to the view that major non-residential uses—particularly those creating heavy traffic and its attendant hazards—should not be permitted in residential zones. If the location is deemed suitable for business or industry, and this use is not damaging to existing residential development, they consider that the proper remedy is to rezone the land by ordinance of the local legislative body

An effective method, now in accepted usage, is to create appropriately located "permissive business (or industrial) zones," within which no major non-residential use may be established except on approval of the board of appeals with appropriate safeguards, and further subject to advisory report by the planning board. As these permissive zones should preferably exclude new residential construction, they should not be spread over extensive areas of the zoning map, far in advance of known or clearly prospective demand.

Transition Zoning. Because of the great divergence between permitted utilization of property in, for example, business and residential zones, modern ordinances customarily provide special transition provisions where such zones abut one another. As a simple example, a corner store in a business district may be prohibited from having an entrance on its side-street frontage within a certain number of feet of a residential zone boundary. Again, a store which has a side lot line in common with a residential zone boundary may be required to provide a side yard, even though no side yards are stipulated in the business district proper

[14] Village of Euclid (Ohio) v. Ambler Realty Co., 272 U. S. 365, 47 S. Ct. 114 (1926).

This subject has a great many possibilities and ramifications. For detailed authoritative treatment, reference may be made to *Transition Zoning*, by Arthur C. Comey, Harvard City Planning Studies, 1933. A caution should be expressed against enactment of too many special transition clauses. The result will be overcomplexity and confusion, with the attendant danger of adverse court decision.

Rural Zoning. This article is pointed toward the problems of municipal—and therefore largely *urban land*—zoning. It would not be complete without passing reference to the application of zoning principles to rural and forest areas. The outstanding contribution in this field has been made under county zoning powers in the State of Wisconsin. The objective has been to prevent the settling and farming of wide areas in which the soil is poor, or which are so isolated that excessive governmental costs would result in the process of providing roads, schools, and other needed improvements.

Another phase of rural zoning is the protection of highways in open country from defacement and exploitation by billboards and commercial roadside enterprises. The protection of highways is also important from the standpoint of reducing the toll of traffic accidents. Both objectives have been successfully accomplished, under state or county authority, by ribbon residential zoning of the frontage on each side of the highway, to a depth of 200 or even 500 ft.

15. CAPITAL PROGRAMS

Need for Long-Range Financial Planning.

The techniques of city and regional planning are now well established. Not planning, but the *carrying out* of plans is the real challenge. . . . While considerable progress has been made in the preparation of master plans, few major cities have set up official financial programs.[1]

The process of long-range financial planning involves estimating the capital cost of the construction projects shown on the master plan, arranging these projects by categories (parks, streets, schools, other public buildings, etc.), and assigning to each its relative order of urgency. Analysis of the municipality's financial capacity will determine the dollar value of work that can be undertaken in each year, and a financing schedule can thus be laid out. Annual expenditures for debt service and operation are then calculated, so as to show the effect of the program upon future tax rates.

In the City of New York, the charter requires the City Planning Commission to prepare a detailed capital budget each year, and also a more generalized capital program for the succeeding five years. These documents are officially adopted by the City Council and the Board of Estimate.[2]

The charter of the City of New York states that subsequent amendments to the capital budget shall require "recommendation in writing by the city planning commission, approved by the affirmative vote of two-thirds of the members thereof plus approval by the board of estimate and certification by the mayor, the city council, the comptroller, and the city clerk." Some other large cities have similar procedures, but their capital budgets are more informal in character and do not bind the administration to adhere to them. Baltimore, San Francisco, Cincinnati, and Cleveland are in this category. Philadelphia occupies a middle ground, having the requirement of adherence, but with less strict provisions than New York City for the authorization of amendments.

The financial program serves to fulfill a customary provision of state planning enabling acts:

In preparation of the master plan, the planning board shall give due consideration to the ability of the municipality to carry out, over a period of years, the various public or quasi-public projects embraced in the plan without the imposition of unreasonable financial burdens.[3]

Every capital improvement of "today" may be said to become a current budget item of "tomorrow." Without the capital program, there is no evidence that the master plan can be financed. With the capital program, the projects comprised in the plan are fitted into the fiscal structure of the municipality, their financial practicability and scheduling are tested; and their effect upon future borrowing capacity and tax budgets is evaluated.

Exclusion of Non-Tax-Supported Projects from the Capital Program. Engineering cost estimates must be prepared for all capital improvements to be included in the long-range financial plan. It is customary to screen out the following types of projects, as they do not affect municipal borrowing capacity or future tax rates.

1. *Proprietary projects* (water supply, municipal electric lighting plants, airport development, etc.) to the extent financed by rates or revenues.

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[1] *Surging Cities*, by T. T. McCrosky, Charles A. Blessing, and J. Ross McKeever, Greater Boston Development Committee, Inc., 1948, p. 150.

[2] *Ibid.*, p. 151.

[3] Quoted from New Jersey Municipal Planning Act of 1953, Section 12.

2. *Assessable improvements* (local sewers, paving, sidewalks, etc.) to the extent financed by benefit assessments on affected properties.

3. *State and federal highways* to the extent financed by non-municipal funds.

4. *Other projects*, federal, state, and county, to the extent financed by non-municipal funds.

5. "*Authority*" *projects* (toll bridges, markets, parking lots, etc.) to the extent financed by revenue bonds not supported by the faith and credit of the municipality.

In connection with item 5, it will be necessary to reach an advance decision as to whether normal municipal financing or authority financing shall be resorted to. The particular advantage of authority financing for such items as off-street parking lots is that the bonds are thereby exempt from the legal debt-incurring capacity of the municipality. If a city is close to its statutory debt limit and is empowered by state law to create a parking authority, it can thus conserve its bonding capacity for other essential projects such as schools and parks. However, a price must be paid. Because authority bonds are not backed by the city's faith and credit, they usually command up to 1% higher interest than general obligation bonds. This differential will bulk large for untried authorities embarking on new ventures. By contrast, the Port of New York Authority can float bond issues at much the same interest rates as the City of New York. Another disadvantage of revenue-bond financing is that the advance estimates must indicate a substantial excess of receipts above expenditures, in order to assure the marketability of the bonds. The Greater Boston Development Committee, Inc., calculated that using public-credit bonds and thereby reducing the interest rate and the excess of receipts would result in sufficient saving in fixed charges on a proposed combination of a vehicular tunnel and a bridge to finance the debt service on $50,000,000 of arterial highway approaches.[4] The State of Connecticut has benefited by this policy: recent toll bridges have been financed by state-credit bonds, even though they were estimated to be self-supporting. The same is true of the New York State Thruway Authority's bonds.

Capital Improvement Schedules. To revert to the subject of municipal capital costs for master-plan projects, after making all deductions listed under items 1 through 5 above, the planner will have arrived at the amount of net capital cost of each project to the municipality, which is payable out of tax budget funds. The next step is to determine which projects, or parts of projects, should be financed on a "pay as you go" basis, directly out of current tax levies. The remaining projects, constituting the balance of annual capital outlays, must be bonded. They will therefore be paid, principal and interest, out of future tax levies.

It is sound policy to finance as much work as feasible by pay as you go, in order to save interest charges and also to avoid hypothecating the municipality's permissible borrowing capacity. The method is particularly applicable to purchases of equipment which will last only a few years and to other capital items that are known to recur nearly every year. For example, a large city may need to replace one school, or build a new school, *every* year for 40 years to come. If each school costs $2,000,000, a city operating on the pay as you go plan would put $2,000,000 for this purpose in its tax budget each year. However, if this city financed its schools by 40-year serial bond issues at an average interest rate of 2%, each school would then cost approximately $2,925,000. At an interest rate of 4%, the cost per school would be $4,042,000. In other words, the city would pay twice over for every school it built, or more than $160,000,000 in interest and amortization to construct 40 schools costing only $80,000,000 to build.

On the other hand, a small city may need one new high school, costing $6,000,000, with indications in the city plan that no other new high school will be required for 40 years. It is preferable to spread the cost of this one school over a long term of years, so as to avoid saddling such an expensive project upon the taxpayers in the current budget of a single year.[5]

In order to determine how much the annual tax budgets can reasonably include in the pay as you go classification, the planning engineer will need to perform the analyses of future assessed valuations, revenues, operating expenses, tax rates, borrowing capacity, etc., described under the next heading of this article. It is best to carry these computations forward simultaneously with preparation of the annual pay as you go and bond issue schedules, by applying cut-and-try techniques. The end result will be a pair of schedules which are considered to fit safely within the municipality's anticipated taxing and borrowing power for each year of the program.

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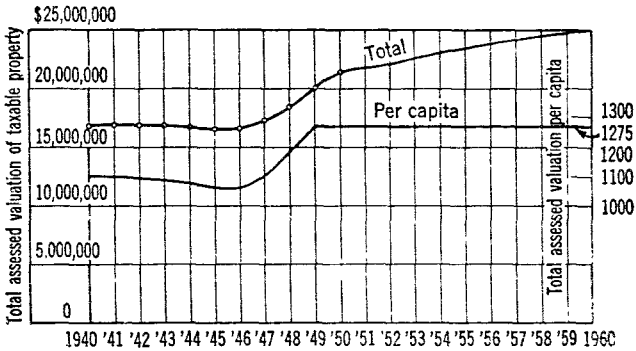
[4] Unpublished study, 1947.

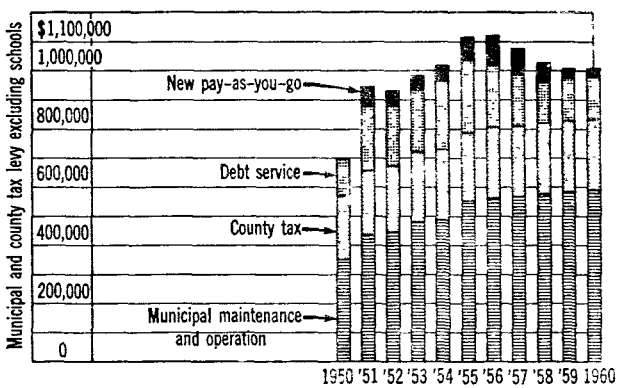
[5] These examples of school costs are abstracted, with modifications, from *Surging Cities*, by T. T. McCrosky, Charles A. Blessing, and J. Ross McKeever, Greater Boston Development Committee, Inc., 1948, pp. 151 ff.

It is necessary to determine, from local precedent or state regulation, the appropriate terms for various categories of bond issues, schools, rights-of-way, paving, trunk sewers, parks, etc. Interest rates must then be chosen on the basis of the recent local credit rating and anticipated long-term trends in the municipal bond market. Prudence requires that moderately higher rates be assumed than the current credit of the municipality commands.

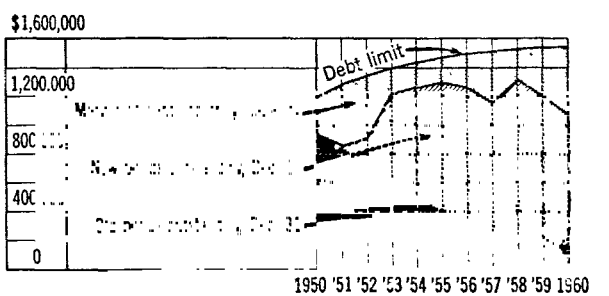
Total and per capita assessed valuation of taxable property



Municipal and county tax levy excluding schools



Relation of outstanding bonds to debt limit



McHugh and McCrosky, Consultants
January, 1951

FIG. 14 Typical capital budget charts.

Most municipal financing is now done by serial bonds, payable annually in equal or substantially equal installments. Sometimes, a bond issue is payable in amounts which will make the annual sum of interest plus amortization equal to a constant debt-service figure. As a third alternative, local practice may call for term bonds supported by periodic payments into a sinking fund.

Table 21. Framework for a Capital Budget

Line	Item	1950 1951 1952 1953 1954 1955 1956 1957 1958 . . . 1965 (or so)
1	Assessed valuation	(for purposes of calculating tax rates). This is usually the total assessed valuation ("grand list") of taxable real and personal property, plus the value of utility franchises and other items as provided by law. See comment on line 3.
2	Population of the municipality	Estimates for the future should be conservative but realistic. (See Art. 2 for estimating procedures.)
3	Per capita assessed valuation	Line 1 divided by line 2. Projection of the curve of assessed valuation per capita furnishes a valid method for determining future assessed valuations to be entered on line 1.
4	Redemptions of "old bonds"	An "old bond" is one issued before preparation of the capital program. The redemption schedule should be obtained from the municipal treasurer or other cognizant official.
5	Interest on "old bonds"	Figures to be obtained as in line 4.
6	Debt service on "old bonds"	Line 4 plus line 5.
7	Per capita municipal operating and maintenance costs	Actual total cost of government figures should be obtained for 10–15 past years, so as to be able to extrapolate the per capita trend into the future. A curve should be drawn. Appropriate allowances must be made for future operating and maintenance costs of master-plan projects.
8	Per capita county (and state) taxes paid by municipality	Procedure similar to line 7, but adjustment must be made for the relative rates of growth of the county and the municipality.
9	Per capita pay-as-you-go costs for "old" improvements	Same procedure as for line 7.
10	Per capita revenues	(from all sources other than general property taxes: e.g., state and county aid, licenses, and fees). Same procedure as for line 7, but care must be taken to avoid overweighting of non-recurring "windfalls."
11	Per capita tax levy, excluding debt service	Sum of lines 7, 8, and 9, minus line 10.
12	Tax levy, excluding debt service	Line 11 multiplied by the population for each year (line 2).
13	Tax levy, including debt service on "old bonds"	Line 12 plus line 6. Line 13 is the future net cost of government paid by real estate taxes, *before* adding in the annual pay-as-you-go and debt-service costs of carrying out the master plan. It is at this point that the planning engineer must start his cut-and-try procedure to determine how many new pay-as-you-go appropriations, and how much debt service on master-plan bonds, can be supported in each future year, without incurring an excessive tax budget and tax rate, and without prejudicing the legal debt-incurring capacity.
14	Future pay-as-you-go appropriations for master-plan projects	These are in addition to the prevailing pay-as-you-go experience recorded on line 9. See previous discussion of the pay-as-you-go principle.
15	Redemptions of "new bonds"	(issued to finance master-plan projects). These are determined from the bonded improvement schedule, and will involve cut-and-try technique to arrive at a considered judgment of the optimum size of bond issues to be scheduled for each year.
16	Interest on "new bonds"	Same technique is involved as for line 15. For purposes of computation, it may be assumed that all bond interest is payable semiannually on January 1 and July 1, and that all redemptions are made on July 1. This simplification will give good average results.
17	Debt service on "new bonds"	Line 15 plus line 16.
18	Annual cost of capital program	Line 14 plus line 17.
19	Total tax levy	Line 13 plus line 18. This is the future annual cost of government borne by general property taxes including the annual cost of the capital program for carrying out the master plan.
20	Tax rate	Line 19 divided by line 1. Some states have placed a limitation on the tax rate for municipal operating and maintenance costs. In such cases, this partial tax rate must be computed separately from lines 7, 10, 2, and 1.
21	Statutory debt limit	(legal borrowing capacity). This is usually a percentage of the taxable assessed valuation, fixed by state law. In many jurisdictions, however, it is a percentage of the average of the taxable assessed valuations for several prior years. Also, in some states, the valuation of personal property, utility franchises, etc., must be excluded in computation of the statutory debt limit.
22	"Old bonds" outstanding in each year	A convenient date, such as December 31, must be used consistently. Accurate figures should be obtained from the municipal treasurer, or derived from the data supplied for purposes of filling in line 4.
23	"New bonds" outstanding in each year	Use same calendar date as for line 22. Figures derive from the schedule of new bond issues and annual redemptions.
24	Total bonds outstanding in each year	Line 22 plus line 23.
25	Unencumbered margin of debt-incurring capacity	Line 21 minus line 24.
26	Per cent of debt limit used	Line 24 divided by line 21 and expressed as a percentage. The capital program should not crowd the debt limit. To protect the credit of the municipality, the percentages shown on line 26 ought not usually to exceed 75–85%. Conferences with local officials and municipal bond specialists should guide the planner's judgment. Much will depend on the expected rate of growth of the municipality. If rapid population increase is predicted, the percentage should be quite conservative, as failure to realize the anticipated growth would reduce actual future debt limits far below their estimated levels.
27	Property tax on a typical house	Line 20 multiplied by the assessed valuation of a typical house. The figures on line 27 enable property owners to compare present taxes with the estimates for future taxes during the years of carrying out the master plan.

The fact that separate improvements are initially financed by small sales of bond notes, which are subsequently redeemed by consolidated bond issues, can generally be ignored. Short-term bond notes carry a lower interest rate than long-term bonds, so any disparity will be on the safe side.

Effect of Capital Program on Future Tax Rates and Borrowing Capacity. It is good practice to carry the studies of the financial plan at least 6 years into the future. Ten to 15 years is preferable, as it is advantageous to show the financing of a substantial proportion of the entire master plan, rather than only the first few years of an orderly program. Also, the peak of annual debt service in support of master-plan projects, superimposed upon peaks in the cost of other governmental services, may produce higher tax rates 10 or 15 years hence than 6 years hence. Calculations should therefore be run out into the future until all foreseeable peaks have been passed.

The list of items in Table 21 is typical of those that need to be computed for each year.[5] The tabulation should begin 1 or 2 years in the past so as to record actual figures from published sources, and then extend 10 or 15 years into the future. Each of these items will therefore be a *line* in a table and the *columns* of the table will be the years. In Table 21, explanatory notes are given for each line, where needed to indicate necessary procedures involved. Figure 14 illustrates many of the items in Table 21, by graphical analysis of a typical capital program.

Modifications and Periodic Revisions of Capital Program. Many different and equally adequate capital programs can be prepared for the same schedule of master-plan improvements. The good judgment of the planning engineer is a ruling factor in all the decisions that must be made in the long step-by-step procedure that has been described. Even more significant, however, will be the policy decisions of municipal government department heads and legislative members. Because of their knowledge of past practice and their understanding of public opinion, the planning engineer should seek their guidance in working out desirable modifications of his tentative schedules.

Properly prepared, the capital program gives both municipal officials and citizens a clear view of the financial road ahead. To keep this view in good focus, it is necessary that the program be adjusted and amended periodically in the light of changing conditions, including new knowledge of such basic factors as assessed valuations, rate of population growth, going interest rates, and dollar value indices. In good practice, the capital program is re-examined once each year. When thus kept up-to-date, it can become a permanent guide to succeeding municipal administrations. Official adherence to the capital program will give proof to the electorate that the master plan is accepted as a living force, and that each year will see further progress toward its goals.

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[5] In actual practice, the list must be varied somewhat to accord with local requirements and availability of data. It may be found that the municipal capital program will have to be set up entirely separately from the board of education's capital program, in cases where their geographic boundaries do not coincide.

SECTION 2

SURVEYING

BY

GEORGE D. WHITMORE

IN COLLABORATION WITH

SOL A. BAUER, WILLIAM T. PRYOR, LANSING G. SIMMONS, MORRIS M. THOMPSON

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SURVEYING INSTRUMENTS AND ADJUSTMENTS

By George D. Whitmore

1. CHAINS AND TAPES

The term chain formerly meant either a Gunter's chain 66 ft (4 rods) long, having 100 links of 7 92 in. (0 66 ft) each; or an engineer's chain 100 ft long, having 100 links of 1 ft each. The Gunter's chain was much used in early land surveys. Chains have been almost entirely displaced by heavy steel tapes, but the use of the term chain persists, and engineers' or surveyors' steel tapes are sometimes erroneously called chains. Also, the measuring of distances with a tape is usually called chaining, rather than taping.

Cloth, metallic, glass fiber, and steel tapes are in common use. Cloth tapes stretch so readily that they are seldom used for surveying. Metallic tapes are cloth tapes with fine bronze wires woven into them to prevent stretching, and are made in lengths from 25 ft to 100 ft. It is not uncommon for metallic tapes, when subjected to rough usage and to alternate wetting and drying, to become from 0 2 to 0.5 ft in error in a length of 50 ft. Woven glass fiber tapes are now frequently used instead of metallic tapes because of their better accuracy and durability.

Steel tapes may be obtained in lengths up to 500 ft, but the most common lengths are 50 ft, 100 ft, 200 ft, and 300 ft. The shorter tapes are usually made of thin ribbons of steel; the longer tapes are made heavier in cross section so that they will withstand rougher usage and will not become kinked so readily. The light tapes are usually graduated throughout their length into feet, tenths, and hundredths; while the heavier tapes are generally marked only at every foot; except that tapes used in building construction are usually graduated in feet, inches, and fractions of inches. In some tapes the zero point is at the outer edge of the ring attached to the end of the tape; while on others the zero point is placed on the tape at 0.2 to 0 5 ft from the end ring. Special steel tapes fitted with a thermometer, and a spring-balance handle for registering the amount of tension, or pull, can be obtained from manufacturers. Tapes that will not be used for extremely accurate work can be mended by riveting to the back of the tape a piece of an old tape of the same width, or by using any of the various splicing clips that are available.

A new 100-ft tape is usually intended to be of standard length at 68°F when under a tension of 10 lb and supported throughout its length. Any steel tape will vary in length with variations in temperature and pull. Also, the method of supporting the tape will cause the distance spanned by the tape to vary. However, in many surveying operations, sufficient accuracy can be attained without regard to any of these factors. For surveys which require greater accuracy, the correct length or span of the tape for various measuring conditions should be determined, and appropriate corrections applied

The change in the length of a steel tape caused by changes in temperature will be significant in precise work. Each change in temperature of 10°F will change the length of the tape by about 1:15,000, or about 0.007 ft per 100 ft. Thus for a 50°F change in temperature, the length of the tape will change by 1:3000, or about 1.7 ft per mi. The change in the length of a steel tape caused by temperature changes is determined by the formula

$$C_t = c(t_m - t_s)_L$$

in which C_t is the temperature correction in feet; c is the coefficient of expansion per °F (average for tape steel = 0.0000065); t_m is the Fahrenheit temperature of the tape at the time of use; t_s is the Fahrenheit temperature at which the tape is considered to be of correct length, or "standard" temperature; and L is the nominal length of the tape in feet. The length of the tape will be longer, and the correction will be plus when t_m is higher than t_s. The same formula can be used to compute the temperature correction for the full length of a measured course, in which case L is the nominal length of the entire course.

The change in a tape length caused by a different tension, or pull, is also significant in precise work. Each change of 10 lb in the tension applied to a lightweight steel tape (say 1 lb for a 100-ft tape) will cause the tape to stretch or shorten by approximately 1:8000 (about 0.012 ft per 100 ft, or about 0 7 ft per mi), the exact amount depending on

the cross-sectional area of the tape and the modulus of elasticity. The pull correction may be determined by the formula

$$C_p = \frac{(P_m - P_s)}{SE} L$$

in which C_p is the pull correction in feet, being plus (longer tape) when P_m is larger than P_s; P_m is the tension in pounds applied during the actual taping operation; P_s is the tension in pounds for which the tape is considered to be of standard length; L_t is the nominal length of the tape in feet; S is the cross-sectional area of the tape in square inches; and E is the modulus of elasticity, average for tape steel being about 29,000,000 lb per sq in. The changes in length of any tape that are caused by applying different tensions can be readily determined by direct test The rear end of the tape should be tightly fastened to a point on a smooth surface, as a floor. Various tensions, measured with a spring balance, should then be applied to the forward end of tape. By noting the position of the graduation mark at the forward end of the tape for each different tension, the corresponding variations in length can be determined

When a tape is not supported throughout its entire length, it sags between points of support, taking the form of a catenary, and the ground distance that it then spans is less than the distance that would be spanned by the same tape when supported throughout, assuming the tension to be the same. For a *heavy* tape (say 3 lb per 100 ft of length) under 20 lb pull, and supported only at the zero and 100-ft points, the sag correction would be approximately 1:1000, or about 5 ft per mi. The correction for sag for any length of unsupported tape is determined by the formula

$$C_s = \frac{W^2 L}{24 P_m^2}$$

in which C_s is the correction in feet to be applied (minus, because the tape is spanning a shorter length) for each unsupported section of the tape; W is the total weight of the tape between supports, in pounds; P_m is the actual tension applied to the tape, in pounds; and L is the unsupported length of tape

The reduction in the span of a tape, due to its sag, may also be determined by direct observation. While supported throughout, the tape should be stretched to a specified tension, as measured by a taping spring balance, and the zero and 100-ft points marked The tape should then be raised off the ground and placed on supports almost directly over the tape marks on the ground. With the tape stretched under the same tension, the zero mark of the tape is plumbed exactly over the zero mark on the ground. The 100-ft mark of the tape is then plumbed to the ground, and the difference between its location and the previous ground mark represents the shortening of the span of the tape for the tension used. This test can also be applied to any fractional part of the tape length.

The pull and sag formulas, combined, may also be used inversely to determine (approximately) the tension required to make the horizontal span of an unsupported tape approximately equal to its nominal length. The combined formula would be

$$P_n \text{ (approx)} = \sqrt[3]{\frac{W^2 SE}{24}} + \frac{P_s}{3}$$

in which P_n is the so-called "compensating pull," in pounds, necessary both to stretch the tape and reduce sag so that the spanned length is approximately the same as the nominal length, and the other terms are the same as in preceding formulas. In the case of an extremely heavy tape, or a long span, it would be impractical to apply sufficient tension to accomplish this compensation. The "compensating pull" may also be determined by trial.

Where unusually precise tape measurements are required, the tape to be used should be tested and "standardized" by the U S. Bureau of Standards, Washington, D. C Unless otherwise requested, the Bureau of Standards will determine the correct length of any 100-ft tape (in good condition) to the nearest 0.001 ft, at 68°F, when supported throughout, and under 10 lb tension. On a project where several tapes are to be used and a high order of accuracy is required, one of the tapes should be standardized, and then set aside and used only for calibrating other tapes A simple procedure for calibrating a tape in the field is as follows. (1) on a smooth, protected surface (as a building floor, or shaded sidewalk) stretch the standardized tape under the same tension as was applied in the Bureau of Standards test, and transfer fine pencil marks or scratches to the supporting surface from the zero and 100-ft points; (2) immediately stretch the uncalibrated tape over the same course, under the same tension, and precisely scale (under a magnifier) in thousandths of a foot, the amount that it is longer or shorter than the standardized tape; (3) add or subtract this small difference to the absolute length of the standardized tape, as given in the Bureau of Standards certificate, to obtain the absolute length

of the second or unstandardized tape, for the same pull, temperature, and conditions of support as obtained at the time the Bureau of Standards test was made on the first tape. For any other conditions of support, tension, or temperature, the correct length of the second tape can be calculated by use of the preceding formulas

Tapes made of a special nickel steel called "Invar" change in length only slightly with change in temperature. The coefficient of expansion for different types of Invar varies somewhat, but an average would be from one-tenth to one-twentieth of that for steel. Invar is a relatively "soft" metal, not as springy as steel, and hence will bend and kink more quickly. Use of Invar tapes is therefore usually limited to extremely precise triangulation base lines and first-order geodetic traverses. Invar tapes are also useful as comparison tapes, for local calibration of steel tapes.

Other causes for variations in tape measurements are: careless plumbing, incorrect alignment, and fluttering caused by wind

Plumbing is the operation of transferring any point on a horizontal tape to the ground under the tape by means of a plumb bob. Inaccuracies due to this process can only be avoided by using great care In the plumbing procedure the work is facilitated by measuring downhill. Even when great care is taken in plumbing, so much error is introduced that for accurate results it is usually better to measure the inclined distance from the horizontal axis of the instrument to the station mark ahead, measure with the vertical arc of the transit the angle of inclination of the tape, and compute the horizontal distances by means of the versine of the angle of inclination; i.e., horizontal distance = inclined distance minus (inclined distance × versine angle). This computation can usually be done with sufficient accuracy by means of a slide rule. This method requires a set-up of the transit at every other tape length. Another way to measure inclined distances is to tape directly from stake to stake in one tape length; and set the instrument up at every other stake and measure the vertical angle to a point above the station point on either side of the instrument equal to the vertical distance of the horizontal axis of the transit above its stake. This will give the inclination of the tape. Still another method is to measure the inclined distance from stake to stake and obtain the difference in elevation by leveling When much slope taping is to be done it is usually more expeditious to use a 200-ft or 300-ft tape.

Alignment errors are not likely to be large; therefore lining in the tape by eye is exact enough for most measurements. If in measuring 100-ft tape lengths a point is 1 ft out of line it will introduce an error of only 0.005 ft in the tape length (1:20,000). The error due to poor alignment may be computed by the formula $c - a = h^2/2c$, where c is the tape length or fractional tape length; h is the offset from the correct line; and a is the correct length. Thus, if one end of the tape is on line and the other 0.8 ft off line the error in that one tape length is $0.8^2/200 = 0.003$ ft. Of course, there will be a similar error in the next tape length, making a total error of 0.006 ft The shorter the tape length the greater is the error due to poor alignment

With a steel tape, if ordinary care is taken in plumbing and alignment, and with rough corrections made for temperature, an accuracy of 1:5000 can be readily obtained. For accuracy of 1:10,000 or better, it is necessary to apply corrections for temperature, pull, and sag, and to keep the tape closely on line.

In all measurements it is of utmost importance to distinguish between errors which tend to balance and those which continually accumulate, the latter being far more important. For those which tend to balance, the number of errors which will probably remain uncompensated, according to the method of least squares, will be the square root of the total number of opportunities for error. For example, if a 100-ft tape is 0.02 ft too long a cumulative error of about 1 ft will be made in measuring a line 1 mile long. If, on the other hand, the tape lengths are not marked closer than 0.05 ft, the total error made by this compensating error of 0.05 ft will be usually about $\sqrt{52} \times 0.05 = 0.36$ ft.

2. VERNIERS

The Vernier is a device for determining the subdivision of the smallest division of a scale more accurately than can be done by simply estimating the fractional part. Its accuracy depends upon the fact that one can judge more accurately when two lines coincide than he can estimate a fractional part of a space. The simplest form of vernier (Fig. 1) is used on some kinds of leveling rods. It is constructed by making the entire length of the vernier equal the length of 9 divisions on the scale, and subdividing this vernier length into 10 equal parts. In this way 1 division on the vernier equals $^9/_{10}$ of a division on the scale, so that ab is $^1/_{10}$ and cd is $^2/_{10}$ of a scale division. If the vernier is raised until a becomes opposite b then the reading will be 701; if raised until c comes opposite d the reading

is 702. The number of the line on the vernier that coincides with some line on the scale is the number of tenths of the smallest division on the scale that the index point lies above the scale division just below it.

Transit Verniers are usually made double, one on each side of each index, so that angles can be read in either direction. For transits reading to minutes the vernier scale is made by dividing a space equal to 29 half-degrees of arc into 30 parts, so that the difference in length of 1 division of the circle and 1 division of the vernier is equal to 1 min of arc. If the vernier is placed so that its zero point (or the index) coincides with the 0° mark on the circle, then the first lines on either side of the zero of the vernier will fail to coincide with the corresponding lines on the circle by just 1 min, the second lines on each side of the zero of the vernier will fail to coincide with their corresponding marks on the circle by just 2 min, and so on. If the vernier is moved along the circle so that the first line next to the zero of the vernier exactly coincides with its corresponding line on the circle the reading will be 0° 01′; and if the vernier is moved 1 min more the second lines will coincide and the reading will be 0° 02′ Therefore in reading an angle on a transit, read the position of the vernier index on the circle, reading the angle in the same direction that the telescope has moved and estimate roughly the number of minutes the index has passed over from the graduation next back of it on the circle, then follow along the vernier in the same direction and find the vernier line which coincides with some line on the circle and add the number of this line to the circle reading. Thus, Fig 2 represents a 1-min transit circle with double vernier which reads, if the telescope (which always moves with the vernier) has moved clockwise, 274° 23′. If it has moved anticlockwise it reads 85° 37′. For a 1-min instrument the circle is divided into 30-min spaces and there are 30 divisions on the vernier; a 30-sec instrument has the circle divided into 20-min spaces with 40 divisions on the vernier, a 20-sec instrument has the circle divided usually into 20-min spaces with 60 divisions on the vernier; and a 10-sec instrument has the circle divided into 10-min spaces with 60 divisions on the verniers.

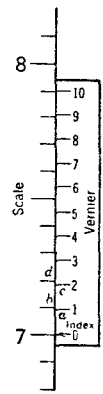

FIG 1

Single Verniers which are to be read in either direction must be numbered from both ends, and the index at either one end or the other is set at 0° on the circle depending upon which direction the angle is to be measured. These single verniers are used on transits reading to 10 sec or 20 sec where a double vernier would be so long as to interfere with the standards. Another type of single vernier called the **folded vernier** is sometimes used on the vertical circle of transits and on alidades of plane tables. It is like any single vernier,

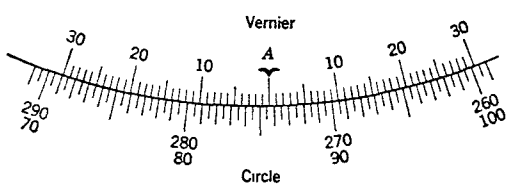

FIG 2 Transit circle with double vernier

except that the index is the middle division of the vernier. In reading it, if a coincidence is not reached by passing along the vernier in the proper direction, it is necessary to pass to the other end of the vernier and continue in the same direction, toward the center, until the coincidence is found

Retrograde Vernier. The verniers described above are all **direct** verniers in which the smallest division of the vernier is smaller than the smallest division of the circle. The **retrograde** vernier is one in which the smallest division of the vernier is larger than the smallest division of the circle. It is seldom used on surveying instruments.

3. THE COMPASS

The Surveyor's Compass is an instrument for determining the difference in direction between any horizontal line and a magnetic needle. The needle is balanced on a pivot in the center of a compass box so that it can swing free in a horizontal plane, the pivot being the center of a circle which is graduated usually to half-degrees and numbered from 0° to 90° in both directions from the N and S end of the compass box. The pivot and the circle

are secured to a brass frame on which are two vertical sights placed on the line passing through the pivot and the N and S points of the compass box. When the compass is being carried or moved in any way, or is otherwise out of use, the needle must be kept raised from the pivot by the lifter; otherwise the finely pointed pivot is sure to be blunted by the pressure of the jeweled cap, rendering the needle sluggish and unreliable. The compass is connected to the tripod by a ball-and-socket joint which is clamped in position after the instrument has been leveled by means of two spirit levels attached to the frame at right angles to each other. The frame has a spindle which fits into the ball-and-socket joint so that after the instrument has been leveled it can be swung around in a horizontal plane. In the northern hemisphere a small counterweight is attached to the S end of the needle, as the N end of the needle would dip downward if not counterbalanced. When a compass is moved from one locality to another, the amount of dip of the needle may be different, and hence the compass needle may be out of balance in the new location; that is, one end or the other may touch the glass cover, and the compass needle would then not swing freely on its pivot. Correct balance of the needle can be restored by sliding the counterweight slightly in the appropriate direction.

In use, the compass is first set up over the proper point and leveled, and then sighted along the line whose direction is desired. Since the needle stands still and the box turns under it, the letters E and W on the box have been reversed from their natural position so that the reading of the needle will give the proper quadrant. To obtain the bearing of the line in the direction it is sighted it is important to follow this rule. When the N point of the compass box is toward the station whose bearing is desired, read the N end of the needle (the end to which the counterbalance is not attached in northern hemisphere). When the S point of the box is toward the station, read the south end of the needle. The term "station" as here used means any point on the survey line. Bearings are usually read to the nearest quarter of a degree. The bearing looking in the opposite direction along the line is called the reverse bearing. To avoid a parallax error the needle should be read by looking along the needle, not across it.

Precautions. Before the bearing is read the glass should be tapped lightly over the end of the needle to be sure that it is not touching the under side of the glass. If the needle appears to cling to the glass it probably indicates that the glass has become electrified, and this difficulty can be removed by placing the moistened finger on the glass.

In the use of the compass great care should be taken that no iron or steel is near the instrument to attract the needle from its true position. The chain, pins, ax, pocket knife are fruitful sources of errors. Stray direct currents in the ground (usually leakage currents from street-railway systems) affect the compass so seriously that it is of little use in cities for obtaining even an approximate magnetic meridian. It is customary to take the forward and reverse bearing of lines so that any local irregularity of the declination may be detected. If the bearing of AB taken from Station A and the bearing of BA from Station B do not agree it indicates that at either A or B there is local irregularity To determine at which station it exists, take the bearing of BC with the compass at B, and then with the compass at C determine the bearing of CB. If these agree it indicates that there is no local irregularity at B.

Adjustments of the Compass. (1) The bubbles are adjusted like those on the horizontal plate of a transit (2) Straightening needle and centering pivot point: Bent needle may be detected by reading both N and S ends when N end is held opposite a given graduation, and then by reading both ends again when S end is held opposite same graduation, take needle out and bend it with pliers at the middle until the end reads the same in either position Pivot may be centered by finding (by trial) position of maximum difference between end readings on different parts of the circle, and then bending pivot so that end readings agree. (3) Remagnetizing needle: rub with a bar magnet from center towards ends, using N end of magnet for S end of needle, and vice versa.

The Pocket Compass is a hand instrument for obtaining roughly the bearing of a line. There are two kinds, the plain and the prismatic. The former is much like the surveyor's compass, except that it has no sights. In the prismatic compass the graduations, instead of being on the compass box, are on a card which is fastened to the needle (like a mariner's compass) and which moves with it. This compass is generally provided with sights. The bearing can be read, by means of a prism, at the same instant that the compass is sighted along the line.

The Brunton Pocket Transit is a pocket-sized compass usually hand-held but sometimes mounted on a Jacob staff. It has peep sights and a hinged mirror cover which may be tilted so that the level bubbles and compass circle may be read while taking a sight. The level tubes are mounted on an auxiliary rotatable segment having a graduated arc in the same plane as the compass circle. When the instrument is turned in a vertical plane one of the level tubes serves to indicate when the segment is leveled on its then horizontal

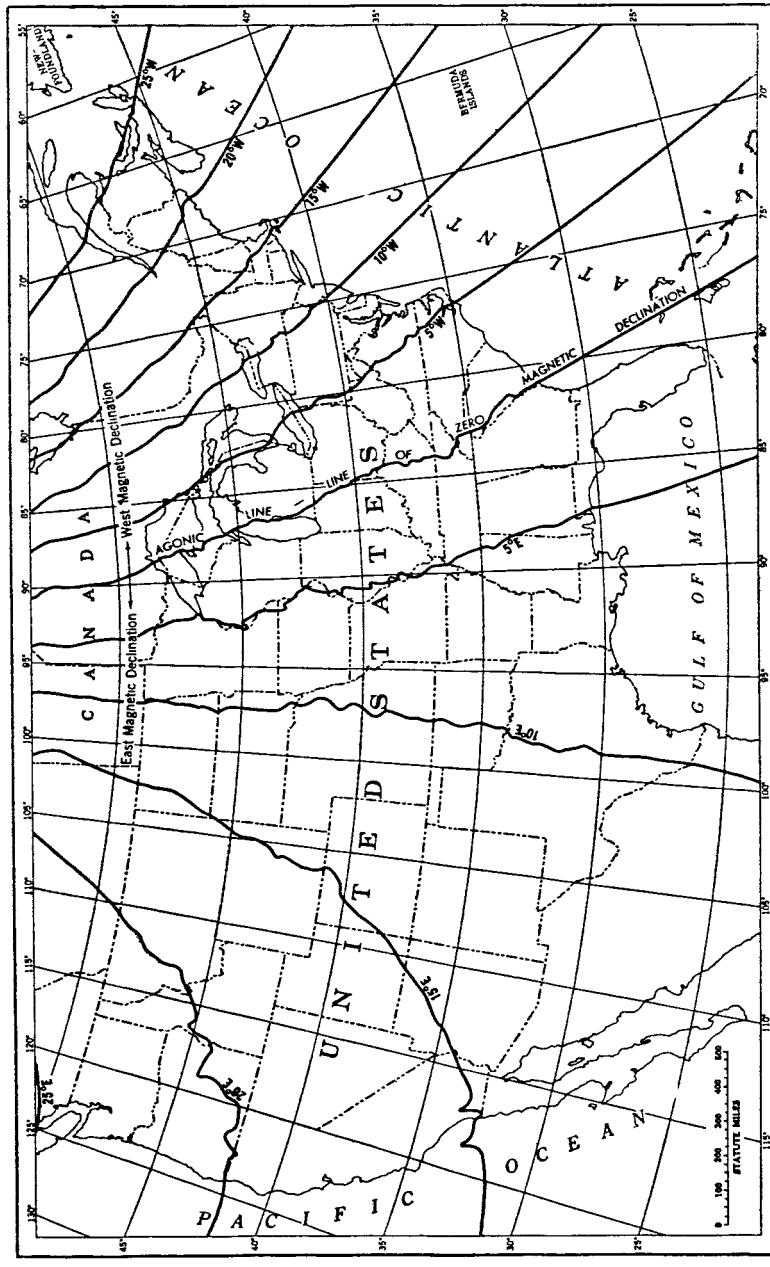

FIG. 3 Isogonic chart of the United States for 1945 *Reproduced by permission of the U. S. Coast and Geodetic Survey*

axis so that the instrument can also be used as hand level or as a clinometer for reading vertical angles. The same sights and mirror system are used for all operations.

The Index Error of a compass, or of the needle of a transit when used as a compass, is the amount by which its sight line deviates from the magnetic bearing shown by readings of the needle. It can be determined approximately by taking a sight on a line having a known true bearing, at a place where the declination has been determined, as at a magnetic station of the U. S. Coast and Geodetic Survey.

The Magnetic Declination is the bearing of magnetic north, east, or west of true north. As of 1945, in the Carolinas and the states lying north and northeast of them, West declination prevails, reaching a maximum of 22° in northern Maine. In other parts of the United States, the declination is generally East, having values as high as 22 $^1/_2$° in the neighborhood of Seattle, Wash. Typical values for a few other regions of possible interest to American engineers are as follows: Fairbanks, Alaska, 29 $^3/_4$° E, San Juan, P. R., 6 $^1/_4$° W; Caracas, Venezuela, 3 $^3/_4$° E; Honolulu, Hawaii, 11 $^1/_2$° E; Manila, Philippines, 1° E. The declination is subject to changes of several kinds.

The Secular Change is a gradual movement which continues for many years in the same direction, but ultimately reverses. It is variable at any place and different in different places. It cannot be predicted or expressed by means of any known empirical rule.

The Daily Variation consists of a swing of the needle from its extreme easterly position at about 8 A.M. to its most westerly position at about 1:30 P.M. The needle is likely to reach its mean position at about 10 A.M. and 5 P.M. This daily variation has an amplitude that varies from 5 to 15 minutes of arc.

The Annual Variation (about 1′ in amplitude) is a yearly fluctuation so small that it need not be considered in surveying work.

Irregular Variations, due to disturbances of solar origin, are uncertain in character and cannot be predicted. Though usually small, these variations sometimes become large, resulting in the so-called magnetic storms associated with auroral activity.

Isogonic Lines are curves drawn on a map so that each one connects all the places where a particular value of the declination is found at a given time. When an exact value of the declination is required, it must be found by a series of observations at the point in question. The validity of such a measurement, however, is restricted to the immediate vicinity of the spot. For other points, or for a generalized value pertaining to an extended area, an approximate value may be scaled from a chart. Isogonic charts are always smoothed to some extent; they cannot show the intricate, local detail characterizing the real distribution. Isogonic charts of the United States are based on observations made by the U. S. Coast and Geodetic Survey at about 8000 stations, and are issued by that agency at 5-yr intervals. A simplified version of the one for 1945 is illustrated in Fig. 3.

4. THE TRANSIT AND ITS ADJUSTMENTS

The Engineer's Transit, a view of which is shown in Fig. 4, has two spindles one inside the other, to each of which is attached a horizontal circular plate, the outer spindle being attached to the lower plate and the inner one to the upper plate. Attached to this upper plate are two standards supporting the horizontal axis of the telescope. The motion of this horizontal axis and of the two spindles is controlled by clamps and slow-motion (tangent) screws. On the upper plate are two spirit levels, which are used in leveling the instrument Most transits have four leveling screws; but those made for triangulation work usually have only three. When the instrument is provided with a telescope bubble and a vertical arc it is called an **engineer's** (or **surveyor's**) **transit;** if it does not have these attachments it is called a **plain transit.**

The Telescope has 2 lenses cemented together for its objective, and an eyepiece that is usually made of 4 lenses if the instrument is an erecting transit, 2 lenses if inverting. An inverting instrument gives a somewhat better illuminated field. The cross-hair ring is located just ahead of the eyepiece. In some instruments, mainly the older models, the telescope is focused by moving the objective away from or toward the eyepiece. Most of the later-model instruments employ a system of interior focusing in which an additional lens between the cross hairs and the objective is moved back and forth This system has a number of advantages The telescope barrel is nearly dustproof and waterproof, there is no appreciable wear in the focusing lens slide, there is less change in telescope balance in focusing on near objects (most noticeable on levels), and the stadia constant (f plus c factor) is zero. Interior lens surfaces are now usually given a special coating to reduce loss of light from lens-surface reflections, thus effecting a 15 to 30% brighter image. The two outer lens surfaces are not usually coated as part of the coating may be worn off, giving the lens an appearance of being partly covered by a film. The line of sight, or line of col-

limation, is the straight line drawn through the optical center of the objective and the point of intersection of the cross hairs. **Focusing** is the adjustment of the eyepiece and of the objective so that the cross hairs and image of the object can be clearly seen at the same time. The **magnifying power** of a telescope is the amount by which an object is increased in apparent size. It is equal to $\tan \frac{1}{2} A / \tan \frac{1}{2} a$ (or nearly A/a), A being the angle subtended by the object as seen through the telescope and a the angle as seen by the unaided eye. It may be found approximately as follows: Hold a rod a short distance from the instrument and view it through the telescope with one eye; at the same time look at it directly with the other eye. It will be noticed that one space as viewed through the telescope will appear to cover several spaces as seen with the naked eye. This number is approximately the magnifying power of the telescope.

Common Special Attachments. (1) Diagonal or prismatic eyepiece for sighting high altitudes. (2) A reflector attached to the telescope for illuminating the cross hairs when working in the dark. (3) A gradienter screw for measuring rates of grades directly. (4) Solar attachment, for determining a meridian by observation on the sun.

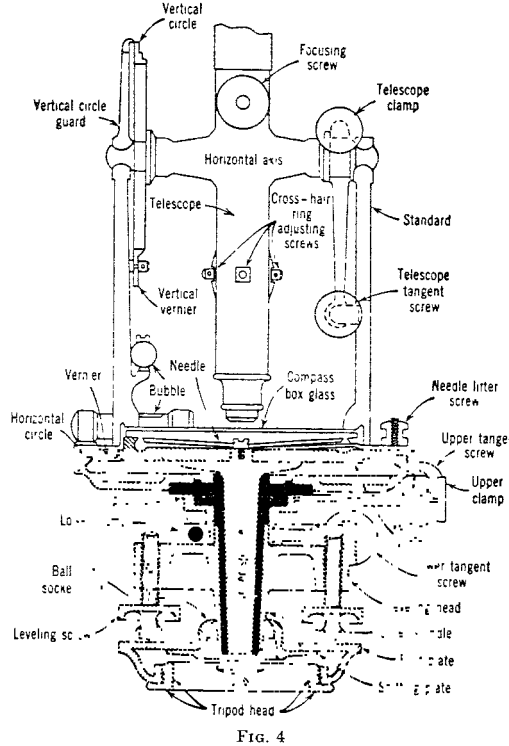

FIG. 4

By Observing a Few Important Precautions much of the wear of instruments and accidents to them can be avoided. Neither the tripod legs nor the shoes should be allowed to become loose. In taking the instrument out of its box always lift it by placing the hands beneath the leveling base. In leveling an instrument, the leveling foot screws must not be tightened excessively. When about to move the transit from one point to another be sure that the four leveling screws are properly bearing, that the needle is lifted, and that the clamps are set just firmly enough so that any slight shock will allow motion. A waterproof bag should be carried at all times to cover the instrument if it rains or when the instrument is exposed to dust. When the transit must stand out in the rain the cap should be put over the object glass; if water should get into an exterior-focusing telescope, take out the eyepiece, cover the open end of telescope tube with cloth, and allow it to dry out. Any parts of the instrument that have been wet should be wiped dry, especially the vertical arc, but be careful not to touch the edges of the arcs. One should always avoid placing the hands on exposed graduations, as it will tarnish the metal. In cleaning the lenses use a fine camel-hair brush, and for screw threads use a stiff clean tooth brush and apply a very little watch oil after cleaning, but do not apply oil to exposed screws, except for cleaning.

Important Adjustments of the Transit. (1) To make the plane of the plate bubbles perpendicular to the vertical axis of the instrument. (2) To make the line of sight perpendicular to the horizontal axis. (3) To make the horizontal axis perpendicular to the vertical axis of the instrument. These three adjustments are made to depend on the principle of reversion, the effect of an error being doubled by a reversal of the instrument. Each adjustment should be repeated to test its accuracy.

1. Adjustment of the Bubble Tubes. Bring plate bubbles in center of respective tubes by means of leveling screws. Turn 180° in azimuth; half the apparent error in bubbles is the real error; so bring bubble halfway back by screws on the bubble tubes. Adjust each bubble independently.

2. Adjustment of the Line of Sight. First adjust the vertical cross hair so it will lie in a plane perpendicular to the horizontal axis. Set up and level the instrument. Leveling the instrument is not necessary for this part of the adjustment, but as it is an essential part of the second portion of this adjustment it is well to level at this time. Sight the vertical hair on a well-defined point, clamp both plates, rotate telescope about horizontal axis. If point does not appear to travel along the vertical cross hair, loosen screws holding cross-hair ring, and, by tapping lightly on one screw, rotate ring until above condition is fulfilled. Then tighten screws and proceed with the second part of the adjustment as follows: sight telescope at point A (200 to 300 ft) and clamp vertical axis; revolve telescope on horizontal axis and set a point B in line of sight and same distance approximately from instrument as A but in the opposite direction. Points A and B should be at about the same elevation. Loosen clamp, turn 180° in azimuth and sight A again, clamp, revolve telescope on horizontal axis, and set point C in line of sight beside point B. Mark or note a point D one-fourth the distance between C and B measured from C. To adjust, move the cross-hair ring laterally until vertical wire falls on D, by loosening the screw on one side of the telescope and tightening that opposite. If the transit has an erecting eyepiece move the ring in the direction D to C; if an inverting eyepiece, move ring in the same direction C to D.

3. Adjustment of the Standards. Set up and level the transit. Sight on some high point A and clamp vertical axis. Lower telescope and set point B in line of sight about level with telescope. Reverse telescope and turn in azimuth 180°, sight on B, and clamp. Raise telescope until point A is visible and note point C in line of sight and at same height as A. If C does not fall on A, loosen pivot cap screws at adjustable end of horizontal axis and raise or lower the end of the axis by means of capstan-headed screw under axis until the vertical wire will fall on a point midway between points A and C. The adjusting screw should be brought into position by a right-hand turn, otherwise the block on which the horizontal axis rests may stick and not follow the screw. The cap screws should then be tightened just enough to avoid looseness of the bearing.

If the instrument is badly out of adjustment it is better that all adjustments be made in proper order, then repeated in the same order until no error is found at any point.

To adjust the vernier of the vertical circle so it will read 0° when the telescope bubble is in the center, loosen the screws holding the vernier, and tap lightly until the zeros coincide. This adjustment eliminates index correction in measuring vertical angles provided the line of sight is parallel to the bubble tube.

The peg adjustment for making the line of sight of the telescope parallel to the bubble tube of the telescope is made in the same manner as for a level instrument, as described in Art. 5

To eliminate effects of errors in adjustment and still obtain accurate results the instrument must be used as follows. To avoid errors in the plate bubble, level up, turn 180° in azimuth and bring bubbles halfway back by means of leveling screws. This makes the vertical axis truly vertical, and the bubbles should remain in the same parts of their respective tubes as the instrument is turned about the vertical axis. Errors in line of sight and horizontal axis are avoided by using the instrument with telescope in its direct and then in its reversed position and taking the mean of the results, whether the work is running lines or measuring angles. Errors of eccentricity in the spindles are eliminated by taking the mean of the readings of the two opposite verniers, and errors of graduation of the circle are nearly eliminated by reading the angle in different parts of the circle or by measuring the angle by repetition. Where only one vernier is read in determining an angle, always read the one that was set.

The Optical-Reading Theodolite has been popular in Europe for some time, and is being increasingly used in the United States. Its principal points of difference from a transit are that the graduated circles are of glass; both the horizontal and vertical circles are seen in an auxiliary reading telescope situated immediately adjacent to the main telescope, the images of the graduations being brought into view by means of an internal system of lenses and prisms; and the two opposite portions of each graduated circle are seen simultaneously in juxtaposition in such a manner that a mean of the two opposite sides of the circle is obtained with one reading. The circles are brightly illuminated and the graduation marks are clear and sharp. For daytime work a small mirror is adjusted so that light is reflected to and projected through the glass circles. For night work light from a small flashlight bulb is used to illuminate the circles and the reticle. An optical plummet is usually provided, this being in effect a vertical collimator built into the instrument, which facilitates exact centering of the instrument over the station, especially in windy weather.

Various sizes of optical-reading instruments are available, the large models giving direct readings to either 0.1″ or 1″, and the smaller models giving direct readings to either 10″

or 1′. On most of these instruments interpolated readings can readily be made to the tenth part of the direct reading. Optical-reading theodolites are much smaller in size and weight than conventional American instruments having equivalent accuracy.

5. THE LEVEL AND ITS ADJUSTMENTS

A Level Instrument is a telescope with a delicate spirit level attached to it so that when the bubble is in the center the line of sight is horizontal, or tangent to a level surface, which is a curve every point of which is perpendicular to the direction of gravity. The two common types are the **wye** and the **dumpy** levels. In both of these instruments the telescope is mounted on a vertical axis about which the telescope can swing, and is leveled by means of four leveling screws. In American precise level instruments, and in most European instruments, three leveling screws are used.

In the Wye Level the spirit level is attached to the telescope which rests in two Y-shaped supports, which in turn are fastened to a horizontal bar to which the vertical axis is attached. The telescope can be taken out of the Y's, turned end for end, and replaced, when testing the bubble for adjustment.

The Dumpy Level has its vertical axis, the horizontal bar, and the supports of the telescope all in one piece, to which the spirit level is attached. The dumpy level will stand much rougher usage than the wye level, has fewer wearing parts, and allows fully as precise work. Practically the only advantage the wye level has over the dumpy is that the adjustment of the line of sight can be more conveniently tested. The precautions in use of the level and its care are similar to those described for the transit in Art. 4.

The Locke Hand Level is simply a metal tube with plain glass covers at its ends and with a small spirit level on top. When looking through the tube the bubble is seen on one side of the tube in a mirror through a lens, on the other side the landscape is viewed. When the bubble is in the center of the tube the observer can note where the horizontal line which appears in the center of the bubble tube cuts a rod and in this way do approximate leveling.

Principal Adjustments of the Wye Level. (1) To make the line of sight coincide with the axis of the wyes, or parallel to it. (2) To make the line of sight and axis of bubble tube parallel. (3) To make the axis of the bubble tube perpendicular to the vertical axis. As in the transit instrument, most of the adjustments depend upon the principle of reversion

1. Adjustment of Line of Sight. First make the horizontal cross hair truly horizontal when the instrument is level. This adjustment is tested by sighting, after leveling, on a point and noting whether the cross hair appears to remain on the point as the telescope is moved in azimuth. If an adjustment is necessary it is done by rotating the cross-hair ring as described in the case of the second adjustment of the transit. After this adjustment is made loosen the clips which hold the telescope in the wyes. Sight the intersection of the cross hairs at a point and clamp. Rotate the telescope 180° in wyes. If the cross hairs do not remain on the point they must be moved halfway back to the point by means of screws on the cross-hair ring. On some older instruments each cross hair must be adjusted separately by means of its proper ring screws, but on modern instruments the vertical wire is set perpendicular to the horizontal wire by the manufacturer, and so both wires are moved together.

2. Adjustment of Bubble by Indirect Method. Bring the bubble in the center of its tube and clamp in that position. Rotate telescope in wyes a few degrees around its horizontal axis; if the bubble moves, correct the entire error by means of the horizontal capstan screws at one end of the bubble tube. Then clamp the telescope over the pair of leveling screws and bring the bubble into the center of the tube, lift the telescope from the wyes, turn it end for end, and replace in the wyes without jarring the instrument. Correct half the apparent error by the vertical adjusting screw of the bubble tube.

3. Adjustment of the Wyes. Level the instrument, then bring the bubble exactly to the middle over a pair of leveling screws. Then turn the telescope 180° about its vertical axis and correct half the apparent error by means of the adjusting screw of the wye support. Since the bubble is brought to the center of the tube at each rod reading, this last adjustment in no way affects the accuracy of the leveling work but is a convenience.

When the two circular rings on the telescope of a wye level are not truly circular or equal, the instrument should be used as a dumpy level. In this case the clips should always be securely clamped, and the adjustments should be performed as for a dumpy level.

The Adjustments of the Dumpy Level are the same in purpose as for the wye level, but because of the construction of the instrument are done in a different order and by a differ-

ent procedure in some cases. They are: (1) To make the horizontal cross hair truly horizontal when the instrument is level. (2) To make the axis of the bubble tube perpendicular to the vertical axis. (3) To make the line of sight parallel to the axis of the bubble.

1. Adjustment of the Cross Hairs. This is done as described under the first adjustment of the wye level.

2. Adjustment of Bubble Tube. Level the instrument and carefully center the bubble over a pair of leveling screws. Turn the telescope 180° in azimuth and correct half the apparent error in the bubble by means of the adjusting screws of the level tube.

3. Adjustment of Line of Sight by Direct or "Peg" Method. Select points A and B about 300 ft apart at approximately the same elevation. Set up instrument between A and B, about 30 ft from A; with bubble centered take reading on near rod A, then on far rod B. Move the instrument and set it up at a point between A and B, about 30 ft from B, center the bubble, and read near rod B and far rod A. These two sets of observations give two independent determinations of the difference in elevation between the two points. The true difference in elevation is the mean of the two results. Knowing the true difference of elevation from B to A, the correct value for the reading on the far rod A can be calculated, as indicated below:

Instrument near Station A		Instrument near Station B	
Reading on rod A	= 3.971	Reading on rod B	= 5.064
Reading on rod B	= 4.937	Reading on rod A	= 4.036
Difference in elevation A to B	−0.966	Difference in elevation B to A	+1.028
True difference in elevation	= 0.997		

Correct reading on rod A for instrument at set-up near B will therefore be 5.064 − 0.997 = 4.067.

To effect the adjustment, leave the instrument set up near B, point to the rod at A, bring bubble to center of tube, and keep it there while raising or lowering the cross-hair ring by means of its adjusting screws until the cross hair falls on correct rod reading.

The "peg" adjustment may be used for adjusting the line of sight of the wye level, except that in viewing distant rod A the horizontal cross hair is placed on the correct rod reading, and the bubble is brought to center by means of its adjusting screws. The "peg" adjustment is also used for the hand level.

In using the "peg" method for transits the adjustment may be made either by moving the cross-hair ring, or by moving one end of the level tube. If the cross hair is moved the adjustment of the line of collimation must be tested; if the bubble tube is moved, the vertical arc must be adjusted.

In a precise instrument having an extremely sensitive bubble, it is not advisable to attempt to adjust the bubble for a very small deviation between the level bubble and line of sight. This deviation is known as C, and is the amount of inclination in feet of the line of sight from true level per 100 ft of horizontal distance. If C exceeds 0.005 ft, the bubble should be adjusted. In the foregoing example, if the distance from rod A to rod B had been 300 ft, and if at each set-up the distances from instrument to near rod and far rod had been 30 ft and 270 ft, respectively, the rate of inclination C would be approximately 0.013. This is derived by dividing the error of 0.031 ft in the reading on far rod A by the net distance of 240 ft, or 2.4 in hundreds of feet.

To eliminate the effect of errors in the adjustment of the line of sight, the bubble tube, or the wyes, the length of the backsight and foresight should be made approximately equal for each set-up. In running a line of benchmark levels, however, it is satisfactory to balance (approximately) the total of all foresight distances and backsight distances for all the set-ups between each two adjacent benchmarks, rather than balancing the sights on each set-up.

Tilting Dumpy Level. In this type of instrument the line of sight can be slowly inclined upward or downward by turning a micrometer screw located at the eyepiece end of the telescope. The pivot may be located either at the center of the instrument or at the forward end under the objective lens. In many of these instruments the cross-hair ring is fixed in position by the manufacturer and should not be disturbed; therefore the principal adjustment is to make the level bubble parallel to the line of sight, using the "peg" method previously described.

Many instruments of this type are provided with either a mirror, or a set of prisms with supplementary viewing telescope, for observing the position of the bubble with the left eye while simultaneously viewing the rod through the main telescope with the right eye. An efficient prism arrangement shows the two ends of the bubble side by side, the bubble being centered when the images of both ends coincide.

6. LEVELING RODS

Two general types of rods, target and self-reading, are in common use. The target rods are intended to be read only by the rodman, while the self-reading rods are read directly by the levelman. In most respects self-reading rods are preferable to target rods.

The Philadelphia rod, commonly used in the United States, is an extendable wooden rod of two sliding sections, about 7 ft in length when not extended (short rod) and about 13 ft in length when fully extended (long rod). It can be used either as a self-reading rod or as a target rod. When the lower section, or "short rod," is used as a target rod, the target is moved up or down on the rod, according to the levelman's directions, and readings are made by noting the position of the target's special scale on the front-face graduations. When the rod is extended, the target is set at a designated graduation near the top of the rod, the upper section of the rod is slid up or down, and the target reading is made by noting the position of a special scale in contact with the graduations on the back face of the rod.

Many special types of graduations for self-reading rods are in use Usually the figures on the face of a self-reading rod are made of some definite height, as 0 06, 0.08, or 0 10 ft, and of a thickness such as 0.01 or 0.02 ft, as an aid in making accurate rod readings Some rods are in three parts, and these usually can be extended or jointed together to make a length of 16 ft. There are also available a variety of rod graduations that are printed on flexible bands or tapes, usually of heavy impregnated fabric. These can be tacked or glued to narrow wooden boards or staffs, and are satisfactory for leveling of low accuracy and for stadia surveys.

When accurate leveling is required, each level rod to be used should be periodically calibrated for over-all length. Wooden rods tend to shorten or lengthen with age and with seasonal humidity changes. For most leveling surveys, it will be sufficient to determine the length of the rod by comparing it with a finely graduated steel pocket tape. Rods that are seriously incorrect for length should be discarded. The errors in leveling caused by using a level rod of incorrect length will be in proportion to the vertical difference in elevation. If a 10-ft rod is short or long by 0.02 ft, then a difference of elevation of 100 ft, as measured with this rod, will be incorrect by 0 2 ft

A self-reading rod for precise leveling is made of a single piece of wood (not extendable), with the fine graduations painted on an Invar tape. The numbering and coarse graduations are painted directly on the wood, on either side of the Invar strip. The Invar tape is fastened to a steel shoe at the bottom of the rod and is kept taut by spring tension at the top of the rod. Thus neither moisture nor temperature will significantly change the length of the rod. A circular spirit level is usually attached to each Invar rod, to insure accurate plumbing.

7. THE ALIDADE AND ITS ADJUSTMENTS

The telescopic alidade and plane table are frequently used on engineering surveys, in addition to their traditional use in mapping. For many kinds of surveys the plane table is a versatile and efficient instrument.

The principal adjustments of the modern telescopic alidade are for leveling and collimation.

The adjustment of the level bubble which strides the telescope (often called the striding level) should be tested frequently. The test is made by clamping the telescope, bringing the level bubble to center by the slow-motion screw, lifting the striding level from the telescope, reversing it end for end, and replacing it on the telescope in reversed position. If the bubble runs off from center, the level is adjusted by bringing the bubble halfway back to center, by means of the vertical adjusting nuts at one end of the tube. The adjustment should be checked, and further adjusted, if necessary, by releveling the alidade by means of the tangent screw, and again reversing the bubble. This test and adjustment should be made frequently, at least once during every day of use.

When a new level vial is installed in the tube, a test for lateral adjustment, or side swing, should be made. The pin, in the top of the telescope, that keeps the level tube attached to the telescope should first be unscrewed and removed. The level tube is then replaced on the telescope, and, after centering, is rocked slowly from side to side through an arc of about 10°. If the level is in good lateral adjustment, the bubble will stay centered throughout all positions. If not, it should be adjusted, by means of the side adjusting screws, so that it will stay centered during the rocking The locking pin must be replaced as soon as this adjustment is completed.

The collimation adjustment of the telescopic alidade is made in two steps. The verticality of the vertical wire is tested first. The alidade is placed on a level surface, and the vertical wire is sighted on a nearby point. The telescope is then raised or lowered by the observer, who notes whether the vertical wire stays in contact with the point sighted. The same test can be made by making the vertical wire coincide with the vertical edge of a building, or a plumb line, and then noting, while raising and lowering the telescope, whether the wire remains parallel to the vertical test line. If the wire is found to be not vertical, the screws for the cross-wire ring should be loosened, and the ring rotated as required to make the vertical wire run truly vertical.

The second step of the collimation adjustment is to make the line of sight parallel to the telescope axis, and is accomplished as follows: Make the intersection of the cross wires coincide with a sharp object some distance away, rotate the telescope 180° through its supporting sleeve, then note the position of the intersection of the cross wires with respect to the object sighted. If the intersection of the cross wires remains on the object sighted, the adjustment is good. If not, the reticle screws should be loosened slightly, and the cross-wire ring moved halfway toward the point sighted. The operation should be repeated until the cross-wire intersection stays on any sighted object when the telescope is rotated 180°.

The circular level, used for leveling the plane-table board, should be kept in good adjustment. It is tested by placing the alidade on a level surface. If the circular bubble is then centered, it needs no adjustment; if not, the bubble should be brought to center by means of any one or two of the three adjusting screws.

Alidades of recent manufacture may have a movable vernier arm with a separate level tube attached. This is called the side level, control level, or vernier level. In reading vertical angles with this type of instrument, the index angle can be made zero, by bringing the vernier level bubble to center before reading the vertical arc. Its adjustment is tested by (1) placing the alidade on a horizontal surface; (2) leveling the telescope by means of the striding level (which has been previously adjusted); (3) setting the vernier of the vertical arc at zero; and (4) turning the adjusting screw or capstan-headed nuts at one end of the vernier level until its bubble is centered.

The compass needle should be tested for proper balance, and the counterweight adjusted as necessary, whenever an alidade is moved from one locality to another.

Manufacturers align the blade of each alidade closely perpendicular to the instrument's horizontal axis, but this is not a necessary requirement.

SURVEYING PROCEDURES

8. TRANSIT SURVEYING

The transit is commonly used for prolonging a straight line, for measuring angles of traverse and triangulation, and for laying off angles in land subdivision and in layout for construction. The angles measured for traverse may be the deflection angle (the angle between the last course produced and the next course), the interior angle, the clockwise angle, or the azimuth angle. A straight line is extended by setting up the transit at any point B on the line, with both clamps tight, backsighting any point A on the line, "plunging" the telescope so that it points forward along the line, and setting point C anywhere on the line AB extended. To eliminate adjustment errors, repeat the process, beginning the backsight with telescope inverted, so that in the second foresight the telescope will be direct. If the two points so established at foresight C are different, take a mean position. The line BC may then be further extended by occupying new station C, backsighting B, etc.

In measuring a deflection angle the telescope has to be inverted, and any error in the line of collimation will therefore be introduced into the deflection angle. This error may be eliminated as follows: measure the angle first with the telescope direct and then, without changing the circle reading, invert the telescope and "double" the angle; half the final angle is the correct angle. To avoid this necessity of reversing the telescope and repeating the angles some surveyors prefer to measure the interior angles directly. Deflection angles should be recorded as R (right) or L (left). The algebraic sum of the deflection angles of a closed traverse should equal 360°. If the interior angles are read, the sum of all the interior angles of a closed traverse should be $(n - 2) \times 180°$, where n is the number of sides.

In measuring clockwise angles the transit is set up at B, the A vernier is set to read 0° 00′, and backsight station A is sighted. The upper-motion clamp is then loosened, the telescope is pointed on foresight station C, and the A vernier is read. To eliminate errors

of adjustment the angle should be measured twice and a mean value taken, the telescope being inverted for the second turning.

For Azimuths, the instrument is first set up at point A with the circle set at 0° and the lower clamp loose. The telescope is turned so as to point toward magnetic south or true south (true meridian is the better reference direction) and lower motion clamped. Then the upper plate is unclamped, B sighted, and the angle read in a clockwise direction. This gives the azimuth of the line AB, an azimuth angle being the total angle read in a clockwise direction from the south around to 360°. The instrument is then taken to B so as to obtain the azimuth of BC. Invert the telescope and backsight on A, the vernier remaining at the same reading it had when azimuth AB was read, clamp the lower motion, turn the telescope to its direct position, loosen the upper clamp, and sight C. This will give the azimuth of BC referred to the same meridian as AB. Evidently this method does not eliminate any error in the line of collimation adjustment. By the azimuth method, when the survey is about to be closed the azimuth of the first line AB can be obtained by setting up again on A and using the last course as a backsight. The difference between the first determination of the azimuth of AB and its final determination gives directly the total error in the angular work.

When great accuracy in angle measurement is required, it is necessary to repeat the measurement several times; this is called the repetition method. To measure an angle by repetition, set up the transit at A, set the verniers at 0°, sight B, and tighten the lower clamp Loosen the upper clamp and sight C; read and record the angle. Leaving the two plates clamped together, unclamp the lower clamp and sight B; unclamp upper clamp and sight C; and read and record the angle. Half this angle should check closely the first angle read, and the result obtained is more exact than the first angle It is evident that by repeating this process six times with a 1′ instrument, and dividing by 6, an angle to about 10″ can be obtained. This method of repetition might be carried even further, but ordinary transits are not good for results much closer than 10″.

This method of repetition can be readily applied to laying off an angle. The angle is first laid off with a single reading and a point is set; then the angle which has been laid off is measured by repetition as described in the previous paragraph. If it is found to be in error, say by 20″, the point on the stake is moved (offset) in the proper direction a distance equal to the distance from the point set to the instrument times tan 20 sec

The precision required in angle measurements depends on the purpose of the survey. In traverse, as well as in most other work, it is desirable to have angular measurements consistent in precision with the distance measurements. In general, if distances are accurate to the nearest 0.01 ft, angles should be measured to the nearest 15″; but, if angles are measured only to the nearest minute, distances to the nearest 0.1 ft (sometimes to the nearest foot) are close enough, depending on the length of the courses. A proper appreciation of the relation of distances and angles may be had if it is borne in mind that for a course of 100 ft an offset of 0.03 ft subtends 1′ of arc, and that this is approximately 1.5 ft per mi.

The error of closure of a traverse in which the angles are measured to the nearest minute and the distances to a tenth of a foot should not exceed 1/5000. But where the angles are measured to 15″ and the distances to hundredths of a foot, a closing error of better than 1/10,000 may be obtained.

Checks on Traverses. In traverses which enclose an area there is a mathematical check on the distances provided the angles are correct, as shown in Art. 25. But in traverses which do not close there is no check on the distances other than by remeasurement of the lines (preferably in the opposite direction from the first measurement), except by cut-off lines which form closed traverses of portions of the survey. The angles can be checked by determining the "true" bearing of the first line of the traverse by solar or stellar observation, and whenever it is desired to check the angles the "true" bearing of any course can be determined by another meridian observation [1] The meridian can be determined to the nearest minute by solar observation and much closer by observation on the polestar.

As a check against large errors in the angles, the magnetic bearing of each line should be read when practicable and compared with the calculated bearing. The calculated bearing of the first line is assumed to be its observed bearing; the calculated bearing of any other line is obtained from the calculated bearing of the line next preceding combined with the measured angle between the two lines.

A traverse is calculated by the method of rectangular latitudes and departures; the area enclosed by a traverse may then be calculated by any of several methods. Specimen calculations for latitude and departure and for area of tract are given in Art 25 and 26.

[1] See Art 11 for astronomic azimuths and convergence of meridians

Supplying Closing Side of a Traverse. If the latitudes and departures of the several courses of a traverse which does not close are computed, the algebraic sum of the latitudes gives the latitude of the closing side and the algebraic sum of the departures gives the departure of the closing side (Art. 26). The square root of the sum of the squares of these two elements gives the length of the closing side, and the tangent of its bearing is its departure divided by its latitude.

To Cut Off an Area by a straight line from a point on a side. Plot the field and the known point, draw a trial line from this point to an angle in the other side of the field so as to lay off approximately the required area. Then the sides and bearings of all the lines except the trial line across the field are known; and its length and bearing can be computed as explained in the previous paragraph, and the area of the portion cut off computed by the D.M.D. method (Art. 26). The difference between this area and the area required is a triangle whose base is the trial line and whose altitude can be readily computed, from which the distance along the side of the field from the end of the trial line to the correct point can be computed. Then the closing side can again be calculated and the area cut off computed by the D M D. method, which should this time give the desired result.

To find the area cut off by a line in a given direction from a given point, let the cut-off line be AC. This problem may be readily solved by drawing a line from A in the traverse to the corner B which lies nearest the other extremity C of the cut-off line The area of the portion of the traverse cut off by AB is then computed (Art. 26), and to this area is added or from it is subtracted the area of the triangle ABC.

9. CROOKED BOUNDARY PROBLEM

Crooked or Curved Boundaries, such as brooks, are often located by measuring the perpendicular offsets from the traverse line which has been run alongside the brook. The curvature of the boundary may be so flat and uniform that it will be feasible to take the offsets at regular intervals, while in cases where the boundary changes in direction abruptly it is necessary to take the offsets only at the points where these changes occur. In either case there will be the areas of several trapezoids to compute to find the area between the traverse line and the boundary, but if the offsets are taken at equal intervals then the computation may be made by one of the following methods:

By the Trapezoidal Rule, Area $= (1/2)d(h_e + 2\Sigma h + h_e')$, where d = common interval between offsets; h_e and h_e' = end offsets of the series of trapezoids; and Σh = sum of intermediate offsets. This rule assumes that the boundary is a straight line between adjacent offsets.

Simpson's One-Third Rule assumes that between adjacent offsets the boundary is a curve (a parabola). By this rule **Area** $= (d/3)(h_e + 2\Sigma h_{odd} + 4\Sigma h_{even} + h_e')$, where d = common interval between offsets, h_e and h_e' = end offsets of the series; $2\Sigma h_{odd}$ = twice the sum of the odd offsets except the first and last (the 3rd, 5th, 7th, etc.); $4\Sigma h_{even}$ = four times the sum of the even offsets (the 2nd, 4th, 6th, etc.). For this rule to apply there must be an odd number of offsets; if there is an even number, compute the area of one end trapezoid separately.

To Straighten a Crooked Boundary is to run a straight line cutting the crooked boundary so as to make the lots on either side of the straight line have the same areas as they have with the crooked boundary for the division line. To do this, first run a trial line AB (Fig. 5), then measure proper offsets to the crooked boundary, and compute the areas X, Y, and Z on each side of AB between it and the crooked boundary. The sum of X and Z should equal Y, for $X + Z$ is the amount taken from lot M, and Y is the amount taken from lot N. If $X + Z$ does not equal Y the difference is the amount the trial line has taken from one lot more than it has taken from the other so this whole difference must be returned. For example, suppose the original area of lot M = 50,000 and lot N = 40,000. When AB is run it is found that the area of X = 300, Y = 1000, and Z = 500; that is, the trial line AB has made lot M = 50,000 − 800 + 1000 = 50,200, and lot N = 40,000 − 1000 + 800 = 39,800. An area 1000 − (300 + 500) = 200 must therefore be taken from lot M and added to lot N by running the final line AD so as to make the area ADB equal to 200.

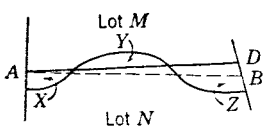

FIG 5.

10. OBSTACLES AND INACCESSIBLE DISTANCES

A Random Line is sometimes required on account of obstacles on line. If it be required to run the line AB (Fig. 6), neither point being visible from the other, a random line AX can be run. Measure the perpendicular distance BC and also AC. The length of AB may then be computed, and any desired point may be set on the line AB as follows: Suppose a point D is to be set on AB and that a perpendicular from D meets AC at E, then $AE = AD \times AC/AB$, and $ED = BC \times AD/AB$ By this method, then, it may not be necessary to run out and actually measure the line AB The angle CAB can be found from CB and AC.

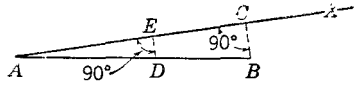

Fig 6

If a straight line such as AX cannot be run, a traverse composed of several straight lines and angles may be made to connect A and B, and, using AB as the closing side of a closed traverse, its length and direction may be computed (Art 25), and also the coordinates of any point in the traverse may be found referred to AB so that distances may be laid off from the traverse which will define any desired points on AB. This traverse method, since it involves the measurement of angles, will not lend itself as readily to accurate results as the case first described where the random line is a straight line In running a straight line where it is necessary to produce the line by reversing the telescope it should always be done by taking the mean result of a double reversal, the telescope being erect in taking the first backsight and inverted when taking the second backsight, so as to eliminate any error in the line of collimation.

An Equilateral Triangular Traverse about an obstacle is a special case of the above method. Let the obstacle be on the line AB (Fig. 7); the instrument is set up at C, an angle of 120° is laid off, and a stake set a sufficient distance away at D, and CD is measured The transit is then set up at D and an interior angle of 60° laid off so as to run a line that will cut AB, this line is made equal to CD and stake E is set. With the instrument at E an angle DEB is laid off equal to 120°, which should be sighting along AB if the work has been done accurately. Evidently $CD = ED = CE$. This method is weak in accuracy owing to its dependence upon angle measurements A slight error in any angle will introduce an appreciable error in CE If CD is short the error in CE is small, but the error in producing EB by an angle laid off from the short side ED is likely to be large.

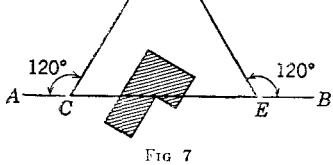

Fig 7

Running Parallel Line Past Obstacle. One of the most exact ways of producing a straight line past an obstacle of limited size, such as a house, is to run an offset parallel transit line past the obstacle as follows The instrument is set up at C (Fig. 8) and a right angle ACC' laid off with the transit. CC' is made any convenient distance which will bring the auxiliary line beyond the obstacle. Similarly, point A' is set opposite point A, and sometimes a second point B' opposite B These points A' and B' need not be set by means of a transit set up at A and B if AA' is short. The instrument is then set up at C' and backsighted on A', the sight is checked on B', the telescope inverted, and points D', E' and F' set on line. Leaving the telescope inverted, another backsight is taken on A', and the process repeated to eliminate any error in the line of collimation Then the transit is moved to point D', and a right angle turned off, and point D set, the distance $D'D$ being made equal to $C'C$. Then by setting up at D and sighting ahead on $F(FF' = DD')$, and checking on point $E(EE' = DD')$, the transit line is again run forward in its original location. The distance $C'D'$ is carefully measured, which gives the distance CD, and thus it appears why it is so necessary that the lines CC' and $D'D$ shall be laid off at right angles by means of the transit. The other offsets AA', BB', EE' and FF' are not in any way connected with the measurement along the line; they simply define the direction of the line, so that if convenient it is often only necessary to show these distances as swing offsets for the transitman to sight on. The swing offset is given by

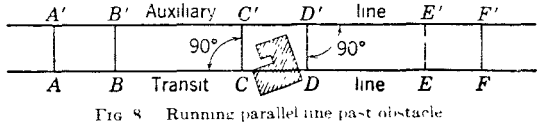

Fig 8 Running parallel line past obstacle

swinging a tape in a horizontal arc about A as a pivot. The zero point of the tape is held at A, and at the distance AA' out on the tape a pencil is held vertically on which the transitman sights when the pencil appears to be swung out farthest from A so that his line of sight will be tangent to the arc. The offsets BB' and EE' are not absolutely necessary, but they serve as desirable checks on the work, and in first-class surveying they should not be omitted. For obvious reasons the offsets AA' and FF' should be taken as far back from the obstacle as is practicable

Should the house be in a hollow so that it is possible to see over it with the instrument at A, the point F, or a foresight of some sort, should be set on line beyond the house to be used as a foresight when the transit is set up again on the original line. The distance may be obtained by an offset line around the house. Sometimes it is possible to place exactly on line on the ridgepole of the house a nail, which gives an excellent backsight when extending the line on the other side of the building.

If the building has a flat roof and it is possible to set a point on the roof exactly on line, move the instrument to this point on the roof and prolong the line in this way. Under these conditions the transitman will have to be extremely careful in the use of his instrument on account of its insecure foundation. If he walks around the transit he will find that it affects the level bubbles and the position of the line of sight; it is therefore well for him if possible to stand in the same tracks while he backsights and foresights. Sometimes two men, one in front and one behind the transit, can carry on the work under these conditions more accurately and conveniently. This method insures an accurate prolongation of the line, but the distance through the building must be found by an offset method, by plumbing from the edge of the flat roof, or by inclined measurements.

To Measure an Inaccessible Distance where the line is visible, as across a pond, several methods may be employed

1 Lay off from the transit line AC (Fig 9) a line AB which passes by the end of the pond so that it can be taped and set stake B; then set up the instrument at B and lay off $ABC = 90°$, point C being obtained by intersecting the main transit line Measure angle CAB and side AB, from which $AC = AB \cos CAB$; or, better, $AC = AB \div AB$ exsec CAB. The measurement of the angle at C and of CB will serve as checks.

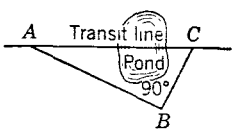

Fig. 9.

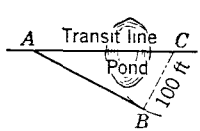

Fig 10.

2. With the instrument at A (Fig. 10) and a swing offset of 100 ft from C (some point on the main traverse line on the other side of the pond) measure the angle between CA and a tangent AB to the swing offset. $AC = 100/\sin CAB$.

3. Any line AB (Fig. 11) may be laid off along the shore of a river, and a point C set on the main traverse line across the river; measure angles A and B, and C as a check, and from AB as a base compute AC by trigonometry If AB is run perpendicular to CA and made some number of hundred feet long the computation is greatly simplified.

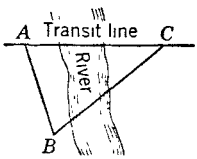

Fig 11

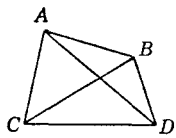

Fig 12.

To Obtain the Distance between Two Inaccessible Points A and B (Fig. 12) by observations from two accessible points C and D, measure DC and angles ADC, ADB, ACB and BCD. Compute CB in triangle CBD, AC in triangle ACD, and then in triangle ACB compute AB.

To Obtain the Inaccessible Distance AB (Fig. 12) between two accessible points by observation on two inaccessible points C and D when distance CD is known, measure angles CAD, DAB, CBD and ABC. Assume $AB = 1$. Then compute CD by the process described in the above paragraph. This gives a ratio between AB and CD, and, since CD is known, the actual length of AB may be computed.

11. OBSERVATIONS FOR AZIMUTH

Astronomical Data Required. In making astronomical observations for surveying purposes it is necessary to have available, from published tables, certain astronomical calendar data. For surveyors whose work will be mainly confined to observations for astronomic azimuth, the *Ephemeris*, a pamphlet prepared annually by the U. S. Bureau of Land Management (published by the Government Printing Office), is satisfactory. It contains all the necessary data for computing either Polaris or solar azimuth observations that are intended to have an accuracy tolerance of about 30″.

It is also necessary to know the latitude and longitude position of the instrument station, but in the United States these can ordinarily be determined with sufficient accuracy by identifying the instrument station on an official government map that shows parallels of latitude and meridians of longitude, and scaling the position. The topographic quadrangle maps of the U. S. Geological Survey are ideal for this purpose, but, if these are not available, any reliable government map or chart on a scale of 1:250,000 or 1:500,000 should suffice for observations that are to be made with an ordinary engineer's transit.

In some astronomic calculations it is necessary to convert mean solar time to sidereal time, and vice versa. As a mean time interval of 12^h0^m is equivalent to $12^h02^m0^s$ of sidereal time, a mean-time interval may be converted to a sidereal interval by multiplying it by 1.003 (approximately); and sidereal time would be multiplied by 0.997 to convert it to mean time.

Polaris at Elongation. In northern latitudes the simplest method of determining the direction of the true meridian is by sighting Polaris when it is at eastern or western elongation. About half an hour before the computed time of elongation the transit should be set up at the point through which the meridian is desired. The cross hairs must be illuminated by means of a flashlight or a lantern. Set the vertical hair on the star and clamp the plates. It will be observed that the star moves very slowly in azimuth, to the left if approaching western elongation, to the right if eastern elongation. As the star nears its elongation it no longer moves in azimuth but will move vertically, downward if at western elongation, upward if at eastern elongation. This shows that the position of elongation has been reached, and the time should be in close agreement (within a few minutes) with the time of elongation as given in the *Ephemeris*. Now lower the telescope without disturbing its azimuth, and set a point on a stake several hundred feet to the north of the transit, in line with the vertical hair. Next quickly reverse the telescope, re-level the plate levels if necessary, point again at the star, and set another point on the stake. If these two points are close but do not exactly agree, take a point midway between them. To obtain the meridian lay off from this last point an angle equal to the azimuth of the star at elongation, which may be taken from a table of Polaris azimuths. The meridian point will be to the right (or east) of the star if the observation was made at western elongation, and to the left (or west) if the observation was made at eastern elongation.

When it is desired to determine the azimuth of a line other than the meridian, the angle may be measured between the star when at elongation and some reference mark on the desired line. The sum or difference of the computed azimuth of Polaris, and the measured angle from star to mark, is the azimuth of the mark from north.

If a table of azimuths of Polaris at elongation is not available, but the star's declination is known, the azimuth of Polaris (or any other circumpolar star) may be found by the equation $\sin Z = \sin D \sec L$ where Z is the star's azimuth from the north, D the declination of the star taken from an *Ephemeris*, and L the latitude of the observer. The time of elongation of Polaris is given in the *Ephemeris*. At northern latitude 40°, western elongation of Polaris will follow upper culmination by $5^h55.4^m$, and eastern elongation will precede upper culmination by the same amount. Lower culmination follows or precedes upper culmination by $11^h58.0^m$.

Culmination of Polaris. If the time of culmination of Polaris is known, a rough determination of meridian may be made by pointing the telescope upon Polaris at that exact instant. Such a determination may be in error 1′ or 2′, however, as the star moves most rapidly in azimuth when at its culmination.

Polaris at Any Hour Angle. If accurate sidereal time at the instant the star is sighted is known (it can be calculated from accurate standard time or local mean time provided longitude position is known), an observation on Polaris for azimuth may be taken at any time. The azimuth of Polaris is then found by solving the PZS triangle. The hour angle t of Polaris is found by subtracting its right ascension from the sidereal time at the instant of observation. The azimuth is then computed by the formula

$$\tan Z = \frac{\sin t}{\cos L \tan D - \sin L \cos t}$$

where L = the latitude, D = the declination, and Z = the azimuth.

Azimuth may be determined with considerable accuracy by this method, if the angle from star to mark is measured with sufficient precision, and if the time of sighting the star is accurately determined. If a standard-time watch is used, the accuracy of the sidereal time of pointing will depend on the accuracy of the longitude position as well as the watch accuracy. It is customary to take several different observations, compute each one separately, and take a mean for final azimuth value. When the *Ephemeris* prepared by the Bureau of Land Management is available, the azimuth of Polaris at any hour angle, for any given latitude and declination, may be taken directly from tables.

By the Altitude of the Sun. If the azimuth of a line AB is to be determined, set the transit over A and set the vernier to read 0°. Sight the vertical hair on B and clamp the lower motion. Loosen the upper clamp, place a dark glass over the eyepiece, and observe the sun as follows In the forenoon sight first on the right and lower edges of the sun (simultaneously) using the vertical hair and the middle horizontal hair, and read the watch, the vertical angle, and the horizontal angle; take two more observations in the same way. Then sight on the left and upper edges, taking the same readings as before and the same number of observations as before If the transit has a complete vertical circle it is advisable to reverse the telescope between these two half sets of observations. The index error should be determined after the sights on the sun are finished. The error of the watch should have been determined previously by comparing with railroad time or by radio. After completing the pointings on the sun the point B should again be sighted and the 0° reading verified (If the observation is being made in the afternoon sight the left and lower edges and then the right and upper edges.) Before calculating the azimuth the intervals between readings should be examined for errors. The change in azimuth per minute of altitude should be nearly the same as found from the four intervals. The means of the six altitudes, the six horisontal angles, and the six watch readings correspond to a single observation on the center of the sun in some position. To obtain the true altitude the mean of the vertical circle readings must be corrected for index error and must be diminished by the amount of the refraction. (See Table 1.) It should also be increased by the parallax, which is but 9″ times the cosine of the altitude. The latitude of the place must be known. This may be scaled from a reliable map to the nearest minute, or found by observation. The declination of the sun must be taken from the *Ephemeris*. The azimuth of the sun's center (from the north) is then computed by the formula

$$\cos \frac{1}{2} Z = \sqrt{\frac{\cos s \cos (s - p)}{\cos L \cos h}}$$

in which $s = (1/2)(L + h + p)$, and L, h, and p stand for the latitude, altitude, and polar distance, respectively. In the forenoon Z will be measured toward the east, in the afternoon toward the west. Avoid taking observations when the sun is near the meridian. Best results are obtained when the sun's bearing is about east or west.

Table 1. Refraction Correction

(In minutes)

True altitude = measured altitude − refraction correction

Altitude	Refraction	Altitude	Refraction	Altitude	Refraction	Altitude	Refraction
5°	9.9	11°	4.9	17°	3.1	35°	1.4
6	8.5	12	4.5	18	3.0	40	1.2
7	7.4	13	4.1	19	2.8	45	1.0
8	6.6	14	3.8	20	2.6	50	0.8
9	5.9	15	3.6	25	2.1	55	0.7
10	5.3	16	3.3	30	1.7	60	0.6

Convergence of Meridians.[1] An astronomic azimuth that is used as a fixed azimuth in a system of plane surveys must first be converted to a grid azimuth, by applying a correction for convergence of meridians This varies according to the latitude of the azimuth observation, and the distance of the observation station east or west of the central meridian of the grid system. It is computed from the formula

$$Cm'' = 52.13 \times d \times \tan \phi$$

in which Cm'' is the angular allowance for convergence of meridians in seconds; d is the east or west distance in miles along the parallel from the observation station to the central meridian of the grid system; and ϕ is the latitude of the observation station. If grid azi-

[1] See also Art. 35

muths are counted clockwise from the south point, astronomic azimuths that are located east of the central meridian should be decreased to convert to a grid azimuth, and increased if located to the west.

12. STADIA SURVEYING

The Stadia Method is one in which distances are measured by observing through the telescope of a transit, on a graduated rod, the space included between two horizontal hairs called **stadia hairs.** If the rod is held at different distances from the instrument different intervals on the rod are included between the stadia hairs, the spaces on the rod being proportional to the distances from the instrument to the rod, so that the intercepted space is a measure of the distance to the rod. This method furnishes a rapid means of measuring distances in filling in details of topographic and hydrographic surveys. It is used either with a transit or alidade. It provides a means of measuring inaccessible distances such as across water surfaces, the errors of measurements are compensating rather than cumulative, and it affords an accuracy sufficient for many kinds of work, being applicable even to surveying for area of such lands as wood lots or farms, for an accuracy of 1 part in 500 may be attained with the stadia. The rod used may be of any desired pattern of self-reading rod, the Philadelphia rod being a good type for short distances. It is well to use a rod with the graduations represented by some diagram which can be seen distinctly a long distance away, 1000 to 2000 ft. Portions of two of these rods are shown in Fig. 13, with the reading of the three cross hairs marked in the figure.

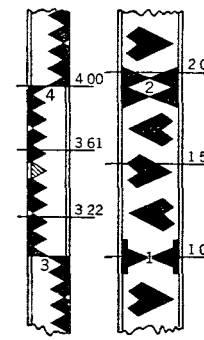

FIG. 13.

The Fundamental Principle of the stadia is the simple geometric proposition that in two similar triangles homologous sides are proportional. The vertex of these triangles is not the center of the transit but is at a point in front of the objective a distance equal to F, the focal length of the objective, so that the distance from the center of the instrument to the rod $= (F/i)(s) + (F + c)$, where i is the distance between the upper and lower stadia hairs; s is the intercepted space on the rod; and c is the distance from the center of the instrument to the objective Evidently $(F + c)$ is practically a constant for any external-focus instrument, for transits it varies from 0.75 to about 1.35, depending upon the style of instrument, so for all ordinary purposes it may be taken as 1 ft. For telescopes with internal focus the stadia constant $(F + c)$ is zero; hence this term should be omitted from the stadia formulas when this type of instrument is used. The other function $(F/i)s$ is a variable, but as it is almost always customary to have the stadia hairs spaced so that $F/i = 100$ this reduces the equation to **Distance = 100s + 1.** Every hundredth of a foot on the rod is then a measure of 1 ft of distance.

Inclined Sights. Since it is the horizontal distance which is required it is necessary to reduce all inclined readings to the horizontal, and when the difference in elevation between the instrument station and the rod station is desired the vertical distance also must be computed from the inclined sight. In practice it is customary to hold the rod plumb rather than perpendicular to the line of sight, so that it is evident that if the line of sight is inclined it will subtend an interval on the rod which is too great depending upon the angle of inclination, α, and the distance the rod is from the instrument. By trigonometric proof, with one slight assumption, it may be shown that the

$$\text{Vertical Distance} = (F/i)s\cdot(1/2)\sin 2\alpha + (F + c)\sin\alpha$$

$$\text{Horizontal Distance} = (F/i)s\cdot\cos^2\alpha + (F + c)\cos\alpha$$

For the ordinary conditions (when $F/i = 100$ and $F + c = 1$) and making the approximations that $\sin\alpha = (1/2)\sin 2\alpha$ and $\cos\alpha = \cos^2\alpha$ in the last terms of the above formulas, which are nearly correct for small angles,

$$\text{Vertical Distance} = (100 \times \text{rod interval} + 1)(1/2)\sin 2\alpha$$

$$\text{Horizontal Distance} = (100 \times \text{rod interval} + 1)\cos^2\alpha$$

In practice the relation of the stadia intercept on the rod to actual horizontal distance on the ground is determined by trial for each instrument. A tape is used to measure accurately, on level ground, a distance of 1000 ft to 1500 ft, each 100-ft station being marked. The stadia rod is then held successively at each of these 100-ft marks, and the stadia intercept on the rod is recorded. These measured values for stadia distance would

Table 2. Reductions of Stadia Observations

Vertical Heights for 100 Ft. Inclined Distance

Minutes	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0	0.00	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45
2	0.06	1.80	3.55	5.28	7.02	8.74	10.45	12.15	13.84	15.51
4	0.12	1.86	3.60	5.34	7.07	8.80	10.51	12.21	13.89	15.56
6	0.17	1.92	3.66	5.40	7.13	8.85	10.57	12.26	13.95	15.62
8	0.23	1.98	3.72	5.46	7.19	8.91	10.62	12.32	14.01	15.67
10	0.29	2.04	3.78	5.52	7.25	8.97	10.68	12.38	14.06	15.73
12	0.35	2.09	3.84	5.57	7.30	9.03	10.74	12.43	14.12	15.78
14	0.41	2.15	3.90	5.63	7.36	9.08	10.79	12.49	14.17	15.84
16	0.47	2.21	3.95	5.69	7.42	9.14	10.85	12.55	14.23	15.89
18	0.52	2.27	4.01	5.75	7.48	9.20	10.91	12.60	14.28	15.95
20	0.58	2.33	4.07	5.80	7.53	9.25	10.96	12.66	14.34	16.00
22	0.64	2.38	4.13	5.86	7.59	9.31	11.02	12.72	14.40	16.06
24	0.70	2.44	4.18	5.92	7.65	9.37	11.08	12.77	14.45	16.11
26	0.76	2.50	4.24	5.98	7.71	9.43	11.13	12.83	14.51	16.17
28	0.81	2.56	4.30	6.04	7.76	9.48	11.19	12.88	14.56	16.22
30	0.87	2.62	4.36	6.09	7.82	9.54	11.25	12.94	14.62	16.28
32	0.93	2.67	4.42	6.15	7.88	9.60	11.30	13.00	14.67	16.33
34	0.99	2.73	4.48	6.21	7.94	9.65	11.36	13.05	14.73	16.39
36	1.05	2.79	4.53	6.27	7.99	9.71	11.42	13.11	14.79	16.44
38	1.11	2.85	4.59	6.33	8.05	9.77	11.47	13.17	14.84	16.50
40	1.16	2.91	4.65	6.38	8.11	9.83	11.53	13.22	14.90	16.55
42	1.22	2.97	4.71	6.44	8.17	9.88	11.59	13.28	14.95	16.61
44	1.28	3.02	4.76	6.50	8.22	9.94	11.64	13.33	15.01	16.66
46	1.34	3.08	4.82	6.56	8.28	10.00	11.70	13.39	15.06	16.72
48	1.40	3.14	4.88	6.61	8.34	10.05	11.76	13.45	15.12	16.77
50	1.45	3.20	4.94	6.67	8.40	10.11	11.81	13.50	15.17	16.83
52	1.51	3.26	4.99	6.73	8.45	10.17	11.87	13.56	15.23	16.88
54	1.57	3.31	5.05	6.79	8.51	10.22	11.93	13.61	15.28	16.94
56	1.63	3.37	5.11	6.84	8.57	10.28	11.98	13.67	15.34	16.99
58	1.69	3.43	5.17	6.90	8.63	10.34	12.04	13.73	15.40	17.05
60	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45	17.10

Horizontal Corrections

Distance	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
100	0.0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	1.9	2.5
200	0.0	0.1	0.2	0.5	1.0	1.5	2.2	3.0	3.9	4.9
300	0.0	0.1	0.4	0.8	1.5	2.3	3.3	4.5	5.8	7.4
400	0.0	0.1	0.5	1.1	2.0	3.0	4.4	6.0	7.8	9.8
500	0.0	0.2	0.6	1.4	2.5	3.8	5.5	7.5	9.7	12.3
600	0.0	0.2	0.7	1.6	2.9	4.6	6.5	8.9	11.6	14.7
700	0.0	0.2	0.8	1.9	3.4	5.3	7.6	10.4	13.6	17.2
800	0.0	0.2	1.0	2.2	3.9	6.1	8.7	11.9	15.5	19.6
900	0.0	0.3	1.1	2.4	4.4	6.8	9.8	13.4	17.5	22.1
1000	0.0	0.3	1.2	2.7	4.9	7.6	10.9	14.9	19.4	24.5

Table 2. Reductions of Stadia Observations—*Continued*

Vertical Heights for 100 Ft. Inclined Distance

Minutes	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
0	17.10	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78
2	17.16	18.78	20.39	21.97	23.52	25.05	26.55	28.01	29.44	30.83
4	17.21	18.84	20.44	22.02	23.58	25.10	26.59	28.06	29.48	30.87
6	17.26	18.89	20.50	22.08	23.63	25.15	26.64	28.10	29.53	30.92
8	17.32	18.95	20.55	22.13	23.68	25.20	26.69	28.15	29.58	30.97
10	17.37	19.00	20.60	22.18	23.73	25.25	26.74	28.20	29.62	31.01
12	17.43	19.05	20.66	22.23	23.78	25.30	26.79	28.25	29.67	31.06
14	17.48	19.11	20.71	22.28	23.83	25.35	26.84	28.30	29.72	31.10
16	17.54	19.16	20.76	22.34	23.88	25.40	26.89	28.34	29.76	31.15
18	17.59	19.21	20.81	22.39	23.93	25.45	26.94	28.39	29.81	31.19
20	17.65	19.27	20.87	22.44	23.99	25.50	26.99	28.44	29.86	31.24
22	17.70	19.32	20.92	22.49	24.04	25.55	27.04	28.49	29.90	31.28
24	17.76	19.38	20.97	22.54	24.09	25.60	27.09	28.54	29.95	31.33
26	17.81	19.43	21.03	22.60	24.14	25.65	27.13	28.58	30.00	31.38
28	17.86	19.48	21.08	22.65	24.19	25.70	27.18	28.63	30.04	31.42
30	17.92	19.54	21.13	22.70	24.24	25.75	27.23	28.68	30.09	31.47
32	17.97	19.59	21.18	22.75	24.29	25.80	27.28	28.73	30.14	31.51
34	18.03	19.64	21.24	22.80	24.34	25.85	27.33	28.77	30.19	31.56
36	18.08	19.70	21.29	22.85	24.39	25.90	27.38	28.82	30.23	31.60
38	18.14	19.75	21.34	22.91	24.44	25.95	27.43	28.87	30.28	31.65
40	18.19	19.80	21.39	22.96	24.49	26.00	27.48	28.92	30.32	31.69
42	18.24	19.86	21.45	23.01	24.55	26.05	27.52	28.96	30.37	31.74
44	18.30	19.91	21.50	23.06	24.60	26.10	27.57	29.01	30.41	31.78
46	18.35	19.96	21.55	23.11	24.65	26.15	27.62	29.06	30.46	31.83
48	18.41	20.02	21.60	23.16	24.70	26.20	27.67	29.11	30.51	31.87
50	18.46	20.07	21.66	23.22	24.75	26.25	27.72	29.15	30.55	31.92
52	18.51	20.12	21.71	23.27	24.80	26.30	27.77	29.20	30.60	31.96
54	18.57	20.18	21.76	23.32	24.85	26.35	27.81	29.25	30.65	32.01
56	18.62	20.23	21.81	23.37	24.90	26.40	27.86	29.30	30.69	32.05
58	18.68	20.28	21.87	23.42	24.95	26.45	27.91	29.34	30.74	32.09
60	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78	32.14

Horizontal Corrections

Distance	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
100	3.0	3.6	4.3	5.1	5.9	6.7	7.6	8.5	9.5	10.6
200	6.0	7.3	8.6	10.1	11.7	13.4	15.2	17.1	19.1	21.2
300	9.1	10.9	13.0	15.2	17.6	20.1	22.8	25.6	28.6	31.8
400	12.1	14.6	17.3	20.2	23.4	26.8	30.4	34.2	38.2	42.4
500	15.1	18.2	21.6	25.3	29.3	33.5	38.0	42.7	47.7	53.0
600	18.1	21.8	25.9	30.4	35.1	40.2	45.6	51.3	57.3	63.6
700	21.1	25.5	30.2	35.4	41.0	46.9	53.2	59.8	66.8	74.2
800	24.2	29.1	34.6	40.5	46.8	53.6	60.8	68.4	76.4	84.8
900	27.2	32.8	38.9	45.5	52.7	60.3	68.4	76.9	85.9	95.4
1000	30.2	36.4	43.2	50.6	58.5	67.0	76.0	85.5	95.5	106.0

then be used in the formulas instead of the theoretical values given by the expression (100 × rod interval + 1).

In most stadia surveys no attempt is made to read the rod intercept closer than the nearest 0.01 ft, thus giving the distance only to the nearest foot. When distances of less accuracy are satisfactory, as in most topographic work, the constant $(F + c)$ may be neglected; also, there would be no need of applying the horizontal correction for vertical angles under 3°.

Distances are read by setting one of the stadia hairs on a whole footmark, by means of the vertical arc clamp and tangent screw, and counting the feet, tenths and hundredths of a foot between the stadia hairs. Great care should be taken not to mistake the middle horizontal cross hair for a stadia hair. This can be avoided by always making a mental estimate of the distance. Occasionally a half-interval may be read when something obstructs the view of the whole interval.

The Vertical Angles are taken by sighting the middle cross hair on a point on the rod whose distance above the foot of the rod is equal to the distance from the horizontal axis of the telescope to the station beneath the transit. The distance is known as the **height of instrument (H.I.)**; it is not the same as the H.I. in ordinary leveling, which is the distance of the telescope above the datum. When the middle hair is sighted at the H.I. point on the rod the line of sight is parallel to a line drawn from the station under the transit to the foot of the rod, and the vertical angle read is the inclination of this line, so that the difference in elevation of the two stations can be directly computed.

It is sometimes impossible to set on the H I. point on the rod owing to obstacles in the line of sight. In such a case the angle is taken when sighting at some convenient footmark on the rod, and this reading of the middle hair on the rod is recorded in the notes with the vertical angle. The H.I. point is sometimes marked by a red cloth band, its position being changed at each new set-up of the instrument.

The Order of Fieldwork is as follows. First read the distance and record it, then set the middle hair on the H.I. point of the rod and the vertical hair on the rod. Signal the rodman to go to the next point, and while he is moving to the next point read the horizontal circle, the vertical arc and its index correction if necessary. Unless elevations are required, the H.I. is not used, and the vertical arc will not be read except for angles of 3° or more.

Beaman's Stadia Arc. A device for simplifying the calculations of elevations in the field consists in using only those vertical angles for which the differences in elevation are simple multiples of the rod interval. This principle is used in the attachment devised by Beaman for the vertical arc of the transit or plane table. On this attachment are marked the numbers giving multiples of the rod interval, the zero graduation being marked 50 instead of 0 so as to make it easier to determine which are plus and which minus angles. In using a transit with this attachment the observer reads the distance, turns the tangent screw of the telescope until the index line is opposite some line on the attached arc, and then notes the rod reading of the middle cross hair. The difference in elevation between the telescope and the reading on the rod is (50 minus the reading of the arc) times the rod interval. From this result must be subtracted the rod reading to give the difference in elevation between the telescope and the foot of the rod. Evidently no stadia tables or slide rule are needed when this attachment is used

Stadia Computations. Reduction of stadia field notes is usually done by means of tables, diagrams, or stadia slide rule, rather than by direct use of the formulas. The stadia slide rule is the most convenient when the reductions are required in the field, as in plane-table surveys. The difference in elevation can be most rapidly obtained by means of a diagram or stadia slide rule, while the horizontal distances can be rapidly computed by use of a table of horizontal corrections which gives the distance correction to be subtracted from the inclined reading to obtain the horizontal distance, to which must be added the instrument constant $(F + c)$ for instruments having external-focus telescopes. The stadia reduction tables presented here give, for vertical angles up to 20°, the difference in elevation for an inclined distance of 100 ft. If the inclined reading is 612, the instrument constant 1 ft, and the vertical angle 4° 42′, the difference in elevation = 6.13 × 8.17 = 50.1 ft. Since the rod intervals are read as a rule to only three significant figures this multiplication can be accomplished with sufficient accuracy by means of the ordinary 10-in. slide rule. The 50.1 ft is recorded in the notes, and this difference in elevation is applied to the elevation of the station over which the transit is set to give the elevation of the point desired It will be seen from an inspection of Table 2 that a half-minute greater vertical angle would have increased the difference in elevation by a little less than 0.1 ft, which shows the importance of reading the vertical angle carefully and of applying its index correction. Even without a special stadia reduction table, vertical heights may be computed if a table of horizontal corrections is available. A condensed table of horizontal

corrections can be made which can be pasted on a leaf in the back of the notebook. After the horizontal distance has been found by use of this table, the vertical height can be computed by multiplying the horizontal distance by the tangent of the vertical angle, which can be conveniently accomplished by use of the tangent scale on an ordinary slide rule.

13. MINE SURVEYING [1]

An underground survey is based on precisely the same principles as a land-surface survey. The methods of reading angles and measuring distances, of computing azimuths, bearings, latitudes and departures, coordinates, and elevations, and of plotting the results are alike in all essentials. In their practical details, however, a number of differences are introduced by certain peculiar conditions under which a mine survey has to be conducted. These conditions are. (1) Working in the dark requires that both the point to be sighted and the cross hairs of a telescope must be illuminated (2) Working in constricted places, and in passages through which traffic must not be interrupted, often requires the use of special devices for mounting the instrument. (3) Stations must be placed in the roof, when possible, to assure their permanence and visibility. (4) Steeply inclined sights are more prevalent, requiring that a transit shall always be in perfect adjustment; with very steep sights, an auxiliary telescope is required, involving a correction in the reading of angles. (5) A higher degree of accuracy is usually required than in ordinary surface surveying; in this respect, mine surveying is comparable with the locating of bridge piers or the boundaries of city real estate This is because, in most of the important problems in mine engineering, aside from mere mapping, an error means loss of time and money in the subsequent mining operations Furthermore, it frequently happens that a complete circuit cannot be made, whereby the error of closure can be determined and distributed, or a mistake located and corrected; hence careful work by refined methods is the only assurance of accuracy. (6) Shaft plumbing is a special problem, peculiar to underground surveying.

Transferring a meridian into the mine is one of the important steps. This is usually done by means of two heavy plumb bobs hung on wires from the staging above the shaft. The transit is set up both above and below on this line, and thus an azimuth line is established between the surface and the workings. In order to have as long a base as possible the plumbs should be hung near the shaft casing Wires used for shaft plumbing are of iron, steel, copper, brass, or Phosphor bronze. In any case, the size of the wire should be as small as will safely carry the weight of the bob; B & S No. 20 or 24 is convenient. Iron wire is easily obtainable, and its ductility permits the bob to stretch out its kinks without breaking.

In the case of an inclined shaft, where the method of two plumb bobs would not be feasible, the meridian may be transferred underground by setting up the transit at the mouth of the shaft, taking a backsight on a surface station, then taking a foresight down the shaft, and measuring the inclined distance. The transit station at the surface, and the foresight point in the mine, should be placed so as to make the line of sight as nearly horizontal as possible. The transit is then set up at the bottom of the shaft, a backsight taken on the top station, and the traverse carried into the mine. The utmost care must be taken to eliminate all errors of adjustment of the transit, for the accuracy of all the underground survey depends upon this process. The telescope should be reversed, and the mean of the direct and reversed sights should be taken as the location for the foresight point. It is well to repeat the procedure several times, for checking purposes

Underground traverses are run through the passages in the mine It is often necessary to introduce very short lines into the traverse, and since the azimuth is transferred to distant parts of the mine through these short lines great care must be taken to eliminate all instrumental errors by reversing the telescope and using the mean of the two results, repeating the operation several times for good accuracy. The positions of walls and workings are located by measurements from the traverse line. The station points are usually placed in the roof, a spad in a wooden plug making a good station mark

Notes of mine surveys are kept in customary form, with provision for a column for vertical angles, as most of the traverse courses are inclined. Sketches are generally used to show details, such as the location and extent of stopes. The different station points of the survey are numbered consecutively. For convenience, stations on the first level are numbered 101, 102, etc., and on the second level, 201, 202, and so on.

If the whole survey system, above and below ground, is based on the same set of coordi-

[1] This article is composed mainly of verbatim extracts from *Mining Engineers' Handbook*, by Robert Peele, 3rd Ed., 1941, John Wiley & Sons, New York

nate axes, nearly every problem in mining engineering can be readily solved by the application of the principles of trigonometry, or analytical or descriptive geometry. To minimize the possibility of error in computation it is well to take as the origin of coordinates a point well to the south and west of the entire workings; negative coordinates are thus avoided. For the same reason elevations should be referred to a plane well below the deepest workings.

Surveying for Patent. By patent proceedings are meant the proceedings necessary to obtain from the government a fee-simple deed of a mining claim. Title to metalliferous land as granted by the United States conveys the right to all minerals included in the downward prolongation of the portion of vein cut off by the vertical end boundaries of the claim. The federal law allows a claim to cover 1500 ft located along the direction of a vein and 300 ft of surface ground on either side of it. These are maximum dimensions and may be reduced by local laws. For further information see the *Manual of Instructions for the Survey of the Mineral Land of the United States*,[2] prepared by the Bureau of Land Management, Washington, D. C.

14. SPIRIT LEVELING

Spirit leveling procedures are described in Art. 38, "Geodetic Leveling," in Art. 71, "Profile Leveling," and Art. 72, "Cross Sections," and in Art. 5, "The Level and Its Adjustment," and Art. 6, "Leveling Rods." The principal requirements for accurate leveling work are: (1) to keep the level bubble in good adjustment, and always to check the centering of the bubble just prior to making each rod reading; (2) to keep the backsight distance and foresight distance in each set-up approximately equal; (3) to prevent the accumulation of a large difference between backsight and foresight distances by keeping the total of both the backsight and foresight distances nearly equal in each section of line between adjacent benchmarks; (4) to keep all sights short enough so that the smallest graduations on the rod, or the target mark, can be seen distinctly; (5) to take suitable check readings for the purpose of eliminating blunders in reading the foot or tenth of foot; (6) to use as turning points and benchmarks only objects having a distinctive high point, making sure that the bottom of the rod rests on the same point for both the backsight and foresight; and (7) to have the rod plumb.

Benchmark Levels. Leveling surveys that are for the purpose of determining the elevations of a series of benchmarks are called benchmark levels. These should never be of less than third-order accuracy,[1] which require a minimum closure of 0.05 ft $\sqrt{M}$, M being the length of the level circuit in miles. When possible benchmark levels should be run between existing benchmarks of equal or higher accuracy, in which case the circuit length M is the distance between the existing starting and closing benchmarks. In other cases the benchmark levels should be double run, in order to eliminate blunders and prove the precision of the work. On some construction projects the later construction surveys, such as profile levels, can be used as a rough check on the bench levels. However, the profile levels are usually run with less accuracy than the benchmark levels, and so cannot be relied on to detect errors of less than 0.1 ft in the benchmark levels.

Whenever benchmarks on mean-sea-level datum are not located near enough to establish sea-level datum for a project, a local datum is used, or an approximate sea-level datum is assumed. An assumed datum is based on the best available information. An aneroid barometer may be used to establish approximate sea-level datum elevation of the first benchmark. At a later time, when practicable, the benchmarks should be connected to sea-level datum.

In the benchmark levels for a construction project, such as a new highway, the new benchmarks should be set at intervals along the route on permanent and substantial objects that will not be disturbed during the construction work to follow. The minimum distance between benchmarks will depend on project conditions and requirements, and may range from about 500 ft to 1 mile. Satisfactory benchmarks for this purpose include a well-marked point on a large boulder or on solid rock outcrop, a well-marked point on a bridge pier or a substantial building, a spike driven in a large tree or telephone pole, and a non-corrodible metal plug set in a concrete post that extends below frost line.

Lengths of Backsights and Foresights. Proper lengths of backsights and foresights depend on the distance at which the rod can be read clearly, according to the accuracy required, as well as on the steepness of the ground slope. Under ordinary conditions,

[2] May be out of print, but it is available in offices of the Bureau of Land Management, in public libraries, etc.

[1] See "Geodesy and Control Surveys" in this section for definitions of all orders of accuracy for level surveys.

sights should be kept under 500 ft, and preferably under 300 ft if rod readings are to be made to the nearest 0.01 ft. On warm days in sunshine, especially in the summertime and in the tropics, heat waves in the air may make it necessary to shorten the sights considerably in order to attain the desired accuracy. In deserts and other hot places, leveling should be done during the early morning hours, or during the night with the use of lights. The length of each backsight and foresight should be determined by stadia or by pacing, and be recorded in the notes so that the total for all the backsight and foresight distances can be approximately balanced for each independent section of the line.

Three-Wire Leveling. When utmost precision in benchmark leveling is required, it is advisable to read all three hairs of the telescope, making sure that the bubble is centered for each of the three readings. A mean of the three readings is then computed, and taken as the correct level-sight reading, providing that it checks closely with the middle-wire reading, and provided also that the interval between the upper and middle hairs agrees satisfactorily with the interval between the middle and lower hairs. The notes should provide a column for the backsight distances and the foresight distances, and their cumulative totals, thus providing the means of equalizing (approximately) the total of the backsight and foresight distances for each section of the line.

Curvature and Refraction Correction. A level line is a curved line, approximately parallel to a sea-level surface. The line of sight of a surveyor's level is a straight line that is tangent to the curved level line at the instrument station. Also atmospheric refraction bends the line of sight upward by a slight amount. A correction should be applied in running precise benchmark levels, especially when an unusually long sight is taken, as for a river crossing. The combined correction for curvature and refraction is, for sights of 300 ft, about 0.002 ft; for 500 ft, about 0.005 ft; and for 1000 ft, about 0.020 ft. For longer sights the amount of the curvature and refraction correction, in feet, may be computed closely enough for most purposes from the expression $0.574\ D^2$, where D is the length of the long sight in miles. The correction theoretically is subtracted from any single rod reading, for either a foresight or a backsight. However, no correction is required if the backsight and foresight distance for each instrument set-up, or if the total of backsight and foresight lengths for a sestion of level line, are kept nearly equal.

Reciprocal Leveling. It is sometimes necessary to determine the difference of elevation between two points so situated that it is not practical to make the foresight and backsight distances nearly equal. A typical case is where a level line must be carried across a river, or over a deep ravine. To overcome the effects of uncertainty in the amount of correction for curvature and refraction that should be applied to the long sight, it is customary, in precise level work, to use what is called reciprocal leveling. In the case of a river crossing, for example, the instrument should be set up a few feet from turning point A on the near bank, a backsight taken on A, then a long foresight taken on the rod at turning point B across the river. The instrument is then moved across the river and set up a few feet from turning point B, a backsight across the river is taken on the far rod at A, then a foresight is taken on the near rod on turning point A. The second set-up should be made as soon as possible after the first set-up, in order to have the vertical refraction conditions on the two long sights very nearly the same. The difference of elevation resulting from the first set-up on the near bank at A may be different from the difference of elevation determined from the second set-up on the far bank at B, but the mean of the two should be nearly a true difference of elevation, and be free from the effects of curvature and refraction, as well as improper bubble adjustment It is customary to reduce the rod reading on each long sight by applying a correction for curvature and refraction, and although this should make the two differences of elevation agree much better, it would not change the value of the mean difference of elevation.

15. TRIGONOMETRIC LEVELING

Finding the Difference in Elevation of two points by means of the horizontal distance between them and the vertical angle is called **trigonometric leveling.** It is used chiefly in determining the elevation of triangulation stations and in obtaining the elevation of a plane-table station from any visible triangulation point of known elevation. In triangulation work the vertical angles are usually measured at the same time the horizontal angles are measured, so as to obtain the elevations of triangulation points as well as their horizontal positions. The vertical angle is measured to some definite point on the signal whose height above the center mark of the station was determined when the signal was erected, and the height of the instrument above its station should be measured and recorded In the most exact work the angles are measured with a special vertical circle instrument. In less precise work an ordinary theodolite whose vertical arc reads by

verniers to 30″ or to 20″ may be used, but with such instruments only single readings can be made. The best results with such an instrument are obtained by taking the average of several independent readings half of which are taken with the telescope direct and the other half with the telescope inverted. In every case the index correction, or reading of the vertical arc when the telescope is level, must be recorded.

Simultaneous Observations. The chief difficulty in obtaining accurate results by trigonometric leveling is due to the uncertainty of the angle of refraction, that is, the angular deviation of the line of sight on account of the refraction of the air. This varies with the locality, the temperature, and the atmospheric pressure, so that the only way its effect can be practically eliminated is by taking simultaneous observations between two stations, in which case

$$h_1 - h_2 = K \tan \frac{1}{2}(\alpha_2 - \alpha_1) \left\{1 + \frac{h_1 + h_2}{2R} + \frac{K^2}{12R^2}\right\}$$

where $h_1 - h_2$ = difference in elevation of stations in feet; K = arc of the earth subtended between plumb lines through the two station points, approximately equal to the distance between the two stations in feet; α_1 and α_2 are the simultaneously observed vertical angles; R = radius of earth at latitude of the stations; for most work it is close enough to use log R (in feet) = 7.32068.

Observation at One Station. If the observation is made at only one station the **refraction coefficient** must be known and applied to the vertical angle. The refraction coefficient is $m = r/c$, whence $r = mc$, where r = the angle of refraction and c = the angle at the center of the earth between the two stations. For a single observation

$$h_2 - h_1 = K \tan \left[\alpha_1 + \left(\frac{1}{2} - m\right) \frac{K}{R \sin 1''}\right] \left[1 + \frac{h_1 + h_2}{2R} + \frac{K^2}{12R}\right]$$

In this formula the letters have the same significance as in the formula for simultaneous observations above. In solving for $h_2 - h_1$, approximate values are first obtained by omitting the last factor; the approximate values are then substituted in the last factor and corrected values are computed.

From a large number of observations the U. S. Coast and Geodetic Survey has determined values of m as follows.

For lines crossing the sea	0.078
Between primary points (high elevation)	0.071
For interior of the country, about	0.065

Clarke, in his *Geodesy*, gives:

For rays crossing the sea	0.0809
For rays not crossing the sea	0.0750

Rough Determination of the difference in height may be found by multiplying the horizontal distance by the tangent of the vertical angle and applying a single correction for curvature and refraction by the formula $h = K^2/1.7426$, where K is the distance in miles between the stations and h is the correction in feet. If K is expressed in units of 1000 ft, then $h = 0.02 \times K^2$ (nearly). This correction is applied so as to increase the difference in elevation if the vertical angle is plus, and it should decrease the difference in elevation if the vertical angle is minus. This method is used for the stadia and plane table except that for ordinary sights or for rough work the curvature and refraction correction is omitted.

16. BAROMETRIC ALTIMETRY

The barometer may be used as a means of measuring differences in elevation, since 1 in. in the height of the mercury column is equivalent to about 900 ft in altitude. The atmospheric pressure varies also with changes of temperature and humidity, so that it is necessary in measuring differences in altitudes with the barometer to determine the amounts of these variations and to make proper allowance for them. The ordinary cistern mercurial barometer and the aneroid are used in surveying. The latter, on account of its compactness and sensitiveness, is more generally employed.

Two types of aneroid barometers are used in surveying. The small or pocket-type barometer is useful in making a reconnaissance or in determining the elevations for small scale maps. On the pocket aneroid are two scales, the inner one corresponding to inches of mercury and the outer one to feet of altitude, the zero of the altitude scale being in most instruments at 31 in. on the mercury scale. The outer scale should not be movable with reference to the inner scale, for the number of feet of altitude corresponding to 1 in. of mercury is different in different parts of the scale. Aneroids marked "compensated" are

supposed to be adjusted so that changes in temperature of the instrument will not affect the readings. The instrument should be handled carefully in order to avoid disturbing its delicate mechanism. When it is to be read, tap the case lightly to be sure the instrument has adjusted itself to the changed pressure. It should stand a few minutes before it is read to allow it to come to the true reading; it should not be allowed to become heated by exposure to the sun's rays or to other conditions of abnormal warmth; it may be held in either a vertical or horizontal position when being read, but as the readings in these two positions are different it should be held in the same position at all stations. The larger and more precise aneroids are used for profile surveys and to establish spot elevations for control of photogrammetric mapping when elevation tolerances of ±5 ft are permitted.

Aneroid altimeters capable of giving good results should have the following characteristics: They should have no inertia due to friction; they should be free from lag and come promptly to the correct reading; they should have locks so that ordinary shocks have no effect and the instrument can be safely transported. They should be fully compensated for temperature.

This type of instrument cannot of course overcome variations in pressure due to atmospheric changes, any more than can any other type of barometer. To obtain the most accurate results from aneroids, it is necessary to provide one or more extra instruments for the purpose of ascertaining the changes in atmospheric pressure that may occur while the work is being done. The extra aneroids are called the base aneroids, and each of these is usually stationed at a point of known elevation within the project area. Frequent readings are made on each base aneroid for the purpose of measuring and recording the changes in atmospheric pressure. The procedures commonly used are known as the two-base and one-base methods.

The general procedures for the two-base [1] and one-base methods are described under "Topographic Surveys and Mapping." It should be noted that temperature and humidity corrections are not required for the two-base method, but are necessary for accurate work in all other aneroid-altimetry procedures. Most surveying aneroids have these corrections listed on a chart attached to each instrument. When using two or more aneroids as a set, it is necessary to calibrate them by comparing their altitude readings taken at the same time on the same point. If there are differences, one of the instruments is regarded as the master instrument, and an appropriate constant or index correction is then applied to all readings made on each of the other instruments The section on topographic surveying also includes a general description of the so-called leap-frog method of aneroid altimetry, a "moving-base" method, which is well suited for determining elevations of a series of points along a route that would extend a considerable distance from a fixed-base station.

When only one aneroid is available, a reading should be taken at the first station when the work is being started, and again on the same station at the end of the trip, or end of the day's work, whichever occurs sooner. The difference between these two readings is assumed to represent a continuously progressive change in atmospheric pressure, at that point and for the area, and interpolated corrections for the times when the aneroid was read at other stations must then be calculated and applied to those readings.

17. PLANE-TABLE SURVEYING

The Plane Table is an instrument by means of which points are located in the field by graphical methods directly on the map, which is fastened to a table top supported on a tripod. The plane table is the only surveying instrument admitting of a rapid solution of the Three-Point Problem in the field; this makes it practicable to locate stations independently of each other, so that errors cannot accumulate as they do in traversing. The most important advantage of the plane-table method over other topographic methods, however, is that all the sketching is done in the field, where the topographer can see the form of the ground that he is mapping. He can sketch details at once in their proper position, without burdening his memory and without making elaborate notes. For this reason the details may be accurately sketched from a much smaller number of located points than would be required, for instance, by the transit-and-stadia method.

The plane-table method has the disadvantage of requiring more time for the field work than other methods, and it is also more dependent upon favorable weather than a method where the map is not exposed. But taking into account both the field and office work the

[1] An instruction pamphlet on the two-base method, titled *Instructions for Altimeter Surveys Used in Supplemental Control*, is available on request to the Topographic Division, U S Geological Survey, Washington 25, D. C. Also, aneroid manufacturers furnish comprehensive instructions for several different techniques of aneroid altimetry.

plane table will prove to be much more economical than the transit and stadia for work in open country, and at the same time the results obtained will be sufficiently accurate for most topographic work.

The Plane Table Itself consists of a board, usually about 24 by 30 in., mounted on a tripod with a device for leveling and clamping the board, the Johnson ball-and-socket leveling device and clamp being the most commonly employed. The **alidade** consists of a telescope mounted on a horizontal axis resting in wye supports, which are connected to a metal column at the base of which is attached a flat piece of metal about 18 in. long having both edges straight and parallel to the telescope. On this base are a circular spirit level for leveling the table and a magnetic needle for orienting by compass. The telescope has a vertical motion only and a vertical arc. The entire instrument is moved about in azimuth on the board and sighted as desired

At every instrument station the initial operations are: (1) level the plane table board; (2) orient the map sheet; (3) if the location of the instrument station is unknown, determine its position on the map sheet; and (4) determine elevation of point occupied and HI (height of instrument). Orientation and instrument location may be determined by any of the methods described in the following paragraphs.

Base-Line Orientation and Control Extension. In order to begin a survey with a plane table it will in general be necessary to have on the map two plotted points corresponding to two points on the ground the distance between which is known and at least one of which can be occupied with the table. The simplest method of locating points by means of the plane table without measuring any distance is as follows: The base-line ab is plotted on the plane-table sheet, representing, to some scale, the measured base AB. The table is set so that a on the map is vertically above A on the ground and the table is leveled; then one edge of the alidade is placed along the base-line ab drawn on the map, the table is then turned in azimuth until the telescope sights the signal B, and the horizontal motion clamped. The line ab is now parallel to AB and the table is said to be oriented. The alidade is then placed so that the straightedge passes through a, the telescope is sighted to some signal C, and a line of indefinite length drawn toward C. The point c on the map (representing the signal C) is somewhere on this line. If the table is now moved to B (b being set vertically over B) and the process of orienting the table and sighting toward C is repeated, the point c is located on an indefinite line through b; hence it lies at the intersection of these two lines ac and bc. The triangle abc is similar to the triangle ABC, and each line on the map is parallel to the corresponding line on the ground. In a similar manner any number of points may be located.

Orientation by Magnetic Compass. To use the needle for orientation of the plane table, it is necessary to have a magnetic north line drawn on the map sheet. This is obtained as follows: (1) accurately orient the map by occupying one control point and sighting another, etc.; (2) with alidade in approximate center of board, swing the alidade until the needle lines up with its north index mark, (3) draw a line along the edge of alidade ruler for its full length; this will be a magnetic north line. In orienting by needle, place the edge of the alidade ruler adjacent and parallel to the magnetic north line; then swing the board in azimuth until the needle is aligned with its north index mark. Orientation by compass should not be attempted near railroads, electric transmission lines, large bodies of steel or iron, or in volcanic regions.

If the map is oriented by compass at an unknown location, the position of the instrument station on the map must next be determined When a control point is located nearby, the alidade's ruler edge is placed adjacent to the control point and is rotated in azimuth around the control point as a pivot until the vertical hair of the telescope bisects the target at the control point, a fine line being then drawn lightly in pencil from the control point toward the new location The distance between the two points is then determined by stadia or taping, and is plotted on the sight line. If two or more such control points are visible, a location for the new station should be determined from each of them, taking a mean position in case of minor discrepancies. In case two distant control points are visible from the new station, the position of the plane-table station on the map sheet may be determined by drawing a sight line from each of the control points, the location of the new station being at the intersection of these lines, called resection lines.

Plane-Table Traverse. In running stadia traverse by plane table the board is first oriented by compass, or by occupying a known station and backsighting to another known station, which is often of necessity the previously occupied station of the traverse. However, the accuracy of the orientation will be greatly increased if a distant control point or previously mapped feature can be used as a backsight rather than a nearby point. The telescope is then pointed to the next forward station of the traverse, a foresight line drawn toward it, and the stadia distance measured and plotted. Each plane-table traverse should be made to close on a previously established control point, so that an indica-

tion of its accuracy can be determined from the size of the closure. If the closure is within satisfactory limits, the positions of the stations of the traverse should be adjusted graphically on the map sheet on a prorata basis

Locating Points by Resection. It sometimes happens that it is desired to locate a plane-table station from a base only one end of which can be occupied by the table. In such a case we may proceed as follows: Let A and B represent the points on the ground at the ends of the base line, C is the signal which is to be located, and ab represents the base line plotted on the plane-table sheet. Set up at A, the end of the base which is accessible, and orient the table by sighting B with the alidade along ab. Then, centering the alidade on a, draw an indefinite line toward C. This line should be drawn the full length of the alidade. The table is then taken to C and oriented by means of the indefinite line just drawn. Since the position of c on the indefinite line is not known it is necessary to estimate its position on the map and to use this point in placing the table over point C. If the alidade is now centered on b and sighted toward B, a resection line may be drawn, and this line will cut the first indefinite line, thus locating the point c desired. Thus, without going to station B, the same line has been drawn on the map that would have been obtained by an intersection from B. It is evident that this resection method may be advantageous even if B could be occupied, for point C has been located without taking the time required to go to station B. The position of c found by this method should be checked if possible by resection lines from other points whose positions are known to be correct.

The Three-Point Problem. One of the great advantages of the plane table is that it may be set up at any place where three triangulation points (plotted on the sheet) can be seen and the position of this plane-table station located on the sheet simply by observations from this point. The position of the plane table is found by means of the so-called Three-Point Problem, which, in this case, is an application of the principle of resection. The two graphical solutions of this problem chiefly used in plane-table surveying are known as (1) **Lehmann's** method, or the **Triangle-of-error** method and (2) **Bessel's** method, or the **Inscribed Quadrilateral.** The former is a trial method, but it is the more rapid of the two for ordinary work and is used in practice far more than the latter method. Bessel's method gives a direct solution and consequently requires less experience than the former. It has the disadvantage, however, that in certain positions of the signals a part of the required geometric construction falls outside the limits of the plane-table sheet, in which case the solution is not practicable.

Lehmann's Method. If three signals A, B, and C have their plotted positions at a, b, and c, and if the table is set up at D and oriented correctly, the resection lines drawn from a, b, and c will all pass through d, the plotted position of D. Since there is no means of accurately orienting the table, the position of d being unknown at the start, the table must be oriented approximately by estimation. If the plane table is not oriented exactly the three resection lines will not ordinarily pass through a common point but will form a triangle known as the triangle of error (Fig 14). From this triangle of error the true position of d may be estimated, and by a second trial a new triangle of error may be obtained which is smaller than the former. By successive trials this triangle may be made so small that it is almost a point. In practice very few trials are necessary, the triangle often being reduced to a point in the second trial, so that the method is in reality rapid

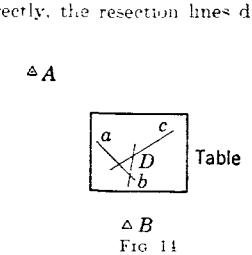

Fig 14

If the table is on the circumference of the circle through the three signals its position is indeterminate. When point D is inside triangle ABC it is in a favorable position for an accurate location. If the table is outside this triangle there are certain positions of the signals which are not favorable, especially when the angles subtended by the sides of the triangle formed by the signals are small and the middle signal is farthest from D, but if the middle signal is near D the location of d is strong.

If D lies inside the triangle ABC then d will lie inside the triangle of error, and vice versa. If a circle is passed through a, b, and the intersection of the resection lines from a and b it will pass through the true position of d, similarly a circle through b, c, and resection lines from b and c may be sketched and in this manner a close estimate of the position of d made for the second trial. The distance of d from any resection line is proportional to the distance of the table from the signal from which that line was drawn.

Locating Points for Mapping. Rod readings for filling in map details may be located on the map by alidade direction and stadia distance in the same manner as a foresight station in plane-table traverse. If the rod is at a well-defined feature that should be located in exact position on the map, the alidade's ruler edge should be placed in contact with the

point representing the occupied station, a light pencil line should be drawn toward the rod location, and the distance should be read and plotted with the same care required for plane-table traverse. However, if the rod reading is only for determining the location of an elevation for sketching contours in gently rolling terrain, slightly less accuracy in determining and plotting direction and distance will be satisfactory. Certain types of features may be located on the map by taking two or more intersection cuts to them. Also, the elevation of "cut-in" features may be determined by reading a vertical angle to the same point on the feature with each cut, the distances required for computing elevations from the vertical angles being scaled from the map, after the position of the feature has been determined.

Elevations by Plane Table. Differences of elevation are determined by using the alidade in the same manner that a transit would be used for this purpose However, because the plane-table outfit is not as stable as a transit, it is necessary to check the centering of the striding level bubble before taking each level shot; and in the case of vertical angles either to read the index angle or center the control bubble before taking an inclined reading.

18. HYDROGRAPHIC SURVEYING

Hydrographic Surveying is the term applied to the processes used in surveying any body of water. The determination of the topography of the bottom of a lake, harbor, or other body of water is a common problem In connection with such surveys the character of the material composing the bottom is often desired The field work is usually done by first establishing certain points on shore, by triangulation or traverse, to which the hydrographic survey may be referred, and then measuring (usually from a boat) the depth of the water at various points and determining the position of these points. Besides the ordinary transit and tape outfit, a sounding pole or lead line and a boat with the necessary equipment will be required. A tide gage should be set up in tidal waters and in lakes where the water level changes rapidly The sounding poles are made like self-reading rods graduated to tenths of a foot. A shoe is sometimes attached to the bottom, provided with a cup-shaped cavity which, if smeared with tallow or soap, enables samples of material to be collected. The lead line consists of a long chain or a hemp or cotton line at the end of which is attached a lead weight. A brass sash chain gives very satisfactory results, with cloth tags of various colors for foot marks. Where there is not much current a 6- to 10-lb weight will suffice for depths up to 40 ft.

Methods of Locating Soundings. There are six general methods of locating soundings: (1) The boat is rowed on a range at a uniform rate of speed and the soundings are located by time intervals (2) The boat is rowed on a range line and the positions of the soundings are "cut in" by a transit angle taken on shore or by an angle taken with a sextant from the boat (3) The boat may or may not be rowed on any definite range, and its position is located by angles taken simultaneously by two transits on shore or by angles taken simultaneously to shore points from the boat by means of two sextants. (4) The positions of the soundings are located by the stadia method. (5) The positions of the soundings are defined by the intersection of fixed ranges. (6) A wire or line is stretched across a stream from shore to shore and soundings are taken at different points along this wire and located by measured distances from one end of it

Locating a Sounding by a Range and an Angle. In still water where there is no difficulty in keeping the boat in any desired position soundings may be conveniently located by keeping the boat on a range line of known position marked on shore by two objects such as range poles, and then "cutting in" the position of the leadsman by means of a transit angle taken from shore at the instant the sounding is made. The ranges may be fan-shaped if the pivot point of the system of ranges is some steeple or other object located far enough back from the shore so that the lines will not diverge too rapidly. The recorder writes in his notebook the depths as they are called off by the leadsman and also the times when the soundings are made He also notifies the leadsman by calling out "Sound about 5 sec before each sounding is desired. The leadsman takes the soundings as quickly as possible, usually within 2 or 3 sec of the desired time When soundings are located by two angles taken with transits on shore, soundings are usually taken at 15-, 20-, or 30-sec intervals, and a location made each minute by the instrument men at the instant the signal is given by the signalman in the boat The chief of party usually acts as signalman and directs the work in the boat, and sees that the boat is kept on the ranges (if any are used) and that the boat is so propelled as properly to cover the area to be sounded. The signal is given by holding up a flag for about 10 sec and dropping it suddenly the instant the sounding is taken, at which moment the transitmen on shore take angles to the leadsman or to his hand if visible White, red, and sometimes black flags are used for signaling.

Both the recorder's and the instrumentmen's notes should show the colors of the signal as well as the time for each located sounding, thus giving two means of identifying the angles and the corresponding soundings. This double check is of particular value where the lines of soundings are long In tidal waters the time record is required also as a means of reducing the soundings to **Mean Low Water** or to whatever other datum is used. The tide-gage readings and times are recorded throughout the day.

Where the soundings are taken with a view to obtaining every slight change in the slope of the bottom, instead of taking the soundings at a given interval of time it is desirable to take them as frequently as the leadsman can conveniently handle the pole or line. In this method the boat is usually rowed on a range and the instrumentmen on shore "cut in" only those soundings that are designated by the signalman in the boat The time in this case is recorded to the nearest second. As a rule every sixth or eighth sounding is located.

In plotting notes like these where the soundings are quite close together the points which were "cut in" are located on the plan and the intermediate readings are interpolated between them; the soundings are assumed to be equally spaced between the located ones.

Locating a Sounding by Two Angles from a Boat. A common method of locating soundings when ranges are not used is by taking two angles simultaneously from the boat to three signals, or any previously determined points A, B, and C, on shore. This is an application of the Three-Point Problem that is frequently used in plane-table work. It is essential that the signals or points should be fixed stations; such points as buoys or floats will not give satisfactory locations.

In measuring angles from a boat it is necessary to employ some instrument which does not require a steady support like the transit. For this reason the angles are usually taken with two sextants. These two angles are sufficient to locate the position of the soundings, except in the one case where the boat happens to be on the circumference of the circle passing through the three signals between which the angles are measured.

This method is less frequently employed than the "range and angle" method or the "two angles from shore" method, because it often happens that the two angles taken with the sextant do not give good intersections; this is especially true when the soundings extend far from shore. Furthermore, if the signal happens to be high above the shore, and consequently not at the level of the boat, the angle measured will be enough different from the horizontal angle to introduce serious error into some parts of the work.

Use of Sextant. The sextant is an instrument adapted to measuring angles in any plane. Because it can be used by an observer who is on a moving object, such as a boat, it is especially valuable for hydrographic work. It is employed not only for taking angles from a boat in locating soundings but is also in common use for making astronomical observations which are necessary in determining the latitude, longitude, and time at sea. The **frame** ABI (Fig. 15) is usually of brass, on the under side of which is attached the wooden handle D. The index arm IE is pivoted at I, the center of the arc AB, and this arm can be swung around I as a center so that the vernier V can pass from A to B on the arc (or limb) and can be set at any position on the arc by means of the clamp C and tangent screw T. At I is a plane glass mirror called the index glass; it is attached rigidly to the index arm and perpendicular to the plane of the sextant, its reflecting surface being over the pivot about which the index arm revolves. Rigidly attached and perpendicular to the frame of the sextant is the plane horizon glass H, the upper half of which is transparent and the lower half is a mirror; this glass is set so that its plane is parallel to the index glass when the vernier is set at 0°. The telescope is at F. K and L are colored glasses which are hinged so that they can be swung around the pivot into the path of the rays of light to protect the eye of the observer in making observations on the sun.

It should be borne in mind that the angle measured with the sextant is not the horizontal angle, but is the angle lying in a plane defined by the two objects and the eye of the observer; the objects sighted should therefore be near the shore level unless the sextant is far from shore.

The vertex of the angle is not a fixed point; it moves farther away from the instrument as the angles grow smaller for the lines defining the angle pass from the two objects through the center of the mirror I and of the glass H

The limb AB is graduated into spaces which are really half-degrees, but on account of the construction of the instrument each of these is marked as a whole degree, so that the scale has an extent of 120°. The graduations are so subdivided that the angles can be read in most instruments to 10″; in some of the smaller instruments the vernier reads only to half-minutes. In the ordinary sextant the arm is from 5 to 8 in. in length. A pocket sextant having an arm about 2 in. long is very convenient for reconnaissance surveys and for filling in the details of more accurate surveys.

To Measure an Angle with the sextant, hold the instrument by its handle in the right hand and turn it so that the plane of the sextant coincides with the plane through the two objects to be observed, with the telescope on the upper side of the sextant if the angle is horizontal, or on the left-hand side in the case of a vertical angle. Without changing the plane of the sextant twist it in the hand so as to turn the telescope toward the left-hand object and observe it through the upper (transparent) portion of the horizon glass. Then, holding the instrument as steady as possible, turn the index arm with the left hand until the other object appears in the silvered portion of the horizon glass opposite the first point.

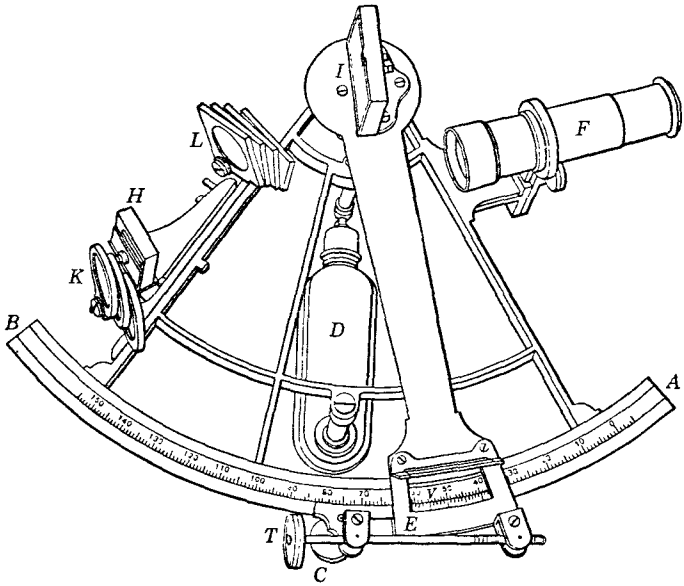

FIG. 15. The sextant.

Bring the second point exactly opposite the first one by means of the clamp and tangent screw of the index arm. The coincidence of the images should be tested by twisting the instrument a little so as to make the reflected image move back and forth across the direct image. Read the vernier and apply the index correction. The telescope is not used for rapid work such as locating soundings; the sight is simply made through the ring in which the telescope fits.

Echo-Sounding Instruments capable of giving good accuracy in measurement of water depths (as shallow as 1 fathom) are now in general use among the larger hydrographic surveying organizations. In echo sounding, depths are determined indirectly by measuring with electric equipment the time required for sound waves to travel, from a point just under the water surface and near the boat to the bottom and back again. The sound waves are emitted in rapid succession, and the time intervals are automatically recorded as depths, in feet, meters, or fathoms, on graph paper as a continuous profile of the bottom. The route of the profile, and position fixes along the profile route, may be determined by the use of surveying methods previously dessribed.

19. STAKE-OUT SURVEYS

Setting Batter-Boards for a Building. One of the most common tasks of the surveyor is to set the batter-boards for the excavation and construction of the cellar of a new building. For a brick or stone building the lines to be defined are the outside lines of the building, and the elevation desired is usually the top of the first floor. For a wooden building the line usually given is the outside line of the brick or stone underpinning, and the elevation given is the top of this underpinning on which the sill of the house is to rest. Sometimes the outside line of the sill is desired instead of the outside line of the underpinning.

There should be a definite understanding in regard to these points before the work of staking out is begun.

The first work is to stake out the location of the building by accurately setting temporary stakes at all of the corners of the building—in Fig. 16 at *A*, *B*, *C*, *D*, *E*, and *F*. A stake should be set at *G* also so that the entire work can be checked by measuring the diagonals *AC* and *BG*, and *FD* and *EG*. These checks should always be applied where possible. Then the posts for the batter-boards are driven into the ground 3 or 4 ft outside the line of the cellar so that they will not be disturbed when the walls are being constructed. On these posts, which are usually of 2 x 4-in. scantling, 1-in. boards are nailed. These boards are set by the surveyor so that their top edges are level with the grade of the top of the underpinning or for whatever other part of the building the grades are required. After the batter-boards are all in place they should be checked roughly by sighting across them; they should all appear at the same level. Sometimes, however, on account of the slope of the ground some of them have to be set a definite number of feet above or below grade.

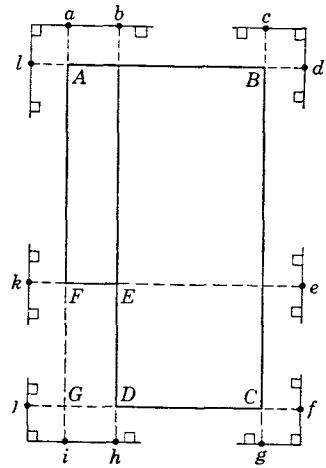

FIG 16

Then the lines are to be marked by nails driven in the top of these batter-boards. The transit is set up on one of the corner stakes of the house at *B* (Fig. 16), for example, and a sight is taken on *C*. This line is then marked on the batter-board beyond (at *g*) and on the one near the transit (at *c*). Then a sight is taken along *BA* and this line is produced both ways and nails set on the batter-boards at *l* and *d*. In a similar manner all of the lines are marked on the batters. These points should be marked with nails driven in the top-edges of the batter-boards, and there should be some lettering on the boards to make clear which lines have been given. It is well for the surveyor also to show these marks to the builder or inspector and have it clearly understood just what parts of the structure these lines and grades govern. See Arts. 68, 78, and 79 for other types of stake-out surveys.

Athletic Grounds

In the layout of athletic grounds several important things must be kept in mind: (1) the orientation, so that the sun will not shine in the eyes of players or the spectators, (2) drainage, (3) seating capacity around the grounds, (4) use of the grounds in different seasons for many types of sports, and (5) access from suitable automobile parking areas. A baseball, football, or tennis ground should be laid out so the afternoon sun will not interfere with the players. Few permanent divisions or obstructions should be installed. Provision should be made for use of temporary but serviceable divisions or markers for each season use. Some of the most permanent combinations are a football field inside a running track, which, if laid out properly, can serve also for soccer, baseball, football, and track meets.

Running Tracks should be as a rule $^1/_8$ or $^1/_4$ mile in length, because on tracks of these lengths a full number of laps (or half laps) are required for the ordinary dashes of 220, 440, and 880 yd, thus bringing both the start and the finish of the race in front of the grandstand A good running track should be not less than 15 ft wide on all portions except in front of the grandstand, where it should be made 20 to 25 ft wide for use in 100-yd dashes, in which race the contestants are usually "bunched." For these short dashes the straightaway portion of the track is usually lengthened beyond the curve so as to allow at least 40 ft beyond the finish line. The curves at the ends of an oval field track should not be sharper than 100-ft radius if it is possible to avoid it. The line for measurement of the length of a running track is 18 in. from the pole, and in a horse racetrack it is 3 ft from the pole. A good running track can be made in three layers: 8 in. of rock, 2 in. of cinders, and 2 in. of 3 parts screened cinders to 2 parts screened clay loam

A quarter-mile track with curves of 99- to 125-ft radius at its ends on the distance measuring line will allow placing a football gridiron inside the oval. The 99-ft radius provides a 17 $^1/_2$-ft and the 125-ft radius a 43 $^1/_2$-ft strip for turf between the sidelines of the football field and the curb line of the running track. Two plans for placing a football field inside a $^1/_4$-mile track are given in Fig. 17 Extension of the straight courses for the

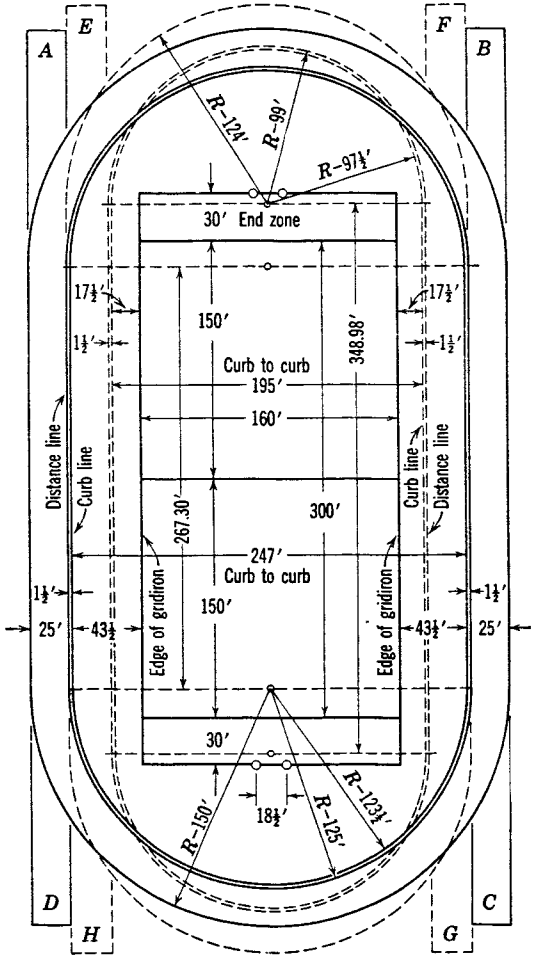

FIG. 17 Football field inside ¼-mile tracks.

Quarter-Mile Tracks

End Radius (R)		Straight Length of One Side (S)	Width of Athletic Field, Curb–Curb Face (W)	Strip Width between Grid-iron Edge and Curb Line
Measurement Line	Curb Line, Track Side			
99	97.5	348.98	195	17.5
100	98.5	345.84	197	18.5
101.5	100 0	341.13	200	20
104	102.5	333.27	205	22.5
105	103.5	330.13	207	23.5
107	105.5	323.85	211	25.5
124	122.5	270.44	245	42.5
125	123.5	267.30	247	43.5

track are A, B, C, and D for the 125-ft radius track and E, F, G, and H for the 99-ft radius. One or more of them may be used for starting lengths in the dash races so that the finish line will be in front of the grandstand. Some $^1/_4$-mile tracks are tabulated on page **2**-36 (distances in feet) Measurement of length is 1.5 ft from the track side of the curb. An end radius shorter than 99 ft does not provide a wide enough strip between the gridiron

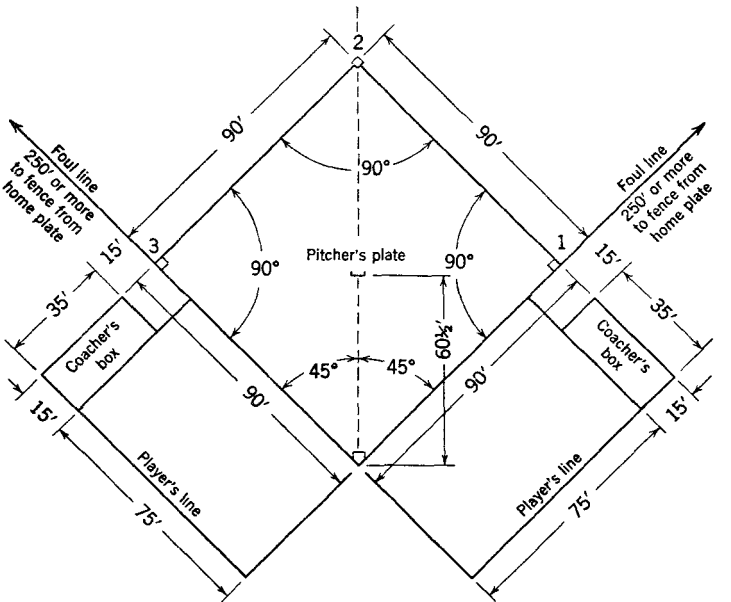

FIG 18 Baseball diamond, home base, and batman's boxes.

and the track, and a radius less than 95 ft is too short for the runners. The best radius to use is about 125 ft, as it provides more width and makes the field area more adaptable for other games.

Football Grounds should be 360 x 160 ft, and on level ground; the goal lines are 300 ft apart, but the goal posts are 360 ft apart. Lines 5 yd apart and parallel to the goal lines are laid out to aid in estimating distances made in the plays. The goals are placed in the center of the ends of the field; the goal poles are 18 $^1/_2$ ft apart, and the horizontal cross bar 10 ft above the ground. It is of importance, if a good football field is to be made, to screen the surface material through a half-inch screen. Also, the field should be mulched to support a good turf. Further details are given in Fig. 17.

A Baseball Diamond is laid out as shown in Fig. 18. The size of the field itself is limited only by the space available, but should be not less than 350 ft square. The regulation field must have a batting range of 250 ft along the foul line, and preferably should have 300 ft. Also, there should be no obstruction within 60 ft of the home-plate corner of the

diamond, a clear space of 90 ft being preferred; a backstop closer than 60 ft to the home plate requires a ground rule and special building for protection of spectators. The ground should be as near level as possible; there must not be more than a 15-in. fall from the

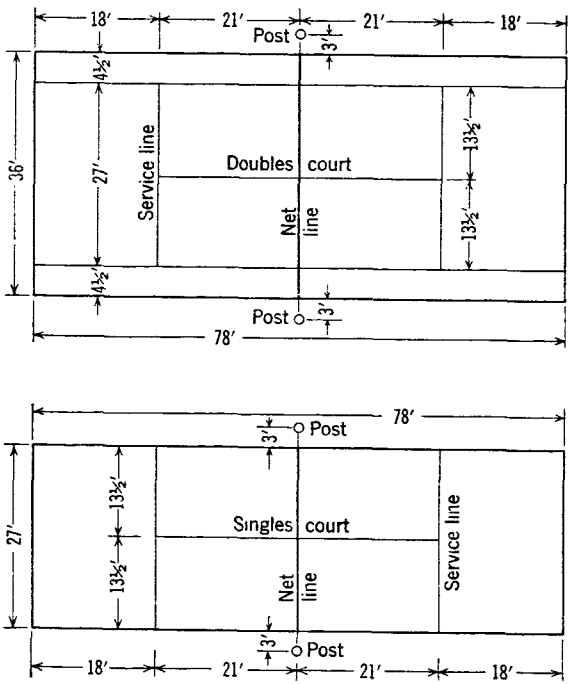

FIG. 19 Tennis courts.

pitcher's box to the base lines or to home plate, and the base lines should be level. The official junior diamond for boys under 16 years is 82 ft square and has a pitching distance of 50 ft, although younger boys frequently use a 75-ft diamond and 45-ft pitching distance, with a batting range of 200–250 ft.

The official softball diamond size is 60 ft square, with a 43-ft pitching distance. Girls, women, and small boys sometimes use a 45-ft diamond and a pitching distance of 35 ft.

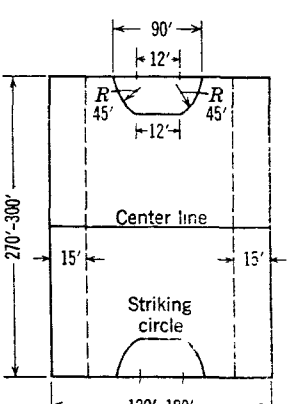

FIG. 20 Field hockey ground.

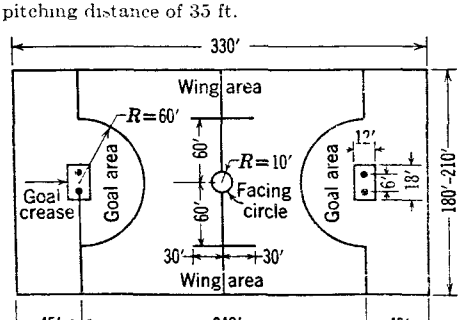

FIG. 21 Lacrosse field

Tennis Courts are laid out as shown in Fig. 19. An area of 120 x 66 ft is about the minimum for a good tennis court. It should have surface drainage provided by grading

the court on a slight slope (about 6 in. per 100 ft) from the center toward the sides. The net posts are placed 3 ft outside the side lines, and are 3.5 ft high; the net should be 3 ft high at the center.

Field Hockey grounds are laid out as shown in Fig 20. The length is 270 to 300 ft, but the width must be not less than 120 ft nor more than 180 ft; always 180 ft, if possible.

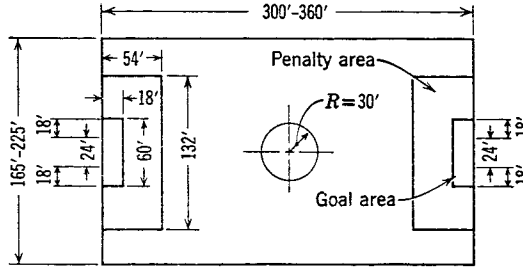

FIG 22. Soccer field

In Lacrosse, as shown in Fig. 21, the field is rectangular, the standard total length being 330 ft and the standard width 180 to 210 ft. Two pairs of goal posts are set symmetrically in the field 240 ft apart. Posts in each pair are 6 ft apart with the cross bar between set at a height of 6 ft.

In Soccer the field is rectangular, not less than 165 ft nor more than 225 ft wide, and not less than 300 ft nor more than 360 ft long. The field is marked and goals are located as in Fig. 22.

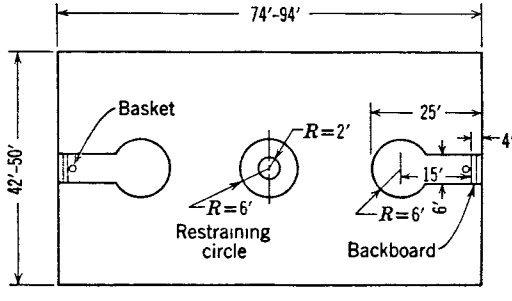

FIG 23. Basketball court

In Basketball the rectangular court is not less than 42 ft nor more than 50 ft wide and not less than 74 ft long. The field is marked and the goals are located as in Fig. 23. The two hoops are 18 in. in diameter, and on each one a net is hung. Top of hoop is 10 ft above the floor.

Badminton Courts are rectangular and are laid out as in Fig. 24. The net extends 10 ft on each side of the central line of the courts and at right angles to it; it is 5 ft high at the center and 5 ft 1 in. at the posts.

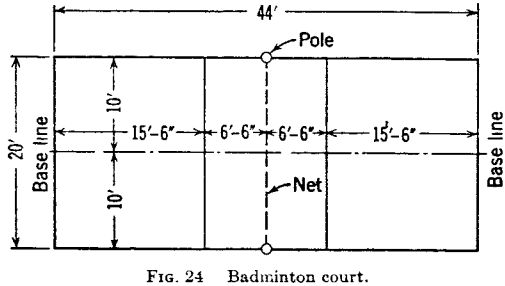

FIG. 24 Badminton court.

In Quoits the rink should be about 80 ft long and 25 ft wide. The pitching distance is 54 ft. Sticky pottery clay should be used around each iron pin so that the quoit will stay where it lands.

PROPERTY SURVEYS [1]

By Sol A. Bauer

For simplification, the term "property surveys" will be used to define this field, also referred to as "land surveys" and "boundary surveys." Although property surveying is based upon the same mathematics, and uses the same instrumentation, as that used in engineering surveying, there is the fundamental difference that property surveying requires, in addition, a thorough knowledge of the laws of boundaries and the location of the various sources of public records, access to, or ownership of, volun records of old surveys; and a knowledge of local history and local customs of prope. irvey practice, both present and past.

20. GENERAL PROCEDURE FOR PROPERTY SURVEYS

Every parcel of land whose boundaries are to be surveyed should be made to conform with the recorded title boundaries of such land. The surveyor, prior to making such a survey, should acquire all available data, including deeds, maps, certificates of title, street center lines, and other boundary-line data in the vicinity. He should compare and analyze all the data he has obtained, and make the most nearly correct theoretical legal determination possible of the position of the boundaries of the parcel in question. He should then make a field survey, traversing and connecting all available monuments appropriate or necessary for the location, and coordinate the facts of this field survey with the predetermined theoretical analysis. After careful adjustment of the various calls of the descriptions and titles to actual conditions as obtained in the field, monuments marking the corners of the parcel in question should be set Since boundary determinations are essentially legal problems, and since laws governing such matters vary considerably in different localities, it is necessary that each boundary survey be given individual study in the light of its own complications, and as affected by the laws and statutes of the locality in which the property is situated.

21. LEGAL ASPECTS OF PROPERTY SURVEYS

The task of the property surveyor is more often the reestablishment of old boundaries rather than the fixing of new ones. In either case a knowledge of the laws of boundaries is essential. This subject can be treated in this book only in the briefest terms, and will necessarily be limited to a listing of a few of the more important legal aspects of property surveying. The laws of boundaries of the United States are in a large measure based upon old English common law, augmented, supplemented, and sometimes contradicted by the precedence of innumerable court cases.

Function of the Land Surveyor. The primary function of the property surveyor is that of fact finder. Under normal conditions, in resurveys of land, he does not have the right to fix disputed boundaries, correct contradictions in deeds, or even interpret the legality of situations. Disputes can be settled only by the courts, but voluntary agreements to correct existing errors can be handled by the legal processes of new conveyances. However, the opinions of a practicing property surveyor of good repute often are sufficiently regarded, in an entirely extralegal sense, to the extent that his opinion is accepted by title companies, other owners, lawyers, and the courts themselves.

Field Notes and Survey Plat. The notes that are the record of the work of the survey, and the plat of survey that constitutes the record of the final result of the survey and the opinions of the surveyor, should in all cases be so complete as to indicate fully the manner in which the work was performed, the degree to which the investigation was carried, and the reasons for the opinions favored by the surveyor.

Right of Entry. The property surveyor ordinarily does not have the right of entry upon property adjacent to that of his client. Where it is necessary to enter upon adjacent lands, greatest care should be maintained to avoid damage to the trespassed lands.

[1] With grateful acknowledgement to David G Knapp, U. S Coast and Geodetic Survey, for suggestions on surveys using the magnetic needle, and to W H Richards, U S Bureau of Land Management, for suggestions used in Art 23, "United States Public Land Rectangular Surveys."

Control of Conflicting Elements. In theory, the problem of the surveyor in a resurvey is "to retrace the footsteps of the original surveyor." Therefore, physical marks left undisturbed by the original surveyor tend to have priority over and above the calls of the deeds drawn from the original survey. Also, the locations of the original monuments become of prime importance in resurveys, and the recorded bearings and distances become secondary. However, because of the increased accuracy of modern instrumentation and procedures, combined with the relative instability of ordinary monuments, there is a growing tendency for courts to give greater weight to course and distance, especially where there is clear indication that the work was performed with care and skill sufficient to justify the change from the historic preference for physical monumentation.

Proration. In theory, one of the first steps of the surveyor engaged in a resurvey is to calibrate his tape to that of the original surveyor, particularly in resurveys of subdivided lands Assuming that a reasonably constant relationship can be determined between the length of lines as measured in an original survey with modern remeasurements, it is entirely proper that intermediate points of that survey or subdivision be reestablished by a uniform proration of any excesses or shortages that may be found. Except for the rules established by the U. S. Bureau of Land Management (formerly General Land Office) for the resurvey of the public lands (see Art. 23), proration should be regarded as a method of calibration rather than as a general method for the elimination of error or discrepancy in measurement. It is generally true that large variations between deeded and actual lengths of lines are not properly subject to proration, because such variations ordinarily result from erroneous monuments, shifted lines, or mistakes that cannot be adjusted or corrected under the theory of calibration and proration. The simplicity of the proration method for the distribution of error has often tempted engineers and surveyors to use it erroneously, thereby compounding property-survey problems.

Intent. When land and boundary records are doubtful or contradictory, the surveyor must not arbitrarily interpret them in a manner favorable to the interests of his client. Instead, he must remain unprejudiced and try to discover, as would all courts of law, the original intent, or "the expressed intention of the parties gathered from all parts of the instrument . . . and read in the light of existing conditions and circumstances." [1] Of all the general legal principles applicable to property surveying, this is perhaps the most important.

Occupation. The underlying theory of "retracing the footsteps of the original surveyer" has given importance to what is ordinarily called "occupation" by surveyors. Occupation can be anything from a precisely monumented line to such things as fences, hedgerows, a swath of trees, or even the edge of a plowed field. Lacking any other indication of the intent of the conveyance, it sometimes becomes necessary, particularly in rural communities, to fall back upon such "occupation" as may exist. It should be emphasized, in cases of boundary-line determination by occupation, the plat or survey should indicate clearly that occupation was the basis of the determination. Also, the careful surveyor will limit his responsibility to the statement that such occupation was the best evidence obtainable and that, in his opinion, land ownership should correspond with the occupation, at the same time carefully explaining the manner in which the occupation differs from the calls of the deed descriptions. It is further advisable that the surveyor, if he fails in every effort to determine the property line by some more precise method, indicate to the neighboring owners on each side of the line in question, the line that the surveyor plans to recommend, and obtain their mutual agreement for his fix of the line. A good method for legalizing such an agreement is to get the adjoining owner to sign the plat or survey, endorsing and agreeing to the lines shown thereon.

The ease with which occupational boundaries can be located, together with the difficulties encountered in making a thorough search of the records of adjoining properties (frequently necessary in ascertaining the true position of a property line), often result in the arbitrary and immediate use of occupation, however hazy and ill-founded it may be. It is a fundamental precept of boundary law that occupation alone should be used only as a last resort, particularly if the occupation is not in agreement with the calls of the deeds.

Adverse Possession. Adverse possession of another person's land, when accompanied by certain legally prescribed acts and circumstances, may result, through court action, in the vesting of title in the name of the possessor. The origin of this form of title acquisition is found in old English common law, where it resulted from a sociological change, marking the end of the feudal system of England There are many requirements that limit and make difficult the matter of acquiring land in this manner. The mere fact of adverse possession does not convey title, transfer of title because of adverse possession can be finally effected only by court action.

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[1] *Penley* v *Emmons*, 117 Maine 108, 102 Atl 972, citing *Perry* v *Buswell*, 113 Maine 399, 94 Atl 483.

Priority of Deed. The increasing use of title companies, for the purpose of searching title and certifying as to the validity of the title to any parcel of land, has brought about an increased importance to the calls of the deeds of record. Because title searches are based primarily on the evidence in recorded, written documents, and are only occasionally supplemented by actual ground surveys, the specific calls of the written document have become increasingly important. It is being increasingly recognized that property surveys should be based mainly on the public records, at the same time recognizing all conditions adverse or contradictory to the deed of record, and publicly noting any physical variations found by the actual ground survey. Conflicts that are the result of faulty records should be corrected by proper legal action that will bring the records into agreement with the accepted physical conditions. The surveyor is not legally qualified to change or contradict the record document.

Riparian Rights. Owners of lands fronting upon water courses, lakes, streams, seas, or other bodies of water have, under certain conditions, riparian rights, sometimes limited to access to and use of the body of water, and sometimes extending to actual title of the lands in the bed of the body of water. The shifting of such bodies of water presents problems of ownership that are too highly specialized to be discussed herein. It is sufficient to state that, in general, there are three types of water-boundary changes: avulsion, erosion, and accretion. Avulsion, in simplest terms, is the change in the area or boundaries of a tract caused by a sudden change in the course of a body of water; accretion is the slow and almost imperceptible action that causes the land in question to be increased in size; and erosion is the slow, almost imperceptible action that causes the land in question to be reduced in size.

Legal Descriptions. Land descriptions in deeds, and other legal documents, are called legal descriptions, although in their present forms many of them have often become highly technical engineering documents. The law recognizes various priorities of control in the calls of deeds, but such priorities are occasionally reversed by modern practice. In any attempt to determine the position of property lines, a thorough study of all related deeds and legal descriptions is essential. This study will often include the chain of title, formed by the succession of deeds, beginning with the original owner, of whose property the one in question was once a part. The adjoining titles should be studied similarly, and due consideration should be given to the priority of the senior title over the junior title out of the original owner in common.

Contradictions. As in most matters of law, the basic theory of property-boundary law often is contradicted by specific court decisions, and no conclusions of importance as to property-line location should be made without a very careful analysis of the facts, weighed against the laws of boundaries applicable to the case. It is further to be noted that the laws of boundaries often vary from state to state, and it is therefore necessary for the surveyor to become familiar with the laws and applicable court decisions of each community in which he operates.

22. TECHNICAL STANDARDS FOR PROPERTY SURVEYS

Property surveying has been practiced in the United States since its settlement, but, until recently, no nationally established standards for property surveying had been developed. In 1946, after three years of work by several of the leading surveyors throughout the United States, a document entitled "Technical Standards for Property Surveys" was approved and adopted by the American Congress on Surveying and Mapping.[1] These standards, which establish general principles of practice, rather than detailed procedures, have been subsequently approved by several state and local engineering and surveying societies.

23. UNITED STATES PUBLIC LAND RECTANGULAR SURVEYS

That part of the United States acquired by the Federal Government by cession from the states, treaty and purchase, now embraced in the states of Alabama, Arizona, Arkansas, California, Colorado, Florida, Idaho, Illinois, Indiana, Iowa, Kansas, Louisiana, Michigan, Minnesota, Mississippi, Missouri, Montana, Nebraska, Nevada, New Mexico, North Dakota, Oklahoma, Ohio, Oregon, South Dakota, Utah, Washington, Wisconsin, Wyoming and the Territory of Alaska, comprised the original public domain. The rectangular

[1] Copies may be obtained from the American Congress on Surveying and Mapping, Box 470, Benjamin Franklin Station, Washington 4, D. C.

system of surveys has been extended over this area as the basis for the identification, administration, and disposal of the public lands.

The rectangular system of surveys was inaugurated by the Ordinance of May 20, 1785, of the Continental Congress. Important constructive modifications of the system were made during the period from 1785 to 1832. While numerous refinements of practice and improvements of methods have been introduced during subsequent years, the basic elements of the system are unchanged. The surveys in the continental United States are largely completed, approximately 116,000,000 acres remaining unsurveyed, mostly in the Pacific Coast and Rocky Mountain states. Basic legislation provided that:

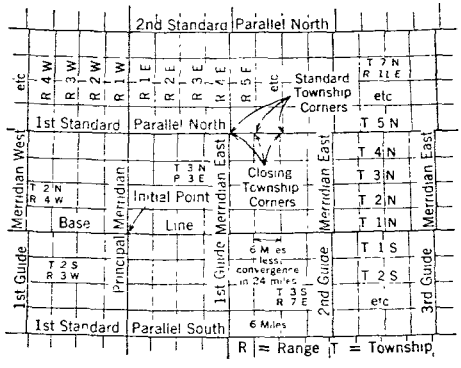


FIG 25 Subdivision of 24-mile tract into townships

First The public lands shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles, so as to form townships of six miles square, unless where the line of an Indian reservation, or of tracts of land heretofore surveyed or patented, or the course of navigable rivers may render this impracticable, and in that case this rule must be departed from no further than such particular circumstances require

Second The corners of the townships must be marked with progressive numbers from the beginning; each distance of a mile between such corners must also be distinctly marked with marks different from those of the corners

Third The township shall be subdivided into sections, containing as nearly as may be, six hundred and forty acres each, by running through the same, each way, parallel lines at the end of every two miles; and by making a corner on each of such lines at the end of every mile [1] The sections shall be numbered, respectively, beginning with the number one in the northeast section, and proceeding west and east alternately through the township with progressive numbers till the thirty-six be completed.

The following is a general outline of the survey procedure.

1. The establishment of independent initial points, each to serve as an origin for surveys to be extended from such points.

2. The survey of principal meridians and base lines originating at the initial points.

3 The establishment of guide meridians initiated at base lines and of standard parallels initiated at principal meridians at intervals of 24 miles. (Earlier practice 30 to 36 miles)

4. The subdivision of the 24-mile quadrangles into townships by the establishment of range lines as true meridians and township lines as east and west lines at intervals of 6 miles. The townships are designated by numbering them according to the plan shown in Fig. 25

5. The subdivision of townships into 36 sections by running parallel lines through the townships from south to north and from east to west at distances of 1 mile, numbered from 1 to 36, according to the plan shown in Fig. 26.

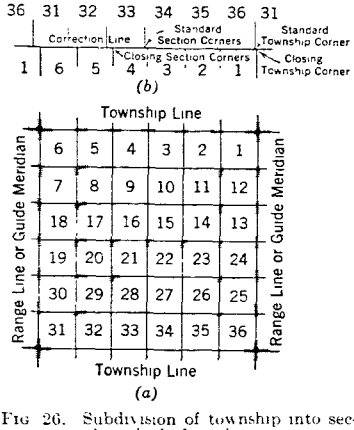


FIG 26. Subdivision of township into sections and method of marking corners

Principal and guide meridians and range lines conform to the true meridian, extending north or south from the appropriate beginning corner Base lines and standard parallels conform to the true parallel of latitude extended east or west from the appropriate beginning corner. The guide meridians and range lines converge. The extent of convergency is kept within workable limits by terminating these lines on the standard parallels normally at 24-mile intervals.

[1] Authority for the establishment of section lines at intervals of 1 mile was contained in the Act of May 10, 1800 (2 Stat 73), and the Act of April 29, 1950

The section lines are usually surveyed from south to north and from east to west in order to place the excess or deficiency of measurement on the north and west side of the township. The meridional lines usually are run parallel to the east boundary for normal distances of 5 miles from the south boundary. The remaining portion of each meridional section line is run as a random line, to connect with corners on the northern exterior boundary of the township. The east-west lines are also run as random lines, connecting with previously established corners on the meridional section lines, and, for the western tier of sections in each township, connecting with corners on the western exterior boundary. A random line is always recorded in the field notes and shown on the official plat as a true line. Also, when a random line through brush or timber deviates significantly from the true line, a re-run is made for the purpose of blazing the true line. This plan results in 25 regular sections containing 640 acres. The sections along the north and west boundaries are subdivided on a plan which provides for lots to absorb the fractional areas resulting from excess or deficiency in measurement and the convergency.

Meandering. All navigable bodies of water and other important rivers and lakes are segregated from the public lands by the traverse of the margin thereof at ordinary high-water elevation. Such a traverse is termed a meander line. It is a recognized legal principle that meander lines are not boundaries but are run for the purpose of defining the sinuosities of the bank or shore line and as a means of ascertaining the quantity of the land embraced in the survey. The actual bank or shore is the boundary, and when the high-water mark changes gradually, ownership changes with it.

Corner Monuments. Permanent marking of the lines and corners of the rectangular surveys has been accomplished by the use of durable native materials for the corner monuments during the period from 1785 to 1910. Stone monuments were used when available In timbered regions, wood posts were set. In prairie regions, charred stakes, stone, charcoal, or distinguishable material were used. In all cases, accessories such as mounds of stones, pits, bearing trees and objects are established. Since 1910, a standard iron post with brass cap for appropriate marking has been used for monumenting the public land surveys.

Resurveys. Under certain conditions Congress has authorized resurveys of the public lands. The legislation contemplated that the necessity for retracement and remarking lines of the older rectangular surveys should be demonstrated and that all land owners and other interested parties should be notified. A resurvey involves the re-running and re-marking of the lines of an original survey. In the words of the U. S. Supreme Court in the case of *Cragin* v. *Powell* (128 U. S. 691):

> The making of resurveys or corrective surveys of townships once proclaimed for sale is always at the hazard of interfering with private rights, and thereby introducing new complications A resurvey, properly considered, is but a retracing, with a view to determine and establish lines and boundaries of an original survey, .

Where federally owned lands are involved, the identification and proper monumentation of the public land survey lines is often required. The rules of resurvey procedure are in harmony with leading court decisions and are intended to protect the valid rights of the owners of privately owned lands as well as to identify the boundaries of the public lands.

Field Notes and Plats. The written record of the public land surveys consists of the field notes and plats. The preliminary field notes are kept by the engineer or surveyor in pocket field tablets. The final field notes are transcribed in standard form and arrangement from the field tablets, and, upon approval by the appropriate supervising officer, become the official record. The plat is prepared from the field notes, and in addition to showing directions, lengths of lines, and topography, provides designation and area for all parcels of public land shown thereon. Ordinarily an original survey of public lands does not ascertain boundaries; it creates them The running of lines in the field and the preparation of the field notes and plat are not in themselves sufficient to constitute official survey. Such survey does not become official and effective for designation and disposal of the public lands until accepted by the Director of the Bureau of Land Management.

The field notes and plats of the public land surveys are on file in the Bureau of Land Management, U. S. Department of the Interior. Photostatic copies of these records may be secured from that agency at established prices The original plats and field notes in the following states have been transferred to the state authorities and in nearly all cases are filed in the state capitals: Alabama, Arkansas, Florida, Illinois, Indiana, Iowa, Kansas, Louisiana, Michigan, Minnesota, Mississippi, Missouri, Nebraska, North Dakota, Ohio, Oklahoma, Wisconsin.

Restoration of Lost Corners and Subdivision of Sections. The sections are not ordinarily subdivided in the field in the execution of the public land surveys, the government's function being fulfilled when the section boundary monuments are established and the

official record in the form of field notes and plat prepared and approved. The function of the private surveyor or engineer is to identify the sectional subdivisions which have passed into private ownership. His work will usually involve the location and recovery of the original official corners and the subdivision of the section into fractional units shown upon the official plat.

The general rules controlling the location of patented lands are as follows:

1. That the boundaries of the public land surveys, when approved and accepted, are unchangeable.
2. That the original corners must stand as the true corners which they were intended to represent, whether or not in the places shown by the field notes.
3. That lost or obliterated corners are to be restored to their original locations.

A lost corner is one whose position cannot be determined, beyond reasonable doubt, either from traces of the original marker or from acceptable evidence or testimony that bears upon the original position, and whose location can be restored only by reference to one or more interdependent corners. The restoration of a lost corner is based upon the principles of proportionate measurement between the nearest known corners of the original survey. This subject is discussed in detail in Chapter V of the *Manual of Surveying Instructions*,[2] and in a pamphlet entitled *Restoration of Lost or Obliterated Corners and Subdivision of Sections*.[2]

The method to be followed in the subdivision of a section into quarter sections is to run straight lines from the established quarter-section corners to the opposite quarter-section corners; the point of intersection of the lines thus run will be the corner common to the several quarter sections or the legal center of the section.

Preliminary to the subdivision of quarter sections, the quarter-quarter- or sixteenth-section corners will be established at points midway between the section and quarter-section corners, and between the quarter-section corners and the center of the section, except on the last half mile of the lines closing on township boundaries, where they should be placed at 20 chains, proportionate measurement, counting from the regular quarter-section corner.

The quarter-quarter- or sixteenth-section corners having been established as directed above, the center lines of the quarter section will be run straight between opposite corresponding quarter-quarter- or sixteenth-section corners on the quarter-section boundaries. The intersection of the lines thus run will determine the legal center of a quarter section.

In applying the rules for the subdivision of sections when the lands are in private ownership, the surveyor or engineer should keep in mind that disputes over land boundaries must come before the local courts unless settled by agreement between the parties. The laws of the state and the decisions of its courts must be considered, particularly in matters of agreement between adjoining owners, acquiescence, and adverse possession.

24. MAGNETIC-COMPASS SURVEYS

Surveys with the magnetic compass, or with the compass needle of a transit, do not meet modern survey standards; they are included in this chapter primarily for an understanding of the proper use of compass bearings in making retracement surveys of old compass surveys. Magnetic-needle bearings are read only to the nearest quarter of a degree, any attempt at higher precision being vitiated by the daily variation. In many places, the local irregularities in magnetic declination cause further discrepancies.

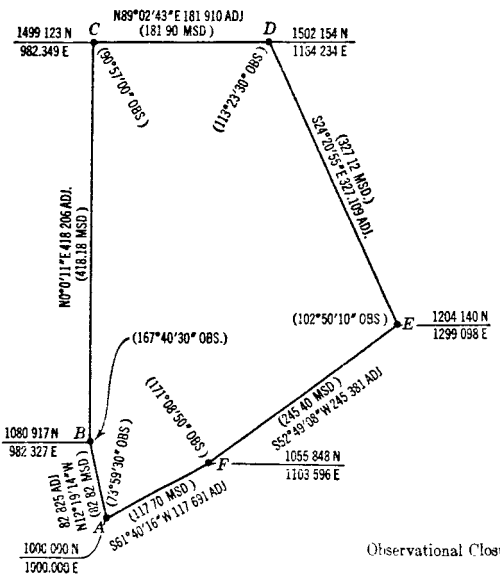
In re-running old lines, the recorded compass bearing theoretically must be modified by the amount of secular change in declination that has taken place since the bearings were first determined. Tables of secular change are issued regularly by the U. S. Coast and Geodetic Survey. The tables extend back to colonial times throughout the United States. However, the difficulty of determining the date of the original compass observation, the unknown quantities of that observation (its accuracy, its application to the particular line, the skill of the observer, etc.), in addition to the fact that many so-called compass bearings in early deeds were often merely assumed bearings and often merely carried over from earlier or neighboring deeds of equally questionable origin—all these factors limit the strict application of the theory of magnetic variations listed above. The surest method of reestablishing old lines with compass bearings is to observe one or more well-defined lines of the old survey, and calculate from those observations the change

[2] Originally prepared by Bureau of Land Management, available from Superintendent of Documents, Government Printing Office, Washington 25, D. C. Also available is a pamphlet called *Specifications for Descriptions of Tracts of Lands for Use in Executive Orders and Proclamations*

of bearing due to the various factors of change, always bearing in mind that an angular accuracy of a quarter of a degree is probably the best that can be expected of old compass readings.

25. TRANSIT AND TAPE TRAVERSE

The transit-and-tape method is ordinarily used for measurement of traverses in property surveys. A traverse may run from one pair of fixed geodetic (or control) monuments to another pair of fixed control monuments, which type of traverse will herein be called a "position closure", or the traverse may run from some point of beginning, in a closed circuit back to the same point of beginning, which type of traverse will herein be called a "loop closure."



	Observed Angles	Adjusted Angles
A	73°59′30″	73°59′35″
B	167°40′30″	167°40′35″
C	90°57′00″	90°57′05″
D	113°23′30″	113°23′35″
E	102°50′10″	102°50′15″
F	171°08′50″	171°08′55″
	719°59′30″	720°00′00″

$180°(N-2) = 720° = \text{true}$

$\text{Angular error} = \frac{30''}{6} = 5'', \text{ angle}$

Observational Closure

			cos	sin	Lat. N	Lat. S	Dep. E	Dep. W
AB	N12°19′25″W	82.82	0.9769577	0.2134399	80.912			17.677
BC	north (assumed)	418.18	1.0	0.0	418.180			
CD	N89°02′55″E	181.90	0.0166041	0.9998621	3.020		181.875	
DE	S24°20′40″E	327.12	0.9110838	0.4122212		298.034	134.846	
EF	S52°49′05″W	245.40	0.6043481	0.7967204		148.307		195.515
FA	S61°40′10″W	117.70	0.4745577	0.8802444		55.855		103.002
		1373.12			502.112	502.196	316.721	316.794
						502.112		316.721
						0.084		0.073

$$\text{Error of closure} \begin{cases} \sqrt{0.084^2 + 0.073^2} = 0.111 \\ \frac{0.111}{1373} = \frac{1}{12{,}400} \end{cases}$$

	Lat Adj	Dep Adj.
	$\frac{0.084}{1373} = \frac{0.00612}{100}$	$\frac{0.073}{1373} = \frac{0.00532}{100}$
×0.83	−0.005	−0.004
×4.18	+0.026	+0.022
×1.82	+0.011	+0.010
×3.27	−0.020	+0.018
×2.45	−0.015	−0.013
×1.18	−0.007	−0.006
	0.084 ✓	0.073 ✓

FIG. 27 Sample calculation, loop closure and area.

Adjusted Traverse

	N	S	E	W	Dep ÷ lat. = tan α	α = Bearing Angle	cos α	sin α	Length = $\frac{\text{lat.}}{\cos\alpha}$ or $\frac{\text{dep.}}{\sin\alpha}$
AB	80.917			17 673	0 2184090	12°19′14″	0 9769691	0 2133809	82 825 = $\frac{\text{lat.}}{\cos\alpha}$
BC	418 206		0.022		0.0000526	0° 0′11″	1 0000000	0 0000533	418.206 = $\frac{\text{lat}}{\cos\alpha}$
CD	3 031		181.885		60.0082481	89°02′43″	0.0166623	0.9998612	181.910 = $\frac{\text{dep}}{\sin\alpha}$
DE		298 014	134.864		0 4525425	24°20′55″	0.9110538	0 4122875	327.109 = $\frac{\text{lat.}}{\cos\alpha}$
EF		148 292		195.502	1.3183584	52°49′08″	0.6043365	0.7967292	245 381 = $\frac{\text{dep}}{\sin\alpha}$
FA		55 848		103 596	1.8549635	61°40′16″	0.4745321	0.8802382	117.691 = $\frac{\text{dep.}}{\sin\alpha}$
	502.154	502.154	316.771	316.771					

Check Closure and Area by Double Meridian Distances

			N	S	E	W	D.M D.	D.A. = D.M.D. × Lat
BC	N0°0′11″E	418.206	418.206		0.022		0.022	− 9.20
CD	N89°02′43″E	181 910	3 031		181 885		181 929	− 551.43
DE	S24°20′55″E	327 109		298 014	134 864		498.678	+148613 03
EF	S52°49′08″W	245 381		148 292		195 502	438.040	+ 64957 83
FA	S61°40′16″W	117.691		55 848		103 596	138 942	+ 7759 63
AB	N12°19′14″W	82.825	80 917			17 673	17.673	− 1430.05
(See above for functions)			502.154	502.154	316.771	316 771		2A = 219339 81 sq ft

A = 109670 sq ft

Area from Coordinates

A	1000 000	1000.000
B	1080.917	982.327
C	1499.123	982.349
D	1502.154	1164 234
E	1204.140	1299.098
F	1055.848	1103.596
A	1000.000	1000.000

2A = 219339 81 sq ft

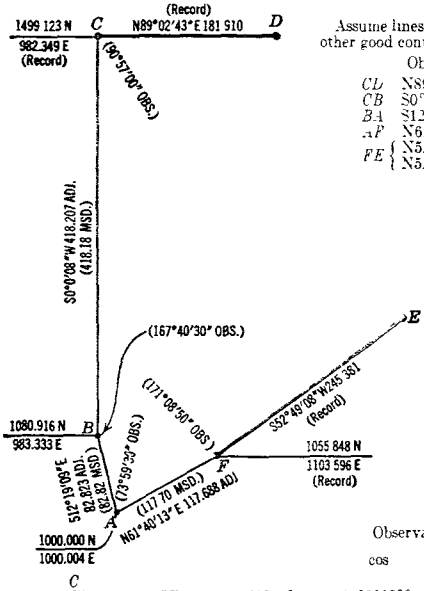
A = 109670 sq ft

Note Figures connected with heavy lines are multiplied and accumulated positively. Those figures connected with dotted lines are multiplied and accumulated negatively. The total of positive and negative accumulations equals twice the area of the figure This entire calculation can be performed on modern automatic calculators requiring only the recording of the final result.

FIG. 27. (*Continued*).

Angles, Position Closure. In position closures, the coordinates of geodetic control monuments and the azimuths between them are held fixed in accordance with the records of the control survey. Angular observations are begun from the first pair of control monuments, and each angle between the first pair of the control monuments and the final pair of control monuments is observed by transit readings, repeated a sufficient number of times to insure accuracies within the limits of the specifications for the work. A minimum angular observation with any instrument should consist of one reading with the telescope in a direct position, and one reading with the telescope in an inverted position, the mean of the two readings being used for the value of the angle, unless the mean differs from the first reading so much as to indicate a mistake in either reading.

The observed angles are added to (or subtracted from, as the case may be) the azimuth of the first pair of control monuments. The difference between the azimuth so calculated for the final pair of control monuments and the fixed azimuth of that pair of control monuments is the total angular error of the traverse. That total error divided by the number of angles observed is the correction per angle to be applied for angular adjustment of the traverse, before final adjustment. (See sample calculation, Fig 28.) This equal distribution of angular error assumes a uniform condition of observation. It is advisable to condu t field observations so as to keep all observations of uniform consistency. In



Assume lines C-D and F-E and points C, D, F, and E are fixed by geodetic or other good control.

	Observed Bearings	Corrections	Adj. Bearings
CD	N89°02′43″E record		N89°02′43″E
CB	S0°0′17″E calc.	− 9″	S0°0′08″E
BA	S12°19′47″E calc.	−18″	S12°19′29″E
AF	N61°39′43″E calc	+26″	N61°40′09″E
FE	N52°48′33″E calc.	+35″	N52°49′08″E
	N52°49′08″E record		

$$\text{Angular error} = \frac{35''}{4} = 8\ 8'' \text{ per angle}$$

Correction is clockwise, i.e., NE and SW = +
NW and SE = −

Observational Closure

			cos	sin	N	E	
C					**1499.123**	**982.349**	record
CB	S0°0′08″E	418 18	1.0000000	0.0000388	−418 180	+0.016	
B					**1080.943**	**982.365**	
BA	S12°19′29″E	32 82	0 9769536	0 2134519	−80 911	+17 678	
A					**1000 032**	**1000.043**	
AF	N61°40′09″E	117.70	0.4745620	0.8802221	−55.856	−103.602	
F					**1055 888**	**1103.645**	
		618 70			**1055.848**	**1103.596**	record
					0.040	**0.049**	

$$\text{Error of closure} \begin{cases} \sqrt{0.040^2 + 0.049^2} = 0.0633 \\ \dfrac{0\ 0633}{618.70} = \dfrac{1}{9{,}770} \end{cases}$$

	Lat. Adj.	Dep. Adj.
	$-\dfrac{0\ 040}{619} = \dfrac{0\ 00647}{100}$	$-\dfrac{0\ 049}{619} = \dfrac{0.00792}{100}$
×4.18	= −0 027	= −0 033
×5.01	= −0 032	= −0 040
×6.19	= −0 040	= −0.049

Note: In the above calculations only the figures in bold type would have to be calculated if modern automatic calculating machines are used. The individual latitudes and departures remain inside the machine where they are algebraically added, to give the resulting coordinates.

Adjusted Closure

	N	E		Lat.	Dep.	$\tan\alpha = \frac{\text{dep.}}{\text{lat.}}$	α	cos	sin	Length
C	1499 123	982.349	record							
CB				−418 207	− 0.017	0.0000406	0°00′08″	1.0000000	0.0000388	418.207
B	1080.916	983.332	calc.							
BA				− 80 916	+ 17 671	0.2183869	12°19′09″	0 9769743	0.2133572	82 823
A	1000.000	1000.003	calc.							
AF				+ 55 848	+103.593	1.8549097	61°40′13″	0 4745449	0.8802313	117.688
F	1055.848	1103.596	calc. and rec.							

Check Closure

			N	E
C			1499.123	982.349 record
CB	S0°0′08″W	418.207		
B			1080 916	983 333
BA	S12°19′09″E	82.823		
A			1000.000	1000.004
AF	N61°40′13″E	117.688		
F			1055.848	1103.596 calc. and record

(See above for functions)

Note: Above calculation indicates full calculation when performed on modern automatic calculating machine. Individual latitudes and departures remain inside of the machine where they are algebraically added to give the resulting coordinates.

FIG 28 Sample calculation, position closure

angular observations, field conditions often limit the length of some of the sights. Where one course of a traverse, for example, is one-half the length of the other courses in the traverse, it is proper to give the angle or angles affected by the short sight twice the correction of the other angles.

Angles, Loop Closure. In loop closure, all angles are similarly observed, and the angular closure is distributed as described above for position closure. In a closed loop the sum of the angles mathematically should equal 180° multiplied by $(n - 2)$, where n is the number of sides in the loop.

Horizontal Distances. Horizontal distances between all angle points are measured generally by steel tape. Usual practice is to measure each line between successive angle points separately, though railroad and highway surveys often carry a continuous measurement throughout the traverse, the length of any single course being obtained by subtracting the station values of the angle points.

Accuracies. Angular and linear measurements should be approximately equal in accuracy. Since high precisions require greater care and more time, it is often found that, in rural and sparsely developed communities, low orders of surveying accuracies are common. The rapid change of land values from low to high, that is characteristic of American communities, has demonstrated the danger of calibrating surveying precisions according to the apparent land value of the moment. The "Technical Standards for Property Surveys" (see footnote 1, Art. 22) recommends a minimum traverse accuracy (closure) of 1/10,000 for any survey involving the conveyance of land or the determination of boundaries. An even higher order of accuracy may be advisable in city areas.

Commensurate with such precision, angles should be observed to an accuracy of at least ±20″; this precision generally requires four to six repetitions of each angle with the ordinary 1′ American transit. The same precision can generally be obtained with from two to four repetitions with optical theodolites which read to 0.1′ by estimation. In general, with a 1′ transit, the mean of angles observed six times should fall within 30″ of the first reading. Optical theodolites reading to 0.1′, and using four repetitions, will generally show a mean varying not over 10″ from the first reading.

Consistent linear accuracy requires alignment of the tape ends within approximately 6 in. of the straight line, both horizontally and vertically. Linear distances should always be corrected for temperature (to the nearest 2°), horizontalness in the event of inclined measurements, and absolute length. If the tape has been calibrated, absolute length can be obtained by calculating and using the tape at a tension that would theoretically bring it to its proper length.

It is to be remembered that, because of the behavior of compensating errors, no one traverse closure, whether position closure or loop closure, is by itself a true indication of accuracy. The theory of probability indicates that, at any fixed precision, closures of more than double the true accuracy will occur in about 25% of the cases, and closures of less than half the true accuracy in approximately 18% of the cases. It is further to be remembered that loop closure is merely a measure of consistency of work rather than a measure of true accuracy, unless care has been taken to eliminate the various accumulating errors. Only an instrumental and procedural method that has been sufficiently tested to guarantee a specific accuracy can be relied upon for true accuracy.

Adjustment of Traverse. For both position and loop closure, the angular error is first distributed as described above. The traverse is then closed by calculation, using the latitude and departure method. (The latitude of the course equals the distance times the cosine of the bearing; the departure is the distance times the sine of the bearing.) Where control monuments are available, the plane coordinates of the angle points should be computed. The original calculations of a traverse will invariably disclose some error of closure, which is best adjusted in accordance with the so-called "compass rule," wherein the errors of both latitude and departure are distributed to the individual courses according to the ratio of individual course length to the total length of the traverse.

Calculations of traverses may, of course, be made by the use of logarithms and logarithmic functions. Preferred practice, however, because of the faster speed, is to use modern calculating machines, with natural seven-place or eight-place trigonometric functions.[1] Such examples of calculations as are herein shown (Fig. 27 and 28) are assumed to have been made in this manner.

After distribution of the latitude and departure errors, the adjusted bearing and distance of each course are calculated from the adjusted latitudes and departures. As a final check, it is advisable to recalculate the traverse, using the adjusted bearings and distances, to insure that both latitude and departure errors of closure have been eliminated

[1] The U. S. Coast and Geodetic Survey has compiled a table of natural sines and cosines to eight decimal places, known as *Special Publication* 231. This is for sale by the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C.

(see Figs. 27 and 28). The length of each course can be calculated either by dividing the adjusted latitude by the cosine of the adjusted bearing, or by dividing the adjusted departure by the sine of the adjusted bearing. It should be noted, however, that greater accuracy will result by dividing whichever value is the larger by its respective function, since this results in the greatest number of significant figures being used.

The problem of adjusting two or more interlocking loop closures that are not tied to control monuments is frequently encountered. It has been found that most consistent results are usually obtained by adjusting the several loops in the order of the size of their respective errors of closure. In other words, having three connected loop closures, with calculated errors of closure of, say, 1/10,000, 1/15,000, and 1/20,000, the loop having the largest error of closure, 1/10,000, is first adjusted as described above. Its bearings, distances, and coordinates are then considered fixed, and the closures of adjacent loops are recomputed as position closures. Using the recomputed closures, the remaining loops are reevaluated; the one having the largest error of closure is adjusted next, and the bearings and distances of its courses are then held fixed while the closures of any adjacent loops are recalculated, and so on. This method will tend to result in an equalizing of each successive closure, but in no case should the adjustment result in any loop being reduced in accuracy to an amount below that which is acceptable for the work.

26. AREAS OF TRACTS

Double Meridian Distance Method. Assuming that calculated latitudes and departures are available, a convenient method of computing the area enclosed by the traverse loop is by the double meridian distances. The double meridian distance is the sum of the departures of the two ends of each course of the traverse. Beginning with the most westerly point of the traverse, the double meridian distance of the first clockwise course from that point is equal to its departure. The double meridian distances of other courses equal the double meridian distance of the preceding course, plus the departure of the preceding course, plus the departure of the course itself. The double meridian distance of the last course should be numerically equal to the departure of the last course. It must be borne in mind that, in calculating double meridian distances with the traverse courses taken in a clockwise direction, east departures are positive, and west negative. All double meridian distances are considered positive.

Next, multiply the latitude of each course by its corresponding double meridian distance, thereby obtaining a series of north products and south products. The difference between the summation of the north products and the south products is double the area of the tract. (See sample calculation, Fig. 27.)

Crisscross Multiplication Method. Another method of calculating areas that is convenient when automatic calculating machines are available, and when the plane coordinates for the angle points of the traverse are available, is known as the "crisscross" method of multiplication. In this method, the north coordinate of a beginning point is multiplied by the east coordinate of the next clockwise angle point. To that is added positively the products of each successive northing coordinate, with its following easting coordinate, until the point of beginning has been reached. Then the northing coordinate of the beginning point is multiplied negatively with the easting coordinate of the next angle point in a counterclockwise motion, and so on, negatively, back again to the point of beginning. The result, which is the difference obtained by subtracting the total of the negative results from the total of the positive results, is the double area of the tract. The advantage of this system lies in the fact that the entire calculation can remain inside the computing machine with no tabulation necessary—until the final answer. If a diagram showing adjusted coordinates is used, the procedure can be followed directly from the diagram, without the tabulation of the coordinates. Its disadvantage arises from the lengthy series of multiplications, all of which must be correctly made or the entire process must be repeated. In the sample calculation given herein, Fig. 27 coordinates connected with the heavy lines are multiplied and added positively, while those connected with the dotted lines are multiplied and added negatively, resulting in the double area.

Areas by Planimeter. Areas can also be determined with relatively good precision by the use of planimeters.

Accuracy of Area Calculation. In all calculations of areas, it is to be remembered that the resultant area is accurate only to the same number of significant figures as there are in the shortest side of the tract.

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27. USE OF STATE PLANE COORDINATES

The permanence of marks or monuments of land boundaries, the finding of such monuments, the replacing of them when they have been moved or destroyed are serious matters to all surveyors and owners of land. No monument has yet been devised which is not subject to movement or destruction by the elements or the works of man. Permanence of position of a land corner can be assured (if the survey work meets prescribed standards of accuracy) by relating that position to the positions of many other monuments by mathematical means, so that, unless all are simultaneously destroyed, those which remain may be used to recover and reestablish the missing points. Such correlation of survey monuments is of particular advantage in those areas which were not originally subdivided by the United States public land surveys. The geodetic control surveys of the U. S. Government form a convenient and dependable local control survey into which the surveyor can tie his property surveys, thereby automatically establishing mathematically fixed relationships between all surveys in the region.

The state plane coordinate systems devised by the U. S. Coast and Geodetic Survey provide a plane-survey procedure for the mathematical location and description of land corners and boundaries. Information on simplified formulas for converting state plane coordinates into geographical coordinates, and vice versa, as well as data on the triangulation control available within any state or region, are available in official government publications.[1] As of 1950, 24 states had adopted enabling legislation permitting the use of state plane coordinates for property description purposes.

GEODESY AND CONTROL SURVEYS

By Lansing G. Simmons

Geodesy is the science that deals with the determination of the size and the shape of the earth. Theoretical geodesy is a complex subject and has no *direct* bearing on engineering. The engineer is concerned, however, with geodetic control surveying, which might be considered a branch of geodesy. The principal difference between geodetic control surveying and precise plane surveying is that account must be taken of the curved surface of the earth in calculating geodetic survey observations. Geodetic surveys include triangulation, traverse, astronomic observations, spirit leveling, and trigonometric leveling. These are the basis of country-wide mapping and large-scale engineering construction projects. Geodetic survey methods are used in controlling and laying out large projects such as dams and reservoir areas, bridges, tunnels, and city surveys. On local projects of this type the curvature of the earth may be neglected, and the triangulation and traverse surveys may be computed on a local plane-coordinate system, or preferably, in the United States, on the official state system of plane coordinates.

Geodetic triangulation and traverse surveys are graded according to accuracies in accordance with the criteria listed below:

Accuracy	Maximum Average Triangle Closure, sec.	Maximum Linear Closure Expressed as Ratio
First order	1.0	1:25,000
Second order	3.0	1:10,000
Third order	5.0	1:5,000
Fourth order		Anything lower than third order

In triangulation one form of linear closure is the discrepancy between the length of a triangle side as computed from a base length through a chain of triangles, and the measured length of that side, this being determinable only when there are two base lines or known lengths in the triangulation system. Another form of linear closure in triangulation is the discrepancy between the computed position and the fixed position of a tie station of established triangulation that is of equal or higher order; or, similarly, it may be the position closure developed in a loop of triangulation. In traverse the closure is the hypotenuse of the latitude and departure closures.

[1] Available from U. S. Coast and Geodetic Survey, Washington 25, D. C. For information on use of state plane coordinate systems, see U. S. Coast and Geodetic Survey *Special Publication* 235, on the state plane coordinate systems.

28. GENERAL PROCEDURE FOR TRIANGULATION

Triangulation is a series of connected, and sometimes overlapping, triangles. At least two of the angles of each triangle must be measured with utmost precision by transit or theodolite. It is preferable to measure all three angles of each triangle whenever possible, as this provides a check on the accuracy of the angle measurements, and permits the three angles to be adjusted to equal exactly 180° for each triangle. The length of one of the triangle sides must be known, and the lengths of all other triangle sides of the scheme are then computed by the sine formula. The known length may be either a newly measured base line, or it may be one of the triangle sides of a previous triangulation scheme which is tied in and used as one of the triangle sides of the new survey.

The successive steps in a triangulation survey are (1) a reconnaissance, for the purpose of determining the location of the vertices (stations) of the triangles and the location of the base lines; (2) measurement of the base lines; (3) measurement of the angles at each station; (4) adjustment of the triangle closures; (5) computation of the azimuth and length of each triangle side; and (6) computation of the latitude and departure of each triangle side, and the plane coordinates of each triangulation station.

Before starting a triangulation survey in the United States, inquiry should be made of federal and state agencies regarding the possibility of existing geodetic triangulation in the area Information on federal control surveys can be obtained from either the U. S. Coast and Geodetic Survey or the Map Information Office of the U. S. Geological Survey, both at Washington, D. C. Although government triangulation stations and lines may not be suitable in every case for use as base lengths, they will usually be helpful in supplying a starting azimuth and a state plane-coordinate position

29. TRIANGULATION RECONNAISSANCE

The purpose of the reconnaissance is to determine the location of the vertices of the triangles, or triangulation stations. In selecting triangulation stations, consideration must be given to the intervisibility of the lines over which observations are to be made; accessibility and strategic location of the station sites for best serving the project at hand; the strength of the angles that will be used in computing the lengths of the triangle sides; and the heights of any observation towers or elevated signals that may be needed. If there is no suitable government triangulation in the area, it will be necessary to provide a base line in the new scheme. The best base-line sites are usually along railroad or highway tangents. In the absence of either it may be necessary to select a site across open and fairly flat ground. Astronomic azimuths, when necessary, are generally observed over the base line.

A generalized outline of the triangulation scheme should first be plotted on the best available map of the area. A field inspection must then be made in order to determine a final location for each triangulation station. It is sometimes necessary to revise the scheme several times in order to find a system of stations that will give strong triangles and at the same time not require excessively high observing towers or sighting signals. In determining signal heights for short lines, only the intervening obstructions, such as buildings, trees, and ridges, need be considered; but for long lines the effect of the curvature of the earth must also be considered. The reconnaissance engineer must also guard against selecting lines passing near smokestacks or other heated obstacles, including large structures and hillsides which may produce differences of temperature. Such lines of sight may be subject to unusual horizontal refraction, which could be the cause of errors in the angle measurements, and hence produce large triangle closures.

The reconnaissance problems are greatly reduced in cleared, hilly country, where all lines may be tested by actually sighting over them. It is in gently rolling and timbered country where ingenuity and perseverance will be required if the best triangulation scheme is to be found. Great economy can result from a well-planned reconnaissance survey therefore the engineer performing this work should be thoroughly familiar with all phases of triangulation.

30. STRENGTH OF FIGURE

It is also necessary that the final scheme be made up of strongly shaped triangles. In computing the length of an unknown side in a triangle the two angles that are used (one opposite the known side, the other opposite the unknown side) are called the distance angles. Where these angles are near 90°, the length computation is mathematically very

strong, as the sine of such an angle changes very slowly. Conversely, when the distance angles approach 0° or 180° the length computation is very weak, as the sine then changes rapidly. As a general rule, therefore, the angles of the triangles should be within the range of 30° to 150°. The ideal scheme of triangles for an area would be made up of nearly equilateral triangles, with all angles being approximately 60°.

In addition, it is desirable to have a scheme of triangles that will make it possible to compute the length of each triangle side by two or more different routes, or chains of triangles, from the original base length. This usually may be accomplished by observing extra diagonal sights, or by adding a few extra stations, as illustrated in Figs. 29 and 30. Figure 29 shows a scheme of simple triangles in which there can be no independent check on the computed lengths of the sides (unless an additional base length is provided, as at

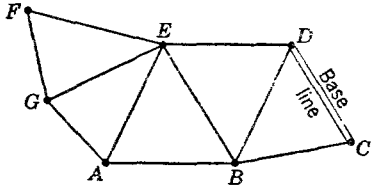

FIG. 29. Chain of single triangles.

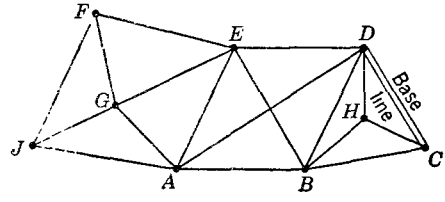

FIG 30 Chain of checked figures.

line FG). In Fig. 30, however, the diagonal sight AD and the new stations H and J, with new sights BH, CH, DH, AJ, FJ, and GJ, provide a scheme in which all lengths can be computed through two independent sets of distance angles. It is obvious that the scheme illustrated in Fig. 30 is much stronger mathematically than that in Fig. 29. The figure of four triangles bounded by stations A, B, D, and E in Fig. 30 is known as a quadrilateral. The figure of four triangles bounded by stations A, E, F, and new unoccupied station J is a four-sided central-point figure (which is not as strong as when the angles at station J are measured). The figure bounded by stations B, C, and D, with new station H in its interior, is called a three-sided central-point figure.

When several different arrangements of stations and observable sights are practical, the scheme of triangles that will result in the mathematically strongest lengths should usually be adopted. The comparison of the strength of one set of distance angles with another through a triangulation scheme can be reduced to a mathematical basis by use of the formula (or table) known as the strength-of-figure formula. Full details on the use of the strength-of-figure formula, and comparisons, are available in any standard text on geodesy.

31. TRIANGULATION BASE LINES

At least two tapes, preferably three, should be used in base measurement. These should be standardized for length and coefficient of expansion by the National Bureau of Standards, or carefully compared with a tape so standardized. To insure utmost accuracy, the tapes should be standardized in the testing laboratory under the same conditions of tension and support as will be employed in the field. Geodetic base tapes are usually 50 m long and made of a nickel and steel alloy, such as Invar or Lovar, which has a low coefficient of expansion in comparison with steel. This reduces errors resulting from poor temperature determination. Good-quality steel tapes of either the 50-m length or 100-ft length will be satisfactory for many local triangulation surveys, particularly if used on cloudy days or at night when the determination of the tape's temperature will be more reliable. A special tape-stretching apparatus, including a spring balance, is employed to bring the tape to the standard tension during the measurement

For bases measured over open ground or along the side of a highway, the first operation is to drive 2 x 4-in. stakes, at tape-length intervals, obtaining alignment by transit. If taping stakes are not convenient, portable taping tripods may be used instead. The stakes (or tripods) are used as supports for the ends of the tape, and for marking the tape-end locations. In taping over stakes or tripods, the middle of the tape is supported on line and grade by a nail driven horizontally into a stake, or by some adjustable device carried by the middle tapeman. Stakes are not necessary when the base is measured along a rail or pavement surface, but care must be taken to eliminate friction between tape and rail.

A base-taping party ordinarily consists of six men: one each for the rear and forward tape stretcher, one each for the rear and forward tape contact, one to tend the support at

the middle of the tape, and one to record. A smooth marking surface, such as a copper strip, should be fixed on the top of each stake or tripod. After tension is applied (usually 20 lb for a 100-ft tape and 33 lb for a 50-m tape) the zero mark of the tape is brought into coincidence with the starting point or contact mark on the rear stake, and a fine scribe is made opposite the mark at the forward end of the tape. Two calibrated thermometers, one being tied to the tape about 1 m from each end graduation, are read and recorded for each tape length The base line should be divided by semipermanent marks into sections from 1/2 to 1 mile in length, and each section should be measured twice, once forward and once backward, using a different tape for each measurement. In first-order work the two measurements over each section should agree within about 1:100,000, and the totals of the forward and backward measurements over the whole base should agree within about 1:300,000 The tolerances for second- and third-order base lines are about two and three times, respectively, those for first-order work.

When the base cannot be laid out in one straight line, the angle stations must be occupied with a theodolite, and the angles should be measured with about second-order-triangulation accuracy (see Art. 32, "Triangulation Angles"). When the end of a measured base line along a highway or railroad is not suitable as an angle observing station, the true station is established at an approximate right-angle offset a short distance from the end of the measured line. It will then be satisfactory to use a transit for measuring the offset angle, and an ordinary steel tape for measuring the offset distance.

An elevation must be taken on each tape support in order to provide elevation differences for reducing the slope measurements to horizontal. An ordinary wye or dumpy level, or a transit equipped with a spirit level, will give sufficient accuracy for this purpose, provided that the grades are low. In no case should the grade exceed 10% if first-order results are sought.

In calculating the base line, the measured lengths must be corrected for slope, temperature, length of tape, and reduction to sea level. For all practical purposes these corrections may be computed as follows.

$$\text{Slope correction} = -d^2/2l$$

where d = difference of elevation of the tape ends and l = length of tape.

$$\text{Temperature correction} = +Cl(t - t_0)$$

where C = coefficient of expansion of the tape, l = the length, t = the mean temperature, and t_0 = the temperature of standardization.

The correction for the length of the tape is the difference between its nominal length (e.g., 50 m or 100 ft) and its actual length at standard temperature as determined by standardization under the same conditions of tension and support as used in the field. This eliminates the need for sag and tension corrections.

In geodetic surveying all horizontal distances are assumed to be on the spheroid at sea level; hence all distances measured above (or below) sea level must be projected to the sea-level datum. This correction $= -S(h/r)$, where S is the length of base at the mean elevation h, and r is the mean radius of curvature of the earth, which may be taken as 20,900,000 ft or 6,370,000 m.

All these corrections can be easily computed with a slide rule, or taken from tables appearing in many textbooks on surveying. The accompanying sample computation includes all necessary corrections:

Computation of Base Section

Tape Lengths	Tape Readings, ft	Diff. Elev., ft	Slope Corr., ft	Temp., °F
N.B.–1	100	0.52	0.001	77
1–2	100	1.44	0.010	75
2–3	100	2.60	0 034	76
3–4	100	0 91	0 004	78
4–5	100	1 24	0.007	78
5–6	100	3.71	0 068	80
6–7	100	0.82	0.003	79
7–8	100	4.29	0 092	79
8–9	100	1.20	0.007	80
9–10	100	2 45	0.030	80
10–*A*	2.767	0.00	0 000	
	1002.767		0.256 Sum	78.2 Mean

Length of tape at std. temp. of 68°F = 99.9943

Tape corr. = −0 0057 per 100 ft

Reduction, North Base to A

	Feet
Nominal length	1,002 767
Tape corr.	$-0\ 057 = 10\ 03 \times -0\ 0057$
Slope corr.	$-0\ 256$
Temp corr.	$+0\ 066 = 0\ 00000645 \times +10.2 \times 1003$
Sea-level corr. (Mean elev. = 867 ft)	$-0\ 042 = \left(\frac{867}{20,900,000}\right) \times 1003$
Reduced length	1 002 475

If the base contains angle points, each straight portion should be projected onto the line joining the base terminal stations by multiplying its length by the cosine of the angle it makes with that line. All angles should be measured and the loop closure distributed equally among the angles

32. TRIANGULATION ANGLES

The angles of a triangulation scheme may be measured with either a good transit or a theodolite, depending upon the accuracy sought. A theodolite is essentially of the same design as a transit, but has a more accurately divided circle, microscopes for reading the circle, and a more powerful telescope. A theodolite is generally employed for first- and second-order triangulation, in which the average triangle closure is kept within 1 and 3 sec respectively. Either a transit or theodolite may be used on third-order (average triangle closure of 5 sec or less) or lower-accuracy triangulation.

A direction theodolite can be read more accurately than it can be pointed, therefore repetition of the angles for the purpose of more precise readings of the circle is not necessary, and no lower motion is provided The circle may be turned about the vertical axis, by hand, independently from the alidade. A tripod may or may not be available. If not, a specially constructed tripod built at each station is used to support the instrument. Where the theodolite must be elevated in order to obtain a clear line of sight, the supporting tripod—usually of 2 x 4-in lumber with 1 x 4-in. bracing—should be independent of the scaffolding supporting the observer

The transit or repeating theodolite should have a vernier reading to 10″ or 20″. Third-order work can be attained with a 30″ transit and first- or second-order work can be attained with a 10″ transit, provided that a sufficient number of repetitions are made on properly designed signal targets.

The design of the targets or signals upon which the theodolite pointings are made is of great importance in obtaining accurate results. The method of supporting the target is not important, but the object to be sighted on must be perfectly symmetrical and centered over the station. Ordinary range poles held in place by special tripods or guy wires will be satisfactory for relatively short lines on second- or third-order triangulation. Intersecting vanes mounted on a straight vertical pole are adequate for second-order triangulation, but the pole should be sighted on whenever possible, resorting to the vanes only for the longer lines. The best color for the target depends upon the predominant background. Yellow and orange have been found best for all types of backgrounds. For night observations an electric signal lamp is by far the best type of target for first-order work It may be a modified automobile spotlight, or even a flashlight for shorter lines.

33. ANGLE OBSERVATIONS BY DIRECTION THEODOLITE

In first-order work as many as 16 observations are made of the directions of each line of the scheme, with the telescope both direct and reversed. In order to balance the errors of graduation of the theodolite circle, measurements are made using several different parts of the circle. Since all modern theodolites have two reading microscopes 180° apart, the circle is advanced $180°/N$ between each set of observations, direct and reverse, where N is the number of sets. For the best-grade theodolites, 16 sets are usually required for first-order triangulation, 8 sets for second-order, and 4 sets for third-order.

The observer should first check the tripod or instrument stand for rigidity. An initial signal is then selected, the telescope pointed on it, and the circle set to read near 0°. Proceeding clockwise, a pointing and circle reading are made on each signal in order, until all signals have been observed upon. The telescope is then reversed, and the pointings and circle readings made in the reverse order, ending on the initial signal. This constitutes one set or position of the circle. The circle is advanced the proper interval and

the second, third, etc., sets are observed until the program is completed. Although it is impossible to eliminate completely the error of "run" of the micrometers, its effect can be substantially canceled from the results by advancing the initial micrometer reading along with the circle reading at the beginning of each set. The advance of the micrometer should be one-fourth the graduation interval, approximately. Thus, if the graduation interval is 5′, the micrometer should be advanced about 1′ 15″ between sets of observations. For example, the first four initial circle settings for a 16-position observing program with a 5′ circle graduation interval would be.

1. 0° 00′ 30″
2. 11° 01′ 45″
3. 22° 03′ 00″
4. 33° 04′ 15″

34. ANGLE OBSERVATIONS BY REPETITION METHOD

The use of a transit or repeating theodolite in triangulation requires that the angle be accumulated on the circle. The usual method is to measure the angle six times with the telescope direct, reading the circle after the sixth measurement, then measure the explement six times with the telescope reversed, again reading the circle after the sixth measurement The last reading should equal the initial reading except for errors due to pointing and imperfections of the instrument. All movements of the alidade, when the upper motion is free, should be from left to right or clockwise. This tends to eliminate the effects of drag on the clamp and vertical axis bearings. Each angle between adjacent signals should be measured, including the one necessary to close the horizon. The horizon closing error is distributed equally among the angles and the result constitutes one set. For a 10″ repeating theodolite, one set will usually give third-order results; two or three sets, second order; five or six sets, first order. The principle of advancing the circle and vernier between sets applies to repeating instruments in the same manner as for direction instruments. For second- and third-order triangulation a proportionate additional number of sets should be taken with 20″ and 30″ transits. For first-order results only a 10″ repeating theodolite should be used

35. TRIANGULATION COMPUTATIONS

It is convenient to list the results of triangulation angle observations as directions that are referred clockwise to some arbitrarily selected target or signal as 0° 00′ 00″. Any angle may then be determined by taking the difference between the two directions involved The listing of directions is done directly from observations that are made with a direction instrument. If a repeating instrument is used, a list of directions may be made up by adding successively the individual angles, choosing any station as the initial, ordinarily the one farthest to the left. The directions must next be corrected for any eccentricity of the theodolite or signal.

Eccentric Reduction. At times it is not possible to set the theodolite or target over the station mark. The observed angles must then be reduced to what they would have been had instruments and targets been centered. The problem is best solved by correcting each line or direction for the "swing" necessary to reduce it to the line between station centers. A sketch should be drawn so that the direction of the swing may be determined without mistake. In Fig. 31, A and B are the station centers and E is the eccentric theodolite position. The observed direction EB is to be reduced to the true direction AB. The amount is equal to angle θ or angle ABE. The distance d between E and A and the angle α at E between A and B are measured. The length AB or s is known approximately by scaling from a map or from a rough computation of triangle-side lengths. From the law of sines, $\sin\theta = d \sin\alpha/s$. Since θ is usually a very small angle, the expression can be written $\theta'' = d \sin\alpha/s \sin 1''$. In this case the correction is clockwise or plus in sign. The problem is the same for an eccentric signal. When all directions have been reduced to center, the corrected angles are computed from the differences of the final directions. The next step is the computation of the triangles.

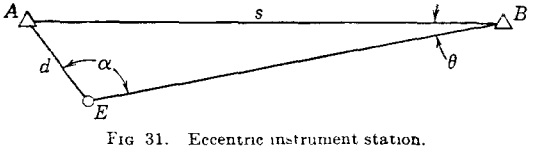

Fig 31. Eccentric instrument station.

Computing the Triangles. The sum of the three spherical angles of a triangle should equal 180° plus the spherical excess of the triangle. The spherical excess is proportional to the area of the triangle and amounts to 1″ of spherical excess per 76 sq mi of area; therefore, unless the triangles are very large, the spherical excess may be neglected. In any case the amount by which the three observed angles fail to sum to 180° is divided by 3 and applied, with the correct sign, to each angle to form a plane triangle whose angles sum to 180° exactly. If one of the stations of a triangle is not occupied by the instrument, as at J in Fig. 30, the angle at that station must be "concluded," by subtracting from 180° the sum of the other two observed angles of the triangle. The plane triangle is used to compute the unknown lengths by the law of sines: that is, the lengths of the sides of a plane triangle are proportional to the sines of the opposite angles. Thus, the lengths of the unknown sides may be computed progressively, starting from the measured base line or from a triangle side of known length from a previous survey.

If it is desirable to increase the strength of a scheme of triangulation by observing extra lines to form quadrilaterals or central-point figures, it is certainly worth while to make some sort of a computational adjustment. A rigid least-squares adjustment is the best, but a simpler method can be employed which will bring about geometric consistency. For example, consider the quadrilateral $ABDE$ in Fig 32. Number the angles as shown in Fig. 32. The numbering can start at any angle, but it must be consecutive and in the direction which completes a triangle with the first four angles.

There are four triangles in the quadrilateral, each composed of four observed angles. The sums of the sets of four angles must be made to add to exactly 180°, spherical excess neglected. The first step is to correct the angles in triangles ABD and AED by applying one-fourth the closing error to angles 1, 2, 3, 4, and 5, 6, 7, 8, as shown in columns 2 and 3. This closes the quadrilateral to exactly 360°, and it remains only to balance the other two triangles, AEB and BED, to exactly 180° by applying one-fourth the remaining closing error with one sign (minus) to angles 1, 2, 7, 8, and the other sign (plus) to angles 3, 4, 5, 6 column 4. The quadrilateral is now closed exactly and is consistent as far as the angles are concerned.

	(1)	(2)	(3)	(4) After Angle Adj.	(5) After Side Adj.
1.	28° 51 26.33″	26.03″		26 01″	29.00″
2.	46 35 08.71	08.41		08.39	05.40
3.	77 58 50.51		50.21″	50 23	53.22
4.	26 34 35.64		35.35	35 37	32.38
	180 00 01.19			00.00	00.00
	4) 1.19				
	−0.30				
5.	20 35 40.14		39.91	39.93	42.92
6.	54 50 54.68		54.45	54.47	51.48
7.	68 53 32.99	32.76		32 74	35.73
8.	35 39 53.10	52.88		52.86	49.87
	180 00 00.91	00.08	59.92	00.00	00.00
	4)00.91	4)00.08	4)00 08		
	−00.23	−00.02	+00.02		

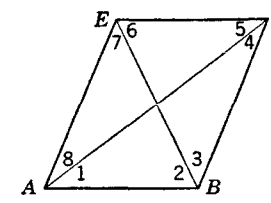

FIG. 32 Quadrilateral adjustment.

		Log Sine		Diff. 1″
1.	28° 51′ 26.01″	9.683	6131	+38.2
3.	77 58 50 23	9.990	3731	+ 4.5
5.	20 35 39.93	9.546	2348	+56.0
7.	68 53 32.74	9.969	8379	+ 8.1
		9.190	0589	106.8
				106.2
				213.0

$$\frac{636}{213.0} = 2.99$$

		Log Sine		Diff. 1″
2.	46° 35′ 08.39″	9.861	1775	+19.9
4.	26 34 35.37	9.650	6884	+42.1
6.	54 50 54.47	9.912	5579	+14.8
8.	35 39 52.86	9.765	698.	+29.4
		9.190	1225	106.2
			589	
			636	

		Adjusted Log Sine	
1	29.00″	9.683	6245
3.	53.22	9.990	3745
5.	42.92	9.546	2515
7.	35.73	9.969	8403
		9.190	0908

		Adjusted Log Sine	
2.	05.40″	9.861	1715
4.	32.38	9 650	6758
6.	51.48	9.912	5535
8.	49.87	9.765	6900
		9.190	0908

From the law of sines, the sum of the logarithms of the sines of the odd-numbered angles should equal that of the even-numbered angles. The discrepancy in the example is 636 in the seventh decimal place, the sum being smaller for the odd-numbered angles. The correction, in seconds, to each angle is obtained by dividing this discrepancy by the sum of the log-sine differences for 1″. The correction is, in this case, plus for the odd-numbered angles and minus for the even-numbered angles. Since each triangle is made up of two odd- and two even-numbered angles, the angle balance is maintained. In case a log-sine difference is minus in sign, as for angles over 90°, it need only to be treated algebraically, and subtracted in obtaining the sum of the log-sine differences.

The same general principle may be applied to central-point and other figures.

Position Computation. In triangulation of limited extent, where position is defined by rectangular coordinates, the positions of the new points may be computed by latitudes and departures as in plane surveying. Azimuths are usually reckoned clockwise from south as zero, west being 90°, north 180°, and east 270°. The coordinate position of each new station can be determined by at least two routes; therefore unless the entire network has been made geometrically consistent by some such device as a least-squares adjustment, there will be slight discrepancies in the two or more sets of coordinates. In this case an arithmetic mean should be taken and held fixed. For example, in Fig. 29, assuming the plane coordinates of stations C and D are known, the coordinates of new station B should be computed by line CB, and also by line DB. Then, after fixing the coordinate position of station B, the position of station E should be fixed by taking the mean of the two values determined by the two lines DE and BE, and so on through the entire scheme.

After the positions of all stations of a network have been so fixed, complete geometric consistency can be attained, if desired, by calculating a final plane azimuth and distance for each triangle side. This is accomplished by making an inverse computation from the final latitude and departure value for each side, which in turn are derived from the final coordinates fixed for each station.

When official governmental or state geodetic control is available within the area, the plane-coordinate system used should be the state-wide system of plane coordinates established by the U. S. Coast and Geodetic Survey.[1] In these state coordinate systems, the surface of the earth is projected mathematically upon cones or cylinders, which are then unrolled into plane surfaces, forming dependable projections or maps of the areas. In this way, a strip of the earth's surface 158 miles wide and of indefinite length can be represented on a single plane surface, and errors of scale (distance) caused by the distortion created by representing the curved surface of the earth upon a plane surface will not exceed 1:10,000, or about 1 ft in 2 miles.

It should be noted that, when working on a plane-coordinate system, no allowance need be made for convergence of meridians, but grid azimuths, and not astronomic or geodetic azimuths, must be consistently employed. On a local grid system, an observed astronomic azimuth, or a true geodetic azimuth, may be converted to a grid azimuth by applying a correction for convergence of meridians, computed (closely enough) from the formula

$$C'' = \Delta\lambda'' \sin \phi$$

in which C'' is the correction for convergence of meridians in seconds of arc, $\Delta\lambda''$ is the difference in longitude in seconds of arc between the location of the astronomic azimuth and the central meridian of the local grid system, and ϕ is the latitude of the point at which the astronomic azimuth is observed.

36. GEODETIC TRAVERSE

Geodetic traverse is essentially the same as plane traverse, the chief difference being the refinement of the equipment and procedure employed. The length-measuring equipment and procedure are identical to those for triangulation base lines. In fact, a triangulation base line is equivalent to one course of a geodetic traverse. However, certain precautions required for base-line measurement are not necessary in traverse. Usually only one precise measurement is made with the calibrated tape and special equipment, an approximate measurement by ordinary taping methods being used as a check for detecting blunders. Also, there is no limit to the size of the angle at the main angle stations in traverse work. The angle-measuring equipment is the same as for triangulation, although the theodolite need not be as large or as precise. The targets or signal lights are generally smaller, as traverse courses are almost always much shorter than triangulation lines. Range rods, or pole targets of 2 x 2-in. lumber, are satisfactory for daylight signals, and

[1] See U. S. Coast and Geodetic Survey *Special Pub.* 235, "The State Coordinate Systems."

flashlights are generally of sufficient size for night-time signals. Taping bucks (portable rigid taping tripods), or stakes driven in the ground, should be used to support the tape ends. Tapes of 100 ft or more in length should be supported in the middle. For traverse of second- and third-order accuracy the tape may be supported throughout, as on a pavement or railroad rail. In all cases the tape end supports must be leveled over to determine slope corrections.

The tape supports, either stakes or taping bucks, are first placed in position along each traverse course, using a transit for alignment, and making a preliminary measurement with a steel tape for position along the course. The precise taping is then accomplished in the same manner as for triangulation base lines. In first-order work, 50-m Invar tapes are much preferred. In second- or third-order work steel tapes may be employed. In all cases the tapes should be frequently checked against a standard tape used for that purpose only.

The observing of the angles of the traverse may follow or precede the taping. Night observations are preferred for first-order work. The number of observations of the angles is usually about half of that for the corresponding order of triangulation. Extreme care is required in centering the signals and instrument, and no eccentric set-ups should be permitted.

Where two main angle stations are intervisible, but the terrain does not permit direct taping between the points, it may be necessary to measure a broken base with supplemental angle stations and project the lengths of the supplemental courses to the main course. The azimuth for the traverse is then carried through the main angle stations. The angle between any supplemental course and the main course should be kept as small as possible, preferably under 20°.

37. TRAVERSE COMPUTATIONS

The measured lengths are corrected in the same manner as triangulation base lines, the only exception being that no sea-level reduction is necessary for surveys to be computed on a *local* rectangular coordinate system. A further correction is desirable, in high-order work, when the computation is based on a state coordinate system, this being the correction for the scale of the projection in the particular area of operation. For limited areas, such as small counties, the scale factor may be combined with the sea-level factor to obtain one combined factor for reducing lengths for these purposes.

The grid azimuths are computed through the main traverse courses, using the known starting azimuth and the observed angles When a tie is made to another known grid azimuth, the error of closure is distributed equally among the measured angles.

The x and y coordinates of a plane rectangular system are based on the conventional latitudes and departures of plane surveying. Distribution of the closure error of a single traverse line should be on a simple prorata basis.

Where the traverse lines form a network of loops, it is best to determine the positions of the junctions by some scheme of weighted means. The coordinate position of each junction should be determined several times by various routes of the traverse survey, with the weight of each determination being inversely proportional to the length of the route. The weighted mean is then held as fixed for the position of the junction. Each link in the net is then adjusted between fixed junctions. The final length and azimuth of each course are then determined from the adjusted coordinates by a simple right-triangle computation.

38. GEODETIC LEVELING

Geodetic leveling differs from ordinary leveling chiefly in the refinement of the equipment. The standard specifications for first-order leveling require that observations be made over each line in both directions, and that the two observed differences of elevation agree within the limits indicated below. These limits apply also to loop closures and to line closures between previously established first-order benchmarks. Closure limitations for the orders of leveling are as follows

Order of Leveling	Closure
First order	$4.0\ \text{mm}\sqrt{K}$, or $0.017\ \text{ft}\sqrt{M}$
Second order	$8.4\ \text{mm}\sqrt{K}$, or $0.035\ \text{ft}\sqrt{M}$
Third order	$12.0\ \text{mm}\sqrt{K}$, or $0.050\ \text{ft}\sqrt{M}$

where K is the distance in kilometers and M the distance in miles.

First- or second-order accuracy is possible only with a leveling instrument possessing a high-power telescope and a sensitive spirit level. Specifications usually call for a magnifying power of at least 40 and a sensitivity of 5″ or better per 2-mm run of the level bubble. The instrument should be provided with a fine-thread tilting screw to control the final leveling, and a device for viewing the position of the bubble at the same time the rod is read.

Two self-reading rods, in either meters or yards, with the small graduations being on Invar strips either to centimeters or to hundredths of yards, are employed in geodetic leveling. The strip is set in the face of a wooden staff, which is generally about 10 ft long.

The back of the staff is usually graduated to tenths of feet. A thermometer is set in the back of the staff, for measuring rod temperatures. A spot level is also attached to the back of the staff, to insure verticality of the rod. Steel pins about 1 ft long, driven into the ground, are generally used as turning points, except that, when leveling along a railroad track, the head of a rail spike serves well for this purpose.

It is required that the level constant C, a function of the angle between the level axis and collimation axis, be kept very small. At the beginning of each day's work this constant is checked by the "peg" method. If the level constant so determined can be reduced by instrument adjustment, the spirit level is adjusted, not the cross hairs.

Lengths of foresights and backsights should be kept very nearly equal, and their sums between benchmarks should not differ by more than 20 yd. With a high-grade leveling instrument, sights up to 150 yd may be taken when atmospheric conditions permit, which seldom occurs except on cloudy days. The normal length of sight ranges between 60 and 110 yd, slope of ground permitting. The reticule is provided with three equally spaced stadia cross hairs, with a stadia constant of about 1 in 300.

The telescope is pointed so that the vertical cross hair coincides with one edge of the leveling rod. Three rod readings are then taken, one for each of the three horizontal cross hairs, interpolation being made to the nearest millimeter or thousandth of a yard. The rod is then turned so that the back of the rod faces the instrument, and a check reading is made to the nearest hundredth of a foot for the middle horizontal cross hair. The upper and lower stadia intervals should be very nearly equal, and the foot reading on the back of the rod should check the mean of the three readings on the face of the rod, after conversion from feet to meters or yards is made. Backsights and foresights are taken first at alternate set-ups of the instrument, as this tends to eliminate any systematic error resulting from settling of the instrument.

The procedure for second-order leveling is the same as for first-order except that each line is run only once. Third-order leveling may be accomplished with the ordinary dumpy or wye level, and wooden rods of either the self-reading or target type may be used. The trend is toward the use of tilting levels and self-reading rods even for third-order work, because of the greater speed and accuracy attained.

PHOTOGRAMMETRY

By George D. Whitmore and Morris M. Thompson

39. APPLICATIONS OF PHOTOGRAMMETRY

Photogrammetry, the science of obtaining reliable measurements by means of photography, has important applications in many fields, including engineering, military intelligence, geology, forestry, and agriculture. In engineering, photogrammetry is used in preparing topographic and cadastral maps; in route-location studies for highways, railways, transmission lines, and pipe lines; in detailed site surveys for dams, bridges, power plants, airports, and other construction projects; in municipal and regional planning; and in property valuations, assessments, and acquisition. An advantage of photogrammetry over traditional ground surveys is the increased speed and facility with which mapping of large-size projects can usually be accomplished.

Photogrammetry techniques are used extensively by several agencies of the Federal Government in their mapping programs. States, municipalities, and many industries also use photogrammetry extensively. Although photogrammetry is used by many different organizations and for many different purposes, the general principles underlying its use remain the same in all cases.

The photographs used in photogrammetric work may be taken with the optical axis of the camera lens horizontal, vertical, or oblique. The utilization of photographs that

are taken from ground stations with the axis of the camera lens horizontal, or nearly so, is known as *terrestrial photogrammetry*. The principal purposes of terrestrial photogrammetry are (1) the construction of contour maps of terrain with steep slopes, where it is difficult to run a ground survey or to obtain satisfactory aerial photographs, and (2) the extension of ground survey control. The utilization of vertical or oblique photographs taken from an airplane is known as *aerial photogrammetry*. The usual purpose of oblique photography is to increase the ground area covered by each exposure. Vertical aerial photographs are used in the great majority of photogrammetric projects.

Engineering uses of aerial photographs are mainly: (1) as simple pictures of the terrain; (2) as matched groups of photographs, or mosaics; (3) as a source of information for the construction of planimetric or plane maps; and (4) as a source of information for the construction of relief or contour maps.

40. SCALE OF AERIAL PHOTOGRAPHS

A vertical aerial photograph differs from a map because (1) the negative, at the time of exposure, is usually not parallel to the ground it represents, resulting in a distorted representation of the terrain, and (2) relief of the terrain causes images on the photograph to be displaced from true map position. If the camera axis were truly vertical at the time of exposure, and if the ground were perfectly flat, then the photograph would be the general equivalent of a map of uniform scale. Assuming that the conditions of true verticality of camera axis and flatness of ground could be obtained, the following relationships would hold (Fig. 33):

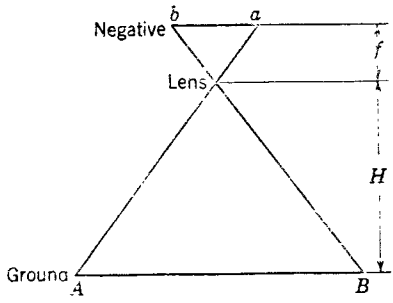

FIG. 33 Vertical photography of level ground

Let f = focal length of camera lens.
H = height of lens above ground.
AB = distance between two points on the ground.
ab = corresponding distance on the photograph.

Then scale of photo = $ab/AB = f/H$ If f is in inches and H is in feet, scale = $f/12H$. For example, if the focal length is 6 in. and the height of lens above ground is 12,000 ft, then

$$\text{scale} = f/H = 6/(12 \times 12{,}000) = 1/24{,}000$$

Ordinarily, the height of the lens above ground is not accurately known, and the scale of the photograph is not uniform because of tilt and ground relief. The approximate scale of the photograph can be determined by comparing the distance between two well-defined points on the photographs with the corresponding distance scaled from a map or measured on the ground. To reduce the effects of tilt and relief, such a scale line should pass near the center of the photograph and the two end points should be at approximately the same elevation and equally distant from the center. As an example of a scale calculation, let ab = the distance between points a and b on a photograph = 8 in., and let $a'b'$ = corresponding distance on a 1/25,000 map = 7.68 in. Then

$$\text{Photo scale} = 8/(7.68 \times 25{,}000) = 1/24{,}000$$

It should be noted that the scale of aerial photographs can be readily changed by making enlargements or reductions, called "ratioed" prints, and that this has the same effect as changing the value of the focal length (f) of the camera lens.

41. AERIAL CAMERAS

The most commonly used type of aerial mapping camera has one lens, or lens assembly, and is known as a single-lens camera. The dimensions of film negatives to be used for mapping purposes are usually 9 x 9, 7 x 7, or 7 x 9 in., these dimensions being referred to as the "format." The lens of each camera has a particular focal length whose precise value is close to but not exactly equal to the nominal value. For the same flight height, cameras having shorter focal lengths produce smaller-scale photographs than those having longer

focal lengths, (photo scale = f/H). Nominal focal lengths of 4, 5.2, 6, and 8.25 in. are commonly used in the United States for mapping purposes.

The extent of ground area photographed by a camera at a given flight height depends on the format, focal length, and angle of coverage of the lens. The angle of coverage is the central angle of the cone of light rays admitted by the lens. Figure 34 shows the coverage obtained with a camera having a 9 x 9-in. format, focal length of 6 in., and angle of coverage of 90°, assuming a flight height of 10,000 ft, no tilt, and no relief. On the plan of the negative, the solid outline indicates the area of the negative which is utilized. On the plan of the ground, the solid outline indicates the ground area recorded on the photograph. On both plans, the dashed circle indicates the total area included in the field of the lens, but only that portion within the solid outline is recorded on the negative. A similar figure can be constructed for any camera whose focal length, format, and angle of coverage are known.

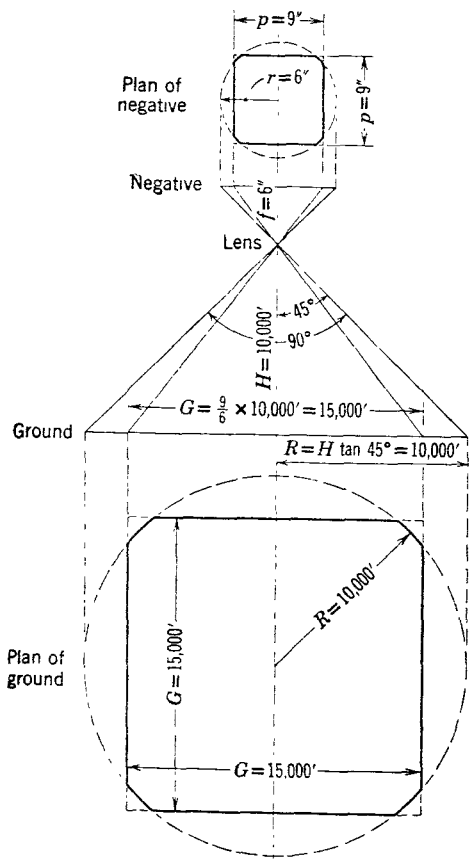

FIG. 34. Relation of ground coverage to flight height, focal length, angle of coverage, and format

Cameras having a longer focal length have, as a rule, either a smaller angle of coverage or a larger format, usually the former, since formats larger than 9 x 9 in. are inconvenient to handle. Thus, for a given flight height, longer-focal-length photographs have a larger scale but usually cover a smaller area.

Cameras having a 9 x 9-in. format and 8 1/4-in. focal length have an angle of coverage of approximately 72° and are known as normal-angle cameras. Cameras having a 9 x 9-in format and 6-in. focal length have an angle of coverage of approximately 90° and are known as wide-angle cameras. Other commonly used wide-angle cameras have a focal length of 5.2 in. with a 9 x 9-in. format, or a focal length of 4 in. with a 7 x 7-in. format.

Single-lens cameras consist of three main parts: the film magazine, the camera body, and the lens cone. In precision-type cameras, the lens cone usually cannot be detached from the camera body in the field, and the lens is locked in place in the lens cone by means of a dowel pin. These nonremovable features serve to insure against any change in the relation of the lens and lens axis to the plane of the negative.

Present-day mapping cameras are instruments of high precision. The lens must meet rigorous standards with respect to distortion, resolution, and other optical properties. The lens and other precision parts of the camera must be manufactured and assembled with great care. Provision must be made to hold the film perfectly flat in the focal plane at the instant of exposure. This can be accomplished either by a vacuum or a pressure system. In the vacuum system, the film is held flat by means of a perforated plate that forms one side of a shallow box and fits against the back surface of the film. The perforated plate is connected to a tubing through which a partial vacuum is formed in the box and the atmospheric pressure on the front of the film pushes it firmly against the plate. In the pressure system, air under pressure is employed to flatten the film against a focal plane backing plate. In either system, the pressure is released after each exposure so that

the correct amount of film for the next exposure may be unwound from one spool and wound on the other. The camera must be so assembled that the focal plane is exactly perpendicular to the optical axis. The shutter is usually mounted between components of the lens so that all parts of the negative may be exposed simultaneously.

Multiple-lens cameras are used when it is desired that each exposure cover a much larger area than could be included with a single-lens camera. The units of a multiple-lens camera are usually arranged so that there is one central unit which takes a vertical photograph while each of the other units takes an oblique photograph, all exposures being made simultaneously. The obliques can be rectified and matched with the vertical so as to give the equivalent of a single vertical view. Three-, four-, five-, and nine-lens cameras have been used successfully for reconnaissance mapping.

42. PHOTOGRAPHIC FLIGHTS

For the guidance of the airplane pilot, it is necessary to prepare a flight map, based on the best available map of the area. On the flight map are shown the flight line for each strip of photographs, and information concerning flight heights, spacing between successive photographs, cameras to be used, and special requirements. A good flight map results from careful planning of the photographic project and helps to insure adequate and efficient photographic coverage.

The main geometrical considerations in planning vertical photography are (1) direction of flights, (2) focal length and flight height, (3) spacing (or overlap) of photographs along the flight line, and (4) spacing (or overlap) of photographs between flight lines.

If possible, aerial photographic flights should be flown (1) parallel to the long dimension of the project, (2) perpendicular to parallel lines of control, and (3) parallel to the ridges and valleys. Usually these conditions cannot be met simultaneously and the choice of direction is determined by the relative importance of these factors in the particular project under consideration. In most cases, the flights are arranged to run north-south or east-west, whichever more nearly meets the above requirements.

The determination of the most suitable focal length and flight height depends on the purpose of the photography. For most uses, the scale of the photography is an important consideration. Since the photographic scale is f/H, the scale increases directly with the focal length and inversely with the flight height. Thus, for such purposes as the detailed study of engineering construction sites, requiring large-scale photography, a relatively long focal-length lens, low flight height, or both, may be indicated. On the other hand, for extensive mapping projects, a relatively small scale is desired in order to increase the area coverage of each photograph, and keep the control-survey requirements at a minimum; standard topographic mapping photography is therefore usually flown at higher altitudes and taken with shorter focal-length cameras than are required for detailed surveys.

43. OVERLAP OF PHOTOGRAPHS

In order that a satisfactory map or mosaic may be prepared from aerial photographs, and in order that the photographs may be examined stereoscopically, the adjacent exposures must cover overlapping areas on the ground. The amount of overlap of adjacent photographs along the line of flight is called forward overlap, while the amount of overlap of photographs in adjoining parallel strips is called side overlap. The forward overlap is usually made about 60% of the area of each photograph, in order that each photograph in a strip, with the exception of the one at each end, will include the principal points (photo centers) of three successive photographs. An important result of the 60% overlap is that every point on the ground photographed, except at each end of the flight line, appears on at least two photographs.

For most photogrammetric work, the side overlap, or overlap between flights, should preferably be about 25% of the area covered by each photograph. The amount of side overlap is subject to great variation because (1) it is difficult to guide an airplane on straight parallel courses, and (2) relief and tilt cause displacements of images.

44. RELIEF DISPLACEMENTS ON VERTICAL PHOTOGRAPHS

The only points that will be in their proper relative positions on an aerial photograph are the points lying in the datum plane, which is the horizontal plane selected as a reference elevation for computing the scale of the photograph. The datum plane for each proj-

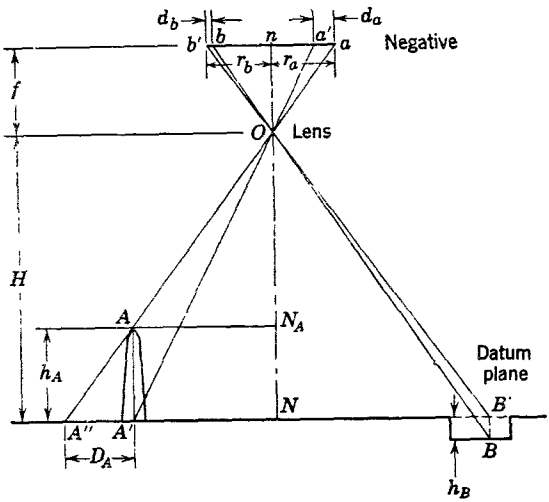

FIG. 35 Displacement due to relief.

ect is usually fixed at any convenient elevation that is approximately the same as average ground elevation for the project.

In a truly vertical photograph the plumb-point image coincides with the principal point, the point at which the lens axis intersects the negative plane. In Fig. 35, h_A represents the elevation of the top of the monument A above the datum plane which contains the plumb point N. Also, h_B represents the depression of the bottom of the cut B below the datum plane. If all objects were in the datum plane, the negative would have a scale of f/H over its entire area. In nature, $AN_A = A'N$, but on the negative these distances are recorded as an and $a'n$, respectively. The scale of distance an is $f/(H - h_A)$, which is inconsistent with the general scale of the negative, f/H. Likewise, the scale of distance bn is $f/(H + h_B)$.

The amount of relief displacement d_a on the negative is evaluated as follows:

$$\triangle a'Oa \text{ is similar to } \triangle A''OA'$$

$$\therefore\ d_a/D_A = f/H \qquad \text{or} \qquad d_aH = D_Af$$

$$\triangle nOa \text{ is similar to } \triangle A'AA''$$

$$\therefore\ r_a/D_A = f/h_A \qquad \text{or} \qquad h_Ar_a = D_Af$$

$$\therefore\ d_aH = h_Ar_a \qquad \text{and} \qquad d_a = h_Ar_a/H$$

In general,

$$d = hr/H \tag{1}$$

The amount of relief displacement of images on an aerial photograph will therefore vary according to: (1) the elevation h of the object above or below the datum plane; (2) the distance r of the image on the photograph from the image of the plumb point, which in turn depends on the focal length f; and (3) the altitude H of the aerial camera. It is readily seen from inspection of Fig. 35 that the displacement is radially outward from the plumb point if the object is higher in elevation than the datum plane, and radially inward toward the plumb point if the object is lower in elevation than the datum plane.

Equation 1 is useful for determining the correct locations on a vertical aerial photograph of images of objects that are displaced by ground relief, providing H and h are known. Equation 1 may also be used to determine the approximate height of any object shown on the photograph, when H is known, by scaling values for r and d directly from the photograph, and solving for the height h.

Relief displacements are the cause of many unusual effects on vertical aerial photographs. For example, the image of a long highway tangent that runs across hilly country, when located near the edge of a photograph, will appear as a series of curves, or as a wiggly line, rather than as a straight line. However, if any point of the same highway tangent happens to lie directly below the camera, so that the image of the highway tangent is in effect a line radiating from the center of the photograph, it will appear on the photograph as a straight line.

45. EFFECT OF TILT ON VERTICAL PHOTOGRAPHS

Because of winds, high speed, and other flight conditions which cause banking and tipping of an airplane, it is difficult to maintain the optical axis of the camera in an exactly vertical position. Practically all aerial photographs are made while the axis of the lens is

tilted from the vertical, and the resulting photographs are not of uniform scale. For many purposes, however, the effect of a small amount of tilt on photographic scale can be ignored.

Figure 36 shows the effect produced on a photographic negative by tilting of the axis of the camera through an angle θ from the vertical. The principal point of the tilted negative is the point p at which the lens axis intersects the tilted negative. The principal point of an untilted negative, for the same exposure station, is the point n' at which the lens axis intersects the untilted photograph: it coincides with the plumb-point image. The vertical plane containing both of these principal points is called the principal plane. The lower

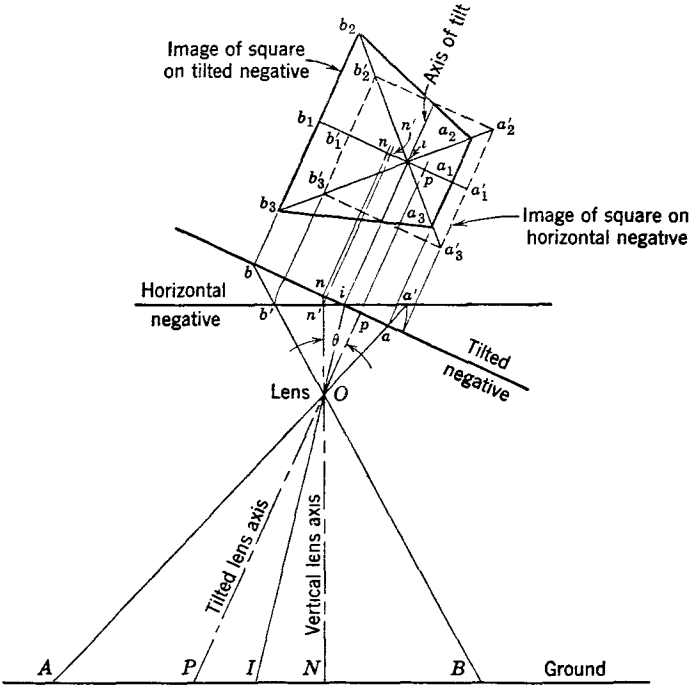

FIG. 36. Effect of tilt.

view in Fig. 36 is a projection on the principal plane. The upper view is an auxiliary view on a plane parallel to the tilted negative, with the horizontal negative revolved into that plane. The unprimed points represent positions on the tilted negative. The plane of the tilted negative intersects the plane of the horizontal negative in a line which is called the axis of tilt. The trace of this axis on the principal plane is the point i, which is known as the isocenter. $\triangle Opi$ and $\triangle On'i$ are congruent triangles, and therefore $ip = n'i$. If the angle of tilt is less than 5°, which is about the practical limit of tilt for vertical aerial photographs, the distances ni and $n'i$ may be considered equal and therefore ni and ip may be considered equal. Consequently, the isocenter may be considered to be equidistant from the principal point and the plumb point on the tilted negative. The displacement of any image due to tilt occurs in a radial direction from the isocenter.

The image of point A appears at a on the tilted negative and at a' on the untilted negative, and the distance ia is less than ia'; however, the distance ib is greater than ib'. It is thus indicated that the scale of the depressed side of a tilted negative is smaller than the scale of a horizontal negative at the same exposure station, while the scale of the raised side is larger than the scale of the horizontal negative. It is also apparent that the *difference* in scales between the tilted negative and the untilted negative increases as the distance from the isocenter increases. Therefore, any area that is a rectangle on the datum plane will appear as a trapezoid on a tilted photograph. Figure 36 shows this effect for a square on the ground whose sides are equal to AB.

Various formulas that have been developed for computing the displacement caused by tilt show that the displacement varies approximately as the square of the distance from the isocenter. However, most aerial photographs now being made for engineering work are so nearly vertical that, for many purposes, they may be assumed to be true verticals. Precise plotting instruments solve the tilt problem by graphical or mechanical means. The analytical solution of tilt problems, a lengthy process, is not required in most of the practical applications of photogrammetry.

46. RADIAL-LINE PRINCIPLE AND APPLICATION

It has been shown that the displacements caused by tilt are radial from the isocenter while those caused by relief are radial from the plumb point. An accurate determination of these two points can be made if the angle of tilt and axis of tilt are known, but it is a laborious procedure. However, if the tilt is held within 3°, and the relief of terrain is not more than 10% of the flight height, the distances of the isocenter and plumb point from the principal point are so small that the principal point may be regarded as the center of radiation for combined tilt and relief displacement This approximation, known as the radial-line assumption, enables the aerial photographs themselves to be used as a means of establishing the map locations of a number of points on each photograph of a strip.

According to the radial-line assumption, the true direction of an object from the principal point of the photograph is a straight line from the principal point through the image of the object. When two photographs are properly oriented with respect to each other and their principal points plotted, radial lines from each principal point through images located in the common overlap area will, at their intersection, establish the true map position of such images at the scale of the line joining the principal points of the photographs. Thus, angles that are measured at the principal point of each aerial photograph may be regarded as true angles, and hence procedures similar to those of plane-table triangulation may be used in extending horizontal control through a strip of photographs. Triangulation by resection is used to establish the map location of the principal point of each photograph, and triangulation by intersection is used to establish the map locations of a number of photographic control points.

The principal points of successive photographs are located on a map or on a control plot that will become the base sheet for a map, by application of the three-point principle. According to this principle, the position of any point can be determined by measuring the angles from it to three other points whose positions are already known. In compiling a photogrammetric control plot, the three-point principle is applied as follows: Three points of known location are plotted accurately on the map; also, these three known points and the principal point are marked on the photograph. The three known points may be ground-survey control points, or photographic control points previously established by radial-line procedures. Two of the three points should be located in the overlap area of the first two photographs of the strip. A sheet of tracing paper or transparent celluloid is placed over the photograph, the principal point is marked on the tracing, and radial lines are drawn on the tracing from the principal point through the images of the three control points, these lines being extended for some distance beyond the control points. The tracing is next placed over the map and is moved about until the radial lines pass through the plotted positions of the control points on the map. When the tracing has been properly oriented, the location of the principal point may be pricked through onto the map, by means of a needle point.

The principal point of one photograph is thus located on the control plot, and it becomes a new known point. Radial lines from it to the selected photographic control points in the forward overlap area are transferred from the photograph to the control plot, by use of a tracing or templet. The principal point of the next photograph along the strip is then determined by a three-point resection, using two of the original known control points and the newly located principal point of the first photograph. When the second principal point has been located on the map sheet, radial lines, or rays, are drawn from it to the photographic control points that were selected on the first photograph, and the points of intersection of these new rays with the previous rays serve to establish the map positions of those new photographic control points. Before removing the tracing or templet from the control plot, rays to the selected new photographic control points in the forward overlap area of the second photograph are drawn. This procedure of resecting by the three-point method in order to locate on the map the principal point of each photograph, and intersecting for the new photographic control points, is continued throughout the several photographs of each strip, until another known control point, or group of points, is encountered. By establishing radial-line intersections for each such fixed point the closure discrepancy for each photographic flight is determined. If the closure is within acceptable

limits, appropriate adjustments along the strip are made, and the control plot for that strip is ready for use as a map base. The accuracy of the photographic control points can be greatly strengthened by carrying the radial line control forward for two or more strips simultaneously.

It should be noted that three photographic control points should be established in the overlap zone that is common to each set of three photographs. One of these should be the principal point of the middle photograph, and each of the other two should be located near the side edge of the photographic strip. It is also noted that the use of the radial-line procedure is comparable, in part, to the use of backsights and foresights in ground surveying. With an overlap of more than 50%, each photograph shows the images of the principal points of the preceding and following photographs as well as its own principal point. The direction on any photograph from its principal point to the principal point of the preceding photograph corresponds to a backsight along the flight line. Similarly, the direction to the principal point of the following photograph corresponds to a foresight along the flight line. These sights along the flight line are very useful for determining the correct orientation of the tracings for successive photographs.

Figure 37 illustrates the method of selecting photographic control points for radial-line work on a strip of four vertical aerial photographs, having three ground control points in the first exposure and one ground control point in the fourth exposure. The map sheet on which it is desired to establish the position of the photographic control points originally has only the ground-survey control plotted on it. By the use of transparent tracings, the principal point, P_1, is located on the map by the three-point principle already described, using the known points A, B, and C. The next principal point, P_2, is located on the map using radials to the three points A, B, and P_1, whose positions have already been established. The map position of a_2 and b_2 can then be determined by the intersections of radial lines from P_1 and P_2. Principal point P_3 can then be located on the map by using radials to a_2, b_2, and P_2. This process is continued until the checkpoint D is reached.

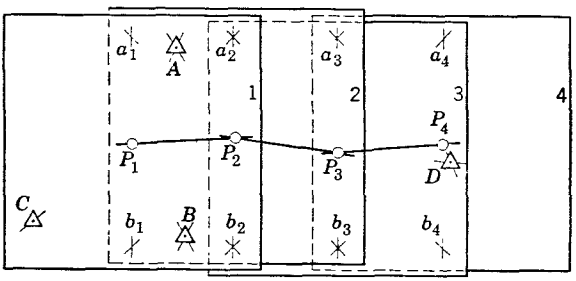

FIG 37 Method of selecting photographic control points for radial-line plot

The application of radial-line methods is facilitated by the use of slotted templets in place of transparent sheets. A commonly used type of templet is made from cardboard. A circular hole, corresponding to the principal point of the photograph, is punched in the center of the templet. Elongated slots, representing radial lines to image points, are cut so as to radiate from the center of the hole. After each photograph is properly marked, it is laid on a templet sheet and the points are pricked through to serve as guides for cutting the slots. The slots are made long enough to provide for the scale differences usually encountered.

The templets are assembled over a map base on which the known control points have been accurately plotted and a fixed stud is fastened at each such known point. A stud is inserted in the center hole of each templet but not fastened to the map base. An unfixed stud is also provided for each photographic control point to be located. Each of the templets containing a slot pertaining to a given photographic control point is placed so that the stud representing that point slides in the appropriate slot. When the templets are laid down on the map base in their proper relationship, the floating studs ride in the slots, causing the templets and unfixed studs to shift into their final assembly. The positions of the photographic control points are then pricked onto the base by pushing a pin through the hollow center of each stud.

47. STEREOSCOPIC PERCEPTION

If two photographs of the same area or object, taken from different exposure stations, are properly oriented and viewed simultaneously, the observer has the impression of viewing a relief model of the terrain covered by the photographs. Stereoscopic vision is facilitated by the use of an instrument called a stereoscope.

Simple stereoscopes are of two general types: direct viewing and mirror. Direct-viewing stereoscopes are generally preferred, but they can only be used when relatively small-sized photographs are being observed. When larger photographs are to be observed, it is necessary that a system of mirrors be used, because larger photographs must necessarily be spread considerably farther apart than normal eye-separation distance. Both types of simple stereoscopes are illustrated in Fig. 38. Use of the stereoscope greatly facilitates correct interpretation of aerial photographs, because the three-dimensional views invariably give a clearer conception of the relief and elevation of natural terrain, and clarify the details of man-made structures. Minor but important differences in elevation, such as embankments, cuts, ditches, and stream banks, are readily detected, and approximate elevation differences estimated, by means of simple stereoscope observation.

In using a stereoscope with aerial photographs, it is necessary to orient a pair of overlapping photographs and the stereoscope so that the flight line on both photographs forms a straight line parallel to the observer's eyes and the axis of the stereoscope, and in such position that the left eye sees only the left-hand photograph and the right eye only the right-hand one.

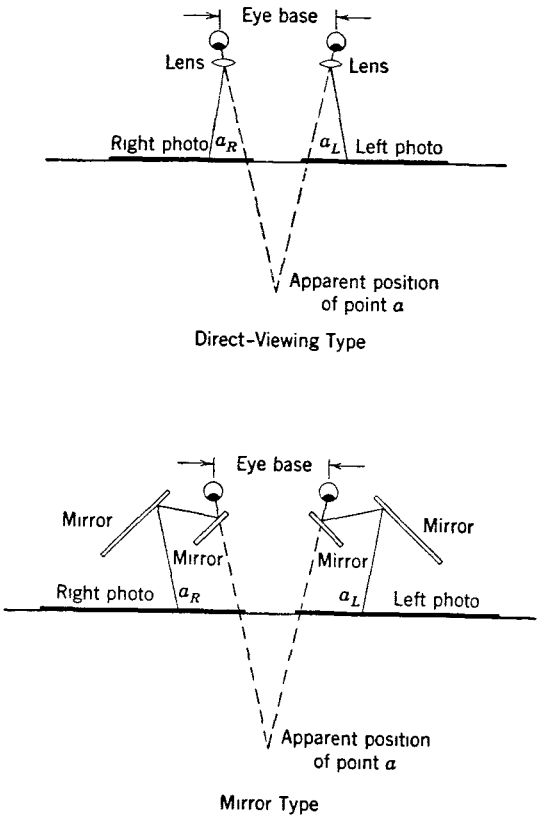

FIG. 38 Simple stereoscopes.

48. INTERPRETING AERIAL PHOTOGRAPHS

The ability to identify visible features of terrain from their photographic images, or to determine the existence of hidden features by the configuration of visible features, is a skill which is acquired through practice and experience. The stereoscope should always be used in interpreting aerial photographs, when overlapping pairs are available. Shadows often serve as useful guides in the identification of features. (It is usually helpful to orient the photograph so that the shadows fall toward the observer.) Objects such as radio towers, water tanks, bridges, church spires, and tall buildings are often clearly revealed by their shadows in the photograph, even though the image of the feature itself does not serve to identify it. A bridge may appear on the photograph as a narrow strip, whereas the shadow may indicate whether it is of arch, truss, cantilever, or suspension construction.

Many cultural features are identified by the regularity of their outline or alignment. Straight or smoothly curved lines indicate roads, railroads, canals, power lines, fences, or other aligned cultural features. Roads usually appear as narrow light-colored bands. Major highways usually are distinguished by long tangents, easy curves, and regularity in width. Unimproved roads appear lighter in color and irregular in width and generally have sharp turns. Railroads usually show narrower than highways and are easily distinguished by their long tangents, regular curves, heavy cuts and fills, stations, round-

houses, sidings, and freight yards. Buildings appear as small rectangles, L-shapes, T-shapes, or other characteristic outlines. The observer can soon learn to identify a wide variety of cultural features by their characteristic appearance.

Natural features are irregular, in contrast to the geometric patterns of man-made features. Streams are easily identified by their winding courses, variable widths, and the pattern of vegetation along their banks. Open bodies of water appear in tones ranging from very light for muddy water to very dark for clear water. The position of the sun also affects the appearance of water bodies. The shore line of water features can be studied in great detail on aerial photographs.

The outline, size, and density of wooded areas can likewise be clearly discerned on aerial photographs. Evergreen forests appear as dark, dense masses, whether the photography is taken in the summer or winter. Deciduous forests also appear dark and dense on summer photography; but, on photographs taken in the winter, when the leaves are off, the ground is clearly visible through the trees. For this reason, most mapping photography of wooded areas is obtained in winter or early spring.

Cultivated fields appear as rectangular areas, usually darker than the uncultivated lands, except that freshly plowed damp ground usually has a light-gray tone. Orchards are easily identified by the regular pattern of the trees.

The interpretation of photographs for special purposes, such as physiographic, geologic, agronomic, and other technical uses, requires detailed study of tones and textures and their relationship to vegetative cover or land forms.

49. MOSAICS AND PHOTO-MAPS

An aerial-photograph mosaic consists of a group of overlapping vertical aerial photographs matched together as well as possible to form a unified picture of the desired area. A photo-map is a mosaic on which certain map features have been added. Mosaics and photo-maps are frequently used as temporary substitutes for conventional maps, but for some purposes they serve better than maps. Mosaics are said to be controlled, or uncontrolled, depending on whether ground-survey control points are available, and the degree of accuracy attained. A mosaic that is compiled from only one or two flights, and covers a long and narrow route, is commonly called a strip mosaic.

Controlled mosaics are compiled on a base sheet of composition board on which are plotted all available ground-survey and photographic control stations. The scale for the base sheet (sometimes called a control plot), is usually selected so as to be any convenient scale that is as near as practicable to the average scale for all of the photographs that are to be used. The locations on the base sheet for the principal point of each photograph, as well as for several photographic control points for each photograph, are then determined by a radial-line templet procedure, as explained in Art. 46. Each photograph to be used in the mosaic is then carefully measured and compared with the control points shown on the base sheet, and from this comparison the scale and tilt of each photograph are determined closely enough for the purpose. Each photograph that does not fit the control points shown on the base sheet is then ratioed (enlarged or reduced) as necessary for changing its scale, and also, when necessary, is rectified for the purpose of eliminating the effects of tilt. Each photograph is then trimmed of excessive overlap and is carefully pasted in place on the base sheet, getting the best fit possible with all control points and at the same time getting the best possible image match with all adjoining photographs. Lettering and other pertinent map data are then added, as desired.

If several copies are required, a film negative of the original mosaic is made by a large map-copying camera, the negative being enlarged or reduced at this time in order to obtain the desired final scale. Contact prints (or enlargements or reductions) are then made from this negative in the usual manner, when and as needed.

Uncontrolled mosaics are made by less rigid methods and are less accurate than controlled mosaics. Nevertheless, a reasonably useful composite picture of the terrain can be obtained by means of an uncontrolled mosaic. The effects of the various distortions can be minimized by using only the net central area of each print.

50. PHOTOGRAMMETRIC MAPPING INSTRUMENTS

Photogrammetric mapping instruments range in character from relatively simple devices, used for the production of maps of relatively low accuracy, to elaborate plotting machines designed for mapping to a high standard of precision. Such instruments may be grouped into the two general categories of monoscopic and stereoscopic. Monoscopic

instruments are used only for transferring planimetric map features from photograph to map, and, because they provide no convenient way of correcting for relief displacements, are not suitable for accurate map work in hilly or mountainous terrain. Stereoscopic instruments utilize the principles of stereoscopy for depth perception and measurement, and hence are suitable for use in topographic mapping. Also, most stereoscopic plotting instruments provide a means of correcting for scale distortion caused by tilt, and image displacements caused by relief, and hence are suitable for accurate mapping

In most of the modern stereoplotting instruments, a so-called stereoscopic model, to true scale, is formed from each pair of overlapping photographs. Then, with a "floating mark" in the field of view, it is possible to follow a contour line in the stereoscopic model and to raise or lower this mark to correspond to different contour elevations. The floating mark is a visible point which can be seen at the same time that the stereoscopic model is being viewed. One of the most widely used types of floating mark consists of a small illuminated hole in the center of a platen which can be adjusted in a vertical direction; as the platen is moved vertically, the floating mark appears to move up or down. By using a drawing device connected to the floating mark, the operator may correctly delineate, in both plan and elevation, all map features appearing in each model.

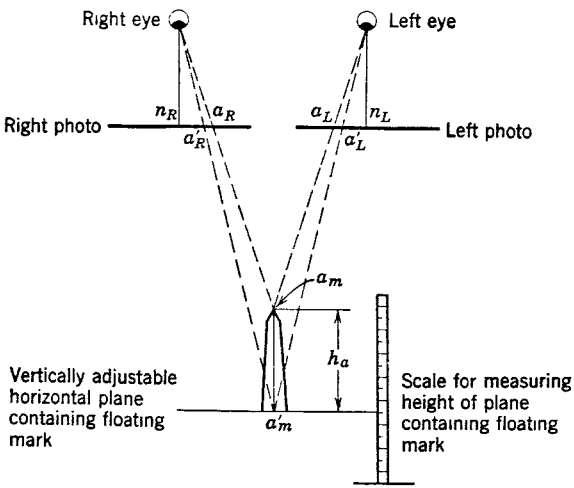

FIG 39 Stereoscopic height measurement.

The principle of stereoscopic height measurement is illustrated schematically in Fig. 39. It is assumed that the photographs are printed on transparent plates so that the floating mark can be seen at the same time the photographs are viewed. The images of the top and bottom of the monument appear on the left and right photos at the positions indicated by a_L and a'_L, and a_R and a'_R, respectively. If the two photographs are viewed stereoscopically, with the horizontal plane containing the floating mark adjusted vertically so that the floating mark is at the intersection a'_m of the rays $a'_L\,a'_m$ and $a'_R\,a'_m$, it will appear that a'_m lies in that plane. Point a_m, however, will appear to be at a higher elevation, and the amount by which the plane of the floating mark must be raised to appear to be at a_m is a measure of the difference in elevation between the top and bottom of the monument.

The effectiveness or general efficiency of stereoplotting instruments is usually expressed in terms of an empirical constant known as the C factor.

$$C \text{ factor} = \frac{\text{Flight height}}{\text{Least contour interval accurately plottable}}$$

If a plotting system is shown to be capable of plotting standard-accuracy 20-ft contours (but not smaller) when the flight height is 12,000 ft, the C factor is 12,000/20 = 600. Conversely, if the C factor is known to be 600 and it is desired to know what the flight height should be for the accurate plotting of 20-ft contours, the flight height is evaluated as 20 × 600 = 12,000 ft. The optimum value of the C factor for a given plotting instrument is sometimes a controversial issue and may vary considerably for different projects, according to the type of terrain (wooded or cleared, grazing or cultivated) and the amount of haze in the atmosphere. In general, C-factor values range from about 100 for the least accurate instruments up to about 1500 for the most accurate instruments.

Photogrammetric mapping instruments of the Multiplex type (including also the Kelsh plotter and the ER-55) are widely used in the United States. In the Multiplex system, the images of two overlapping photographs are projected through colored light filters, one red and one blue, onto a white platen mounted on a movable tracing instrument. The

operator is able to view the resultant stereoscopic model by use of colored spectacles, one lens being red, the other blue, to match the colors of the filters. A floating mark of the type previously described is mounted in the center of the platen; as the elevation of the platen is adjusted up or down the floating mark appears to move up or down. A vertical scale is provided on the platen's support for measuring changes of elevation. A pencil mounted vertically below the floating mark can be brought into contact with the map sheet for the delineation of detail. Planimetric features are compiled by pushing the tracing table along the feature—raising and lowering the platen so that the floating mark appears always to be on the ground. A contour is traced by locking the platen at the correct elevation and pushing the tracing table around the topographic feature so that the floating mark is always just tangent to the ground. Proper delineation of detail is contingent on the correct orientation of the Multiplex projectors. Each projector has adjustments so that its orientation can be made to duplicate the aerial-camera orientation at the instant the corresponding negative was exposed. These adjustments include three linear motions, x, y, and z, for recovering the space coordinates of the exposure station. The x motion is generally parallel to the flight course, the y motion is generally horizontal and perpendicular to the flight course; the z motion is generally vertical. Three rotational motions about the x, y, and z axes (x tilt, y tilt, and swing) serve to recover the orientation of the lens axis at the instant of exposure. The photographic images used in the Multiplex projectors are printed on small glass plates called diapositives. The diapositives are made from the original negatives in a reducing printer specifically designed for the purpose.

Photogrammetric mapping instruments of the "universal" type, such as the Zeiss Stereoplanigraph and the Wild Autograph, provide a means of carrying out, with a high degree of efficiency, mapping projects that have unusually exacting requirements. Projection and viewing in these instruments are accomplished by means of elaborate optical or optical-mechanical trains. Because of the complexity and high initial cost of the universal type of instruments, they are not as widely used in the United States as the Multiplex type.

In addition to their use for the compilation of map detail, both the Multiplex type and the universal type of instrument are used for the extremely important function of aerotriangulation, or extension of control. In aerotriangulation with a photogrammetric instrument, successive pairs of photographs in a strip are oriented to form a continuous bridge and the entire bridge is adjusted to ground-survey control located at the ends of the strip; control required for intermediate models is then obtained from the adjusted bridge.

51. PHOTOGRAMMETRY IN TOPOGRAPHIC MAPPING

One of the most important applications of photogrammetry is in the preparation of topographic maps. By using any of the more accurate photogrammetric map-plotting instruments, in combination with low-altitude and large-scale aerial photographs, it is possible to produce maps of standard accuracy with 1-ft or 2-ft contours, at scales such as 1 in. equals 50 ft or 100 ft. Many of the detailed topographic maps of cities and metropolitan areas are prepared by photogrammetric procedures. The topographic quadrangle maps of the United States Government, usually published at scales of 1:24,000 and 1:62,500, are prepared by procedures making extensive use of aerial photogrammetry. Aeronautical charts with 500-ft and 1000-ft contours, at scales of 1:250,000, 1:500,000, and 1:1,000,000, are prepared from aerial photographs, usually oblique photographs. There are few topographic mapping projects covering extensive areas in which photogrammetry is not utilized in one way or another. This does not mean, however, that ground surveys can be entirely eliminated. On the contrary, the ground-survey phases of photogrammetric-topographic mapping usually cost more than the photogrammetric phases. For example, control surveys must still be done largely by ground-survey methods, and ground surveys are also required for field checking the photogrammetric compilations, and for completing those portions of each map sheet that cannot be done, or are not economically done, by means of aerial photographs. Map details of the type that are necessarily procured by ground surveys include, for example, names, features hidden within forests or behind tall buildings, and cadastral information. Also, where the terrain is extremely flat, the contours usually can be obtained more economically by means of conventional ground surveys. In general, however, the use of photogrammetry will prove to be an economy for most topographic mapping projects. It is true that a considerable investment for specialized photogrammetric equipment is required, and that specially trained key personnel are necessary. Where equipment and trained personnel are not immediately available, the services of the private-practice photogrammetric organizations should be investigated.

52. PHOTOGRAMMETRY FOR THE CIVIL ENGINEER

Although the more advanced applications of photogrammetry usually require costly equipment and the services of photogrammetric specialists, there are a number of ways in which the general civil engineer can make profitable use of this powerful implement without the need of extensive specialized training or a large investment in equipment. He should at least be aware of the following facts about photogrammetry:

(*a*) Most of the area of the United States has been photographed by the United States Government. These photographs, with the exception of those covering certain restricted areas, are available at a nominal cost. An inquiry to the Map Information Office, U. S. Geological Survey, Washington, D. C., will enable the engineer to obtain complete information regarding the availability and character of aerial photography covering any area in which he is interested.

(*b*) Simple stereoscopic observation of pairs of overlapping photographs affords a quick and useful means of reconnaissance or of rough physical inventory of the terrain, cover, geology or culture of an area. With practice, it is not difficult to acquire considerable skill in stereoscopic photointerpretation. A pocket stereoscope is a simple and inexpensive device that should be standard equipment in every civil engineering office.

(*c*) An extensive literature in the field of photogrammetry is available, notably the publications of the American Society of Photogrammetry. The Society's quarterly journal, *Photogrammetric Engineering*, is the principal medium for the presentation of current developments, while the *Manual of Photogrammetry* is a comprehensive reference work covering practically all phases of photogrammetry.

(*d*) Although precise photogrammetric plotting instruments are costly, there are available several relatively inexpensive instruments capable of performing useful, if limited, objectives.

(*e*) In the absence of a map, aerial photographs or a mosaic can serve as a temporary map substitute.

When the civil engineer's practice includes surveying and mapping problems that can be economically solved by advanced photogrammetric techniques beyond his own experience, he may find it profitable to use photogrammetry in one of two ways:

(*a*) If the occurrence of advanced problems is only occasional, the engineer can employ a firm specializing in photogrammetry, thereby eliminating the need to purchase equipment or hire additional personnel. Before entering into such an arrangement, the engineer should be reasonably sure that the photogrammetric solution of the problem will be cheaper or better than a field-survey solution. The cost aspect can be decided by comparing his own estimate of the cost of a field-survey solution with the bid or estimate submitted by the photogrammetric firm. The quality of the survey can be assured by providing adequate specifications to govern the project. The accuracy of a map or survey obtained by photogrammetric means is stated in exactly the same form as if the product were obtained by field-survey means, for the accuracy statement applies to the product, not to the means of obtaining it. A photogrammetric survey can also be better than a field survey in another sense: time of completion. Thus, even if it is neither cheaper in direct cost nor more accurate, a photogrammetric survey may save money by permitting earlier completion of the project.

(*b*) If advanced problems occur frequently and are broad enough in extent to warrant the investment, the engineer can purchase photogrammetric plotting equipment and employ photogrammetric personnel. The choice of the equipment to be purchased depends on the nature of the work, the accuracy required, and economy of operation; this choice is sometimes a rather difficult one and may entail extensive investigation. With the aid of excellent instructional literature now available, the engineer's technical employees usually can be readily trained in the operation of photogrammetric equipment.

TOPOGRAPHIC SURVEYS AND MAPPING

By George D. Whitmore

53. GENERAL PROCEDURES

The procedures ordinarily employed in topographic surveying may be grouped into four basic techniques, briefly described as follows:

(*a*) Cross-section surveys, in which the area to be mapped is covered by parallel profiles, spaced at appropriate intervals, as 25, 50, or 100 ft, with elevations determined for regular stations along each profile at suitable intervals, as 25, 50, or 100 ft, and with extra elevations being taken as plus points at all abrupt changes in slope For irregular terrain an additional set of cross profiles might be required. This method is widely used for obtaining topography of a narrow strip of terrain, as for a proposed highway or railroad location, but it probably is not economical otherwise except for site-survey maps of small areas

(*b*) Transit-stadia surveys, in which transit-tape traverse lines with elevations are run at random throughout the mapping area, with stadia "side shots" being taken from each transit station as necessary to cover the adjacent areas. The traverse stations and side shots are later plotted on the map sheet, and used as guides in sketching the planimetric features and contours. A disadvantage of this and the cross-section procedure is that the sketching usually is done in the office and the sketcher therefore is handicapped by not being able to view the terrain directly. To offset this disadvantage, a combination of transit and plane table is sometimes used, in which both the plane table and the transit are set up near each other at each instrument station. The transitman reads the stadia distance and determines the elevation of each side shot in the usual manner, but does not read the azimuth angle. The plane-table operator simultaneously plots each transit reading, using the alidade to obtain azimuth to the rod, and sketching the map details for each set-up while directly viewing the terrain.

(*c*) The plane-table method, described in Art. 57

(*d*) The photogrammetric method, in which both planimetric features and contours are, as much as feasible, identified, measured, and plotted from aerial photographs. This procedure greatly reduces, but does not by any means eliminate, the need for conventional ground surveys. For example, ground surveys are still necessary for horizontal and vertical control, for locating contours in flat terrain, and for locating features not visible in the photographs by means of field-completion surveys. These latter include names of places and features, political-subdivision boundaries, public-land lines, property corners, pipe lines and wire lines, road and building classifications, features hidden within wooded areas, and several very small features such as springs, private cemeteries, and wells. On some projects it is satisfactory to do field-completion work in advance of the photogrammetric compilation, in which case the missing features are delineated on a set of aerial photographs. In most cases greater efficiency is obtained by doing the field-completion surveys after the photogrammetric compilation sheet is available. In that case the compilation sheet is reduced or enlarged to a suitable field scale, by use of a map-copying camera, and is then printed as a blueline print on a scale-stable plane-table sheet. This is taken to the field, and is checked, corrected, and completed, as may be necessary. This is the general method used by the U. S. Geological Survey on its standard quadrangle maps of United States areas, the principal phases being: (1) procurement of aerial photographs; (2) basic control; (3) supplemental (picture-point) control; (4) compilation of photogrammetric manuscript sheet; (5) field-completion surveys by plane table (which may include complete contour surveys in flat areas); (6) map finishing operations, including final inking and editing.

54. CONTROL SURVEYS FOR MAPPING

Control surveys for topographic mapping are necessary on all projects which cover more than a few acres. The cost of control surveys is substantial, in relation to the total mapping cost, and may range from a few to 50%. Significant savings may be achieved by finding the most favorable locations for the control surveys, and by utilizing those procedures which are most appropriate for the accuracies required.

Horizontal-control surveys provide a series of base lines or base stations on which the less accurate surveys for locating map details are originated and closed. The most eco-

nomical locations and distribution for horizontal-control surveys will depend on both the map scale and the mapping procedures to be used. The minimum accuracy required in the horizontal-control surveys, if used only for mapping, depends mainly on the map scale.

Vertical-control surveys provide benchmarks and other accurate elevations on which the less accurate surveys for contouring are originated and closed. As a general rule, more control elevations are required for photogrammetric procedures than for conventional ground-survey procedures. The most economical locations and distribution for control elevations will depend on both the contour interval and the mapping procedure. The minimum accuracy required for vertical control is directly related to the contour interval.

Control surveys are further differentiated according to the accuracy attained as first order, second order, third order and fourth order. Local control surveys that are to be used only for map preparation seldom require any accuracy higher than third order, and usually fourth order will suffice. The control-survey methods that will be most efficient or economical for any mapping project will depend on several factors, including the type of terrain and the accuracy required.

Third-order horizontal control must have accuracy and closures of 1:5000 or better, with fourth order including all accuracies of less than 1:5000. Horizontal-control surveys for mapping should contain no blunders or errors larger than the least plottable distance on the map, and this criterion can be used to determine the accuracy necessary and the techniques that will be satisfactory in the surveys for horizontal control. Usually a combination of a main scheme of control of third-order accuracy, with fourth-order accuracy for a supplemental scheme, will be found economical. Horizontal-control surveys of third-order or better accuracy are usually executed by the conventional methods of triangulation in mountainous terrain, and transit-tape traverse in non-mountainous terrain, or by modifications and combinations of triangulation and traverse. When the horizontal control is established by means of transit traverse, it is customary to run the vertical control over the same routes and use the traverse monuments as benchmarks.

Third-order vertical control must have accuracy and closures of better than 0.05 ft $\times \sqrt{M}$, in which M is the length in miles of the level line or circuit, all work of lower accuracy being classed as fourth order. Vertical control of third-order accuracy can be accomplished only by conventional spirit-level surveys. For mapping purposes, elevations that are correct within one-tenth of the contour interval will usually be satisfactory; hence, unless the lines are long, fourth-order accuracies will ordinarily suffice.

55. FOURTH-ORDER CONTROL METHODS

Transit traverses, with distances determined either by taping or stadia, are commonly employed in obtaining fourth-order horizontal control for large-scale maps. Fourth-order elevations of the traverse stations are then conveniently established by trigonometric levels (vertical angles) over the traverse courses, or, in flatter terrain, by fly levels. If the terrain is such that traverse taping is difficult, stadia distances are not practical, and conventional triangulation is not economical, a procedure known as the subtense-base method is sometimes efficient. In this procedure the length of each course of the traverse is determined by (*a*) measuring an accurate but short base line at each traverse station, laid out perpendicular (or nearly so) to the traverse course; and (*b*) occupying the station at the other end of the course with theodolite, and precisely measuring (to within a few seconds) the small angle subtended by the base line. The length of the traverse course is then computed by the tangent formula. An Invar bar of fixed length, called a subtense bar, and usually 4 m in length, may be used instead of the measured base line, but, because the subtense bar is relatively short, the traverse courses must also be kept shorter than when tape-measured base lines are used. For reliable fourth-order work, the ratio of the short perpendicular base to the traverse course should average to be of the order of 1:20, and should seldom exceed 1:40.

In hilly terrain, when a main-scheme triangulation survey has been completed and the stations are flagged, the three-point method of locating new stations, with angles measured by transit or theodolite, is usually efficient. If vertical angles are observed at the same time, and appropriate corrections applied for curvature and refraction, station elevations reliable from within a few tenths of a foot to a few feet can usually be obtained, the accuracy being mainly dependent on the length of sights and size of vertical angles. In this procedure, as in all types of trigonometric leveling, the simultaneous measurement of reciprocal vertical angles will usually increase the elevation accuracy considerably, because the errors caused by changing refraction conditions are largely eliminated.

The intersection method of locating a number of new control stations is economical in

certain types of terrain. New stations are "cut in" by measuring horizontal and vertical angles to each such intersection station with theodolite or transit from two or more of the basic triangulation stations. Plane-table triangulation, including intersected "cut in" stations and resected three-point stations, is an efficient method for small-scale mapping, elevations of the stations being determined at the same time by means of vertical angles measured with the alidade.

When elevation control only is required, a variety of procedures are available, particularly when a planimetric map, or base map, has been previously compiled by photogrammetry. It is then feasible to determine the horizontal distance between any two planimetric features by scaling on the base sheet, and to use such scaled distances in connection with field-measured vertical angles for determining elevation differences. This combination has led to the development of several economical procedures. For example, a line of trigonometric levels can be carried across country by measuring only the vertical angles from one well-defined planimetric feature to another. It is necessary, of course, that the stations of such a line all be "well-defined" planimetric features or other definite points that can be positively identified on the aerial photographs, and hence can be shown in accurate location on the base map.

56. CONTROL ELEVATIONS BY ANEROID [1]

There are several procedures available for utilizing the precision-type aneroid barometers, on projects where occasional elevation errors of up to 4 or 5 ft are permissible. In all procedures utilizing aneroids it is necessary to determine a continuous record of the changes in atmospheric pressure which occur within the project area during the period of survey. This may be determined with fair reliability by either the one-base or the two-base method. In the one-base method one aneroid is left at a base station within the area to be covered by each day's work, and a dial reading is made at regular 5′ or 10′ intervals during the entire time that the roving field aneroids are in use. It is then assumed that the changes in atmospheric pressure which occurred at the base aneroid occurred simultaneously at each field aneroid

The two-base method requires two base aneroids within the project area, one located as near to the lowest elevation for the project as may be convenient, and the other located at as high an elevation within the project as may be convenient. Simultaneous readings are made of each base aneroid at regular intervals throughout the working day. The atmospheric pressure change to be applied to each roving aneroid reading is determined by a simple prorate method, depending mainly on the altitude of the field aneroid with respect to the high-base and low-base aneroids.

It is claimed that maximum accuracy with aneroids is achieved by the so-called "leap-frog" method. In this procedure two aneroids are used to carry forward a line of elevations. Both aneroids, A and B, are read at the same time at the starting point. Aneroid A then remains at the starting elevation while aneroid B advances to the first new station. Both A and B are then read at the same instant. Aneroid B remains at its station until aneroid A arrives, then both are again read, at the same time. Aneroid A then advances to the next station on the line, and the whole procedure is repeated for each section of the line. This technique gives, in effect, a measurement and record of the atmospheric pressure changes at the points where elevations are being determined. It should furnish unusually reliable elevations when the line can be terminated on a fixed elevation, with closure distributed linearly, as this would eliminate the inaccuracies caused by a sloping isobaric plane.

57. PLANE-TABLE METHODS

When topographic maps are to be prepared by a field-survey method, the plane table and alidade are the preferred equipment among experienced topographers. This preference seems to extend even into the field of large-scale surveys, including those of city areas, subdivisions, and industrial developments. The accuracy of the plane table in surveying for large-scale maps is limited mainly by the limitations of stadia-distance accuracy.

There are three general ways in which the plane table is used, depending on the type of base map, or plane table sheet. The first is known as the "white-paper" method. It is the traditional plane-table procedure, in which the field topographer is supplied with a plane-table sheet showing only control points. This method is now used mainly on large-scale, small-interval maps of relatively small projects, but it is also an efficient method of making preliminary investigations and reconnaissance surveys of proposed developments.

[1] See also Art 26, p. 2-28, on Barometric Altimetry

The second general method is to prepare first an accurate planimetric base sheet, by photogrammetry, and to use this as a plane-table field sheet. The field surveyor's job is thus greatly reduced, his principal task then being, in theory, the addition of contours and spot heights to the map. Actually he must also check all the features taken from the aerial photographs, in order to ascertain that they exist as shown and are properly classified and symbolized. Also, all planimetric features that were not visible on the aerial photographs, such as boundary monuments, unmarked property lines, and features hidden within wooded areas, must be searched out and located on the map sheet. Even so, the advantage of a planimetric base sheet is considerable, because the planimetric features that are shown can be used in the same manner as horizontal control stations. Thus the need for long plane-table traverses, or extensive plane-table triangulation, is eliminated or greatly reduced. This method is most advantageous in flat or gently rolling terrain. In hilly or mountainous terrain, particularly if clear of forests, it would usually be more economical to use photogrammetric procedures for the contours as well as for the planimetry.

The third general method is similar to the planimetric base method, except that either accurate aerial-photograph mosaic sheets, or ratioed prints of single photographs, are used as plane-table base sheets. This method is practical only when the terrain is quite flat, otherwise the displacements of photographic images caused by ground relief become troublesome As soon as relief displacements become serious, a compiled planimetric base sheet becomes a more satisfactory method. Also, because it is difficult to do fine-line, pin-point drafting on photographic-emulsion surfaces, the photographic base sheets must usually be made to a considerably larger scale than is needed for the final map.

58. PREPARING ORIGINAL MAP SHEETS

Distances (and azimuths) from topographic maps are determined by scaling, rather than by reference to written dimensions, as is so with such items as subdivision plats and building plans It is therefore desirable that each original topographic map sheet be prepared by such procedures and with such care as will insure consistent scaling accuracy. The original projection of rectangular-coordinate grid lines, which are used as guide lines for the plotting of all horizontal control points, must be plotted with pinpoint precision, using beam compass, steel straightedge, and best-quality scales. Grid lines are usually spaced at such convenient distances as will represent about 5 in. on the map sheet. The usual requirement is that both the grid projection lines and the horizontal control points (having positions expressed in terms of rectangular x and y coordinates) shall be plotted in correct relative position within $^1/_{200}$ in., the same tolerance also being required for the over-all dimensions of the map sheet. Also, when a mapping project requires more than one map sheet, hairline accuracy is necessary in matching features which cross the margin of adjoining maps.

Because of the emphasis on scale accuracy in topographic maps, the preferred practice is to use scale-stable drafting materials throughout as much of the mapping procedure as is feasible. In plane-table mapping, it is especially important to use scale-stable field sheets, because of the extremes of dampness and dryness that are encountered during field work. The present preference is for metal-mounted map sheets, these being made of good-grade drawing paper mounted on both sides of sheet metal or heavy foil, usually aluminum. A drawing material of coated glass fiber is scale stable and takes both pencil and ink fairly well Plastic sheets are reasonably scale stable, but most of these materials do not take or retain pencil work as well as drawing paper Materials that have shrinkage or expansion that is uniform in both directions are not so troublesome as those that have differential distortion.

It is standard practice in many mapping organizations to prepare the original map sheet (called work sheet, compilation sheet, or manuscript sheet) at a scale that is from 20 to 50% larger than will be required for the final map. In that case the accuracy tolerances that are permissible on the final map can be enlarged proportionally for the larger scale used on the original map sheet.

59. MAP FINISHING PROCEDURES

The operations that follow the completion of the original manuscript sheet are called the map finishing procedures. These include all the operations necessary to prepare the map for reproduction, i.e., the final drafting and lettering, the checking, and the map editing.

The conventional linen tracing, with hand-drafted lettering, may be satisfactory for some of the large-scale special-purpose maps, especially when only a few copies are required, or only a few users are involved. Tracing linen has the disadvantage of undergoing sizable differential shrinkage and expansion with changes of atmospheric humidity. These difficulties can be largely eliminated by using transparent plastic sheets (as vinylite or acetate film) instead of linen, but such materials do not yet offer good drafting surfaces as ordinary drafting inks do not apply readily and uniformly, and tend to chip off easily. Also, the hard surfaces of plastic materials tend to dull drafting pens very rapidly. A translucent sheeting of impregnated glass fiber expands or contracts very little with humidity changes, and takes drafting inks better than most plastics.

When a large-size map-copying camera is available, hand tracing of the original manuscript sheet may be eliminated entirely. The procedure is briefly explained in a report dated January 1946, by the Committee on Topographic Surveys, Surveying and Mapping Division, American Society of Civil Engineers, as follows:

> Much of the accuracy of a map may be lost through its manner of reproduction. Why go to the cost of time and money to make a map whose limit of accumulated error throughout is $1/50$ inch on the map scale and then lose this accuracy many times over through reproducing it by tracing linen and blue printing, in which the over-all error may be as much as $1/4$ inch. Tracing linen alone may change scale between morning and night by more than 0 10 inch.
>
> A film positive, made on low shrinkage film or on glass, at map scale may be made by photography, from any properly pencilled map sheet Prints from these film positives made by the ozalid or other dry reproduction processes will be much nearer to exact scale than blue prints from tracings on linen. The cost will be less than preparing the linen tracing Maps of large and important projects, especially where many copies are needed, should be reproduced by photo-lithography or by engraving.

When a map is to be printed by photolithography in only one color, the lithographer's copy may be the original manuscript sheet, neatly inked and lettered. For some purposes, the original manuscript of a one-color map need not be inked, but can be prepared neatly enough in pencil.

When the map is to be in several colors, each separate color must be carefully drafted on a separate sheet, called a color-separation guide sheet. "Stick-up" lettering, printed on sheets of thin celluloid or rice paper in a conventional printing press, is affixed in place on the map drawing with wax or paste. Printed "stick-up" is also used for symbols that are repeated many times on the map, including such as tufts depicting marshland and a variety of patterns that indicate different types of woodland. In the reproduction process, a glass-plate negative at publication scale is made of each color-separation drawing, by means of a large-size copying camera. Only the inked features and "stick-up" will show on these negatives, the camera exposure being timed so that the uninked blueline features do not photograph. For each negative a lithographic press plate on sheet metal (zinc or aluminum, coated with an albumen emulsion) is made, by contact printing in a vacuum printing frame. Each press plate is separately installed in an offset-lithographic press, and the final map sheets are run through the press once for each separate plate or color.

60. TOPOGRAPHIC FEATURES AND SYMBOLS

The features that are commonly shown on topographic maps may be grouped into four principal categories, these being: (*a*) cultural features, or works of man, including names and other descriptive information; (*b*) water and drainage features; (*c*) vegetative cover and land uses; and (*d*) hypsography, or the depiction of terrain relief and elevations. Maps that do not show hypsography are called planimetric maps or plans

Cultural features include all land-surface transportation ways and communications utilities; all structures of sufficient size to be appropriate on the map scale, including buildings, bridges, dams, piers, and wharves; political-subdivision boundary lines; and monuments, such as benchmarks, traverse stations, and boundary stones. Large-scale maps may show also such details as private-property boundaries, water hydrants, manholes, telephone and electric-service poles, street curbs, lamps, sidewalks, and outside stairways. As much as possible all features to be shown on the map should be drawn to correct scale and in complete detail, use of artificial symbols being held to the minimum. When the nature of any symbol is not self-evident, an explanatory note (called label) should be shown adjacent to the symbol. Where special and unfamiliar symbols are required with such frequency that individual labeling would be impractical, the map should carry a legend, or explanatory notes. A few typical symbols for planimetric and topographic maps are illustrated in Fig. 40.

Water and drainage features include all natural and constructed waterways, and all bodies of water, as lakes, ponds canals, ditches, marshes, and swamps. Such features as

water tanks, standpipes, conduits, and swimming pools are also classified as water features.

Vegetative cover and land-use features vary according to the type of map and its scale. On the topographic quadrangle maps of the United States Government only certain types

Topographic Signs

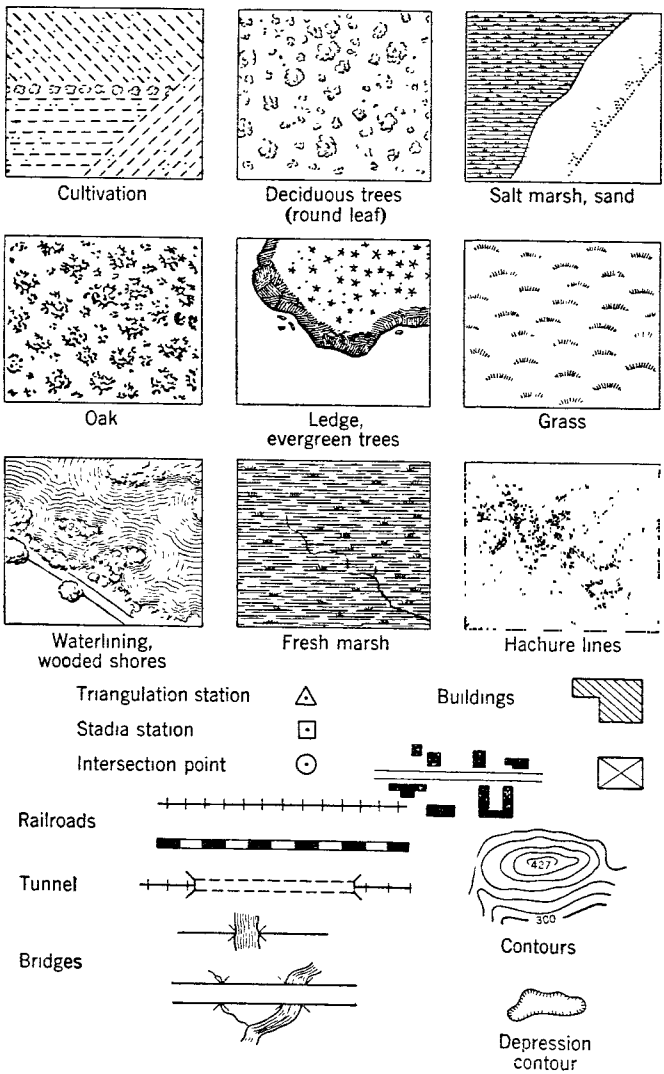

FIG 40. Representative signs for topographic maps

of woodland cover are shown, as woods, brush, orchards, and vineyards. On special-purpose maps the use of each land unit may be indicated, as grazing land, cropland, forestland, and wasteland. On very large-scale maps, such as those for architectural studies or city subdivisions, each individual tree and bush should be located and described

Hypsography may be depicted by hachures, hill shading, form lines, contours, and spot elevations, singly or in combination. Related features such as levees, spoil banks, sand dunes, sand and gravel beaches, dry lake bottoms, and mine-waste areas are usually depicted by symbols, heights being indicated by means of spot elevations. Individual spot heights or elevations are customarily shown for such features as hilltops, saddles, bridge floors, water surfaces, and intersections of streets, roads, and railroads.

Contours are ordinarily used on topographic maps made for engineering purposes. When the area being mapped includes both hilly and flat terrain, it is customary to show auxiliary or supplementary contours across the flat areas, these being lines of short dashes or dots, so that they may be readily distinguished from the conventional full-line contours. The interval for supplementary contours is usually one-half the regular interval for the map, although supplementary contours at one-fourth interval and one-fifth interval are also utilized when there are extreme differences between hill slopes and flat areas.

Approximate contours are those that cannot be shown within the tolerance required for conventional contours, but nevertheless are reasonably reliable in indicating approximate elevations of topographic features. Approximate contours are usually shown in lines of medium-length dashes, so that they can be distinguished from supplementary contours.

Form lines, which resemble contours in appearance, are used on reconnaissance-type maps to depict the more significant topographic features. No attempt is made to maintain a uniform vertical distance or interval between adjacent form lines, and therefore form lines cannot be relied upon for yielding even approximate elevations of topographic features, although spot elevations of prominent topographic features, when available, should be shown. Form lines are shown by means of long dashes, and are thus distinguishable from conventional, supplementary, and approximate contours.

61. MAP ACCURACY SPECIFICATIONS

The minimum accuracy to be attained in any topographic map can be specified rather definitely and in simple terms, and tests for determining the compliance of any map sheet with the accuracy specifications can be readily made. The "standard" map accuracy specifications of the United States Government, for maps of all scales and contour intervals, are briefly as follows:

Horizontal accuracy. For maps on publication scales larger than 1:20,000, not more than 10 per cent of the points tested shall be in error by more than $^1/_{30}$ inch, measured on the publications scale; for maps on publication scales of 1:20,000 or smaller, $^1/_{50}$ inch. These limits of accuracy shall apply in all cases to positions of well defined points only. "Well defined" points are those that are easily visible or recoverable on the ground, such as the following: monuments or markers, such as bench marks, property boundary monuments; intersections of roads, railroads, etc.; corners of large buildings or structures (or center points of small buildings); etc.

Vertical accuracy, as applied to contour maps on all publication scales, shall be such that not more than 10 per cent of the elevations tested shall be in error more than one-half of the contour interval. In checking elevations taken from the map, the apparent vertical error may be decreased by assuming a horizontal displacement within the permissible horizontal error for a map of that scale.

Similar specifications are recommended by the American Society of Civil Engineers in Manual 10, *Technical Procedure for City Surveys.*

With slight modifications, the so-called standard specifications can be applied to almost any topographic mapping project, regardless of the map scale or contour interval. However, the horizontal-accuracy tolerances may become impractical when the map sheets are prepared on tracing linen, vellum, unmounted paper, or other materials that will shrink and expand unevenly with changes in humidity, and that do not permit pinpoint plotting and fineline drafting. In such cases, the tolerance for horizontal displacement might well be increased to $^1/_{20}$ in., and even to a larger amount in unusual circumstances. There is not much to be gained, however, by liberalizing the usual half-interval tolerance for standard contours. Actually, this tolerance is not difficult to obtain, provided that appropriate procedures and equipment are used, and provided also that the contour interval is in proper relation both to the map scale and the type of terrain involved. In considering accuracy specifications for special-purpose maps, it is well to bear in mind that a small-scale map of standard accuracy may be as useful as a larger-scale map of substandard accuracy; and that large-interval contours of standard accuracy may be as useful as smaller-interval contours of substandard accuracy.

Topographic maps are usually tested for accuracy compliance by means of traverse-and-profile surveys. The horizontal accuracy of a map may be sample tested by running an

accurate transit-tape traverse at random across any section of the mapped area, originating and closing on stations of the control surveys. All "well-defined planimetric features" lying along or near the traverse route are tied in, so that the plane coordinates of each such feature can be computed. Each test feature is then plotted on the map sheet, and the distance on the map, to the nearest $^1/_{200}$ in., is scaled from the plotted test point to the map location for the same feature. Contour accuracy may be conveniently tested by determining an accurate elevation at each significant change of slope along the traverse route. The location of each such test elevation is then plotted on the map sheet, and the value of the test elevation is compared with the elevation of the plotted test point as interpolated from the contours of the map. In steep terrain, if the elevation difference at any test point exceeds the tolerance, either or both of the contours adjacent to the test point may be temporarily shifted or displaced in any direction on the sheet, by the same amount that is permitted for horizontal displacement of the planimetric features. It should be noted, however, that this provision for an imaginary contour "shift" will be helpful only where the contours are very closely spaced on the map.

ROUTE SURVEYS

By William T. Pryor

62. ROUTE LOCATION METHODS

The location for a highway, railroad, transmission line, canal, or pipe line is determined from information supplied through three types of surveys, i.e., reconnaissance (usually in two stages), preliminary survey, and location survey. The first stage of reconnaissance will determine whether a proposed project is feasible, and if so, whether more than one location is available. The second stage of reconnaissance will determine the best route for more detailed survey.

The preliminary survey will involve a thorough investigation of a band of territory that may vary in width from a few hundred feet to a mile or more. The end result may be a strip topographic map covering the route selected during the reconnaissance. The variations of alignment, grade, and cross section may then be studied, and the line selected which is most suitable and economical.

The location survey includes the staking of the selected location on the ground for construction. As the work progresses, minor adjustments in line and grade are made, to obtain most advantageous location.

It should be noted that on many projects there will be no definite subdivision into types of surveys, e.g., reconnaissance and preliminary surveys may be simultaneous operations, and preliminary surveys may become location surveys where the problem is relatively simple. Conversely, where the problem is complicated, as for a new highway location in a well-developed area, the first operations of the location survey may be mainly for the purpose of obtaining more accurate and detailed information for refinement of design, and thus would be in reality a second-stage preliminary survey. New surveys would then be required along the actual location zone, including alignment, profile, cross sections, right-of-way surveys, and soil and foundation data. In these cases the center line may be significantly changed during the process of redesign, and the staking or restaking of the adjusted location, as well as slope staking, structure staking, and right-of-way staking, would necessarily be delayed until the redesign had been completed.

63. RECONNAISSANCE SURVEYS

In all route surveys, before most field work is started, maps of the area should be assembled and studied. Topographic maps, such as the quadrangle sheets of the U. S. Geological Survey, are especially useful. Geologic maps, soil maps, county highway maps, and other maps of the area will provide valuable supplemental data. Aerial photographs are especially useful for reconnaissance purposes, as they will provide supplemental topographic and land-use information, or can be used instead of topographic maps when none are available. When no maps of any sort are available, the photographs can be assembled into an uncontrolled mosaic, for use as a map substitute. Also, the individual photographs should be examined stereoscopically for best identification and interpretation of information that may not be represented on maps.

The first stage of a reconnaissance is to lay out on the best available map each general route that appears to be feasible, giving due regard to the fixed controls. The locating engineer should then traverse the area by car, on horseback, or on foot, to become familiar with the area and its special conditions, and to note the changes that have occurred since the map was prepared, as well as features that were omitted from the map

When neither maps nor photographs are available, the entire investigation must be made in the field, and survey data for preparing a reconnaissance map must be obtained. Approximate field measurements will suffice for this purpose; hence distances may be measured by pacing or by odometer; bearings of courses may be obtained by compass; and elevations may be determined by aneroid barometer and/or Abney level. The end result of the first-stage or area reconnaissance should be a map, mosaic, or other exhibit showing each feasible route in correct relation to all others, and to the principal controls; or the location of each route may be marked on the ground.

When it is found that two or more routes of nearly equal desirability are available, a second-stage reconnaissance of the alternatives, more complete and detailed, should be made. This is accomplished by carefully examining each route in the field. If large-scale photographs for each route are available, however, or can be provided, an intensive stereoscopic examination of them will be the best means of obtaining comparative information for each route as to topography, soils, drainage areas, land use, and other required data. In order to compare the alternatives fully it may be necessary to prepare a reconnaissance estimate of the construction cost for each feasible route. In that case, it will be desirable to obtain data on directionality, length, rise and fall, serviceability, accessibility, land values, characteristics of subsurface materials, elevations of high-water marks, erosion tendencies and treatments, local sources of suitable construction materials, and all other qualities that will aid in comparing the alternatives.

If the reconnaissance investigations are not adequate for the selection of the best route, additional surveys by transit, level, tape, or photogrammetry will be required. These more detailed surveys are called the preliminary survey, and their main purpose is to obtain all the information needed for determination (design) of the best position for the works on the selected route, although they sometimes must be used also to complete the comparison of routes of nearly equal qualities.

64. PRELIMINARY SURVEY

The preliminary survey of a highway or railroad location supplies information that is used in designing alignment and grade. It includes (1) a transit-tape traverse, called the *P* line, that follows as near as practicable the route selected during the reconnaissance; (2) benchmark levels; (3) profile levels along the same line; and (4) topographic and property-ownership surveys and map of a strip area on both sides of the traverse. On those projects where the traverse stations and benchmarks will be used later as position controls for the location survey and other purposes, it is advisable to make the preliminary surveys of third-order accuracy or better, and to describe and reference the traverse stations and benchmarks for future recovery. It is desirable to use permanent-type benchmarks, and to include these as instrument stations in the traverse whenever practicable.

The traverse should be based on a grid-coordinate system, preferably the official state plane-coordinate system, and the topographic map should be plotted on a projection of coordinate lines. Level datum should be mean-sea-level whenever practicable.

The strip of topography on either side of the *P* line may vary greatly in width, depending on the topography and land use. For a very narrow strip the cross-section method of topographic surveying may be used. For a strip several hundred feet in width the transit-stadia or plane-table method may be best. For a very wide strip extending over a long route, it may be economical to prepare the topographic map from aerial photographs. Regardless of the method used, the final map should be on a scale within the range of from 1:600 (50 ft to 1 in.) to 1:6000 (500 ft to 1 in.), the general preference being for the scales of 1:1200 or 1:2400. The largest scales and smallest contour intervals are used where land values are high. The contour interval should be in the range of from 1 ft to 10 ft, the general preference being for 2 ft or 5 ft. The strip topographic map should be of standard accuracy, and should show complete planimetry and contours, all *P* line stations and benchmarks, and relevant features such as property ownership and land use. The traverse-control surveys that are necessary for maps prepared by photogrammetric methods should be preserved by marking some of the stations, at intervals of 1/2 mile to 1 mile, with permanent monuments, as these will later be used as reference points for the location survey.

Certain types of projects, as pole-and-wire lines, require only a narrow strip of topography, as the profile of the *P* line and parallel lines a short distance away will furnish nearly all required information; also the profile elevations need be only accurate enough to insure proper clearance of wire above ground, and suitability of vertical angles at towers.

65. PROJECTING LOCATION ON TOPOGRAPHIC MAP

On the strip topographic map prepared during the preliminary survey, or on any similar map that may be available, all possible alignments are developed and studied, the objective being to find the location that will provide satisfactory grades and alignment at the least cost. Each alignment that appears feasible should be marked on the map. Trial lines for an improvement that is made up mainly of tangents (as a pole line, cable line, and pipe line) may be marked by straight edges, or by a thread stretched between pins located at proposed angle points. Trial lines for an improvement that requires numerous curves (as a highway, railroad, or canal in hilly topography) may be laid out with a flexible spline.

In areas of considerable relief a trial position for the alignment is determined by setting the scale distance for 100 ft on a pair of dividers and stepping off the maximum grade on the contours. If a 5% grade is specified and the contour interval is 5 ft, beginning at a saddle in the topography, the divider will be pivoted on each contour, up or down a hillside, suitable allowance being made for curvature in the alignment. After selection of the alignment a profile is plotted by scaling the distance to each contour that crosses the proposed line. The proposed grade line is then established, considering all sight distance and clearance requirements. When necessary for a complete study, cross sections based on the contours may be plotted at selected stations.

Study of the profile, grade line, cross sections, adjacent topography, and horizontal alignment features will indicate better sites for structures and places where minor adjustments will improve the alignment or grade, and decrease quantities; also where balance of cut-and-fill quantities is practicable. The procedure is repeated as often as necessary, until the best compromise of cost vs. acceptable alignment and grade has been found.

Other features that may restrict or affect the location of a highway include proper alignment for approaches to bridge sites, connections and intersections, providing height of grade that will be adequate for culverts and will be above recurring high-water levels; and providing fill slopes that are practicable and will "catch" without requiring costly retaining walls.

A description of the alignment finally selected is computed by means of the coordinate grid system on the map. When the alignment contains curves this will include the type and length of all curves, with their beginning and ending points, and the coordinate position, length, and bearing of the joining tangents. The station value of each angle point is then computed, after which the location of each 100-ft station is plotted on the map for the entire alignment.

For improvements requiring grading, a preliminary estimate of cut-and-fill quantities is necessary. For this purpose, cross sections are plotted from the contours of the map at each 100-ft station and intermediate point. The grading computations may indicate where adjustments in alignment, grade line, or both, will be advantageous in attaining a better balance[1] of cuts and fills, or in decreasing length of haul.

A preliminary estimate of total cost is prepared, which should include the cost of right-of-way, severance damages, structures, grading, drainage and surfacing. When there are two or more different locations available within the mapped area, a similar complete study and cost estimate will be required for each alignment, to be certain that the best alignment has been selected.

66. LOCATION SURVEY

The purposes of the location survey for a highway, railroad, or canal are: (1) to provide location stakes and grade information for the center line; (2) to provide base-line survey stations for cross sections and for setting slope stakes, structure stakes, and right-of-way stakes, and (3) to provide data for final design and computation of grading and other pay quantities. The first operations are the staking of the selected center line, and the running of profile levels.

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[1] See Article 77.

The location line, called the L line, is staked by standard transit-tape traverse methods, the preliminary P line being used as a reference line for the staking of key stations on the L line. The L line may not in all cases be in the exact location of the final center line, but it is intended to serve as the survey base line for the project during the entire time of construction. It is therefore expected to be located within such narrow limits as to insure construction within the width of any ordinary right-of-way. The L-line instrument-station stakes should be well referenced to points outside of the construction area so that they may be reestablished as construction progresses, and for use after the construction has been completed for running as-constructed line, profiles, and cross sections to determine final payment quantities.

In running the L line, stakes should be set at all 100-ft stations, at all intermediate points where there is a break in profile, and at all required points on curves. A cross section is taken at each 100-ft station, and at each intermediate plus point. When the final grade line is determined, grade stakes and slope stakes are set for each 100-ft station and plus point. Stakes are also set for all structures, and for right-of-way boundaries.

For a project that does not require grading, as a transmission line, the location survey may be limited to staking the locations for the towers or poles.

67. THE ELEMENTS OF A CIRCULAR CURVE

Points A and B, Fig. 41, where straight lines (tangents) are tangent to the circular curve, are called: P.C., point of curvature, for the beginning of the curve; and P.T., point of tangency, for the end of the curve. The intersection of the two tangents of the circular curve is called the P.I., point of intersection. The angle at the P.I. between the two tangents of the circular curve, Δ, is the same as the angle subtended between the radial lines, OA and OB, that are perpendicular to the two tangents at the P.C. and P.T., respectively. That part of each tangent which extends between the P.C. and the P.I., and between the P.T. and the P.I., is called the semitangent T. Arc ApB is known as the length of curve, usually designated L. The straight line from point A to point B, connecting the P.C. and P.T., is the chord C of the circular curve. The length of line from the P.I. to the circular curve at point p is called the external distance E, and the length of line from point p to the chord is called the middle ordinate M. The distance from the P.C. along the semitangent, or the semitangent extended, to a point K is the tangent distance t_d; and K is the point at which a perpendicular to the tangent from the circular curve intersects the tangent. The length of line from point K to the circular curve is the tangent offset t_o.

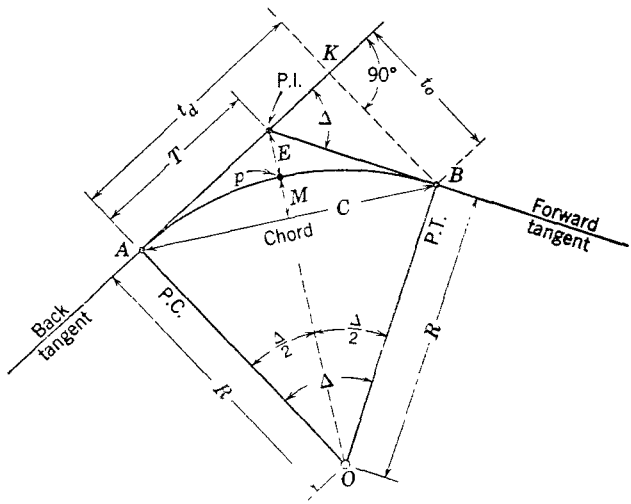

FIG. 41. Elements of a circular curve

Circular curves of different length radius curving in the same direction, when contiguous with each other, are called compound circular curves. Fig. 42. The point where compound circular curves touch a common tangent is called the point of compound curvature P.C.C. Other parts of such curves, except for the P.C. and P.T. points, are separately identified by use of subscript numbers.

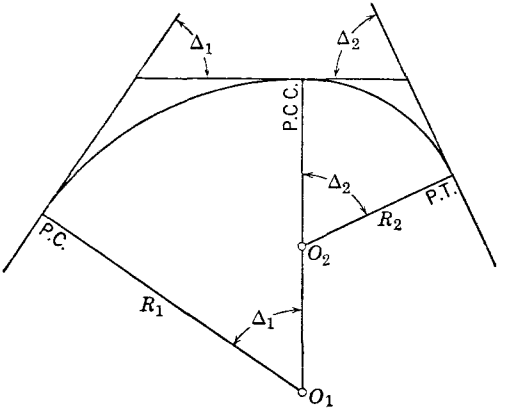

FIG. 42 Compound circular curves.

Circular curves, whether of the same length radius or not, that are curving in opposite direction and are joined at a common point of tangency, are called reversed circular curves, Fig. 43. The point of reversal is called the P.R C., point of reversed curve. Other parts of reversed curves, except for the P.C. and P.T. points, are separately identified by use of subscript numbers in the sequential order of the curves.

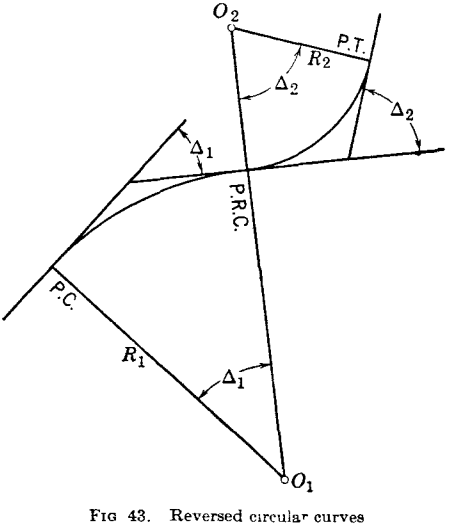

FIG 43. Reversed circular curves

There are three systems for expressing the rate of curvature of a circular curve: (1) Fig. 44, by the length of the radius; (2) Fig. 45, by the number of degrees subtended by an arc of the curve 100 units long; and (3) Fig. 46, by the number of degrees subtended by a chord of the curve 100 units long. In all three systems, distances may be in feet, meters,

or other units. In the metric system of measurement, the degree of curve is sometimes defined as the number of degrees subtended by an arc with a chord that is 20 m long rather than 100 m.

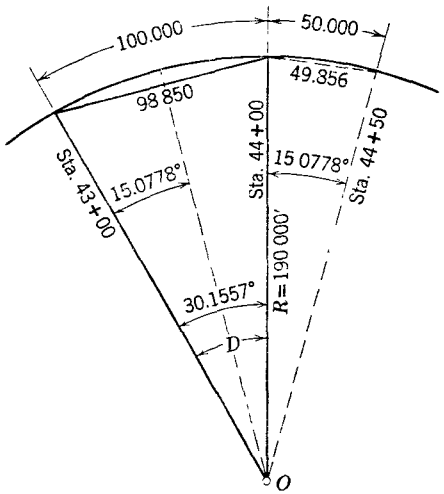

FIG 44 Radius definition $D = \frac{5729.57795}{R}$

The chord definition of circular curves is used most in railroad engineering, for which curve tables were prepared at an early date. The inaccuracies of the chord definition and other difficulties arising in the engineering work (such as the lack of proportionality

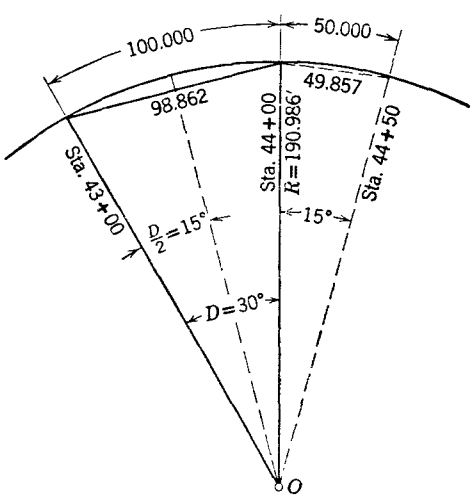

FIG 45. Arc definition. $R = \frac{5729.57795}{D}$

between the degree and radius of the curve) and especially for sharp curves have caused many highway engineers and highway departments to discontinue use of that system. Instead, the radius definition and arc definition have been adopted, with the latter being preferred by many engineers.

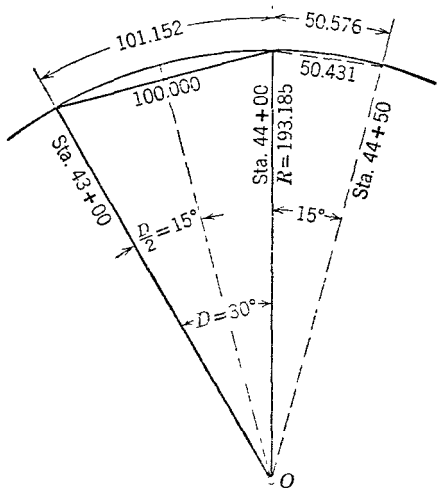

FIG. 46 Chord definition $R = \dfrac{50}{\sin D/2}$

Selected elements of the circular curve, Fig. 41 may be computed by use of the proper formula.

The delta angle, $\Delta = DL_c/100$ and $\Delta = \text{Arc length} \times 360°/2\pi R$

$$\sin\frac{\Delta}{2} = \frac{C}{2R} = \frac{E+M}{T} = \frac{T}{\sqrt{T^2+R^2}} \qquad \text{vers}\,\frac{\Delta}{2} = \frac{M}{R}$$

$$\tan\frac{\Delta}{2} = \frac{T}{R} = \frac{E+M}{C/2} \qquad \text{exsec}\,\frac{\Delta}{2} = \frac{E}{R}$$

$$\tan\frac{\Delta}{4} = \frac{2M}{C} = \frac{E}{T} = \sqrt{\frac{E-M}{E+M}} \qquad \cot\frac{\Delta}{2} = \frac{R}{T} = \frac{C/2}{E+M}$$

$$\cos\frac{\Delta}{2} = \frac{C}{2T} = \frac{R}{R+E} = \frac{R-M}{R} \qquad \Delta = 4\,\text{arc cotan}\,\frac{C}{2M} = 4\,\text{arc tan}\,\frac{2M}{C}$$

The radius, R

Radius definition $R = \dfrac{360L_c}{2\pi\Delta} = \dfrac{TC}{2(E+M)}$

Arc definition $R = \dfrac{5729.57795}{D} = \dfrac{TC}{2(E+M)} = \dfrac{EM}{E-M} = \left(\dfrac{M}{2} + \dfrac{C^2}{8M}\right)$

Chord definition $R = \dfrac{50}{\sin D/2} = \dfrac{C/2}{\sin \Delta/2} = T\,\text{cotan}\,\dfrac{\Delta}{2}$

Degree of curve, D

Radius definition $D = \dfrac{5729.57795}{R}$

Arc definition $D = \dfrac{5729.57795}{R}$

Chord definition $D = \dfrac{100\Delta}{L_c} = 2\,\text{arc sin}\,\dfrac{50}{R} = \dfrac{5730}{R}$ (approx.)

Length of curve, L_c

Radius definition $L_c = \frac{\pi R \Delta}{180°} = R$ (length of arc for radius 1)

Arc definition $L_c = \frac{100\Delta}{D} =$ arc

Chord definition $L_c = \frac{\Delta}{D} =$ number of chords 100 units long

Chord C

$$C = 2R \sin\frac{\Delta}{2} = 2T \cos\frac{\Delta}{2} = 2M \cot\frac{\Delta}{4} = 2R \sin\left(\frac{DL_c}{200}\right)$$

$$C = 2\sqrt{M(2R - M)} = \frac{2E \sin \Delta/2}{\text{exsec}\, \Delta/2} = 2(E - M) \cot\frac{\Delta}{2}$$

Semitangent, T

$$T = R \tan\frac{\Delta}{2} = E \cot\frac{\Delta}{4} = \frac{C}{2} \sec\frac{\Delta}{2} = \frac{2R(E + M)}{C}$$

$$T = \sqrt{E(2R + E)} = E\sqrt{\frac{E + M}{E - M}} = M\frac{\tan \Delta/2}{\text{vers}\, \Delta/2} = 6M \text{ (approx.)}$$

Tangent distance, t_d

$$t_d = R \sin\left(\frac{DL_c}{100}\right) = C \cos\left(\frac{DL_c}{200}\right)$$

$$t_d = \left[R \tan\left(\frac{DL_c}{200}\right)\right]\left[1 + \cos\left(\frac{DL_c}{100}\right)\right]$$

Tangent offset, t_o

$$t_o = \text{vers}\left(\frac{DL_c}{100}\right) = C \sin\left(\frac{DL_c}{200}\right) = \frac{C^2}{2R} = 4M \text{ (approx.)}$$

External distance, E

$$E = R\, \text{exsec}\frac{\Delta}{2} = \frac{CT}{2R} - M = \frac{C}{2}\tan\frac{\Delta}{2} - M = \sqrt{T^2 + R^2} - R$$

$$E = \frac{C}{2}\,\text{cosec}\frac{\Delta}{2}\,\text{exsec}\frac{\Delta}{2} = M \sec\frac{\Delta}{2} = \frac{RM}{R - M} = T \tan\frac{\Delta}{4}$$

Middle ordinate, M

$$M = R\,\text{vers}\frac{\Delta}{2} = \frac{RE}{R + E} = R\,\text{vers}\left(\frac{DL_c}{200}\right) = R - \sqrt{(R + C/2)(R - C/2)}$$

$$M = E \cos\frac{\Delta}{2} = \frac{C}{2}\tan\frac{\Delta}{4} = T \cot\frac{\Delta}{2}\,\text{vers}\frac{\Delta}{2} = t_o/4 \text{ (approx.)}$$

$$M = \frac{C^2}{8R} \text{ (approx.)} = \frac{T}{6} \text{ (approx.)}$$

Table 3. Length of Circular Arcs, Radius 1

To determine the length of a circular curve, multiply the tabular value for the Δ angle of the curve by its radius. Whenever the Δ angle contains minutes, seconds, and degrees, add the appropriate value for the respective minutes, seconds, and degrees to ascertain the figure to be multiplied by the radius. This table is most useful whenever the curve is of even length radius, resulting in an odd degree of curve. It will also give exact results when the degree of curve is even and the length of the radius is odd, but in such a case the curve length is more easily computed by use of the usual formula $L_c = 100\Delta/D$.

Δ Angle, seconds	Length for $R = 1$	Δ Angle, minutes	Length for $R = 1$	Δ Angle, degrees	Length for $R = 1$
0	0.00000000	0	0.00000000	0	0.00000000
1	0.00000485	1	0.00029089	1	0.01745329
2	0.00000970	2	0.00058178	2	0.03490659
3	0.00001454	3	0.00087266	3	0.05235988
4	0.00001939	4	0.00116355	4	0.06981317
5	0.00002424	5	0.00145444	5	0.08726646
6	0.00002909	6	0.00174533	6	0.10471976
7	0.00003394	7	0.00203622	7	0.12217305
8	0.00003879	8	0.00232711	8	0.13962634
9	0.00004363	9	0.00261799	9	0.15707963
10	0.00004848	10	0.00290888	10	0.17453293
11	0.00005333	11	0.00319977	11	0.19198622
12	0.00005818	12	0.00349066	12	0.20943951
13	0.00006303	13	0.00378155	13	0.22689280
14	0.00006787	14	0.00407243	14	0.24434610
15	0.00007272	15	0.00436332	15	0.26179939
16	0.00007757	16	0.00465421	16	0.27925268
17	0.00008242	17	0.00494510	17	0.29670597
18	0.00008727	18	0.00523599	18	0.31415927
19	0.00009211	19	0.00552688	19	0.33161256
20	0.00009696	20	0.00581776	20	0.34906585
21	0.00010181	21	0.00610865	21	0.36651914
22	0.00010666	22	0.00639954	22	0.38397244
23	0.00011151	23	0.00669043	23	0.40142573
24	0.00011636	24	0.00698132	24	0.41887902
25	0.00012120	25	0.00727221	25	0.43633231
26	0.00012605	26	0.00756309	26	0.45378561
27	0.00013090	27	0.00785398	27	0.47123890
28	0.00013575	28	0.00814487	28	0.48869219
29	0.00014060	29	0.00843576	29	0.50614548
30	0.00014544	30	0.00872665	30	0.52359878
31	0.00015029	31	0.00901753	31	0.54105207
32	0.00015514	32	0.00930842	32	0.55850536
33	0.00015999	33	0.00959931	33	0.57595865
34	0.00016484	34	0.00989020	34	0.59341195
35	0.00016968	35	0.01018109	35	0.61086524
36	0.00017453	36	0.01047198	36	0.61831853
37	0.00017938	37	0.01076286	37	0.64577182
38	0.00018423	38	0.01105375	38	0.66322512
39	0.00018908	39	0.01134464	39	0.68067841
40	0.00019393	40	0.01163553	40	0.69813170
41	0.00019877	41	0.01192642	41	0.71558499
42	0.00020362	42	0.01221730	42	0.73303829
43	0.00020847	43	0.01250819	43	0.75049158
44	0.00021332	44	0.01279908	44	0.76794487
45	0.00021817	45	0.01308997	45	0.78539816
46	0.00022301	46	0.01338086	46	0.80285146
47	0.00022786	47	0.01367175	47	0.82030475
48	0.00023271	48	0.01396263	48	0.83775804
49	0.00023756	49	0.01425352	49	0.85521133
50	0.00024241	50	0.01454441	50	0.87266463

Table 3. Length of Circular Arcs, Radius 1—*Continued*

Δ Angle, seconds	Length for $R = 1$	Δ Angle, minutes	Length for $R = 1$	Δ Angle, degrees	Length for $R = 1$
51	0.00024725	51	0.01483530	51	0.89011792
52	0.00025210	52	0 01512619	52	0.90757121
53	0.00025695	53	0 01541708	53	0.92502450
54	0 00026180	54	0.01570796	54	0.94247780
55	0.00026665	55	0.01599885	55	0.95993109
56	0.00027150	56	0.01628974	56	0.97738438
57	0.00027634	57	0.01658063	57	0.99483767
58	0.00028119	58	0.01687152	58	1.01229097
59	0.00028604	59	0.01716240	59	1.02974426
60	0.00029089	60	0.01745329	60	1.04719755
Δ Angle, degrees		Δ Angle, degrees			
61	1.06465084	91	1 58824962	121	2.11184839
62	1.08210414	92	1.60570291	122	2 12930169
63	1.09955743	93	1.62315620	123	2 14675498
64	1.11701072	94	1.64060950	124	2 16420827
65	1 13446401	95	1.65806279	125	2.18166156
66	1.15191731	96	1.67551608	126	2.19911486
67	1.16937060	97	1.69296937	127	2 21656815
68	1.18682389	98	1 71042267	128	2 23402144
69	1.20427718	99	1.72787596	129	2.25147474
70	1.22173048	100	1.74532925	130	2.26892803
71	1.23918377	101	1.76278254	131	2.28638132
72	1.25663706	102	1.78023584	132	2.30383461
73	1.27409035	103	1 79768913	133	2.32128791
74	1.29154365	104	1.81514242	134	2.33874120
75	1.30899694	105	1 83259571	135	2.35619449
76	1.32645023	106	1 85004901	136	2.37364778
77	1.34390352	107	1.86750230	137	2.39110108
78	1.36135682	108	1.88495559	138	2.40855437
79	1.37881011	109	1.90240888	139	2.42600766
80	1.39626340	110	1.91986218	140	2.44346095
81	1.41371669	111	1.93731547	141	2 46091425
82	1.43116999	112	1.95476876	142	2.47836754
83	1.44862328	113	1.97222205	143	2 49582083
84	1.46607657	114	1.98967535	144	2.51327412
85	1.48352986	115	2.00712864	145	2 53072742
86	1.50098316	116	2.02458193	146	2.54818071
87	1.51843645	117	2 04203522	147	2 56563400
88	1.53588974	118	2 05948852	148	2 58308729
89	1.55334303	119	2 07694181	149	2.60054059
90	1.57079633	120	2.09439510	150	2.61799388

Table 4. Circular Curve Table (Arc Definition) *

This table is the field engineers' ready reference. For each degree of curve used most is given the radius, logarithm of the radius for calculation purposes, minutes deflection for an arc of 1, and the chord for arcs of 25, 50, and 100. To determine the deflection in minutes, multiply the curve length for which the deflection is desired by the minutes for an arc of 1. This principle is and the chords for these different arc lengths are applicable in both the foot and metric systems of measurement.

Degree of Curve	Radius of Curve	Logarithm of Radius	Minutes Deflection for Arc of 1	Chord for Arc of 25	Chord for Arc of 50	Chord for Arc of 100
0° 00′	Infinite	Infinite	0 00	25 000	50 000	100 000
30′	11 459 156	4 05915263	0.15	25 000	50.000	100.000
1° 00′	5 729 578	3 75812263	0 30	25 000	50 000	99.999
30′	3 819 719	3 58203137	0.45	25 000	50 000	99 997
2° 00′	2 864 789	3 45709264	0 60	25 000	49 999	99 995
30′	2 291 831	3 36018262	0.75	25 000	49 999	99.992
3° 00′	1 909 859	3.28100138	0.90	25 000	49 999	99.989
30′	1.637 022	3 21405459	1 05	25 000	49 998	99.984
4° 00′	1 432 394	3 15606264	1 20	25 000	49.997	99 980
30′	1 273.240	3 10491012	1 35	25 000	49 997	99 974
5° 00′	1 145 916	3 05915263	1.50	25 000	49 996	99 968
30′	1 041 741	3 01775994	1.65	24 999	49 995	99.962
6° 00′	954 930	2 97997138	1 80	24.999	49 994	99.954
30′	881.474	2 94520928	1 95	24.999	49.993	99 946
7° 00′	818 511	2 91302459	2 10	24.999	49.992	99.938
30′	763.944	2 88306137	2 25	24 999	49.991	99.929
8° 00′	716 197	2 85503265	2.40	24 999	49.990	99.919
30′	674 068	2 82870371	2.55	24 999	49 989	99.908
9° 00′	636 620	2 80388012	2 70	24 998	49 987	99 897
30′	603 113	2 78039903	2 85	24 998	49 986	99.885
10° 00′	572 958	2 75812263	3.00	24 998	49 984	99.873
11°	520 871	2 71672995	3 30	24 998	49 981	99 846
12°	477 465	2 67894139	3.60	24 997	49 977	99.817
13°	440 737	2 64417928	3.90	24 997	49.973	99.786
14°	409 256	2 61199460	4.20	24 996	49 969	99.751
15°	381 972	2 58203137	4 50	24 996	49 964	99.715
16°	358 099	2 55400265	4.80	24 995	49 959	99.675
17°	337 034	2 52767371	5 10	24 994	49 954	99.634
18°	318 310	2 50285013	5.40	24 994	49.949	99 589
19°	301 557	2 47936903	5 70	24 993	49 943	99 542
20°	286.479	2 45709264	6.00	24.992	49 937	99.493
21°	272.837	2 43590334	6 30	24 991	49.930	99.441
22°	260.435	2 41569995	6 60	24 990	49.923	99.387
23°	249.112	2 39639480	6 90	24 990	49 916	99.330
24°	238 732	2 37791139	7 20	24 989	49.909	99.271
25°	229.183	2 36018262	7.50	24 988	49.901	99.209
30°	190 986	2 28100138	9.00	24 982	49 857	98.862
35°	163 702	2 21405459	10.50	24 976	49 806	98.452
40°	143 239	2 15606264	12.00	24 968	49 747	97 982
50°	114 592	2 05915263	15.00	24 950	49 604	96 857
60°	95.493	1.97997138	18 00	24 929	49 431	95 493
70°	81 851	1.91302459	21 00	24 903	49 226	93 896
80°	71.620	1.85503265	24 00	24 873	48 991	92 073
90°	63 662	1 80388012	27 00	24 840	48.725	90 032
100°	57 296	1.75812263	30 00	24 802	48 428	87 782

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Table 5. Degree of Curve and Radius in the Metric System and the Nearest Equivalent Degree in the Foot System

(The degree of curve is the angle at the center subtended by a 100-meter arc.)

This table is useful to the engineer who has the design speed and other relevant design data given in the foot system and is charged with the responsibility of designing and staking curves on the ground in the metric system. Once the curve is determined in one system of measurement, its nearest equivalent can be easily selected for use from this table.

Degree	Radius, meters	Equivalent Degree, foot system	Degree	Radius, meters	Equivalent Degree, foot system	Degree	Radius, meters	Equivalent Degree foot system
0°	Infinite	0° 00′	26°	220 368	7° 55 489	150°	38.1972	45° 43.205′
1°	5729.578	0° 18 288′	27°	212 207	8° 13 777′	160°	35.8099	48° 46 086′
2°	2864 789	0° 36 576′	28°	204 628	8° 32 065′	170°	33.7034	51° 48 966′
3°	1909 859	0° 54.864′	29°	197.572	8° 50 353′	180°	31 8310	54° 51 847′
4°	1432.394	1° 13.152′	30°	190.986	9° 08 641′	190°	30.1557	57° 54.727′
5°	1145.916	1° 31.440′	32°	179 049	9° 45.217′	200°	28 6479	60° 57.607′
6°	954 930	1° 49 728′	34°	168.517	10° 21.793′	220°	26 0435	67° 03.368′
7°	818.511	2° 08.016′	36°	159 155	10° 58 369′	240°	23.8732	73° 09.129′
8°	716.197	2° 26.304′	38°	150.778	11° 34 945′	260°	22.0368	79° 14.890′
9°	636.620	2° 44.592′	40°	143 239	12° 11.521′	280°	20 4628	85° 20 650′
10°	572.958	3° 02.880′	45°	127.324	13° 42.962′	300°	19.0986	91° 26 411′
11°	520.871	3° 21 168′	50°	114 592	15° 14.402′	320°	17 9049	97° 32 172′
12°	477.465	3° 39.456′	55°	104.174	16° 45.842′	340°	16 8517	103° 37.932′
13°	440.737	3° 57.744′	60°	95 493	18° 17.282′	360°	15.9155	109° 43.693′
14°	409.256	4° 16.033′	65°	88.147	19° 48 722′	380°	15.0778	115° 49 454′
15°	381.972	4° 34.321′	70°	81 851	21° 20 163′	400°	14.3239	121° 55.215,
16°	358.099	4° 52.609′	75°	76.394	22° 51.603′	420°	13.6419	128° 00.975′
17°	337.034	5° 10.897′	80°	71.620	24° 23.043′	440°	13 0218	134° 06 736′
18°	318.310	5° 29.185′	85°	67.407	25° 54.483′	460°	12 4556	140° 12.497′
19°	301 557	5° 47 473′	90°	63 662	27° 25 923′	480°	11.9366	146° 18.258′
20°	286.479	6° 05 761′	95°	60 311	28° 57.363′	500°	11.4592	152° 24 018′
21°	272 837	6° 24 049′	100°	57.296	30° 28.804′	520°	11.0184	158° 29.779′
22°	260.435	6° 42.337′	110°	52 087	33° 31 684′	540°	10 6103	164° 35 540′
23°	249 112	7° 00 625′	120°	47 746	36° 34.564′	560°	10.2314	170° 41 300′
24°	238.721	7° 18 913	130°	44 074	39° 37.445′	580°	9.8786	176° 47.061′
25°	229.183	7° 37.201′	140°	40.926	42° 40.325′	600°	9.5493	182° 52 822′

Table 6. Tangents, Externals, and Lengths to a 1° Curve *

This table is applicable in the foot and metric systems of measurement. To compute the tangent, external or length of a circular curve of any other degree and for any Δ angle in the table, divide the tabular value in the table for the Δ angle of the curve by the degree of curve. The 1° curve values for intermediate Δ angles can be interpolated to three places right of the decimal for lengths and two places for the tangents and externals. When using the interpolated values, do not raise the second-place figure when the third-place figure is 5 and larger. For example, interpolation on a straight-line basis for the external is 2305.7752, if the Δ angle is 89° 02′. Do not raise the second-place 7 to an 8; simply drop the 0.0052 and use the correct result of 2305.77. Similarly, for a Δ angle of 89° 06′ the interpolated value is 2310.3776 and the correct value for the external to two places is 2310.37, not 2310.38. Results of interpolation for tangents should be handled in the same manner.

Δ	Tangent	External	Length	Δ	Tangent	External	Length
0° 01′	0 833	0 000	1.667	8° 00′	400.651	13.991	800.000
10′	8 333	0 006	16 667	10′	409 026	14.581	816.667
20′	16.667	0 024	33.333	20′	417 403	15 184	833.333
30′	25.000	0 055	50.000	30′	425 781	15.799	850 000
40′	33.334	0 097	66.667	40′	434.161	16 426	866.667
50′	41.667	0 151	83.333	50′	442 544	17.065	883.333
1° 00′	50 001	0.218	100 000	9° 00′	450 928	17.717	900.000
10′	58 335	0.297	116.667	10′	459 314	18.381	916.667
20′	66 670	0.388	133.333	20′	467.701	19.057	933.333
30′	75.004	0 491	150 000	30′	476 091	19.746	950 000
40′	83.339	0 606	166 667	40′	484.483	20 447	966 667
50′	91 675	0.733	183.333	50′	492.877	21.160	983.333
2° 00′	100.010	0.873	200.000	10° 00′	501 273	21.886	1000 000
10′	108.346	1.024	216.667	10′	509.671	22.624	1016 667
20′	116.683	1 188	233 333	20′	518 072	23.374	1033.333
30′	125 020	1.364	250 000	30′	526.474	24 137	1050 000
40′	133 357	1.552	266 667	40′	534 879	24 912	1066 667
50′	141.696	1.752	283.333	50′	543.286	25.700	1083 333
3° 00′	150.034	1.964	300.000	11° 00′	551.696	26.500	1100 000
10′	158 374	2 188	316 667	10′	560 107	27.312	1116 667
20′	166 714	2 425	333.333	20′	568.522	28.137	1133 333
30′	175.054	2.674	350.000	30′	576.938	28 974	1150 000
40′	183.396	2.934	366 667	40′	585 357	29.824	1166.667
50′	191.738	3 207	383.333	50′	593.779	30.686	1183.333
4° 00′	200 081	3.492	400 000	12° 00′	602.203	31.560	1200.000
10′	208 425	3.790	416 667	10′	610.630	32.447	1216.667
20′	216.770	4.099	433 333	20′	619 059	33.346	1233.333
30′	225 116	4.421	450 000	30′	627 491	34 258	1250 000
40′	233.462	4 754	466 667	40′	635.925	35.183	1266.667
50′	241.810	5.100	483 333	50′	644 363	36 119	1283.333
5° 00′	250.159	5.458	500 000	13° 00′	652 803	37 069	1300 000
10′	258 509	5.829	516 667	10′	661.246	38.031	1316 667
20′	266 859	6.211	533 333	20′	669 692	39.005	1333.333
30′	275 211	6.606	550 000	30′	678 140	39.992	1350 000
40′	283 565	7.013	566 667	40′	686.592	40 991	1366 667
50′	291.919	7 432	583 333	50′	695 046	42.004	1383.333
6° 00′	300 274	7.863	600 000	14° 00′	703.504	43.028	1400 000
10′	308 631	8 306	616 667	10′	711.964	44 065	1416.667
20′	316 989	8.762	633 333	20′	720.428	45.115	1433.333
30′	325 349	9 230	650.000	30′	728.894	46 177	1450.000
40′	333.710	9 710	666.667	40′	737 364	47 252	1466 667
50′	342 072	10.202	683.333	50′	745 837	48.340	1483.333
7° 00′	350.436	10 707	700 000	15° 00′	754 313	49.440	1500 000
10′	358 801	11 224	716 667	10′	762.793	50 553	1516 667
20′	367.168	11.753	733.333	20′	771 275	51 679	1533.333
30′	375 536	12.294	750 000	30′	779.761	52.817	1550.000
40′	383.906	12 847	766.667	40′	788 251	53.968	1566.667
50′	392.278	13.413	783 333	50′	796.743	55 132	1583.333

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Table 6. Tangents, Externals, and Lengths to a 1° Curve—*Continued*

Δ	Tangent	External	Length	Δ	Tangent	External	Length
16° 00′	805 240	56.308	1600.000	26° 00′	1322.777	150.711	2600.000
10′	813.739	57.497	1616.667	10′	1331 558	152.693	2616.667
20′	822.243	58.699	1633.333	20′	1340.344	154.688	2633.333
30′	830 749	59.913	1650 000	30′	1349 137	156.697	2650.000
40′	839 260	61 140	1666 667	40′	1357 935	158 720	2666.667
50′	847.774	62.381	1683.333	50′	1366 739	160.757	2683.333
17° 00′	856.291	63 633	1700 000	27° 00′	1375 550	162.807	2700.000
10′	864.812	64 899	1716 667	10′	1384.367	164 872	2716.667
20′	873.338	66 178	1733.333	20′	1393 190	166.950	2733 333
30′	881.866	67.469	1750.000	30′	1402.019	169 042	2750 000
40′	890.399	68.773	1766.667	40′	1410 854	171.149	2766 667
50′	898 936	70.090	1783 333	50′	1419 696	173.269	2783.333
18° 00′	907 476	71.420	1800.000	28° 00′	1428.544	175 403	2800 000
10′	916 020	72.763	1816.667	10′	1437 399	177.552	2816.667
20′	924 569	74.118	1833.333	20′	1446 260	179.714	2833.333
30′	933.121	75.487	1850 000	30′	1455 127	181 891	2850.000
40′	941 677	76.868	1866 667	40′	1464.002	184 081	2866 667
50′	950 238	78 263	1883 333	50′	1472.882	186 286	2883 333
19° 00′	958 803	79.670	1900 000	29° 00′	1481.770	188.505	2900.000
10′	967.371	81.091	1916 667	10′	1490.664	190.738	2916 667
20′	975 944	82.524	1933 333	20′	1499 564	192 985	2933 333
30′	984 522	83.971	1950.000	30′	1508 472	195 247	2950.000
40′	993 103	85 430	1966 667	40′	1517 386	197 523	2966.667
50′	1001.689	86.902	1983.333	50′	1526 308	199 813	2983 333
20° 00′	1010 279	88 388	2000.000	30° 00′	1535 236	202.118	3000 000
10′	1018 874	89 887	2016 667	10′	1544.171	204 436	3016 667
20′	1027 473	91 398	2033 333	20′	1553 113	206 770	3033 333
30′	1036 076	92.923	2050.000	30′	1562.062	209 117	3050 000
40′	1044.685	94.461	2066.667	40′	1571.019	211.479	3066.667
50′	1053.297	96.012	2083 333	50′	1579.982	213 856	3083 333
21° 00′	1061 915	97 576	2100 000	31° 00′	1588 953	216 247	3100 000
10′	1070 536	99.154	2116 667	10′	1597.930	218 652	3116.667
20′	1079 163	100 744	2133 333	20′	1606 916	221.073	3133.333
30′	1087.794	102 348	2150 000	30′	1615 908	223 507	3150.000
40′	1096.430	103.965	2166 667	40′	1624 908	225.956	3166.667
50′	1105 071	105.595	2183 333	50′	1633 915	228 420	3183 333
22° 00′	1113.717	107.239	2200.000	32° 00′	1642 930	230.899	3200.000
10′	1122 368	108 896	2216 667	10′	1651.952	233 392	3216.667
20′	1131.023	110 566	2233 333	20′	1660 982	235.900	3233 333
30′	1139 684	112 249	2250 000	30′	1670 020	238.423	3250 000
40′	1148.349	113 946	2266 667	40′	1679 065	240 960	3266 667
50′	1157 020	115.656	2283 333	50′	1688 118	243 512	3283.333
23° 00′	1165 696	117.379	2300 000	33° 00′	1697 178	246.079	3300.000
10′	1174 377	119.116	2316 667	10′	1706 247	248.661	3316 667
20′	1183 063	120 867	2333 333	20′	1715 323	251 258	3333.333
30′	1191 754	122 630	2350 000	30′	1724.407	253 870	3350.000
40′	1200.450	124 407	2366 667	40′	1733 499	256 496	3366 667
50′	1209 152	126.198	2383 333	50′	1742 600	259 138	3383 333
24° 00′	1217 859	128 002	2400 000	34° 00′	1751 708	261 794	3400 000
10′	1226 572	129 820	2416 667	10	1760 824	264.466	3416 667
20′	1235 290	131 651	2433 333	20′	1769 949	267 153	3433.333
30′	1244 013	133 496	2450 000	30′	1779 081	269.855	3450.000
40′	1252.742	135.354	2466 667	40′	1788 222	272.572	3466 667
50′	1261 477	137 226	2483 333	50′	1797 371	275 304	3483 333
25° 00′	1270.217	139 111	2500 000	35° 00′	1806 529	278.051	3500 000
10′	1278 963	141 010	2516 667	10′	1815 695	280 814	3516 667
20′	1287 714	142.923	2533 333	20′	1824 869	283.592	3533.333
30′	1296 471	144.850	2550 000	30′	1834 052	286 385	3550 000
40′	1305 234	146.790	2566 667	40′	1843.244	289 194	3566 667
50′	1314.003	148 744	2583 333	50′	1852 444	292 018	3583.333

Table 6. Tangents, Externals, and Lengths to a 1° Curve—*Continued*

Δ	Tangent	External	Length	Δ	Tangent	External	Length
36° 00′	1861.653	294.857	3600.000	46° 00′	2432.062	494.809	4600.000
10′	1870.870	297.712	3616.667	10′	2441.902	498.660	4616.667
20′	1880.096	300.582	3633.333	20′	2451.756	502.530	4633.333
30′	1889.331	303.467	3650.000	30′	2461.621	506.418	4650.000
40′	1898.575	306.369	3666.667	40′	2471.499	510.323	4666.667
50′	1907.828	309.286	3683.333	50′	2481.389	514.247	4683.333
37° 00′	1917.090	312.218	3700.000	47° 00′	2491.291	518.189	4700.000
10′	1926.361	315.166	3716.667	10′	2501.207	522.150	4716.667
20′	1935.641	318.130	3733.333	20′	2511.134	526.128	4733.333
30′	1944.930	321.109	3750.000	30′	2521.075	530.125	4750.000
40′	1954.228	324.104	3766.667	40′	2531.028	534.140	4766.667
50′	1963.535	327.115	3783.333	50′	2540.994	538.174	4783.333
38° 00′	1972.852	330.142	3800.000	48° 00′	2550.972	542.226	4800.000
10′	1982.178	333.185	3816.667	10′	2560.964	546.297	4816.667
20′	1991.513	336.243	3833.333	20′	2570.969	550.386	4833.333
30′	2000.858	339.318	3850.000	30′	2580.987	554.494	4850.000
40	2010.213	342.408	3866.667	40′	2591.018	558.620	4866.667
50′	2019.576	345.515	3883.333	50′	2601.062	562.765	4883.333
39° 00′	2028.950	348.637	3900.000	49° 00′	2611.119	566.930	4900.000
10	2038.333	351.776	3916.667	10′	2621.190	571.112	4916.667
20′	2047.726	354.931	3933.333	20′	2631.274	575.314	4933.333
30′	2057.129	358.102	3950.000	30′	2641.372	579.535	4950.000
40′	2066.541	361.289	3966.667	40′	2651.483	583.775	4966.667
50′	2075.964	364.492	3983.333	50′	2661.608	588.034	4983.333
40° 00′	2085.396	367.712	4000.000	50° 00′	2671.746	592.312	5000.000
10′	2094.838	370.947	4016.667	10′	2681.898	596.609	5016.667
20′	2104.290	374.200	4033.333	20′	2692.064	600.926	5033.333
30′	2113.753	377.468	4050.000	30′	2702.244	605.261	5050.000
40′	2123.226	380.753	4066.667	40′	2712.438	609.616	5066.667
50′	2132.708	384.055	4083.333	50′	2722.646	613.991	5083.333
41° 00′	2142.201	387.373	4100.000	51° 00′	2732.868	618.385	5100.000
10′	2151.705	390.708	4116.667	10′	2743.105	622.799	5116.667
20′	2161.219	394.059	4133.333	20′	2753.355	627.232	5133.333
30′	2170.743	397.427	4150.000	30′	2763.620	631.685	5150.000
40′	2180.278	400.811	4166.667	40′	2773.900	636.157	5166.667
50′	2189.823	404.213	4183.333	50′	2784.194	640.649	5183.333
42° 00′	2199.379	407.631	4200.000	52° 00′	2794.502	645.162	5200.000
10′	2208.946	411.066	4216.667	10′	2804.825	649.694	5216.667
20′	2218.523	414.517	4233.333	20′	2815.163	654.246	5233.333
30′	2228.111	417.986	4250.000	30′	2825.515	658.818	5250.000
40′	2237.710	421.472	4266.667	40′	2835.883	663.410	5266.667
50′	2247.320	424.974	4283.333	50′	2846.265	668.022	5283.333
43° 00′	2256.941	428.494	4300.000	53° 00′	2856.662	672.655	5300.000
10′	2266.573	432.030	4316.667	10′	2867.075	677.307	5316.667
20′	2276.216	435.584	4333.333	20′	2877.502	681.981	5333.333
30′	2285.870	439.155	4350.000	30′	2887.945	686.674	5350.000
40′	2295.535	442.743	4366.667	40′	2898.403	691.388	5366.667
50′	2305.212	446.348	4383.333	50′	2908.877	696.123	5383.333
44° 00′	2314.900	449.971	4400.000	54° 00′	2919.366	700.878	5400.000
10′	2324.599	453.611	4416.667	10′	2929.870	705.654	5416.667
20′	2334.310	457.268	4433.333	20′	2940.391	710.450	5433.333
30′	2344.032	460.943	4450.000	30′	2950.927	715.267	5450.000
40′	2353.766	464.635	4466.667	40′	2961.478	720.106	5466.667
50′	2363.512	468.345	4483.333	50′	2972.046	724.965	5483.333
45° 00′	2373.269	472.073	4500.000	55° 00′	2982.629	729.845	5500.000
10′	2383.038	475.817	4516.667	10′	2993.229	734.746	5516.667
20′	2392.819	479.580	4533.333	20′	3003.845	739.668	5533.333
30′	2402.611	483.361	4550.000	30′	3014.477	744.612	5550.000
40′	2412.416	487.159	4566.667	40′	3025.125	749.577	5566.667
50′	2422.233	490.975	4583.333	50′	3035.790	754.563	5583.333

Table 6. Tangents, Externals, and Lengths to a 1° Curve—*Continued*

Δ	Tangent	External	Length	Δ	Tangent	External	Length
56° 00′	3046.471	759.570	5600.000	66° 00′	3720 831	1102.160	6600 000
10′	3057.168	764.600	5616.667	10′	3732.690	1108 627	6616 667
20′	3067 882	769 650	5633 333	20′	3744.572	1115 119	6633 333
30′	3078.613	774.722	5650 000	30′	3756.476	1121.639	6650.000
40′	3089.361	779 816	5666 667	40′	3768.403	1128.186	6666 667
50′	3100.125	784.932	5683.333	50′	3780.352	1134.759	6683 333
57° 00′	3110 907	790.070	5700.000	67° 00′	3792.325	1141.360	6700.000
10′	3121.706	795.229	5716.667	10′	3804.321	1147.988	6716 667
20′	3132.521	800.411	5733.333	20′	3816.339	1154.644	6733 333
30′	3143.354	805.614	5750 000	30′	3828.382	1161.327	6750 000
40′	3154.204	810 840	5766.667	40′	3840.447	1168.037	6766.667
50′	3165.072	816.088	5783.333	50′	3852.536	1174.776	6783.333
58° 00′	3175.957	821.358	5800.000	68° 00′	3864.649	1181.542	6800 000
10′	3186.860	826.651	5816 667	10′	3876 786	1188.336	6816.667
20′	3197 780	831.966	5833.333	20′	3888.946	1195.158	6833.333
30′	3208.718	837.304	5850.000	30′	3901.131	1202.008	6850 000
40′	3219.674	842.664	5866.667	40′	3913.339	1208.887	6866 667
50′	3230.647	848.047	5883.333	50′	3925.572	1215.793	6883.333
59° 00′	3241.639	853.452	5900.000	69° 00′	3937.830	1222.729	6900.000
10′	3252 649	858.881	5916.667	10′	3950 112	1229 693	6916.667
20′	3263.677	864.332	5933.333	20′	3962.418	1236.685	6933.333
30′	3274.724	869.807	5950.000	30′	3974 750	1243 707	6950 000
40′	3285.788	875.304	5966.667	40′	3987 106	1250.757	6966 667
50′	3296.872	880.825	5983.333	50′	3999.487	1257.837	6983 333
60° 00′	3307.973	886 369	6000.000	70° 00′	4011 894	1264 945	7000 000
10′	3319 094	891.936	6016.667	10′	4024.325	1272 083	7016 667
20′	3330 233	897.527	6033.333	20′	4036.783	1279.251	7033.333
30′	3341.391	903.141	6050.000	30′	4049 265	1286 447	7050 000
40′	3352.568	908.779	6066.667	40′	4061 774	1293 674	7066.667
50′	3363.764	914.440	6083.333	50′	4074 308	1300.930	7083.333
61° 00′	3374 979	920.125	6100.000	71° 00′	4086 868	1308.217	7100.000
10′	3386.214	925.834	6116.667	10′	4099.455	1315.533	7116 667
20′	3397.468	931.567	6133.333	20′	4112 067	1322 880	7133 333
30′	3408 741	937.324	6150.000	30′	4124 706	1330 257	7150.000
40′	3420 033	943.105	6166.667	40′	4137 371	1337.664	7166 667
50′	3431.346	948.910	6183 333	50′	4150.063	1345.101	7183 333
62° 00′	3442 678	954 739	6200.000	72° 00′	4162.782	1352 570	7200 000
10′	3454 030	960 593	6216.667	10′	4175.528	1360.069	7216 667
20′	3465.401	966.471	6233.333	20′	4188.300	1367.599	7233.333
30′	3476.793	972.374	6250.000	30′	4201.100	1375.160	7250.000
40′	3488.205	978.301	6266.667	40′	4213.928	1382.753	7266 667
50′	3499.637	984.253	6283.333	50′	4226 782	1390.377	7283.333
63° 00′	3511 090	990.230	6300.000	73° 00′	4239.665	1398 032	7300.000
10′	3522 563	996.231	6316.667	10′	4252.575	1405 718	7316.667
20′	3534.056	1002.258	6333.333	20′	4265.513	1413 437	7333 333
30′	3545.570	1008.310	6350.000	30′	4278.479	1421 187	7350.000
40′	3557.105	1014.387	6366.667	40′	4291.473	1428 970	7366.667
50′	3568.661	1020.489	6383.333	50′	4304.496	1436 784	7383.333
64° 00′	3580.238	1026.617	6400.000	74° 00′	4317.547	1444.631	7400 000
10′	3591.835	1032.770	6416 667	10′	4330 626	1452.510	7416.667
20′	3603 454	1038.948	6433 333	20′	4343.735	1460 422	7433 333
30′	3615.094	1045.152	6450.000	30′	4356.872	1468 366	7450 000
40′	3626 756	1051 382	6466.667	40′	4370 039	1476 343	7466 667
50′	3638 439	1057.638	6483 333	50′	4383.235	1484 354	7483 333
65° 00′	3650.144	1063.920	6500.000	75° 00′	4396 460	1492 397	7500.000
10′	3661.870	1070 228	6516.667	10′	4409 714	1500.474	7516 667
20′	3673 618	1076 562	6533.333	20′	4422 999	1508.584	7533.333
30′	3685 388	1082.922	6550.000	30′	4436 313	1516.727	7550 000
40′	3697.181	1089 308	6566.667	40′	4449 658	1524.905	7566.667
50′	3708 995	1095 721	6583.333	50′	4463 032	1533 116	7583.333

Table 6. Tangents, Externals, and Lengths to a 1° Curve—*Continued*

Δ	Tangent	External	Length	Δ	Tangent	External	Length
76° 00′	4476.437	1541.361	7600 000	86° 00′	5342.918	2104.631	8600.000
10′	4489 872	1549.640	7616.667	10′	5358.519	2115.280	8616.667
20′	4503.338	1557.954	7633.333	20′	5374.162	2125.973	8633 333
30′	4516.835	1566.302	7650.000	30′	5389.849	2136.713	8650 000
40′	4530.363	1574.685	7666.667	40′	5405.578	2147.499	8666 667
50′	4543.922	1583.102	7683.333	50′	5421.350	2158.331	8683.333
77° 00′	4557.512	1591.554	7700.000	87° 00′	5437.166	2169.209	8700.000
10′	4571.134	1600.042	7716.667	10′	5453.026	2180.135	8716.667
20′	4584.787	1608.565	7733.333	20′	5468.930	2191.107	8733.333
30′	4598.472	1617.123	7750.000	30′	5484.878	2202.127	8750.000
40′	4612.190	1625.717	7766 667	40′	5500.870	2213.195	8766.667
50′	4625 939	1634.346	7783.333	50′	5516.907	2224.310	8783.333
78° 00′	4639.721	1643.011	7800 000	88° 00′	5532.989	2235.473	8800.000
10′	4653 535	1651.713	7816 667	10′	5549.116	2246.684	8816.667
20′	4667.382	1660.450	7833 333	20′	5565.289	2257.944	8833.333
30′	4681.262	1669.224	7850.000	30′	5581.508	2269.253	8850.000
40′	4695.174	1678.035	7866.667	40′	5597.772	2280.610	8866.667
50′	4709 120	1686.882	7883.333	50′	5614.083	2292.017	8883.333
79° 00′	4723.100	1695.766	7900 000	89° 00′	5630.441	2303.474	8900.000
10′	4737.112	1704.687	7916 667	10′	5646.845	2314.980	8916.667
20′	4751.159	1713.646	7933 333	20′	5663.296	2326.537	8933.333
30′	4765.240	1722.641	7950.000	30′	5679.795	2338.144	8950 000
40′	4779.354	1731.675	7966 667	40′	5696.341	2349.801	8966.667
50′	4793 503	1740.746	7983.333	50′	5712.935	2361.509	8983.333
80° 00′	4807.687	1749 855	8000.000	90° 00′	5729.578	2373.269	9000.000
10′	4821.905	1759.002	8016.667	10′	5746.269	2385.080	9016.667
20′	4836.158	1768.187	8033.333	20′	5763.009	2396.942	9033.333
30′	4850.446	1777.411	8050.000	30′	5779.797	2408.857	9050.000
40′	4864 769	1786 674	8066.667	40′	5796.635	2420.824	9066.667
50′	4879 128	1795 975	8083.333	50′	5813.523	2432.843	9083.333
81° 00′	4893.522	1805.315	8100 000	91° 00′	5830.461	2444.915	9100 000
10′	4907.952	1814.695	8116 667	10′	5847.449	2457.040	9116 667
20′	4922.418	1824.114	8133 333	20′	5864.487	2469.219	9133.333
30′	4936 920	1833.573	8150 000	30′	5881.576	2481.451	9150.000
40′	4951.459	1843.071	8166 667	40′	5898.717	2493.738	9166.667
50′	4966.034	1852.609	8183.333	50′	5915.908	2506.078	9183.333
82° 00′	4980 646	1862.187	8200 000	92° 00′	5933.152	2518.473	9200 000
10′	4995.295	1871 806	8216.667	10′	5950.447	2530.923	9216.667
20′	5009 981	1881.465	8233.333	20′	5967.795	2543.429	9233.333
30′	5024.705	1891.165	8250 000	30′	5985.195	2555.989	9250.000
40′	5039.466	1900.906	8266 667	40′	6002.649	2568.606	9266.667
50′	5054.265	1910.688	8283.333	50′	6020.155	2581.278	9283 333
83° 00′	5069.102	1920 511	8300.000	93° 00′	6037.715	2594.007	9300 000
10′	5083.978	1930.376	8316.667	10′	6055.329	2606.793	9316 667
20′	5098 891	1940.282	8333 333	20′	6072.998	2619.636	9333.333
30′	5113.844	1950.231	8350.000	30′	6090.721	2632.536	9350.000
40′	5128 835	1960.221	8366.667	40′	6108.499	2645.493	9366 667
50′	5143.865	1970.254	8383 333	50′	6126.332	2658.509	9383 333
84° 00′	5158.935	1980.330	8400.000	94° 00′	6144.220	2671.583	9400 000
10′	5174.044	1990.448	8416 667	10′	6162.165	2684 716	9416.667
20′	5189 193	2000.609	8433.333	20′	6180.165	2697.907	9433.333
30′	5204 382	2010 813	8450 000	30′	6198.223	2711.158	9450.000
40′	5219 611	2021.061	8466 667	40′	6216 337	2724.469	9466.667
50′	5234 881	2031.352	8483.333	50′	6234.508	2737.839	9483.333
85° 00′	5250.191	2041 689	8500 000	95° 00′	6252.737	2751.270	9500.000
10′	5265 542	2052.067	8516 667	10′	6271.024	2764.762	9516.667
20′	5280.934	2062.490	8533 333	20′	6289.369	2778.314	9533.333
30′	5296.367	2072.958	8550 000	30′	6307.773	2791.928	9550 000
40′	5311.842	2083.471	8566 667	40′	6326.236	2805.604	9566 667
50′	5327.359	2094.029	8583.333	50′	6344.759	2819.342	9583 333

Table 6. Tangents, Externals, and Lengths to a 1° Curve—*Continued*

Δ	Tangent	External	Length	Δ	Tangent	External	Length
96° 00′	6363.341	2833.142	9600 000	106° 00′	7603.407	3790.919	10600.000
10′	6381.983	2847.005	9616 667	10′	7626.460	3809.340	10616.667
20′	6400.686	2860.931	9633 333	20′	7649 603	3827.853	10633.333
30′	6419 450	2874.921	9650.000	30′	7672 835	3846 458	10650 000
40′	6438.275	2888.974	9666 667	40′	7696.159	3865.156	10666 667
50′	6457.161	2903.092	9683.333	50′	7719 574	3883.948	10683.333
97° 00	6476.110	2917 275	9700 000	107° 00′	7743.080	3902.834	10700.000
10′	6495.121	2931.522	9716 667	10′	7766 679	3921 814	10716.667
20′	6514 195	2945 835	9733.333	20′	7790 372	3940.890	10733.333
30′	6533.332	2960 214	9750 000	30′	7814 158	3960 063	10750.000
40′	6552.532	2974.659	9766.667	40′	7838 039	3979.331	10766.667
50′	6571.797	2989.170	9783.333	50′	7862 015	3998 698	10783.333
98° 00′	6591.125	3003.749	9800.000	108° 00′	7886.088	4018.162	10800 000
10′	6610.519	3018.395	9816.667	10′	7910 256	4037 725	10816 667
20′	6629 978	3033.108	9833.333	20′	7934 522	4057 388	10833 333
30′	6649.503	3047.890	9850.000	30′	7958.886	4077 151	10850 000
40′	6669.093	3062.741	9866.667	40′	7983 348	4097.014	10866.667
50′	6688 750	3077.660	9883.333	50′	8007.910	4116.979	10883.333
99° 00′	6708.474	3092.649	9900.000	109° 00′	8032.572	4137.046	10900.000
10′	6728.265	3107 707	9916.667	10′	8057.335	4157.217	10916.667
20′	6748.124	3122.836	9933 333	20′	8082.199	4177.490	10933 333
30′	6768 051	3138.036	9950.000	30′	8107.165	4197.868	10950 000
40′	6788.046	3153.307	9966 667	40′	8132.235	4218 352	10966.667
50′	6808.111	3168.649	9983 333	50′	8157.408	4238.941	10983.333
100° 00′	6828.245	3184.063	10000 000	110° 00′	8182.685	4259.636	11000 000
10′	6848 449	3199.550	10016 667	10′	8208 068	4280.439	11016.667
20′	6868 723	3215.109	10033.333	20′	8233 557	4301.350	11033.333
30′	6889.069	3230.742	10050 000	30′	8259.152	4322 370	11050 000
40′	6909.485	3246.448	10066.667	40′	8284.856	4343 500	11066.667
50′	6929 937	3262.229	10083.333	50′	8310.667	4364 740	11083 333
101° 00′	6950.534	3278.085	10100.000	111° 00′	8336.588	4386.091	11100 000
10′	6971.167	3294 015	10116.667	10′	8362.618	4407 554	11116 667
20′	6991.873	3310 021	10133.333	20′	8388.759	4429 130	11133.333
30′	7012.653	3326.104	10150.000	30′	8415 012	4450 820	11150 000
40′	7033.507	3342.262	10166.667	40′	8441.378	4472 624	11166 667
50′	7054.436	3358.498	10183.333	50′	8467.856	4494.543	11183.333
102° 00′	7075.440	3374 812	10200.000	112° 00′	8494.449	4516.579	11200 000
10′	7096.519	3391 203	10216 667	10′	8521 156	4538.731	11216.667
20′	7117.674	3407.673	10233 333	20′	8547.979	4561.001	11233.333
30′	7138.906	3424 222	10250.000	30′	8574.919	4583.390	11250 000
40′	7160.215	3440.850	10266.667	40′	8601.977	4605.898	11266.667
50′	7181.602	3457.558	10283.333	50′	8629.153	4628.527	11283.333
103° 00′	7203.067	3474 347	10300 000	113° 00′	8656.448	4651 277	11300.000
10′	7224 610	3491.217	10316.667	10′	8683 863	4674 150	11316 667
20′	7246 233	3508 168	10333 333	20′	8711.400	4697 145	11333.333
30′	7267.935	3525.201	10350.000	30′	8739.059	4720.265	11350.000
40′	7289.717	3542 317	10366.667	40′	8766.840	4743.509	11366.667
50′	7311.581	3559.517	10383.333	50′	8794.746	4766 879	11383.333
104° 00′	7333 525	3576 799	10400 000	114° 00′	8822 776	4790.377	11400.000
10′	7355.552	3594.166	10416 667	10′	8850.933	4814.002	11416.667
20′	7377.661	3611 618	10433 333	20′	8879.216	4837 755	11433.333
30′	7399 852	3629.155	10450.000	30′	8907 627	4861.639	11450.000
40′	7422.128	3646 778	10466.667	40′	8936 166	4885.653	11466.667
50′	7444.487	3664.487	10483.333	50′	8964 836	4909.799	11483.333
105° 00′	7466 931	3682.283	10500 000	115° 00′	8993.636	4934.078	11500.000
10′	7489.461	3700 167	10516 667	10′	9022.568	4958.490	11516.667
20′	7512.076	3718 139	10533 333	20′	9051 633	4983.037	11533.333
30′	7534.778	3736.199	10550.000	30′	9080.831	5007.720	11550.000
40′	7557.566	3754 349	10566.667	40′	9110.165	5032.539	11566.667
50′	7580.442	3772.589	10583 333	50′	9139 635	5057.497	11583.333

Table 6. Tangents, Externals, and Lengths to a 1° Curve—*Continued*

Δ	Tangent	External	Length	Δ	Tangent	External	Length
116° 00′	9169.241	5082 594	11600.000	118° 00′	9535.619	5394.994	11800.000
10′	9198 986	5107.830	11616.667	10′	9567.110	5421.999	11816.667
20′	9228.870	5133.208	11633.333	20′	9598.755	5449.159	11833 333
30′	9258.895	5158.728	11650.000	30′	9630.555	5476.476	11850.000
40′	9289.061	5184 391	11666.667	40′	9662.510	5503.951	11866 667
50′	9319 369	5210.198	11683.333	50′	9694.622	5531.584	11883 333
117° 00′	9349.821	5236.152	11700 000	119° 00′	9726.893	5559 378	11900.000
10′	9380.419	5262.252	11716 667	10′	9759.324	5587 333	11916.667
20′	9411.162	5288 499	11733.333	20′	9791.915	5615.451	11933.333
30′	9442.052	5314.896	11750.000	30′	9824.670	5643.733	11950.000
40′	9473 091	5341.444	11766 667	40′	9857.587	5672 180	11966 667
50′	9504 280	5368.142	11783.333	50′	9890.671	5700.795	11983 333
				120°00′	9923.920	5729 578	12000.000

68. STAKING CIRCULAR CURVES ON THE GROUND

There are five methods used in staking circular curves on the ground for location and construction purposes. Choice of method depends upon topography, ground cover, and engineering equipment available.

Deflection Method, Instrument Point on the Curve. The deflection-angle method, Fig. 47, is used most in staking alignment for railroads and highways, where the instrument can be set up on the curve. First set the transit up at the P.C. or the P.T., whichever is the most convenient. If on the P.C., set the vernier of the transit at 0°, sight on the P.I. or a P.O.S.T. (point on the semitangent which had been set previously). Turn the deflection angle a and measure the chord from the P.C. to A, a station or plus point on the curve. The deflection angle is turned after loosening the upper clamp while keeping the lower clamp set. Once turned, the upper clamp is set to hold position until the head chainman has the line rod where the stake should be driven. Deflection angles b,

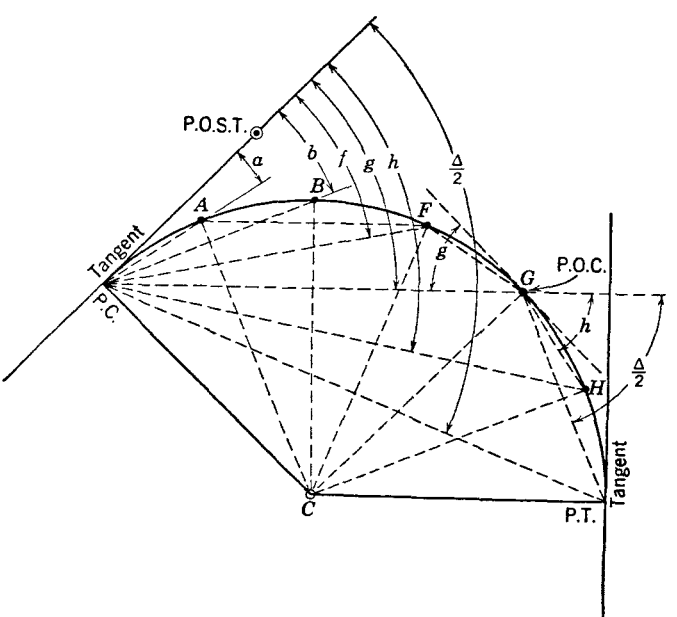

FIG 47. Deflection angles in staking a circular curve.

f, and g are likewise turned in succession to curve points B, F, and G. Chords are measured in convenient lengths from one curve station to another. When the chords are longer than the engineers' tape being used, as chords P.C.–F and P.C.–G, chords of convenient length can be measured from chaining points on previously set stakes, as from A to F when deflection angle f is turned, and from F to G for when deflection angle g is turned. When it is not possible to sight from the P.C. to all curve points, as points beyond G, a tack is placed in the stake set at G for an intermediate instrument set-up, called a P.O.C., point on curve. When the transit is on a P.O.C., the vernier is set to read the deflection angle of the backsight point, and the telescope, inverted, is sighted on that point. Should backsight be made on the P.C., the vernier is set to read 0°. If the deflection angle of the P.O.C. (angle g) is turned, the telescope of the instrument will be pointing along a line that is tangent to the curve at the P.O.C. Curve points beyond the P.O.C., as points H and the P.T., can be staked by turning the deflection angles h and Δ 2, respectively.

For arc- and chord-definition circular curves, the deflection angle in minutes per unit of curve length is 0.3 of the degree of curve, in degrees. As an example, for a 12° circular curve, the deflection is 3.6′ per foot of curve length, and 3.6′ per meter. (The 12° curve in the foot system of measurement would be sharper, however, than the 12° curve in the meter system, inasmuch as 1 ft equals 0.3048 m.) For radius-definition circular curves, the deflection angle in minutes per unit of curve length is 1718.8734 divided by the radius.

Tangent-Offset Method. The only time angles are measured in this method is when extreme accuracy is required, or when, for some reason, the transit cannot be set up initially on the P.C. or P.T. to line-in a P.O.C. for staking other curve points by the deflection method. Then a right angle is measured at the offset point on the semitangent (opposite the P.O.C. to be set) and the tangent offset distance a–A, or b–B, etc., is measured from the semitangent, Fig. 48, to set a P.O.C. where a transit set-up can be made.

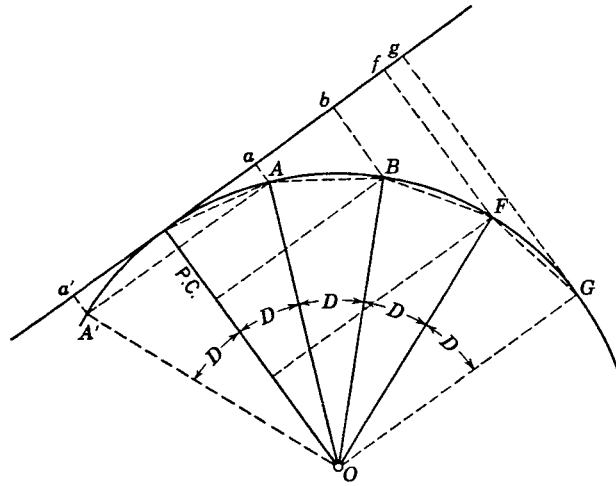

FIG. 48. Tangent offsets and tangent distances.

Before tangent offsets can be measured, both the tangent offset and tangent distance for each curve point to be staked must be computed. The tangent distances are the distances from the P.C. to points on the semitangent, as points a, b, f, and g, opposite the curve points A, B, F, and G, respectively. The tangent distance $t_d = R \sin (DL_c/100)$, and the tangent offset $t_o = R \text{ vers } (DL_c/100)$, considering L_c as the length of the curve from the P.C. to each curve point to be staked. Should the curve points A, B, F, G, etc., be at even stations from the P.C. the angles for which the sine and versine are ascertained would be D, $2D$, $3D$, $4D$, etc.

To stake the curve by tangent offsets, measure tangent distances from the P.C. to each offset point and drive stakes at those points. In doing this, line the offset points in by the transit, sighting along the semitangent from a set-up on the tangent. Curve points A, B, etc., may then be staked by taping only.

Middle-Ordinate Method. This method is used in rough setting or checking of the setting of forms, and in the staking of short-length sharp circular curves, where conditions

or time do not warrant the use of a transit. In Fig. 49, B–b, F–f, G–g, and H–h are the middle ordinates, and A–F, B–G, F–H, and G–K are the chords from which the middle ordinates are measured. The middle ordinates and chords are computed. For the first segment of the curve, the chord A–F is measured while the middle ordinate b–B is measured perpendicularly outward from the midpoint of the chord. For the second and subsequent

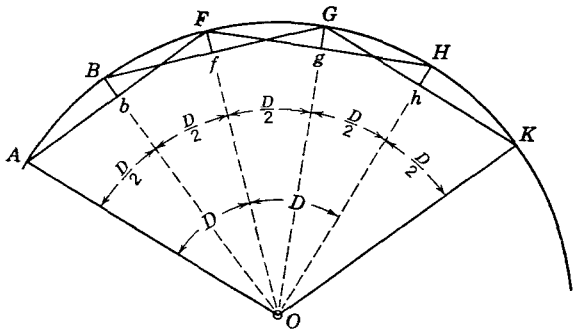

FIG 49 Middle ordinates.

segments, the middle ordinate is measured inward from the point that was set at the end of the chord on the previous measurement, as F–f, to establish the line B–f–G for the next chord. The curve point is staked at end of the chord measured while the tape is lined over point f measured from F perpendicularly to the midpoint of the chord measured from the curve point B, and so on to the end of the curve. To make this method easy in application, it is customary to set stakes at equal-length increments of $^1/_2$ tape length or shorter.

Chord-Offset Method. This method has two principal applications: (1) To aid the head chainman in quickly getting on line whenever a curve is being staked by the deflection method, and (2) to rapidly lay out a curve on reconnaissance surveys in which relative relationships are essential but definitive accuracy is not required. In Fig. 50, the initial offset bB from a tangent to the curve, as line A–b, is the same as the middle ordinate for

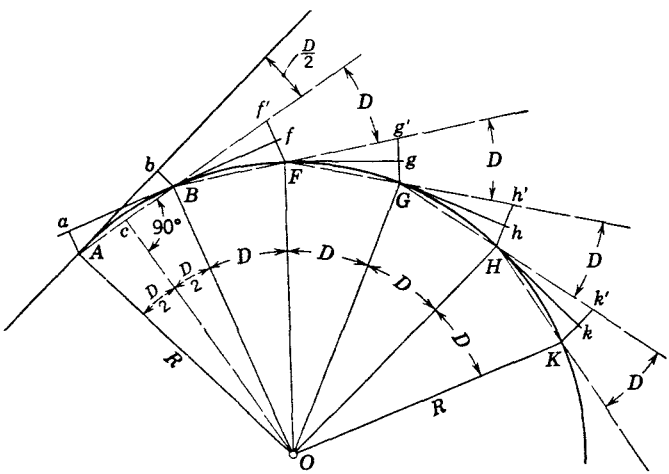

FIG 50. Chord offsets.

an arc of the curve twice as long as the curve length from point A to point B. Thus, the initial offset $bB = R$ vers $D = t_o$. Subsequent chord offsets ($f'fF$ for curve point F; $g'gG$ for curve point G; and so on) equal $2R$ vers D, $= 2t_o$ for all station points at uniform distances apart. Any chord-to-chord offset, as $f'F = C^2/R = 174.533C^2D10^{-6}$ (in terms of D instead of R). Wherever the beginning chord A–B is not equal to subsequent chords, a tangent to the curve at point B is established by measuring offset aA equal to offset bB

at distance Ba from point B equal to distance Ab, as initially used in setting point B on the curve from the tangent through point A. Then point F is set by measuring $^1/_2$ the chord offset for uniform chord lengths from point f on the tangent to point F on the curve; while this is done, the exact location of point F is determined by simultaneously measuring chord B-F from previously set point B. Subsequent curve points, as G, H, and K, are set by measuring the chord offset from the respective points g', h' and k' on the line of the chords produced while the chord is simultaneously measured from the curve point previously set.

Deflection Method, Instrument Point on Tangent. This method is used when it is not possible to make an instrument set-up on the circular curve. The instrument is set up at a point on the semitangent or semitangent produced from which all curve points can be

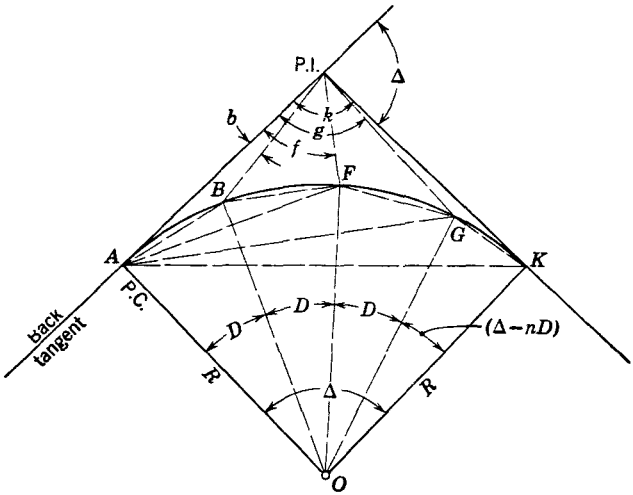

FIG. 51. Deflections from P I or a point on the tangent.

seen. It is an accurate method adaptable to many situations, wherever positioning of stakes on the curve must be accurate. The deflection angles for an instrument set up on the semitangent are determined from the tangent, considering the instrument point as a P.I. whether it is inclusive of the entire circular curve, a segment of it, or would be so far from the P.C. as to include a curve longer than the one to be staked. The deflection angles are computed in a step process. First, each of the deflection angles, b, f, g, etc., Fig. 51, is determined from:

$$\text{cotan deflection angle} = \left[\frac{(\tan \Delta/2) - \sin nD}{\text{vers } nD}\right]$$

In the formula, n is the number of D angles from the P.C. of the curve to the curve point for which the deflection angle is required; Δ is the supplement of the deflection angle from the back tangent that would be turned to establish a tangent to the curve in the forward direction; and D, in degrees, equals 100Δ divided by the length of curve L_c. To stake the curve, the instrumentman lines the line rod held by the head chainman into position while the chainmen successively chain the chords of the curve, beginning at the P.C. and ending at the P.T., whether the P.T. is at points F, G, K, or beyond K.

69. CIRCULAR CURVE PROBLEMS

The Length and Bearing of a Tangent Joining Two Circular Curves. The circular curves are usually laid out on a topographic map by means of a spline line, circular curve templates, or compass. The tangents are then computed to join the curves. This is good procedure, whether the curves are curving in the same direction or in opposite directions.

Figure 52, with curves in opposite directions, is an illustration of one type of problem. The curves were laid out on the coordinate grid, so as to fit the local controls. Measure or compute the coordinates of the centers of both circular curves, points O_1 and O_2. Line O_1O_2 equals the square root of the sum of the squares of lines O_1p and pO_2, the difference

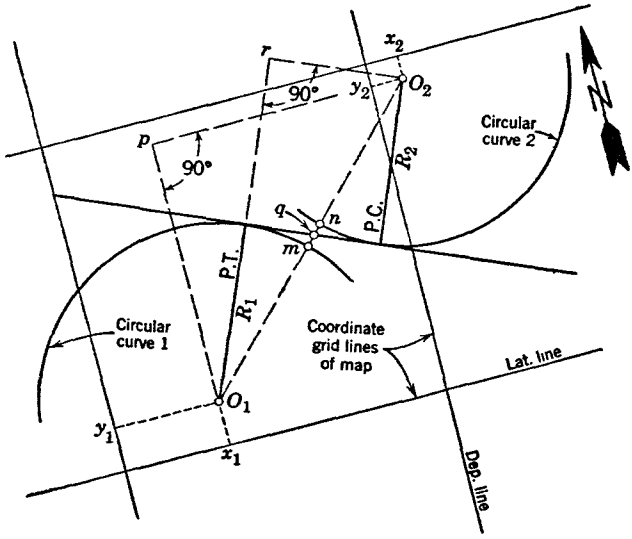

FIG. 52 Tangent joining two circular curves, its bearing and length.

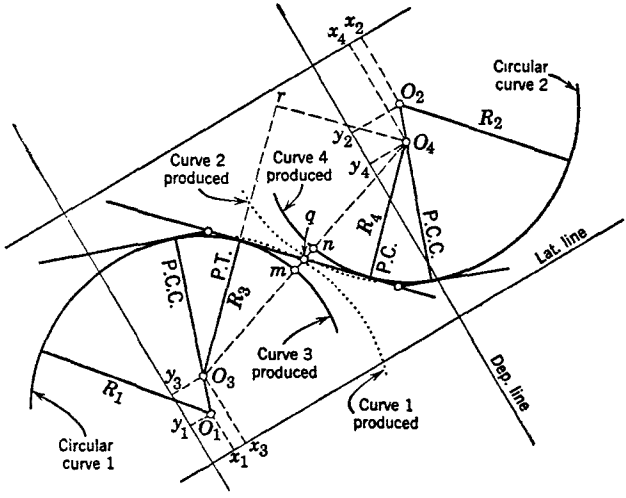

FIG 53. Use of a compound curve on each end of two fixed-position circular curves to provide sufficient joining tangent distance between the curves for change of direction, and computation of the bearing and length of the tangent

$$O_3O_4 = mn + R_3 + R_4 = (R_3 + R_4) \sec (O_4O_3r)$$

$$mn = R_3 \operatorname{exsec} (O_4O_3r) + R_4 \operatorname{exsec} (O_4O_3r)$$

and

$$\text{angle } (O_4O_3r) = \arctan \frac{O_4r}{R_3 + R_4}$$

distance O_4r = tan distance from P.T. to P.C. required for the change of direction

in latitude and departure of the centers. The tangent of the bearing angle pO_1O_2 of line O_1O_2 equals the length of the departure line pO_2 divided by the length of the latitude line O_1p. Line $O_1r = (R_1 + R_2)$. Line O_1r divided by line O_1O_2 equals the cosine of angle rO_1O_2. Knowing that angle, compute the length of line rO_2 by the use of this formula.

$$rO_2 = (R_1 + R_2) \tan \text{angle } rO_1O_2$$

which equals the length of the tangent from the P.T. of curve 1 to the P.C. of curve 2. Knowing the bearing of the line O_1O_2 and the angle rO_1O_2 of the right-angle triangle O_1rO_2, compute the bearing of line rO_2, which is parallel to the tangent from the P.T. to the P.C.

Sometimes, it is necessary to keep the length of the line rO_2 (equal to the distance from the P.T. to the P.C.) not less than a specific length, sufficient for a transition to the circular curves. When that is the case, trial solutions to the problem are resorted to, using appropriate circular-curve templates on the topographic map. Then the offset between the circular curves mn cannot be less than a fixed amount. $mn = (R_1 - R_2)$ exsec angle O_2O_1r, and the tangent of angle $O_2O_1r = O_2r/(R_1 + R_2)$. Once the length of line mn is known, the curve templates are adjusted on the map (held the mn distance apart), while controls are fitted as closely as possible. When the curve templates are in position, the curves are redrawn on the map and the computations are repeated. Wherever the radius of circular curve 1 or circular curve 2 or both cannot be shortened, it is necessary to compound curves 1 and 2 with sharper circular curves in order to provide the required offset distance mn. The method is illustrated in Fig. 53.

The Inaccessible P.I. The most usual case is illustrated in Fig. 54. The P.I. at E is not accessible. Instrument points on the semitangents at the triangulation points A and B

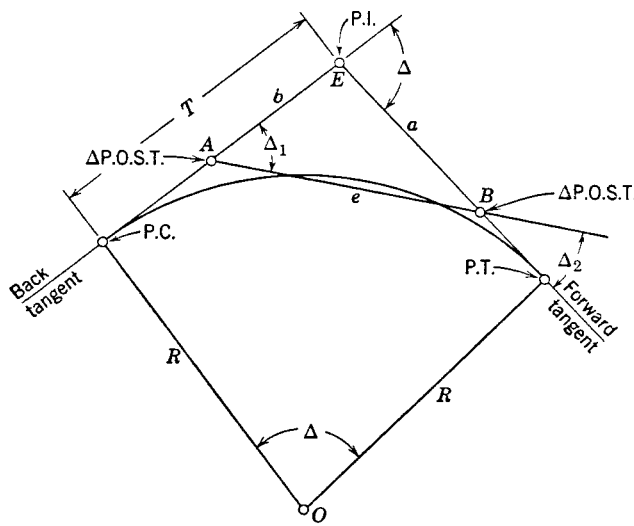

FIG. 54. The inaccessible P.I. and triangulation points on the semitangents, ΔP.O.S.T.'s.

are intervisible. Both points A and B are called a △P.O.S.T. The distance e, from A to B, is measured. Distances from A to E and from E to B must be computed. The angles Δ_1 and Δ_2 are measured with a transit. The sum of these angles equals the Δ of the curve. The distance from the P.I. to each △P.O.S.T. is calculated by use of the sine law:

$$a = \frac{e \sin A}{\sin E} \quad \text{and} \quad b = \frac{e \sin B}{\sin E}$$

A general case is illustrated in Fig. 55. A traverse has to be run from one tangent to the other, because they are not intervisible. The traverse may begin anywhere on the back tangent, end anywhere on the forward tangent, and have two or more courses in it. In the example, the traverse begins at A on a △P.O.T., has three courses, and ends at G on a △P.O.S.T. The angles Δ_1, Δ_2, Δ_3, and Δ_4, and the distances b, f, and g of the traverse are measured. The delta angle, Δ, of the curve is the algebraic sum of the traverse angles Δ_1, Δ_2, Δ_3, and Δ_4. To calculate the required distances K and M, compute closure of the traverse A, B, F, G, P.I. (E), and back to A. This is done by assuming that one of the unknowns K or M has a north-south or an east-west bearing. To compute the closure, after

setting the bearing of K or M in the north-south or east-west direction (1) determine the bearing of all lines in the traverse in relation to the assumed course; (2) multiply each traverse distance by the cosine and sine of its bearing to compute its latitude and departure; (3) algebraically add the latitudes and departures of the known courses; (4) compute the latitude and departure of the unknown course K or M, whichever does not have a zero cosine or sine of its bearing; (5) the K or M course completes the closure of the latitude or departure in the traverse, whichever distance has a cosine or sine value of one. As an example, assume the traverse angles and distances in Fig. 55 had these values: $\Delta_1 = 27°$, $b = 184'$; $\Delta_2 = 29°$, $f = 159'$; $\Delta_3 = 75°$, $g = 269'$; $\Delta_4 = 26°$, and $\Delta = 99°$.

The completed tabulation in computing the distances K and M is given below. Distances computed by use of the tabulation are in square brackets. The bearing of each

Inaccessible P.I. Calculation

Point	Distance	Bearing	Cosine	Sine	Latitudes N	Latitudes S	Departures E	Departures W
A								
	184 (b)	N 27° E (1)	0.8910065	0 4539905	(2) 163.945		(2) 83.534	
B								
	159 (f)	N 2° W (1)	0.9993908	0.0348995	(2) 158.903			(2) 5.549
F								
	269 (g)	N 73° E (1)	0.2923717	0.9563048	(2) 78.648		(2) 257.246	
G								
	(3) [339 410] (M)	N 81° W (1)	0.1564345	0.9876883	(4) [53.095]			(3) [335 231]
P I.								
	(5) [454.591] (K)	South (1)	1	0		(5) [454.591]		
A								

traverse course was computed assuming line K from the P.I. to point A had a north-south direction. The actual bearing of line K may be any value, and to assume its bearing as

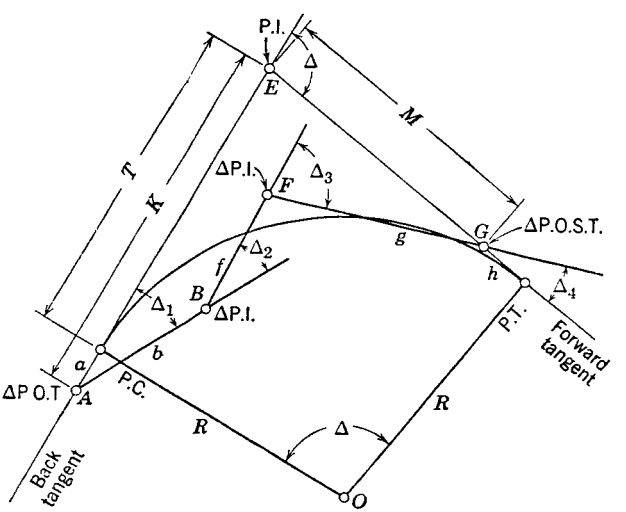

FIG 55 The inacessible P.I and inside P I 's on a traverse from ΔP O T. to ΔP O.S.T

north and south in the solution of the inaccessible P.I. problem is called rotation of bearings. After the distances K and M are computed, all lines of the traverse and curve are mathematically rotated back to their true bearing. In computing the unknown distances of Fig. 55 (bracketed figures in table above), each sequential step is numbered in parentheses.

Whether the circular curve has spiral transitions or not, the solution of the inaccessible P.I. problem is the same. The tangent distance of the curve system fixes the distance to be measured from the triangulation or traverse points to set the beginning and ending curve points on their respective tangents.

Parallel Tangent Problems. In Fig. 56, both curves are of same degree, and $AA' = VV' = BB' = d/\sin\Delta = d \operatorname{cosec}\Delta$. In Fig. 57, both curves have the same P.C., and $R - R' = d/\text{vers}\,\Delta = d \cot \Delta/2 \operatorname{cosec}\Delta$. In Fig. 58, both curves have P.T.'s on same radial line, and $R - R' = d/\text{exsec}\,\Delta$ and $AA' = (R - R') \tan\Delta$.

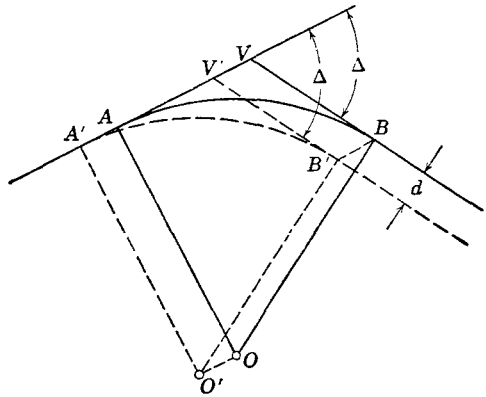

FIG. 56. Parallel tangents.

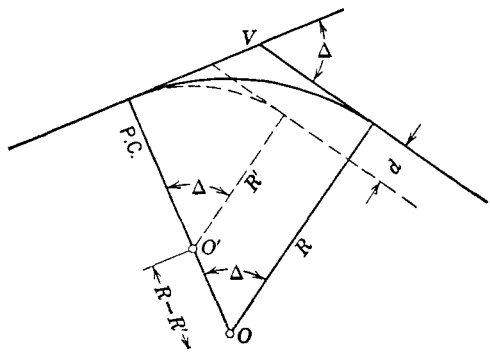

FIG 57. Parallel tangents, same P.C.

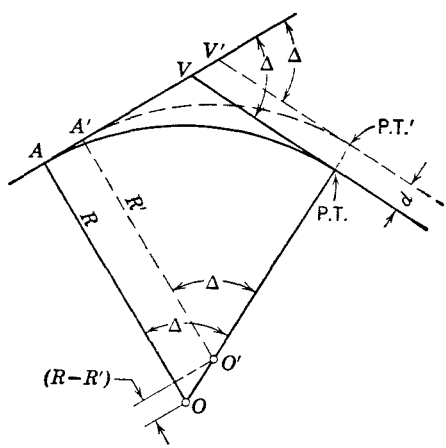

FIG. 58. P.T. on same radial line

Miscellaneous Problems. The direction of the forward tangent to be changed at the P.T., Fig. 59, $FA = F'A' = R$ vers $\Delta = R'$ vers Δ', and $R' = R$ vers Δ/vers Δ', and $AA' = R \sin \Delta - R' \sin \Delta'$.

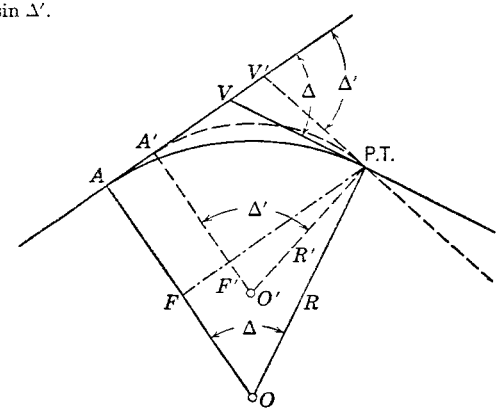

FIG 59 Direction of forward tangent changed at P T

Figure 60 To find the station of B on a circular curve from which a tangent will pass through C, the radius being known. Measure Δ' and AC. In triangle ACO, compute OC and AOC In right-triangle OBC, compute BOC. Then $AOB = AOC - BOC$, and Sta. B = Sta A + 100 AOB D.

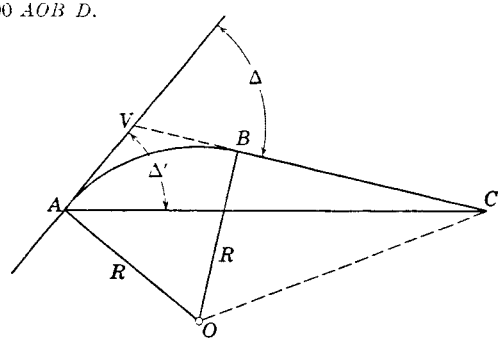

FIG. 60. Tangent to pass through a point.

Figure 61. To find R of curve which shall join tangents AV and BV and pass through a given point C. Measure angle f and VC. Then $\sin VCO = \cos (\Delta/2 + f)/\cos \Delta/2$. In

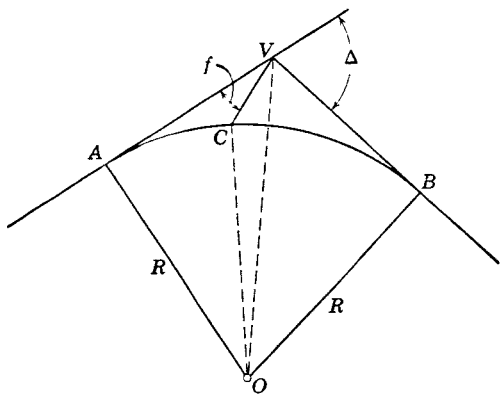

FIG. 61. Curve through a point and tangent to two lines.

triangle VOC, $CVO = 90° - (\Delta/2 + f)$, VCO and VC are known; solve for radius OC. If angle VOC is very small find radius by $VC \sin f/\text{vers } AOC$.

Figure 61. To find distance VC, given R, Δ, and f. Solve triangle VOC for VC; $OC = R$; $OV = R/\cos \Delta/2$; $OVC = 90° - (\Delta/2 + f)$.

Compound Circular Curves. The following apply only to compound curves composed of two circular curves. Where there are more than two curves the formulas become quite long. In Fig. 62 all the elements marked with the subscript 1 refer to the curve of shorter

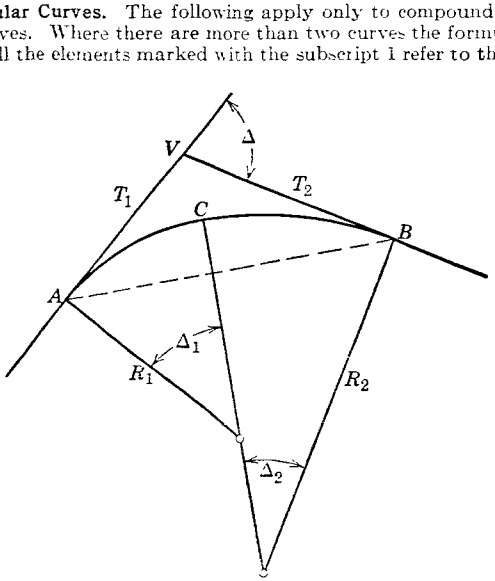

FIG 62 Compound circular curves

radius, and those marked 2 refer to the curve of longer radius. Any compound curve composed of two circular curves may be defined by any four of the seven elements marked on Fig. 62. With four elements, the remainder may be computed:

Given R_1, R_2, Δ_1, Δ_2, required Δ, T_1, T_2 Here $\Delta = \Delta_1 + \Delta_2$, and

$$T_1 = R_1 \tan \Delta_1/2 + \frac{(R_1 \tan \Delta_1/2 + R_2 \tan \Delta_2/2) \sin \Delta_2}{\sin \Delta}$$

$$T_2 = R_2 \tan \Delta_2/2 + \frac{(R_1 \tan \Delta_1/2 + R_2 \tan \Delta_2/2) \sin \Delta_1}{\sin \Delta}$$

Given T_1, R_1, R_2, Δ; required T_2, Δ_2, Δ_1.

$$\text{vers } \Delta_2 = \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{R_2 - R_1} \qquad \Delta_1 = \Delta - \Delta_2$$

$$T_2 = (R_2 - R_1) \sin \Delta_2 + R_1 \sin \Delta - T_1 \cos \Delta$$

Given T_2, R_1, R_2, Δ; required T_1, Δ_2, Δ_1.

$$\text{vers } \Delta_1 = \frac{R_2 \text{ vers } \Delta - T_2 \sin \Delta}{R_2 - R_1} \qquad \Delta_2 = \Delta - \Delta_1$$

$$T_1 = R_2 \sin \Delta - (R_2 - R_1) \sin \Delta_1 - T_2 \cos \Delta$$

Given T_1, R_1, Δ_2, Δ, required T_2, R_2, Δ_2 Here $\Delta_2 = \Delta - \Delta_1$, and

$$R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } \Delta_2}$$

$$T_2 = (R_2 - R_1) \sin \Delta_2 + R_1 \sin \Delta - T_1 \cos \Delta$$

Given T_2, R_2, Δ_2, Δ; required T_1, R_1, Δ_1 Here $\Delta_1 = \Delta - \Delta_2$, and

$$R_1 = R_2 - \frac{R_2 \text{ vers } \Delta - T_2 \sin \Delta}{\text{vers } \Delta_1}$$

$$T_1 = R_2 \sin \Delta - (R_2 - R_1) \sin \Delta_1 - T_2 \cos \Delta$$

Given T_2, T_1, R_1, Δ; required R_2, Δ_2, Δ_1.

$$\tan \Delta_2/2 = \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{T_2 + T_1 \cos \Delta - R_1 \sin \Delta} \qquad \Delta_1 = \Delta -$$

$$R_2 = R_1 + \frac{T_2 + T_1 \cos \Delta - R_1 \sin \Delta}{\sin \Delta_2}$$

Given T_2, T_1, R_2, Δ; required R_1, Δ_1, Δ_2.

$$\tan \Delta_1/2 = \frac{R_2 \text{ vers } \Delta - T_2 \sin \Delta}{R_2 \sin \Delta - T_2 \cos \Delta - T_1} \qquad \Delta_2 = \Delta - \Delta$$

$$R_1 = R_2 - \frac{R_2 \sin \Delta - T_2 \cos \Delta - T_1}{\sin \Delta_1}$$

Given AB, VAB, VBA, either R_1 or R_2; required either R_2 or R_1, Δ_2, Δ_1, Δ. Solve the triangle AVB for AV and VB, which gives T_2 and T_1. $\Delta = VAB + VBA$. Then the four elements T_2, T_1, Δ, and either R_1 or R_2 are given. The remaining elements may be computed by formulas given previously.

Compound Curve Problems. Given a circular curve between two tangents. The curve is to be compounded so as to end in a parallel tangent. In the case of Fig. 63, the new

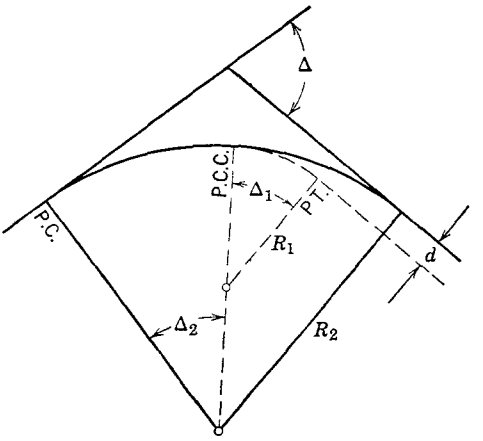

FIG. 63. Compound, using shorter radius to a parallel tangent.

tangent is inside the old tangent, the radius of the original curve becomes R_2, and vers $\Delta_1 = d/(R_2 - R_1)$. If the new tangent falls outside the old tangent, then the radius of the original curve becomes R_1, and vers $\Delta_2 = d/(R_2 - R_1)$.

Given a compound curve between two tangents. Required to retain the same radii but to change the P.C C. so as to end in a parallel tangent:

(*a*) When the new tangent lies inside the old tangent and the curve of the larger radius is at the P.T. end, Fig. 64.

$$\text{vers } \Delta'_2 = \text{vers } \Delta_2 - d/(R_2 - R_1)$$

(*b*) When the new tangent lies outside the old tangent and the curve of the larger radius is at the P.T. end.

$$\text{vers } \Delta'_2 = \text{vers } \Delta_2 + d/(R_2 - R_1)$$

(*c*) When the new tangent lies inside the old tangent and the curve of shorter radius is at the P.T. end.

$$\text{vers } \Delta'_1 = \text{vers } \Delta_1 + d/(R_2 - R_1)$$

(*d*) When the new tangent lies outside the old tangent and the curve of the shorter radius is at the P.T. end.

$$\text{vers } \Delta'_1 = \text{vers } \Delta_1 - d/(R_2 - R_1)$$

In the above formulas, the angles with the prime mark are the new angles. In laying out compound curves in the field, each curve is laid out separately, from the P.C. to the P.C.C., and from there to the P.T.

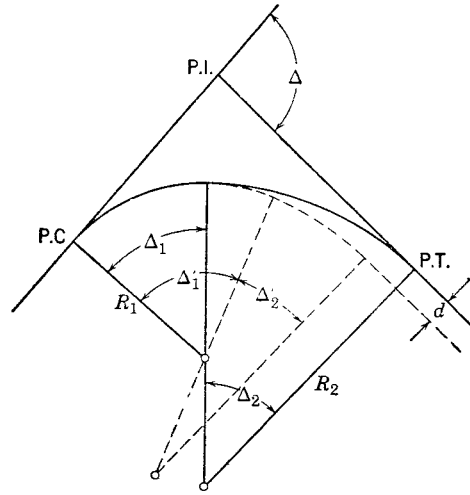

FIG. 64 Compound, moving long radius to a parallel tangent.

Reversed Circular Curves. (*a*) Connecting Parallel Tangents. In this case, Fig. 65, the central angles are equal, the P.R.C. lies on the straight line AB, and $BAE = \Delta/2$.

$$\text{vers } \Delta = \frac{d}{(R_1 + R_2)} \qquad \text{and} \qquad AB = \sqrt{2d(R_1 + R_2)}$$

If both curves have the same radius, then

$$\text{vers } \Delta = \frac{d}{2R} \qquad \text{and} \qquad AB = 2\sqrt{dR}$$

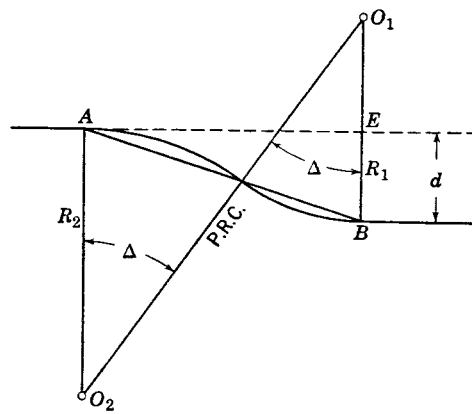

FIG. 65. Curves reversed to parallel tangents.

(*b*) Connecting Non-Parallel Tangents (Fig. 66). As in the compound curve, the choice of four of its elements defines the reversed curve. Given T_2, Δ, R_1. R_2; required Δ_1, Δ_2, and T_1.

$$\text{vers } \Delta_1 = \frac{R_2 \text{ vers } \Delta + T_2 \sin \Delta}{R_1 + R_2} \qquad \Delta_2 = \Delta_1 - \Delta$$

$$T_1 = T_2 \cos \Delta + R_2 \sin \Delta - (R_2 + R_1) \sin \Delta_1$$

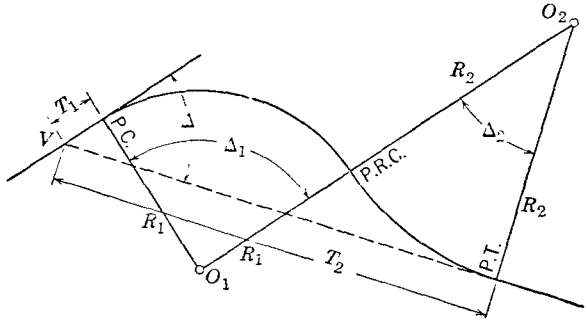

FIG. 66 Curves reversed to non-parallel tangents

Given T_1, Δ, R_1, R_2; required Δ_1, Δ_2, T_2.

$$\text{vers } \Delta_2 = \frac{R_1 \text{ vers } \Delta + T_1 \sin \Delta}{R_1 + R_2} \qquad \Delta_1 = \Delta_2 + \Delta$$

$$T_2 = T_1 \cos \Delta + R_1 \sin \Delta + (R_1 + R_2) \sin \Delta_2$$

(*c*) Fixed P.I.'s The two tangents through the P.C. and P.T. are frequently so nearly parallel that the intersection V cannot be readily found, in which case a line V_1V_2, Fig. 67,

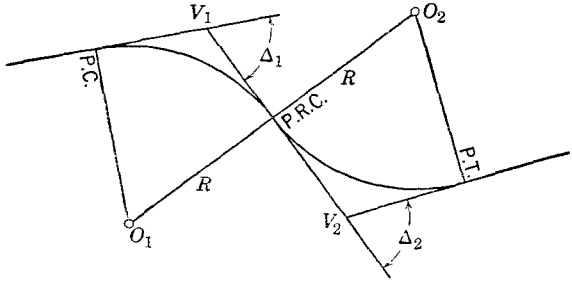

FIG. 67 Reversed curves, fixed P.I.'s and nearly parallel tangents

may be chosen as the common tangent, with angles Δ_1 and Δ_2, and the line V_1V_2 being measured. To connect these tangents by curves having equal radii:

$$R = \frac{V_1V_2}{\tan \Delta_1/2 + \tan \Delta_2/2}$$

(*d*) Fixed P.C. and P.T. Another way to define a reversed curve, having chosen the P.C. and P.T., Fig. 68, is to measure the distance AB between them and also angles a and b. To connect these tangents by curves having equal radii:

$$\sin \Delta = \frac{\cos a + \cos b}{2} \qquad R = \frac{AB}{\sin a + \sin b + 2 \cos \Delta}$$

In reversed curves with non-parallel tangents at the P.C. and P.T. the line AB, Fig. 68, does not pass through the P.R.C.

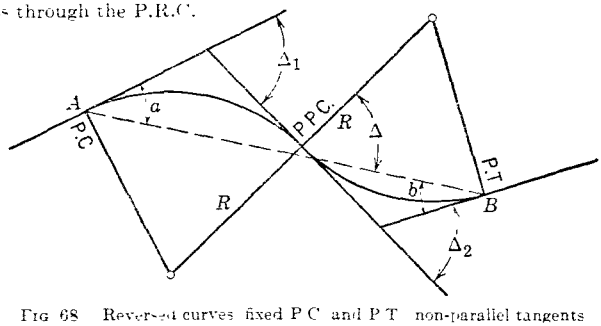

FIG. 68 Reversed curves fixed P.C. and P.T. non-parallel tangents

Obstacles on Curves. When obstacles occur, as frequently happens, so that the entire curve cannot be laid out by deflection angles from one position of the instrument, the difficulty can often be overcome by running out the curve as far as possible and then setting up the transit at the last located point on the curve and running the curve ahead from this intermediate set-up. This method is applicable when the obstacle is not large or when it does not lie directly on the curve If a large obstacle lies on the curve proper, the curve may be run out until it meets the obstacle, both from the P.C. and P.T., or such obstacles may be passed by use of the offset-from-tangent method, the offset being measured from the main tangent or from an auxiliary tangent of some intermediate point on the curve. Sometimes an obstacle on a curve can be passed by laying off the deflection angle for a point beyond the obstacle, computing and measuring the long chord to that point, thus locating a station without making a new set-up of the transit, and regular chord measurements may start again from this station.

If the P.C. is inaccessible lay off the curve from the P.T as far as possible toward the P.C. and check the last point by the offset-from-tangent method. In applying this latter method, compute the distance along the tangent from the P.C. and subtract this distance from T, which gives the distance from V to the point on the tangent from which the tangent offset is laid off perpendicular to the tangent

70. SPIRAL CURVES FOR TRANSITION TO CIRCULAR CURVES

Purpose and Basic Formulas. On curves the outer rail of a railroad should be elevated to overcome the effect of centrifugal force on the locomotive and railroad cars, and the roadbed of a highway must be radially superelevated.

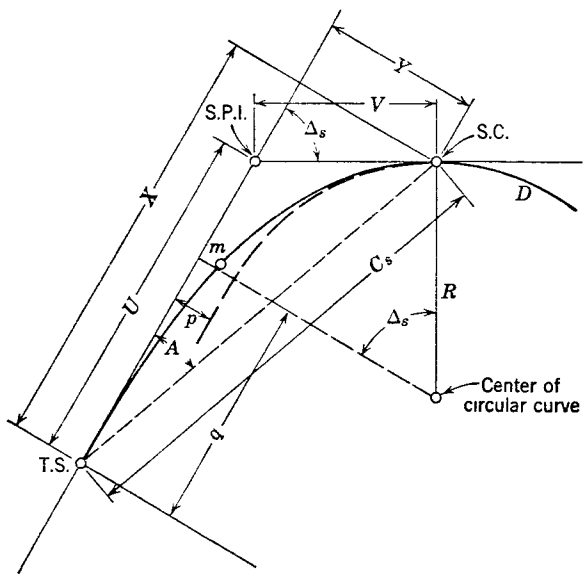

FIG 69 Seven functions of the clothoid spiral

m = midpoint of spiral, the spiral angle to this point is $\frac{\Delta_s}{4}$

T S = tangent to spiral
S P I = spiral P I
S C = spiral to circular curve

A transition curve is necessary for the acceleration and compensating superelevation to be gained at a uniform rate as direction is changed from a straight line to the circular curve. The clothoid is a spiral that is equally useful for those purposes in railroad and highway alignment

The seven functions of the clothoid spiral are shown in Fig. 69. The length of the spiral L_s is the curve distance from the T.S , where the spiral leaves the tangent to the S C., where the spiral joins the circular curve D

Equations for X and Y of the clothoid spiral are:

$$X = L_s(1 - 3.0461742\Delta_s^2 10^{-5} + 4.2959154\Delta_s^4 10^{-10} - 3.0198708\Delta_s^6 10^{-15} + \cdots)$$

$$Y = L_s(5.81776417\Delta_s 10^{-3} - 1.26585165\Delta_s^3 10^{-7} + 1.22691057\Delta_s^5 10^{-12} - \cdots)$$

For the other functions, the equations are:

$$q = X - R \sin \Delta_s \qquad V = Y \operatorname{cosec} \Delta_s$$

$$p = Y - R \operatorname{vers} \Delta_s \qquad C_s = Y \operatorname{cosec} A = X \sec A = \sqrt{X^2 + Y^2}$$

$$U = X - Y \cot \Delta_s = X - V \cos \Delta_s \qquad A = \arctan X/Y$$

The spiral angle can be computed by use of this formula:

$$\Delta_s = \frac{DL_s}{200} = \text{degrees}$$

Table 7. Functions of the Unit-Length Spiral *

To compute spiral parts of the spiral being used in the alignment, select the unit-length value of each function opposite the spiral angle Δ_s in the table and multiply it by the spiral length L_s. This gives correct results in any system of measurement. Unit-length functions for intermediate Δ_s angles may be obtained by straight-line interpolation to an accuracy of five places right of the decimal. For the Y, q, and p, however, interpolation usually gives results to six places.

Spiral Angle Δ_s	Spiral Coordinates: Abscissa X	Spiral Coordinates: Ordinate Y	Curve Coordinates: Abscissa q	Curve Coordinates: Ordinate p	Long Tangent U	Short Tangent V	Long Chord C_s
0 00°	1 0000000	0.0000000	0.5000000	0 0000000	0.6666667	0.3333333	1.0000000
.50°	0.9999924	.0029089	.4999987	.0007272	.6666693	.3333358	0.9999966
1 00°	.9999695	.0058176	.4999949	.0014544	.6666773	.3333430	.9999865
50°	.9999315	.0087262	.4999886	.0021816	.6666906	.3333551	.9999695
2 00°	.9998782	.0116345	.4999797	.0029088	.6667092	.3333720	.9999458
.50°	.9998096	.0145424	.4999683	.0036359	.6667332	.3333938	.9999154
3.00°	.9997259	0174499	.4999543	.0043629	.6667624	.3334204	.9998782
.50°	9996269	.0203567	.4999378	.0050899	.6667970	.3334518	.9998342
4 00°	9995127	.0232630	.4999188	.0058168	.6668369	.3334881	.9997834
.50°	.9993833	.0261684	.4998972	.0065435	.6668822	.3335293	.9997259
5 00°	.9992387	.0290730	.4998731	.0072702	.6669328	.3335753	.9996616
.50°	.9990789	.0319766	.4998465	.0079968	.6669887	.3336261	.9995905
6 00°	.9989039	.0348793	.4998173	.0087232	.6670500	.3336819	.9995127
.50°	.9987138	.0377807	4997856	.0094495	.6671166	.3337424	.9994281
7.00°	.9985084	.0406810	4997513	.0101757	6671886	.3338079	.9993368
.50°	.9982879	0435799	4997146	.0109016	.6672660	.3338783	.9992387
8.00°	.9980522	.0464773	.4996753	.0116274	.6673487	.3339535	.9991338
50°	.9978014	.0493733	4996334	.0123530	.6674369	.3340336	.9990222
9.00°	.9975354	.0522677	.4995890	.0130784	.6675303	.3341186	.9989038
.50°	.9972543	.0551603	.4995422	.0138036	.6676292	.3342086	.9987787
10 00°	.9969581	.0580512	.4994927	.0145286	.6677335	.3343034	.9986468
.50°	.9966468	.0609401	.4994408	.0152533	.6678432	.3344032	.9985082
11.00°	.9963204	0638271	.4993863	.0159778	.6679584	.3345079	.998362$_8$
.50°	.9959789	.0667120	4993293	.0167020	6680789	.3346176	.9982107
12.00°	.9956224	.0695947	.4992698	.0174260	.6682049	.3347322	.9980518
.50°	.9952508	.0724752	.4992078	.0181496	.6683364	.3348518	.9978862
13 00°	.9948642	.0753533	.4991432	.0188730	.6684733	.3349763	.9977139
.50°	.9944626	.0782289	.4990762	.0195961	.6686157	.3351059	.9975348
14 00°	.9940460	.0811020	.4990066	.0203188	.6687636	.3352405	.9973490
.50°	.9936144	.0839725	.4989345	.0210412	.6689170	.3353800	.9971564
15 00°	.9931678	.0868402	4988599	.0217633	.6690759	.3355246	.9969571
50°	.9927063	.0897051	.4987827	.0224850	.6692404	.3356743	.9967511
16.00°	9922299	.0925670	.4987031	.0232064	.6694103	.3358290	.9965384
50°	9917386	.0954260	.4986210	.0239273	.6695859	.3359888	.9963190
17.00°	.9912324	.0982818	.4985363	.0246479	6697670	.3361537	.996092$_8$
.50°	.9907113	.1011345	.4984492	.0253681	.6699537	.3363236	.9958599
18 00°	9901754	.1039838	.4983596	0260879	.6701461	.3364987	.9956204
.50°	.9896247	.1068298	.4982674	.0268072	.6703440	.3366790	.995374$_1$
19.00°	.9890592	.1096723	.4981728	.0275261	.6705477	.3368643	.995121$_1$
50°	.9884789	.1125112	.4980757	.0282446	.6707569	.3370549	.994861$_4$
20.00°	9878838	1153465	4979761	.0289626	.6709719	.3372507	.994595$_0$
.50°	.9872741	.1181781	.4978740	.0296801	.6711926	.3374516	.994322$_0$
21.00°	.9866497	.1210057	.4977694	.0303971	.6714189	.3376578	.9940422
.50°	.9860106	.1238295	.4976623	.0311137	.6716511	.3378693	.9937558
22.00°	.9853568	.1266492	.4975528	.0318297	.6718889	.3380860	.9934627
.50°	.9846885	.1294649	.4974408	.0325453	.6721326	.3383080	.9931629
23.00°	.9840055	.1322763	.4973263	.0332603	.6723821	.3385353	.9928564
.50°	.9833080	.1350834	.4972093	.0339747	.6726374	.3387680	.9925433
24.00°	.9825960	1378862	.4970899	.0346887	.6728986	.3390060	.9922231
.50°	.9818695	.1406844	.4969680	.0354020	.6731657	.3392494	.9918975
25 00°	.9811285	.1434782	.4968436	.0361148	.6734386	.3394982	.9915642
.50°	.9803731	.1462672	.4967168	.0368270	.6737175	.3397525	.9912240

Table 7. Functions of the Unit-Length Spiral—*Continued*

Spiral Angle Δ_s	Spiral Coordinates Abscissa X	Spiral Coordinates Ordinate Y	Curve Coordinates Abscissa q	Curve Coordinates Ordinate p	Long Tangent U	Short Tangent V	Long Chord C
26 00°	.9796032	.1490515	.4965875	.0375386	.6740023	.3400122	.9908778
.50°	.9788191	.1518310	.4964558	.0382496	.6742931	.3402774	.9905248
27.00°	.9780205	.1546056	.4963216	.0389599	.6745900	.3405481	.9901652
.50°	9772077	.1573752	.4961850	.0396697	.6748928	.3408243	.9897989
28 00°	9763806	.1601396	4960460	.0403788	.6752018	3411061	.9894260
.50°	9755393	.1628989	.4959045	.0410872	.6755168	.3413935	.9890465
29.00°	9746837	.1656529	.4957605	.0417950	.6758379	.3416866	.9886603
50°	.9738140	1684016	.4956142	.0425021	.6761652	.3419852	.9882676
30.00°	.9729302	.1711448	.4954654	.0432085	.6764987	.3422896	.9878683
50°	9720323	.1738825	.4953142	.0439142	.6768384	.3425997	.9874624
31 00°	0.9711203	0.1766145	0 4951605	0.0446192	0.6771844	0.3429155	0.9870498
.50°	9701944	.1793409	.4950045	.0453235	.6775366	.3432371	.9866308
32.00°	.9692544	.1820615	.4948460	0460270	.6778952	.3435645	.9862051
50°	.9683005	.1847761	4946851	.0467298	.6782601	.3438978	.9857729
33.00°	9673327	.1874849	.4945219	0474319	.6786314	.3442369	.9853341
.50°	.9663511	.1901875	.5943562	.0481332	.6790091	.3445820	.9848887
34.00°	9653557	.1928841	.4941881	.0488337	.6793933	.3449330	.9844368
50°	.9643464	.1955744	.4940176	.0495334	.6797839	.3452900	.9839784
35 00°	9633235	.1982584	.4938448	.0502324	6801811	.3456530	.9835134
.50°	9622869	.2009361	.4936696	.0509305	.6805849	.3460221	.9830419
36.00°	.9612366	.2036072	.4934920	.0516278	.6809953	.3463973	.9825638
.50°	.9601727	.2062718	.4933120	0523243	.6814123	.3467786	.9820793
37.00°	.9590953	.2089298	.4931296	.0530199	.6818361	.3471662	.9815882
.50°	.9580044	.2115811	.4929449	0537147	.6822665	.3475599	.9810907
38 00°	.9569000	.2142255	.4927578	0544086	.6827038	.3479599	.9805866
.50°	.9557821	.2168631	.4925684	.0551016	.6831479	.3483663	.9800761
39.00°	.9546510	.2194937	.4923766	.0557938	.6835988	.3487789	.9795591
.50°	9535065	.2221173	.4921824	0564850	.6840567	.3491980	.9790356
40.00°	9523487	.2247337	.4919860	0571754	.6845215	.3496236	9785056
.50°	.9511777	.2273429	.4917872	.0578648	.6849933	.3500556	.9779692
41 00°	.9499935	.2299448	.4915860	.0585533	.6854722	.3504941	.9774263
.50°	.9487962	.2325394	.4913825	.0592409	6859582	.3509393	9768770
42.00°	.9475858	.2351265	.4911767	.0599275	.6864514	.3513910	9763213
.50°	.9463624	.2377061	.4909686	.0606131	6869517	.3518495	.9757592
43.00°	.9451260	.2402781	.4907582	.0612978	.6874593	.3523147	.9751906
50°	.9438767	.2428423	.4905455	.0619815	.6879742	.3527867	9746156
44.00°	.9426145	.2453989	.4903305	.0626643	.6884965	.3532655	.9740342
.50°	.9413395	.2479475	.4901132	0633460	.6890262	.3537513	.9734464
45.00°	.9400517	.2504883	.4898935	0640267	.6895634	.3542349	.9728523
.50°	.9387512	.2530211	.4896716	.0647064	6901081	.3547436	.9722518
46.00°	.9374381	.2555457	.4894475	.0653850	6906604	3552504	9716449
.50°	.9361123	.2580623	.4892210	.0660626	6912204	.3557642	.9710316
47.00°	.9347740	.2605706	.4889923	.0667392	.6917880	.3562853	.9704120
50°	.9334232	.2630706	.4887613	0674147	.6923635	.3568136	.9697861
48 00°	9320600	2655622	.4885281	.0680891	.6929467	.3573492	.9691538
.50°	.9306844	.2680454	.4882926	.0687624	.6935379	.3578921	.9685152
49.00°	.9292965	.2705200	.4880549	.0694346	.6941370	.3584425	.9678704
.50°	.9278963	.2729861	.4878150	.0701058	.6947442	.3590004	.9672192
50.00°	.9264839	.2754434	.4875728	.0707758	.6953594	.3595658	.9665617
.50°	.9250593	2778920	.4873283	.0714447	.6959828	.3601389	.9658979
51.00°	9236227	.2803318	.4870817	.0721124	.6966145	.3607197	.9652279
.50°	.9221741	.2827627	.4868328	0727791	.6972544	.3613082	.9645516
52.00°	.9207134	.2851847	.4865818	0734445	.6979027	.3619046	.9638690
50°	9192409	.2875976	.4863285	.0741088	.6985595	.3625088	.9631802
53 00°	.9177565	.2900014	.4860731	0747720	6992248	.3631211	.9624852
.50°	.9162603	.2923960	.4858154	.0754339	.6998987	.3637413	.9617840
54.00°	.9147524	.2947814	.4855556	.0760946	.7005812	.3643698	.9610765
50°	.9132329	.2971574	.4852936	0767542	.7012725	.3650064	.9603628
55 00°	9117017	.2995241	.4850294	0774125	.7019727	.3656514	.9596430
.50°	.9101590	.3018813	4847631	0780696	7026818	.3663047	.9589170
56 00°	.9085049	.3042290	.4844946	.0787255	.7033998	.3669664	.9581848
.50°	.9070393	.3065670	.4842239	.0793801	.7041270	.3676367	9574464
57.00°	.9054624	.3088955	.4839511	.0800335	.7048633	.3683156	.9567019
.50°	.9038742	.3112142	.4836762	.0806857	.7056089	.3690032	.9559513
58.00°	.9022748	.3135231	.4833991	0813365	.7063638	.3696996	.9551945
.50°	.9006643	.3158221	.4831200	.0819861	.7071282	3704049	.9544316
59.00°	.8990426	.3181112	.4828387	.0826344	.7079021	.3711192	.9536626
.50°	.8974100	.3203904	.4825553	.0832814	.7086856	3718425	.9528876
60.00°	8957664	.3226595	.4822698	.0839270	.7094789	.3725750	.9521064

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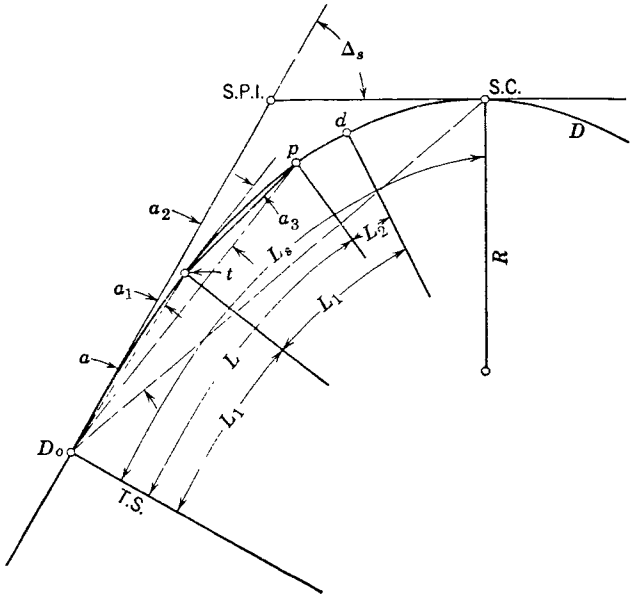

FIG 70 Deflection angles on a spiral curve.

Spiral Deflection. The deflection angle a, Fig. 70, from the tangent at the T.S. to the S.C., is equal to $(\Delta_s/3) - c$, where c is a small correction. Usually, c is considered too small to apply unless Δ_s is larger than about 20°, as is indicated in Table 8. This means

Table 8. Minutes Correction for Spiral Deflections *

Spiral Angle Δ_s	Minutes Correction
0°	0.000
5	0.006
10	0.051
15	0.174
20	0.413
25	0.809
30	1.401
35	2.231
40	3.340
45	4.771
50	6.568
55	8.776
60	11.44

The correction is subtracted for deflections from the T.S. or S.T. and is added for deflections from the S.C. or C.S.

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that the deflection angle from the tangent at the T.S. to any other points on the spiral curve, as a_1 for t and a_2 for p, are also equal to one-third ($^1/_3$) the spiral angle at those points minus a correction. The backward deflection to the T.S. from the tangent at any point on the spiral is two-thirds ($^2/_3$) of the spiral angle at that point plus a correction. In making deflection corrections, remember that the spiral angle to any point p is:

$$\Delta_p = (L^2/L_s^2)\Delta_s$$

Using those principles and applying them to the basic equation, any deflection angle in minutes, angles a, a_1, a_2, and a_3, can be computed by use of these formulas:

Transit at T.S. $a = (DL_s/10) - c =$ minutes

$a_1 = (DL_1^2/10L_s) - c =$ minutes

$a_2 = (DL^2/10L_s) - c =$ minutes

Transit at t $a_3 = (DL^2/10L_s) + (DL_1L_2/10L_s) - c =$ minutes

The deflection angles are more easily computed when the degree of curve D, in degrees, is divisible by the length of spiral L_s, in feet. Consequently, when the spiral length is computed, a length is chosen that will divide into D evenly. The constant $(D/10L_s)$ is first computed. It is a measure of the sharpness of the spiral curve. The L lengths are in feet:

L = distance from T.S. to p, the point being set by deflection from the T.S. or from an intermediate transit point as t.
L_s = length of spiral from T.S. to S.C.
L_1 = distance from the T.S. to t, any intermediate transit set-up point, and from t to the distance point d.
L_2 = distance from point d to the deflection point p.

When L_2 is measured toward the T.S. it is negative, and it is positive in direction away from it.

Circular Curve with Equal-Length Spirals. Equal-length spiral curves on each end of a circular curve makes a curve system that is the most common combination of spiral-

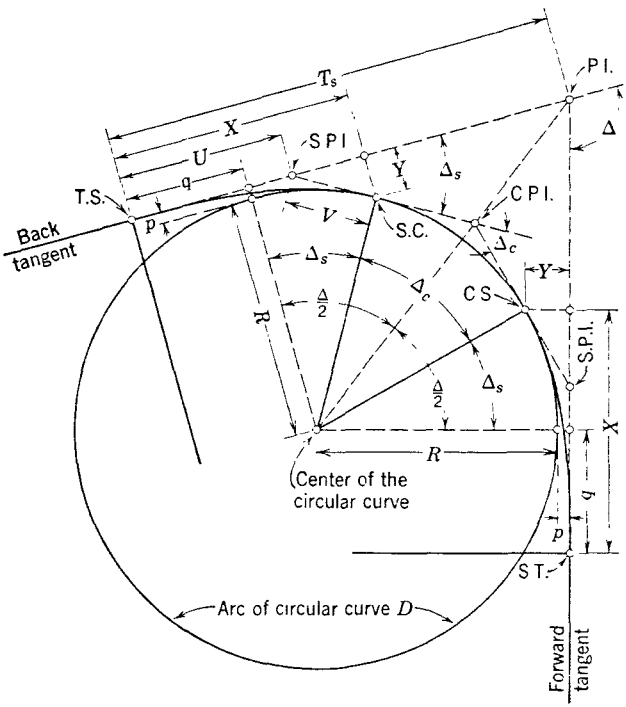

FIG. 71. Circular curve with equal-length spirals

circular curve-spiral, Fig. 71. The T_s is the distance from the T.S. to the P.I. and from the P.I. to the S.T.

$$T_s = (R + p) \tan (\Delta/2) + q$$

The E_s is the external distance from the P.I. to the circular curve, measurable along the line drawn from the P.I. to its center.

$$E_s = (R + p) \text{ exsec } (\Delta/2) + p$$
$$= (R + p) \text{ secant } (\Delta/2) - R$$

The central angle Δ equals $(2\Delta_s + \Delta_c)$, and $\Delta/2$ equals $[\Delta_s + (\Delta_c/2)]$. Formulas for other geometric parts (X, Y, q, p, U, V, C_s) are the same as given in the beginning of this article.

Circular Curve with Unequal-Length Spirals. There are situations where controls of topography or of land use or both prevent the use of equal-length spirals. The semi-

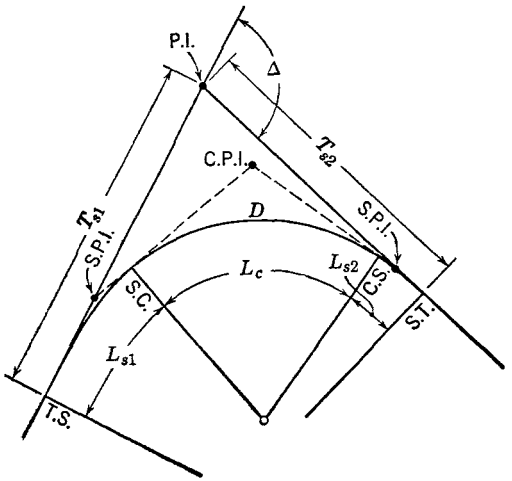

FIG. 72. Unequal-length spirals and circular curve.

tangents, Fig. 72, are designated T_{s1} from the T.S. to the P.I. and T_{s2} from the P.I. to the S.T. They are computed by use of these formulas:

$$T_{s1} = q_1 + (R + p_2) \tan (\Delta/2) - (p_1 - p_2) \cot \Delta$$
$$T_{s2} = q_2 + (R + p_2) \tan (\Delta/2) + (p_1 - p_2) \text{ cosec } \Delta$$

Those formulas apply in all cases, even though there may not be a spiral on one end of the circular curve. In such a case, the corresponding q and p values would be zero where there is no spiral. In using the formulas, it must be remembered that the cotangent is positive in the first and third quadrants and is negative in the second and fourth, and that the cosecant is positive in the first and second quadrants and negative in the third and fourth.

Revising Circular Curve Alignment to Introduce Spirals. In introducing spirals into existing circular-curve alignment, many existing conditions will limit the choice of spiral and introduce special problems. One of the most common is illustrated in Fig. 73, showing half a circular curve AB. One of the methods of introducing a spiral is to shorten the radius from R_1 to R_2. In that way the new circular curve is moved slightly outward in the middle and its sharpening allows it to fall inside the tangent making $ED = p$. Calling the outward movement $BC = k$, then

$$R_2 = R_1 + k - \left(\frac{k + p}{\text{vers } \Delta/2}\right)$$

and

$$AF = q - (R_1 + k - R_2) \sin \Delta/2$$
$$k = (R_1 - R_2 - p) \sec (\Delta/2) + (R_2 - R_1)$$

If the track of a railroad or center line of a highway cannot be moved more than 1 ft along the line between the P.I. (V) and center of the curve (O_1), assume $k = 1$ and p about $2k$. Using the first equation compute R_2: then AF sets the approximate location of the T.S., and D_2 and p may be used to determine the length of spiral L_s.

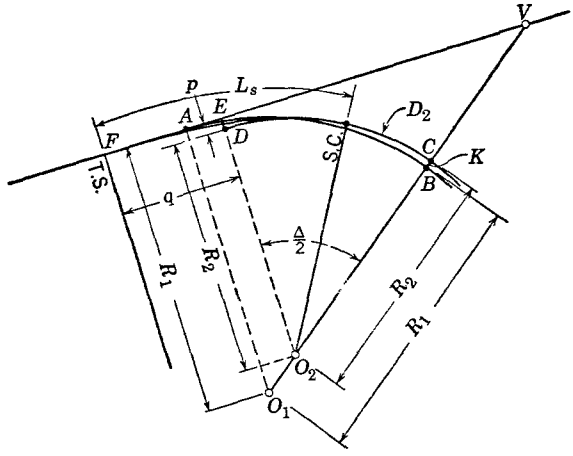

FIG 73 Revising a circular curve to introduce a spiral curve transition

Another method of introducing the spiral curve is to shift the original curve inward a few feet toward the center, Fig. 74. The amount of the shift is represented by $k = CB = O_1O_2$, and $k = p \sec \Delta/2$. The external distance $VB = E_s = [Y + R(\cos \Delta_s - \cos \Delta/2)] \sec \Delta/2$.

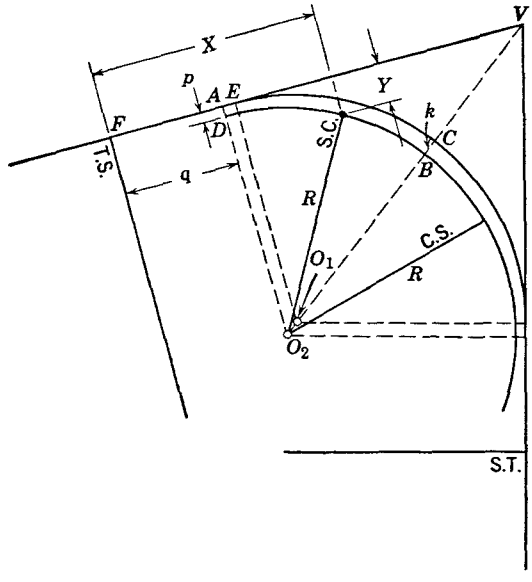

FIG. 74 Shift of circular curve to join it to tangent with a spiral.

$$\text{C.B.} = k$$

$$= P \sec \Delta/2$$

$$E_s = VB = \frac{Y + R\left(\cos \Delta - \cos \frac{\Delta}{2}\right)}{\cos \frac{\Delta}{2}}$$

71. PROFILE LEVELS

Levels for obtaining a profile of a line on the ground are called profile levels. Ordinarily a transit party works ahead of the profile party and stakes the line to be profiled. The profile party obtains an elevation at each 100-ft station of the line, at each intermediate plus point that represents a distinct break in slope of the ground line, at all required points on curves, and at other points that may be marked

In the field, the instrument is set up in any convenient location near a benchmark (BM), a backsight (BS) reading is taken, and the height of instrument (HI) is computed. Rod readings for ground elevation are then taken successively at as many of the line stakes as can be clearly seen [As the ground point at each center-line stake becomes a sort of benchmark for cross-section surveys at that station, the rod should always be held on the face side (on which stationing is marked) of each such stake] A foresight (FS) reading is then taken on any conveniently located turning point (TP). The instrument is then moved forward is set up any convenient location beyond the TP, a BS reading is taken, and the procedure is repeated Rod readings for the ground profile are usually to the nearest 0 1 ft, with readings on BM's and TP's to the nearest 0.01 ft; but, if profile readings are required to the nearest 0 01 ft, the TP readings should be to the nearest 0 001 ft, etc. Ties should be made to all benchmarks along the route that are within convenient distances

Double-Rodded Lines. When benchmark levels have not been run in advance of the profile work it is good practice to utilize two different sets of TP's while running the profile levels. This is called double rodding. It provides two independent HI's for each instrument set-up, and thus helps to prevent errors. On the first instrument set-up the first backsight is taken in the usual manner. When the party is ready to move forward, two forward TP's are set instead of one. These are set near each other, but at different elevations, preferably different by more than 1 ft. On the second instrument set-up a backsight is read on each of the two TP's, and two HI elevations are computed. These should be in close agreement; the limit of discrepancy allowed depends on the class of levels All of the precautions necessary for bench-work leveling should be observed, particularly that the sum of all backsight and foresight distances be kept nearly equal This method is particularly applicable to long *P* lines for canals, highways, and railroads in undeveloped regions

Profile-Level Notes. The field notes should be kept in a standard-type notebook having 6 columns ruled on the left-hand page for the level notes. The right-hand page is ruled in squares for topography and cross-section notes, for descriptions of benchmarks, and for other remarks In profile notes it is good practice to record all rod readings in sequential order up the page, from bottom to top of the notebook The notes are then in orientation as the levelman works and looks forward along the line, and mistakes in recording cross-section readings and distances on the wrong side of the center line are less likely to occur. An example of profile level notes is given in Fig. 75 When cross sections are recorded on the right-hand page, the notes on the left-hand page are double spaced in order to avoid crowding of the cross-section notes.

72. CROSS-SECTION SURVEYS

Cross sections are required for earthwork computations of borrow pits, highways, railroads, canals, etc. A cross section is actually a short profile, running approximately at right angles to, and extending on both sides of, the center line. A cross section is usually taken at each 100-ft-station stake and plus point. Cross-section surveys may be accomplished by several methods depending on the character of the topography, the type and density of ground cover, and the accuracy required

When an elevation accuracy of 0.01 ft is required in the cross-section surveys, dumpy or wye levels are used, for an accuracy of 0.1 ft, a transit (as either a level or vertical-angle instrument), a Rhodes arc, or a hand level will suffice, and for less than 0.1 ft (as for many types of preliminary-survey cross sections), the angle-reading Abney hand level may be used.

With Wye and Dumpy Level. The level is set up for taking rod readings along one or more cross sections, and HI elevation is determined by a BS on a BM or TP, as in profile leveling The rod is first held on the ground at the line stake (on face side) where the cross section is to be taken. The rodman then moves outward either to the right or to the left in a direction approximately at a right angle to the line, holding the rod for a reading at each break in ground slope. At each rod position the horizontal distance is measured from the line stake. Both the rod reading and the horizontal distance are recorded concurrently in the cross-section notebook, as shown in Fig. 75. Profile and cross-section

Adams—Jonesboro Highway
Preliminary Survey

March 21, 1951
Weather: Windy and Cold

Turner—Notes
Frost—Level

Poulsen—Rod
Bacon—Axe

Lowe—Tape

PROFILE LEVELS (Wye or Dumpy Level)						CROSS SECTIONS 1. Wye or Dumpy Level						
Station	Plus +	HI	Minus −	Rod	Elevation	(Cross-section rod subtracts from the HI elevation, the same as profile rod.)						
811+90				1.4	6277.7	−0.7 / 167	HL +2.6 / 130	HL +1.8 / 102	−1.4 / 0	−3.7 / 52	−8.0 / 114	HL −16.0 / 167
	4.67	6279.07										
TP			10.83		6274.40	(HL = Hand-level reading above HI, or extension of rod on minus readings.)						
811+50				7.2	6278.0	−0.6 / 150	−1.6 / 118	−5.7 / 27	−7.2 / 0	−10.1 / 50	−12.0 / 114	HL −16.7 / 150
	10.11	6285.23										
BM 101					6275.12	Spike 3′ from ground in 30″ spruce 85′ rt. of 810+13						
						2. Hand Level						
+60	(All heights and horizontal distances are measured from the ground at the center-line stake.)				5814.2	+3.0 / 172	+1.8 / 92	−0.7 / 27		−1.1 / 14.6	−3.8 / 39	−7.4 / 114
703+15					5810.6	+4.1 / 117	0.0 / 56	−1.4 / 24.2		−1.9 / 18.4	−8.2 / 47	−7.6 / 123
						3. Rhodes Axe						
302	(⊼ = Instrument point. The heights and horizontal distances for all breaks beyond this point are measured from it; others from the ground at the center-line stake.)				3729.8	+1.0 / 89	⊼ +7.5 / 105	+1.4 / 38		−3.6 / 27	⊼ −13.0 / 102	−3.2 / 73
301+45					3724.1	+1.6 / 89	⊼ +8.3 / 118	+1.0 / 71.4		−7.4 / 75	⊼ −17.0 / 137	−4.1 / 67
						4. Abney Level						
+50	(⊼ = Instrument point. The angle readings in degrees and slope distances beyond it are measured from that point; others from ground at the center-line stake.)				5141.1	−3.0 R * / −27 / 101	⊼ −16 / 107	−13.5 / 84		+17.5 / 74	⊼ +16 / 100	+16 / 141
146+10					5148.4	+2.0 R * / −19 / 110	−21.5 / 78	⊼ −13 / 90		+15 / 28	⊼ +19 / 146	+6.7 R * / +14 / 100

* (−) or (+) R indicates that the point read on the rod to get the slope angle is (−) below or (+) above the HI by the amount shown. In plotting the cross sections, points must be lowered or raised by the amount the rod reading is above or below, respectively, the point that should have been read.

FIG. 75. Sample notes. Explanations are in parentheses

leveling with the wye or dumpy level are often accomplished at the same time, especially where the topography is not rugged and ground cover is not a serious handicap.

On steep and irregular cross sections, an extra instrument set-up may sometimes be eliminated by using a hand level as a supplementary instrument to determine differences of elevation when the rod is higher or lower than HI. For each rod position above HI, the hand level is used to determine the difference in elevation between the HI and the zero point of the rod; this difference is recorded in the notes as a plus (+) rod reading. The arithmetic operations are handled mentally, or on separate paper. When the top of the rod is lower than HI, the hand level is used to determine the difference of elevation from HI to top of rod, which difference is mentally added to the regular rod length; the total is recorded as a minus (−) rod reading.

With Hand Level. In using the hand level, the levelman stands at any convenient place and takes a plus (+) reading on the rod at the center stake, to establish HI above ground at the center-line stake. Each minus (−) rod reading at a ground break is then mentally added algebraically to HI, the algebraic difference being recorded in the notes as the difference in elevation between the ground at the center stake and the cross-section point. (See Fig. 75.) On hillsides it is good practice to use a stick of convenient height on which to hold the hand level when the rod readings are taken. The distance from the center stake to each cross-section point is measured horizontally, usually with a metallic tape.

With Rhodes Arc. The Rhodes arc is held at any convenient height above the ground by an instrumentman standing at the center stake. At each rod position the instrument is pointed to a height on the rod that is equal to HI above ground, and the cosine and sine of the slope angle are read on the graduated arc. At the same time the slope distance is measured along the line of sight, or on a line parallel to it, using a metallic tape or steel tape. The cosine and sine values are then multiplied by the slope distance to get, respectively, the horizontal and vertical distance from the center stake to the rod position, the vertical distance being plus or minus as the sine is plus or minus. Notes are recorded in the same manner as for cross sections taken with a hand level.

It sometimes happens that steep slopes or dense ground cover make it impractical to sight the rod at the desired HI reading. In that case the instrument is sighted to any convenient reading on the rod, and the slope distance and vertical angle are measured to that point. The difference between the actual rod reading and the HI is then mentally calculated and shown in the notes above the cross-section record for that point, preceded by a plus or minus sign to show whether the reading was higher or lower than the HI.

When it is necessary to move the Rhodes arc outward from the line stake to a new position, the subsequent cross-section readings beyond the new instrument point are measured from that point, and are so recorded in the notes; the ground point at which the new set-up is located is indicated in the notes by the tripod symbol, as shown in Fig. 75.

With Abney Level. The procedure in using the Abney level is the same as with the Rhodes arc, except that the vertical angle to the nearest 0.5°, or the nearest per cent of grade (whichever is most convenient) is read on the graduated arc. The vertical-angle reading, with appropriate plus or minus sign, is recorded in the notes instead of the computed differences of elevation in the numerator position, the slope distance is recorded in the denominator position.

Cross-Section Notes. These may be kept in the profile level book, or in a separate book, as convenient Cross-section notes are susceptible to many errors, and must therefore be carefully recorded, special attention being paid to the use of correct plus or minus signs. The data for each cross-section point are shown in the notes in the form of a fraction, the elevation data being in the numerator position, and the distance figures in the denominator position. (See Fig. 75.) As in profile leveling, the notes in this example begin at the bottom of the page and proceed to the top of the page. The vertical line in the center of the right-hand page of the book represents the center line of the survey. Each rod position is recorded in sequence to the left or to the right of this center line, corresponding to the left or right side of the recorder while he is looking forward along the line.

73. GRADES

Grades are expressed in per cent, i e., the ratio of the vertical rise in feet per 100 ft of horizontal distance, or the rise in meters per 100 m. A 1% grade is a straight-line rise of 1 in 100. Actually, the per cent of grade is the tangent of the vertical angle multiplied by 100.

In most engineering work, the rise is so small in proportion to the horizontal distance that it is customary to exaggerate the rise on profile drawings at a ratio of 10:1. On

drawings of cross sections and structures, however, grades are always drawn to natural scale, i.e., the same scale is used both horizontally and vertically.

On a vertical curve the grade is considered to be a rate of change. This may be expressed by the formula

$$r = (g_1 - g_2)/L$$

in which r is the average change in per cent per 100 ft station; g_1 and g_2 are the grades, in per cent, of the intersecting straight slopes, such as BA and AE in Fig. 76; $(g_1 - g_2)$ is

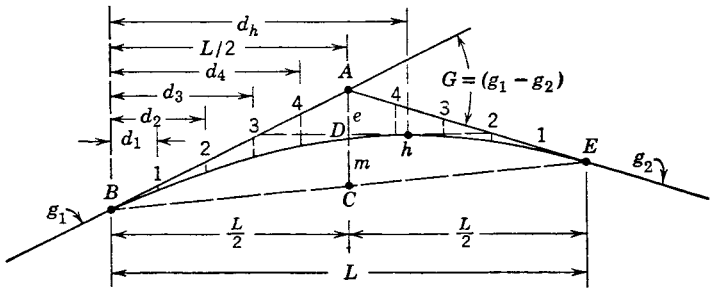

FIG 76. Symmetrical vertical curve, equal horizontal length each side of vertex of grades.

the algebraic difference of the grades in per cent; and L is the vertical curve length in 100-ft stations from one grade point to another. Thus the grade at any point, such as B, Fig. 76, is changed at any other point, such as E, according to the formula $g_2 = g_1 - rL$.

74. VERTICAL CURVES

Vertical curves are used in grades and in cross sections. They are used at the vertex to avoid the sudden change of vertical direction in passing from one grade to another. On cross sections they are used in crowns and to eliminate abruptness in joining the ground line at top of cut slopes and at the toe of fills, providing a pleasing appearance to the cross section and aiding in the prevention of erosion.

Vertical curves are parabolic instead of circular. There are two types of vertical curves: (1) symmetrical, having equal horizontal length each side of the intersection of grades; and (2) unsymmetrical, having unequal horizontal lengths each side of the vertex. Most vertical curves are of the symmetrical type. Unsymmetrical curves are used wherever controls, appearance, and other conditions require it.

In Fig. 76, the horizontal length L is halved by a vertical line from A, the vertex of grades g_1 and g_2. In Fig. 77, L is divided into lengths L_1 and L_2 on each side of the vertex.

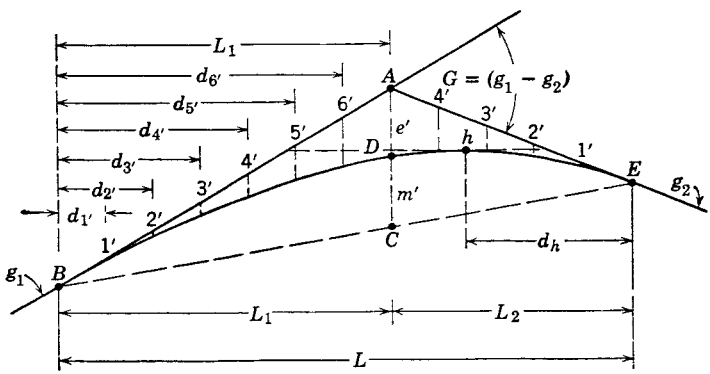

FIG 77. Unsymmetrical vertical curve, unequal horizontal length each side of vertex of grades.

L_1 and L_2 are not equal. In both curves, the chord BE is intersected at C by a vertical line from the vertex, A. Point D at the intersection of this vertical line and the curve is

midway between A and C, thus the external, e, equals the middle ordinate, m. The elevation of

$$D = {}^1/_2\,(\text{elev. } A + \text{elev. } C)$$

and

$$e = \left(\frac{g_1 - g_2}{8}\right) L = \frac{GL}{8} = \text{feet} \qquad e' = \frac{G}{2}\left(\frac{L_1 L_2}{L}\right)$$

In the formulas, $G = (g_1 - g_2)$, the algebraic difference of the grades in per cent. All numbered offsets from the grade line to the vertical curve vary as the squares of the horizontal distances from B or from E. Therefore each offset, numbered 1, 2, 3, and so on, equals:

$$\text{Both sides equal} \quad e_n = d_n^2 \left[\frac{e}{(L/2)^2}\right]$$

$$\text{Long side} \quad e'_n = d_n^2(e' \cdot L_1^2) \qquad \text{Short side} \quad e'_n = d_n^2(e'/L_2^2)$$

The d_h distance to the high point or the low point is often required for use in determining sight distances, to mark the place for a storm sewer inlet, and the like.

For symmetrical curves:

$$d_h = g_1 L/G = g_1 L^2/8e \qquad (L - d_h) = g_2 L/G = g_2 L^2/8e$$

For unsymmetrical curves, where g_2 and L_2 apply to the short part of the curve: $d_h = g_2 L_2/2e'$

A grade line tangent to point h is always level.

75. AREA OF CROSS SECTIONS

The area of a cross section between the original ground line and the section of roadway or other earth grading may be determined by planimeter for preliminary design purposes, and where accurateness is essential, as for contract payments, computed by forming a series of triangles between the ground line and the final section, the triangle dimensions being horizontal and vertical. The vertical distances may be determined by either of two methods (1) to roadbed grade between the center line and the shoulder, and beyond it to the level of the shoulder; or (2) to the level of the center-line grade In forming the triangles, it is necessary to begin at the center line, to keep the horizontal distances sequential as the triangles are formed working outward to the right and the left, and always to close on grade of cut zero for a cut section and fill zero for a fill section.

Method 1 Figure 78 shows the triangles of a section of fill (embankment) and cut (excavation) where heights are to roadbed grade. The areas formed by triangles in cut are

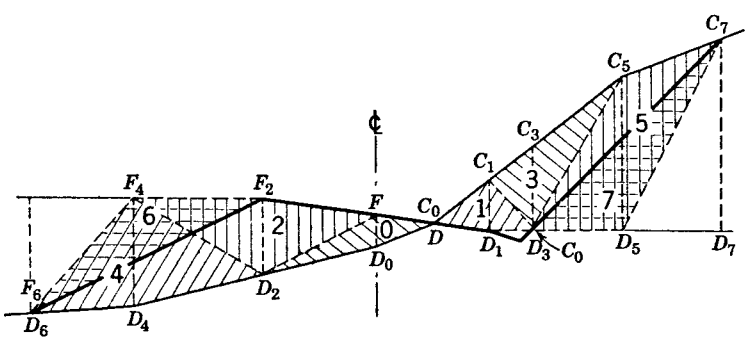

FIG. 78. Section of fill and cut, heights to roadbed grade Triangles in end-area computations formed by fill and cut at sequential distances from center line

given odd numbers, and those in fill even numbers; the distances from center line are similarly designated. The additive areas are cross hatched with vertical or sloping lines, and the subtractive areas are cross hatched with horizontal lines. Equations for the double areas in terms of applicable vertical and horizontal distances are given below.

Area Formulas

Area Number	Double Area	Area Number	Double Area
0	$F[(D_2 - D_0) + (D - D_3)]$	1	$C_1(D_3 - D)$
2	$F_2(D_4 - D_0)$	3	$C_3(D_5 - D_1)$
4	$F_4(D_6 - D_2)$	5	$C_5(D_7 - D_3)$
6	$F_6(D_2 - D_4)$	7	$C_7(D_3 - D_5)$

By observing signs, double areas 6 and 7 will be subtractive, because D_4 is farther from the center than D_2, and D_5 than D_3. The algebraic sum of the double areas is divided by two, and the area of the ditch is added.

Method 2. This method is illustrated in Fig. 79, which shows the part of a superelevated cross section that is in cut, with ground-line "overhang" and with all breaks referred to

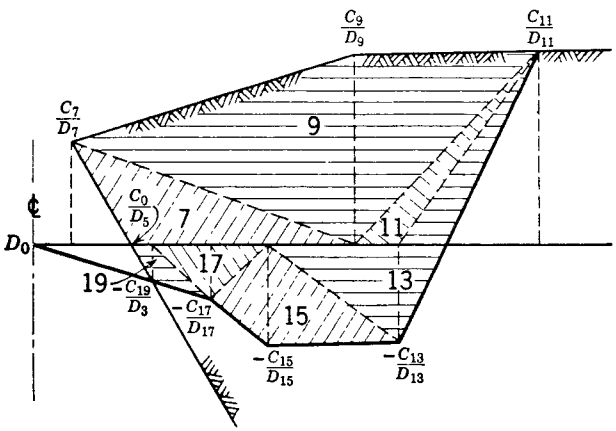

FIG. 79 Heights to center-line grade. Triangles in end-area computations formed by cut at the sequential distance from center line for overhang cross section

the level of center-line grade. Notes for the cross section, for either side of the center line, should be arranged to begin at the center line where the distance is zero, D_0. For the cut, all breaks in the ground line as far as the slope stake are successively recorded, then the breaks in the final cross section back toward the center line to grade C_0 at distance D_5 are similarly recorded. Similar steps are followed for a fill. If all breaks in the fill section that are higher than the center-line grade are considered to be minus fills, and all breaks in the cut section that are lower than the center-line grade are considered as minus cuts, the use of corresponding algebraic signs will insure that the corresponding double areas are appropriately added or subtracted as they are computed in sequence.

The disadvantage of method 2 is that the slope stakes do not mark cut or fill to the level of shoulder grade. Its principal advantage is in computing end areas from contours.

Regardless of the method of computation, the survey for each staked cross section should be extended to include sufficient ground-line points beyond the slope stakes. These are for possible later use in computing the end areas of (1) overbreak and earth slides that may occur during construction, (2) fill widening that may become necessary in the disposal of waste caused by slides, overbreak, or swell of solid rock, and (3) side borrow to provide additional material that may be required. The volumes of overbreak and of unpreventable slides are calculated after final cross sections are taken, and the differences in area between the staked and final cross sections are obtained.

76. VOLUME COMPUTATIONS

From Cross Sections. To compute the volume of earthwork between two parallel cross sections, let A_1 and A_2 be the end areas, A_m the area of a section of roadbed base b midway between the ends, c_1 and c_2 the center heights, and h_L and h_R slope-stake heights at the end sections, D_1 and D_2 the sums of the side distances d_L and d_R, L the length of the

solid, and V its volume. The terms c, d, and D, used in computing the prismoidal correction, are illustrated in Fig. 80.

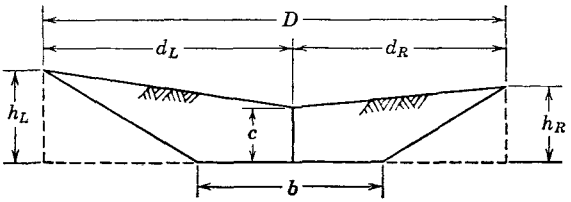

FIG. 80 Cross section dimensions, prismoidal correction.

By average end areas,

$$V = (A_1 + A_2)L/2$$

By prismoidal formula,

$$V = (A_1 + 4A_m + A_2)L/6$$

By average end areas with prismoidal correction,

$$V = (A_1 + A_2)L/2 - (c_1 - c_2)(D_1 - D_2)L/12$$

Table 9. Calculation Sheet for Grading

Stations	Cross-Section Areas, sq ft			Length	Cubic Yards					Mass Balance
	Excavation		Embankment		Excavation		Embankment	Excavation Makes Cubic Yards of Embankment		
	Rock	Common			Rock	Common		Rock r	Common c	
+50		175	32							
				50		356	30		267	100,257
403		210	0							
				30		454	□0		341	100,494 *
+30		608	0							
				40	△0	538	0		404	100,835
+70	0	118	0							
				30	39	44	△26	49	34	101,239
404	104	0	70							
				50	270	△0	227	338		101,296
+50	188		175							
				50	338		260	423		101,407
405	177		106							
				35	143		191	179		101,570
+35	44		188							

r = multiplication factor = 1.25 for swell of 25%.
c = multiplication factor = 0.75 for shrinkage of 25%.
△ = pyramid in volume computations.
□ = wedge in volume computations.
* The mass balance at station 403+00 is 100,257 (the mass balance at station 402+50) plus 267 minus 30, equaling 100,494. For mass balance at subsequent stations, proceed in a similar manner.

When the given quantities are in feet, V is in cubic feet and must be divided by 27 to obtain cubic yards. The method of average end areas is used most, without applying the prismoidal correction. An example of this method is given in Table 9. The mass balance is arbitrarily started large enough to prevent the algebraic summation from becoming negative; also, stationing points are interpolated for the centers of gravity in the mass and for the position of the free haul line, as explained in Art. 77 and illustrated in Figs. 85 and 86.

Correction for Curvature. The field surveys for cross sections on curves are taken along radial lines, such as AB, CD and EF, Fig. 81. In computing the earthwork each solid is assumed to have a length equal to the center-line arc, as GH or HJ, and to have end sections whose planes are perpendicular to their chords. In computing the two solids KGL–MHN and OHP–QJR, however, it will be seen that the solid PMH has been included twice, and solid NOH has been omitted. If the cross section DHC is symmetrical about the center line, these discrepancies balance; but if the cross section through H is unsymmetrical, a correction must be applied. In Fig. 82, DF has been drawn so as to form a symmetrical figure with DE. For the cross section $EDFBA$ there is no correction, but for the portion FDG a correction must be applied. This correction may be found by an application of Pappus' theorem, i.e., "If a plane area lying wholly on the same side of a straight line in its own plane revolves about that line and thereby generates a solid of revolution, the volume of that solid thus generated is equal to the product of the revolving area and the path described by the center of gravity of the plane area during the revolution." For the case illustrated in Fig. 82, the correction is $(b/2 + sc)(h_1 - h_2)(d_1 + d_2) \times 0.002909\Delta$, in which Δ = sum of half the central angle (in degrees) under the chords GH and HJ, so that if arcs GH and HJ are each 100 ft, Δ = degree of curve. In the formula, s is the slope, being equal to $(d_1 - b/2)/h_1$. The correction is in cubic feet if the distances are in feet, and it is added if the greater area is outside the curve, and subtracted when the greater area is inside the curve.

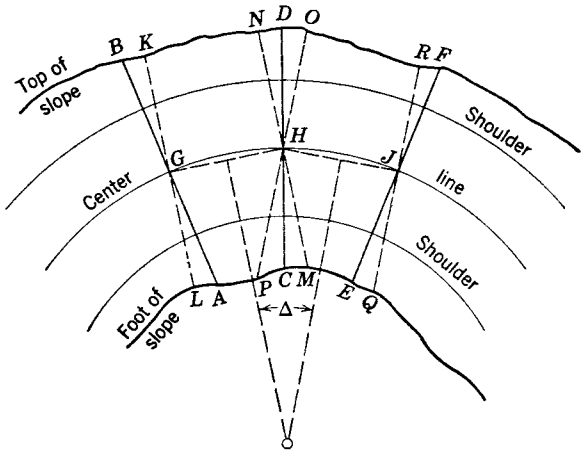

FIG. 81. Correction for curvature.

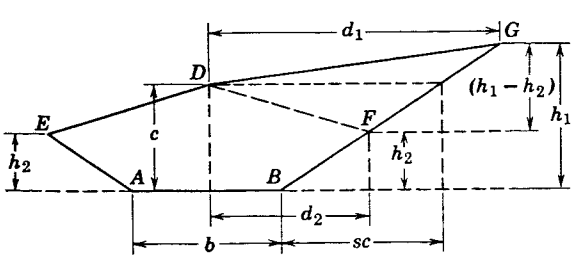

FIG. 82. Correction for curvature.

Borrow Pits. In railroad, highway, and similar types of construction, it is often more economical, when the length of haul becomes great, to obtain material from nearby borrow pits for construction of embankments. Borrow pits are sometimes necessary also as a source of special materials for the subbase and other purposes, when regular excavation is not suitable. The customary method of providing data for calculating the amount of material taken from a borrow pit is first to lay out a grid of squares over the entire potential borrow-pit area, then to determine the elevation at each corner of each square, and at each intermediate point between corners that represents a significant break in slope of ground surface. When the excavation is completed the same system of squares is again laid out, and elevations determined for the corners and intermediate points. The differ-

ence between the original and final elevations at the same points provide a simple means of determining the volume of borrow.

Volume Computations from Contours. Volume computations from contours are regularly used for solving many problems in landscape grading and similar work. The principles are sound, and accurate determinations may be made by this method, provided the contours are accurately located. However, the use of contours in earthwork computations for highways and railroads has heretofore been confined almost exclusively to preliminary designs and estimates. For this purpose, a contour map or plan of the entire project area is advantageous, as approximate quantities of earthwork can be determined for any part of the project area without additional field work. In highway work the cross-section method for computing earthwork for payment of contracts is usually more accurate and economical, and more acceptable to contractors.

There are four general methods of computing earthwork from contours, briefly described as follows

Cut and Fill between Contours. The simplest application of this problem is illustrated by Fig. 83, in which the full lines are contours representing the elevation and conformation of the original ground surface, the dash lines are contours representing the proposed surface, and the dotted lines are for the purpose of connecting points of zero excavation on

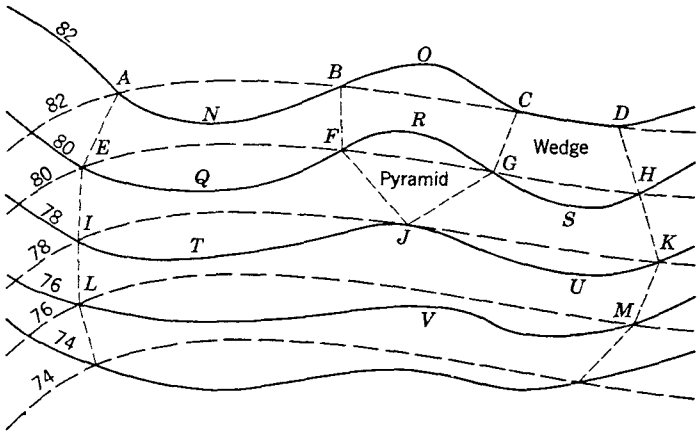

FIG 83. Earthwork volumes from contours

adjacent contours and marking the limits of the solids to be computed. Such solids as *ANB–FQE*, *EQF–JTI*, *ITJCK–MVL*, in which the final grade is below the original surface, represent excavation; conversely, solids like *BOC–GRF* and *FRG–J* represent embankment. The solid *ANB–FQE* is essentially an inclined prismoid having two parallel horizontal bases whose areas are *ANB* and *FQE*, the perpendicular distance between these being the contour interval, in this case 2 ft The volume of solid *ANB–FQE* is computed by first determining the area of each of the parallel bases *ABN* and *EFQ*, then multiplying the mean of these two areas (considered to be end areas) by the contour interval. The other excavation and embankment solids in Fig. 83 are computed in a similar manner. It should be noted that solid *CD–HSG* is a wedge, and that solid *FRG–J* is a pyramid with a base of *FRG* and an altitude of 2 ft.

In this and other methods of computing volumes from contours, the required areas of surfaces (end areas) are best determined by planimeter

Volume between Contours and a Plane. Sometimes the problem is so complicated that the quantities of earthwork cannot be readily separated into prisms, wedges, and pyramids. A convenient solution in that case is to assume a horizontal plane below the lowest part of any of the earthwork, compute the volume between that plane and the original surface, then compute the volume between that plane and the proposed surface The difference between these two volumes will be the amount of cut or fill

Cut-and-Fill Contours. This method is particularly applicable when the original ground is very irregular and the proposed surface is to be of an entirely different configuration, with the result that the proposed final grade contours do not cut many of the original contours of the same elevation. The first step is to mark the amount of cut (or fill) at every intersection of a new-grade contour with an original-surface contour, and then to connect

by means of a smooth line the successive points of equal cut and of equal fill. It will be seen that these new lines mark the limits of the horizontal projections of a series of irregular surfaces that are parallel to the proposed final surface If the contour interval is 2 ft, these lines would mark, progressively, the line of no cut or fill, the limit of the 2-ft cut or fill, 4-ft, etc. The solids included between these irregular surfaces may be visualized as layers of earth, each 2 ft thick, the volumes of which are computed by determining the area of each different surface and applying conventional end-area methods For example, the volume of the solid between the surfaces marking the 2-ft cut and the 4-ft cut is found by first determining the area of the surface within the 2-ft cut line, next determining the area of the surface within the 4-ft cut line, then multiplying the mean of these two end areas by the vertical height between the surfaces, which in this case is 2 ft The solids at the top or bottom of these layers will be either wedges or pyramids

Cross Sections from Contours. This method is as accurate as the contours, but the locations of the cross sections that are to be scaled from the contours must be carefully selected. The method is particularly applicable to the solution of design problems involv-

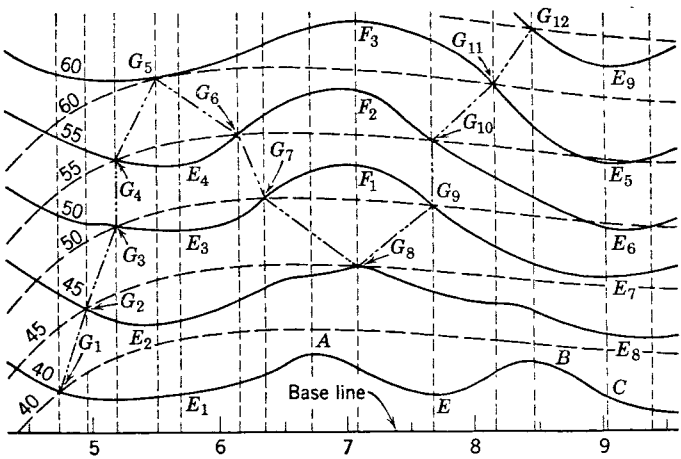

FIG 84 Cross sections for volumes from contours

ing balanced grading or preparation of alternate plans, as it is relatively easy to make trial changes of the proposed new grade and cross sections. The principles of the method can be visualized by reference to Fig. 84, in which the solid lines represent the contours of the original ground; the dash lines represent the proposed new grade; the dotted lines, extending perpendicularly from the stationed base line, mark the cross sections that should be scaled from the contours; and the dash-two-dot line marks the grade-line contour (line marking change from cut to fill) Each of the points marked G_1 to G_{12} on the grade-line contour represents the intersection of an original ground-surface contour with the proposed-grade contour of the same elevation. Cross sections should be laid out to pass through all G points, and through certain other points to be marked on the contours, including: points of maximum excavation marked E_1, E_2, etc ; points of minimum cut marked A and B; points of maximum fills, marked F_1, F_2, etc , and points of change in direction of the contours, marked C. Usually a cross section taken for one point such as G_5, serves another point, such as E_2, similarly the cross section for G_9 serves G_9 and E The area of each cross section is then determined, either by planimetering or by forming triangles for computation of the area, and the method of averaging the end areas is applied in the volume computations.

The prismoidal formula for volumes is sometimes utilized in problems of this nature. The middle area, Am, is interpolated by eye, or every other contour-plane area (or cross-section area) is treated as the middle area. The errors in sketching the contours, in scaling the map, in the shrinkage of the map paper, and in the uncertainty of the amount of shrinkage in the earthwork, however, are sufficient to offset any advantage in accuracy which the prismoidal formula may have over the end-area method

Dredged Material. Measurements of quantities of dredged material are taken in two ways, i.e., (1) measurement in place, and (2) scow measurement. For the method of

measurement in place, soundings are taken both before and after the dredging work is done, the volume of the removed material being computed by one of the methods previously described.

When scow measurements are used, the capacity of each pocket of the scow is carefully measured and computed. After the scow has been loaded, the number of full pockets is noted. The unused portion of each pocket that is not full is then determined by measuring vertically downward from the top of the coaming of the scow to the line where the material would come if it were leveled off. The volume of this small rectangular prism is then calculated and deducted from the known capacity of the corresponding pocket. For convenience, tables giving the quantity in each pocket at various distances below the top of the coaming are usually prepared for each scow in use. Scow measurements should be taken before the tow starts for the dumping ground. When scows remain moored for a day or so before being towed to the dumping ground some of the material in the pockets may leak out through loose-fitting bottom doors, and much material may find its way back into the dredged portion of the channel. On deck scow, where the material is piled on deck, any convenient method may be used to determine the volume, the measurements depending upon the shape of the pile. The amount of rock excavation can be calculated by obtaining its weight, which is ascertained by determining the displacement of the scow before and after loading.

77. MASS DIAGRAM AND OVERHAUL

The mass diagram (Figs. 85 and 86) is a graphic curve of the algebraic summation of the volume of grading along a highway, railroad, or other route improvement. Its purpose is to provide a convenient means of measuring haul, and comparing costs of different schemes of grading and haul. Grading includes both cut and fill. A descending line on the diagram denotes fill, which is considered minus; an ascending line denotes cut, considered plus. The diagram is prepared by plotting the algebraic cumulative total of grading at each successive cross section along the route, adding the excavation volumes, and subtracting the embankment volumes.

In computing volumes for the mass diagram, allowance must be made for swelling or shrinkgage of material that is excavated. Rock excavation usually swells. Common earth excavation usually shrinks when placed in a fill and properly compacted, and there is also a loss in transportation. Each excavation volume must therefore be multiplied by a factor for swell or shrinkage, as illustrated in Table 9, before use in the computation of the mass balance. These factors vary according to the type of materials. Actual values may be determined by taking final cross sections in cut and on fill. For solid rock the swell ranges from about 10 to 25%; for common earth the shrinkage ranges from about 10 to 30%. (If 1000 cu yd of common earth excavation make 750 cu yd of embankment, the shrinkage factor would be $^1/_4$, if 800 cu yd of rock excavation make 1000 cu yd of embankment, the swell factor would be $^1/_4$.) The factors will vary according to the dimensions (depth and width) of cuts and fills, type of material, slopes on which the fills are placed, methods of excavation and haul, and the compaction.

After the mass diagram is plotted, a balance line is drawn to establish the most economical method of hauling and disposing of the excavated materials. The objective is to make the cuts after shrinkage and swell factors are applied, balance the fills, so as to eliminate (as much as practicable) the need for waste or borrow. With motorized equipment, it is customary to require that materials be hauled free for a limited distance. Free-haul distances might be 400 or 500 ft on small projects, and 1000, 1320, or 2000 feet on large projects.

The mass diagram in Fig. 85 illustrates the use of the balance line and the method of determining free-haul yardage and overhaul yardage. For example, a balance section for the project, established by line WZ, between stations 50 and 67+50, contains sufficient excavation materials after shrinkage and swell factors are applied, to balance the embankments. Line FH, from station 52+40 to 65+60, is at the free-haul distance of 1320 ft, and the material between these stations will be hauled free. To compute the overhaul in terms of station-yards, a yardage line YX is drawn at a point such as Y, where there is a marked change in the slope WF. This divides the overhaul computations into two divisions, each bounded on both sides by fairly uniform slopes of the mass curve, as fill slope WY and cut slope XZ for division 1, and, similarly, slopes YF and HX for division 2. Other similar divisions would be made for this purpose, when and as necessary. Points e and f midway between the yardage lines W–Z and Y–X, and points m and n midway between lines Y–X and F–H, are marked; the station-and-plus value of each such point is calculated by interpolation for use in the overhaul computation, as noted below. All

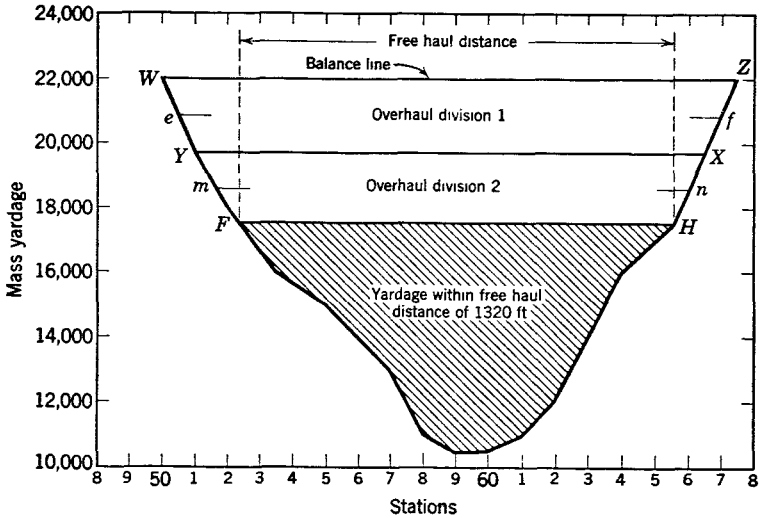

FIG 85. The mass diagram, free haul and overhaul divisions.

points in a mass curve like e and f, and m and n, are the centers of gravity in the mass referred to in Art. 76. For division 1, the overhaul in station-yards, if the excavation material shrinks, would be computed by the formula:

$$[(\text{Sta. } f - \text{Sta. } e) - (\text{Sta. } H - \text{Sta. } F)]\left[\frac{(\text{Cu yd fill})(W - Y)}{1 - \text{Shrinkage factor}}\right]$$

If the excavation material swells the denominator of the second term would become 1 + swell factor.

In Fig. 86 are shown a profile and mass diagram with certain of their principal features and applications. GG is the grade line across the profile. The adopted balance lines are

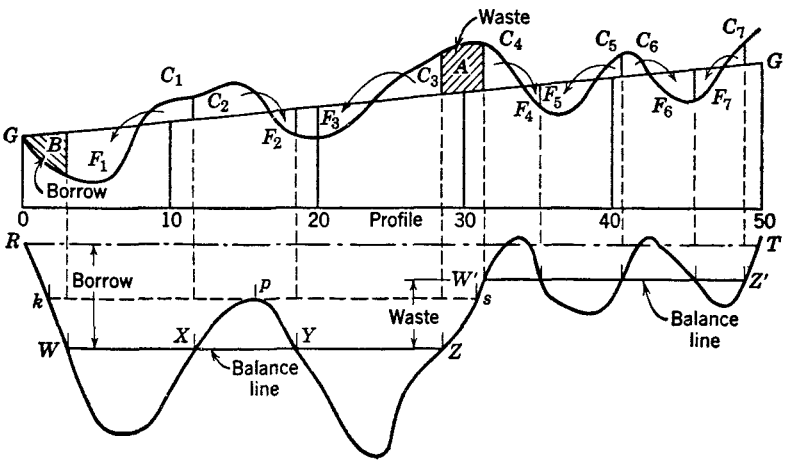

FIG 86 Profile, mass diagram, balance line, borrow, waste and haul.

WZ and $W'Z'$. Lines extended vertically from the points where the balance lines intersect the curve of the mass diagram are used to mark on the profile the limits of borrow (B) and waste (A), and the limits of each section within which the fills and cuts balance.

Arrows along the profile indicate the direction in which cuts, C_1, C_2, C_3, etc., are to be hauled to make the adjacent fills F_1, F_2, F_3, etc. The YZ portion of balance line WZ is equal to the adopted free-haul distance of 1000 ft, and WX is less. If overhaul is introduced by moving the balance line upward to k–p–s the borrow can be nearly halved and the waste decreased nearly three-quarters. If the balance line is drawn from R to T all borrow and waste would be eliminated. However, as this would require overhaul to move material from the cut between Stations 31+70 and 35+70 to the fill between Stations 0+00 and 3+40, a balance line such as RT should not be adopted until it is determined that the overhaul is less costly than the waste and borrow. There are certain circumstances, however, in which such a criterion would not be applicable, as for example: (1) when cut material is not suitable for embankment construction, the balance lines such as WZ and $W'Z'$ are placed so the unsuitable material will be wasted; and (2) when a special bridge would be necessary to haul the grading over in advance of completion of the regular bridge. Another possible solution in the case illustrated would be to extend line k–p–s to Station 48+25, which would eliminate waste between points s and W' and introduce waste between Stations 48+25 and 50+00. A method of eliminating borrow and waste, while keeping the overhaul to a practical limit, is to adjust the curvature or the grade line so as to decrease or increase as desirable cut or fill, the amount of such adjustment and the place where it is required being determined by an inspection of the mass diagram, after decision has been made as to the best place for the balance lines.

78. SLOPE STAKES

The purpose of slope stakes is to mark on the ground the edge of cut and toe of fill on the cross section at each 100-ft station and at such intermediate plus points as may be necessary. On a side-hill cut-and-fill section, a 0.0 stake is also set to mark the point where cut ends and fill begins.

At the time slope stakes are set conventional cross sections are taken for the purpose of computing pay quantities of excavation and fill. Slope stakes, however, are not always set at all of these cross sections. Topography that is extremely rugged and irregular may

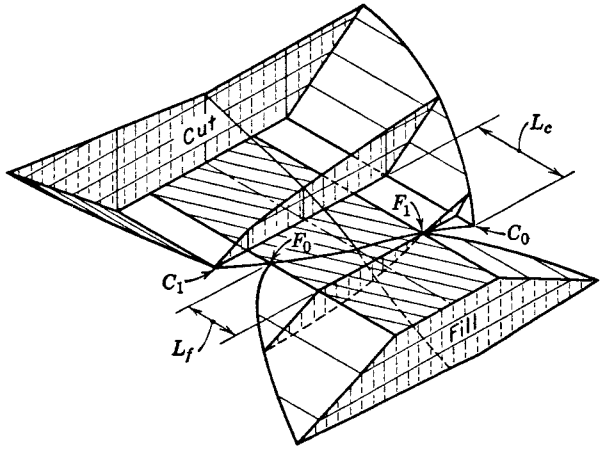

FIG 87 Perspective view, roadway cut-and-fill section Cross sections must be taken across four points (C_1, F_0, F_1, C_0) in change from all cut to all fill

require several fully surveyed cross sections per 100-ft station for the purpose of measurement of excavation and embankment; in such cases the staked sections are usually spaced at regular intervals of 25, 50, or 100 ft, depending on needs of the construction force. Each cross section of the original ground line is extended a sufficient distance beyond the normal grading limits to provide measurement data for unexpected situations, such as slides, side borrow, overbreak from rock blasting, fill widening, etc. The rod readings for these cross sections are taken in the usual manner, but before being recorded in the notes each reading is converted to a depth of cut or height of fill, and noted whether cut or fill by the letter (C) or (F) at the slope stakes, and by a plus or minus sign at all breaks in

ground slope. Otherwise the notes for staked cross sections are similar to other types of cross-section notes, as illustrated in Fig. 75. On superelevated grades the cut or fill for points beyond the shoulder may be computed either from the elevation of shoulder grade or from the elevation of center-line grade. Both methods have advantages, but the first is better for the builder and therefore is generally preferred. However, only one method should be used on a project.

Where there is a change from cut to all fill, or from fill to all cut, certain intermediate cross sections must be taken. For example, in Fig. 87, cross sections would be taken (1) where the cut runs out on the downhill side at C_1 on the outside of the ditch; (2) where the fill begins on the downhill side at F_0 on the shoulder of the roadbed; (3) where the cut runs out on the uphill side at F_1 on the shoulder; and (4) where the cut runs out on the uphill side at C_0 on the outside of the ditch. Each point, C_1, F_0, F_1, and C_0, is a 0.0 grade point, whether or not the cross section is superelevated.

The position of slope stakes and 0.0 stakes on the ground is found by trial. The procedures for setting the slope stakes on these typical cross sections are described below, and are illustrated in Fig. 88, in which the position of the instrument (level, transit, etc.) is indicated by the tripod symbol.

Figure 88*A* represents a fill with no superelevation. The center-line fill k is determined by subtracting the profile elevation at the center-line stake from the center-line grade elevation. The rod is held on the ground at the stationing side (face) of the center-line stake, and a level sight h is taken. The center-line fill k is subtracted from the rod reading h to get the "grade rod" g. Cross-section rod readings are taken in the usual manner, g being subtracted from each reading to obtain height of fill (or depth of cut) for each rod point. When the rodman has passed the shoulder point, a trial distance d for trial location of the slope stake is computed from $\frac{1}{2}$ base + s, s being height of fill at shoulder times the fill-slope rate. The rate of fill slope is expressed as a ratio, the first figure being the horizontal distance required for each vertical foot of fill or cut. The first trial distance is slightly increased or decreased, depending on whether the ground is sloping downhill or uphill. The rod is then held at the trial distance from the center-line stake, a rod reading r is made, and g is subtracted from r to find fill height F. Wherever there is a crown in the roadbed, the amount of the crown is added algebraically to g before F is computed. Using F, a new trial distance for d is computed from $\frac{1}{2}$ base + F times the slope rate. Then another rod reading is taken at the new location of the new trial distance, a new F is obtained, a new trial s is computed, etc. Trials are thus continued until the computed distance d equals the measured distance d, and the slope stake is then set. The difference between the measured and computed d distance is an aid in judging where the rod should be held on the next trial: experienced rodmen often find the correct point on the first trial, and seldom use more than two trials.

Figure 88*B* illustrates the method on a side-hill cut, where the finished subgrade is to be superelevated. Where the superelevation is upward from the center line, as the cross-section party works outward, and the grade rod g is positive; the grade rod g' is determined by subtracting the amount of superelevation e from the value of g; where the superelevation is downward, e is added to g to obtain g'. For points inside the shoulder the amount to be added or subtracted is the superelevation rate times the distance of the point from the center line, for points beyond the shoulder, the superelevation allowance is the fixed quantity e.

Figure 88*C* illustrates a high fill that is superelevated, and with the HI falling below center-line grade. In this case h is subtracted from k to determine the center-line grade rod $-g$; and e is algebraically added to $-g$ to get the grade rod g'.

Wye and dumpy levels are used for slope staking when a high degree of accuracy is required. When a tolerance of $\frac{1}{4}$ ft in vertical accuracy is satisfactory, a hand level, Rhodes arc, or a slope board may be used, although with careful work an accuracy of about 0.1 ft can be obtained with these instruments.

During construction operations, slope stakes are often knocked out or covered, and need replacing. To facilitate replacement, reference-point (RP) stakes are set on the cross-section line extended far enough beyond the slope stake to avoid being disturbed. The distance between each RP stake and the slope stake it references is called the offset. Offset distances should all be of the same amount throughout each project, as far as feasible. Figure 89 shows the relative positions of a full set of stakes on a typically staked cross section.

Slope-Stake Markings. The system of markings on center-line stakes, slope stakes, and reference-point (RP) stakes should be uniform throughout each project. One such system in common use includes markings as follows.

Center-Line Stakes. Face of stake shows station number and plus distance; back of stake shows letter C or F to designate whether cut or fill, and the amount of C or F in feet.

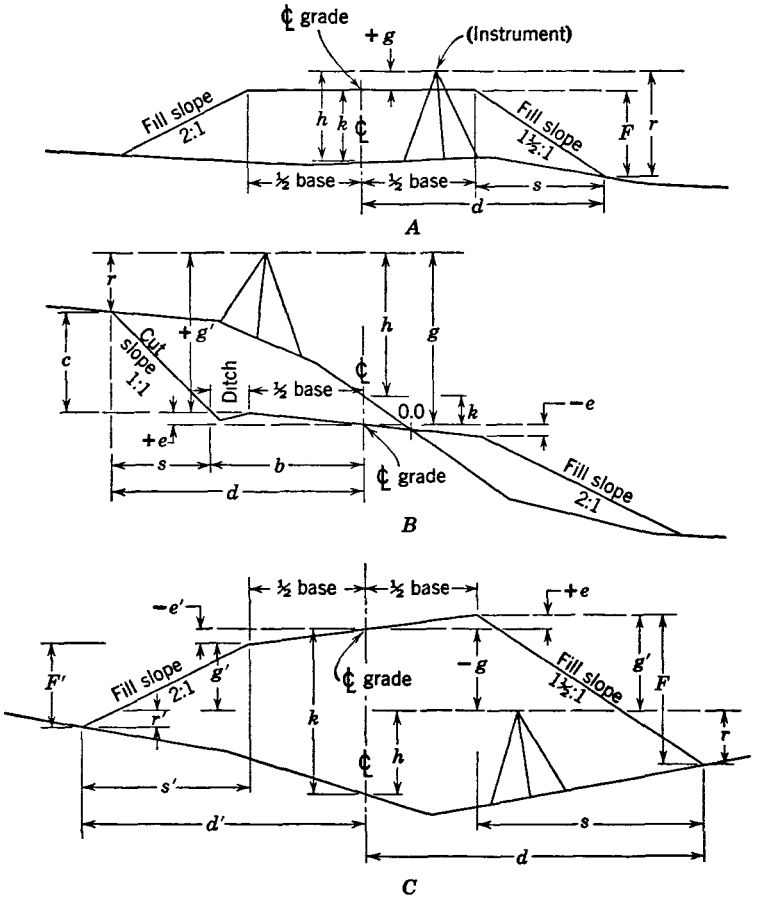

FIG. 88. Slope staking cross sections: *A*. Fill, no superelevation. *B*. Cut on side hill, superelevated. *C*. Fill, superelevated

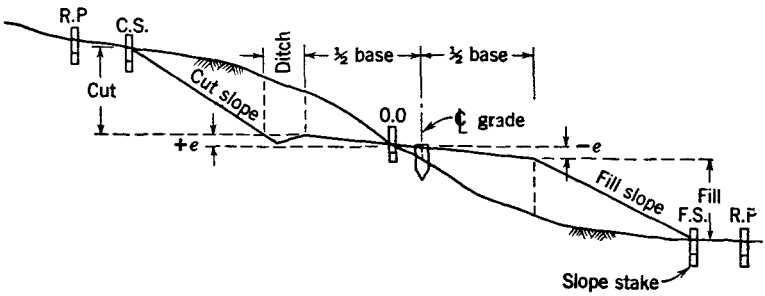

FIG. 89. Cross section; slope staked and referenced; cut-and-fill section superelevated on a curve.

R P. = reference-point stake
F.S. = fill-slope stake
C.S. = cut-slope stake
e = superelevation
0.0 = grade stake
℄ = center-line stake

Slope Stakes. Face of stake shows letter C or F, the difference in elevation between shoulder grade and ground at stake, the distance from center line, and the rate of slope; back of stake shows station number and plus distance, and letter L or R for left or right; right edge of stake shows rate of crown slope per foot for cross sections on tangent, or, for curves, the superelevation rate per foot; left edge of stake shows $1/2$ base distance.

RP Stakes. Each RP is marked on all four sides with the same information as its corresponding slope stake, except that the distance from center line and the amount of cut or fill are given for the RP-stake position instead of the slope-stake position, and the offset distance from the slope stake is also shown.

79. SETTING STAKES FOR STRUCTURES

Stakes for structures have two principal purposes: (1) to establish grade and line for the guidance of construction crews in excavating and in building forms for the structure, and

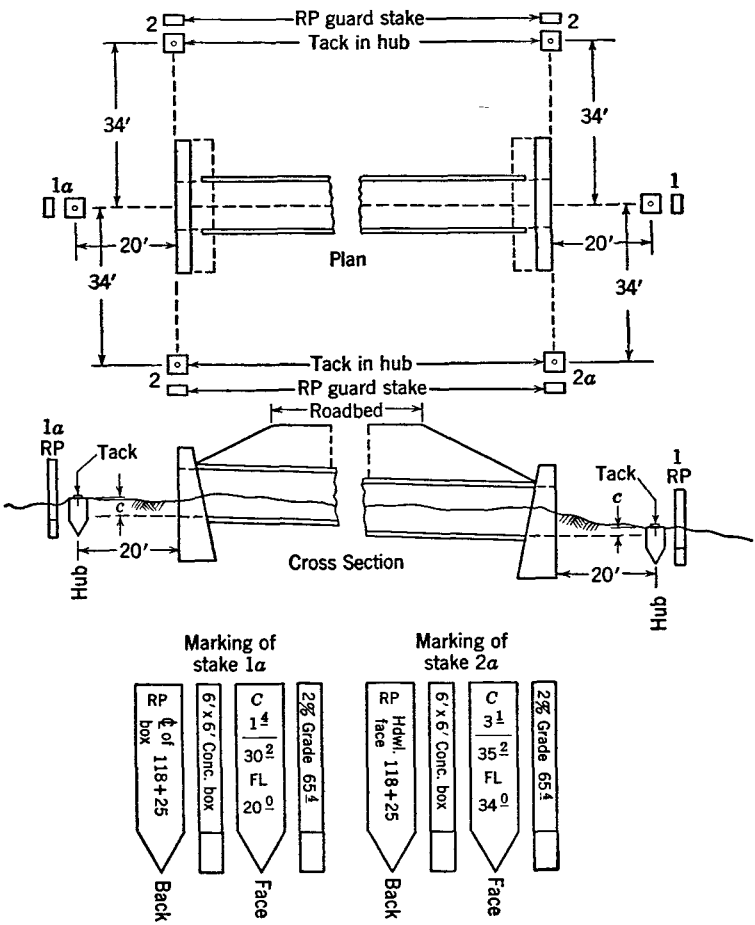

FIG 90. Culvert reference stakes and markings

(2) to remark line and grade as required during construction operations. This means that two types of stakes are required, somewhat as in slope staking. The first set of stakes are set near the edges of the structure, offset a few feet so they will not be knocked out as soon as the construction work is started. These stakes will mark both line and grade in

reference to a tack in a 2-in. square hub driven flush with the ground. A guard stake is driven near the hub, but far enough away so as not to disturb it. On the guard stake are marked the essential data, i.e., grade of structure above or below top of hub, and offset distance of tack in the hub from the particular part of structure to which it applies. The second set of stakes are reference stakes, which, like RP stakes for cross sections, are set far enough away to preclude disturbance. Reference stakes for structures must be accessible for occupancy by a surveying instrument, and in many cases must be intervisible until the structure is built.

In Fig. 90 are shown reference stakes and markings for a culvert. The set of stakes labeled No. 1 on the diagram are on the center line of the culvert, and those labeled No. 2 are on the face line of the headwall. If the inlet and outlet had winged walls, stakes would also be set on the face line of each wing. The construction stakes, not shown, are similarly set within a foot or so of the end of the culvert and the wing walls, before construction begins. Once the construction stakes are loosened, moved, or knocked out, all subsequent construction work must be checked from the reference stakes.

For bridges and other large structures, reference points are tacks lined on a row of hubs driven flush with the ground, called a reference leg, a guard stake being set near each hub to mark its position. On the guard stake are markings to show the part of the structure to which it is a reference, the height of the hub above or below that part, the offset distance of the RP from the structure and from adjacent RP's, and the RP number. In the structure stake-out notes, a full record diagram of all RP's is made to scale. Usually three RP's are established on each reference leg, so that if one is destroyed the remaining two will establish the leg. For a major structure, the intersection of two legs mark the point that is referenced. The angles between reference legs are measured and recorded, for possible later use in recovering the hubs by measurement should the guard stakes become misplaced or destroyed. The angle of the intersection of each pair of RP legs should be as near 90° as conditions will permit; for strong intersections the angle should be not smaller than 60° or larger than 120°. When it is not possible to occupy with an instrument the point referenced, as a pier to be built in the center of a stream, triangulation methods must be used. For elevation control one or more permanent benchmarks should be set near each end or side of a large structure, in locations where they will not be disturbed.

SECTION 3

TRAFFIC ENGINEERING

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TRAFFIC ENGINEERING

1. DEFINITION

In the highway engineering field traffic engineering includes the analyses of traffic characteristics, the planning of regulatory measures, the design and application of control devices, and the geometric design and functional planning of routes and terminals.

CHARACTERISTICS OF TRAFFIC FLOW

2. COMPONENTS OF TRAFFIC

The unit of traffic which is of primary concern is the man-machine combination of driver and vehicle. While using the highway facilities in numbers, these units develop characteristics of speed, volume, origin-destination, stream flow, intersection flow, and accident occurrence, all of which are influenced by regulatory directives, control devices, physical designs, route conditions, and environments. An understanding of the laws of cause and effect in the human element of traffic and of the capabilities of performance in the mechanical element leads to the accurate prediction of traffic performance and to effective planning and control.

3. THE ROAD USER

The time, place, route, origin, and destination of travel are fundamentally controlled by the driver. Every detail of traffic performance is influenced or modified by him. His reactions and capabilities are basic considerations in traffic-engineering planning.

Vision. The driver must have sufficient visual perception to respond quickly to conditions requiring positive action and to ascertain sign legends, markings, and other details. Conversely, the designs and positions of signals and instructions should conform to minimum acceptable driver-perception abilities. For clear distinct vision, the image of objects should fall on the retina within a 5° cone. The acuity at the outside edges of a 10° cone is about one-half that at the center of the eye and, outside of that cone, accuracy in letter determination drops off rapidly particularly in the vertical direction. As a rule of thumb, the field of vision within which the driver will be able to identify details correctly is covered by the hand held at arm's length.

Reaction Time. The time lapse between the instant a driver perceives the need for a decision on his part and his response varies with his age and physical condition, the character and strength of the stimulus, and the number of choices of action. The movement of the vehicle during this time lapse is significant in the determination of capacities of intersections, safe stopping and passing distances, and safe approach speeds at obstructions. The PIEV[1] time of drivers under laboratory test conditions has been found to vary from $1\frac{1}{4}$ to $1\frac{1}{2}$ seconds with different combinations of stimuli and required responses. For regulatory and design purposes allowance must be made for indecision and inattention of drivers. The total time required for a driver to form a judgment and to act may amount to 3 seconds or more in complex situations.

Habit Factors. Major changes required in vehicle operation necessitate gradual transition and strong stimuli. Thus a curve of ordinarily accepted radius may actually constitute a hazard if it is preceded by a lengthy section of highway conducive to high speeds.

---

[1] Perception, intellection, emotion, and volition

4. THE VEHICLE

Predominant Types. Current manufacture of motor vehicles includes over 200 model types of passenger automobiles and taxicabs, 300 types of trucks,and 60 types of buses. The numbers registered in the United States in 1954 were:

Type	Number Registered in U. S. (1954), thousands
Passenger cars	48,000
Trucks	9,800
Buses	250

Design Vehicle Dimensions. The range of dimensions for passenger cars manufactured in the United States in 1954 were:

	Width, in.	Length, in.	Height, in.	Minimum Turning Radius,* Curb to Curb, ft
Minimum	66	124	59	17.5
Maximum	82	238	69	27 5

* Radius of path described by outer front wheel when the vehicle is making the sharpest possible turn at very slow speed.

Within legal limits there is also considerable variation in truck and bus dimensions. Suggestd design vehicle dimensions are given in Table 1.

Table 1. Suggested Design Vehicle Dimensions, ft

Design Vehicle	Wheel Base	Front Overhang	Rear Overhang	Overall Length	Overall Width	Height	Minimum Turning Paths: Inside Curb Radius	Minimum Turning Paths: Outside Clearance Radius
Passenger car	12	3	4	19	6.5	...	19.3	29.6
Single-unit truck	20	4	6	30	8	...	32.3	47.0
Semitrailer combination, intermediate	14 + 21 = 35	4	4	43	8	12.5	19.8	41 6
Semitrailer combination, large	18 + 26 = 44	4	2	50	8	12.5	21.0	45.6

Source: A Policy on Geometric Design of Rural Highways, AASHO, 1954.

Resistance to Motion. A moving vehicle may be acted upon by rolling resistance, air resistance, engine resistance, gradient resistance, inertia, and braking resistance. The rolling resistance on most smooth, hard-surfaced roads ranges from 20 to 25 lb/ton. The air resistance is proportional to the frontal area and the square of the velocity and ranges from 2.5 to 5.0 lb per sq ft of frontal area at speeds of the order of 50 mph. Engine resistance, produced by the friction of moving engine parts and the compression of vapors in the cylinders, may vary considerably. The gradient resistance may be plus or minus and amounts to 20 lb/ton of vehicle for each per cent of grade. The inertia effect may also be plus or minus and is proportional to the mass and acceleration.

Stopping. Rolling, air, and engine resistances combined will develop sufficient retarding force to cause significant deceleration when the power is cut off on the level, particularly at speeds above 50 mph, as shown in Table 2.

Table 2. Vehicle Deceleration without Brakes

(Instantaneous deceleration in feet per second per second)

Speed, mph	Vehicle in Gear	Vehicle out of Gear
70	3.20	1.75
60	2.79	1.40
50	2.36	1.07
40	1.94	0.82
30	1.62	0 61
20	1.26	0.40
10	...	0.37

Source: Deceleration Distances for High Speed Vehicles, by Ernest E. Wilson, 1940 Proceedings, Highway Research Board.

The maximum braking resistance of a vehicle is dependent on the friction of the tires on the road and the design and performance of the brake system. The coefficients of friction required by various rates of deceleration are shown in Table 3.

Table 3. Coefficients of Friction Required by Various Average Rates of Deceleration

(Employed in bringing a vehicle to a stop from 70 mph)

Description of Condition	Deceleration, ft/sec/sec	Coefficient of Friction
Comfortable for passengers	8.55	0.26
Uncomfortable for passengers; driver would rather not use	11.05	0.34
Severely uncomfortable for passengers, objects slide off seats, driver classes this as emergency stop	13.90	0.43
Maximum stop, skill required to stop in 12-ft lane; brakes in best condition	19.50	0.60

Source: Deceleration Distances for High Speed Vehicles, by Ernest E. Wilson, 1940 Proceedings, Highway Research Board.

5. SPEEDS

Spot Speeds. The term "spot speed" refers to the speed of a vehicle at a given instant in a specific location. Figures 1 and 2 show typical curves of spot-speed frequency distribution and cumulative frequency. The percentile values of cumulative frequency are of special value; the 85 percentile is used as a basis for speed regulations and the 97–98 percentile is used as a criterion in highway design.

Operating Speeds. In 1954 nationwide observed operating speeds on rural highways averaged 49.7 mph for all vehicles, 51.1 mph for passenger cars, 45.2 mph for trucks and 51.8 mph for buses. On two-lane rural highways where speed is not limited by the characteristics of the highway or other conditions, maximum speeds are usually slightly more than 60 mph. On express highways where the only limitation on speed stems from the desires of the drivers, maximum spot speeds extend to 80 mph. In general, average speeds decrease linearly with increases in volume.

Speed characteristics are not materially influenced by various hard-surface pavements, lane widths in excess of 9 ft, or curves when the coefficient of friction, acting radially, remains less than 0.16. Vertical curbs tend to reduce average speeds in adjacent lanes by 2 to 3 mph. Open-type bridges with pavement widths of 24 ft or more for two-lane operation do not influence speeds, but long through-type bridges with similar pavement widths tend to reduce speeds by 7 to 8 mph. The influence of moderate gradients is shown in Table 4.

Table 4. Observed Average Motor-Vehicle Operating Speeds on Various Grades

	Trucks, mph	Buses, mph	Passenger Cars, mph
Level	36.9	45.9	45.8
Downgrade			
3%	37.6	46.4	46.5
5%	38.9	41.4	42.1
7%	34.1	37.4	40.2
Upgrade			
3%	34.3	37.0	43.5
5%	26.6	29.3	39.5
7%	24.6	26.6	34.4

Source: *Public Roads*, June 1942.

It should be realized, however, that the length of a sustained grade has a significant influence upon vehicle speeds, especially where heavy commercial vehicles are involved.

Measuring Spot Speeds. Speeds may be computed with moderate accuracy from stop-watch measurements of the time required for vehicles to traverse a fixed distance. The determination of speeds to a high degree of accuracy may be accomplished by several methods including pressure-contact strips and timing mechanisms, photoelectric meters and timing mechanism, photographs exposed at constant time intervals and radar meters.

Speed and Delay Studies. Surveys to determine average traffic speeds, the effectiveness of existing signs, signals, and controls, and the principal causes and durations of delays are usually made by driving a test car over the route at the median route speed. The test

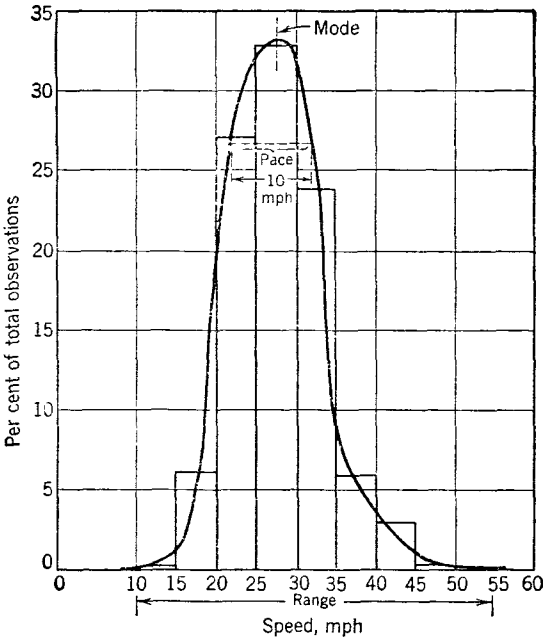

FIG. 1 A characteristic histogram and frequency-distribution curve of spot speeds.

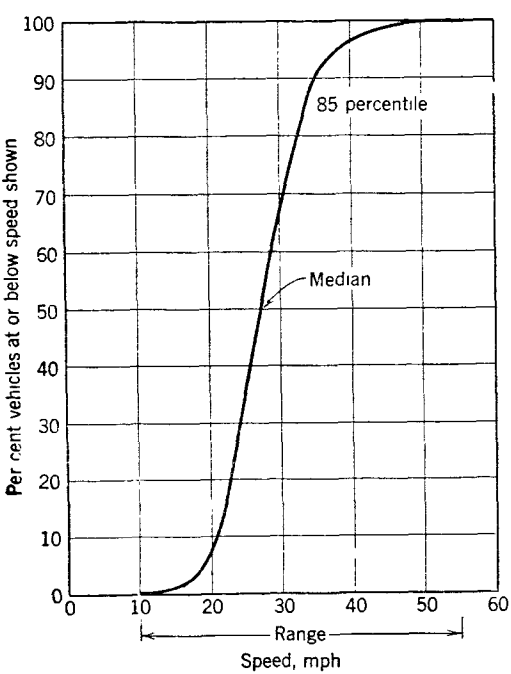

FIG. 2 A characteristic cumulative-frequency curve of spot speeds.

car does this, in general, by passing a vehicle for every one that passes it. An observer in the test car records the time and distance at control points and the locations, durations, and causes of stops and delays.

Analysis of Speed and Delay Data. Graphical presentations of speed and delay data may be made to show the overall and running speeds by zones, the distribution of the total running time and the total delay time by zones, the distribution of total delay time by causes, and the frequency and durations of delays by causes (Fig. 3). Zones of travel time from a given focal point can be determined from a series of speed and delay runs made through the focal point.

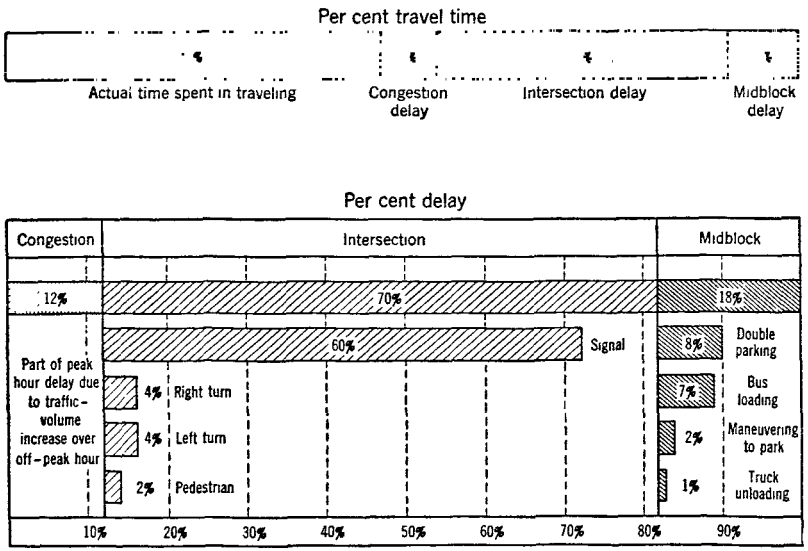

FIG. 3. Typical distribution of traffic delays. *Traffic Survey of Spokane, Washington*, Wash. State Dept. of Highways

6. VEHICLE VOLUMES

Traffic Volume is a time rate of vehicle flow. Volume data are basic for highway planning and are needed in nearly every phase of traffic analysis and administration, including the determination of trends, the determination of justifications for stop signs and traffic signals, the estimation of traffic loads which would use new facilities, and the determination of desirable routings on existing highway networks. Figure 4 shows a graphical analysis of volume data.

Composition of Traffic Volumes. The proportions of various types of vehicles in a traffic stream are affected by locations, grades, scenic values, and restrictions on the use of parallel routes. Illustrative compositions are as follows:

Vehicle Type	Usual Range in Per Cent of Total Traffic
Passenger automobiles	74–86
Local	60–90
Foreign (out of state)	10–40
Trucks	12–24
Single unit	11–18
Combinations	4–6
Transit vehicles	0–7
Buses	1–7
Streetcars	0–5

Volume Counting. Traffic volumes vary from a low of 1 or 2 vehicles per day on highways in sparsely settled areas to over 120,000 vehicles per day on metropolitan thoroughfares. In rural areas counts are usually made at strategic locations throughout the network

of routes under consideration. In urban areas counts are made at intersections in the central business district as well as at all other points where special problems or concentrations of traffic are encountered. Where patterns of cyclical variations are known and

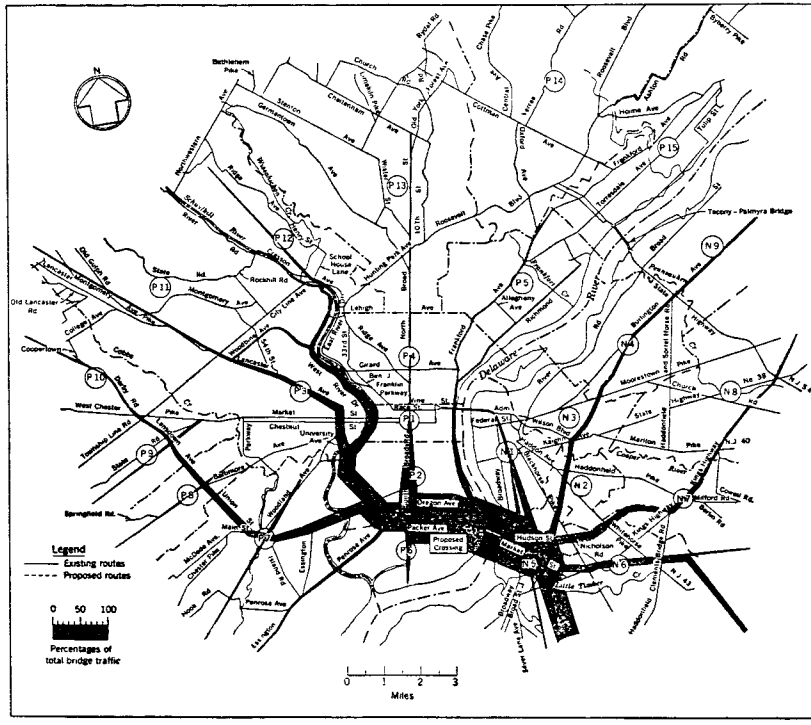

FIG. 4. Traffic-flow analysis. Zones shown are those used in study for proposed crossing.

are dependable, counts of 15 min to 1 hr may be expanded with sufficient accuracy for some uses. More often, however, counts are necessary for periods of 8, 12, 16, or 24 hr. The basic methods employed in recording volumes are:

1. Manual counts by observers, usually with the use of hand counters for recording totals of various classifications of vehicles.
2. Automatic counts with the use of devices which register the number of vehicles or axles passing a given point. These counters render cumulative totals for given time intervals. Some types are designed to record volumes by directions.

Cyclical Variations in Traffic Volume. Three cyclical variations are ordinarily found in the volume of flow: the diurnal cycle, the weekly cycle, and the seasonal cycle. The diurnal cycle, illustrated in Fig. 5, is generally found to be highly repetitive and stable for weekdays. The volume passing a point during a peak hour in a given day usually corresponds to 7 to 10% of the 24-hr volume.

The weekly cycle, illustrated in Fig. 6, reflects the community habits of employing certain days for work and others for recreation. There is a tendency for volumes on Mondays and Fridays to rise above the weekday average. Sunday volumes on rural roads during fair weather ordinarily range from 130% to 200% of the average daily volume for the week.

The seasonal cycle, illustrated in Fig. 7, varies considerably with the latitude. April and November volumes are usually near the annual average; midwinter volumes range about 30% below average; and midsummer volumes range about 30% above average. The wider range is found in latitudes of inclement weather.

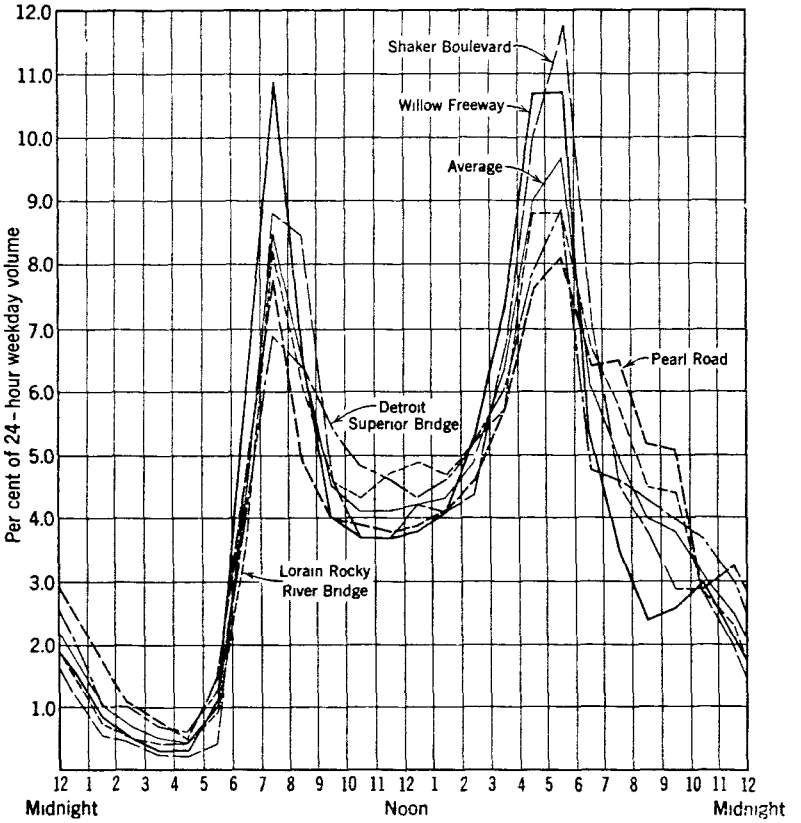

FIG. 5 Typical hourly variations in traffic flow, % of 24-hour weekday volume, Cleveland, Ohio, area

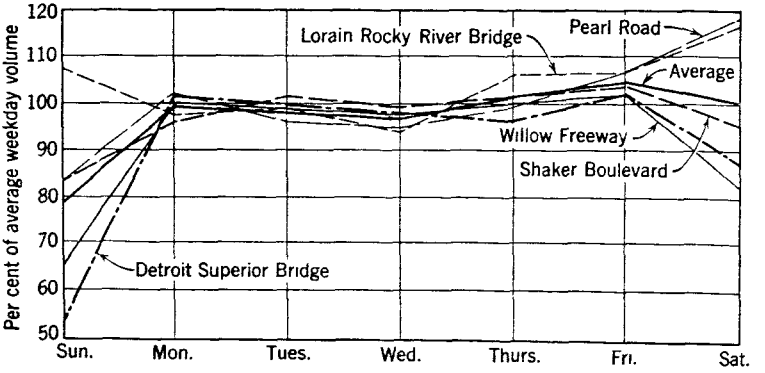

FIG 6. Typical daily variations in traffic flow, % of average weekday volume, for Cleveland, Ohio, area

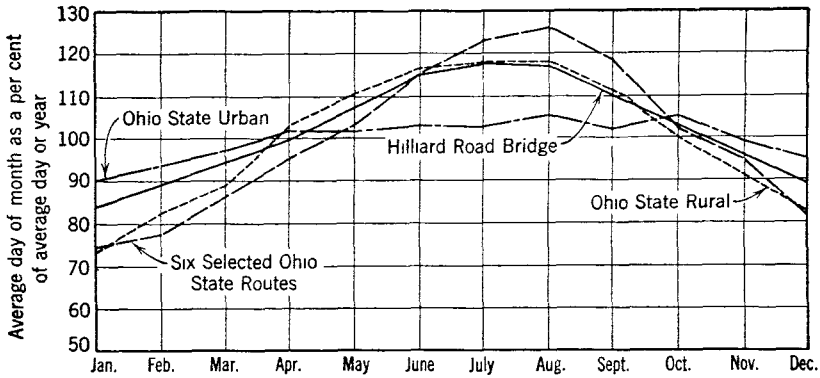

FIG 7 Typical monthly variations in traffic flow

Hourly Volumes for Design. It is rarely justifiable to design a highway for the highest expected hourly peak. Where congestion is not acute, the magnitudes of peak hourly flows decrease rapidly from the single highest hour of the year to the 20th to 50th highest hours and thereafter decrease only gradually (Fig. 8). For the National System of Inter-State Highways, the AASHO recommends the design of traffic lanes for the volumes anticipated during the 30th highest hour of the year for which the design is made. That volume, on a percentage basis, generally varies little from year to year. Average percentage values for 30th highest hourly volumes are given in Table 5.

Table 5. Variations in Traffic Flow on Main Rural Highways Averaged for Various Census Regions, 1953

Census Region	Number of Locations Studied	Average 24-Hour Volume	Percentage 30th Highest Hourly Volume Is of Average 24-Hour Volume
New England	37	5919	15.8
Middle Atlantic	7	6230	11.7
South Atlantic			
North portion	30	6720	13.7
South portion	16	4248	11.8
East North Central	25	5897	12.6
West North Central	42	2855	13.2
East South Central	15	3653	12.7
West South Central	127	3229	12.1
Mountain	27	3977	12.8
Pacific Coast	15	3917	12.8
Total	341	4206	12.9

Source: U. S. Bureau of Public Roads.

Directional Distribution of Traffic. During peak hours the traffic volumes in one direction on radial streets in urban areas generally range from 125% to 175% of the volumes in the opposite direction. For routes in outlying urban and rural areas, the distribution of peak-hour traffic is usually about two-thirds in the direction of major flow and one-third in the opposite direction.

Trends in Traffic Volumes. Between 1930 and 1954 the annual travel on U. S. highways increased approximately 170%, as shown in Fig. 9. An average long-term nationwide growth of at least 5% per annum, compounded, can reasonably be expected. The actual growth will, of course, be materially greater where the physical environment is conducive to the ownership and operation of private vehicles. On the other hand, in congested areas where terminal and roadway facilities are overloaded, significant growth cannot be expected to take place unless relief capacity is provided.

Induced Traffic. New traffic facilities, such as bridges and expressways, develop traffic which cannot be accounted for on the basis of cyclical variations, long-term trends, diversion from other routes, or conversion from other modes of transportation. This traffic is created as a direct result of the increased convenience of travel and is referred to as induced traffic. In urban areas, where there are large "reservoirs" of traffic, limitations of

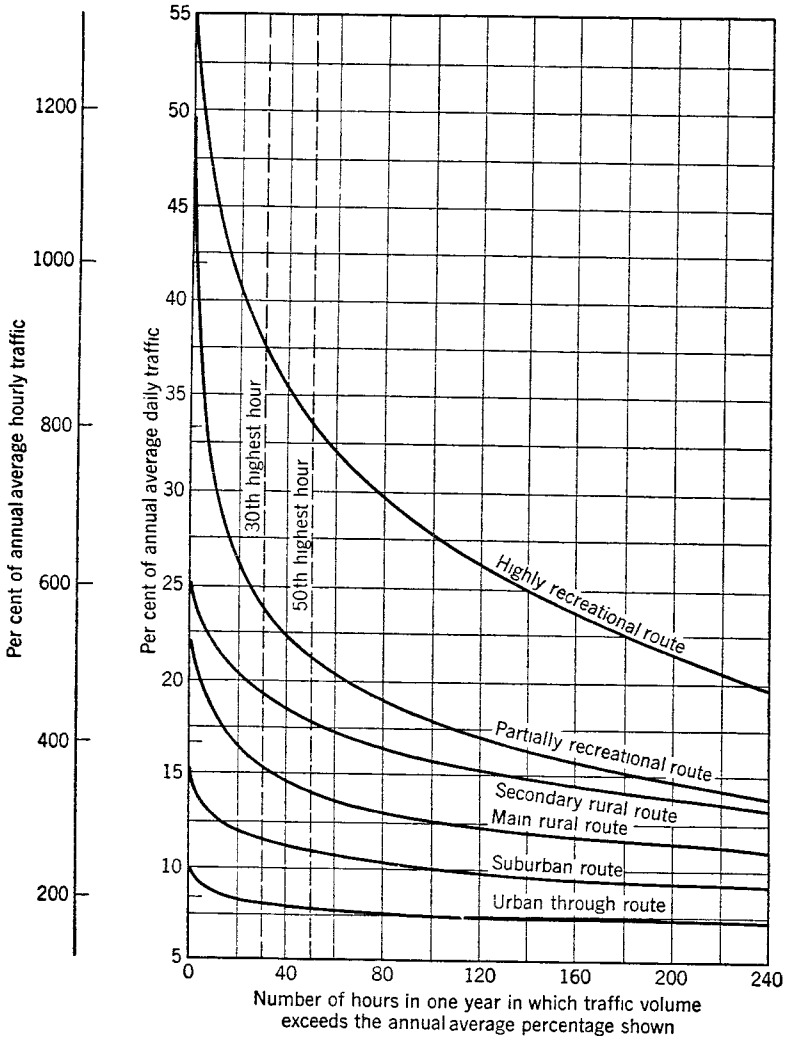

FIG 8 Frequencies of occurrence of high hourly volumes on various type routes Adapted from *Hourly Traffic Volumes as a Basis for Design*, Conn State Highway Dept

capacity, and inconvenient parallel facilities, this new traffic may range up to 250% of the traffic diverted from other routes. By contrast, in rural areas where difficulties of travel are not great, the percentage of induced traffic may be negligible. Tables 6 and 7 illustrate the extent to which traffic was generated by the opening of several well-known toll crossings and toll highways.

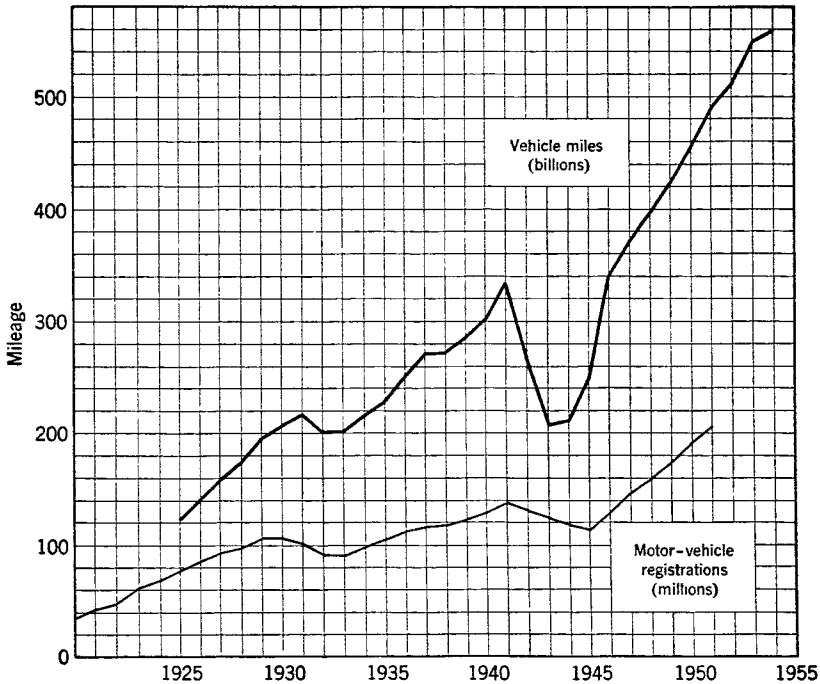

FIG. 9. Annual motor-vehicle miles and registration.

Table 6. Induced Traffic Factors for Selected Toll Crossings

New Crossing	Opening Date	Competing Crossings' Traffic †	Estimated Annual Volume of Induced Traffic	Induced Traffic Expressed as Percentages of: Traffic Diversion	Competitive Crossings' Traffic ‖	New Crossings' Traffic in First Year	Average Toll on New Crossings, cents
(a) Crossings in the New York Area							
Holland Tunnel	Nov. 13, 1927	11,686,300	4,250,000	125	39.3	55.5	51.5
Goethals Bridge	June 29, 1928	409,973	365,422	187	89.1	64.5	52.0
Outerbridge Crossing	June 29, 1928	480,751	184,632	58	38.4	36.5	52.0
George Washington Bridge	Oct. 25, 1931	4,987,000	2,225,000	65	45.0	39.5	52.5
Bayonne Bridge	Nov. 15, 1931	583,160	336,420	244	57.7	70.9	51.5
Triborough Bridge	July 11, 1936	12,214,000	5,115,000	105	42.0	51.0	26.0
Lincoln Tunnel	Dec. 22, 1937	5,245,000 ‡	1,190,000	195	22.7	66.1	55.0
(b) Crossings Elsewhere in the United States							
Philadelphia-Camden Bridge	June 30, 1926	5,600,000	4,446,000	131	80.0	56.7	28.0
Carquinez Bridge	May 21, 1927	625,000	389,000	53	62.0	34.9	90.0
Tacony-Palmyra Bridge	Aug. 15, 1929	376,228	875,900 §	234	232.8 §	70.0	36.0
Oakland Bridge	Nov 12, 1936	6,087,000	8,798,000	239	144.5	70.5	50.0
Golden Gate Bridge	May 27, 1937						
(c) Crossing in Europe							
Queensway Vehicular Tunnel *	July 1934	1,150,334	1,971,763	145	171.4	59.1	55.5

* Between Liverpool and Birkenhead, England.

† Competing crossings' traffic in the year preceding the opening of the new crossing.

‡ Three nearby ferries only.

§ Includes an amount diverted from the lower Delaware River ferries and Camden Bridge that was difficult to determine.

‖ Percentage of traffic of competitive crossings for the year preceding the opening of the new crossing.

Source: N. Chermack, *Trans. ASCE*, 1941.

Table 7. Induced Traffic for Selected Toll Highways

Highway	Opening Date	Principal Parallel Routes	Year	Daily Volume of Traffic on the Toll Highway: Total Traffic	Estimated Diverted Traffic	Estimated Induced Traffic	Induced Traffic Expressed as Percentage of Diverted Traffic	Induced Traffic Expressed as Percentage of Total Traffic
Pennsylvania Turnpike	Oct 1, 1940 (origin of section)	U. S. 30, U. S. 22	1941	4,200	3,000	1,200	40	29
Maine Turnpike	Dec. 13, 1947	U. S. 1	1948	3,400	2,600	800	31	23
New Hampshire Turnpike	June 24, 1950	U. S. 1	1950	6,300	5,430	870	16	14
New Jersey Turnpike (at Newark)	Nov. 5, 1951 (northern portion) Jan. 15, 1952 (southern portion)	U. S. 1	1952	30,000	16,000	14,000	88	47

Note. Passenger-car tolls on turnpikes range in general between 1 and 1.5 cents per vehicle mile except for short distances where tolls are as high as 5 cents per vehicle mile. The rate for the northern end of the New Jersey Turnpike is 4.3 cents per vehicle mile. The rate for truck-trailer combinations is 6.3 cents per vehicle mile on the Pennsylvania Turnpike and 4.3 cents per vehicle mile on the New Jersey Turnpike.

Source. *Research Rept.* 11, The Institute of Transportation and Traffic Engineering, University of California.

7. ORIGIN AND DESTINATION

Origin and destination (O.D.) data are of prime importance in determining the locations and demands for new thoroughfares, bridges, tunnels, and parking garages. Its analysis leads to a determination of the desired lines of travel in a given area and provides bases for estimating the volumes which would use new or improved routes and terminals at selected locations.

The Collection of O.D. Data. The methods used in determining the origins and destinations of traffic movements range from simple observations to intensive home interviews. The observations should be made or the data obtained for representative periods, generally normal weekdays, unaffected by inclement weather conditions or other unusual events. The period covered will depend on the purpose of the survey; the 16-hr period from 6 A.M. to 10 P.M. is frequently adequate. For most surveys it is generally necessary at the outset to decide upon the locations and boundaries of zones of O.D., resolving the complete area of investigation into a mosaic of subdivisions which lend themselves to desirable groupings of trips.

It is neither necessary nor generally possible to secure data on all movements, the trip information obtained being expanded in proportion to the total volume of traffic recorded at key counting stations during the survey. Four of the most common methods of obtaining O.D. data are:

License-Plate Observations. Limited data are obtained by stationing observers at selected points where simultaneous observations can be made of entries and exits of the route, intersection, or area being surveyed. License-plate numbers, recorded in a time series, are matched to determine the routes traversed by vehicles. Dictating machines may be employed in the field, or field entries may be made on business-machine cards for subsequent punching and mechanical matching. In a variation of this method, colored cards are affixed to bumpers or door handles of vehicles at points of entry into the cordon area and are removed at points of exit.

Post-Card Questionnaires. Prestamped or business-reply questionnaire post cards are handed to road users at toll stations or other convenient points, are placed on parked vehicles, or are distributed at focal points of O.D. such as places of employment or business. The road user is requested to answer the questions on the post card and to mail it. In a variation of this method, post-card questionnaires are mailed to registered vehicle owners in the survey area (Fig. 10). Supplementary roadside interviews must be made at cordon lines around the area or at screen lines cutting through the area in order to account for "foreign" vehicles and to provide a check on questionnaire data. Some bias of sample may be expected due to classes of road users who do not return the cards. Responses range from 10 to 60%.

Route Interviews. Carefully selected and trained personnel, suited to meet the public, are stationed at points where the sight distances, gradients, prevailing speeds, and roadway widths are suitable. At interview stations where traffic does not ordinarily stop, traffic should be controlled by uniformed highway police. A survey of long duration may be undertaken in shifts on different days. Data may be recorded on individual cards for each vehicle, or entered on a field sheet listing 20 or more trips.

Home Interviews. Trained interviewers are sent to selected dwellings in the survey area to collect travel data from members of the households. Information is usually sought at about 10% of the homes, selected on some arbitrary basis designed to minimize bias in the results. Similar interviews are made for trucks and taxicabs. In addition, roadside interviews are generally required to provide data on through traffic. Although this method permits the collection of much related data, the inherent comprehensiveness required usually limits its use to public authorities such as state highway departments and the U. S. Bureau of Public Roads.

Analysis of O.D. Data. Where large numbers of movements must be analyzed business-machine methods may be desirable for sorting and classifying the data. Several forms of graphic portrayal easily and quickly show the significant characteristics of O.D. An example of a "desire" line analysis, involving the plotting to scale of hypothetical straight-line movements or "desired" lines of travel, is illustrated in Fig. 11. Analysis by means of a flow map may also be employed (Fig. 4).

In addition to the tabular and graphical methods it is sometimes useful to analyze the O.D. data by a *method of moments* similar to that used in mechanics for the determination of centroids and neutral axis of physical bodies and areas. This method may be used to show the optimum locations of various facilities such as terminals, straight-line routes, bridges, or ferry crossings.

Front

COLUMBUS—FRANKLIN CO. TRAFFIC SURVEY

Authorized by Columbus City Council, Franklin Co. Commissioners, and Participated in by Ohio Department of Highways

Please list below ALL the trips made by your vehicle "On the Day After You Receive This Card," but DO NOT use Saturday or Sunday trips. If your vehicle makes no trips, write "NONE" across the face of the card. As soon as you have the card filled out, DROP IT IN A MAIL BOX. No postage stamp is required.

Trip No.	(Origin—Place where trip started)	(Destination—place where trip ended)
1.	from	to .....
2.	from	to .....
3.	from	to .....
4.	from	to .....
5.	from	to .....
6.	from	to .....
7.	from	to .....
8.	from	to .....

DO NOT SIGN YOUR NAME

Back

READ INSTRUCTIONS CAREFULLY—FILL OUT CARD—TEAR OFF AND MAIL AT ONCE

1—You and your car, and the trips you make each day, are important to the solution of our traffic problems. Columbus and Franklin County are elegible for State and Federal funds to help in street improvements **If We Meet Their Requirements.** These funds are from taxes you have already paid. By returning this card you will aid materially in getting these funds for Columbus and Franklin County. PLEASE FILL OUT THE CARD—AND MAIL.

2—What Shall I Consider As A Trip? Answer: Mrs. Smith takes her husband to his work—that is a trip. From there Mrs. Smith goes shopping—that is another trip Mrs. Smith then goes home—that is another trip. In other words, anyplace that Mrs. Smith stops for some purpose, is the end of one trip and the beginning of another.

3—"Origins" (Place where trip started) and "Destinations" (Place where trip ended) may be listed as an address, such as 500 North High Street, or an intersection, such as Fifth Street and High Street, or as a place, such as the Court House, State Office Building, or any of the well known stores, banks, buildings, etc. Do not say you went from home to work; we don't know where you live or where you work.

4—You will have done a good job of filling this card out if, after it is finished, any other person could read it and quickly tell where the trips had started and where they ended.

5—If you have any difficulty filling the card out, call any of the following telephone numbers for assistance: Ma. 1265, Ext. 780; Ad. 7474; Ma. 2211; Ad. 4281; Ma. 1321

6—If you have more than 8 trips, list them on another sheet and mail, or bring them to the Survey Office.

7—DON'T FAIL TO FILL OUT THE ATTACHED CARD AND MAIL. Thank you,

Director of Survey

FIG 10 Return postcard interview form.

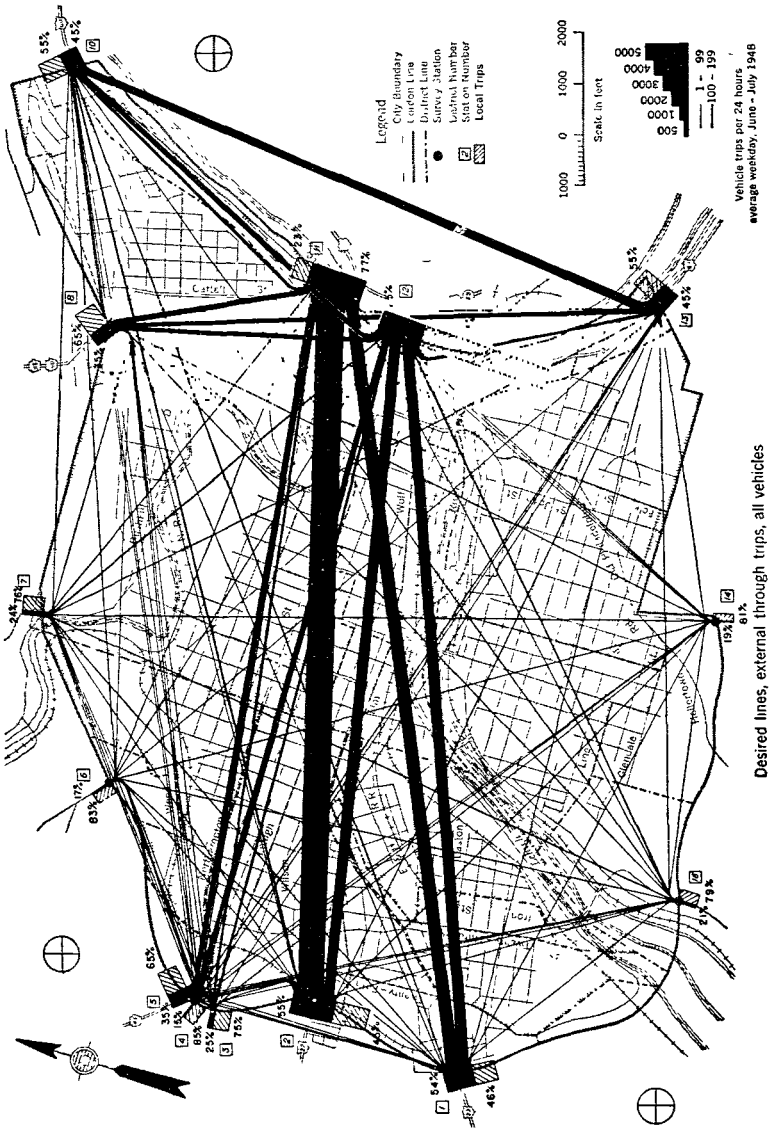

FIG. 11. Desire line map Easton metropolitan area traffic survey, 1948.

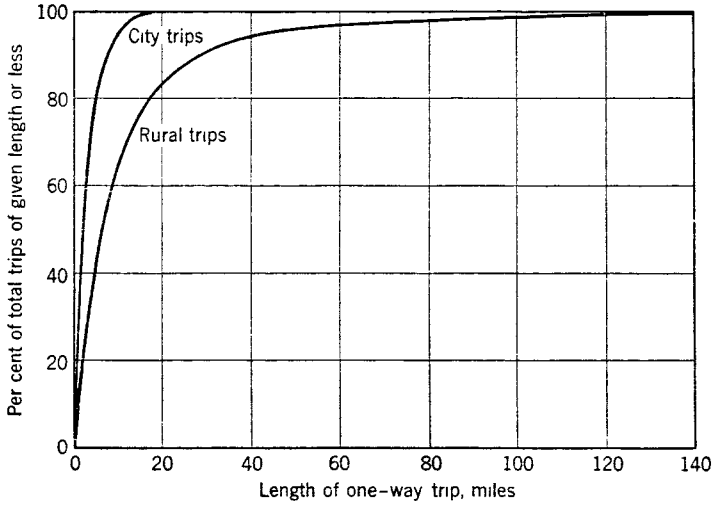

FIG 12 Typical distribution of trip lengths *Public Roads*, May 1939. *Traffic Survey of the Sacramento Area*, Dept of Pub Works and Pub Roads Adm, Sacramento, Calif, 1948.

Table 8. Resident Trip Generation in Various Cities

(Based on home-interview data)

City	Approximate Population	Date of Survey	Automobile per Dwelling Unit	Persons per Dwelling Unit	Trips by All Modes of Transportation for Typical 24-Hour Day: Persons Making Trips per Dwelling Unit	Trips per Dwelling Unit	Trips per Capita	Automobile Trips per Capita *
St. Louis, Mo.	1 480 000	1945	0 49	3 31	1.38	5 81	1.76	0 39
Baltimore, Md.	912 900	1945	0 45	3.32	1 77	4 50	1 35	0.65
Seattle, Wash	518 600	1946	0 64	2 77	1.58	4.60	1 68	1 09
Portland, Ore	422,700	1947	0.68	2.97	1.77	6.10	2.07	1.48
Indianapolis, Ind.	411,600	1945	0.60	3.22	1 48	3.56	1 11	1 02
Denver, Col.	360,000	1944	0 72	3.10	1.44	5.10	1 64	1 00
Memphis, Tenn.	278 500	1944	0.46	3.58	1.49	3.24	0 91	0 66
Nashville, Tenn.	195 000	1945	0.42	3.46	1.52	3 20	0 92	0 74
Tacoma, Wash.	138 700	1948	0 73	2.89	1 61	5.40	1.85	1 68
Spokane, Wash.	138 400	1946	0 61	2 85	1.32	3.40	1 18	1.34
Little Rock, Ark.	130,300	1944	0 56	3 34	1.37	2.62	0.78	0.99
Tucson, Ariz	126 900	1949	0 85	3 28	1.88	7.50	2 28	1.74
Chattanooga, Tenn.	126 100	1946	0.63	5 42	2.14	4.48	0 83	0 81
Reading, Pa	119 800	1946	0 45	3.16	1 27	3 49	1 10	1 26
South Bend, Ind	119 400	1945	0.69	3 27	1 36	3.40	1 04	1 56
Ft Wayne, Ind	114 900	1944	0.76	3 34	1 55	3 84	1 15	1 16
Charlotte, N C	100 600	1945	0 51	3 51	1 57	3 80	1 10	1 39
Harrisburg, Pa	95 000	1946	0 52	3.27	1 59	8 30	2 54	1 88
Columbus, Ga.	79 200	1947	0 43	3.90	1 52	3.70	0 95	1.97
St Joseph, Mo	75 700	1945	0 48	3 02	1 10	2 60	0 85	1 08
Port Huron, Mich.	33,800	1945	0 87	3 20	1 61	7 10	2 22	1.97
Ottumwa, Iowa	29,500	1945	0 61	3 48	1 56	4 92	1 41	1 26
Mason City, Iowa	23 200	1945	0 71	3.47	1 32	6 10	1 76	1 60
Average			0 60	3.35	1 53	4 64	1 41	1 25
Range			0 42–0 87	2 77–5.42	1 10–2 14	2 60–8 30	0 78–2 54	0 39–1 97

* Based on all automobile trips regardless of origin or destination or residence of driver (all other columns based on trips by internal residents only).

In the use of any of these methods as aids in the assigning of volumes of traffic to a new route or facility a number of factors of practical concern must be considered.[1] Among these are:

1. Travel distance and time on existing road net in comparison with similar values on the proposed route.
2. Trip lengths and relative amounts of each trip length which may be accommodated on the proposed route.
3. Comparison of travel conditions on the existing network with those on the proposed route.

Trip Lengths. As shown in Fig. 12, 85% of all rural trips are 20 miles or less in length and 40% are 5 miles or less in length. The average trip length of urban traffic varies with the size of the city but is usually about 3 miles.

Trip Generation. The average family dwelling unit in the city generates 4.64 trips per day by all modes of highway travel, as shown in Table 8. Those persons making trips average 3.29 trips each.

Trip Mode. Table 9 shows the average percentage of passenger trips made by private automobile, taxicab, and mass transit for typical cities. Generally speaking, the mass carrier is of greater importance as the population size of the city increases.

Table 9. Trip Mode *

(Based on home-interview data)

City	Approximate Population	Date of Survey	Per Cent of Persons Using Each Mode of Travel: Auto Drivers	Auto Passengers	Taxi Passengers	Sub-total	Mass Transit
St. Louis, Mo.	1,480,000	1945	26.6	5.3	0.2	32.1	67.9
Baltimore, Md.	912,900	1945	27.4	14.7	1.3	43.4	56.6
Seattle, Wash.	518,600	1946	43.4	21.5	0.5	65.4	34.6
Portland, Ore.	422,700	1947	47.1	24.5	0.4	72.0	28.0
Indianapolis, Ind.	411,600	1945	47.0	17.6	0.5	65.1	34.9
Denver, Col.	360,000	1944	38.4	23.1	1.2	62.7	37.3
Sacramento, Calif.	201,300	1947	59.0	25.7	... †	84.7	15.3
Spokane, Wash.	138,400	1946	46.7	21.5	1.4	69.6	30.4
Little Rock, Ark.	130,300	1944	36.5	20.3	...	56.8	43.2
Tucson, Ariz.	126,900	1949	57.2	29.8	0.3	87.3	12.7
Reading, Pa.	119,800	1946	39.0	16.4	..	55.4	44.6
South Bend, Ind.	119,400	1945	63.9	17.9	0.3	81.1	17.9
Ft. Wayne, Ind.	114,900	1944	49.3	13.9	0.5	63.7	36.3
Harrisburg, Pa.	95,000	1946	37.3	17.9	0.5	55.7	42.3
Lincoln, Neb.	88,200	1944	47.0	27.4	0.7	75.1	24.9
Columbus, Ga.	79,200	1947	39.7	19.8	...	59.5	40.5
St. Joseph, Mo.	75,700	1945	39.0	14.9	0.5	54.4	45.6
Port Huron, Mich.	33,800	1945	51.6	30.0	0.3	81.9	18.1
Ottumwa, Iowa	29,500	1945	44.1	20.4	...	64.5	35.5
Mason City, Iowa	23,200	1945	55.8	27.3	...	83.1	16.9
Average			44.8	20.5	0.4	65.7	34.3

* Internal trips only—excluding truck drivers and passengers and taxi drivers.
† Taxi passengers unreported or less than 0.1%.

Trip Purpose. About 66% of all trips are made for the purpose of reaching work or returning home. Social and recreational trips account for about 12%, and business and shopping trips account for about 14%. See Table 10.

Table 10. Trip Purpose *

(Based on home-interview data for 15 cities)

	Per Cent of Total Trips: Home	Work	Social Recreational	Business	Shopping	Medical Dental	School	Serve Passenger	Eat Meal	Other
Average	42.7	23.0	11.6	6.5	7.6	1.2	2.3	3.1	1.8	2.1

* Includes only those trips made by persons accounted for in the Internal Home-Interview Survey.

[1] See Art. 24 for further discussion of the distribution of traffic on alternative routes.

Bypass Traffic. Fifty to sixty per cent of the vehicles passing through the central business districts of most cities do not desire to stop there. On the other hand, the percentages of approach traffic that may be bypassed around the entire city instead of merely around the central business district vary considerably with the population size, as shown in Table 11. The usual distribution of metropolitan area traffic by major areas of origin and destination is shown in Fig. 13.

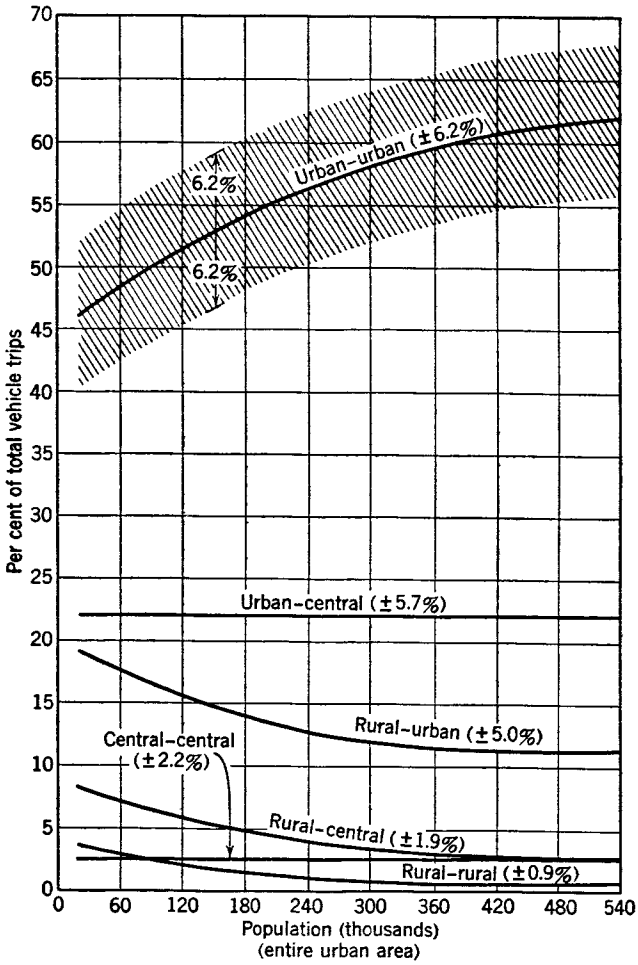

FIG. 13. Characteristic distribution of basic classes of traffic in cities of various sizes, based on analysis of data in O.D. surveys made in 28 cities of 23,000 to 540,000 population from 1944 to 1949. Note: $\pm x\%$ = standard error of estimate.

Table 11. Proportions of Traffic Approaching Cities of Various Population Groups Which Are Bound beyond the City, to the City, and to the Central Business District *

Population Group	Number of Cities	Number of Roads	Traffic Bound beyond the City, %	Traffic Bound to the City, %	Traffic Bound to the Central Business District, % †
Less than 5,000	15	68	52.0	48.0	27.8
5,000 to 10,000	34	191	38.6	61.4	33.7
10,000 to 25,000	48	339	35.0	65.0	28.3
25,000 to 50,000	27	281	22.7	77.3	27.2
50,000 to 100,000	26	375	17.4	82.6	24.7
100,000 to 250,000	23	373	15.5	84.5	23.3
250,000 to 500,000	6	157	11.9	88.1	22.1
500,000 to 1,000,000	8	222	6.8	93.2	16.1
Total	187	2,006			

* Based on interviews obtained on roads at the limits of the urban area. All roads entering the area are included.

† Based on interview data for 162 of the cities.

Source: U. S. Bureau of Public Roads, June 1952.

8. TRAFFIC-STREAM FLOW

Elements of the Traffic Stream. The traffic stream is a dynamic flow which is vitally affected by the width, arrangement, and other design features of the roadway; the conditions of the road surface; gradients; sight distances; frequency and forms of intersections; drainage structures; and roadway appurtenances. The behavior of individual units of traffic is affected by other units of the stream; signs; signals; markings; pedestrians; the weather; lighting; and other conditions. Ordinarily the traffic stream has flow and counterflow along a common route. It may, however, by design or regulation, be separated into a pair of one-way flows.

Transverse Distribution of Vehicles. In a one-lane one-way stream, road users steer their vehicles so that there is central tendency of their paths about some optimum line within the lane. This characteristic is of importance in road design and marking. In multilane one-way streams, the left lane naturally develops higher speeds and volumes, even though the right lane may appear to have the heavier flow. An illustrative distribution of vehicles in each lane of a two-lane, one-way stream is shown in Fig. 14.

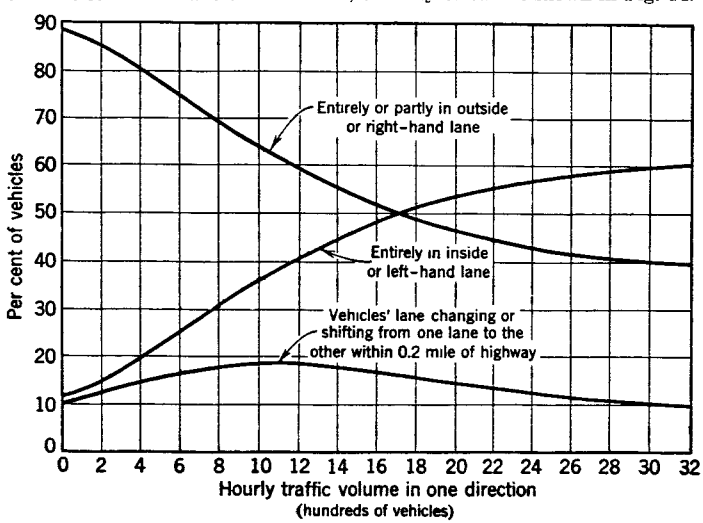

FIG. 14. Distribution of vehicles using lanes of a four-lane highway at various volumes. O. K. Normann, *Public Roads*, June 1942.

In a two-lane, two-way stream the distribution is dependent on the proportions of flow in opposite directions. On level tangent sections in rural areas the percentage of vehicles of a given flow which are transgressing in the lane of counterflow, in order to overtake and pass, decreases as the volume of counterflow increases until, at about 1050 vehicles per hour in the direction of counterflow, all vehicles in the given direction will be confined to single-lane operation as shown in Fig 15.

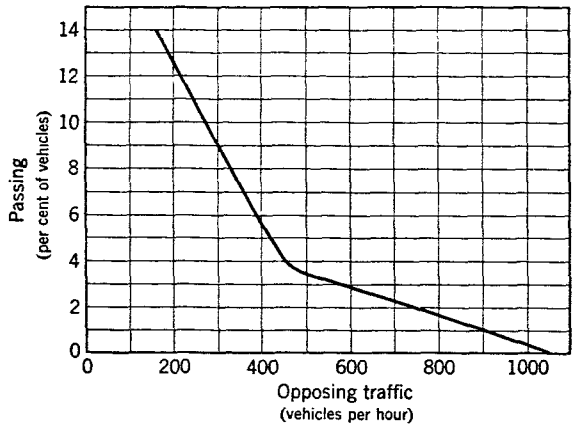

FIG 15. Per cent of vehicles traveling in one direction performing passing maneuvers against various volumes of opposing traffic on a two-lane, two-way tangent section. O. K. Normann, *Public Roads*, February 1939

The distribution of vehicles in a three-lane, two-way traffic stream on a level tangent, when about two-thirds of the traffic movement is in one direction, is illustrated in Fig. 16. Even with unbalanced demand, the center lane of such a stream is not a very efficient carrier of traffic.

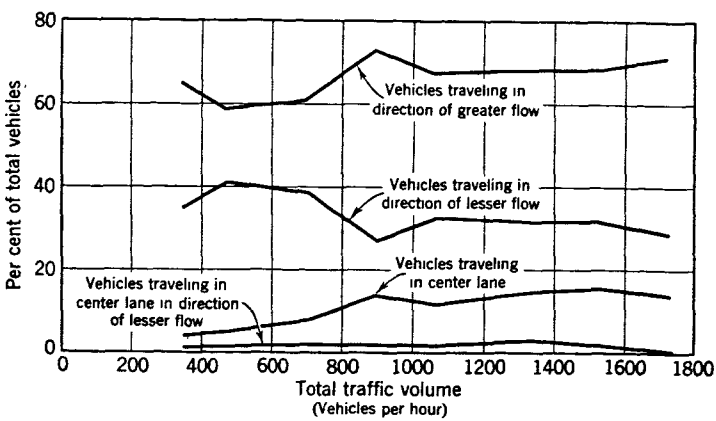

FIG 16. Distribution of vehicles on lanes of a level, straight three-lane highway. O K Normann, *Public Roads*, June 1942.

Longitudinal Distribution of Vehicles. At every instant, the longitudinal arrangement of vehicles in the traffic stream gives to the individual road user a sense of safety with freedom of movement or a sense of hazard and congestion. Each road user drives with a certain amount of free road space ahead of his vehicle. These road spaces, or gaps, are basic indications of an inherent, unique quality of each stream. The variation of gap length ranges from a few feet to great distances and is dependent on the speed and the overall density of the traffic stream.

The time interval of a gap, from head to head of successive vehicles, is referred to as *headway*. The relation between headway and the relative speeds of closely following vehicles is shown in Fig. 17. When one vehicle is overtaking another on a rural road free from other restraining influences, the absolute speed of the rear vehicle begins to fall rapidly to approach the speed of the vehicle ahead as the headway decreases to 9 sec. This initial interference headway value marks a sharp break in the relative speeds of the successive vehicles through a wide range of absolute speeds found on rural highways. It may be concluded that the average road user senses some degree of interference or congestion when his headway is less than 9 sec although it is clear that road users do not ordinarily demand that much headway.

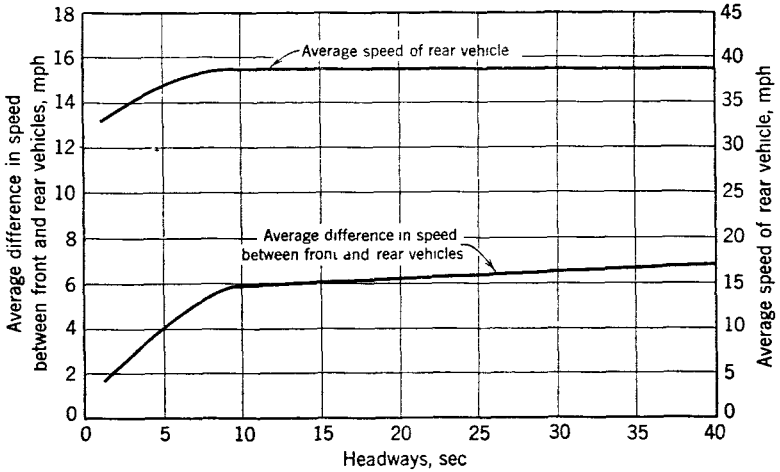

FIG 17 Speed characteristics of vehicles traveling at given time spacings O. K Normann, *Public Roads*, February 1939

The average minimum headways which develop when a queue of stopped or very slowly moving vehicles is suddenly released, as at a signal, are given in Table 12.

Table 12. Average Minimum Headway from Queue Formation

Vehicle Position in Queue	Average Minimum Headway, sec
1st	3.8
2nd	3.1
3rd	2.7
4th	2.4
5th	2.2
6th	2.1

Source: *Tech Rept. 1*, Bureau of Highway Traffic, Yale University.

Random Distribution in the Traffic Stream. The distribution of any set of events which meet the criteria of randomness can be predicted through the use of the theory of probability. The two basic requirements of randomness in a traffic stream are that at a given point the arrival of each vehicle be independent of the arrival of every other vehicle, and that the number of vehicle arrivals in a unit of time does not affect or influence the number of vehicle arrivals in any other equal unit of time. The distribution may be stated as follows:

$$p(x) = \frac{e^{-m}m^x}{x!}$$

wherein $p(x)$ = probability of the arrival of x vehicles in a given length of time (x is always a whole number).
m = average number of vehicles arriving in the given length of time.
e = Napierian base of logarithms = 2.71828.
$x!$ = factorial x, i.e., $x(x-1)(x-2)\cdots(3)(2)(1)$.

Numerous tests have demonstrated that, when a traffic stream is free of congestion, the longitudinal distribution of vehicles follows closely this *Poisson's law* of randomness. Typical observed relationships of headways and volumes for one direction of flow on two-

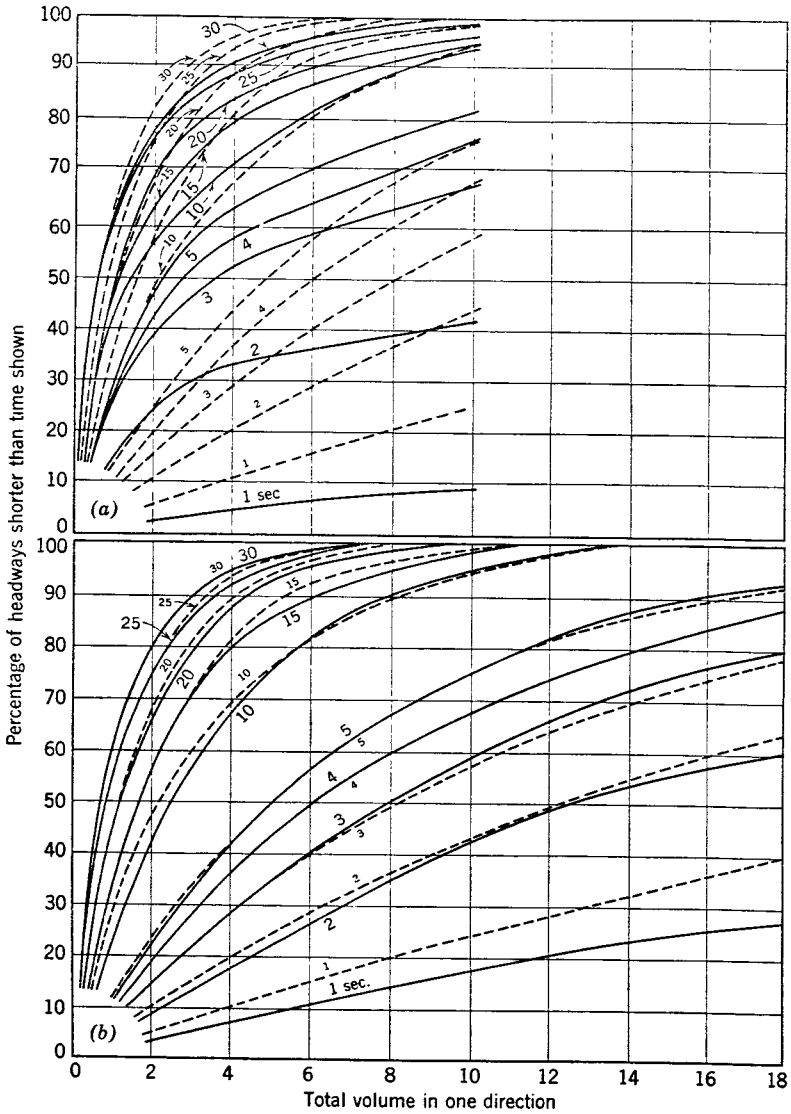

FIG 18. Headway distribution on typical two-lane and four-lane highways. (*a*) Typical two-lane rural highway. (*b*) Typical four-lane rural highway Solid lines = observed headways Dashed lines = theoretical headways From *Public Roads*, June 1942.

lane and four-lane highways are compared with relationships computed by the use of Poisson's law in Fig. 18.

Lane Changes. Within the same stream of traffic, the single transfer of a vehicle from one lane to the next adjacent lane is defined as "*lane change*." The basic cause of this

maneuver is the inherent range of speed found among vehicles in the traffic stream. The greater the range of speed, the greater will be the frequency of demand for lane change. The volume of the stream is also of great significance. As the volume increases, the frequency of demand for lane changes also increases, but the opportunity to make changes decreases. When lane change is prevented or inhibited there is a natural tendency for the traffic to develop queues behind slow-moving vehicles. Such queue formations compel inefficient utilization of the route and are extremely annoying to most road users.

When the headway of the vehicle changing lanes is rapidly approaching zero, the lane change is *forced*. Any other lane change is classed as *optional*. The gap or headway between the vehicles in the adjacent lane may be either (*a*) so short that it is not acceptable to the lane-change driver; (*b*) acceptable to the lane-change driver but so short that both the lead and trailing vehicles influence his maneuver; or (*c*) so great that only the lead vehicle or only the rear vehicle or neither vehicle will influence the lane-change driver in any significant way. When the lane change is made under condition *b*, it is termed a *gap* lane change; when made under condition *c*, if influence stems from the lead vehicle only, it is termed a *retarded* lane change; when influenced by the rear vehicle it is called a *conflict* lane change; and when neither lead nor rear vehicle impose any influence it is called a *free lane change*. When the average speed is 30 to 40 mph, the average length of roadway required to accomplish a lane change varies from about 125 to 260 ft, depending on the type of lane change made, as shown in Table 13.

Table 13. Average Length, Duration, and Speeds of Various Types of Lane-Change Maneuvers

Type of Lane Change	Length, ft	Duration, sec	Speed, mph Start	Speed, mph End
Optional				
Free	222	4.1	35 8	36.0
Retarded	167	3.3	33.3	33.3
Conflict	261	4.3	40 6	41 0
Gap	218	3.9	37.1	37.1
Average	218	4.0	36.1	36.2
Forced				
Free	132	2.7	34.4	32.7
Retarded	124	2.6	33.4	32 7
Conflict	169	2.9	39 3	43.7
Gap	145	3.0	34.9	32.1
Average	137	2.8	34 8	33.2

Source: *Tech. Rept. 4*, Bureau of Highway Traffic, Yale University.

9. INTERSECTION FLOW

Intersection Operation. The two basic movements which road users execute at intersections are transfers from one route to another and crossings of intersected traffic streams. As each road user traverses an intersection, he finds it necessary to (1) diverge from, (2) merge with, or (3) intersect and cross the paths of other road users. Although all intersection movements may be analyzed on the basis of the three elemental maneuvers, in many instances these maneuvers are so closely interdependent as to require seriation in their development. Because of the complexities which arise from these circumstances, the prediction of intersection performance is difficult.

Conflicts. Whenever a basic maneuver of intersection occurs, potential if not actual conflict between two or more road users arises. This conflict may concern only the two proximate road users whose paths are joined, crossed, or divided, or may be reflected back in each of the flows approaching the area of potential collision. *Conflict areas* and *collision areas* occur for each of the elemental maneuvers. The integration of all of these areas in a given intersection forms the conflict area and the collision area for that intersection as a whole.

The normal right-angled intersection of 2 two-way streams develops 32 conflict points, 16 of which are of the more severe crossing type. If one stream is removed by design so as to make a T or Y intersection, there occur only 9 conflict points and of these only 3 involve crossing maneuvers. If two of the flows are stopped simultaneously by signal control, there remain but 8 conflict points, of which only 2 involve the more severe crossing type of conflict.

The frequency with which interference may arise at conflict points is dependent on the volume of units found in each path of flow. Thus in the standard right-angled intersection, if each entering stream carries 200 vph and 10% in each stream turn right and 10% turn left, there will develop 1200 potential conflicts per hour, each representing a movement wherein one vehicle departs from, enters in front of, or crosses the path of another vehicle. It does not indicate, however, the proximity of any of the vehicles involved in such maneuvers nor the degree of hazard or delay encountered.

The Diverging Maneuver. The time-space relationships of the diverging maneuver are illustrated in Fig. 19. A potential area of conflict is developed as the diverging vehicle reduces its speed, in order to prepare for a comfortable turn, and continues until that vehicle leaves the lane of original travel. In the instance illustrated, the collision area is nearly congruent with the entire conflict area of the maneuver. Generally, diverging maneuvers are not a serious interference to vehicular stream flow.

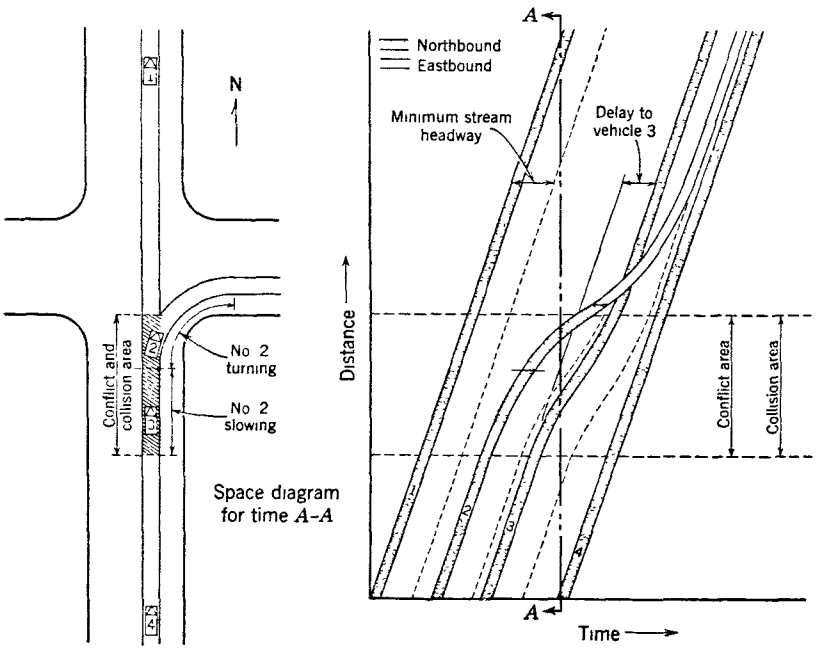

FIG. 19. Time-space relationships of diverging maneuver.

The Merging Maneuver. If an adequate *lag* does not exist between the arrival at the collision area of the merging vehicle and the arrival of the next approaching conflict unit, the merging maneuver must be deferred until an adequate *gap* occurs. Exploratory studies point to acceptable gap sizes of from 2.5 to 6 sec, or more, in length.

Typical time-space relationships of the merging maneuver are shown in Fig. 20. In this case the conflict area begins at a safe distance back of the collision area and extends to a point beyond, where the merging vehicle has achieved approximately normal speed. The collision area extends from the point of entry of the merging vehicle to the forward limits of the conflict area. Although most delay is suffered by the merging vehicle, there is also delay to the vehicles in the lanes of flow into which merging takes place.

The Crossing Maneuver. Typical time-space relationships found in the crossing maneuver are shown in Fig. 21. Limited observations indicate that, in non-stop crossing maneuvers, at right-angled crossings, the average minimum time interval between the passage of conflicting vehicles through the collision area is about 2.8 sec. When one of the conflicting vehicles is at rest, just prior to the right-angled crossing maneuver, the average driver of the stopped vehicle will require a lag of about 4.5 to 8 sec before undertaking the maneuver. Left-turning drivers who are delayed usually refuse lags less than 3.75 sec and accept lags of 4.75 sec or greater. Smaller gaps are generally acceptable for

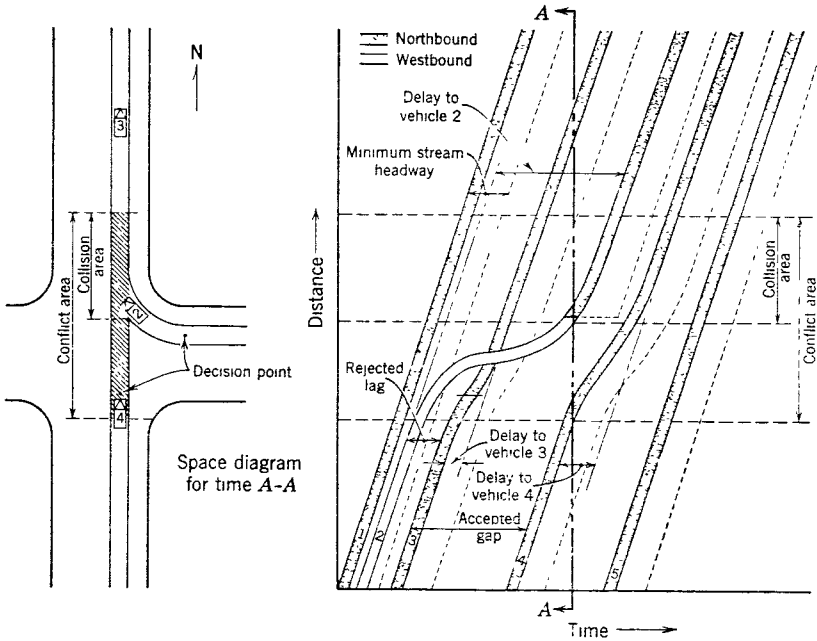

FIG 20. Time-space relationships of merging maneuver

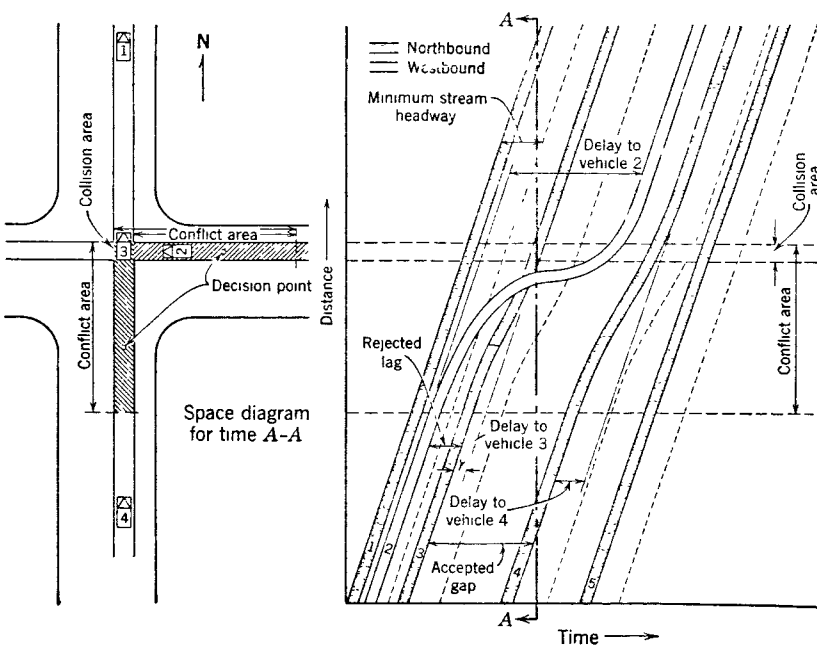

FIG. 21 Time-space relationships of crossing maneuver

oblique crossings of the non-stop weave type at a channelized intersection. Average drivers may accept gaps of 1.8 sec for such non-stop oblique crossings. The median acceptable gap or lag required for a pedestrian to cross a vehicular flow is 5.7 sec for a one-way vehicular stream and 7.3 sec and 7.7 sec for the near and far flows, respectively, for a two-way vehicular stream.

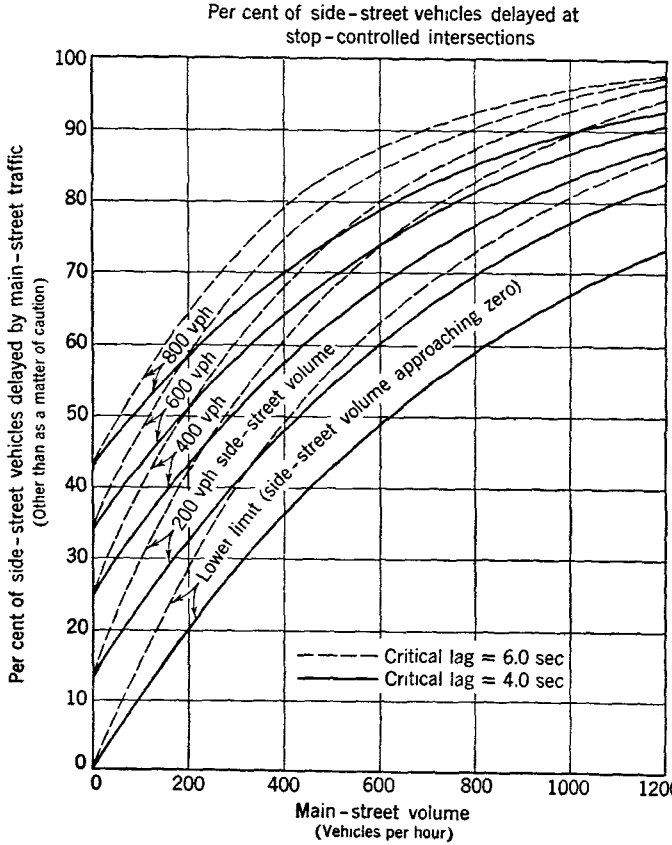

FIG 22 Per cent of side-street vehicles delayed at stop-controlled intersections

Limited experimental studies indicate that the percentage of vehicles delayed at a crossing, for various volumes of flow, other than as a matter of caution, are as shown in Fig. 22. On the assumption that the traffic flow to be crossed is a random series, the average waiting time per vehicle for all crossing traffic is as given in Fig. 23 in terms of the critical lag as a unit of time measure. For example, if a critical lag of 6 sec is required for the crossing maneuver, any vehicle which is to cross a flow of 600 vehicles per hour would be crossing a flow averaging one vehicle per critical lag. Although not all such crossing vehicles would be stopped, the average waiting time would be expected to be 0.8 of a critical lag or about 5 sec.

Table 14. Time Loss Due to Stopping of Passenger Vehicles, in Seconds

(In addition to waiting time)

Normal Speed	Decelerating	Getting under Way	Accelerating	Total
20–30	2.5	2.0	2.9	7.4
30–40	3.9	2.0	3.6	9.5
40–50	4.8	2.0	4.9	11.7

Characteristic values of time lost by passenger vehicles in decelerating, getting started, and accelerating are given in Table 14. The average passenger vehicle which is stopped will suffer from 7 to 12 sec of delay in addition to the delay incurred while waiting for opportunity to use a collision area. This time value is of a magnitude which is as great or greater than many of the blockades found in the conflicting stream.

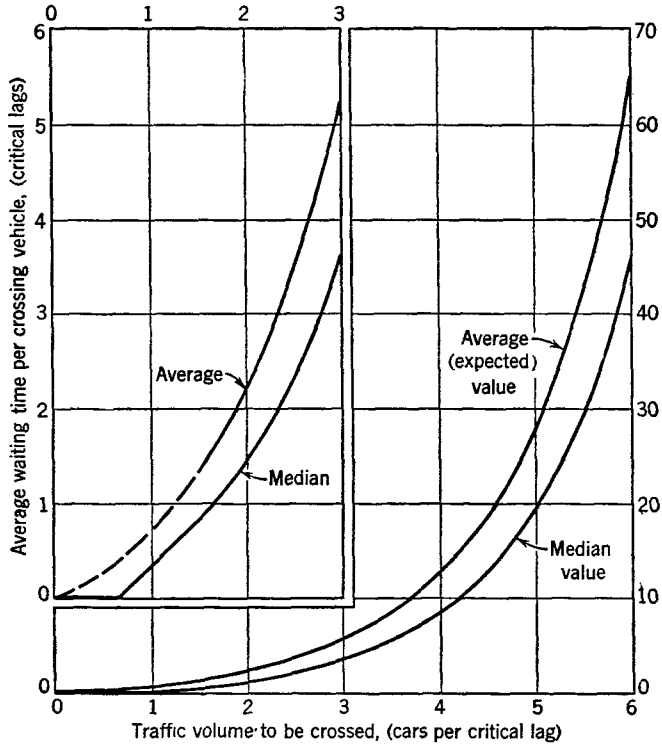

FIG 23. Average and median waiting time as a function of volume.

10. TRAFFIC ACCIDENTS

Trends. There is a degree of hazard in every traffic movement. Not by design nor intent, yet with suddenness, events occur in the traffic stream which result in death, injury, and property damage. Since the advent of the motor vehicle, over 1 million persons have been killed in traffic accidents in the United States. In 1954, 36,300 fatalities and 1,250,000 injuries were incurred. The peak year of accident occurrence was 1941, when nearly 40,000 persons were killed and about 1.4 million were injured.

As shown in Fig. 24, the death rate on a vehicle basis stabilized at about 12 deaths per 10,000 registered vehicles in the 1920's but decreased to about 7 in the decade following World War II. The death rate on a population basis stabilized at about 25 deaths per 100,000 population since 1930. The trend on a vehicle-mileage basis has been steadily downward since 1925.

In the interpretation of these values it is significant to note that, in spite of the drastic changes which have been made in the mode and amount of travel, the accidental death rate on a population basis has remained roughly in the magnitude of 20 to 30 deaths per 100,000 population for all forms of transportation since the turn of the century.

Significance and Uses of Accident Data. Traffic accidents are symptoms of actual failure of the road user, the vehicle, or the fixed facilities to discharge properly their respective functions in traffic movement. Near failures of these elements are equally significant but are extremely difficult to capture.

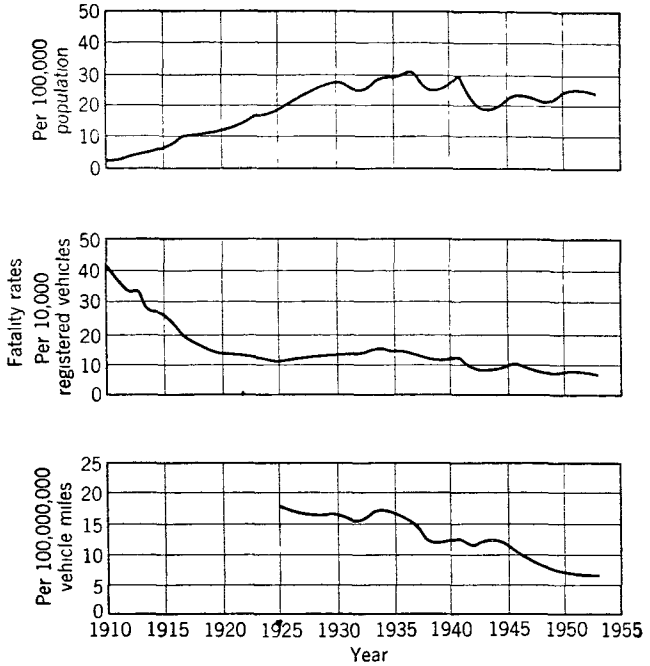

FIG. 24. Trends in motor-vehicle fatality rates.

Analysis of accident facts leads to an understanding of the conditions surrounding the functional failure of a traffic facility and the causes of the failure. Among the items requiring analysis in the light of accident experiences are:

1. Highway design standards.
2. Warrants for traffic-control devices.
3. Channelization schemes.
4. Street and highway lighting.
5. Pedestrian safety facilities.
6. Traffic regulations.

Accident Reporting. Accurate records of traffic accidents are essential for an intelligent approach to the solution of traffic problems. They are valuable not only to traffic engineers but also to all other groups dealing with traffic. From the standpoint of collecting information to be used in accident prevention work, reports on minor accidents are just as valuable as those for serious accidents. Circumstances leading to the accident and the violations and conditions involved might be the same in a very slight property damage case as in a fatal mishap. Because of this, there has been a marked trend toward the enactment of compulsory reporting laws which require a greater percentage of accidents to be reported. A basic system of accident report forms has been adopted by the National Conference on Uniform Traffic Accident Statistics. For engineering uses, original reports of accidents should be filed by locations, but some authorities prefer to file reports chronologically, with cross references of locations.

Accident Analyses. Traffic accidents are usually analyzed for trends in rates per hundred thousand persons, ten thousand vehicles, or hundred million vehicle miles, seasonal and daily variations and types. Analyses of primary concern to the engineer are those indicating the accident record at specific locations or sections of a highway. By correlation of the accident history with traffic flow and roadway geometry and conditions, underlying factors of accident causation may become apparent and remedies for their correction may be suggested. The general influences of roadway types, lane widths, and roadside structures are given in Table 15.

Table 15. Accident Rates on Various Roadway Types (Tangents)

Total Reported Accidents per Million Vehicle Miles

Average Daily Volume (vehicles per day)	Two-Lane	Three-Lane	Four-Lane Undivided	Four-Lane Divided	Four-Lane Controlled Access
Under 5000	3.5	3.8	2.3	*	3.4 †
5000 to 9000	4.5	4.1	4.0	3.2	.
9000 and over	2.6	11.1	3.8	4.1 ‡	1.5
All volumes	3.7	5.0	3.7	3.8	2.3

* Sample considered too small for reliability.
† Very high-speed roadway.
‡ Includes a considerable mileage with a median strip delineated by paint and no other physical separation of traffic moving in opposite direction.
Source: National Safety Council and U. S. Bureau of Public Roads Study.

Accident Rates on Two-Lane Tangents by Widths of Pavement

Pavement Width, ft	Total Reported Accidents per Million Vehicle Miles
Less than 18	5.2
18 to 19.9	3.8
20 to 22.9	3.5
23 and over	3.4

Source: National Safety Council.

Accident Rates at Structures on Two-Lane Roadways Less Than 30 Ft Wide, by Width of Structure Relative to Approach Roadway (8 States)

Width of Structure Compared with Approach Roadway, ft	Total Reported Accidents per 10 Million Vehicles per Year
Narrower by 1 or more	10.0
1 narrower to 5 wider	5.8
Wider by 5 or more	1.2

Source: National Safety Council.

Accident Costs. Although losses in lives and in personal injuries are not reducible to monetary values, some measure of the economic cost is indicated by the resulting wage loss, medical expense, overhead cost of insurance, and property damage. Average values employed in 1954 by the National Safety Council are $22,600 for each fatality, $1,250 for each non-fatal injury, and $190 for each property-damage accident. Since the ratio of property-damage and personal-injury accidents to fatalities is relatively constant, the same authority employed a rounded figure of $110,000 per fatality as the economic loss to the community covering all traffic accidents. Typical out-of-pocket expenses are given in Table 16.

Table 16. Estimated Traffic Accident Losses, Lansing, Michigan, 1945–1946

	Salary Loss *	Medical Bills	Auto Repairs	Other Costs	Total Costs	Job-Time Loss *
Average non-injured driver	$ 4.50	...	$ 66.00	$ 2.50	$ 73.00	6 hr
Average injured driver	131.00	$ 41.00	220.00	16.00	408.00	15 3/4 days
Average injured passenger	114.00	55.00	..	37.00	206.00	29 1/4 days
Average injured pedestrian	262.00	180.00	...	26.00	468.00	35 days

* In many cases job-time loss was not accompanied by a salary loss; hence average value per hour is low.
Source: Association of Casualty and Surety Companies.

REGULATION OF TRAFFIC FLOW

11. TRAFFIC LAWS

Uniform Codes and Ordinances. To aid in the adoption of adequate and uniform traffic laws, the National Conference on Street and Highway Safety prepared a Uniform Model Vehicle Code and a Model Traffic Ordinance. The work of revising and keeping these documents up to date is now a function of the National Committee on Uniform Traffic Laws and Ordinances.

Registration and Licensing. All states require motor vehicles to be registered, and many states have enacted certificate of title laws which, in theory, provide for the maintenance of a complete record of liens and encumbrances on motor-vehicle registrations. All states but one had driver licensing laws in 1951, although in many states such laws fell short of the needs. Many states have financial-responsibility laws which require persons against whom judgments have been rendered in automobile accidents to show financial responsibility in order to retain their operators' permits.

12. VEHICLE CONTROLS

Vehicle Inspection. It has long been recommended by international traffic and transportation bodies and motor-vehicle authorities that compulsory inspection be a part of motor vehicle regulations and, by 1949, inspection legislation of some type had been enacted by 23 states and the District of Columbia.

Sizes and Weights of Commercial Vehicles. The most generally used loading standards are those prepared by the AASHO in 1946. These standards and the limits in effect in the various states in 1954 are shown in Tables 17 and 18. In states where truck traffic is heavy, major problems have developed in administering the laws on sizes and weights. Violations are frequently so common as to make it difficult, if not impossible, for the police to cope effectively with the situation.

Table 17. AASHO Vehicle-Loading Standards (1946)

Distance between First and Last Axles in Group, ft	Maximum Load on Group, lb	Distance between First and Last Axles in Group, ft	Maximum Load on Group, lb	Distance between First and Last Axles in Group, ft	Maximum Load on Group, lb
4	32,000	22	45,700	40	60,800
5	32,000	23	46,590	41	61,580
6	32,000	24	47,470	42	62,360
7	32,000	25	48,350	43	63,130
8	32,610	26	49,220	44	63,890
9	33,580	27	50,090	45	64,650
10	34,550	28	50,950	46 *	65,400
11	35,510	29	51,800	47	66,150
12	36,470	30	52,650	48	66,890
13	37,420	31	53,490	49	67,620
14	38,360	32	54,330	50	68,350
15	39,300	33	55,160	51	69,070
16	40,230	34	55,980	52	69,790
17	41,160	35	56,800	53	70,500
18	42,080	36	57,610	54	71,200
19	42,990	37	58,420	55	71,900
20	43,900	38	59,220	56	72,590
21	44,800	39	60,010	57	73,280

* 50 ft being maximum length limit (U.V.C. recommendations), all values in excess of 45 ft are for cases in which special permits are granted for oversize vehicles.

Source: AASHO, Policy Concerning Maximum Dimensions, Weights, and Speeds of Motor Vehicles to be Operated Over the Highways of the United States, Washington, D. C., 1946.

Table 18. Maximum Legal Dimensions and Weights of Motor Vehicles (1954)

State	Width, in.	Height, ft	Length, ft	Gross Weight, 1000 lb
Alabama	96	12 1/2 [b]	45 [c]	53 9 [d]
Arizona	102	13 1/2	65	76.8
Arkansas	96	12 1/2 [b]	60	74
California	96	13 1/2	60	76.8
Colorado	96	12 1/2	60	73.6 [d]
Connecticut	102	12 1/2	45 [c]	60
Delaware	96	12 1/2 [b]	60	60
Dist. of Columbia	96	12 1/2	50	65.4
Florida	96	12 1/2 [b]	50	64.6
Georgia	96	13 1/2	45	53.9 [d]
Idaho	96	14	65	72
Illinois	96	13 1/2	45	72
Indiana	96	12 1/2 [b]	50	72
Iowa	96	12 1/2 [b]	45 [c]	65.4
Kansas	96	12 1/2	50	63.8
Kentucky	96	12 1/2	45 [c]	42
Louisiana	96	12 1/2 [b]	60	82
Maine	96	12 1/2	45	50
Maryland	96	12 1/2 [b]	55	65
Massachusetts	96	NR	45	50
Michigan	96	12 1/2 [b]	55	120-W
Minnesota	96	12 1/2	45	66.5
Mississippi	96	12 1/2 [b]	45	55.9
Missouri	96	12 1/2	45	60
Montana	96	13 1/2	60	76 8
Nebraska	96	12 1/2	50	64.6
Nevada	96	NR	NR	76.8
New Hampshire	96	13 1/2	45	50
New Jersey	96	13 1/2	50	60
New Mexico	96	12 1/2	65	76.8
New York	96	13	50	60.5
North Carolina	96	12 1/2 [b]	48	58.8
North Dakota	96	12 1/2	45	57.7 [d]
Ohio	96	12 1/2 [b]	60	78
Oklahoma	96	13 1/2	50	60
Oregon	96	12 1/2	50	60
Pennsylvania	96	12 1/2 [b]	50	62
Rhode Island	102	12 1/2	50	88
South Carolina	96	12 1/2	50	68 3
South Dakota	96	13	50	64.6
Tennessee	96	12 1/2	45	55.9
Texas	96	13 1/2	45	58.4
Utah	96	14	60	79.9
Vermont	96	12 1/2	50	50
Virginia	96	12 1/2 [b]	45	50
Washington	96	12 1/2	60	72
West Virginia	96	12 1/2 [b]	45	60.8
Wisconsin	96 [a]	12 1/2 [b]	50	68
Wyoming	96	13	60	73.9

[a] 104 in. for urban buses.
[b] Automobile transporters allowed 13 1/2 ft height.
[c] Full trailers prohibited.
[d] Computed for typical vehicles.
NR = no restriction.
W = maximum possible on maximum axle-load basis.
Source: National Highway Users Conference, *State Motor Vehicle Size and Weight Laws*, Washington, D. C., August 1, 1954.

Brakes. It is usually specified that standard passenger cars be equipped with service brakes on all wheels. Two separate means of applying brakes must be provided, and each must affect at least two wheels. Tractors and semitrailers, excepting the lightest types, are required to have brake mechanisms which permit the operation of the brakes thereon in conjunction with the actuation of the brakes on the tractor. Where brakes are provided on all wheels, the vehicle should be capable of being stopped in a distance of 30 ft from a speed of 20 mph; when the vehicle does not have brakes on all wheels, it should be capable of being stopped in a distance of 40 ft from the same speed

Headlights. Headlight illumination is generally considered adequate if, under normal atmospheric conditions on a level roadway, persons and objects are visible at a distance of 350 ft with the upper beam, and at a distance of 100 ft with the lower beam. In order to avoid serious glare, restrictions are imposed on the upward aiming of the beams The maximum apparent candlepower[1] for an unloaded vehicle must not exceed 8000 at 1° above the horizontal, and the maximum intensity of the light beam must not exceed 75,000 candlepower at any point. The lower light beam, which is required to be used when approaching within 500 ft of an oncoming car, should be so regulated that when the vehicle is unloaded the high-intensity portion to the left of the vehicle at a distance of 25 ft must be 8 in. or more below the center of the lamp, and the high-intensity portion to the right of the lamp must be 3 in. or more below the center of the lamp. It is further provided that in no instance is the high-intensity portion to be more than 32 in. above the ground at 75 ft.

13. RULES OF THE ROAD

Driving on Right. In the United States the practice of driving on the right side of the roadway has always prevailed, and overtaking on the left is conventional except on one-way streets and on multilane roads having at least two unobstructed lanes for travel in each direction. Although many enforcement agencies do not permit legal passing on the right, maximum fluidity cannot otherwise be obtained on multilane roadways It is recognized, of course, that overtaking on the right should always be permitted, when physically possible, if the overtaken vehicle is in the process of negotiating a left turn.

Right-of-Way. The logic of the traditional rule giving the right-of-way to vehicles approaching from the right at an uncontrolled intersection is questionable. However, the rule has been so generally adopted and accepted by motorists that the advantages to be gained by changing it would probably not offset the difficulties of reeducating the motorists.

Vehicles turning left at intersections are usually given the right-of-way over vehicles approaching from the opposite direction after they have first yielded and ascertained that it is safe to negotiate the turn. Right-of-way for turning vehicles further requires the proper hand or mechanical signal.

Where the basic right-of-way rule is altered by the erection of stop signs, a motorist is allowed to proceed into or through the intersection after he has first stopped, yielded the right-of-way to any vehicles approaching on the artery, and ascertained that it is safe to enter or cross the intersection.

General Pedestrian Rules. Legal controls applicable to persons afoot are difficult to enforce. In all plans for pedestrian control, it must be borne in mind that pedestrians want to exercise "inherent" roadway rights; they are difficult to restrain; and they often do not realize traffic dangers which are obvious to motorists At unsignalized intersections, pedestrians are usually given the right-of-way when they are in crosswalks and have already begun a crossing of the traveled portion of the roadway. When pedestrians are not crossing at an intersection or at officially designated crosswalks, they are required by law in most states to yield the right-of-way to motorists, but the same laws also carry general provisos that motorists must exercise due caution; in other words, motorists are not entirely relieved from blame in case of an accident involving a pedestrian crossing at some place other than a designated crosswalk.

14. ONE-WAY STREETS

Under suitable circumstances, one-way traffic regulations offer a simple and inexpensive means of facilitating traffic flow, as shown in Table 19. At the outset, after such regulations are put into effect, psychological resistances of drivers and the persistence of driver habits result in some difficulty and, sometimes, in temporarily increased accident rates. This is usually of short duration, lasting only while the motorists become familiar with the new regulations.

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[1] Apparent candlepower is a relative term, used to relate the intensity of light at any point as received from some source to the intensity of light which would be required if a point source were used If the bulb of the headlight unit were placed in the position of the headlight without reflectors or lenses, the intensity of light which would result would be the bulb candlepower, which in most cases today is 32

Table 19. Effect of One-Way Traffic Regulations on Volume and Speed

Location	Volume			Average Speed, mph		
	Before	After	% Increase	Before	After	% Increase
Chicago, Ill.						
Warren & Washington Blvd.	29,000/24 hr	47,500/24 hr	64	14	28	50
Philadelphia, Pa.						
Walnut Street	4,052/12 hr	12,098/12 hr	198	17	25	47
Chestnut Street	7,137/12 hr	12,938/12 hr	82	17.6	22	25
Kansas City, Mo.						
Thirteenth Street	500/hr	525/hr	5	13.2	19.6	48.5
Fourteenth Street	460/hr	510/hr	11	...	..	...
Fort Wayne, Ind.						
Jefferson Street	1,774/hr	2,741/hr	55	9	11.9	32.2

Requirements. The following points should be considered in the establishment of one-way streets.

1. Traffic should be equally accommodated in each direction on adjacent parallel streets not more than 500 ft apart.
2. Entrance and exit points should be available on cross streets at intervals of 300 to 500 ft.
3. One-way streets should "drain" traffic away from congested areas.
4. The one-way system should meet the transit routing and loading demands.
5. Standard signs, readily visible both day and night, should be provided. During the initial period of operation it may be necessary to use police to assist in the direction of motorists.

Advantages. Among the advantages of one-way traffic regulations are the following.

1. Street capacity is increased through the reduction of conflicts, the utilization of odd traffic lanes, and the facilitation of signal timing.
2. Traffic delays are reduced. It is not uncommon to find 25% increases in average speed after converting from two-way to one-way traffic movements, while volumes increase 50 to 75%.
3. Accident rates are reduced. Reductions are found in practically all types of accidents. One-way streets are generally considered to be about twice as safe as similar two-way streets.
4. Traffic flow is improved at complex intersections by making one or more streets one-way away from such intersections.
5. Signal operations are simplified. Regardless of block lengths, an efficient progressive signal timing plan can be developed when traffic operates only in a single direction.
6. Transit services are generally made much more attractive to transit riders and more economical to transit companies.

15. THE STOP RULE AND THROUGH STREETS

Applications. Properly used, stop signs can bring about a reduction of accident hazards and a facilitation of traffic movement. Although fixed warrants have not been established for the installation of stop signs, the *Manual on Uniform Traffic Control Devices for Streets and Highways* [1] suggests the following general applications:

1. Intersection of a less important road with a main road where application of the normal right-of-way rule is unduly hazardous.
2. Intersection of a country road, city street, or township road with a state route.
3. Intersection of two main highways where no traffic signal is present.
4. Street entering a through highway or street.
5. Unsignalized intersection in a signalized area.
6. Railroad crossing where a stop is required by law or by order of the authority having jurisdiction over the highway or street.
7. Other intersection where high speed, restricted view, or serious accident record indicates a need for control by the stop sign.

[1] Prepared by a joint committee of the AASHO, the ITE, and the National Conference on Street and Highway Safety—PRA, 1948

Four-Way Stop. The application of stop signs to all legs of an intersection is becoming common in some cities. Such treatments are frequently effective for intermediate conditions between those which justify traffic signals and those where standard stop-sign treatments would be in order. Inasmuch as the four-way stop regulation supersedes the normal right-of-way precedent, it has not met with general favor.

Yield. In many instances it is feasible to direct the drivers of the entering or secondary traffic stream to yield the right-of-way to the traffic being entered or crossed. When a yield sign is used it is presumed that the traffic directed to yield will stop unless it is safe to enter or cross the main traffic stream without stopping.

16. TURNS

Methods of Turning. The position of the vehicle in the roadway is of primary importance in negotiating any turn. From a safety standpoint the position is far more important than signals. It is recommended in the Uniform Motor Vehicle Code, and most states have adopted the recommendation, that a left turn on a two-way traffic street be started from the lane nearest the center line of the street and shall be completed in the lane nearest the center line of the entered street. Many of the state laws go so far as to provide that left turns must not only be made in the lanes nearest to the center lines, but that "whenever possible such turns shall be made to the left of the center of the intersection." This method of turning avoids conflicts between left turns from opposite directions, permits the negotiation of the turn with less delay, and makes it possible to complete the turn in a legal manner at all types of intersection.

Because of the need for altering the method of making turns at certain irregular types of intersections, or at locations where peculiar traffic patterns occur, it is important that state laws grant local authorities the right to vary basic practices by placing markers or by appropriate signing.

Prohibition of Turns. Factual guides on which to base turn regulations have not been determined. All too frequently turns are prohibited where traffic signals with turning intervals or channelizing islands would make it possible to allow the turns but still correct interference problems. Pending the development of objective warrants, turns should be regulated on the basis of traffic and accident investigations at individual intersections, evaluating the effect of vehicular volumes, pedestrian volumes, turning radii, traffic lanes, and approach speeds.

17. SPEED CONTROL

Although the restriction of movement is not the objective of highway construction, it is necessary to regulate speeds so that excessive velocities will not prevail on highways of established design. This is accomplished through legislation, speed-control devices, and speed zoning.

Legislation. The Uniform Motor Vehicle Code proposes prima facie numerical limits[1] of 25 mph in business and residential districts and 50 mph (45 mph at night) in rural areas. Many deviations are found among the states although all have a basic speed rule which provides "that no person shall drive at speeds greater than are reasonable and prudent under conditions prevailing." Some states have top absolute limits[2] of 50 mph; others have top limits of 60 mph. In some cases the limits for commercial vehicles are different than those for passenger cars but these limits are difficult to enforce and develop excessive passing practices if they are observed.

Speed-Control Devices. It is possible to control the average speed over a given route by establishing a progressive signal system. However, the use of standard-type traffic signals at non-intersection locations should be avoided. Most flashing signals are ineffective for speed-control purposes except where they are mounted in such a manner as to reveal a roadway condition which necessitates a speed reduction. Vehicle-actuated signal equipment may be utilized to control speeds on the approaches to intersections or roadway hazards.

Education and Enforcement. Many traffic educational programs have been directed primarily at the control of driving speeds. It is generally found that the pleas are unheeded except when they are recognized as reasonable and proper. The motoring public cannot be persuaded to drive at speeds below those which are generally considered safe

[1] Any speed in excess of the prima facie limit is presumed to be unlawful, and proof to the contrary rests with the driver.

[2] An absolute limit refers to the speed above which it is always illegal to drive under any conditions.

and reasonable. On the other hand, there is little evidence to support contentions that a large percentage of drivers want to, or actually do, drive at speeds higher than those which are safe for existing roadway and traffic conditions.

Speed Zoning. Speed zoning involves the determination and posting throughout the highway network of safe and reasonable speeds for favorable driving conditions. It is aimed at promoting traffic safety, obtaining public cooperation in speed regulations, providing the greatest flexibility in speed limits, and warning drivers of impending needs for speed reduction. The legislative authority for zoning usually provides that the limits to be posted must be determined on the basis of engineering studies and investigations. To meet this requirement it is necessary to undertake field surveys of routes to be zoned, including:

1. Spot speed studies. A minimum of 100 passenger cars should be checked under favorable driving conditions for each direction of travel at each location.
2. Running-time studies. The speed ranges of typical vehicles in the traffic stream should be determined at each critical location.
3. Ball-bank indicator tests. Safe speeds for curves should be determined by conventional or especially designed bank indicators.
4. Inspection of physical hazards.

TRAFFIC-CONTROL DEVICES

Uniformity in all traffic-control devices is most desirable. Non-standard devices are breeders of confusion and accidents. For uniform national standards, see *Manual on Uniform Traffic Control Devices for Streets and Highways*, U. S. Bureau of Public Roads, Washington, D. C.

18. LEGAL RESPONSIBILITY

Traffic-control devices are needed on all highways to guide, warn, and regulate traffic movement. Good practice demands uniformity in their design, application, and installation. To insure this uniformity state agencies are frequently given broad authority over their use on state highways and on streets used as state highways. The failure of a state to provide adequate traffic control devices has been the subject of many court cases. In a noteworthy decision [1] the court held that "the traffic signs involved in this action were confusing, improperly located, insufficient in height, insufficient in number, not reflectorized or improperly reflectorized, misleading and an invitation to disaster at night time. . . . The State and the Traffic Commission were negligent in failing to order the removal of the signs in question and in failing to replace those with signs which would conform with nationally accepted standards. . . ."

19. TRAFFIC MARKINGS AND MARKERS

Colors. Standardized colors for markings are either white or "highway yellow." White is used for center lines, lane lines, pavement edges, streetcar clearance lines, turn guides, stop lines, crosswalks, parking stalls, route markings, and word markings. Yellow is recommended, but white is permitted, for double (no-crossing) center lines and barrier (no-passing) lines, including no-passing zone lines, pavement width transition lines, approaches to obstructions, and approaches to railroad crossings. Curb markings for parking prohibitions should be yellow. Vertical surfaces of obstacles should be crosshatched with alternate black and white stripes sloped downward 45° from the vertical toward the side of the obstruction on which traffic passes. Delineators are usually clear white. Reflectors for obstructions are usually red.

Materials. Markings or markers may consist of or be constructed with plain or reflectorized paint, metal, or plastic pavement inserts, bricks or other blocks, glass beads, retrodirective reflectors of plastic or glass, and raised studs, buttons, or strips of metal or concrete.

Longitudinal Pavement Lines. Longitudinal lines are either broken or solid. Broken lines (about 60% gaps) are used for guide purposes and may be crossed at the discretion of the driver. They should be used for lane markings on all rural highways, on roadways where passing is permitted, and for lane markings at congested or irregular intersections.

[1] Ziehm v. State of New York—270 App. Div 876, 1946.

Solid lines are used where they are not to be crossed. The "barrier line" is a solid line of limited length used to prohibit overtaking and passing in areas of low sight distance. All longitudinal lines should be 4 to 6 in. wide.

Transverse Pavement Lines. Transverse lines are used to mark stop lines, crosswalk lines, parking stalls, and railroad crossings (Fig. 25). Because of the low angle with which such lines are viewed they are usually made 1 to 2 ft in width, except for parking stalls.

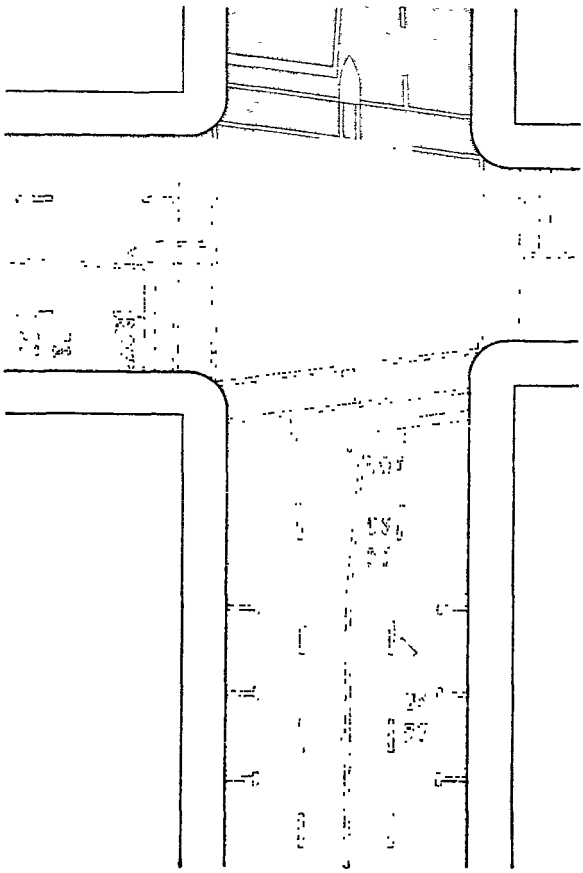


FIG 25. Typical urban pavement markings From Manual on Uniform Traffic Control Devices, U S. Bureau of Public Roads

Symbol Word and Number Markings. Often pavement markings are employed to indicate route numbers, school zones, turning directions, and similar information. Such markings are constructed to outline the shadow cast on a horizontal plane by standard letters normal to the rays of a point source of light located at the relative position of the driver's eyes. The markings are usually 6 to 10 ft in height with individual symbols not over 2 ft in width.

Roadside Delineators. Reflecting elements of glass, plastic buttons, glass beads, and other suitable materials are used to delineate the highway for night driving. Such delineators are ordinarily installed at a height of approximately 42 in. above and from 2 to 10 ft transversely from the pavement edge. Normally, such delineators are placed 200 ft apart on tangents. Due to perspective, however, spacing of delineators must be altered for vertical and horizontal curves.

20. TRAFFIC SIGNS

Traffic signs are of the regulatory, warning, or guide types (Fig. 26). Regulatory signs imposing legal restrictions applicable at a particular location are usually required by law and must be in place before the regulations become enforceable. Such signs include "stop," "one-way," "speed limit," "parking," etc. Warning signs are intended to call attention to hazards or changing roadway or environmental conditions which the motorist is approaching. Guide signs include route markings and directional and informational signs. They carry state and local route numbers, detour information, destinations, etc. Special care should be exercised to avoid the overuse of regulatory and warning-type signs. Excessive uses create a general disregard for the signs and a loss in overall effectiveness. Because of the large amount of travel on almost every highway by non-local motorists, uniformity of design and application is highly desirable.

Shapes. Regulatory signs are usually rectangular with the longer dimension vertical. The two notable exceptions are the stop sign, which is octagonal, and the one-way sign, which is rectangular. Warning signs are "diamond" shape except advance warnings for railroads, which are circular, and the railroad-crossing signs which are marked with a crossbuck.

Colors. Regulatory and guide signs usually have a white background with message or symbol in black. However, route markers are sometimes given distinctive colors for different routes or alternates. The stop sign has a "highway yellow" background with black message, sometimes in a highly reflectorized panel. Parking signs have red lettering for parking prohibitions and green lettering for time-limit regulations. Warning signs have a "highway yellow" background with black letters or symbols.

Dimensions. The width of stop signs measured perpendicular to any flat side should be 30 in. except that 24-in. widths may be used in urban areas where speeds and volume are low. Regulatory signs should be 18 by 24 in. for usual locations and 24 by 30 in. on high-speed and high-volume routes. Urban parking signs are ordinarily 12 by 18 in. Warning signs should be 24 by 24 in. except where lengthy words or messages require 30 by 30 in. for equivalent legibility. The dimensions of destination signs should be based on the highway speed. Good practice requires at least 6 in. of letter height. For daylight purposes and most letter standards, 1 in. of letter height will provide 35 to 50 ft of visibility. Oversized signs should be used when required by traffic speeds, volumes, hazards, and intersection complications. In the enlargement of signs, standard shapes and colors should be used and standard proportions should be retained.

Messages. Conventional symbols, such as curve arrows and intersection diagrams, provide means of simplifying sign design and interpretation. Standard wordings have been adopted for most of the important regulatory and informational signs requiring the use of word messages. Details of symbols and letter shapes are available from the U. S. Bureau of Public Roads.

Illumination and Reflectorization. Visibility of signs is extremely important under conditions of artificial illumination. This is especially true of signs carrying messages of warning and important regulations. Warning signs, stop signs, guide signs on important routes and at critical intersections, and all other regulatory signs, with the exception of those controlling pedestrians and parking, should be illuminated or reflectorized.

Mounting. All signs should be so positioned as to assure visibility. Each should display only one message, which should be definite and specific. Signs for different purposes should be placed at least 100 ft apart where possible. As a general rule signs should be located on the right-hand side of the roadway but under certain conditions they may be advantageously suspended over the roadway or placed on channelizing islands. In rural areas a sign on the roadway shoulder should normally be placed 6 to 10 ft from the right-hand edge of the pavement with its bottom edge at least 2 1/2 ft above the pavement. In urban areas or where there are raised curbs, a similar sign should normally be placed adjacent to the roadway with its nearest edge at least 1 ft from the curb line and its bottom edge approximately 7 ft above the grade of the roadway. An overhead sign should be placed centrally over the travel lanes to which it applies. The clearance from the bottom of the sign to the pavement should be at least 14.5 ft. Warning signs are usually placed in advance of the condition or hazard justifying them. Stop signs are erected as near as practicable to the point where the stop should be made.

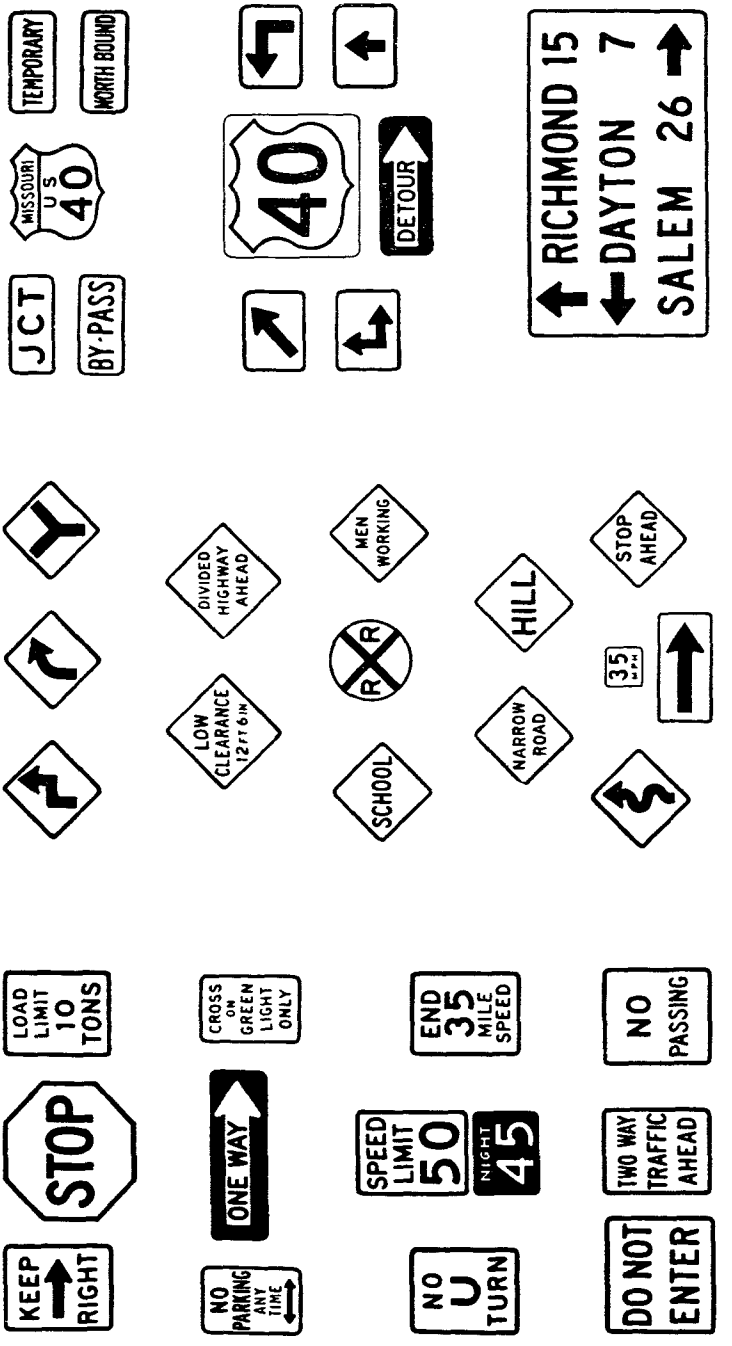


FIG 26 Typical standard highway signs and standard sign lettering From Manual on Uniform Traffic Control Devices, U. S. Bureau of Public Roads.

21. TRAFFIC SIGNALS

Stop-and-go traffic signals are used to give positive time control to vehicular traffic movements at intersections and other critical locations, to give positive time control to pedestrian traffic, and to assign lane-flow directions where those directions are interchanged to accommodate unbalanced flows. Flashing signals are used for warning and regulatory purposes at hazardous intersections which do not require positive time control. Flashing signals, wigwags, and automatic gates are used at railroad grade crossings.

Standards. Each signal face should have at least three lenses—red, yellow, and green. Arrow indicators and pedestrian signals are warranted under special conditions. The color and position of signal lenses in each signal face, reading from top to bottom or left to right, should be as follows:

Position	Indication
1	Red
2	Yellow
3	Green
4	Straight through arrow
5	Left-turn arrow
6	Right-turn arrow
7	Wait (or don't walk)
8	Walk

The standard shape of a traffic-control signal lens is circular, the lens having a visible diameter of at least 8 in. No lettering should be used on the lens. Each signal indication when illuminated should be clearly visible to the traffic controlled at all distances from 10 to 300 ft, except in dense fog. Each signal face should be shielded by visors and hoods so that an approaching driver can see only the signal indications intended for his observance. When mounted at the curb, the bottom of the signal housing should be not less than 8 ft nor more than 10 ft above the sidewalk or grade of the roadway. When mounted above the roadway, the bottom of the signal housing should be not less than 14 1/2 ft nor more than 15 1/2 ft above the roadway surface.

Fixed-Time Signals. Fixed-time signals operate in recurring cycles of equal length. Within each cycle the proportion of each interval to the cycle as a whole is constant and the seriation of intervals is fixed. Such signals are used in independent "isolated" or "coordinated" interrelated operation. Usually, little benefit to progressive movement is gained through time coordination of signal installations which are more than 2000 feet apart. On the other hand, signals within 1200 feet of one another should be coordinated. Time coordination of fixed-time signals requires constant cycle lengths (or low multiples) at all intersections in the "system." The maintenance of a fixed-time "lag" on "lead" from one intersection to the next may be achieved by "non-interconnected" or "interconnected" coordination. In the *non-interconnected coordination*, reliance is placed on the use of synchronous motors to drive each intersection controller. Time coordination between controllers is secured through phase relationships existing in the a-c system of electric-power distribution. In *interconnected coordination*, a master controller is employed and cable connections are made to each intersection. Through this interlocking supervisory circuit each intersection controller is kept "in step." Other circuits for remote shutdown, flashing operation, alternate "resets" or time relationship, etc., are frequently employed in interconnected systems.

Warrants. Good practice requires that the installation and operation of fixed-time and flashing signals be dictated by the warrants given in Table 20. Signal installation is also indicated when a new signal installation will reduce distance between signalized points to 1200 ft so as to provide progressive movement through "platoon" formation, or where five or more reported accidents occur per year of a type which will be eliminated by signals, provided that in addition there exist volumes of vehicles and pedestrians of not less than 50% of the requirements specified under the warrants listed. Signals may also be installed where two or more warrants are satisfied to 80% or more of the values given, and in addition (1) sudden changes from rural to urban conditions occur; (2) extremely wide roadways make pedestrian crossing hazardous and safety islands cannot be installed; or (3) other high hazards exist. For a period of 2 or more consecutive hours, when the vehicle volumes drop below 50% of those specified for minimum vehicular volume warrants, flashing operation may be substituted for fixed-time operation for the duration of those periods of reduced volume, but not more than three "flashing" periods should be used per day.

Table 20. Warrants for Fixed-Time Traffic Signal Installation *

Type of Warrant	Area	Number of Vehicles per Hour on All Approaches or Main Thoroughfare as Indicated	Cross Flow during Same Period	Conditions
Min volume	Urban	750 †	175 veh hr	. .
Min volume	Rural	500 †	125 veh hr	
Continuous flow	Urban	750 †	75 units hr ‡	Undue hazard or delay—20 mph avg speed
Continuous flow	Rural	500 †	50 units hr ‡	Undue hazard or delay—35 mph speed
Continuous flow	Urban	800 §	200 ped. hr or 300 units/hr	‖
Continuous flow	Rural	400 §	100 ped hr or 150 units/hr	‖
Continuous flow	Urban	800 §	30% left turns to or from major flow	‖
Continuous flow	Rural	400 §	30% left turns to or from major flow	‖
Continuous flow	Urban	...	...	‖ and 4 lanes on main thoroughfare with avg speed of 40 mph
Continuous flow	Urban	...	...	‖ with low sight distance or sharp vertical or horizontal curve
Pedestrian	Urban	600 †	250 ped. hr	15 mph avg
Pedestrian	Rural	300 †	125 ped hr	30 mph avg

* Approved by The American Standards Association.
† Average number of vehicles per hour for any 8 hr of an average day on all approaches.
‡ Pedestrians or vehicles
§ Vehicles per hour on main public way for any period of time.
‖ Entrance to and exit from industrial plants, stadia, parks, etc, where surges of traffic occur for relatively short periods.

Cycle Length and Division. As a general principle, cycle lengths should be short and the number of phases should be the minimum consistent with safety and facility. Two-phase operation can ordinarily be achieved in a 35- to 50-sec total cycle length, whereas three-phase operation requires 55 to 70 sec. No phase should be less than 12 sec, including clearance interval. The allotment of time to each phase should be based on the peak demand unless program devices are used to readjust automatically the division of the cycle.

Where pedestrians are to be protected by traffic signals, each phase should provide sufficient time for pedestrians to cross the roadway at speeds of 3.5 to 4 ft per sec. A minimum of 3 to 5 sec should be allowed to get the pedestrians started, and clearance intervals should be of such length that the last vehicle to enter the intersectional area will (1) be able to stop short of the intersection at comfortable, safe rates of deceleration, or (2) be cleared of conflict with released cross flow. In urban practice, minimum clearance intervals of about 2 sec are used. For high-speed rural intersections, 5 to 7 sec may be required. Excessively long clearance intervals lead to accidents.

Signal Systems. Series of signal installations operating in a fixed-time cycle of coordination become a signal system. There are four types of system (1) simultaneous, (2) alternate, (3) simple progressive, and (4) flexible progressive. In the *simultaneous system* the division of cycle is constant throughout the system, and all green intervals for the main arterial street begin at the same instant at all intersections and are of equal duration In the *alternate* system, the cycle is usually divided into two equal phases of equal duration. The main arterial flow is given the green indication at every alternate intersection at the same instant. In the *progressive system*, the green interval for the main street at each intersection is set for the optimum lag or lead from the adjacent or the base intersection of time reference The *flexible progressive system* provides, by remote or automatic control, diverse patterns of coordination, cycle-length change, and other features.

System Coordination. Time, speed, and distance are the three fundamental factors to be dealt with in signal coordination. A commonly accepted method of portraying these relationships is shown in Fig 27

Either the width of band (capacity) or the speed of progression or both may be made to favor a particular direction of flow When this is done, however, it is always at the

expense of traffic movement in the counterdirection. The progressive systems of coordination thus may provide for two or three patterns of coordination (Fig. 28).

In signal timing and coordination work, it is essential to base timing plans upon sound factual measurements of traffic and physical data. In preparing time-space diagrams, consideration should be given to diverse features of operation which are offered in modern signal controllers, including scheduled changes in cycle length and division, special adjustments, and intervals to accommodate heavy turning movements, or pedestrians, and the use of master and program timers for supervisory control including flashing operation,

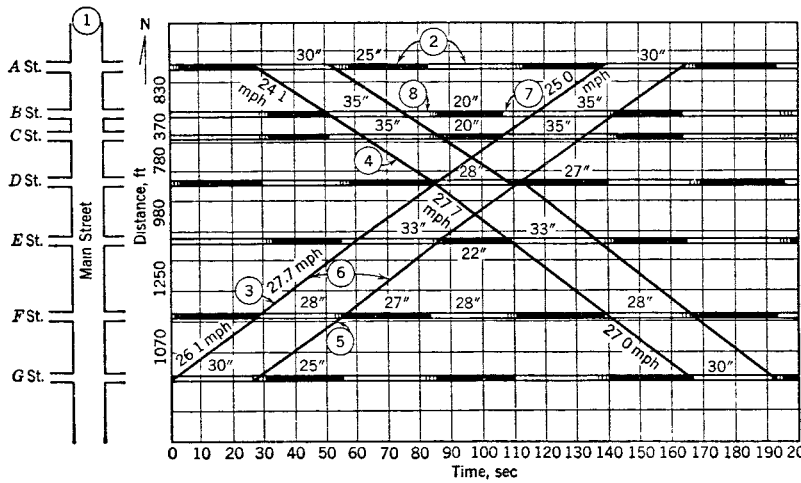


FIG. 27. Typical time-space diagram for traffic signals From Traffic Engineering Handbook, Institute of Traffic Engineers, 1950.

1 Plan of street drawn to scale
2 "Go" intervals for main street designated by open space "Stop" intervals for main street designated by solid line "Yellow" intervals for main street designated by crosshatching.
3 Slope of this line indicates the speed and represents the first vehicle of a group or platoon moving progressively through the system from *G* Street to *A* Street
4 As in 3 above, this line represents the first vehicle moving from *A* Street to *G* Street in the southbound direction.
5. This line is roughly parallel to line marked 3, and represents the last northbound vehicle in a group to go through all intersections
6 The space between lines 3 and 5 on the time scale is the width of the "through band" in seconds This width may vary over different sections of the street
7 "Leading green" where green is given to main street ahead of the "through band."
8 "Lagging green" on main street after the "through band"

Note Speed signs for the illustrated system would be "25 mph" throughout.

shutdown, and other refinements. In general, it is advisable to provide adequate flexibility in controllers in order to permit and accommodate future refinements in time-cycle sequence and coordination.

Traffic-Actuated Signals. With traffic-actuated signals the diverse intervals are initiated by and are dependent on the demands of traffic flow, registered by detectors or push buttons. Because traffic-actuated signals conform more closely to the instantaneous demands of traffic movement, they do not normally delay traffic as much as fixed-time signals, especially at isolated installations. Because of this characteristic, they do not require fixed warrants for their proper use. Their utility is especially directed to complicated multiphase intersections and to those locations where wide fluctuations in traffic flow occur between the diverse phases of movement during different periods of the day. Traffic-actuated signals are especially useful also for the occasional interruption of continuous traffic by very light or highly fluctuating movements of pedestrians or vehicles which must cross major thoroughfares. They are applied sometimes at non-intersection points to control speed, such as at bridges or sharp curves. They are also used to alternate the direction of movement in one-way restricted zones at narrow bridges, tunnels, etc.

Types of Actuated Control. Traffic-actuated signals may be classified into two general categories: semiactuated and full actuated. *Semiactuated signals* are controlled by vehicle detectors in the lanes of cross traffic, or by pedestrian push buttons. This type of control

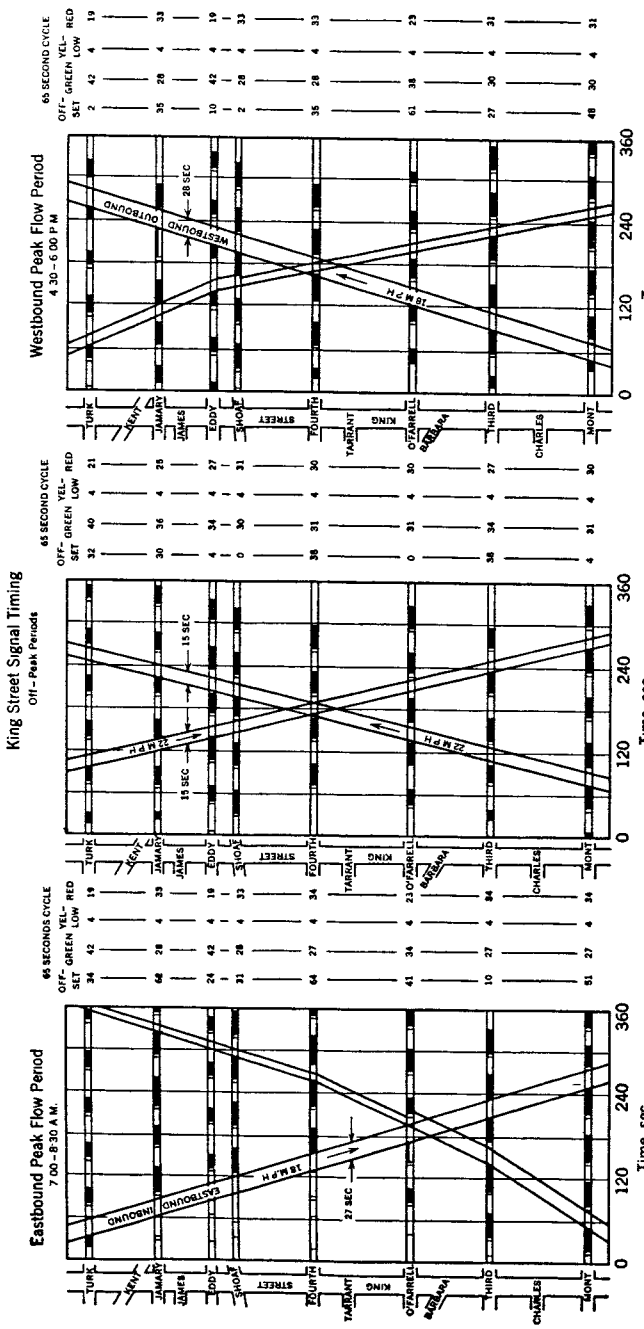


FIG. 28. Three time-space diagrams for a hypothetical street from Traffic Engineer's Handbook, Institute of Traffic Engineers, 1950

is especially suited to those locations where heavy streams require interruption at irregular intervals to accommodate cross traffic.

Full-actuated signals are sensitive to the demands of traffic streams on every street entering an intersection. Thus two-, three-, and four-phase intersection operations can be achieved by this type of control. In addition some full-actuated controllers will distribute the time available at an intersection in accordance with the "volume-density" demands of traffic flow providing thereby an optimum degree of coordination. Full-actuated signals are sometimes used at irregular "problem" intersestions within a coordinated system.

Pressure-sensitive, magnetic, sound-sensitive, light-sensitive, and pedestrian push-button detectors are used for actuation. Special devices are used on trolley wires and on tracks for detection of streetcars and trains.

PLANNING AND FUNCTIONAL DESIGN OF HIGHWAYS

22. TRAFFIC ENGINEERING, HIGHWAY ENGINEERING, AND CITY PLANNING

The planning of traffic facilities requires the application of fundamentals of traffic engineering, highway engineering, and city planning. An ideal highway alignment from a traffic service point of view may present design or construction difficulties or may create planning problems. On the other hand, the ideal location from a planning or construction point of view may be a very poor location insofar as the manifest purpose of the highway, the service of traffic, is concerned. Some of the factors in the planning of traffic facilities are discussed in this section. Related subjects are discussed in Section 4, Highway Engineering, and in Section 1, Metropolitan and Community Planning.

23. THE SELECTION OF HIGHWAY LOCATIONS

Highway networks should conform wherever possible with the land-use pattern of the area served. In urban districts it is desirable to locate main highways on the transitions between different land uses, such as the boundaries of residential, business, industrial and recreation areas. In laying out new routes, care should be taken to minimize interference with closely knit residential and business centers, school districts and other distinct communal and social neighborhoods. Highways should also integrate with other civic improvements, community centers, and prominent groups of buildings. When possible, the highways should be esthetically pleasing so as to be a source of pride in civic accomplishment.

Basic tools for planning highway routes are traffic desire-line maps, traffic flow maps, physical data regarding existing streets and their capacities, characteristics of peak traffic flows, land-use maps, topographic maps, and aerial photographs. With the aid of these tools, the planner selects the most likely alternative alignments which would serve effectively the traffic shown by the desire-line bands, dismissing routes which can be readily seen to be unfeasible. While keeping in mind the reason for the construction of the highway, which is to carry traffic efficiently and safely, the planner endeavors to select alignments which will make maximum use of low-value property, and avoid physical barriers and untouchable properties, such as large churches, schools, and cemeteries. Important objections to a location arise also from encroachments on public parks or historic areas, noise, loss of taxable properties, elimination of housing, and removal of shade trees. In this phase of the work, it is necessary to guard against extreme deviations for sentiment, politics, or other influences or for the accomplishment of other objectives such as slum clearance, park development, and land reclamation, when those objectives can be undertaken more economically as separate projects. It is sometimes unfortunate that an influence may be exaggerated far beyond its true significance, and the planner may be hard pressed to counter the demands of a voceriferous or influential minority.

24. THE DISTRIBUTION OF TRAFFIC ON ALTERNATIVE ROUTES

Factors which may influence choice of routes are travel time, travel distance, length of trip, ability to keep moving, safety, convenience, economy, and habit. Motorists do not hesitate to change routes in order to avoid one which has become congested or otherwise unattractive. They will go considerable distances out of their way in order to reach attractive, free-flowing arterials of modern design.

Recent investigations in regard to the distribution of passenger vehicles on alternative routes lead to the following generalizations.

1. Travel time is the most important single factor in a motorist's choice of routes.
2. The actual time that is saved or lost on a given route provides the best basis for estimates of distribution on alternative routes. A good basis is also provided by travel-time ratios.
3. A superior highway will attract some traffic which saves neither time nor distance in using that highway.
4. Some traffic will use inferior routes even at the expense of time and distance.

Typical traffic diversion curves for passenger vehicles are shown in Fig. 29 for freeways and boulevards. The limited information now available with regard to diversion of commercial vehicles indicates that travel distance is more important for commercial vehicles than for passenger vehicles and may be the controlling factor in the distribution of commercial vehicles on alternative routes.

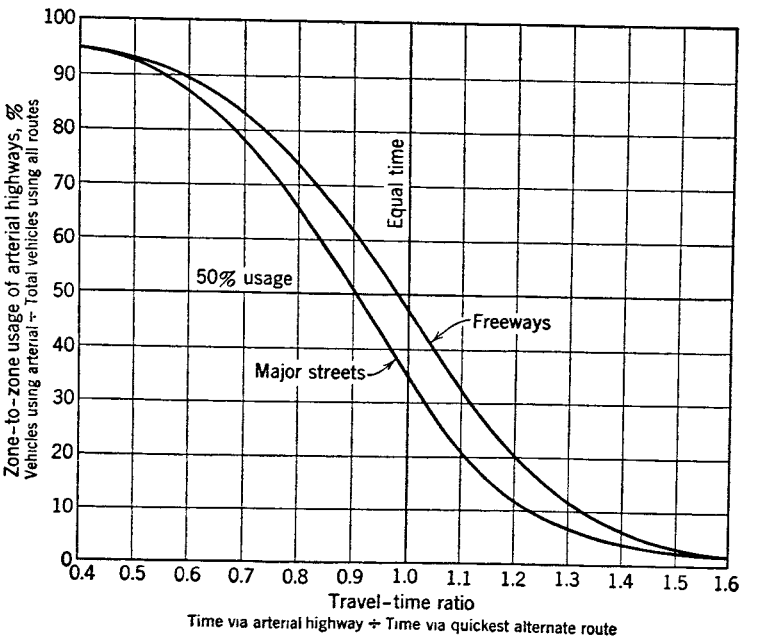


FIG. 29. Traffic diversion curves for urban arterial highways. Prepared by the Subcommittee on Factual Surveys of the AASHO Committee on Administration, and published in the AASHO Convention Group Meetings, December 1952, but not, in January 1955, officially a part of AASHO policy.

25. CONTROL OF ACCESS

The rights of light, air, and access which abutting owners enjoy in regard to public highways under accepted interpretations of common law are essential to local streets and highways but can lead to serious impairment of capacity, safety, beauty, and convenience in the case of facilities designed for through traffic. Uncontrolled access can develop hazards and reduce road capacities to such points that highways of the most modern design become obsolete in terms of ability to accommodate traffic long before they are worn out physically. Where traffic volumes are heavy, especially in the vicinity of urban areas, strip developments of businesses and residences grow rapidly. Numerous intersections and driveways result. Traffic congestion follows, accidents are increased in number, scenic values are destroyed, and blight and decadence of property values are fostered.

To insure that a through highway facility will retain its efficiency, the number of connections to it should be limited to the minimum number and the appropriate type suited

to the highway. This can be accomplished by physical means, such as the provision of parallel frontage roads, or the right of access may be acquired from abutting owners when legal authority exists. For legislative purposes a *limited-access* facility is defined in a model law recommended by the U. S. Bureau of Public Roads as a highway or street especially designed for through traffic, and over, from, or to which owners or occupants of abutting land or other persons have no right or easement or only a limited right or easement of access, light, air, or view by reason of the fact that their property abuts upon such limited-access facility or for any other reason. Such highways or streets may be parkways, from which trucks, buses, and other commercial vehicles shall be excluded; or they may be freeways open to use by all customary forms of street and highway traffic.

About two-thirds of the states now have legal authority to control access. Some states are now building limited-access highways on new locations for anticipated present-day traffic volumes of 2000 vehicles per day. Property rights for limited-access facilities are usually acquired in fee simple.

26. PROPERTY VALUES

There is no clear-cut answer to the question of the effect which a major highway improvement will have on property values. Factors which appear to have important bearing in a particular instance are the type of highway improvement, the nature of the land use, the degree of congestion prior to the improvement, and the increased accessibility afforded by the improvement.

Studies have shown that the construction of freeways has resulted in increased property values in affected areas:

A study made in 1933 of property values along the route of the Bronx River Parkway in Westchester County, N. Y., indicated that, during the period between 1910 and 1932, property values in affected areas along the parkway appreciated almost 800% more than property values in adjacent unaffected areas. This parkway is a slow-speed highway with park-like treatment of the right-of-way. The adjacent areas are mostly residential

A survey by the Texas Highway Department showed that, within 3 years of the opening of the first section of the Gulf Freeway in Houston, the value of property within the zone of influence of that section increased about 65% more than the value of other properties in the city.

When bypasses of congested areas are constructed with unrestricted access from abutting property, there is frequently a tendency for the value of the business property in the bypassed area to decline, since many businesses move to new locations along the bypass route. The net result, of course, defeats the purpose of the bypass inasmuch as the bypass itself becomes congested with the local traffic which was to be bypassed, the investment in the highway improvement is wasted, and the bypassed property suffers a loss. On the other hand, there is ample evidence that the construction of controlled access bypasses of congested business areas promotes business and increases real-estate values in those areas by providing greater safety and convenience for local customers

The California Division of Highways investigated the retail volumes of bypassed businesses and of businesses generally in the vicinity at several bypass locations before and after the construction of the bypasses. The findings of volume changes of retail business for six small cities investigated in 1950–1951 are given in Table 21

Table 21. Effects of Bypasses on Business Volumes

	Population	Number of Businesses Bypassed	Net Effects on Business by Type, %		
			Cafes, Bars	Service Stations	Other
North Sacramento	6016	224	+11.5	+25.5	+21.5
Auburn	4577	74	+5.0	+21.0	0.0
Fairfield	3607	70	−24.4	−23.0	+14.1
Folsom	1706	36	+7.4	+5.0	+1.0
Imperial	1500	21	+1.5	+2.7	+0.5
Escondido	6500	67	+11.9	+25.6	+12.7

The California Division of Highways also found that for single and multiple dwellings no diminution in value occurred by the development of cul-de-sac streets created by expressway construction and, in many cases, increased values resulted. This finding was verified by sales of properties situated on cul-de-sac streets, and also by the determinations of juries in condemnation cases.

27. ROAD CLASSIFICATION FOR DESIGN

According to English common law, a public way is a strip of land, open to the public for purposes of travel as a matter of right and over which abutting property owners have the right of light, air, and access. Highways are now classified according to administrative needs, according to traffic volumes, and according to the extent to which access is controlled through regulation and design. Classifications in the latter category include the following:

A *through facility*, or *throughway* is a preferential street or highway for through traffic which may be entered or crossed by a vehicle only after that vehicle executes a full stop, except where directed to proceed without stopping by an officer or traffic signal.

An *expressway* is a divided, multilane roadway for through traffic with partial control of access, having important crossroads separated in grade, but with some grade crossings, consistent with the traffic and development conditions involved.

A *freeway* is a divided, multilane roadway for through traffic with all crossroads separated in grade and with full control of access.

In the absence of limited-access legislation, strips of land many miles long have been procured and designated, through the powers of state agencies, to be *parkways* dedicated as parks, open to the public, to which abutting owners do not have rights of light, air, or access. Thus, a road which is included as an incidental part of the park area may be an expressway or a freeway except that its use by commercial vehicles is generally prohibited.

28. ROAD SURFACE

Friction. The friction provided by a road surface sustains the braking force of vehicles and resists centrifugal force on curves. For practical purposes, the critical frictional requirement is the ability to resist straight forward-sliding skids. Most hard (but not bumpy) road surfaces offer about equal resistances to skidding when they are dry but some become dangerously slippery when wet. A significant reduction in friction results from increased speed. Ice and sleet, of course, have low frictional coefficients, especially

Table 22. Approximate Coefficients of Friction Determined by Sliding Tests

Speed	10 mph				20 mph		25 mph	40 mph			
Condition of tires	New *		Smooth		New *		New *	New *		Smooth	
Condition of pavement	Dry	Wet	Dry	Wet	Dry	Wet	Wet	Dry	Wet	Dry	Wet
Type of Surface											
Concrete (Va.)					0.625	0.505					
Concrete (Mo.) (broomed)	0.810	0.705	0.830	0.700				0.625	0.535	0.785	0.525
Concrete (Kan.) (smooth belted)	0.710	0.575	0.820	0.600				0.595	0.460	0.725	0.335
Asphalt (D. C.)					0.645	0.505					
Macadam (D C.)					0.600	0.398					
Bituminous (Kan.) (sandy, non-bleeding)	0.915	0 525	0.975	0.430				0.640	0.365	0.645	0.300
Bituminous (Kan) (glazed)	0.845	0.400	0.900	0 400				0 600	0.260	0.645	0.115
Bituminous (Kan) (coarse chip)	0.760	0.630	0.790	0.520				0.520	0.360	0.570	0.510
Bituminous (Wyo) (limestone chip, bleeding)	0.700	0.500	0.670	0.435				0.520	0.290	0.460	0.340
Gravel					0.580			0.550			
Packed snow							0.220				
Ice	0.110	0.070			0.110	0 070					
Mud on concrete (D. C.)						0.548					
Oil on concrete (D. C.)						0.228					

* Tires with tread pattern or a "non-skid" tread are termed "new."

Source: Data assembled by Highway Research Committee, Society of Auotmotive Engineers, 1947.

if the surfaces are wet, clean, and smooth. Friction on snow depends on the compaction of the snow. Mud or wet leaves may be as slippery as ice. Typical coefficients are given in Table 22.

Pavement Smoothness. Traffic may be induced to follow desirable paths by varying the smoothness of the road surface. Roughening to warn traffic at critical locations also has considerable merit. Of course, all roughness designed in the traveled portion of the roadway should be only severe enough to cause vibration and rumbling sound sufficiently strong to gain the driver's attention. However, even small irregularities in the pavement surface become filled with dirt, which renders them ineffective and they are a problem for snow-removal equipment.

Light-Reflecting Characteristics of Pavements. Glare caused by the reflection of oncoming headlights is negligible on a dry pavement, but it is an important factor in safe seeing when the pavement is wet. A film of water on a smooth impervious pavement surface creates a mirror effect, called *specular reflection*. The same characteristics which reduce surface friction also increase the specular reflective power of the pavement. The amount of reflection is determined by the surface texture and the drainage. Light-covered surfaces of rough, well-drained texture do not become specularly reflective but aid silhouette seeing and provide more pavement luminosity under street-light illumination. Eyestrain may be caused by the brilliant reflection of sunlight on white roadway surfaces when the sun is near the horizon. Lampblack is sometimes used to reduce the brightness of cement concrete, but when that is done the color of the pavement should retain much of its light appearance for good night visibility.

Contrasting pavement colors may be utilized to indicate preferential use of traffic lanes and to emphasize acceleration and deceleration lanes, turning lanes, and shoulders.

29. LANE WIDTHS

Although there are no absolute standards, lane widths of 12 ft are considered ideal for all high-speed roads and for low-speed roads with a high percentage of trucks. On low-speed roads in urban areas where the cost of right-of-way and other difficulties are major considerations, an 11-ft lane width may be satisfactory. Lanes should not be so wide as to offer so much freedom of movement that drivers tend to make improper maneuvers. For instance, a wide two-lane road may tempt some drivers to use it as a three-lane road. On the other hand, lanes 12 $^1/_2$ ft to 13 ft in width outside of the zones of influence of curbs, guard rail, etc., have been used on some high-speed, multilane facilities and have seemed to afford ease of operation. Lanes wider than those are sometimes located next to the median to facilitate passing or next to the shoulder to accommodate slower-moving trucks. In neither case, however, has their value been factually demonstrated.

Narrow lanes restrict the capacity of a highway by limiting the freedom of movement of traffic. On two-lane roads, there is a definite correlation between lane widths and capacity since on those roads capacity is primarily a function of the frequency of passing opportunities and the fluidity of the passing maneuvers. On multilane roads with narrow lanes, capacity is reduced by the tendency of drivers to straddle the lanes.

30. NUMBER OF LANES

The number of lanes required by a highway is a function of the design volume and capacity of each lane. The capacity of any lane is related to the opportunities afforded for faster vehicles to overtake and pass the slower ones. By providing a common center lane for passing, the capacity of a road is increased, but it is believed that a high accident potential results although this has not been clearly demonstrated. At any rate, the three-lane road does not lend itself well to unbalanced flow; it cannot be efficiently expanded to a four-lane divided roadway when a raised divider is to be employed; and, if it is to accommodate traffic volumes substantially greater than would be accommodated by a good two-lane road, passing sight distance must be almost continuous over the length of the road.

On urban arterials, traffic peaks are sometimes extremely directional, as evidenced by the inbound traffic rush to the business district in the morning and the outbound rush in the afternoon. To accommodate this unbalanced flow, multilane roads may be arranged to provide for reversing the flow on certain lanes, as required. When this is done traffic signals and neon or other signs should be used to indicate to motorists the direction of flow on the reversible lanes. Lake Shore Drive in Chicago is a particularly interesting example of reversible flow on certain lanes with the physical aid of movable dividers. Dividers

placed between pairs of lanes on the eight-lane facility can be raised and lowered by means of hydraulic jacks. Although the theoretical advantages of this system are quite tempting to the traffic designer, mechanical difficulties and costs discourage its use.

31. TRAFFIC SEPARATORS

Traffic separators are used to prevent collisions between vehicles traveling in opposite directions, to prevent U turns and other dangerous maneuvers, and to channelize traffic into efficient streams. The methods used may be markings on the road, physical dividers, or area dividers. The application of traffic markings is discussed in Art. 19. It should be noted, however, that neither painted nor permanent lines are recommended for centerline marking on multilane highways of four or more lanes. Opposing traffic on such highways should be separated by physical or area dividers.

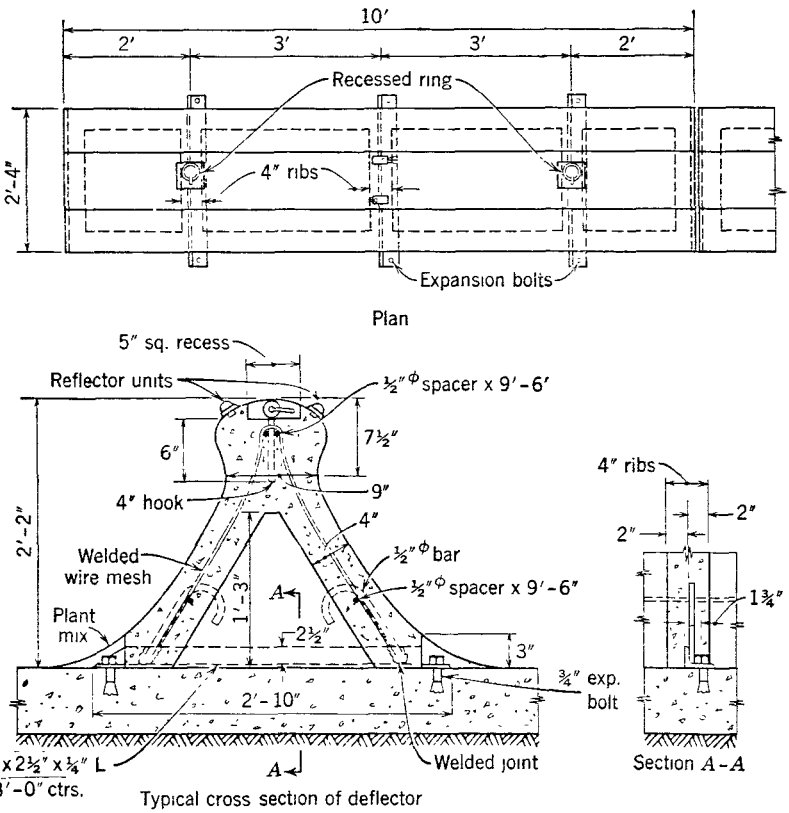


FIG. 30. Precast concrete deflector designed by the California Highway Commission. *Engg. News Record*, July 10, 1947.

Mechanical Dividers. Mechanical dividers prevent head-on collisions with a minimum use of road width, being particularly applicable for installation on existing four-lane undivided roads where wider dividers would be very costly. They should be designed to return an encroaching vehicle to the roadway without allowing any part of the vehicle's body to scrape and without causing overturning (Fig. 30). Disadvantages arise from their appearance; their narrowness, which does not provide an adequate haven for crossing and turning vehicles; and their influence upon vehicle placement, which results in a less effective lane width.

Area Separators. Area separators provide a neutral zone between opposing streams of traffic in which vehicles out of control may be safely brought to a halt. They are satisfactory with regard to ordinary driver comfort and pleasing to the eye. Planting may be utilized to reduce headlight glare. At intersections, wide separators provide a zone of safety for pedestrians, facilitate channelization, and provide space for the "shadowing" of crossing, and left-turn traffic. Area separators should be at least 25 ft wide to "shadow" crossing passenger cars and at least 45 ft wide to "shadow" crossing trucks. Less width is required for vehicles turning left from the divided road but considerably more width is required for U turns moving from inside to inside lanes.

32. GUARD RAILS

Experiments have been made with a guard rail of the parabolic deflector type, similar to one-half a mechanical divider. If properly designed, it may be unsurmountable and yet be able to restrain vehicles without deflecting them back into traffic or causing excessive damage.

33. CURBS

Curbs may be classified as mountable, low-speed barrier, and high-speed barrier (Fig. 31). The *mountable curb* encourages traffic to remain on the through traffic lanes and yet allows drivers to enter the shoulder area with little difficulty in case of emergency. It is also useful in the control of drainage. It may be reflectorized, constructed of white concrete, or ribbed to delineate the edge of the pavement at night. Mountable curbs have only a very slight effect on the transverse placement of vehicles, corresponding to a reduction in effective pavement width of 1 ft during the daytime and no reduction at night.

The low-speed barrier curb is used wherever it is desired to prevent sidewalk encroachment or to provide a parking barrier. Like other curbs, it may be designed to delineate the edge of the road, and although it does control drainage, it should not be used when this is its only function. It has an effect on transverse placement much greater than the mountable curb because of its greater zone of influence. It is therefore recommended that widths of lanes adjacent to low-speed barrier curbs be increased 1 to 2 ft. Curbs of design intermediate to the mountable and the low-speed barrier types should not be used because their purpose is not clear and drivers are reluctant to cross them for emergency parking.

The high-speed barrier curb is intended to prevent vehicles from being driven off the roadway under any circumstance. They are used at unusually dangerous locations where the damage inflicted by holding the vehicle on the road by physical means would be less than that suffered if the vehicle were to leave the road. The best design for a barrier of this type would be a parabolic deflector similar to a half-section of a mechanical traffic separator. The effects of high-speed barrier curbs on placement are similar to those caused by other obstructions along the road.

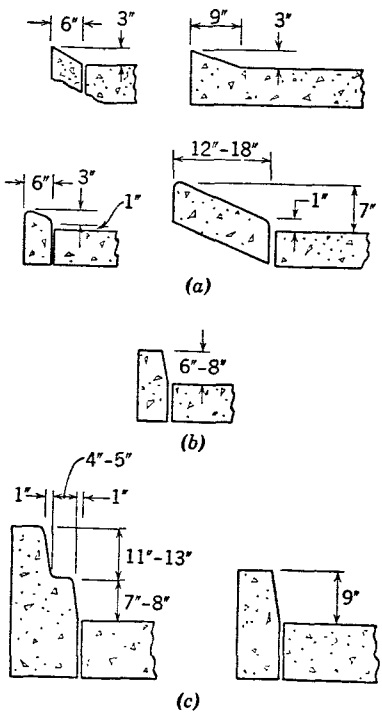


FIG 31. Examples of highway curb types. (*a*) Mountable; (*b*) low-speed barrier; (*c*) high-speed barrier

34. ROAD MARGINS

Shoulders. Shoulders are provided along the edges of roads to serve as emergency lanes for vehicles that are forced or driven off the roadways and to act as service lanes for vehicles that are disabled. Shoulders should be at least 13 ft wide on primary roads serving truck traffic and 10 ft wide on roads serving only passenger vehicles. When a continuous shoulder is not possible, turnouts should be provided frequently to enable vehicles to stop without interfering with the traffic flow.

The riding surface of shoulders should not be as smooth as that of through traffic lanes, since, if they are used as travel lanes, their function as emergency lanes is destroyed and maintenance problems are created. However, the appearance of a shoulder should reflect its ability to support a loaded truck, even in wet weather.

The color of the shoulder should contrast with that of the road surface for better day and night delineation of the edge of the road and to indicate to the driver that the shoulder is not a through lane.

Sidewalks. Where sidewalks are constructed, curbs should be used to prevent vehicle encroachment. Along high-speed roads in rural areas, appropriate barriers should be constructed between the outside edge of the shoulder and the sidewalk, unless the distance separation is sufficiently great for pedestrian safety. From a functional point of view, sidewalks should be designed with a surface smoothness equal to or better than that of the roadway. If the surface is rough or does not drain properly, pedestrians will use the roadway instead.

Roadside Landscaping. Appropriate plantings on islands and separators may be used to block headlight glare and to mark crossovers, curves, and intersections. It is possible, but not demonstrated, that a thick vigorous growth of shrubbery and vines matted together may cushion the impact of vehicles with greater safety and less damage than guard rails. On the other hand, roadside plantings may cause hazards if they are not properly located and maintained. Shrubbery and trees on the inside of curves and at intersections may block vision if they are allowed to grow tall and are not trimmed. Plantings too close to the traveled way may affect vehicle placement, and trees may become lethal obstructions however small they may have been at the time of planting. For these reasons plantings should be limited to shrubbery and held to a height of 3 to 3 $^1/_2$ ft above the surface of the pavement.

Frontage Roads. Frontage roads are local roads used to furnish access to properties along an expressway or freeway facility and to serve traffic desiring to cross or enter the facility. They may be designed for one-way or two-way operation. The standards of cross-section design for frontage roads are lower than those for the through roadway because low speeds and congestion caused by parking and pedestrians are usually to be expected.

Driveways. Driveway locations near intersections should be in keeping with the speed of through traffic, the type of intersection control, and the sight-distance requirements. Between intersections, driveways should not be located where sight distances are restricted or where vehicles parked in the driveways will restrict sight distance. Where more than one driveway is needed to serve a single establishment, such as a filling station, they should be separated by as much distance as possible. The separating island should be raised to prevent vehicle encroachments and should extend into the property far enough to permit adequate sight distance for pedestrians on the sidewalk and for motorists leaving the property. The width of driveway should be held to a practical minimum to reduce the length of crosswalks, but curb radii should be long enough to allow vehicles to turn into the property from the outside lane of the roadway and to return to the roadway without encroaching on other lanes.

35. SIGHT DISTANCES

The sight distance at any point on a road is the distance along the road that drivers can see. Restricted sight distances may stem from a variety of causes, such as parking, billboards, shrubbery, vertical alignment, or horizontal alignment. The sight distance required depends upon a number of driver and vehicle factors, including: (1) the PIEV[1] time of drivers, (2) the time required to avoid a hazard or hazardous situation, and (3) speed.

[1] Perception, intellection, emotion, and volition.

Safe Stopping Sight Distances. In determining braking distances, it is assumed that road-surface friction, and not the friction in the brake mechanism, is the most critical. The total stopping distance, composed of the distance traveled during the driver's reaction time and the distance traveled while the brakes are applied, may be computed approximately as follows:

$$S = \frac{V^2}{30(f \pm p)} + 1.47VT$$

wherein S = stopping distance in feet.
V = initial speed in miles per hour.
f = the average coefficient of friction between tires and pavement.
p = gradient expressed as a decimal.
T = PIEV [1] time in seconds.

The AASHO recommends a minimum value for PIEV time of 2.5 sec. The AASHO also recommends a maximum value of 0.36 for f when V is 30 mph, ranging to 0.29 when V is

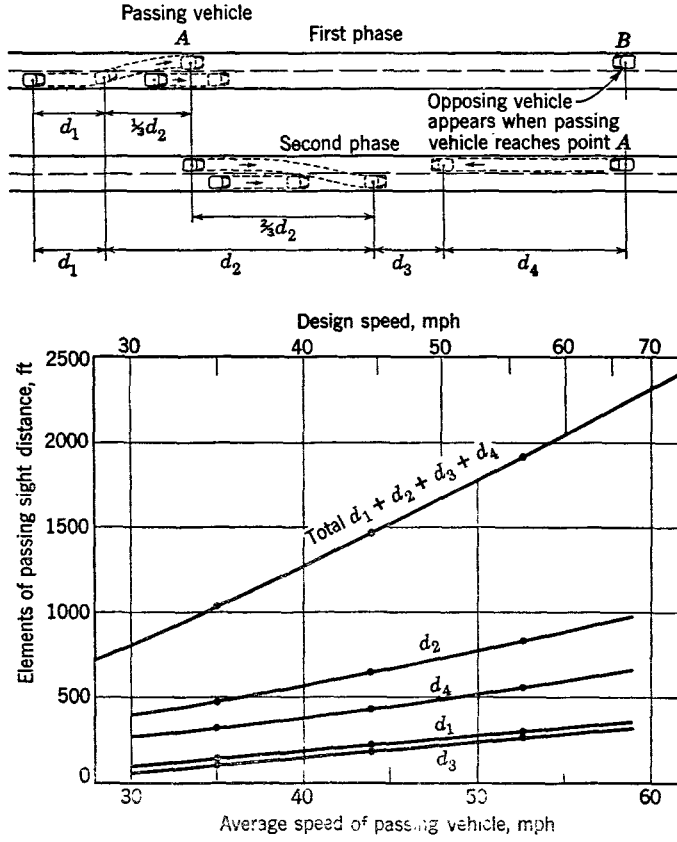


FIG. 32. Elements of and total passing sight distance, two-lane highways. From A Policy on Geometric Design of Rural Highways, AASHO, 1954.

70 mph. These values were assumed for conditions of straight-forward skid on wet, clean pavements. They represent deceleration rates which are severely uncomfortable for the occupants of vehicles, dangerous for trucks, and damaging to truck braking mechanisms. Stopping distances computed on these assumptions are shown in Table 23. Many engineers believe that safe stopping sight distances should be 20 to 30% higher than indicated

on the basis of the AASHO assumptions, and even higher than indicated by the Nationa Interregional Highway Committee.

Table 23. Recommended Minimum Safe Stopping Sight Distances (Daytime)

Design Speed, mph	Level Road Distances, ft AASHO *	NIHC †
30	200	...
40	275	...
50	350	...
60	475	525
70	600	700

* A Policy on Geometric Design of Rural Highways, AASHO, 1954.
† National Interregional Highway Committee, Interregional Highways, House Document 379, Government Printing Office, 1944.

Gradient has a significant effect on safe stopping distances. For example, the braking distance on a 6% upgrade from 60 mph is 160 ft as compared with 213 ft on a 6% downgrade from the same speed.

Safe Passing Sight Distances. Safe passing sight distance applies only to two- and three-lane roads where common lanes are used by traffic moving in opposing directions. Although it is standard practice to provide safe stopping sight distances throughout the length of all improved roads, this same policy for safe passing sight distances, which are from three to five times longer, would present a severe economic problem.

The most critical passing situation occurs when the passing vehicle must be slowed to the speed of the passed vehicle before starting the maneuver and is met by an oncoming vehicle at the finish of the maneuver. The total theoretical safe passing sight distance is shown for various speeds in Fig. 32. The various components of the passing sight distance are:

d_1 = distance traversed during the preliminary delay period (the distance traveled during perception and reaction time and during the initial acceleration to the point of encroachment on the left lane).
d_2 = distance traveled while the passing vehicle occupies the left lane.
d_3 = distance between the passing vehicle, at the end of its maneuver, and the opposing vehicle.
d_4 = distance traversed by an opposing vehicle for two-thirds of the time the passing vehicle occupies the left lane, or $^2/_3$ of d_2 above.

Typical values for the elements of the maneuvers are shown in Table 24.

Table 24

Speed group, mph	30–40	40–50	50–60
Average passing speed, mph	34.9	43.8	52.6
Preliminary delay			
a = average acceleration rate, mphps *	1.40	1.43	1.47
t_1 = time, seconds *	3.6	4.0	4.3
d_1 = distance traveled, ft	145	215	290
Occupation of left lane			
t_2 = time, seconds *	9.3	10.0	10.7
d_2 = distance traveled, ft	475	640	825
Clearance length			
d_3 = distance traveled, ft *	100	180	250
Opposing vehicle			
d_4 = distance traveled, ft	315	425	550
Total distance, $d_1 + d_2 + d_3 + d_4$, ft	1035	1460	1915

* For consistent speed relation, observed values adjusted slightly.
Source: A Policy on Geometric Design of Rural Highways, AASHO, 1954.

Minimum passing sight distances for design are illustrated in Table 25.

Table 25. Minimum Passing Sight Distance for Design of Two-Lane Highways

Design speed, mph	30	40	50	60	70
Assumed passing speed, mph	30	40	48	55	60
Minimum passing sight distance, ft					
Figure 32	810	1265	1675	2040	2310
Rounded	800	1300	1700	2000	2300

Source: A Policy on Geometric Design of Rural Highways, AASHO, 1954.

Intersection Sight Distances. Sight-distance requirements at intersections may be reduced by traffic signals, stop signs, and advance warning devices but, for the purpose of design, it is generally assumed that these controls will not be used. Drivers on all approaches of intersecting roadways should be provided a clear view across the corners created by adjacent approaches for a sufficient distance from the point of intersection to avoid collisions. This area of unobstructed sight is called "*the sight triangle.*" As a practical matter, both drivers seldom stop, but rather the one with the time advantage proceeds through the intersection. Table 26 shows minimum suggested approach distance-speed relationships for roadways of various assumed design speeds when:

1. The potential time of arrival is the same.
2. The vehicle on the preferential road continues at the assumed design speed of that road.
3. The vehicle on the minor road stops from the assumed design speed of that road within the minimum comfortable stopping distance.

Table 26. Suggested Minimum Dimensions of Sight Triangle at Intersections of Minor and Preferential Roads

Minor Road			Preferential Road	
Design Speed, mph	Comfortable Stopping Sight Distance (Dimension of Sight Triangle along Minor Road), ft	Potential Time to Intersection, sec	Design Speed, mph	Dimension of Sight Triangle along Preferential Road, ft
30	195	4.4	30	194
			40	258
			50	323
			60	388
			70	454
40	308	5.2	30	230
			40	305
			50	382
			60	458
			70	535
50	445	6.1	30	268
			40	358
			50	449
			60	538
			70	629
60	606	6.9	30	305
			40	405
			50	509
			60	610
			70	711
70	790	7.7	30	340
			40	453
			50	565
			60	679
			70	792

The critical situation at intersections exists when a vehicle stopped on the crossroad must accelerate across the preferential road. Minimum sight distance for this situation involves the PIEV time of the crossing driver, acceleration time, length of crossing, length of crossing vehicle, and the speed of the vehicle approaching on the preferential road. Acceleration practices of unknown drivers measured at three intersections is shown in Table 27. The normal acceleration characteristics of buses resemble very closely the values for passenger cars.

When two one-way traffic streams unite to form a single one-way stream (Fig. 33), drivers must be able to see gaps in the stream being entered in sufficient time to accept or reject them. When a gap is rejected, there must be sufficient time, also, for the merging vehicle to stop before arriving at the collision point. Empirical studies have indicated that the average driver will merge with the other stream when he arrives at the point of stream conflict approximately 3 sec in advance of an approaching vehicle in the other stream—provided that his speed is about the same as that of the approaching vehicle.

Table 27. Average Acceleration Rates from a Stop, Expressed in Distance and Time

(Does not include PIEV time)

Passenger Vehicles and Buses		Trucks	
Distance, ft	Time, sec	Distance, ft	Time, sec
30	3.5	30	3.9
50	4.7	50	5.4
70	5.7	70	6.5
90	6.8	90	7.6
110	7.8	110	8.8
130	8.8	130	9.7

As the differential in speed of the two streams is increased, a longer time-gap requirement is indicated. This points toward the importance of acceleration lanes, adequate radii of turns, and other design elements to reduce the speed difference of merging streams at intersections. Minimum dimensions of sight triangle for various assumed design speeds are shown in Table 28. These dimensions will allow the merging vehicle to proceed safely into the intersection at constant speed when no interfering vehicle is visible within the minimum sight distance along the preferential road. At some merging intersections where neither of the intersecting streams is inclined to give way, the minimum sight triangle dimension for each road should be the comfortable stopping distance plus the distance equivalent of a 3-sec time gap, computed on the basis of the assumed design speed of each road.

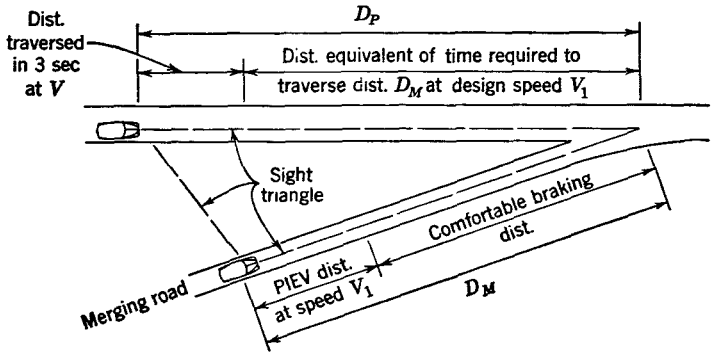


FIG. 33. Suggested sight-triangle distances for merging traffic. V = assumed design speed of preferential road in mph; V_1 = assumed design speed of merging road in mph.

Table 28. Minimum Dimensions of Sight Triangle at Merging Intersections

Assumed Design Speed of Merging Road	D_m, ft	Assumed Design Speed on Preferential Road, mph	D_p, ft
30	195	30	327
		40	436
40	308	40	484
		50	605
50	445	50	666
		60	799
60	606	60	871
		70	1016
70	790	70	1099

Night Sight Distances. Visibility distances available for safe stopping during nighttime conditions are much less than during daylight. Night visibility is the least when drivers are subjected to glare from approaching headlights or other extraneous light sources. Not much is known about the time required for a driver to perceive and react to a hazard

at night. Eyestrain and glare along with the physical and mental fatigue experienced at the end of the day probably is responsible for a longer reaction time at night than during the day. Visual stimuli under low levels of illumination are much weaker than during daytime and therefore do not compel as much attention.

Minimum specifications of the Society of Automotive Engineers for sealed-beam headlights require at least 40,000 total candlepower. The distribution of light within the headlight pattern is controlled by both the Uniform Vehicle Code and specifications set forth by the Society of Automotive Engineers.

The Effect of Vertical Curves on Sight Distances. Daytime safe stopping sight distances are an important factor in the design of crest vertical curves. Current design practice assumes the average height of the driver's eye as 4.5 ft and the height of a hazardous object on the road at 4 in. or more. It is reasoned that lower objects could be struck without much hazard. When grades are short and sight distances extend over several vertical curves, adequate curve lengths are selected by trial and error, usually by plotting the profile to a suitable scale and aligning a straightedge between pairs of points corresponding to the respective heights of eye and object. A chart, such as that illustrated in Fig. 34, may be conveniently used for the determination of minimum length of a single-crest vertical curve when the daytime safe stopping sight distance and grade are known.

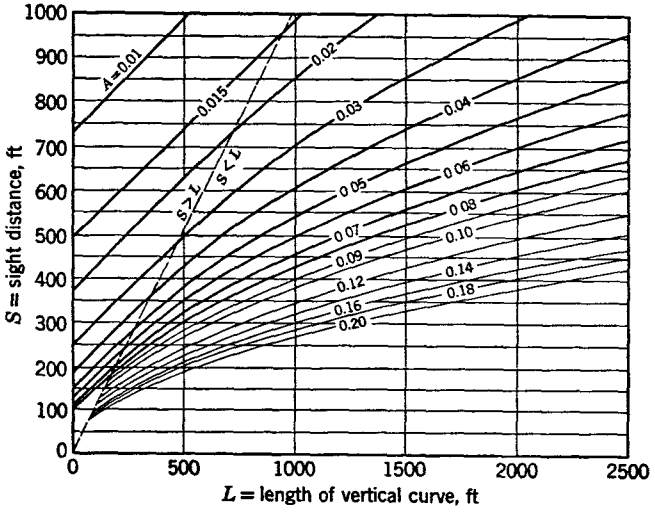


FIG. 34. Non-passing sight distance; relation between sight distance, highway grades, and length of vertical curve. A = algebraic difference of grades − per cent ÷ 100. When $S > L$, $L = 2S - \frac{14.56}{A}$. When $S < L$, $L = \frac{AS^2}{14.59}$. Height of eye, 4.5 ft. Height of object, 4 in.

When passing on a crest, the passing driver first sees the top of an approaching vehicle. Most passenger vehicles have an overall height of about 5 1/2 ft while the height of trucks ranges up to 14 ft. How much of the top of a vehicle must be visible before its presence is detected is uncertain. It is assumed that vision requirements are satisfied when both drivers see each other at the eye height for passenger vehicles, i.e., 4.5 ft above the road surface. Recent studies, however, show that, when sight distance is restricted, drivers seldom pass until they reach the point where they can see the pavement surface ahead for a sufficient distance to permit safe passing. Minimum safe passing sight distances are related to crest curve design by the procedures described for daytime safe stopping sight distances, the only difference being that the vertical height of the object is 4.5 ft instead of 4 in. Where only one vertical curve is involved, the chart shown in Fig. 35 may be used.

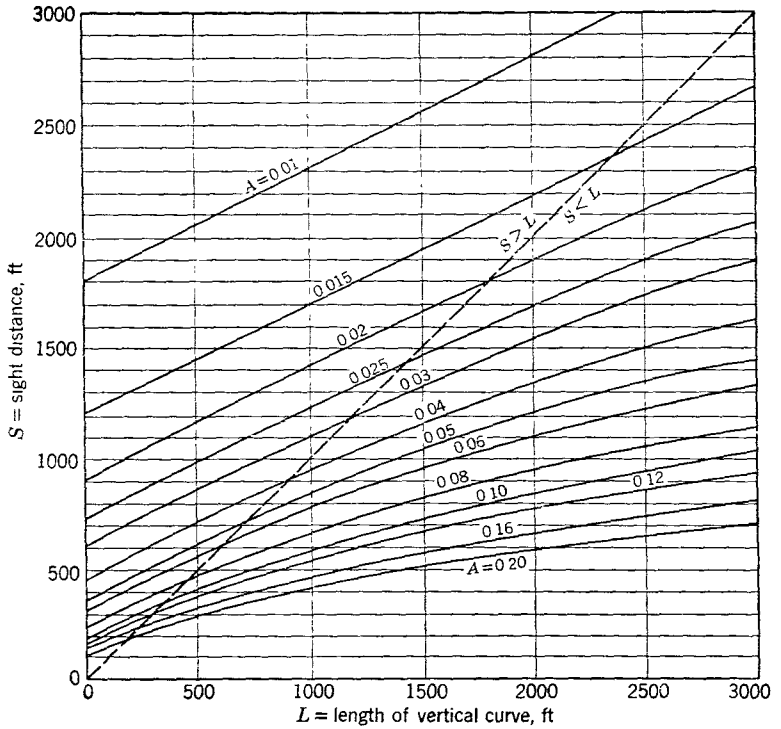


FIG 35. Passing sight distance, relation between sight distance, highway grades, and length of vertical curve A = algebraic difference of grades ± per cent ÷ 100 When $S > L$, $L = 2S - \frac{36}{A}$. When $S < L$, $L = \frac{AS^2}{36}$. Height of eye and height of object, 4.5 ft.

36. VERTICAL ALIGNMENT

Speeds on Grades. Modern passenger vehicles are adequately powered to ascend grades up to 15% without appreciable reduction in speed. Medium and heavy trucks, however, are slowed substantially on grades of 3% and over. Most truck drivers enter ascending grades at the highest speed considered safe under prevailing conditions in order to utilize as much momentum as possible in overcoming the resistance of the grade. On long grades, the momentum is eventually lost and a vehicle operating at full throttle reaches a constant speed. This is called the vehicle's *sustained speed* (Fig. 36).

Highway designers may improve traffic movement by limiting the steepness and/or length of grades, and by providing extra lanes for trucks on steep grades. Most design standards for high-type roads carrying truck traffic specify 3% maximum grades wherever it is economically feasible, except that steeper grades may be used for short distances as for ramps at grade separations.

Effects of Grades on Deceleration and Acceleration. Combined gradient and horizontal curvature should be avoided or minimized because safe stopping distances are greater than normal when vehicles decelerate on curves While no factual data are available, it is believed that gradient may also account for errors in braking-distance judgment. This is particularly hazardous on long steep downgrades. On upgrades, grade resistance is added to tractive resistance and longer distances are required for acceleration. Corresponding allowances must be made in the design of acceleration lanes, passing sight distances, and other features of the road.

Vertical Curves. Within the grade angles employed in highway work, the parabola and circular arc are practically the same and therefore have the advantages of both. The parabola is the simplest to lay out and is commonly used in the United States in both crest and sag vertical curves.

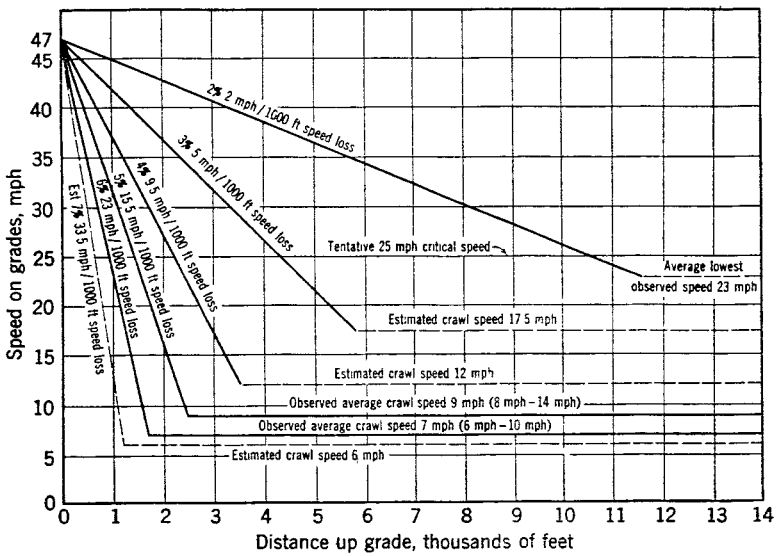


FIG. 36. The effect of length and steepness of grade on the speed of medium trucks with a gross load of 30,000 lb Indicated average speed pattern of heavily loaded vehicles on various percentages of grade Rate of speed loss on various percentages of grade and distance from initial point to start of crawl speed Plotted from 1948–1949 field data Source: 1951 *Proceedings*, Western Assn of State Highway Officials, from *Truck Speeds on Grades*, by W. E Willey, p 163.

37. HORIZONTAL ALIGNMENT

Equilibrium of Forces. To hold a vehicle on a curve, a force must be introduced to equal the centrifugal force acting on the vehicle. This opposing force is the resultant of the superelevation and the friction developed between the tires and the road surface. The equilibrium formula expressing this relationship is:

$$e + f = \frac{0.067\,V^2}{R}$$

where e = superelevation in feet per foot.
f = coefficient of friction in pounds per pound.
V = speed in mph.
R = radius in feet.

Superelevation. Where spiral transitions are provided, superelevation along the entire central circular curve should be the maximum value attained, with all superelevation runoff placed along the transitions. Where spirals are not provided, about two-thirds of the superelevation is placed on the tangent and one-third on the curve. In northern climates, the maximum practicable amount of superelevation is limited by the tendency of slow-moving vehicles to slip to the inside of curves when the pavement is icy. In such climates, a maximum value of 0.08 ft/ft is recommended and a value of 0.06 has been found desirable in some locations. Where ice is not a problem, superelevation is excessive where there is a tendency for loads on trucks to shift toward the inside of the curve at low speeds or where drivers experience an off-balance sensation which may lead to improper steering. The maximum value recommended for southern climates is 0.12 ft/ft although values up to 0.16 ft/ft have been used on interchange ramps.

When the curvature to be employed on a horizontal curve is less than the maximum allowable, there is a leeway as to how the force opposing centrifugal force is to be dis-

Table 29. Side-Thrust Coefficients ($e + f$), Their Distribution to Superelevations (e), Using the Three-Quarters Design-Speed Method, and the Remaining Unbalanced Friction Factors (f)

Degree of Curve *	Radius, ft	20 mph			30 mph			40 mph			50 mph			60 mph			70 mph		
		$e+f$	e		$e+f$	e	f	$e+f$	e		$e+f$	e	f	$e+f$	e	f	$e+f$	e	f
1-30	3820	0 007	†		0.016	†		0.028	†		0.044	†		0.063	0.035	0.028	0.086	0.048	0 038
2-00	2865	.009	†		.021	†		.037	†		.059	0.033	0.026	.084	.047	.037	.115	.064	051
2-30	2292	.012	†		.026	†		.047	†		.073	.041	.032	.105	.059	.046	.143	.081	.062
3-00	1910	.014	†		.032	†		.056	0.031	0.025	.088	.049	.039	.126	.071	.055	.172	.097	.075
3-30	1637	.016	†		.037	†		.066	.037	.029	.102	.058	.044	.147	.083	.064	.201	.113	.088
4-00	1433	.019	†		.042	†		.075	.042	.033	.117	.066	.051	.168	.095	073			
5-00	1146	.023	†		.053	0.030	0 023	.094	.053	.041	.146	.082	.064	.210	.118	.092			
6-00	955	.028	†		.063	.036	.027	.112	.063	.049	.175	.099	.076						
7-00	819	.033	†		.074	.041	.033	.131	.074	.057	.205	.115	.090						
8-00	717	.037	†		.084	.047	.037	.150	.084	.066									
9-00	637	.042	†		.095	.053	.042	.168	.095	.073									
10-00	574	.047	†		.105	.059	.046	.187	.105	.082									
11-00	522	.051	0.029	0.022	.116	.065	.051	.205	.116	.089									
12-00	478	.056	.031	.025	.126	071	.055												
13-00	442	.061	.034	.027	.136	.077	.059												
14-00	410	.065	.036	.029	.147	.083	.064												
15-00	383	.070	.039	.031	.157	.089	.068												
16-00	359	.075	.042	.033	.168	.095	.073												
17-00	338	.079	.045	.034	.178	100	078												
18-00	320	.084	.047	.037	.188	106	.082												
19-00	303	.088	.050	.038	.199	.112	.087												
20-00	288	.094	.053	.041	.209	.118	.091												
21-00	274	.098	.055	043															
22-00	262	.103	.058	.045															

* Based on chord definition.

† Include up to the full value of ($e + f$) in the transverse pavement slope. In any case the slope should be made adequate to provide suitable drainage

tributed between superelevation and "unbalanced" side thrust. With the "three-quarters" method of accomplishing this, superelevation is applied to counteract centrifugal force for a speed of three-quarters of the assumed design speed up to the point where the maximum allowable superelevation is reached. Superelevations computed by this method and the corresponding friction factors for the unbalanced side thrusts are given in Table 29.

Maximum Comfortable Side Friction. Road tests made by the U. S. Bureau of Public Roads indicated that the maximum comfortable side-thrust sensation is experienced with a side friction factor of 0.16 for speeds of 60 mph or less, with a decrease in the factor of 0.01 for each 5-mile increment in speed above 60 mph. Side-friction factors of these magnitudes are considerably less than the maximum that can be developed between normal road surfaces and tires, and their use provides a factor of safety for vehicles traveling at speeds in excess of the design speed. However, wherever possible, the side-friction factor should be kept well below the range in which discomfort is experienced.

Minimum Allowable Radius. Radii corresponding to maximum values for comfortable side friction and maximum values for superelevation are shown in Table 30. Such radii would not ordinarily be used for design purposes.

Table 30. Radii Corresponding to Maximum Allowable Coefficients of Friction

Design Speed, mph	Maximum Side-Friction Factor, ft/ft	Maximum Superelevation, ft/ft	Minimum Allowable Radius, ft
20	0.16	0.08	112
20	.16	.12	96
30	.16	.08	251
30	.16	.12	215
40	.16	.08	446
40	.16	.12	383
50	.16	.08	698
50	.16	.12	598
60	.16	.08	1005
60	.16	.12	862
70	.14	.08	1492
70	.14	.12	1263

Transition Curves. If a vehicle were to enter a circular arc from a tangent, the full effect of "unbalanced" side thrust would be developed abruptly and the vehicle occupants would be subjected to proportionate shock. However, the paths described by drivers when traveling on horizontal curves at speeds high enough to develop "unbalanced" side thrust, are transitional regardless of the curve alignment of the road. For maximum safety and comfort the alignment should follow those transitional paths. This may be effected by widening the roadway or by providing spiral transitions. Details of the geometry of spiral transitions are given in Section 2.

Spiral transitions should be of sufficient length for driver comfort and good appearance. The minimum radius employed should be equal to, and preferably greater than, the minimum allowable radius for a horizontal curve. The spiral length may be computed by the following formula:

$$L_s = \frac{3.2V^3}{aR}$$

where L_s = length of spiral in feet.
V = speed in mph.
R = radius of the circular curve in feet.
a = rate of change of radial acceleration in ft/sec^3.

It is assumed that the rate of change of radial acceleration a is the critical factor and that the value employed by drivers increases on sharp turns, requiring decreased speed. A value of 2 ft/sec^3 is recommended for through roadways and values ranging from 3 ft/sec^3 to 7 ft/sec^3 for speeds of 50 to 20 mph are recommended for turns at intersections.

The formula above is recommended by the U. S. Bureau of Public Roads. In close harmony with the National Interregional Highway Committee recommendation of superelevating all curves where the degree of curvature is sharper than 2°, the Bureau recommends that all curves should be transitioned when they are of such radii that the offset from a non-transitioned circular curve of the same radius is greater than 1 ft for a transition of required length determined by the formula.

An advantage of the spiral transition curve is its suitability to the application of superelevation. Since the radius of a spiral transition is inversely proportional to the distance from the beginning of the curve, the increase in centrifugal force developed by a vehicle

traveling along the transition at constant speed is directly proportional to the distance from the beginning of the curve. Thus, attainment of full superelevation slope at a uniform rate throughout the length of the transition provides an increasing resistance to side skid as centrifugal force increases. Methods of attaining superelevation are discussed in Section 4.

Widening of Curves. There is a certain amount of controversy over the justification of pavement widening on transitioned horizontal curves when lane widths of 11 or 12 ft are used. Widening is not ordinarily indicated for curves of less than 5°. When the transition is designed properly, motorists should have little difficulty remaining within their respective lanes at normal width when traveling at, or below, the speed for which the curve was designed. On the other hand, if motorists are to enjoy the same degree of comfort and safety on sharp curves as they enjoy on tangents, widening on the sharp curves is necessary.

The two factors which limit clearances on horizontal curves and which determine the amount of widening necessary are: (1) the off-tracking of rear wheels of vehicles at low speeds and (2) the slippage of the rear wheels toward the outside of the curve at high speeds. Since highway curves are traveled by fast and slow drivers alike, the width required for both inside tracking and outside slipping should be combined to give the total widening. The following formula is employed for determining the amount of widening required

$$W = n(R - \sqrt{R^2 - L^2}) + \frac{V}{\sqrt{R}}$$

where W = pavement widening in feet.
n = number of lanes.
R = radius of curvature in feet.
L = wheelbase in feet.
V = speed in mph.

It is suggested that for through routes this formula be considered applicable to 11-ft lanes and that the widening computed by the formula be increased when the normal lanes are narrower, or decreased when the normal lanes are wider. Suggested pavement widths for intersection curves are given in Art. 38.

38. INTERSECTIONS

Design Precepts. The layouts to accommodate characteristic maneuvers of road users in crossing, merging, and diverging at intersections may be right-angle, oblique-angle, or rotary types and may have roadways separated in grade. The design for a specific location is influenced by the traffic density, the composition of the traffic stream, the assumed design speed, physical conditions, right-of-way costs, construction costs and benefits. Among the fundamental precepts of functional design are the following:

1. Only movements with which motorists are familiar should be required. Preferably the maneuvers should be established by habit.
2. Traffic lanes, islands, signs, and markings should guide the motorist into his proper lane.
3. Rapid changes in direction or speed should be avoided.
4. Adequate time should be allowed for motorists to make decisions. At any time a motorist should not be faced with more than one decision nor with more than one conflicting movement.
5. The area in which traffic streams intersect should be as small as practicable.
6. Crossings at or near right angles and merging and diverging maneuvers at small angles are desirable.
7. To reduce conflicts and to provide desirable angles of crossing and intersection, traffic streams may be channelized.
8. Overtaking and passing maneuvers in conflict areas may be eliminated by funneling traffic streams through throats of one-lane width.
9. The maximum practicable distance should be provided between consecutive points of conflict. As a "rule of thumb," the minimum weaving distance in feet should be at least equal to the product of the design speed in miles per hour and the lane width in feet.

Radii of Intersection Curves. Minimum curve-radius design at intersections is developed by the centrifugal force formula assuming higher rates of side friction than in the case of roadway curves. Superelevation values employed vary from no superelevation on sharp turns where warping of the pavement surface at the ends of the curve is impracticable to

maximum allowable values on longer curves. Minimum radii for various speeds on intersecting curves are given in Table 31.

Table 31. Minimum Radii for Intersection Curves

Design (turning) speed (V), mph	15	20	25	30	35	40
Side friction factor (f)	0.32	0.27	0.23	0.20	0.18	0.16
Assumed min superelevation (e)	.0	.02	.04	.06	.08	.09
Total $e + f$	.32	.29	.27	.26	.26	.25
Minimum safe radius, feet						
Calculated	47	92	154	231	314	426
Suggested (rounded)	50	90	150	230	310	430
Suggested max degree of curvature (rounded)	...	64	38	25	18	13

Note: For design speeds of 40 mph and over, values same as for open highway conditions.
Source: A Policy on Geometric Design of Rural Highways, AASHO, 1954.

Pavement Widths for Intersection Curves. The additional pavement width required for inside tracking may be computed from the formula on p. 3-60 for any given radius and wheelbase. Additional pavement width is required on intersection curves for outside overhang, clearance to edge of pavement, and clearance between moving vehicles. Unless shoulders are provided, extra pavement width should also be allowed for passing stalled vehicles. Suggested pavement widths are given in Table 32 for one- and two-lane operation under three assumed compositions of traffic: (*a*) where traffic is predominantly passenger vehicles with some single-unit trucks or buses; (*b*) where the traffic includes sufficient single-unit trucks and buses to govern design but consideration must be given also to semitrailer trucks; and (*c*), where the traffic includes sufficient semitrailer trucks to govern design.

Table 32. Design Widths of Pavements for Turning Roadways

Radius on Inner Edge of Pavement, ft	Pavement Width, ft								
	One-Lane One-Way Operation, No Provision for Passing			One-Lane One-Way Operation, with Provision for Passing a Stalled Vehicle			Two-Lane Operation, either One-Way or Two-Way		
	Design Traffic Condition								
	a	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
50	16	17	20	21	24	27	30	33	37
75	15	16	18	20	22	25	28	31	34
100	14	16	17	19	21	24	27	30	33
150	13	15	16	18	20	23	26	29	31
200	13	15	16	18	20	22	26	28	29
300	12	15	15	17	19	21	25	27	28
400	12	14	15	17	19	21	25	27	28
500	12	14	15	17	19	21	25	27	27
Tangent	12	14	14	16	18	20	22	24	24
Width Modification Regarding Edge of Pavement Treatment									
No stabilized shoulder	None			None			None		
Mountable curb	None			None			None		
Barrier curb									
One side	Add 1 ft			None			Add 1 ft		
Two sides	Add 2 ft			Add 1 ft			Add 2 ft		
Stabilized shoulder, one or both sides	None			Deduct shoulder width; minimum pavement width as for no-passing operation			Deduct 2 ft where shoulder is 4 ft or wider		

Source: A Policy on Geometric Design of Rural Highways, AASHO, 1954.

Speed-Change Lanes. The need for acceleration and deceleration lanes should be dictated by the speed differentials involved. Most drivers will merge with or diverge from through traffic at speeds below the design speed of the highway. The deceleration of a vehicle as it approaches an intersection is generally accomplished in two steps: (1) the vehicle slows in gear without braking, and (2) brakes are applied. Lengths of acceleration lanes and deceleration lanes, and suggested lengths of tapering sections to effect the necessary pavement widenings, are given in Table 33.

Table 33. Design Lengths of Speed-Change Lanes: Flat Grades, 2% or Less

Design Speed of Turning Roadway Curve, mph		Stop Condition	15	20	25	30	35	40	45	50
Minimum Curve Radius, ft		..	50	90	150	230	310	430	550	690
Design Speed of Highway, mph	Length of Taper, ft	Total Length of Deceleration Lane, Including Taper, ft								
		All Main Highways								
40	175	300	250	250	200	175	*	...	...	...
50	200	400	350	350	300	250	250	200	*	...
60	225	450	400	400	350	350	300	250	225	*
70	250	500	450	450	400	400	350	350	300	250
		Total Length of Acceleration Lane, Including Taper, ft								
		Case I: High-Volume Highways								
40	175	...	450	400	350	250	*	...	...	...
50	200	...	700	650	600	500	400	250	*	...
60	225	...	1000	950	900	800	700	550	400	250
70	250	...	1300	1200	1100	1000	900	800	700	500
		Case II: Other Main Highways								
40	175	...	300	250	200	*	...	...	...	...
50	200	...	500	450	350	300	*	..	...	...
60	225	...	750	700	600	500	400	250	*	...
70	250	...	950	900	800	750	600	500	350	*

Ratio of Length of Speed-Change Lane on Grade to Length on Level

Design Speed of Highway, mph	Deceleration Lanes	
All	3 to 4% upgrade 0.9	3 to 4% downgrade 1.2
All	5 to 6% upgrade 0.8	5 to 6% downgrade 1.35

	Acceleration Lanes				
	Design speed of turning roadway curve, mph				
	20	30	40	50	All speeds
	3 to 4% upgrade				3 to 4% downgrade
40	1.3	1.3	...	...	0.7
50	1.3	1.4	1.4	...	0.65
60	1.4	1.5	1.5	1.6	0.6
70	1.5	1.6	1.7	1.8	0.6
	5 to 6% upgrade				5 to 6% downgrade
40	1.5	1.5	. .	...	0.6
50	1.5	1.7	1.9	...	0.55
60	1.7	1.9	2.2	2.5	0.5
70	2.0	2.2	2.6	3.0	0.5

Lengths of Taper for Speed-Change Lanes

	Highway design speed, mph				
	30	40	50	60	70
Minimum length, ft	125	150	175	200	225
Average length recommended for design, ft	150	175	200	225	250
Desirable length, ft	150	200	225	250	300

* Less than length of taper, use compound curve or partial taper.
Source. A Policy on Geometric Design of Rural Highways, AASHO, 1954.

Islands and Channels. Where the paths of vehicles are not clearly defined, numerous unpredictable conflicts result. At intersections these conflicts may be avoided or reduced in number and intensity by the use of channelization islands, generally elongated or triangular in shape, located in areas not necessary nor desirable as vehicle paths. The islands may be used to divide opposing traffic, provide shelter for vehicles waiting to cross or enter a traffic stream, and provide refuge for pedestrians. They are generally raised above the pavement surface and should be well marked, reflectorized, or otherwise designed to be sufficiently distinct to command the attention of drivers. Where sufficiently large, island areas may be planted with grass or low shrubs. The approach pavements should be well marked and may be roughened to guide traffic to the proper channels (Figs. 37, 38, and 39). When doubt exists as to the most suitable channelization, temporary layouts may be made using sandbags or traffic stanchions.

Funneling. It is natural for motorists to reduce speed when the channel width becomes narrower. Necessary reductions in speed on ramps at intersections may be accomplished by gradually funneling the ramp width to one lane. This also reduces conflicts by preventing passing attempts at the junction of the intersecting streams.

Rotary Intersections are those in which traffic flows in one way around a central island. They are specifically suitable for intersections with five or more radial roadways where volumes entering from all legs are approximately equal (Fig. 40). Ordinarily the combined traffic volume to be handled by all the intersecting roadways of a high-type rotary should not exceed 3000 vehicles/hr. The central island should have a curved outline of adequate radius at all points. The outer edge of the traffic channel should be designed to avoid reverse curves wherever possible. Full superelevation is not usually applicable because opposite slopes are usually indicated on inner and outer edges of the curves. Maximum angles of crossing should not exceed 40°. Pavement widths should at least be equivalent to half the

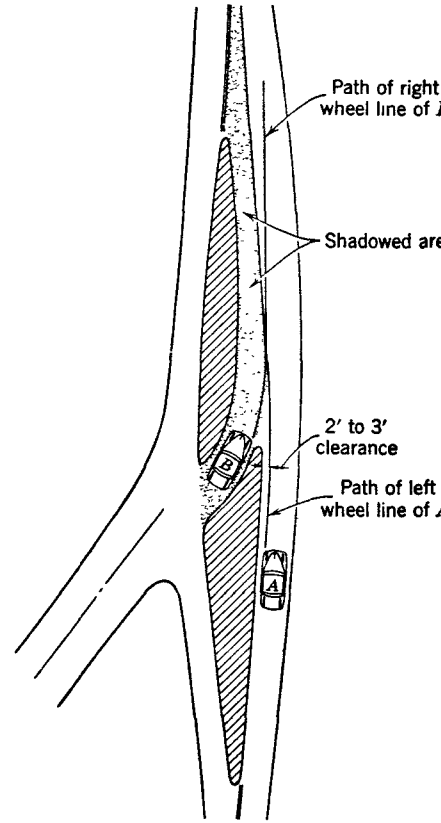


FIG. 37 Shadowing of one movement at a Y intersection. Yale Univ. Highway Traffic Bur.

Table 34. Suggested Design Data for Rotary Intersections

Number of Radial Roads	Sum of Lanes on Radial Roads: Minimum	Sum of Lanes on Radial Roads: Probable Maximum	Required Width of Rotary Roadway, ft
3	6	12	24 to 36
4	8	14	24 to 48
5	10	16	30 to 48
6	12	18	30 to 48

Highway design speed, mph	...	...	30	40	50 or more
Desirable rotary design speed, mph	20	25	30	35	40
Minimum weaving length, ft	120	150	180	210	240

Sources: A Policy on Rotary Intersections, AASHO, 1942 (lanes and widths). A Policy on Geometric Design of Rural Highways, AASHO, 1954 (speed relations).

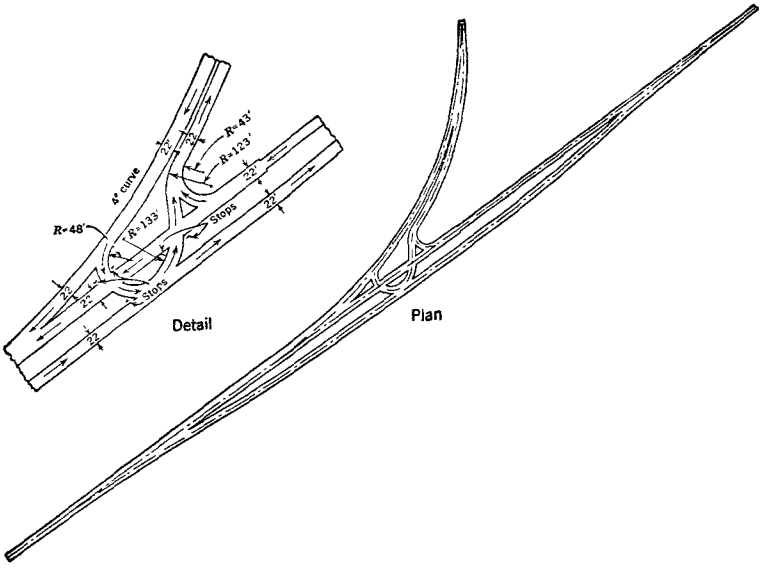


FIG. 38 Channelized Y intersection on two-lane roads. Source: A Policy on Intersections at Grade, AASHO, 1940.

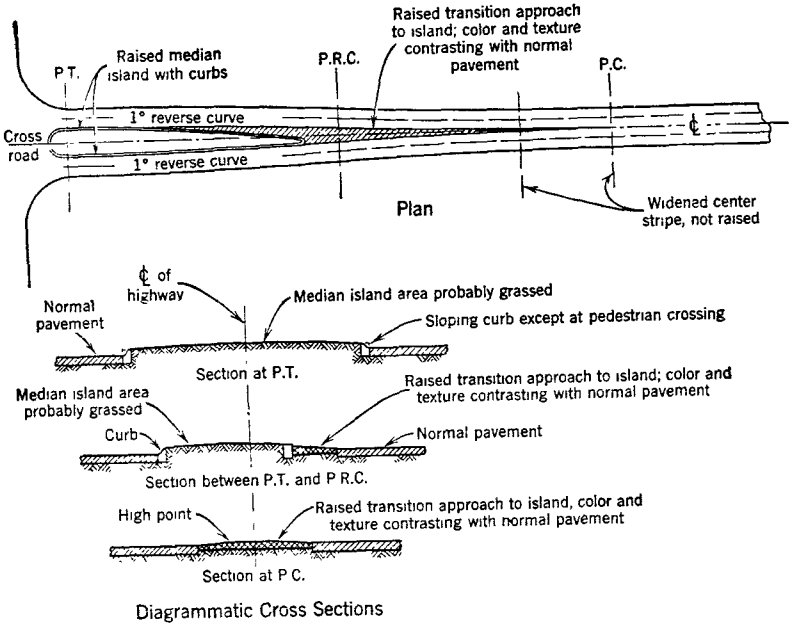


FIG. 39. Dividing island at rural intersection, highway on tangent Source: A Policy on Intersections at Grade, AASHO, 1940

width of the widest radial road plus the width of one lane. The maximum number of lanes should be four. Suggested design data are shown in Table 34.

Reflectorized or lighted signs should be provided well in advance of the rotary to warn traffic. Wherever feasible the entire rotary should be illuminated. Entrance and directional islands should be designed to assure safety in the reduction in speed from the highway design speed to the rotary design speed.

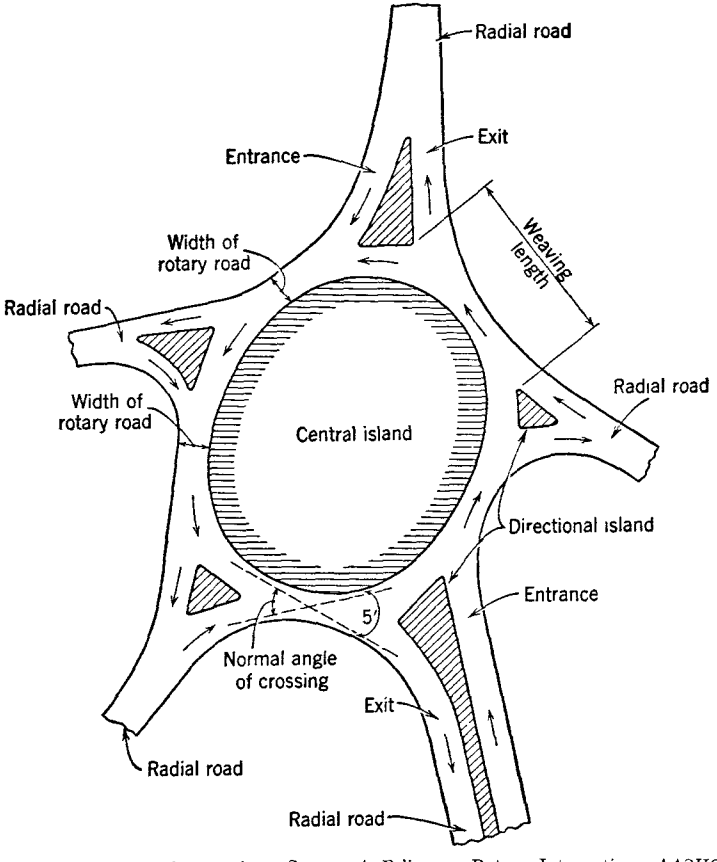


FIG. 40. Typical rotary intersection. Source: A Policy on Rotary Intersections. AASHO, 1942.

Grade Separations. The need for placing the roadway of intersecting highways at different levels to allow uninterrupted through movement on the intersecting roadways may be justified by the improved traffic flow and the reduction of accidents, or it may be indicated by topographic features. Typical grade separation layouts are shown in Fig. 41.

Grades, alignments, and sight distances on the highway approaches should conform to the standards of the intersected highways. The vertical clearances should be at least 14 ft above pavement areas and at least 13 1/2 ft above shoulders. Where curbs are provided on the approach highway, they should be continuous, without pavement widening, through or over the separation structures. If barrier curbs are used they should be set back 4 ft beyond the normal pavement edge and the minimum clearance from the barrier curb face to the structure should be 2 to 3 ft. Where curbs are of the non-barrier type and where curbs are not used, the minimum clearance from the normal curb line or pavement edge to the structure should be 4 to 6 ft. On short spans the full shoulder width should be maintained across the structure or through the structure. Where the full shoulder width is not provided, adequate deflectors should be installed to prevent collisions with the bridge structure. Gradient on ramps should be limited to 4 to 6% except where the proportion

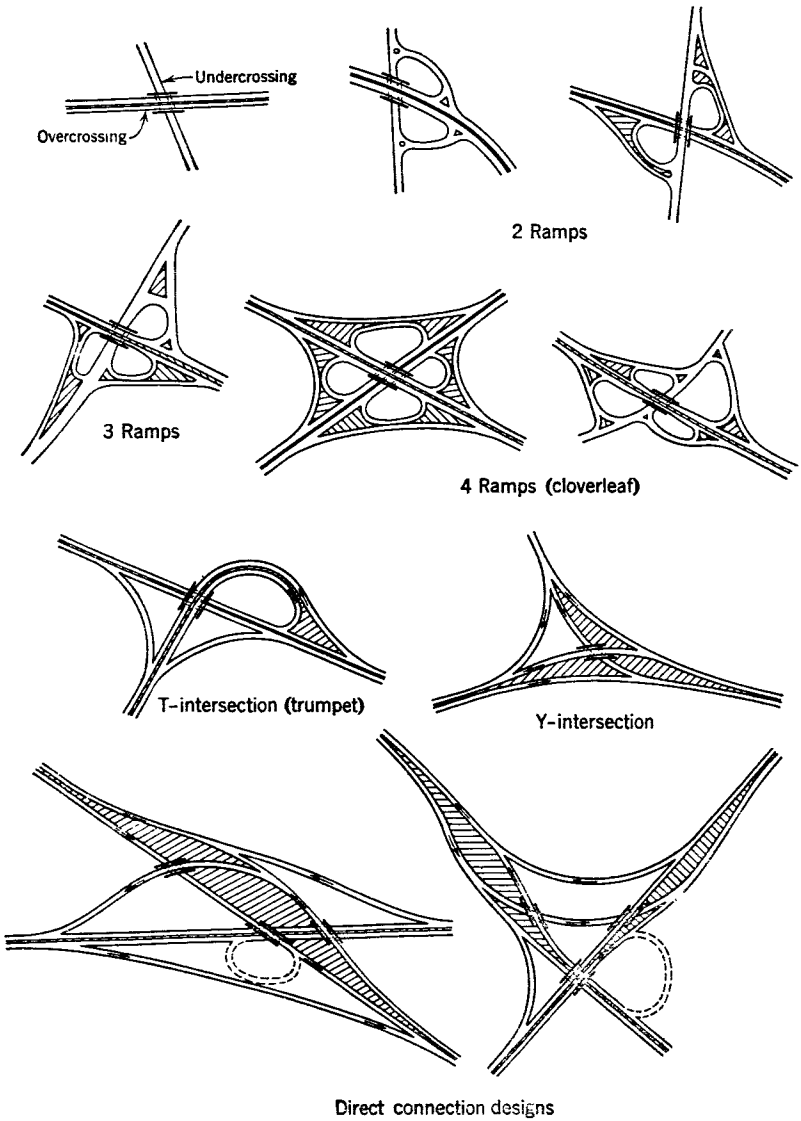


FIG. 41. Types of grade separations. Source: A Policy on Grade Separations for Intersecting Highways, AASHO, 1944.

of truck and bus traffic using the ramps is high, in which cases ramp grades should be limited to 3 to 4%.

39. ROADWAY CAPACITY

Definitions. The capacity of a traffic lane is the ability of the lane to accommodate traffic under specified operating conditions. Among the factors which affect capacity are: design elements, environmental conditions incidental to stream flow, and inherent stream characteristics which decrease the fluidity of traffic movements.

Table 35. Maximum Observed Hourly Volumes on Two-Lane Roads

Route and Location	Average Lane Width, ft	Vehicles per Lane per Hour: In One Lane	Vehicles per Lane per Hour: Average for Both Lanes
Colorado St. Bridge, Pasadena, Calif.	14.0	1710	1215
Sumner Tunnel, Boston, Mass.	10.8	...	1200
Posey Tube, Alameda, Calif.	11 5	1400	1350
U. S. 3, Manchester, N. H.	10.0	.	944
U. S. 45, Libertyville, Ill. (31 mph)	9.0	1510	904
Lincoln Tunnel (1940), New York	10.8	1152	..
U. S. 1, Old Lyme, Conn.	11.5	1135	887

Table 36. Maximum Observed Hourly Volumes on Three-Lane Roads

Route and Location	Average Lane Width, ft	Vehicles per Hour: In One Direction	Vehicles per Hour: Average for All Lanes
State Route 35, Laurence Harbor, N. J.	10.0	2250	1021
State Route 27, Rahway, N. J.	10.0	1905	761
State Route 27, West Scott Ave., Rahway, N. J.	10.0	1814	724
U. S. 1, Danvers, Mass.	10.0	1526	664
State Route 29, Union County, N. J.	10.0	1511	757
State Route 42, Camden County, N. J.	10.0	1391	664

Table 37. Maximum Observed Hourly Volumes in One Direction on Multilane Roads

Route and Location	Width of Road in Lanes	Average Lane Width, ft	Vehicles per Lane per Hour in One Direction: In One Lane	Vehicles per Lane per Hour in One Direction: Average for All Lanes
U. S. 1, Newark Airport, N. J.	4	12.4	.	2275
Grand Central Pkwy., west of Parsons Blvd., N. Y.	4	11.0	2275	2194
U. S. 73 and 75, Omaha, Neb. (34 mph)	4	11.0	.	1763
Cross Bay Blvd., Queens, N. Y.	4	8.0	.	1566
Express Highway, St. Louis, Mo.	4	...	1956	1543
Fourteenth St. Bridge, Washington, D. C. (27 mph)	4	10.0	1620	1508
Queensboro Bridge, upper roadway, New York, N. Y.	4	11.25	1861	1488
Sunrise Highway, at Cross Bay Blvd., N. Y.	4	10.0	.	1350
Outer Drive, Chicago, Ill.	4	11 25	1395	1349
Southern State Pkwy., Troopers Lodge, N. Y.	4	10 0		1264
Holland Tunnel, New York, N. Y. (28 mph)	4	10.0	1404	1253
Superior Bridge, Cleveland, Ohio	5	10.0	.	1557
Queensboro Bridge, New York, N. Y.				
Lower roadway, two lanes one-way	5	10.2	1720	1292
Lower roadway, three lanes one-way	5	10.2	. .	1195
Henry Hudson Pkwy., New York, N. Y.	6	11.0	...	1666
Oakland Bay Bridge, San Francisco, Calif.	6	9.7	...	1538
Leif Eriksen Drive, Chicago, Ill.	6	11.6	...	1435
Memorial Bridge, Washington, D. C. (30 mph)	6	10.0	...	1409
Arroyo Seco Pkwy., Los Angeles, Calif.	6	10.7	...	1367
George Washington Bridge, New York, N. Y.	6	8.0	...	1320
Delaware River Bridge, Camden, N. J. (four lanes one-way)	6	9.4	.	1285
Superior Bridge, Cleveland, Ohio (12.5 mph)	6	10.0	1530	1241
Outer Drive, Chicago, Ill. (37 mph)	8	12.1	2155	1958
Curb lane (32 mph)	8	12.1	1519	...
Second lane (35 mph)	8	12 3	2155	..
Third lane (40 mph)	8	12.5	2081*	..
Fourth lane (40 mph)	8	11.7	2077	..
Six lanes one-way	8	11.25	...	1640
Figueroa Freeway, Los Angeles, Calif.	8	10.0	...	1463

* A maximum of 2174 vehicles/hr in this lane occurred when the average lane carried only 1681 vehicles per hour.

Source: *Highway Capacity Manual*, Highway Research Board, 1955

Table 38. Base Values of Practical Capacity

(Pertaining only to roads of ideal design with no restrictions to uninterrupted movement caused by design or environmental or stream-flow conditions)

Number of Lanes, each 12 ft wide	Operating Speed, mph	Base Capacity in Passenger Vehicles/Hr
2 (two-way)	35–40	1500 both lanes, regardless of distribution by direction
2 (two-way)	40–45	1200 both lanes, regardless of distribution by direction
2 (two-way)	45–50	900 both lanes, regardless of distribution by direction
2 (two-way)	50–55	600 both lanes, regardless of distribution by direction
3 (two-way)	35–40	2000 all three lanes
3 (two-way)	45–50	1500 all three lanes
4 (two-way)	35–40	1500 per lane in direction of heavier flow
4 (two-way)	45–50	1000 per lane in direction of heavier flow

Source: *Highway Capacity Manual*, Highway Research Board, 1950.

Table 39. Effect of Lane Width on Capacity

(Percentage of 12-ft lane capacity)

Lane Width, ft	Two-Lane Rural Roads: At Possible Capacities	Two-Lane Rural Roads: At Practical Capacities	Two-Lanes for One Direction of Travel on Divided Highways at Practical Capacities
12	100	100	100
11	88	86	97
10	81	77	91
9	76	70	81

Table 40. Combined Effect of Lane Width and Edge Clearance on Capacity

(Percentage of 12-ft lane capacity)

Clearance from Pavement Edge to Obstruction, ft	Obstruction on One Side: 12-ft lanes	11-ft lanes	10-ft lanes	9-ft lanes	Obstruction on Both Sides: 12-ft lanes	11-ft lanes	10-ft lanes	9-ft lanes
Possible Capacity of Two-Lane Highway								
6	100	88	81	76	100	88	81	76
4	97	85	79	74	94	83	76	71
2	93	81	75	70	85	75	69	65
0	88	77	71	67	76	67	62	58
Practical Capacity of Two-Lane Highway								
6	100	86	77	70	100	86	77	70
4	96	83	74	68	92	79	71	65
2	91	78	70	64	81	70	63	57
0	85	73	66	60	70	60	54	49
Possible and Practical Capacities of Two Lanes for One Direction of Travel on Divided Highways								
6	100	97	91	81	100	97	91	81
4	99	96	90	80	98	95	89	79
2	97	94	88	79	94	91	86	76
0	90	87	82	73	81	79	74	66

Table 41. Effect of Passing Sight Distance Restriction on Practical Capacities of Two-Lane Roads

Percentage of Total Length of Road on Which Sight Distance Is Restricted to Less than 1500 Ft	Practical Capacity in Passenger Cars per Hour for Both Directions: Operating Speed, 45–50 mph	Operating Speed, 50–55 mph
0	900	600
20	860	560
40	800	500
60	720	420
80	620	300
100	500	160

Table 42. Effect of Commercial Vehicles on Practical Capacities of Multilane Facilities *

Commercial Vehicles, %	Capacity Expressed as a Percentage of Passenger-Car Capacity on Level Terrain: Level Terrain, %	Rolling Terrain, %
None	100	100
10	91	77
20	83	63

* The effect on two-lane highways is about 25% greater than the effect on multilane highways.

Table 43. Effect of Commercial Vehicles and Grades on the Capacity of Two-Lane Highways with Uninterrupted Traffic Flow

(When the grade does not cause a restriction in the passing sight distance)

Length of Grade, miles	Equivalent of One Dual-Tired Commercial Vehicle, in Terms of Passenger Cars, on a Grade Averaging: 3%	4%	5%	6%	7%
0.1	3.9	4.1	4.2	4.2	4.4
0.2	4.1	4.3	4.5	4.7	5.1
0.4	4.3	4.6	4.9	5.3	5.5
0.6	4.4	4.8	5.2	5.8	6.5
0.8	4.6	5.1	5.7	6.4	7.1
1.0	4.6	5.3	6.0	6.7	7.4
1.5	4.8	5.6	6.3	7.0	7.7
2.0	5.0	5.8	6.5	7.2	8.0
3.0	5.0	6.0	6.6	7.3	8.2
4.0	5.1	6.0	6.7	7.4	8.3
5.0	5.1	6.0	6.7	7.6	8.3
6.0	5.1	6.0	6.8	7.6	8.3

The *practical capacity* is the maximum number of vehicles that can pass a given point on a roadway or in a designated lane during one hour without the traffic density being so great as to cause unreasonable delay and hazard, or restrictions to the driver's freedom to maneuver under the prevailing roadway and traffic conditions.

The *possible capacity* is the maximum number of vehicles that can pass a given point on a roadway or in a designated lane during 1 hr. Volumes approaching that magnitude are attained only with a degree of traffic accommodation which is below the standard considered satisfactory.

Maximum Observed Hourly Volumes. Exceptional recorded hourly volumes, which probably approach possible capacities, are given in Tables 35, 36, and 37.

Base Values of Practical Capacity. The many variables and the wide ranges in conditions presented by each highway prevents the development of design capacities which may be arbitrarily applied to any design problem. On the other hand the practical capacity, representing ideal design, and environmental and stream-flow conditions may be determined with reasonable accuracy and used as a base to be modified to meet particular conditions. Practical capacities are greatly influenced by passing opportunities and by prevailing speeds and, therefore, should be expressed in terms of the number of lanes considered and in relation to speed. Typical values of practical capacities are given in Table 38.

Reductions in Capacity. The designer starts with an appropriate base capacity and grades it downward for the specific operating conditions to be expected on a proposed roadway. Capacity with uninterrupted flow is affected principally by lane widths, lateral clearances, sight distances, proportions of commercial vehicles, and grades. Typical influences reported by the Highway Research Board in the *Highway Capacity Manual*, 1950, are given in Tables 39–43.

Typical average street-intersection capacities per hour of green signal are shown in Fig. 42 for one-way and two-way operation in downtown and intermediate areas.

Although no significant data are available, it has been noticed that freedom of passing is appreciably reduced on curves having a radius of 500 ft or less. This factor and driver reluctance to pass on curves may lower the overall capacity of a highway having a large number of horizontal curves.

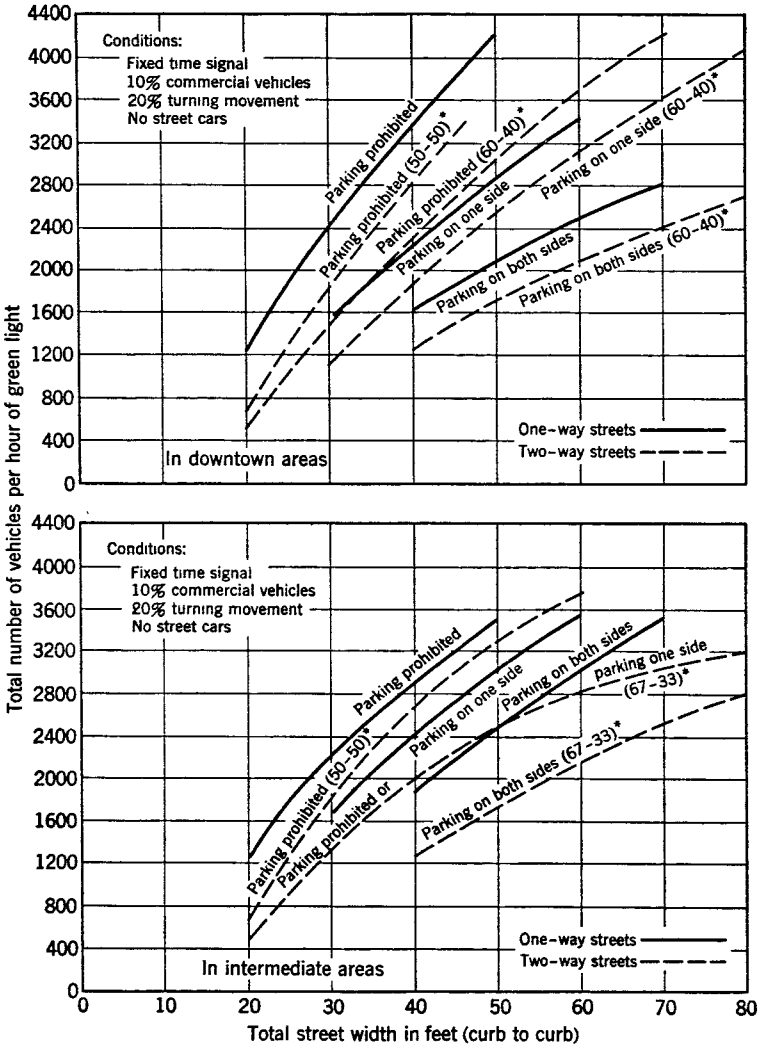


FIG. 42. Comparison of average street-intersection capacities with one-way and two-way operation, in downtown and in intermediate areas * Indicates distribution of traffic flow by direction on two-way streets. *Highway Capacity Manual*, U. S Bur. of Pub Roads, 1950.

40. HIGHWAY DESIGN STANDARDS

Design standards adopted for use on interstate highways, or prepared for the guidance of the various states, must necessarily reflect compromises of practices. Such standards are intended to produce the features considered desirable on an overall basis, but in individual locations they may be minimum or inadequate.

Standards for National System of Interstate Highways. The standards adopted by the National Interregional Highway Committee [1] are intended as long-range standards for the National System of Interstate Highways. Principal features of the standards are:

[1] Interregional Highways, 78th Congr., 2d Sess., House Document No. 379, January 12, 1944.

Control of Access. All sections of the system should be established as limited-access highways.

Grade Separations. There should be no crossings at grade of railways, streets, or highways except that, on rural sections of the system expected to carry an average daily traffic of less than 5000 vehicles, crossing structures may be omitted initially, if infeasible, but all necessary provisions should be made for future conversion to grade-separated design.

Service Streets. In urban areas wherever necessary for the service of property, local streets should be provided at each side of the section of the interregional system.

Design Speed. The speed assumed for design purposes should be as high as practicable consistent with the topography and the expected traffic volume. Under urban conditions the assumption of a design speed higher than 50 mph will usually be impracticable.

Horizontal Curvature. Horizontal curvature should be of the lowest practicable degree and should not exceed the absolute maximum values shown in Table 44.

Table 44

Design Speed, mph	Curvature Limits: Absolute Maximum, degrees	Curvature Limits: Desirable Maximum, degrees
75	3	2.5
70	4	3
65	5	4
60	6	5
55	7	6
50	9	7

Transition Curvature. All horizontal curves sharper than 2° should be approached by transition curves of a length consistent with the design speed and sufficient to obtain the full superelevation within the length of transition.

Superelevation. All curves sharper than 1° should be superelevated. The maximum superelevation should be 0.12 ft per ft in rural areas and 0.10 ft per ft in urban areas except that, where snow and ice may be expected to cause frequent slippery conditions, the maximum should be 0.08 ft per ft.

Sight Distance. Non-passing sight distances, from a height of 4.5 ft above any point on the road surface to the top of an object 4 in. high placed on the road surface, should not be less than the following:

Design Speed, mph	Minimum Non-passing Sight Distance
75	800
70	700
65	600
60	525
55	450
50	400

Wherever feasible, the following passing sight distances, between points 4.5 ft above the road surface, should be provided on two-lane sections of the system.

Design Speed, mph	Minimum Passing Sight Distance
65 to 75	2600
60	2200
55	1800
50	1500

Lane Widths. Lane widths should be 12 ft.

Separation of Roadways. In rural areas where four or more traffic lanes are to be constructed, two distinct one-way roads should be provided wherever advantages of alignment, construction cost, or traffic facility would result. The median strip of divided highways in rural areas should be at least 15 ft wide. On sections which permit crossing or entering at grade at intersecting highways, it should be at least 40 ft wide. In urban areas a raised median strip 4 ft wide may be used.

Shoulders. Shoulders should be at least 10 ft wide except that under extreme conditions they may be 4 ft wide.[2] In urban areas where mountable curbs are provided, an area 10 ft

[2] In the opinion of the authors, shoulder widths should be at least 10 ft wherever practicable.

wide should be reserved outside the curb lines for the temporary accommodation of disabled vehicles.

Side Slopes. Cuts should ordinarily have slopes not steeper than 2:1 except in solid rock. Embankments of 10 ft or less should ordinarily have slopes of at least 4:1.

Gradient. In rural areas, gradients should not exceed those shown in Table 45.

Table 45

Average Daily Traffic (All Vehicles)	Surrounding Topography	Maximum Desirable Grade, %
Less than 1000	Relatively level	3
	Rolling	4
	Mountainous	6
1000 to 2000	Relatively level	3
	Rolling	4
	Mountainous	6
2000 to 3000	Relatively level	3
	Rolling	6
	Mountainous	6
3000 to 5000	Relatively level	3
	Rolling	6
	Mountainous	6
5000 and more	Relatively level	3
	Rolling	4
	Mountainous	5

Gradients in urban sections of the system should preferably be not steeper than 3% and should not exceed 5%.

Width of Right-of-Way. The width of right-of-way should be sufficient for all features of the highway estimated to be needed within a period of 20 yr. In rural areas public control should ordinarily be obtained over a strip of land not less than 224 ft wide in lightly traveled sections and not less than 288 ft in other cases.

Vertical Clearances. Clearances should be at least 14 ft over paved areas and at least 12 1/2 ft over shoulders.

Ramps. All ramps and connections should enable vehicles to leave and enter the through-traffic lanes at 0.7 of the highway design speed except where "stop" control is necessary. All ramps and connections should have a width of at least 16 ft if designed for one-lane operation and at least 26 ft if designed for two-lane operation. Additional width should be provided on ramps and connections of sharp curvature. Shoulders at least 6 ft wide and preferably 8 ft wide should be provided along the right side of all ramp movements. Gradients on ramps should not exceed 6% on upgrades and 8% on downgrades.

Design Standards Recommended by AASHO for Interstate Highways.[3] Principal features of the design standards adopted August 1, 1945, by the American Association of State Highway Officials for use in design projects on the National System of Interstate Highways follow.

Traffic Basis. The design should be based on traffic volumes estimated for the 30th highest hour of the 20th year from the date of construction.

Controlled Access. Limitation of access is recommended for all new construction. Where legal restriction exists, additional right-of-way should be obtained for frontage roads.

Grade Separations. Separation structures should be provided at railroad crossings of two or more main-line tracks and may be justified at single-track crossings where six or more regular train movements are accommodated each day or where indicated by economic analysis of the hazards and delays. Separation of crossroads should be made where the traffic density of the interstate highway exceeds 3000 vehicles per hour. At both railroad crossings and crossroads where separation is not provided, adequate protective devices should be installed.

Design Speed.

	Minimum, mph	Desirable, mph
Rural sections		
Flat topography	60	70
Rolling topography	50	60
Mountainous topography	40	50
Urban sections	40	50

[3] Source: A Policy on Design Standards, AASHO, 1945

Superelevation. Curves in excess of 1° should be superelevated, and curves in excess of 2° should be provided with transitions.

Horizontal Curvature.

Design Speed, mph	Degree of Curvature	
	Maximum	Desirable
70	4	3
60	6	5
50	9	7
40	14	11

Sight Distance. The minimum sight distance should be commensurate with type of highway and the design speed.

Grades. Gradients should be limited to 5% where possible and should not exceed 6%, except for short lengths where 7% may be used.

Lane Widths. Lanes should be 12 ft wide in urban areas and where traffic density exceeds 200 vehicles per hour.

Separation of Roadways. Highways should be divided where the present traffic density is 800 or more vehicles per hour. Widths of median strips should be at least 15 ft and preferably 40 ft in rural areas and at least 2 ft and preferably 12 ft in urban areas. The 2-ft width should be used only in exceptional circumstances; otherwise the minimum should be 4 ft.

Shoulders. Shoulders should be capable of supporting standing or disabled vehicles and should be at least 10 ft wide, except in mountainous topography where 4-ft widths may be used.[4]

Right-of-Way Widths. In rural areas right-of-way widths for two-lane highways should be at least 120 ft and preferably 220 ft; and right-of-way widths for divided highways should be at least 200 ft and preferably 250 ft.

Slopes. In cuts, side slopes should be not steeper than 2:1 except in solid rock or other special soils. In embankments of 10 ft or less side slopes should be not steeper than 4:1.

Vertical Clearance. The vertical clearance should be 14 ft over pavement and 12 1/2 ft over shoulders.

Typical Design Factors for a State Highway System. Table 46 shows typical controlling design factors of the Connecticut State Highway Department, as revised in 1954.

Typical Design Criteria for Turnpikes. Typical design criteria used for the New Jersey Turnpike, the New York State Thruway, and the Ohio Turnpike are shown in Table 47.

[4] In the opinion of the authors, shoulder widths should be at least 10 ft wherever practicable.

Table 46. Highway Classification and Basic Design Standards *

Rural

	A	B	C	D	E	F	G	H
Design traffic volumes †	Under 750 daily	750 to 1490 daily	1500 to 2990 daily	3000 daily to 510 hourly	520 to 790 hourly	800 to 1490 hourly	1500 to 2290 hourly	2300 and over hourly
Design speed (mph)	40	45	50	50	60	60	60	60
Control of access	None	None	None	Relocations only	Yes	Yes	Yes	Yes
Crossroad treatment	At grade	At grade	At grade	At grade	(i): as determined; (u): grade separated	Grade separated	Grade separated	Grade separated
Lane arrangement	2 lanes	2 lanes	2 lanes	2 lanes	(i): 2 lanes; (u): 4 lanes divided	4 lanes divided	(i): 4 lanes divided; (u): 6 lanes divided	6 lanes divided
Median width	None	None	None	None	(i): none; (u): 45′	45′	(i): 54′; (u): 30′	30′
Median type	None	None	None	None	(i): none; (u): 41′ depressed plus 2 2′ paved borders	41′ depressed plus 2 2′ paved borders	(i): 50′ depressed plus 2 2′ paved borders; (u): 26′ depressed plus 2 2′ paved borders	Raised 26′ plus 2 2′ gutters
Right-of-way width	80′	80′	100′	120′	250′	250′	300′	300′
Lane width	11′	11′	12′	12′	12′	12′	12′	12′
Shoulder width	3′	6′	8′	8′	10′	10′	10′	10′
Bridge width (curb to curb) ‡	30′	34′	40′	40′	44′	113′	122′	122′
Curvature (maximum)	8°	6°	5°	5°	2° 30′	2° 30′	2° 30′	2° 30′
Gradient (maximum)	8%	7%	6%	5%	4%	4%	3%	3%
Stopping sight distance	275′	300′	350′	350′	475′	475′	475′	475′
Passing sight distance §	1100′	1200′	1400′	1400′	1400′	...	...	...
Pavement type ‖	*a*	*a*	*a, b, c*	*b, c, d*	*d*	*d*	*d*	*d*
Shoulder type ¶	*e*	*e*		*f, g*	*g*	*g*	*g*	*g*

Urban (Residential)

	A	B	C	D	E	F	G	H
Design traffic volumes †	Under 750 daily	750 to 1490 daily	1500 to 2990 daily	3000 daily to 560 hourly	570 to 850 hourly	860 to 1940 hourly	1950 to 2890 hourly	2900 and over hourly
Design speed (mph)	40	45	45	45	50	50	50	50
Control of access	None	None	None	Relocations only	Yes	Yes	Yes	Yes
Crossroad treatment	At grade	At grade	At grade	At grade	(i). as determined; (u): grade separated	Grade separated	Grade separated	Grade separated
Lane arrangement	2 lanes	2 lanes	2 lanes	2 lanes	(i): 2 lanes; (u): 4 lanes divided	4 lanes divided	(i): 4 lanes divided (u): 6 lanes divided	6 lanes divided
Median width	None	None	None	None	(i): none; (u): 45′	45′	(i): 54′; (u): 30′	30′
Median type	None	None	None	None	(i): none; (u). 41′ depressed plus 2 2′ paved borders	41′ depressed plus 2 2′ paved borders	(i): 50′ depressed plus 2 2′ paved borders, (u). 26′ depressed plus 2 2′ paved borders	Raised 26′ plus 2 2′ gutters
Right-of-way width	70′	70′	90′	100′	250′	250′	300′	300′
Lane width	11′	11′	12′	12′	12′	12′	12′	12′
Shoulder width	8′	8′	8′	8′	10′	10′	10′	10′
Bridge width (curb to curb) ‡	38′	38′	40′	40′	44′	113′	122′	122′
Curvature (maximum)	8°	6°	6°	6°	3° 30′	3° 30′	3° 30′	3° 30′
Gradient (maximum)	8%	7%	6%	5%	4%	4%	3%	3%
Stopping sight distance	275′	300′	300′	300′	350′	350′	350′	350′
Passing sight distance §	1100′	1200′	1200′	1200′	1200	...	...	...
Pavement type ‖	*a*	*a*	*a, b, c*	*b, c, d*	*d*	*d*	*d*	*d*
Shoulder type ¶	*f*	*f*	*f*	*f, g*	*g*	*g*	*g*	*g*

Table 46. Highway Classification and Basic Design Standards * *Continued*

	A	B	C	D	E	F	G	H
			Urban (Commercial or Industrial)					
Design traffic volumes †			1500 to 2990 daily	3000 daily to 600 hourly	610 to 900 hourly	910 to 2390 hourly	2400 to 3490 hourly	3500 and over hourly
Design speed (mph)			40	40	40	40	40	40
Control of access			None	Relocations only	Yes	Yes	Yes	Yes
Crossroad treatment			At grade	At grade	(i): as determined, (u) grade separated	Grade separated	Grade separated	Grade separated
Lane arrangement			2 lanes	2 lanes	(i). 2 lanes, (u): 4 lanes divided	4 lanes divided	(i): 4 lanes divided, (u): 6 lanes divided	6 lanes divided
Median width ‡**			None	None	(i): none; (u): 32′	32′	(i): 32′; (u). 8′	8′
Median type			None	None	(i): none; (u): Raised 28′ plus 2 2′ gutters	Raised 28′ plus 2 2′ gutters	(i): raised 28′ plus 2 2′ gutters, (u): raised 4′ plus 2 2′ gutters	Raised 4′ plus 2 2′ gutters
Right-of-way width			80′	80′	200′	200′	250′	250′
Lane width			12′	12′	12′	12′	12′	12′
Shoulder width			8′	8′	10′	10′	10′	10′
Bridge width (curb to curb) ‡			40′	40′	44′	100′	100′	100′
Curvature (maximum)			8°	8°	6°	6°	6°	6°
Gradient (maximum)			6%	5%	4%	4%	3%	3%
Stopping sight distance			275′	275′	275′	275′	275′	275′
Passing Sight Distance §			1100′	1100′	1100′	...	...	...
Pavement type ‖			*a, b, c*	*b, c, d*	*d*	*d*	*d*	*d*
Shoulder type ¶			*f*	*f, g*	*g*	*g*	*g*	*g*

* From Connecticut State Highway Department. Revised August 1954. These standards apply only to routes which are maintained by the state. They are for use in the design of main roadways and do not apply to interchange and intersection designs or city street designs. They represent minimum values. Higher standards may be used within reasonable economic limits. Lower standards may not be used without written approval of the Chief Engineer.

† The annual average daily traffic or 30th highest hour of one direction 20 years after construction. On recreational routes where the 30th highest hour of one direction is over 18% of the annual average daily traffic, a planning study will determine the directional hourly volume for design.

‡ Clear width on bridges:

(a) On bridges over 300′ in length shoulders to be reduced to 3′.

(b) Lateral clearance of center pier of underpass structures, 4 1/2′ minimum from face of pier to edge of pavement.

§ At least one passing sight distance per mile on two-lane highways for A, B, C classifications. At least two passing sight distances per mile on two-lane highways for D and E classifications.

‖ Standard pavement types:

(a) 2 1/2″ bituminous macadam on 6″ broken stone base.

(b) 2 1/2″ bituminous concrete on 6″ broken stone base.

(c) 2 1/2″ bituminous concrete on 3″ penetrated stone base on 3″ broken stone base.

(d) 8″, 9″, or 10″ reinforced concrete depending on heavy truck volume.

¶ Standard shoulder types:

(e) 8 1/2″ surface treated gravel.

(f) 2 1/2″ bituminous macadam on 6″ gravel base (with pavement type a, b, c).

(g) 2″ bituminous concrete on 6″ or 8″ gravel base (with pavement type d).

** On Urban (Commercial or Industrial) where center pier is used, median to be widened an additional 5′.

Definitions

(i) = initial.

(u) = ultimate.

Vertical clearance on bridges:

14′ 3″ at outside edges of pavement.

Superelevation:

Classes A, B, C, and D: maximum 1 in. per foot.

Classes E, F, G, and H: maximum 3/4 in. per foot.

Table 47. Typical Design Criteria for Turnpikes

	N. J. Turnpike	N. Y. State Thruway Absolute	N. Y. State Thruway Desirable	Ohio Turnpike
	Route Design			
Design speed	70 mph	(50 mph) *	70 mph (60 mph) *	Commensurate with safety
Capacity per lane per hour	750	1500	1000	...
Design hour (one-way)	30th highest	10% of estimated two-directiona 12-hr traffic	10% of estimated two-directional 12-hr traffic	...
Minimum sight distance	600′ (daylight) 1000′ (headlamp)	(600′) *†	1000′ (800′) *†	900′ (daylight) 1000′ (headlamp)
Minimum radius of horizontal curvature	3000′	2800′ (1500′) *	(2000′ or over) *	2292′
Maximum superelevation	3%	1/2″ per ft (3/4″ per ft) *	1/2″ per ft (3/4″ per ft) *	1″ per ft
Length of transition curves	Radius 3000–3200 250′ 3200–3500 200′ Over 3500 None	...	...	Degree of Curvature 0° 15′ 200′ ‡ 2° 30′ 500′ ‡
Minimum length of curve including transition	600′	...	...	1000′
Minimum distance between reverse curves	1000′	Sufficient to run out superelevation	Sufficient to run out superelevation plus at least 500	...
Minimum distance between same-direction curves	2500′	...	...	2500′
Maximum grade	...	3%	..	2% upgrade 4% downgrade
Traffic lane width	12	12′-13′-12′	...	.
Minimum vertical clearance	15′	14.2′	...	14′ 6″ and 15′ 0″
Fill slopes				
Over 10′	2:1	...	...	2 1
Under 10′	4:1	...	...	4:1

Cut slopes	2:1	...	...	2:1
Width of center dividing strip (edge to edge of pavement)	20′	20′ (6′ and in special cases 4′) *	(20′) *	...
		Ramp Design		
Design speed	20–45 mph	40 mph (30 mph) *	40 mph	...
Capacity per lane per hour	1000	1200	800	...
Minimum radius	125′	(150′) *	300′	...
Minimum sight distance	155′–535′	500′ vertical † 275′ horizontal	...	Based on safe stopping distances
Maximum superelevation	8%	...	3/4″ per ft	1″ per ft
Maximum grade	5%	...	..	4% upgrade 5% downgrade
Length of acceleration lane	1200	800′ (500′ for 50 mph) * (700′ for 60 mph) *	1200′	1200
Length of deceleration lane	1200′	800′ (250′ for 50 mph) * (400′ for 60 mph) *	...	1050
Acceleration and deceleration lane width	12′	12′	...	...

* Figures in parentheses apply to restricted areas.
† Vertical sight distance, based on 4′ 6″ driver's eye to 4′ 6″ height of object.
‡ Plus 75′ tangent runouts.

41. HIGHWAY SUFFICIENCY RATINGS

A Highway Sufficiency Rating is a comparative value which reflects the adequacy of a road section as determined by inspection and comparison with established standards. Sufficiency ratings were developed with the view to minimizing the personal factor. The three major elements usually rated are (1) condition or structural adequacy, (2) safety, and (3) service. The weights given to these components in various states are shown in Table 48. There is considerable variation in the subdivisions of the ratings used by each state.

Table 48. Summary of Sufficiency Rating Procedures

Rating Elements	Ariz.	Col.	Conn.	Del.	Ida.	Ill.	Ky.	La.	Mo.	Neb.	N. H.	Ore.	Wash.
Condition or structural adequacy *	35	40	Not used	40	40	350	Not used	40	35	60	40	40	40
Safety †	30	30	50	30	30	350	43	30	30	26	30	30	30
Service ‡	35	30	50	30	30	300	57	30	35	14	30	30	30
Basic rating	100	100	100	100	100	1000	100	100	100	100	100	100	100

* Structural adequacy, observed condition, remaining life, maintenance economy, maintenance cost, foundation, pavement, shoulders.

† Roadway width, surface width, sight distance, stopping sight distance, consistency, accident rate.

‡ Alignment, passing opportunity, surface width, sway in cross section, roughness, surface driving condition, maintenance rating.

Source. *Highway Research Board Bull.* 53.

When ratings are determined for an entire highway network, the relative needs for improvement of various parts of the network are apparent and a clear-cut basis is provided for allocating highway funds. The ratings also provide a means which can be demonstrated to the public in explanation of construction priorities. Graphical presentations are frequently used.

42. FINANCING HIGHWAY FACILITIES

Sources of Funds for Highways. Appropriations for highway and street purposes are ordinarily made by all subdivisions of government. Local funds for highway purposes are usually derived from property taxes of rural agencies, counties, villages, and cities. State funds for highway purposes are derived chiefly from imposts upon highway users, including motor-fuel taxes, vehicle-registration taxes, and motor-carrier taxes. Federal funds can be considered to be derived from federal taxes on motor fuel and excise taxes on the sale of vehicles.

Credit Financing. It is frequently necessary and desirable to accelerate the construction of highways or terminals in order to provide benefits which would not be achieved otherwise. Some of the obvious advantages are the earlier realization of savings to road users and benefits to the community, and the reduction of the need for stop-gap improvements of existing inadequate facilities. It was calculated, for example, that an accelerated highway program for the state of Ohio could be financed through the issuance of bonds during a 10-year construction period, and the retirement of the bonds with interest at 2% during the following ten years, without increasing the annual revenue requirements.

Loans may be *general obligations* guranteed by the faith and credit of the borrowing agency or *limited obligations* guranteed by pledges of user taxes, toll revenues, or other income.

In some states where constitutional limits restrict borrowing for highway financing or where other hindrances exist, special authorities have been created for highway construction. These authorities may be concerned specifically with toll highways and bridges or may be authorized to construct toll-free facilities. An example of the latter is the State Highway and Bridge Authority of Pennsylvania, which is empowered to acquire highways, bridges, and right-of-way, to construct highway facilities, and to rent these facilities to the Department of Highways.

Toll Roads and Toll Bridges. Tolls are levied to finance special highway facilities or to supplement other tax revenues. In the face of popular opposition the tendency to impose tolls for the latter purpose is diminishing. On the other hand there is now occurring a dramatic increase in the development of toll-road projects. The repayment of loans for

many of these projects is predicated solely on revenues from tolls. In other instances the full faith and credit of the state or other government agency having jurisdiction is pledged to supplement toll revenues if needed. Where this is done the interest rates on the loans are significantly lower, the difference is estimated to be as high as 2% in many cases.

When revenues alone are pledged for the amortization of bonds, it is commonly necessary, in order to sell the bonds, for the estimated earnings to exceed the interest and amortization requirements by a sizable margin.

The first step in the development of a toll facility is to secure legislative authorization. Estimates must then be made of the traffic which would use the facility, of the revenue which would be derived, and of the annual costs. If the project is found to be feasible, bond indentures are prepared and means are found to market the bonds. A unique plan was used for financing the New Jersey Turnpike in order to minimize interest charges during construction and to guarantee the sale of bonds at par. By this plan a group of insurance companies and other institutional investors advanced funds only as fast as needed, receiving bonds at face value to cover the advances. A small fee was made to cover the commitment of funds prior to the actual advance.

PARKING

43. THE PARKING PROBLEM

As a city grows the competition for the use of land increases at an accelerated rate, creating the problem of adequate space for parking purposes. In central business districts of metropolitan areas shortages of space are particularly critical because of the importance of those areas as traffic generators. The problem must be approached through the most practicable use of curb spaces, supplemented where feasible by the development of off-street facilities, and it must be related to the capacities of the highway approaches.

44. PARKING ACCUMULATION

The number of vehicles which are parked in a specific area at a given instant is the *parking accumulation*. A survey of this accumulation is facilitated by an inventory of curb space, showing locations and the regulations in effect, and by an inventory of off-street space showing locations, types, and fees. In the case of garages and lots, the observer

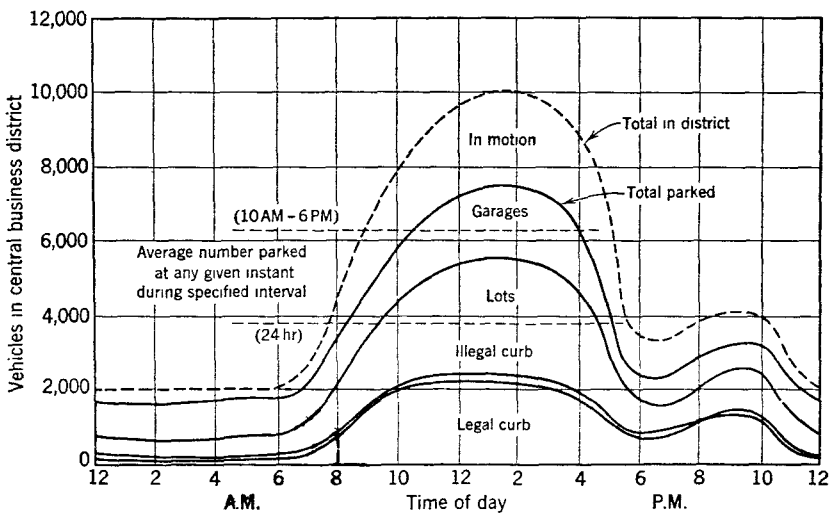


FIG. 43. Typical vehicle accumulation curves for a central business district.

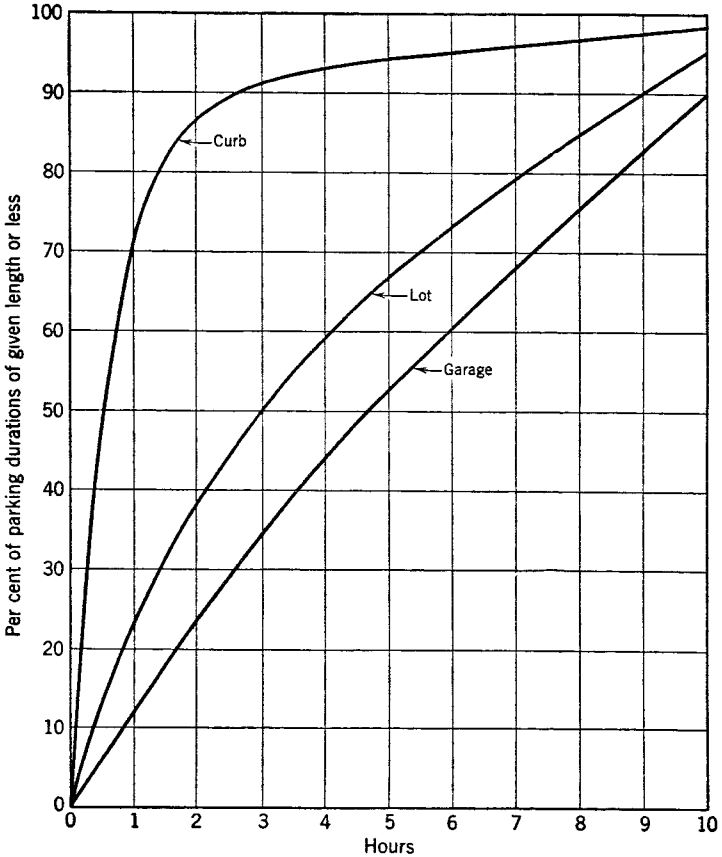


FIG. 44. Parking durations in various types of facilities.

may count the number of marked stalls or consult a layout plan. A lot should be capable of accommodating 1 car for each 300 sq ft, more or less, of the overall area including aisles. Observations as to the use of such space by parked vehicles usually include the type of vehicle, time of arrival, time of departure, and violations incurred.

The parking accumulation within a central business district during peak hours of parking is ordinarily from 70% to 90% of the amount of traffic accumulated. The ratio is slightly less during those times when the rate of change in accumulation is high. Characteristic curves for a typical central business district are shown in Fig. 43. The general shape of the curve for a given district is significant. The peak value attained is a measure of the capacity required for the accommodation of vehicles within the district. The total area under the curve is a measure of the vehicle hours which are accommodated during the period of study.

45. PARKING VOLUMES AND LOADS

The actual number of different vehicles which park is the *parking volume*. The integration of the vehicle hours of parking over a specified period of time is the *parking load*. The magnitude of the parking load in vehicle hours and the manner in which the load is distributed throughout the day are measures of the overall usage of the space provided. Parking loads and parking volumes reflect the cyclical and trend variations which are associated with traffic volumes, but, of course, there is a difference in time between peak flows and peak parking accumulations. Both parking loads and parking volumes per 1000

population are many times more severe in the central business districts of smaller cities than of larger ones. However, this is an indication of actual practice rather than of potential demand and reflects the differences in available space, density of land use, and modes of transportation.

46. PARKING DURATION

The length of time a vehicle remains in a parked position is the parking duration. Characteristic curves are shown in Fig. 44. Durations are usually shorter for curb parking than for off-street parking. The influence of trip purpose and city size on parking durations and the percentage occurrence of curb parking durations of various lengths are shown in Tables 49 and 50.

Table 49. Effect of Trip Purpose and City Size on Parking Durations

Population Group, thousands	Number of Cities	Average Time Parked for Each Trip Purpose, hr				
		Work	Shopping	Business	Other	All Purposes
Less than 25	5	3.1	0.7	0.7	1.1	1.1
25–50	3	2.9	0.7	0.8	0.9	1.3
50–100	2	3.3	0.8	0.7	0.9	1.3
100–250	5	4.0	0.9	1.0	1.5	1.7
250–500	3	4.5	1.4	1.2	1.5	1.8
500 and over	2	5.1	1.4	1.4	1.2	2.5

Source: *Highway Research Board Bull.* 19, Parking July 1949.

Table 50. Length of Time Vehicles Parked at Curb in Central Business Districts of Selected Cities

City and Population	Per Cent of Total Number of Curb-Parked Vehicles						
	0–15 min	15–30 min	0–30 min	30–60 min	0–60 min	1–2 hr	0–2 hr
Bakersfield, Calif. 30,000	...	...	67	16.4	83.4	10.5	93.9
Denver, Col. 325,000	41	21	62	20.8	82.8	12	94.8
New Haven, Conn. 175,000	39.5	26	65.5	21.2	86.7	9.2	95.9
Wichita, Kan. 115,000	51.6	10.4	62	15.7	77.7	...	...
Portland, Ore. 305,000	28.1	20.4	48.5	24.2	72.7	17.3	90.0
Harrisburg, Pa. 90,000	...	...	52.5	15.3	67.8	9.6	77.4
Reading, Pa. 110,000	26	25.8	51.8	23.2	75.0	13.4	88.4
Providence, R. I. 250,000	31.5	25	56.5	22.6	79.1	13.1	92.2
Richmond, Va. 200,000	36.1	17.1	53.2	18.2	7[illegible].4	12.7	84.1
Spokane, Wash. 150,000	17.6	30	47.6	20.4	68.0	14.4	82.4
Average 191,000	33.7	20.3	54	20	74.0	13.3	87.3

47. PARKING TURNOVER

The *parking turnover* is the ratio of the number of cars parked during a period to the stall capacity. High turnover is desirable, both for the revenue and because it relieves the parking problem. On a daily basis, the turnover in garages may range from less than unity to 10 or more. When curb regulations are rigidly enforced curb stalls may have average turnovers of 8 or more.

48. GENERATORS OF PARKING DEMAND

Few definite and reliable criteria have been developed for the parking demand created by various land uses Each new situation must be analyzed separately, considering all the local conditions which affect the demand. Illustrative parking space requirements are given in Table 51.

Table 51. Effect of Land Use on Required Parking Space *

Land Use	Gross Floor Area per Parking Stall, sq ft	Other Units of Measure per Parking Stall
Industrial plant	4223	6.9 employees
Bus terminal	1917	
Retail mail-order house	1816	
Office Building *A*	1628	
University	1401	7.6 students
High school	1263	7.5 students
Hotel	1013	2.7 guest rooms
Hospital (private)	934	1.9 beds
Office Building *B*	818	
Neighborhood shopping center	813	
Department Store *B*	686	518 sq ft selling area
Department Store *A*	475	283 sq ft selling area
Theater	318	19.1 seats
General market	199	
Department Store *C*	180	
RR passenger station	95	

* Actual demand in a city where 43% of all trips are made by automobile.
Source: Adapted from Thompson and Stegmaier, Effect of Building Space Usage on Parking Demand, *Highway Research Board Bull.* 19, 1948.

49. DISTANCE WALKED

The location and amount of parking space is not generally coincident with the location and amount of parking demand in central business districts. Figure 45 typifies this lack of congruity and emphasizes the frequent necessity for road users to leave their vehicles at points removed from their ultimate destinations and to complete their trips as pedestrians. Motorists do not like to walk more than 500 ft from where they are able to park, but under certain conditions they will walk 1000 feet, or more They will walk farther in large cities than in small cities, and they will walk farther when they plan to park longer. However, even in the largest cities only a minority of road users will park more than 2 or 3 blocks from their ultimate destinations, as shown in Fig. 46.

50. SURVEYS OF PARKING DEMAND

Parking demand surveys are usually made by the methods used for origin and destination studies of traffic, but they require greater localization of origins and destinations than ordinarily needed for traffic movement studies. These survey methods include the questioning of an adequate sample of parkers at the curb and in off-street facilities and the mailing of return-addressed, prestamped or business-reply questionnaire postcards to the residences of road users or the placing of such cards on vehicles at the curb and in off-street locations When postcards are placed on parked vehicles, the spaces should be checked at regular intervals during the survey period to insure coverage of all or most of the vehicles using the spaces. The survey data may be analyzed as for traffic movement studies.

51. REGULATION OF CURB PARKING

The parking of automobiles on main thoroughfares and the use of those thoroughfares for loading and unloading purposes provide conveniences for only a few users at the expense of greatly reduced capacities of the thoroughfares, increased accident frequency, and the aggravation of other municipal problems. In intensely developed areas, only a relatively small portion of the parking demand can be handled at the curb. Nevertheless, there are few urban centers where curb parking can be prohibited without serious disruption of the

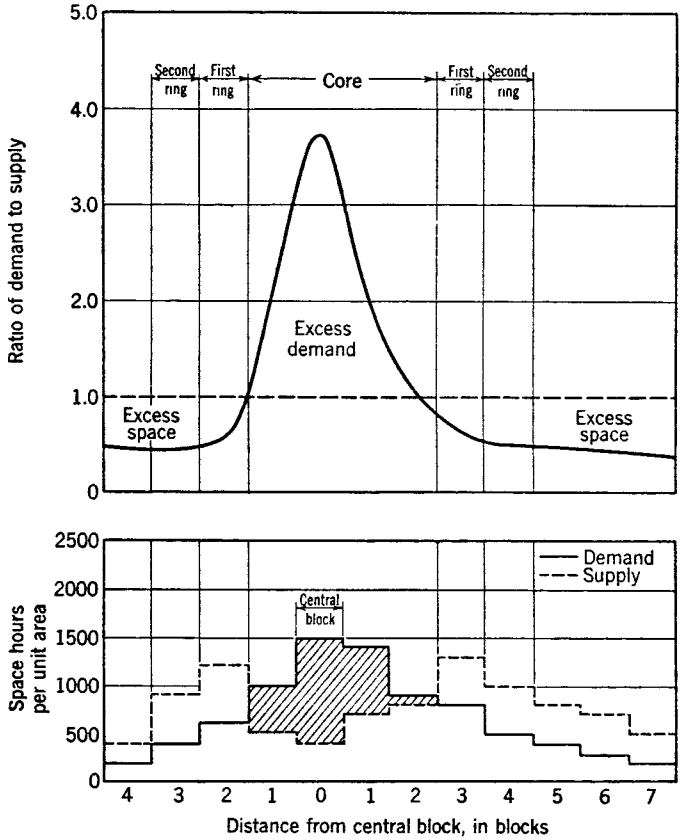


FIG. 45. Effect of location on space-demand relationships.

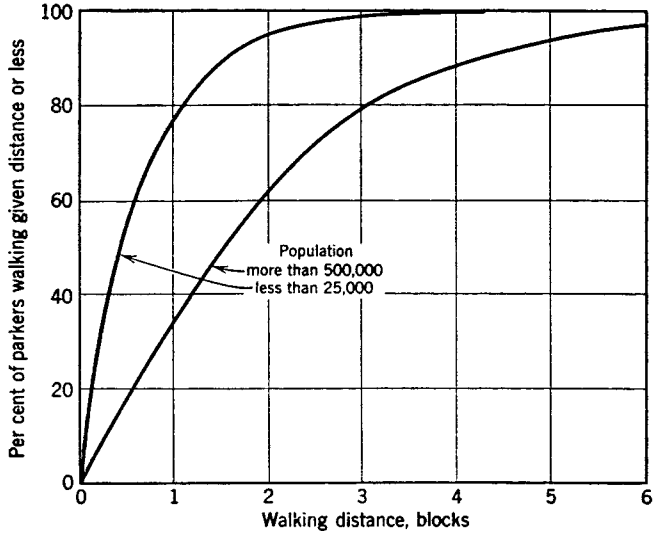


FIG 46 Walking distance of parkers.

business and commercial activities of the area unless equivalent parking spaces are simultaneously provided.

The Effect of Parking on Street Capacity. Curb parking seriously affects street capacity through the narrowing of the roadway available for moving traffic, coupled with frictions created by cars maneuvering into and out of parking spaces. The decrease in capacity varies with street widths, the character of the community, enforcement policies, and other factors, but in all cases marked reductions in capacity develop when curb parking is allowed. Angle parking requires greater street area for maneuvering, and therefore decreases the capacity still further. Curb parking also tends to add to the street traffic load by the cruising of motorists in search of curb parking spaces; in some cases more than 7% of the total number of cars on a retail shopping street were found to be in this category.

The Effect of Parking on Accidents. A high percentage of accidents on downtown city streets involves vehicles which are in the process of parking or unparking or result from maneuvers necessary to avoid collisions with parking or unparking vehicles. In 1949 a parked car was itself in collision in almost 16% of all urban accidents reported. The accidents caused by parking maneuvers, but not involving a parking or unparking vehicle, are difficult to identify from accident records but may range from 2% to 20% of all accidents reported. More than one-third of the pedestrian injuries involve persons struck when walking from behind parked cars. The hazard created by curb parking practices is considerably greater for angle parking than it is for parallel parking. On streets with heavy curb parking demands significant reductions in accident frequency have been reported by several cities following the replacing of angle parking with parallel parking.

Space Control. Legal authority for space control, whether for public safety or public convenience, is covered in state motor vehicle codes. Included are such things as prohibiting parking on sidewalks, in driveways, within intersections, near fire hydrants, on the immediate approach to traffic-control devices, opposite pedestrian safety zones, on the traveled portion of rural roadways, near railroad grade crossings, and in similar places. Space restrictions which are apt to vary with local conditions are usually referred to municipal authorities for legislative enactment.

Parking regulations ordinarily provide that parking shall be parallel to the curb unless parking at an angle or other types of parking are specifically designated by signs or markings. In many downtown districts it is necessary to set aside curb spaces to be used exclusively for loading and unloading of commercial vehicles. Specific curb zones are also designated for exclusive use of taxis. Other zones are designated as bus stops to permit transit vehicles to leave the lane of travel for the loading and discharging of passengers. Minimum desirable lengths of bus stops are shown in Table 52.

Table 52. Minimum Desirable Bus-Stop Lengths

(In feet)

Approximate Bus Seating Capacity	Approximate Bus Length	One-Bus Stop			Two-Bus Stop		
		Near Side	Far Side	Mid-block	Near Side	Far Side	Mid-block
25 passengers or less	25	60	50	85	90	80	115
30 passengers	25	70	50	95	100	80	125
35 passengers	30	75	55	100	110	90	135
40 passengers and more	35	80	60	105	120	100	145

To find the space necessary for additional buses, add the bus length plus 5-ft clearance between buses.

Source: *Traffic Engineering Handbook*, Institute of Traffic Engineers, 1950.

Time Controls. Time controls usually limit maximum parking durations to 30 min in central business districts and to 1 hr in retail shopping areas. Limits of 10 to 15 min may be desirable for banks, post offices, and other special traffic generators. On the fringes of retail districts 2-hr limits may be applicable. On streets where peak-hour traffic flows are significantly unbalanced, the volumes may warrant the prohibition of parking during peak hours on the side of the street bearing the predominant flow. Most cities have regulations controlling all-night parking. The Model Traffic Ordinance contains the following suggested regulations:

> No person shall park a vehicle on any street for a period of time longer than 30 min between the hours of 2 A.M. and 5 A.M. of any day, except physicians on emergency calls.

The intent is not to forbid any parking at the curb during the night hours but to prevent the use of the curb for all-night storage of vehicles, which interferes with street cleaning and snow removal, fire-fighting operations and the like.

Parking Meters. The right to charge a fee for the use of curb space for parking is now generally attributed to the fact that such parking is considered to be a privilege as opposed to the right to travel on the roads or streets. The parking meter was developed as an aid to the enforcement of parking regulations; it was first used in Oklahoma City in 1935 and since has become commonly accepted. It is held to be a legal device to aid the enforcement of curb parking regulations, but sizable sums are collected from its use. Typical city meter receipts are shown in Table 53.

Table 53. Parking Meter Receipts, 1949

City and Population Group	Number of Meters	Receipts	Receipts per Meter	Fees
Over 500,000				
Washington, D. C.	3089	$218,829	$ 71.00	Nickel—1 hr
Detroit, Mich.	3407	207,978	61.00	Nickel—1/4, 1/2, 1, 2 hr
250–500,000				
Oakland, Calif.	4910	398,393	81.14	5 pennies or nickel—1 or 2 hr
St. Paul, Minn	1331	144,618	108.65	Nickel—1/2, 1, 2 hr
100–250,000				
Fort Worth, Tex.	2721	201,061	73.90	Nickel—20 min, 1 or 2 hr
Hartford, Conn.	1358	65,483	48.20	——
50–100,000				
Augusta, Ga.	960	79,500	82.80	5 pennies or nickel—1 or 2 hr
Portland, Me.	853	83,661	98.10	Penny—12 min or nickel—1/2, 1, 2 hr
25–50,000				
Kenosha, Wis.	618	46,063	74.50	5 pennies or nickel—1 hr
Muncie, Ind.	387	35,543	91.80	5 pennies or nickel—1 hr, nickel—2 hr
10–25,000				
Selma, Ala.	572	33,706	58.90	5 pennies, nickel—1 or 2 hr
Hickory, N. C.	400	35,614	89.00	5 pennies, nickel—1 hr

The use of parking meters on public ways was banned in North Dakota in 1948, and the ban was upheld by the Supreme Court of North Dakota. A legislative act of 1949 in Montana specifies that no ordinance providing for the use of parking meters can be enacted in any town of 2500 population or less, unless the qualified electors of the town shall vote in favor of such an ordinance.

52. CURB PARKING IN URBAN AREAS

Parallel parking interferes least with the movement of traffic and permits direct access between the curb and parked vehicles. Center street parking is occasionally permitted where streets are very wide. Although angle parking at usual angles accommodates more vehicles at a given length of curb, it is generally undesirable because of the width of the roadway which is required for maneuvering and because of the interference created in the traffic stream. The unparking maneuver from an angle parking position must be partially undertaken with restricted vision; the driver usually eases his vehicle into the traffic stream until he can see the approaching traffic. This causes deflection of moving traffic and increases the frequency of collisions. Consequently, angle parking should be used only on low-speed urban streets where parking requirements take precedence over through-traffic movements.

53. PARKING ON RURAL ROADS

Parking control along rural roads requires carefully designed engineering plans which reduce hazards to a minimum. Turnouts for commercial buses, school buses, and mail carriers are highly desirable features. Specially designed parking areas should be provided at scenic observation areas, public picnic grounds, wayside business sites, and other points where parking may be expected

54. OFF-STREET LOADING

The stopping of buses and trucks for loading and unloading passengers and cargo is a type of parking insofar as its effects are concerned. Unfortunately little consideration has been given in the past to off-street loading, with the result that such facilities are now usually inadequate or non-existent. Suggested designs for loading docks for trucks are shown in Fig. 47. The distance from the pavement to the floor of the transport bed averages 51 in. for heavy equipment, varying within a 6-in. range, depending on the vehicle design, the tire sizes used, and the deflection of the transport springs under load. Dock heights of 48 to 54 in. are suitable for most heavy-duty equipment. It is preferable have to

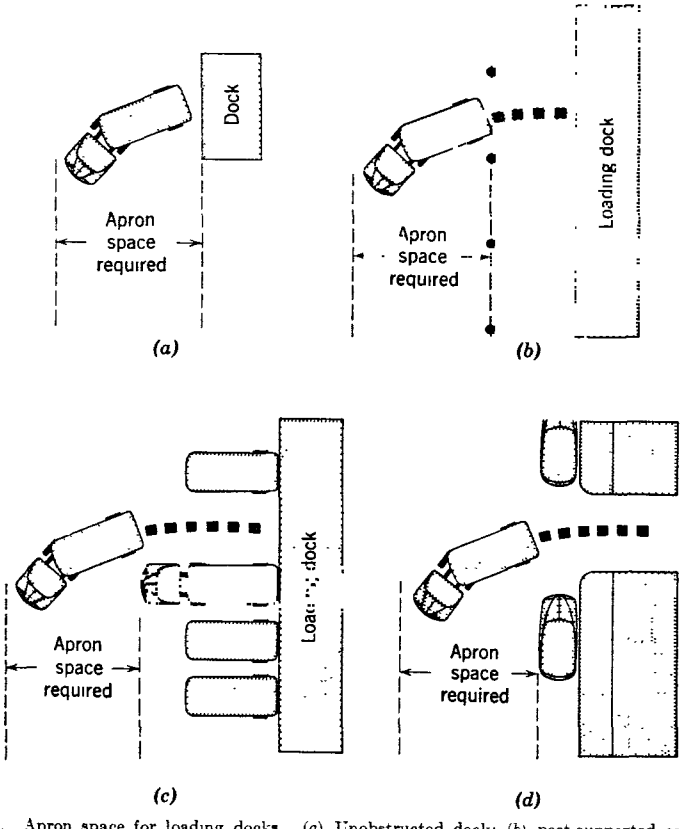


FIG. 47. Apron space for loading docks. (*a*) Unobstructed dock; (*b*) post-supported canopy, (*c*) alongside other vehicles, (*d*) driveways and stalls. Source: Fruehauf Trailer Co.

"Apron" Space Required by Tractor-Trailer for One Maneuver into or out of Position (Measured from Outermost Vehicle or Other Obstruction)

Overall Length of Tractor-Trailer	Width of Position	"Apron" Space Required
35′	10′	46′
	12′	43′
	14′	39′
40′	10′	48′
	12′	44′
	14′	42′
45′	10′	57′
	12′	49′
	14′	48′

the dock level lower than the transport bed to avoid interference with the opening and closing of rear doors of the transport. Widths of dock positions should be 12 ft, and the minimum vertical clearance should be 14 ft.

55. DIMENSIONS OF PARKING STALLS

The layout of parking stalls should provide for convenient movement of cars entering and leaving the spaces and for convenient alighting and boarding by passengers and drivers For vehicle design dimensions of 6 ft 4 in. width and 18 ft length, parking stalls parallel to the curb should be 8 by 20 ft. Recommended dimensions of stalls and widths of parking aisles for various parking angles and back-in and drive-in parking directions are given in Table 54. A parking stall width of 8 ft is adequate when vehicles are parked by trained attendants, but a width of 8 ft 6 in. is recommended when vehicles are parked by their drivers.

Table 54. Recommended Stall and Aisle Dimensions

Angle of Parking	Direction of Parking	Width of Stall	Depth of Stall Perpendicular to Aisle	Width of Aisle	Unit Parking Depth	Width of Stall Parallel to Aisle	Area per Car, sq ft
90°	Back-in	8′ 0″	18′ 0″	22′ 0″	58′ 0″	8′ 0″	232
60°	Back-in	8′ 0″	18′ 10″	17′ 4″	55′ 0″	9′ 3″	254
45°	Drive-in	8′ 0″	17′ 2″	12′ 8″	47′ 0″	11′ 4″	266
90°	Back-in	8′ 6″	18′ 0″	22′ 0″	58′ 0″	8′ 6″	247
60°	Back-in	8′ 6″	18′ 10″	17′ 4″	55′ 0″	9′ 10″	270
45°	Drive-in	8′ 6″	17′ 2″	12′ 8″	47′ 0″	12′ 0″	282

56. LAYOUTS OF PARKING AREAS

Where parking and unparking is performed by attendants, stalls at 90° to the aisles are preferred because they require the least space and can be fitted better into rectangular areas. Although layouts requiring the parking of cars in double rows permit more cars to be stored in a given area than would be possible if each car were parked directly off an aisle, such layouts require attendants to move front-row cars in order to unpark rear-row cars.

Customers prefer parking at angles of 75° or less but are not averse to 90° parking where adequate aisle and stall widths are provided. At times, acute-angle parking is dictated by the dimensions of the parking area. Two methods of intermeshing stalls at acute angles to conserve space are shown in Fig. 48.

Complete circulation should be allowed within the confines of large parking areas. The travel on all aisles of an area should be one way, and the aisle system should require only the minimum number of turns and the minimum travel.

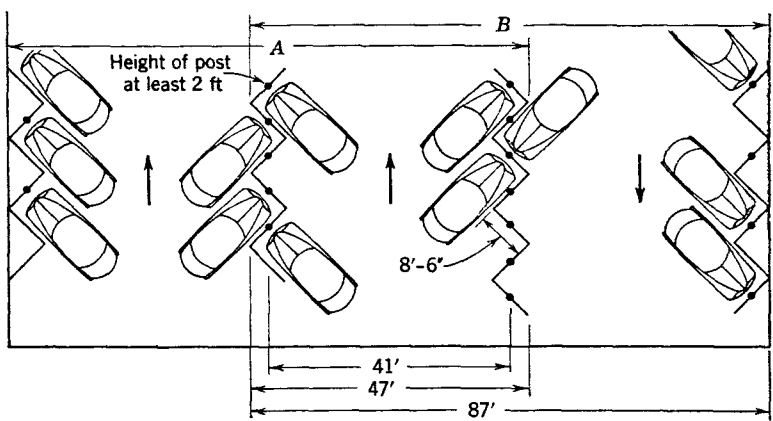


FIG. 48. Parking-lot layouts Herringbone pattern, 45°. Adapted from AAA *Parking Manual*, 1946.

57. PARKING LOTS

The most common type of off-street parking facility is the parking lot. Many municipalities require that lots be enclosed with appropriate fences or other barriers so that vehicles can be moved only through designated entrances and exits. Most cities specify minimum distances permitted along the sidewalk between driveways on the same street. Frequently the locations of driveways with reference to street intersections are also limited. It is desirable to provide landscaped areas, hedges, and other aesthetic treatments where possible.

Lots may sometimes be located advantageously in the interior of blocks where the property is comparatively less valuable than the frontages of the same blocks. In some places where block interiors are used, merchants have installed display windows for the attention of parking-lot users and have constructed entrances for their convenience.

When sites are provided in the fringe areas of business districts, bus service is desirable between the lots and downtown areas. However, such sites are more valuable for long time parking than for short time parking.

In many cases parking lots are developed temporarily on sites which are to be used subsequently for more profitable purposes. The impact of the ultimate withdrawal of such parking spaces should not be overlooked.

58. PARKING GARAGES

Types of Parking Garage. The *ramp* garage utilizes inclined driveways between floors. The *inclined parking floor* garage combines the ramp feature with the parking floors.

Improved elevators in combination with other mechanical devices are finding ingenious applications in *mechanical systems* for parking. Some systems use *elevator-and-crane* devices Each device moves laterally in a central bay flanked by tiers of storage cubicles. While the car is being raised or lowered, the elevator is guided horizontally. The car may be moved to and from the elevator automatically. A fully *automatic parking* device employs elevators equipped with car positioners, which move the cars to and from the elevator. A proposed mechanical system employs independent cages, each capable of holding one car, which are moved vertically in two adjacent columns and interchanged between the two columns at their tops and bottoms.

Ramp Types and Layouts. Hazards, conflicting movements, economy of space, efficiency of operation, and costs of construction are important considerations in the design and layout of ramps. *Clearway ramps* are those used exclusively for interfloor travel. *Adjacent parking ramps*, as the name implies, are inclined parking floors. Although adjacent parking increases the conflicts, it requires less area per car than if clearway ramps are provided and it has been found to operate satisfactorily where the garage sites are small. Ramps may be constructed with either *straight* or *curved* alignments, the actual paths of vehicle travel being curved in either case. With straight ramps, turning must be performed on the floors at the tops and bottoms of the ramps. Floors may be continuous or staggered. With staggered floor systems each ramp is usually one-half the length required for normal floor to floor clearance, but, of course, twice as many ramps are required.

Warped surfaces should be used to ease the transition between the ramps and the floors. Superelevation cannot be provided satisfactorily with straight ramps. Although curved ramps are better suited to vehicle movements and are, therefore, more comfortable for drivers, their use is generally limited because they are more costly than straight ramps.

The travel paths of vehicles moving up from floor to floor may be *concentric* about the paths of vehicles moving down or they may be *tandem*. If the up-and-down ramps are part of the same or *parallel* surfaces the paths of travel for up-and-down movements are in opposite directions. If the ramp surfaces are sloped in opposite directions, *opposed*, vehicles traveling up and down rotate in the same direction. Some of the more common geometric ramp arrangements are shown schematically in Fig. 49.

Parking-Garage Design. The most economical structural type of garage is the *open deck*, which is equivalent to a vertical series of parking lots, each with a low parapet around its periphery to serve as a barrier and to give a sense of security. Typical floor layouts are shown in Fig. 50. The architectural appearance of an open-deck garage may be rendered pleasing by various treatments, but such beautifications should not enclose the garage to the extent that forced ventilation is necessary. Areas that are enclosed, such as levels below the surface of the ground, should be adequately ventilated to avoid carbon monoxide poisoning.

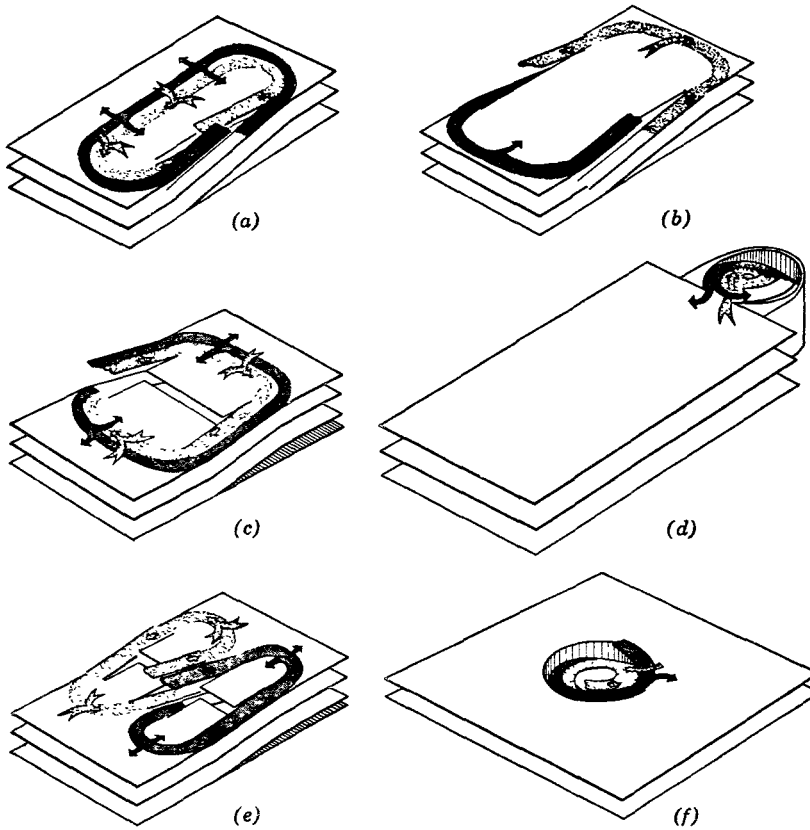


FIG 49 Typical ramp arrangements for parking garages (*a*) Opposed straight ramp system, adjacent parking type, (*b*) opposed straight ramp system, clearway type, (*c*) two-way staggered floor ramp system; (*d*) parallel circular ramp system; (*e*) one-way tandem staggered floor ramp system, (*f*) opposed circular ramp system From. *The Traffic Design of Parking Garages*, by Edmund R Ricker, The Eno Foundation for Highway Traffic Control, 1948.

Structures may be designed for stage construction, permitting the addition of levels as required. The ground floors ordinarily contain entrances, exits, reservoir space for incoming and outgoing cars, cashier's booth, attendants' office and customers' waiting room. Along desirable frontages a portion of the ground floor might be laid out for stores or other commercial uses. Entrances and exits should be few in number, located as far as possible from street intersections and oriented to favor right turns. Entrances should be ample in width to avoid the impression of restriction. Exits should allow adequate sight distances at sidewalk crossings. The reservoir space required for various vehicle arrival rates is shown in Fig. 51. Elevators and stairs should be provided for interfloor travel. Endless-belt elevators may be installed for attendants' use in attendant parking garages. Where ramps are used they should lead directly from the reservoir space to upper levels and to the basement. A heating plant should be provided for heating the cashiers' booths, offices, stores, and customers' waiting rooms. Blower systems should be provided for basements or other areas where an aid to ventilation may be needed. In attendant parking garages interfloor call systems should be installed. Other suggested criteria are listed in Table 55.

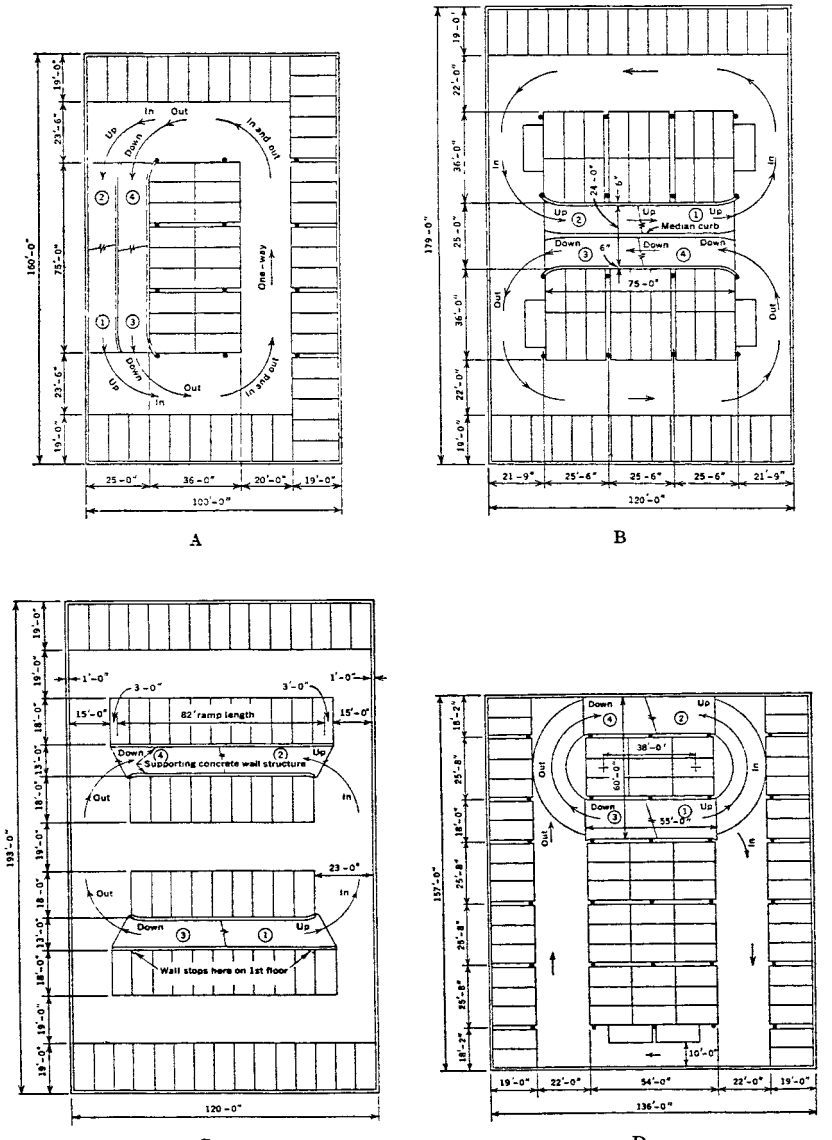
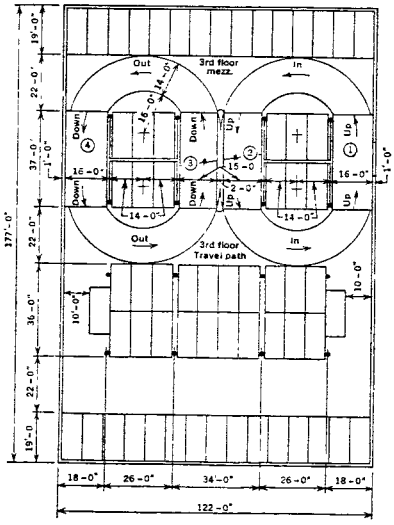

A
In
Out
Up
Down
In and out
One-way
160'-0"
19'-0"
23'-6"
75'-0"
25'-0"
36'-0"
20'-0"
19'-0"
100'-0"
B
179'-0"
22'-0"
36'-0"
25'-0"
10'-0"
24'-0"
6"
Median curb
75'-0"
21'-9"
25'-6"
120'-0"
C
193'-0"
1'-0"
3'-0"
15'-0"
82' ramp length
Subporting concrete wall structure
23'-0"
Wall stops here on 1st floor
120'-0"
D
137'-0"
16'-2"
25'-8"
18'-0"
38'-0"
60'-0"
59'-8"
10'-0"
19'-0"
22'-0"
54'-0"
136'-0"

FIG 50



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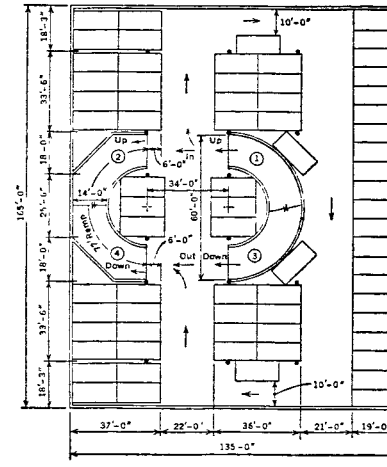

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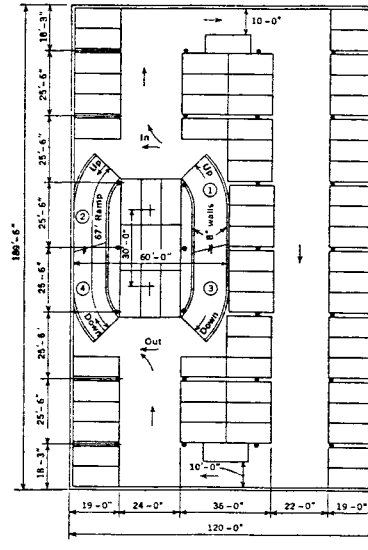

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Fig 50. Examples of floor layouts for various parking garage ramp systems (A) Opposed straight concentric ramps; (B) parallel straight tandem ramps, (C) opposed straight clearway ramps, (D) opposed straight, mostly clearway, ramps; (E) tandem ramps, staggered floor system; (F) circular opposed clearway ramps, (G) elliptical opposed ramps Source: *R/C Bulletin*, published by Portland Cement Assn.

Table 55. Suggested Criteria for Parking Garage Design

Minimum size of unit	200
Maximum size of unit	Dependent on parking demand but not so large as to overtax the street system during peak flows.
Column spacing (when clear span not practicable)	27′ 6″
Ceiling height for first floor	12′
Maximum ceiling height, other floors	7′ 6″
In-bound reservoir space	Four 12-ft lanes
Out-bound reservoir space	Two 12-ft lanes
Maximum grade for ramps	
Attendant parking	15% preferred; 20% absolute
Self-parking	10%
Minimum width straight ramp	
Direct approach	11′
Sharp turn approach	18′
Minimum clear size for stalls	
Attendant parking	8′ x 18′
Self-parking	8′ 6″ x 18′
Minimum width of parking aisles (for 90° parking)	22′
Minimum inside turning radius for aisles and ramps (curves on level portions only preferred)	20′
Maximum curb height	6″
Minimum curb width	
Side curbs	12″
Straight center section	12″
Curved center section	18″
Fireproofing required	Yes
Service facilities	Car service or minor adjustments
Number of attendants	Enough to deliver cars at the rate of customer arrivals in average peak hour

Source: Institute of Traffic Engineers, *Traffic Engineering Handbook*, 1950.

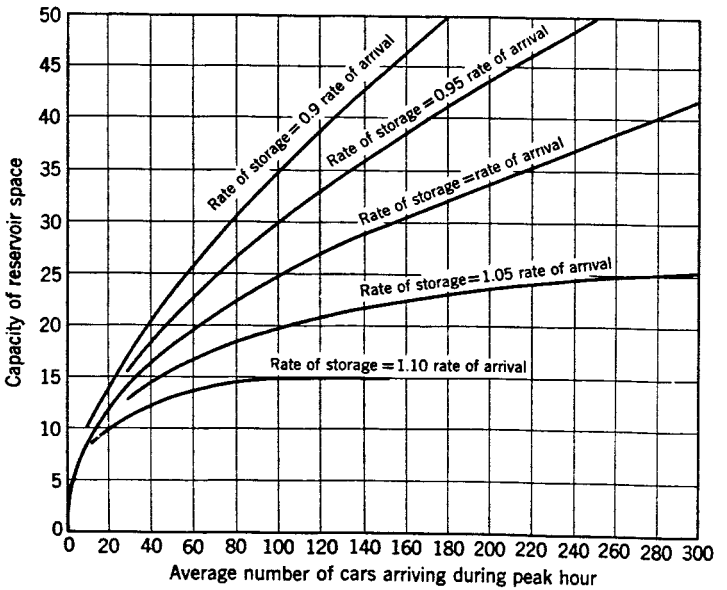

FIG. 51. Parking garage reservoir space required for various vehicle arrival rates, if overloaded less than 1% of time. From: *The Traffic Design of Parking Garages*, by Edmund R Ricker, The Eno Foundation for Highway Traffic Control, 1948

59. THE OPERATION OF PARKING FACILITIES

Parking-Lot Operation. Parking in some types of lots may be administered effectively with the use of parking meters. Where meters are not used the operation of parking lots is generally similar to the operation of parking garages.

Parking Garage Operation. Parking in garages may be performed by attendants or customers. With attendant parking, a minimum of space is used per car and consequently more cars can be stored in a given area than with customer parking. Customers are not involved in the time-consuming process of parking. On the other hand, attendants may not be able to deliver the cars as fast as customers call for them and delays may result.

With customer parking, fewer attendants are used; the customer handles his car entirely and has the protection of locking his car. There is less possibility of delay in picking up cars but there is a danger of discharging cars from the garage at a rate which would interfere with the movement of traffic on the streets.

A *combined system* similar to customer parking involves the driving of cars to and from the storage floors by the customers, the actual parking and unparking on the storage floors being performed by attendants. With this system, space-saving features are gained, attendants' time is not required driving on ramps, and the temporary storage space at the entrance is reduced.

SECTION 4

HIGHWAY ENGINEERING

BY

WILLIAM J. COX AND ARTHUR W. BUSHELL

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HIGHWAY ENGINEERING [1]

HIGHWAY LOCATION

1. BASIC CONSIDERATIONS

Introduction. The "location" of a highway is the position occupied by the center line of the road surface. The horizontal component of the location is called "line"; the vertical component, "grade." Except in flat areas, the best line for a road cannot be established without considering the grade which a road so placed will have. However, the final line is generally established before the final grade, the consistency of the proposed line with satisfactory grades having been determined. Determination of the exact grade is a part of highway design rather than location.

No road can be better than its location. An inadequate pavement can be strengthened or widened. Excessive grades can often be bettered. But alignment, once established, is frequently difficult or impossible to change without prohibitive disturbance of land uses developed along the new road. The use that will be made of a road will ordinarily dictate standards of line (as well as of grade, pavement type, and width, and so on) that can be afforded. When design standards have been adopted for any particular project, it is imperative to select the best location consistent with those standards. If funds are limited, the least amount of compromise from desirable standards should be made with respect to horizontal alignment.

Traffic studies and availability of funds will jointly determine geometric characteristics of the road and also the type of surface to be built. The geometric standards to be incorporated into the road must be agreed upon before location surveys are commenced.

Governing Factors in Highway Location. Ideally a road location should offer the following advantages:

1. Directness.
2. Low grades
3. Easy curves.
4. Good sight distances.
5. Good foundation.
6. Proper drainage.
7. Easy excavation, where required.
8. Available road-building materials.
9. An adequately wide right-of-way.
10. Suitable stream-crossing sites.
11. Desirable sites for intersections with railroads and other highways.
12. Freedom from slides and from bad snow and ice conditions.

The factor of directness, i.e., minimum length, is very important in highway economics.[2] No general relationship between center-line and air-line distance between two termini can be established, and directness cannot be considered independently of other factors. However, directness should always be considered

[1] This section is a companion to Sect. 3, Traffic Engineering, which includes highway systems, planning of highway projects, the traffic studies which should precede location and design, and the development of geometric standards (of sight distance, horizontal curvature, grade, banking and widening of curves, limitation of access from abutting property, etc.) for any particular road.

[2] If a road carries 5000 vehicles a day, an extra mile of length will result in an additional 5000 vehicle miles of travel per day. If the operating cost of this extra travel is 5 cents a vehicle mile, the added cost of travel is 5000 × 0.05 = $250 per day, or $91,250 per year. This can be conservatively capitalized and still produce a very large figure for the additional right-of-way and construction outlay that can economically be made to shorten the route. On the other hand, of course, if the road will serve only 50 vehicles per day, shortening the route by a mile will reduce travel costs by only $913 per year, and this will afford much less justification for a high initial cost

Where truck traffic is important grades not exceeding 3% are desirable. Snow and ice hazards limit safe grades for passenger cars to 6 or 7%. The satisfactoriness of a grade is affected by its length as well as its steepness. The maximum allowable degree of curve for a road is dependent on the design speed and is usually limited by the maximum allowable bank. Whatever maximum degree of curve is specified, the locating engineer should strive for reasonable consistency. A curve much sharper than others on the same road will almost invariably be a place of accident occurrence. Sight distance is a function of design speed and curvature (or difference in grades), jointly. All the foregoing geometric controls on location are discussed in Sect. 3.

No road surface is better than its foundation. The locator should, insofar as practicable, avoid areas of poor soil and poor drainage. He must, moreover, give careful attention to drainage conditions—and sometimes to soil conditions also—in establishing a sufficiently high grade line. The original construction cost may be considerably affected by the nature of the material to be excavated. Avoidance of boulders or ledge rock may govern the location, or, alternatively, may determine the proper grade line. The nearness of suitable embankment material to the road location will affect construction cost. Good stream-crossing sites must be found, and also suitable spots for intersecting railways and other highways. Public safety will be enhanced and maintenance costs reduced by a location free from slides and where ice will melt rapidly and snow will not drift badly. Finally, since roads are built to serve traffic, the location should facilitate the development of the area traversed.

Only rarely can all of the desirable features be found in one location. Consequently the location selected will usually represent a compromise.

Choice between Alternative Routes. There are two kinds of cost in a road: the cost—to the public—of building and maintaining the road; and the cost—to individual motorists—of using it. Where more than one possible location exists, each location should be considered from both standpoints.

"Benefit-Cost Ratio" Method of Comparison. A period of years is selected as being the length of time within which the cost of building and maintaining the road should be returned to road users in the form of benefits received by them through having the proposed road to use in place of the best existing road. Thirty years is often considered a reasonable period. For the 30-yr period, the average annual volume of traffic to be anticipated on each of the proposed routes is estimated.

Each proposed location is then compared, in length and in estimated travel time and sometimes in probable accident frequency also, with the best existing road that connects the termini of the proposed route. The miles of travel, the time and the hazard saved by a car using the proposed route, as compared with the existing route, are estimated, and an appropriate valuation is placed on each saving. Sometimes savings from better grades, less vehicle wear and tear from frequent stops, etc., are also included. This per-car-trip saving is then multiplied by the number of cars that have been estimated to use the proposed route during the year of average traffic volume. The product of the savings per car trip times the annual number of trips is considered to be the benefit received annually by the users of the road as a result of having the improvement.

The cost of right-of-way and of construction of the road is then estimated and is divided by 30 to reduce it to an annual basis. To this is added the estimated annual cost of maintaining the road. This cost should include maintenance and eventual replacement of the pavement, drainage structures, etc., the care of the roadsides and shoulders, removal of snow and protection against ice, and the maintenance of signs, signals, and markings for the guidance and protection of traffic. The sum of these costs is taken as the annual cost of the road.[3]

The ratio of the annual benefit to the annual cost is the "benefit-cost" ratio for that particular route. Of the several possible routes, the one which provides the greatest benefit in proportion to its cost is presumptively the most desirable route to use.

"Least Total Cost" Method. This method has applicability in undeveloped regions where no road exists between the termini. The annual cost of constructing and maintaining the road is determined for each possible route. An estimate is also made of the aggregate annual cost of using each route. This is the product of the number of cars using the route times the cost of each trip, taking into consideration mileage and the time consumed. Vehicle wear, gasoline wastage through excessive rise and fall, etc., should be considered in estimating per mile travel costs. Accident frequencies may also be considered where sufficient data permit. For each route, the annual construction, maintenance and use costs are added. The route for which the sum of these costs is least is presumptively the best.

[3] An interest charge on the cost of each route is sometimes included also, especially when the proposed project is to be financed with borrowed funds.

Either of these methods is an aid to the choice between available routes; neither should be followed too rigidly. Road location is a problem in which no element can be exactly known in advance. Also, there are many imponderables that cannot be considered mathematically—for example, the beneficial effect of opening to useful development an area suffering from inadequate access. However, imperfect as they must be, these methods of analysis, used with good judgment, are useful tools in making a wise choice between possible alternative routes. The first method, particularly, emphasizes the vital fact that there are benefits as well as costs to be considered in a road improvement project. The more costly of alternative locations may prove the more desirable in the long run, provided that the excess of benefit (including both tangibles and intangibles) outweighs the increase in cost.

Special Location Problems of Divided Highways. When traffic necessitates four or more lanes, safety requires the separation of opposing traffic streams, whether urban or rural, for even moderately high traffic speeds.

In locating a divided highway or expressway, the center lines of the two separate roadways may be maintained at a fixed distance apart, or at a variable distance. They may also have the same elevation at any section, or different elevations.

Where the terrain traversed is flat or gently rolling, it will generally be preferable to locate and design the entire highway as a unit. In this case, the center line to be established will be the line midway between the centers of the opposing roadways. However, in broken or mountainous regions, it will often be desirable, and sometimes necessary, to locate roadways for opposite directions of travel largely independent of each other, both as respects line and grade. The width of the dividing strip may vary widely to fit the terrain, and the grades of the roads may be different also. On a side hill location, it will generally be advantageous to bench in at different elevations. Under these circumstances, a location is worked up individually for each of the two roads. Where bridge structures are employed, either at streams or at intersecting roads, the two roads may be brought together onto a single structure, or separate structures may be used. The choice will depend on economy and, in the case of traffic interchanges, upon the requirements of traffic.

In modern expressway design it is the practice to separate grades at intersecting roads and railroads. Attention must be given to locating the intersection at a point that lends itself to the separation. If no satisfactory point can be found, it may be desirable to relocate a short length of the existing road in order to create one. In every case where an expressway is allowed to intersect an existing road at grade, care must be given to the design of the intersection to control hazard. In thickly populated areas, the problem of intersections becomes serious and every advantage offered by the terrain—such as utilizing a stream valley for the expressway—should be taken. Where intersecting roads or streets are close together, the expressway will ordinarily have to be continuously elevated or continuously depressed to avoid an objectionably "rolling" grade. In general, a depressed location is less injurious to abutting property but may increase drainage difficulties.

In the construction of a divided road, it is sometimes feasible to utilize an existing two-lane road for one direction of travel. This can be done even in cases where some geometric characteristics of the existing road are inferior, since the one-way road will require only non-passing sight distances.

In planning a major road it may sometimes be desirable to provide for immediate construction of a two-lane road, with provision for the later addition of two lanes. A disadvantage is that the initial two lanes must meet "passing" sight-distance requirements while traveled in both directions, whereas, in the four-lane stage, they will require only non-passing sight distances. In broken country, the additional grading required to secure an adequate number of passing sight distances may cost enough to make it desirable to build the four lanes initially. This is one of the problems that should be considered before location is attempted.

The location of a limited-access road should not deprive substantial areas from access to any highway.

Control Points. In the location of every road, certain points exert a controlling influence. These may be points (or areas) that must be reached by the road, or that must be avoided, if possible. In the former category are the two terminal points of the project. If the two termini are separated by mountains, a pass through them is likely to be a control point. Suitable bridge sites on large streams are control points. A cemetery is a control point to be avoided. So is a swamp, if it can be avoided; but if it must be crossed a good crossing site is a control point. So is a town, en route, to be served by the road. The identification of control points is one of the very early and important steps in highway location.

When a highway of considerable length is to be built in successive sections, it cannot be too strongly emphasized that the location *of the entire road* should be studied in the very

beginning. It is not enough merely to work from one control point to the next. Reconnaissance should *always* be extended beyond the temporary termini to assure that the later sections of the over-all project will fit properly with the earlier units.

Steps in Highway Location.[4] The process of highway location is frequently carried out in four fairly distinct stages. These are: Area reconnaissance; route reconnaissance; the preliminary survey; the location survey. Each of these stages has its own specific purpose.

Area reconnaissance seeks to identify all possible routes worthy of further examination.

Route reconnaissance seeks to eliminate all but the best one of these routes.

The preliminary survey undertakes to secure the information necessary for a location horizontally of the highway center line. Most of this information can conveniently be recorded on a topographic map; hence, the principal product of the preliminary survey is a topographic map of a strip of land within which the entire highway can be located, together with a "paper location" of the proposed center line, drawn on the map.

The location survey has two purposes: (1) to stake on the ground, with permanent references, the approved center line of the proposed highway; (2) to collect accurate and complete information to enable the designer to prepare the final plans and right-of-way maps for the highway, to estimate all construction quantities and to make an estimate of the cost of the work.

The dividing lines among these four stages are often—perhaps usually—not very sharp. Sometimes, two stages merge into one. On all but minor jobs, however, the four stages can usually be distinguished.

Ground- and Aerial-Survey Procedures. All the steps in highway location can be carried out by ground-survey procedures alone. Aerial-survey methods (photogrammetry) have such advantages in speed and economy, however, that their use (in conjunction with a continuing use of ground surveys for some purposes) will generally be economical except on small and simple projects. The greater the difficulties of location, the greater are the advantages to be had from aerial methods.

2. RECONNAISSANCE

Area Reconnaissance.[1] Area reconnaissance starts with the assembly of the best existing topographic maps if any are to be had; otherwise with such planimetric maps and aerial photographs, "mosaics," etc., as can be secured from governmental units, public utilities, etc. In rural areas, if recent U. S. Geological Survey maps are available,[2] these will generally provide the necessary information to identify all possible routes and perhaps also to eliminate all but one or two. County soil-survey maps made by the U. S. Bureau of Chemistry and Soils may also be helpful, if available. Control points can be spotted on the maps and, with the design standards of the projected route known, general locations can be worked out. It may sometimes be desirable to have the map sheets enlarged photographically.

If adequate topographic maps are not available, existing aerial photographs should be sought.[3] If photographs are not available, it will usually be worth while to have them taken. From these a mosaic is made, and the photographs are also studied stereoscopi-

[4] The problem of location may involve a completely new road through unoccupied wilderness, or an eight-lane expressway through the heart of a city. The road may pierce dense jungle or cross a treeless prairie; traverse a level plain or penetrate rugged mountains. It may be intended to serve fifty vehicles a day, or fifty thousand There may be accurate topographic maps of the area, or no maps at all With a problem of such diversity, no one pattern of procedure will always apply. In the paragraphs which follow, it will be assumed that the area is one presenting considerable, though not extreme, difficulties of terrain, one in which there is a rather intensive land use, with correspondingly high land values and a fairly large amount of previous development to be taken into account. Unless otherwise noted, it will be assumed that the projected road is several miles or more long, that it is expected to carry enough traffic to warrant a design speed representative of most primary highway mileage in the United States at the present time—say 60 or 70 mph—and that the geometric characteristics of design are to be consistent with such a volume of traffic and such a speed. Roads of a more complex nature will sometimes be encountered and will require more refinement of detail than is presented here More frequently, road problems will be of a simpler sort, either because of lighter traffic, a simpler terrain, or less highly developed land use In such cases, procedures somewhat less refined than those outlined may well suffice

[1] See also Sect 2, Arts. 62–79, Route Surveys.

[2] These are obtainable at nominal cost from the U S Geological Survey, General Services Administration Building, Washington, D C Unfortunately, a large part of the United States has not yet been mapped

[3] Consult Map Information Office, U S Geological Survey, Washington, D. C., for available coverage

cally. In these ways, possible routes can be identified. The photographs may also give valuable information about soils and the location of suitable materials of construction. The region to be photographed at this stage should usually have a width of 40 to 60% of its length, unless intermediate control points or mountain ranges or rivers narrow the area

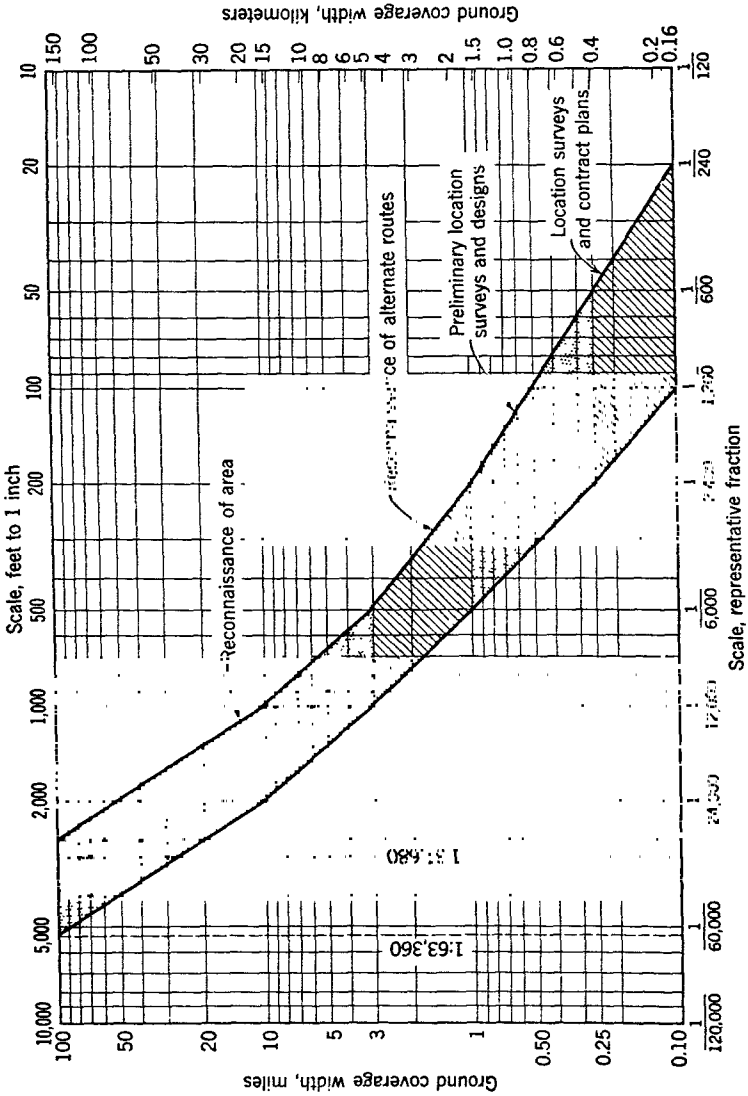

FIG. 1. Scales for aerial photographs and maps. From W. T. Pryor, "Aerial Surveys in Highway Location," *Photogrammetric Engineering*, December 1946.

requiring investigation. The photographic scale may be 1 mile to the inch for large areas of comparatively simple and undeveloped terrain to 500 ft to the inch where the area is smaller and more complex. Recent California Highway Department practice in the design of an expressway has employed photographs ranging from 1600 to 2000 ft per in. in rural areas and to as large as 50 ft per in. in urban areas. The smaller scales were used in reconnaissance. Figure 1 suggests appropriate ranges of aerial photographic scale for the successive stages of location.

If neither maps nor aerial photographs of the region can be secured, area reconnaissance merges with route reconnaissance, since the only way in which the area can then be examined is through the investigation of different specific routes, after control points have been identified as fully as possible.

Route Reconnaissance.[4] Area reconnaissance (with maps or photographs available) should have resulted in the identification of one or more relatively narrow strips of territory within each of which it appears possible to locate a road fully consistent with the specified design standards. Feasibility of each of these suggested routes should next be checked on the ground. Aerial photography does not reveal subsurface conditions such as the depth of swamps, nor does it show, for example, the condition and value of a house. Moreover, when the photographs were not especially made for the project, conditions may have changed importantly since they were taken. The ground investigation should note soil conditions, any adverse drainage situations (including possibilities of flooding or of slides), the availability of good road-building materials, and the effect of the proposed road construction on existing land uses. Special attention should be paid to the limiting values of grade and curvature at critical points to be sure that a road having the required standards can be built economically. All stream crossings of any importance should be studied to verify the suitability of the sites, and all intersections of the proposed route with existing roads and railroads should have thorough examination.

Unless ground reconnaissance rules out all but one route, it is often desirable to continue the aerial reconnaissance by having new photographs made to show the alternative routes in greater detail. These photographs will not only be of larger scale but also will be taken at a lower altitude. This will make them more valuable for stereoscopic study, and they will also be appropriate for making contour maps, if those are desired. Scales for this second set of photographs should usually be between 1000 and 200 ft per in. The width of the strip photographed will be about 25 times the photographic scale; i.e., at a scale of 200 ft per in., a single flight will cover a strip about a mile wide, which will ordinarily be adequate. If alternative routes lie comparatively close together, the single flight may cover both. In other cases, a separate flight may have to be made for each route.

These larger photographs are studied individually, are assembled into a mosaic and are studied stereoscopically. If some one route is materially better than any other, that fact can often be developed from the photographs.

If the routes lie close enough together to be covered in a single flight (as may be the case in intensively developed areas), it will usually be desirable to have a topographic map made at this stage. When this is done, route reconnaissance merges with the preliminary survey.

Even though alternate routes are too far apart to be photographed in a single flight, it may still be desirable to get a contour map of each route. This will be true when the project is of substantial cost and choice between routes is difficult. Each mapped route should be carefully studied from the standpoints of right-of-way cost, construction cost, and benefits to users. A benefit-cost ratio can be established for each route much more accurately with the topographic map than without it.[5]

If reconnaissance by means of maps or aerial photographs is not found feasible, such probable routes as can be surmised from overlooking the country must each be investigated by ground surveys. In some cases, the possible route can be adequately investigated by traveling over it and examining it with such simple instruments as compass and hand level—flagging the route on lath or trees for later identification. At the other extreme, a stadia or other quick instrument survey may be necessary.

Route reconnaissance, by whatever means conducted, should secure enough data to permit at least a rough estimate of right-of-way and construction costs of each route, as well as its approximate length and profile. As the cost of the project and the difficulty of choosing between routes increase, the reconnaissance studies should be made with increasing refinement, even though this may require substantial expenditures.

Reconnaissance Report. In general, it is desirable that before a road project progresses from the reconnaissance to the preliminary survey stage, a detailed report of the reconnaissance should be made. This will enable the recommendations resulting from the reconnaissance to be evaluated accurately by the proper engineering authority. It will also record the considerations which dictated the choice of location, should the wisdom of

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[4] See also Sect. 2.

[5] All the aerial photographs referred to above are taken vertically. A subsidiary but valuable application of photography is to take oblique views of the selected route. The proposed road is then drawn on prints of these. This technique has particular advantage in demonstrating the route.

the location ever be questioned. The proper scope of this report will vary. It should include as many of the following topics as apply:

1. Date of the report.
2. Identification of the project, including termini and approximate length of the proposed road, the larger project or road system of which it forms a part, its location within that system, etc.
3. Traffic conditions and corresponding design standards as given to or assumed by the locating engineer.
4. Maps, aerial photographs or mosaics and all similar data used in the reconnaissance, with the selected route and any discarded routes suitably shown or described.
5. Control points to be met and to be avoided.
6. Nature, extent, and date of photogrammetric and other office studies, and by whom made.
7. Nature, extent, and date of field studies, and by whom made.
8. General character of terrain, soil, drainage, etc.
9. Existing development and use of land.
10. Availability of local road materials and transportation facilities to site.
11. Recommended type of road surface and shoulders to meet conditions of traffic and terrain.
12. Major structures required, both by drainage and by traffic.
13. Discussion of reasons for the selection made.
14. Discussion of any alternate routes, with recommendation of any further investigations, if considered desirable.
15. Approximate cost and cost per mile of each suggested line, based on recommended design standards and road surface type.
16. Name of engineer responsible for report.

3. THE PRELIMINARY SURVEY

Purpose of the Survey. The objective of the preliminary survey is to establish the horizontal location of the center line of the proposed road. This will necessitate consideration of the grade also. To accomplish this purpose an accurate and sufficiently detailed topographic map must be produced

The Topographic Map. Whatever methods are used in making this map, it (alone or with accompanying profiles and cross sections) should show the following information, when complete:

1. Ground elevations, to the extent needed (it may be unnecessary to show complete contours).
2. All houses, barns, sheds, commercial, industrial, and other buildings.
3. All fences, walls, and other boundary lines, according to their type, together with the names of property owners.
4. Pole lines, of all kinds, together with their ownership.
5. All wells, hydrants, manholes, and any other evidences of underground sewers, conduits, and gas or water mains, together with their ownership.
6. All roads and driveways, with notation as to whether improved or not, and type of improvement.
7. Railroads.
8. All sidewalks, crosswalks, curbs, etc.
9. All existing drainage structures.
10. Valuable trees and cultivated shrubs.
11. All natural water courses.
12. All ponds, swamps, etc.
13. All outcroppings of rock.
14. All other features of the site which might affect the final location.

Aerial Maps: Limitations and Advantages. A topographic map cannot be made satisfactorily by aerial means if the ground surface is obscured by the leaves of trees or when it is covered with snow. Consequently, the aerial method will be impractical in heavily wooded tropical areas and in any areas heavily wooded with evergreens. In temperate areas with deciduous cover, the securing of photographs adequate for mapping will have to wait on the proper season, and this may be disadvantageous.

However, aerial mapping, when it can be used, is cheaper under most conditions of terrain and land development than mapping by ground surveys. It will be substantially

cheaper if both types of survey are worked up for the same width of area and with the same amount of detail—especially since the photographs will generally have already been secured for the purposes of route reconnaissance. Moreover, a map can usually be produced in substantially less time by photogrammetry than if produced by ground surveys, and this may offset any seasonal delay in taking the photographs.

The scale of the aerial topographic map is determined by conditions of the terrain and of its development. If the land is flat and a contour interval as small as 2 ft is necessary, the map scale will have to be as large as 100 ft per in., since the scale can be only about fifty times the contour interval. A scale this large is useful in urban and suburban areas; and where the development is very intensive, scales as large as 50 or even 20 ft per in. may be desirable. In rolling or broken areas, where a contour interval of 5 ft will be satisfactory, the map scale can be 200 ft per in., and this will usually be large enough in open country. This scale and contour interval were used by the New York Department of Public Works in developing alignment for 300 miles of the New York Thruway, and also in relocating existing roads through rugged terrain.

Supplementary Ground Investigations. If an aerial map is used, it will have to be taken to the field and checked for accuracy and completeness. Any missing data, such as property lines and the names of property and utility owners, the character of underground structures, the type and condition of houses, etc., will have to be added. Frequently, this may best be done after a tentative center line has been put on the map. The center line will define the areas where completeness and accuracy are most necessary.

Ground Survey: Traverse, Topography, Profile, and Cross Sections. If the topographic map is to be made by ground-survey methods, the first step is the running of a tangent and angle traverse to serve as a base line from which to take topography. This base line should be laid down as close to the final anticipated center-line location as feasible.

This requires consideration of the principles that will control the establishment of the center-line location. The most important of these are:

1. To secure a location desirable from the standpoint of traffic. It should not only conform to the minimum standards of line, grade, sight distances, and road cross section that have been specified for the proposed road, but should also be as much better than the minimum standards as is feasible. The location, at all points, should minimize the possibility of flooding by streams crossing the road or paralleling it.
2. To secure a location reasonable in construction cost. This means that, insofar as possible, areas of swamp, ledge rock, possible landslides, and other foundation troubles must be avoided; that amounts of cut and fill be held within reasonable limits; that good sites be secured for stream crossings, railroad grade crossings (or separations) and intersecting roads, etc.
3. To secure a location that will have the least harmful effect on land use. This means avoidance, as much as possible, of expensive existing development, residential, industrial, agricultural or other; avoidance of unnecessary severance of valuable properties; and of encroachment on cemeteries, playgrounds, parks, and golf courses. In the reconstruction of an existing road, particular study should be given to the possibility of limiting the widening of the right-of-way to one side of the road only.
4. Subject to the above controls, to secure the most direct line.

Knowing the general route to be followed (usually it will have been flagged on the ground or plotted on a map or moasic for his guidance) the locator runs his traverse to conform as fully as possible to the above principles. Where they are in conflict he attempts to develop the best compromise.

The topography is ordinarily taken as the traverse is carried forward. Topography should extend far enough either side of the base line to include the entire right of way of the road when built, making due allowance for possible shifting of the center line away from the traverse when the final line is plotted. Hence the topography need not cover a band of uniform width, but should be widened at points where there is less certainty as to the exact position of the eventual center line. Topographic features are of two general sorts: some, such as rock ledges, important buildings, etc., will affect the location selected for the road; others, such as driveways, will not influence the location but may have to be given consideration in working out the details of the design. At the time of the preliminary survey, it is frequently desirable to map only topographic features of the first sort. After the center-line location has been established on paper and when it is being staked on the ground in the location survey, the topographic map is completed by adding the necessary secondary items. These can be more accurately located with respect to the chosen center line by plotting them from the center-line transit stations than from the original traverse. Also, better judgment can then be exercised as to what topography should be included in the map.

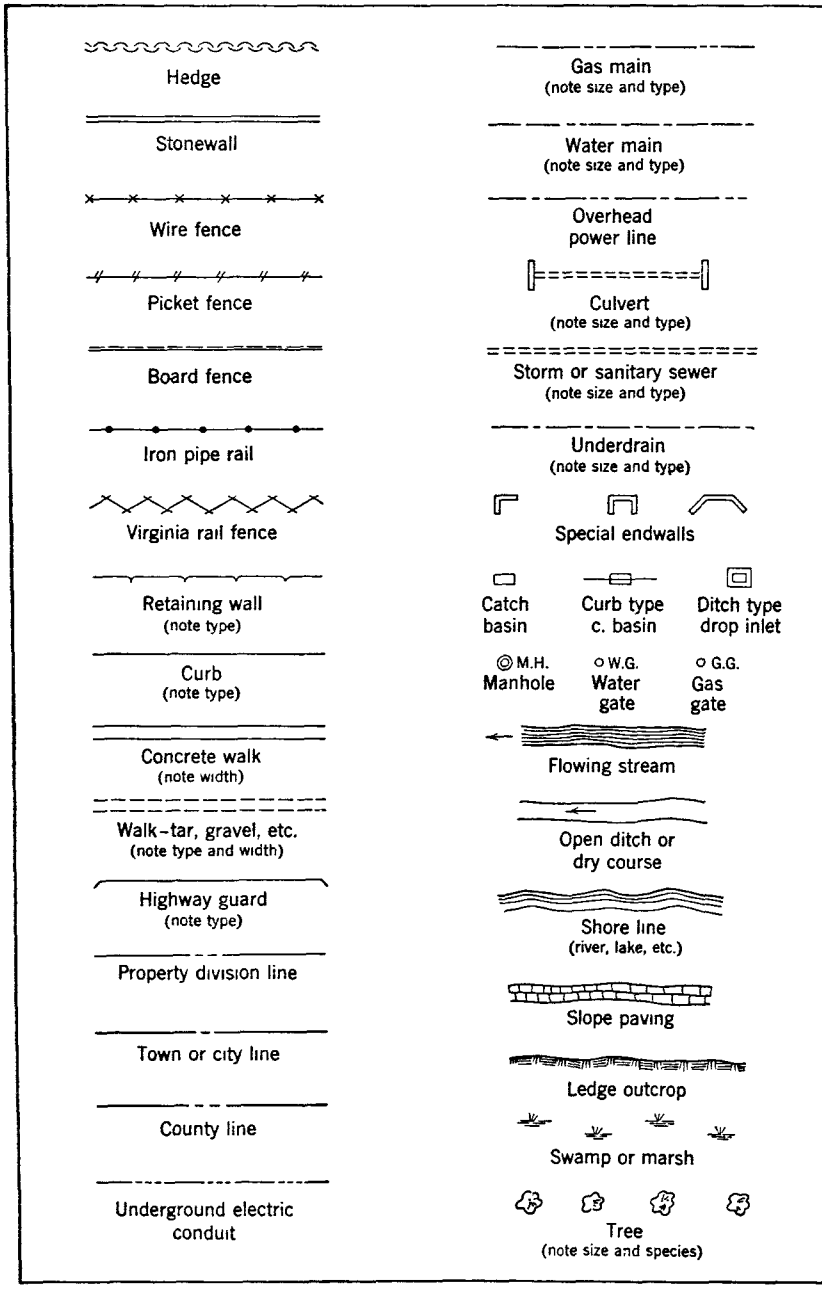


FIG. 2. Topographic map conventions and symbols—Sheet 1.

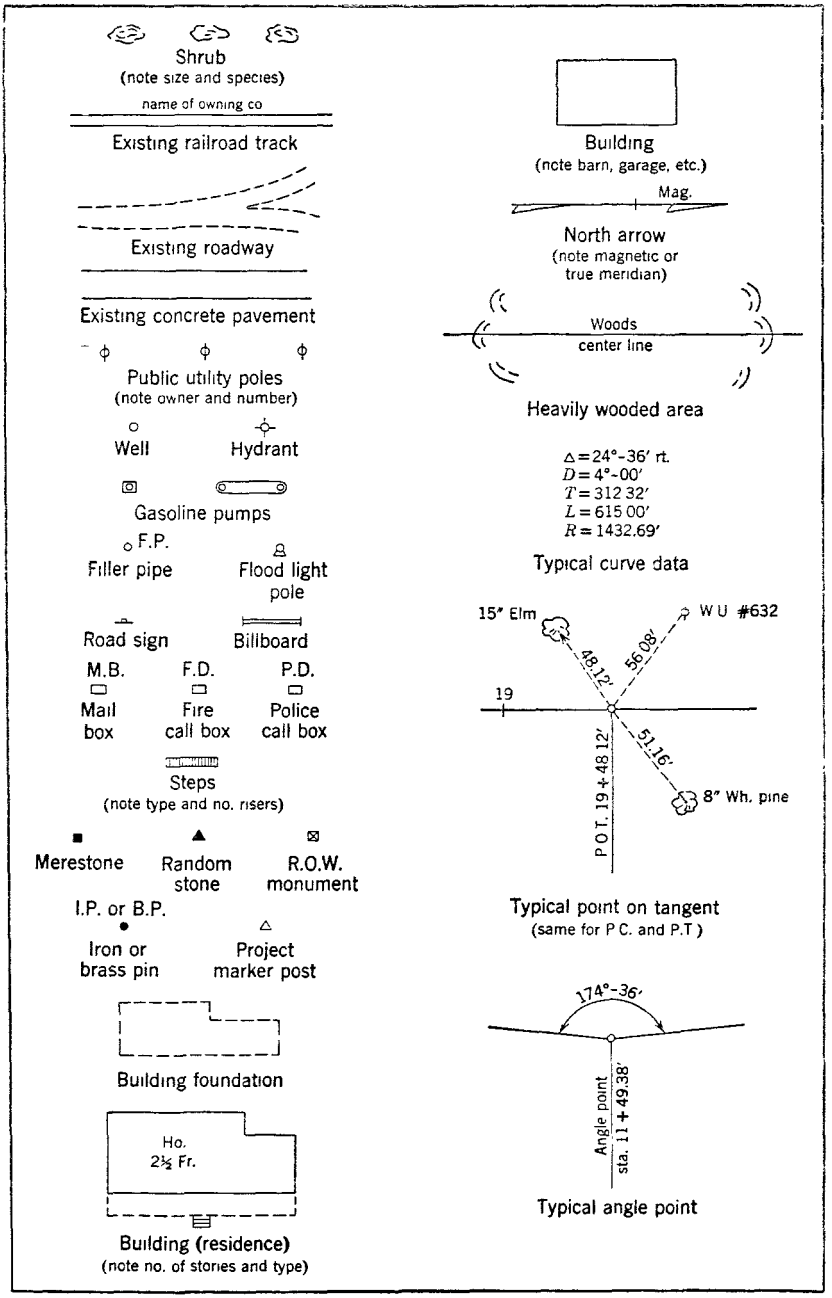


FIG. 3. Topographic map conventions and symbols—Sheet 2.

Levels are run over the traverse to establish benchmarks at intervals of about 1/4 mile. These are carefully marked and referenced. A ground-surface profile of the traverse is run. This may give sufficient elevations for establishment of the preliminary center line, when that profile is fairly representative of the surrounding ground. Cross sections should be taken, however, at points where relative elevations appear necessary or helpful in establishing the center-line location. In occasional areas where neither profiles nor cross sections adequately depict the ground and where the shape of the ground may affect the center-line location, contours at appropriate (often 5-ft) intervals may be mapped.

In making the preliminary survey, the traverse and the bench levels should be tied to existing monuments and benchmarks as fully as possible. Wherever available, USGS datum should be used for elevations. If a grid-coordinate system exists in the area, the map should be accurately tied to it. All levels should be back checked, if reference benchmarks are lacking.

Connections with Existing and Future Roads. The ends of a road project are often points where particular care should be taken in location. This is especially true in reconstructing successive lengths of an existing road. It is then most important to make certain that the center line of the immediate unit is capable of being extended effectively as reconstruction of the route is continued. In such cases, it will usually be desirable to continue the traverse, and to secure important topography for extension of the center line a substantial distance—a quarter or half mile—beyond one or both ends of the immediate project.

Plotting Topographic Map. The field party which makes the transit and level surveys has the additional responsibility of plotting the topographic map. (See Figs. 2 and 3 for conventional signs, etc., in ordinary use.) This map is customarily plotted as the survey progresses, and shows the traverse and all topography taken from it. The plotting of the traverse must be done with care, since it forms the base of all subsequent paper work. As in the case of photogrammetric maps, scales for preliminary ground-survey maps may vary from 200 to 20 ft per in. depending on the complexity of the area represented. If the topographic map is drawn to the scale that will be used later for the construction plans, all data developed on the map which is to be shown on the construction plans can be directly traced. The topographic map is best plotted on a continuous roll of heavy translucent tracing cloth, from which prints will be made.

After the transit line has been carefully checked to eliminate any errors in plotting, it is inked. It may sometimes be desirable to ink other lines of the map also. Inks of different colors are sometimes used to differentiate between different features of the map.

Plotting the Preliminary Center Line. When the topographic map (and supplementary profiles and cross sections) have been plotted, the establishment of a tentative center line is in order. This is sometimes called the "paper" location.

If the preliminary survey was made by ground methods, the engineer who made the survey is usually the person who works out an initial and tentative center line on the topographic map, since he will have the most intimate knowledge of the terrain and the problems to be solved. He will be guided by the traverse line. Usually, however, progress of the traverse will have revealed conditions of topography or land use that suggest some desirable differences between the traverse courses and the center-line tangents. The tangents will then be plotted *near*, rather than on, the traverse courses. In locating the center line, the principles that governed location of the traverse should be kept in mind. If, as center-line plotting progresses, uncertainties arise in the mind of the locator, he can reexamine the site.

The Michigan Highway Department enumerates the following policies to be considered in locating the tentative center line.

"1. Preserve and save, where possible, all roadside shade trees located outside the shoulder of a highway.
2. Use a minimum number of curves.
3. Use existing adequate bridges and large culverts.
4. Secure, as nearly as possible, right angle crossings of intersecting roads, railroads and streams.
5. Locate the center line in stable soils, if possible.
6. Coordinate horizontal curves with vertical curvature for good appearance.
7. Leave local land drainage in as good condition as it is found.
8. Consider the safety of children at all schools.
9. Leave all cemeteries undisturbed.
10. Hold all right of way, building-moving and property damage costs to a minimum.
11. Avoid successive horizontal curves (flat backs) in the same direction. Make longer single curve, compound if necessary.

12. Avoid short reversed (dog leg) alignment.
13. When reverse curves are necessary, distance between curves (P.T. to P.C.) should be 300 feet minimum, and length of curves 600 feet minimum."

The center line is plotted as a succession of tangents and circular curves. A more pleasing effect is secured by the use of long, flowing "directional" curves rather than a frequent alternation of short curves and short tangents. When the center line has been plotted, the bearings of tangents, the curve data (central angle, degree, tangent distance, length, and radius) and stationing are added. Stationing is carried on from the beginning to the end of the project.

Field Review of Preliminary Center Line: Location Center Line. When the tentative center line has been established and plotted, the "paper" location should be reviewed in the field. The entire traverse line should be walked over, and the tentative center line compared with the ground. Particular attention should be given to all points where the center line approaches limiting values of the geometrical standards. Attention should also be given to any conditions which might affect the design or construction, such as areas of unstable, water-bearing soil. The attempt should be made to adjust the location in all feasible ways to simplify problems of right-of-way acquisition. It may be that all questions raised by the field inspection cannot be immediately resolved, but that more extensive investigations will have to be ordered. These should be made and all controversial points settled. The topographic map is then revised to incorporate all additional information and any adjustments that have been made in the tentative center line. The center line as adjusted is then approved as the location center line, and the preliminary survey is considered complete.

4. THE LOCATION SURVEY

Divisions of the Survey. The location survey has two main divisions The first of these is the staking of the center line on the ground exactly as it has been finally established on the map. The second is the collection of all remaining items of information that will be needed in completing the highway design; preparing construction drawings, specifications, and estimated quantities; and purchasing the right-of-way.

Staking the Center Line: Additional Topography. When the center-line location on the topographic map has been approved, the map is returned to the field so that from it the location center line can be staked on the ground. As already mentioned, it is a frequent practice to include in the preliminary survey only the topography essential to center-line location. All additional topography is measured from the staked center line as staking of the line progresses. In showing buildings, all steps and porches should be shown, as well as the buildings themselves. If any part of the building will apparently lie within the right-of-way, all dimensions representing and locating it should be to the nearest tenth of a foot. For intersecting roads, the center line and right-of-way lines should be shown, as well as the travel path. A station plus should fix the intersection of the center lines. The same thing is true of an intersecting railroad line. In addition, the type, dimensions, and condition of any existing railway crossing should be shown, and the nature of any protective devices. The sizes and species of important trees should be shown, as well as their location. There will be special points, such as bridge or important culvert sites and the sites of railway grade crossings or of highway or railway grade separations or traffic interchanges where the designer may need very detailed information.

Important items of topography that were taken from the original traverse should frequently be checked for accuracy of position by measuring their location from center-line stations and checking the measurements on the map This should reveal any errors that may have been made in plotting the center-line location from the traverse.

Cross Sections and Profiles. When the center line has been staked, cross sections of the ground surface are taken, at right angles to the center line, at each station and half station and at all points where there is a significant break in grade. Enough readings are taken at each cross section to establish the shape of the ground, and the section is carried far enough on each side of the center line to cover anticipated construction. Ground-surface readings are usually to the nearest tenth of a foot, and turning-point readings to the nearest hundredth. The benchmarks that were established in the preliminary survey form the reference points for the cross sections.

In addition to cross sections, elevations should be determined wherever they will supply information needed by the designer. The center-line profile for each intersecting road should be obtained for sufficient distances to establish its existing grade line, and to show the effect of any change in the grade of the intersection. Top-of-rail elevations should

similarly be secured for all intersecting railroads. Comparable information should be secured for roads and railroads that parallel the center line or approach it closely. In such cases, the regular cross sections should be extended frequently to include the road or railroad. Inlet and outlet elevations at all existing culverts carrying stream flows should be secured, as well as flood-water elevations and flow-line elevations in the bed of each stream for a reasonable distance upstream and down. All other existing drainage structures should have significant elevations noted. The ground elevation of all trees at critical distances from the center line should be recorded. The first-floor elevation and all other pertinent elevations such as the top and bottom treads of flights of steps should be recorded for all houses near the anticipated right-of-way.

Plotting the Location Survey. All additional topography taken in the location survey should be plotted on the map by the survey party. The survey party should also plot the ground-surface profile of the location center line on a continuous roll of profile paper, using the same horizontal scale as used for the topographic map. The vertical scale should normally be ten times the horizontal. When this profile has been completed, a tentative grade line is put on it. This is done for two reasons: first, to check the satisfactoriness of the center-line location; second, to assist in the plotting of cross sections.

It is a frequent and sound practice for the survey party, in plotting the ground-surface cross sections, to put them directly on the tracing sheets that will form part of the final set of construction drawings. These sheets are often to a scale of 10 ft per in., both horizontally and vertically in rolling country, 5 ft (both ways) where the terrain is flatter. Cross sections are plotted one above another, commencing at the bottom of each sheet and running consecutively on successive sheets, from beginning to end of the project. Offset distances from location center line are marked along top and bottom margins of the sheets, elevations along the vertical margins. The ground-surface cross section for each station or half station (or other point) is plotted in a heavy, solid, inked line. Exposed rock is indicated by conventional hatching under the ground line, and all occurrences of rock as determined from the subsurface survey are plotted in proper location. If extensive rock formations have been determined by more than one boring at the section, the several points are connected by a dashed line marked "rock surface." Swamp bottoms are similarly indicated and labeled. If soundings have been made but have failed to reach rock or hard bottom, the depth of each sounding is indicated. Existing stream beds, buildings, roads, drainage structures, and any similar objects are also plotted.

As many cross sections are plotted on a sheet as can be included without overlapping or crowding when the road templet has been added above or below the ground line, as the case may be. The tentative grade line, as worked up on the field profile, is a useful guide in spacing sections on the cross-section sheets, since it shows a close approximation to the center-line pavement elevation at each section. The station or station and plus for each section is written on the center line and just above or below the ground line, according to whether it is expected the section will be in cut or on fill.

Surface Drainage Survey. All well-defined drainage courses that cross or closely approach the center line should be located by station plus and bearing, whether they carry water continuously or intermittently. In addition, enough information must be recorded to enable the designer to provide adequately for the drainage flow. This will ordinarily require an approximate determination of the drainage area (acres or square miles) and the nature of runoff, whether fast, medium, or slow, according to principles and procedures outlined in Sect. 2 of Vol. II. The drainage area is determined from topographic maps or by stereoscopic study of aerial photographs. If neither of these are available, it will sometimes be necessary to run a quick traverse around the area. Flood elevations in all streams that cross or parallel the location should be determined by noting twigs and other debris lodged in trees along their banks. Examination should be made of drainage structures at the nearest road or railroad crossings upstream or downstream, if there are any, noting the sizes of openings and any evidence of adequacy or inadequacy. Valuable information in regard to the adequacy of culverts can often be obtained from local residents, from highway and railway personnel, etc. Where an existing road is to be reconstructed, the size, type, condition, and adequacy or inadequacy of any drainage structures on the existing road at any points within or near the probable area of the projected highway should be noted. All private drains, field tile, house drains, etc., that cross the center line or affect the design of the future highway must be noted, together with the purpose they serve. If data on any stream channel location appears necessary, detailed information concerning the nature of the channel and maximum flow should be secured. A special drainage survey will often be necessary for this, with traverses and elevations tied in to the center line and benchmarks.

Soils and Other Subsurface Conditions. The kind and amount of investigation of soils will depend upon the geology of the region. In many areas, particularly those once

glaciated, soils are generally good but rock excavation is a serious problem. In other areas, rock is not found but some soils may be very inferior. The investigations to be made must be adapted to the locality. Very generally, bar soundings, test pits, or auger borings should be made at approximately every station in proposed cuts and swamp crossings. The number at each station will depend upon the road cross section and upon soil and rock conditions. Auger borings, by hand or power driven, can be used if the soil is not too bony. Geophysical methods (seismic and electric resistivity) can be used instead of, or supplementing, the above. These explorations, aided by tests of samples and field inspection, enable the soils engineer to make analyses and recommendations on quantities of earth and rock and the probable percentage of large boulders in cuts. They also support recommendations on safe slopes, subgrade treatment, required subbase thickness, subsurface drainage under shoulders and slopes, shrinkage and swell from excavations to embankments, suitability of material for filling in open water, and treatment of swamp and marsh deposits under embankments, such as by excavation, by displacement, or by slow filling with or without vertical sand drainage wells.

In regions where soils are inferior or where successive layers of soil vary greatly in their characteristics and suitability, it may be desirable to make borings rather continuously along the highway center line to secure a soil profile that will assist the designer in establishing his grade line and will apprise him of any troublesome conditions encountered anywhere on the job. Especially at proposed bridge and culvert sites, ample soils data should be secured. Section 7 gives methods, equipment, etc.

Careful records should be made during the location survey of all seepages of water and all vegetational evidences of wet or springy conditions that may require corrective treatment. In road relocation or reconstruction, past history of maintenance trouble due to subsurface water or to frost should be sought and carefully noted.

Road-Materials Investigation. One of the purposes of the soils investigation is to establish whether the material that will be excavated in cuts will be adequate in quality and quantity to furnish the yardage required to complete the embankments and to provide any necessary subbase and backfill for use around drainage structures. If so, no additional material—borrow—will be required. If, however, borrow will be needed, available sources of such material should be investigated. It may be that borrow can be secured within highway limits, either by widening cuts (especially on the insides of curves, which will improve sight lines) or by increasing the cross sections of ditches ("side borrow"). Otherwise, it will have to be found outside of right-of-way lines at points where "borrow pits" can be established. The investigation should establish the location and accessibility of adequate supplies of available borrow; and also approximately the expenditure that will have to be made for temporary roads to bring the material to points where it will be needed on the job. Borrow pits should, of course, be so located as to require no longer haul than necessary. Sometimes separate pits may have to be established for different materials, as when suitable gravel for subbase and backfilling is more remote than material acceptable for the body of embankments.

Occasionally the most available borrow will be sand or other granular material dredged from the bottom of a lake or other body of water near the right-of-way. Good material of this sort, placed hydraulically by pumping, makes an excellent embankment, provided that care is taken to see that no muck or other rubbish overlying the sound material is drawn into the embankment, and that the dredge intake is not moved beyond the limits of acceptable fill. Where use of dredged fill is contemplated, investigation should be made of the quality and quantity of available material, the thickness of overburden, and other factors that might influence the cost of this borrow.

Transportation Facilities. To enable the designer to estimate construction costs more closely, the survey should also locate the rail sidings or other facilities through which machinery and materials other than those locally available may be brought to the job.

Rights-of-Way Investigation. The topographic map and the survey notes, between them, should supply the designer with enough information to enable him to determine the controls that are exerted by abutting property. In open country, these may not amount to much. In closely built-up areas, however, the designer's decision as to the elevation of the roadway in relation to abutting houses or buildings, the width of right-of-way to be taken, and other features of design may be influenced by the amount of damage that may be done thereby to abutting owners. Hence, it is not sufficient to inform the designer of only the shapes, sizes, and horizontal and vertical locations of buildings within or near the probable right-of-way; he should also know their condition, the uses to which they are put, and any other items that will throw a light on the variations in right-of-way costs that will probably result from possible variations in his design. The location survey should develop all such information. In addition, it will, of course, provide accurate information concerning the location of all property boundary lines and property improve-

ments. Where boundaries that affect the highway have been lost, it may be a part of the location survey to reestablish them on the ground, or it may be preferable to establish them later, when needed rights-of-way are to be bought.

Surveys of Bridge Sites. Whenever the highway project includes construction of a bridge, a large culvert, or a grade-separation structure, greater detail of topography is required. Cross sections along the center line of the proposed bridge should be taken at 10-ft intervals, and should show all breaks in the stream bottom for a distance at least 50 ft beyond proposed highway right-of-way lines both upstream and downstream. Enough sections should be taken to include the length of the probable structure and 100 ft of approach at each end. Care must be taken to see that the cross-sectioned area is large enough to include all wingwalls, abutments, areas requiring slope paving, riprapping, etc. A profile of the stream bed should be obtained along the center of the channel for at least 500 ft upstream and downstream, taking elevations every 25 ft or oftener if the irregularity of the bottom indicates. Care should be taken to note holes in the stream bed, or other evidences of scouring action. If there seems to be more than one channel, or if the width of the stream is considerable, additional stream profiles should be secured. Careful record should be made of the stationing of the intersections of these profiles with the center line, and of the approximate angle between them and the center line. Data should be secured from which to calculate the required waterway, all available evidence as to this being sought. Soundings, borings, and other soil investigations should be carried out, as previously described. If a channel relocation is contemplated, complete information for the area of the new channel must also be secured. When an existing structure is located on or close to the proposed center line, the following information should be recorded:

1. Number of spans and length of each.
2. Width of roadway and total width of structure.
3. Size of waterway of existing structure and all available evidence of its adequacy or inadequacy.
4. Stream elevation and velocity at time of survey.
5. Type of bridge superstructure and substructure.
6. Condition of superstructure and substructure.

If the proposed structure is to be for the separation of highway grades, the area mapped in detail must be large enough to include such interchange ramps as may be required. In addition, the profile and cross sections of the intersecting road are taken for a sufficient distance to include any revisions that may be made in that road. Consideration must be given to the possible desirability of relocation of the intersecting road.

For grade-separation structures at railroads, a profile of the tracks for 1000 ft on either side of the highway center line should be secured. The point of intersection between the highway and railway center lines should be established by means of a station and plus on the highway center line. The angle between the center lines should be measured. If the railroad is on a curve, pertinent curve data should be obtained. Cross sections of the railroad should be taken at 100-ft intervals, or oftener if needed, for several hundred feet either side of the center line.

For any projected bridge site, a large-scale topographic map (scale 1 in. = 10 ft, contour interval 2 ft or 1 ft) should be prepared. Profiles and cross sections should be drawn to similarly enlarged scales.

Survey Report. It is frequently desirable to complete the surveys of a highway project with a survey report. This will summarize the more important items of information that have been developed. It will aid the designer studying it in conjunction with the topographic map and tentative grade line and sections to gain a quick, over-all idea of the nature and difficulties of the project and of the proposals of the locating engineer for dealing with those difficulties. It will also serve as a check for the locating engineer, enabling him to discover if he has failed to secure any significant items of information that the designer will need. A suggested outline for such a report, based generally on practice of the Connecticut Highway Department, follows.

I. Identification and history of survey.
   - *A.* Name, number, termini, or other identification of road.
   - *B.* Description and stationing of beginning and ending points; equalities and reasons therefor, length of project; ties with any existing coordinate systems and with established datum; closure errors.
   - *C.* Dates of commencement and completion of various surveys, interruptions and delays in work, etc.
   - *D.* Record of conferences with local officials, utilities, etc.

II. Traffic. Summary of traffic survey findings and resulting geometric requirements as supplied to locating engineer, including:
   A. Design speed or speeds
   B. Type and width of pavement, number of lanes, width or widths of center division, if any
   C. Width and character of shoulders.
   D. Maximum allowable grade
   E. Maximum degree of curvature
   F. Vertical and horizontal sight-line requirements, required frequency of passing sight distances.
   G. Treatment of intersections, grade separations, interchanges, etc.
   H. Extent of any control of access
   I. Any other imposed requirements

III. Topography, existing land use, and utilities (identified by approximate stationing).
   A. General description of topography of area, with particular reference to swamps, rock, and other natural obstacles
   B. General description of property along proposed right-of-way and present use with special note of cemeteries, public parks, golf courses, etc.
   C. Location, size, type, profile, and ownership of water, gas, sewer, and power lines and any other utilities.

IV. Proposed alignment and grades.
   A. Relationship to any existing road or roads.
   B. Alignment control points and their influence on location.
   C. Alternate locations considered, with advantages and disadvantages of each and reasons for selection of line adopted.
   D. Critical points (list) controlling the establishment of the final grade, with upper and lower possible limits, reasons for the grade line recommended, possibilities of adjusting grade, etc.
   E. Discussion of any weak points in proposed line and grade and reasons why these were not improved.

V. Foundation conditions (identified by approximate stationing)
   A. Classification of soil materials as determined by observation and investigation.
   B. Statement of rock occurrences as revealed by inspection or borings, suggestions for rock or earth slopes in cuts, recommendations for any further subsurface investigations.
   C. Description of swamps or marshy areas with statement of depth to hard bottom
   D. Comment on critical points, such as probable slide formations, areas of inferior soil, etc.
   E. Recommendations for use of subbase, removal and replacement of inferior foundation soil, etc.

VI. Drainage
   A. Summary of weather conditions just prior to and during soils and drainage surveys.
   B. Description of suggested general drainage scheme for project, including treatment of all special conditions such as marshes and contiguous streams.
   C. A list of existing culverts and bridges showing location, size, length, and condition, and a statement as to their adequacy. This should include installations on an existing road being reconstructed, and also on parallel roads, railroads, etc., within a reasonable distance, both upstream and downstream.
   D. A list (by stations) of proposed culverts and bridges, giving recommended type, size, length, skew, end walls, etc.
   E. Description of existing sewer systems, ditches, gutters, and other drains.
   F. Recommendations for sewers, underdrain, paved gutters, intercepting ditches, leak-offs, etc
   G. Notation of any locations requiring high grade line to prevent flooding.

VII. Construction materials.
   A. Complete statement on location, quality, available quantity, and haul of local materials.
   B. Recommendations for securing borrow within highway limits
   C. Recommendations for securing borrow outside highway limits.
   D. Description of available sidings with location and distance from project.
   E. Availability of water.

VIII. Right-of-way.
   A. Description of right-of-way to be acquired, giving general condition of houses, barns, and other buildings along route, with approximate valuation where valuation has affected location.

B. Statement of anticipated damages to be incurred because of necessity for moving buildings, replacing trees or shrubs, etc.
C. Right-of-way difficulties involved in setting grade.
D. Any other pertinent data.

IX. Maintenance of traffic (where necessary).
A. Recommendations for handling traffic, whether via project or detours
B. Proposed detour routes, including length, type, and condition of detour roads and statement as to capacity and condition of critical bridges.

X. Special considerations, including.
A. Amount and character of clearing and grubbing required, indicating sizes of all large trees.
B. Statement of road and driveway approaches that must be constructed, with type and width of present surface.
C. Recommended treatment of all intersections with railroads.
D. Recommended treatment of all intersections with highways.
E. Retaining walls.
F. Private wells, springs, water-supply pipes, drains, etc., with recommended treatment in each case.
G. Topsoil locations, with depth to which stripping is possible.
H. Amount and condition of existing guard rail, single posts, etc.
I. Type and condition of existing property-line fences.
J. Station location of transverse joints at ends of project if connection is to be made to existing concrete pavement.
K. Complete list of survey data—maps, profiles, notebooks, cross-section sheets, etc.—accompanying report.
L. Names and positions of engineer making report and of all personnel employed on surveys.
M. Date of report.

HIGHWAY DESIGN: PLANS, SPECIFICATIONS,[1] AND ESTIMATES

5. GENERAL

The design of a highway can be divided into eight parts as follows:

1. Reviewing and plotting survey data.
2. Preparing preliminary construction plans.
3. Reviewing preliminary plans in field.
4. Preparing final construction plans.
5. Preparing specifications.
6. Preparing construction cost estimates.
7. Preparing right-of-way maps (if needed).
8. Preparing right-of-way cost estimate (if needed).

The principal end products of the design process will generally be:

1. A set of construction drawings.
2. A set of construction specifications.
3. A construction cost estimate of the project.

In addition, there may be:

4. A set of right-of-way maps.
5. A right-of-way cost estimate of the project.

A complete set of construction drawings for a typical project will normally include:

1. Title sheet.
2. Quantity estimate sheet.
3. Index plan sheet.

[1] Aspects of highway design involving specifications for materials and construction methods for road surfaces, bases, and embankments are treated under the general heading Highway Construction, p. 4-58.

4. Index profile sheet (may sometimes be combined with item 3).
5. Typical road cross-section sheet.
6. Plan sheets.
7. Profile sheets (where possible items 6 and 7 are combined on plan-profile sheets).
8. Cross-section sheets.
9. Bridge design or other structural sheets, if the project includes bridges or other major structures.
10. Miscellaneous standards and details sheets.

A convenient size for these sheets is 22 x 36 in. The U. S. Bureau of Public Roads specifies this size for all Federal aid highway work, and it has consequently come into wide use.

6. SURVEY DATA

Survey-Map Prints. If the survey map was plotted in the field on suitable material, prints are made directly from it. Usually one set of prints is used in tracing the map onto the separate final construction sheets; a second set is utilized in connection with rights-of-way acquisition; the preliminary highway design is worked out on a third set.

The field map and the notebooks should be carefully checked and compared for accuracy and to assure the completeness of the map. The map may then be traced onto the separate sheets. The proposed construction will be added to these sheets later.

Tracing the Plan Sheets. Whenever a straight alignment and a flat terrain make it possible, combined plan-profile sheets are used; otherwise separate sheets must be used for plan and profile. In either case, the plan sheet or half sheet and the corresponding profile sheet or half sheet should include the same length of road. Each plan should be oriented to make the center line as nearly horizontal as practicable. The plan should be centered in the top half of the sheet, with even stations above the heavy vertical lines of the lower (profile) half insofar as possible. The plan should not be "broken" on a sheet except where absolutely unavoidable. The plan sheets should show the following survey data:

1. All pertinent topography, including all property lines and the names of land and utility owners.
2. The original tangent and angle traverse (if one was run) with the lengths and bearings of all tangents, and witnesses to all transit stations.
3. The locations and elevations of all benchmarks, clearly identified and labeled (usually in a "box" with an arrow).
4. The location center line as staked, with stations, P.C.'s, P.I.'s and P.T.'s marked, and with tangent bearings, and curve data shown.
5. All political boundary lines.
6. A north point.

The center line is shown as a heavy, solid ink line. Light ink lines are used for all other survey data (except that political boundary lines are usually a little heavier). Where a curve extends onto more than one sheet, curve data should be repeated on each sheet. Conventional signs are given in Figs. 2 and 3.

Tracing Profile Sheets. When checked, the center-line ground-surface profile is transferred from the continuous field roll to the separate profile (or plan-profile) sheets. The horizontal scale is the same scale used for the plans, and the vertical scale is ordinarily ten times the horizontal. Besides the center-line profile, supplementary profiles are plotted for nearby portions of intersecting or parallel roads, drives, and railroads as well as for existing sidewalks, curbing and other significant objects. Sill and ground elevations of nearby buildings should be shown. Streams, bridges, culverts, drains, sewers, and any utilities that cross or lie adjacent to the center line of the proposed construction are also shown.

It is convenient to show profiles of roads, drives, etc., that intersect the center line as follows, left or far side by a light dashed line; right or near side by a light solid line. Those extending backward from the intersection are shown backward, those extending forward are shown forward, and those intersecting at 90° are shown extending backward on the left and forward on the right. The same conventions are used for intersecting stream channels whenever it may be desirable to show short lengths of them.

Where rock profiles and swamp-bottom profiles have been approximated by borings, pits, or other soundings, they are shown in light, solid lines. Since these profiles are only approximate and sometimes may turn out to be quite different from what was expected,

sheets showing them should always carry a notation disclaiming any warranty of correctness.

In the case of dual roadways, a separate ground-surface profile is plotted for the center line of each road. All ground-surface profiles on the proposed center lines should be shown in light, solid lines.

When transferring profiles to the final plan-profile (or profile) sheets, they should be placed so that the heavy horizontal lines of the grid coincide with 10-ft intervals (810, 820 ft, etc., and the heavy vertical lines coincide with stations. Stationing should be shown at the top and bottom of the grid, and elevations at the ends. All profiles and other features that form part of the survey data should be inked in solid or dashed lines.

Cross Sections. If cross-section sheets were not plotted as part of the location survey, they are plotted at this time from the cross-section notes.

7. PREPARATION OF PRELIMINARY PLANS

Procedure. As the first step in preparing preliminary plans, the initial decision as to the number and width of lanes and the width of shoulders (and median, if used) must be reviewed and confirmed or revised A decision must then be reached as to all details of the typical road cross sections—on tangents and curves, in cuts, and on fills. These decisions are prerequisite to setting the grade line

A final grade line is then worked out, first as a series of tangents; then with the addition of vertical curves. The profile is carefully studied and adjusted until it is completely acceptable.

With line and grade fully known, the pavement details of superelevation and widening of individual curves (see Sect. 3) can be established.

Drainage is then worked out, both in its surface and subsurface divisions. If bridge structures are required, certain basic data for them is prepared and bridge approaches are tentatively designed and made available to the personnel who will design the bridge.

Decision is made as to locations for guard rail, posts, construction and other signs, and similar auxiliaries

Finally (where this is necessary) "taking lines"—the boundary lines for the right-of-way—are decided upon.

Each part of the design, as it is worked out and accepted, is recorded on the appropriate plan, profile, and cross-section sheets, on which the survey data have already been shown. Consequently, the design gradually builds up on these sheets. The cross-section sheets on which the growing design is recorded will usually be those from which construction prints will be made. For plans and profiles the design will ordinarily be worked out on the prints made from the field maps and on the roll of profile paper that was used in the field These plans and profiles usually should not be traced onto the final construction sheets until the field inspection and any changes resulting from it have been made.

Typical Road Cross-Section Sheet. The important geometric elements of a road cross-section include the width and cross slope of the pavement or travel path, the width

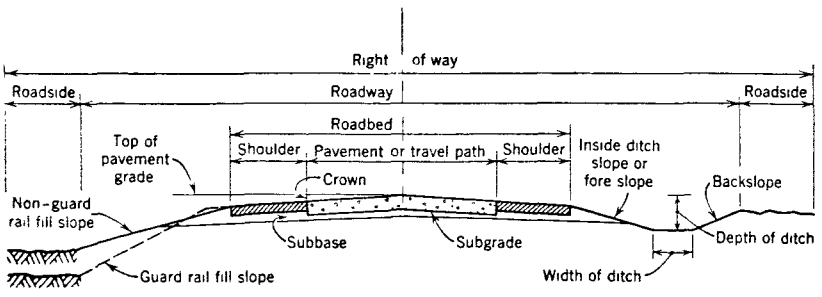


FIG. 4 Elements of road cross section

and cross slope of shoulders, the location and cross section of ditches and gutters, the slopes of cuts and fills, and the width and other details of a median strip where used. Figure 4 illustrates the conventional terms that apply to highway cross sections. Figures 5, 6, and 7 show typical sections of roads suitable for various traffic volumes.

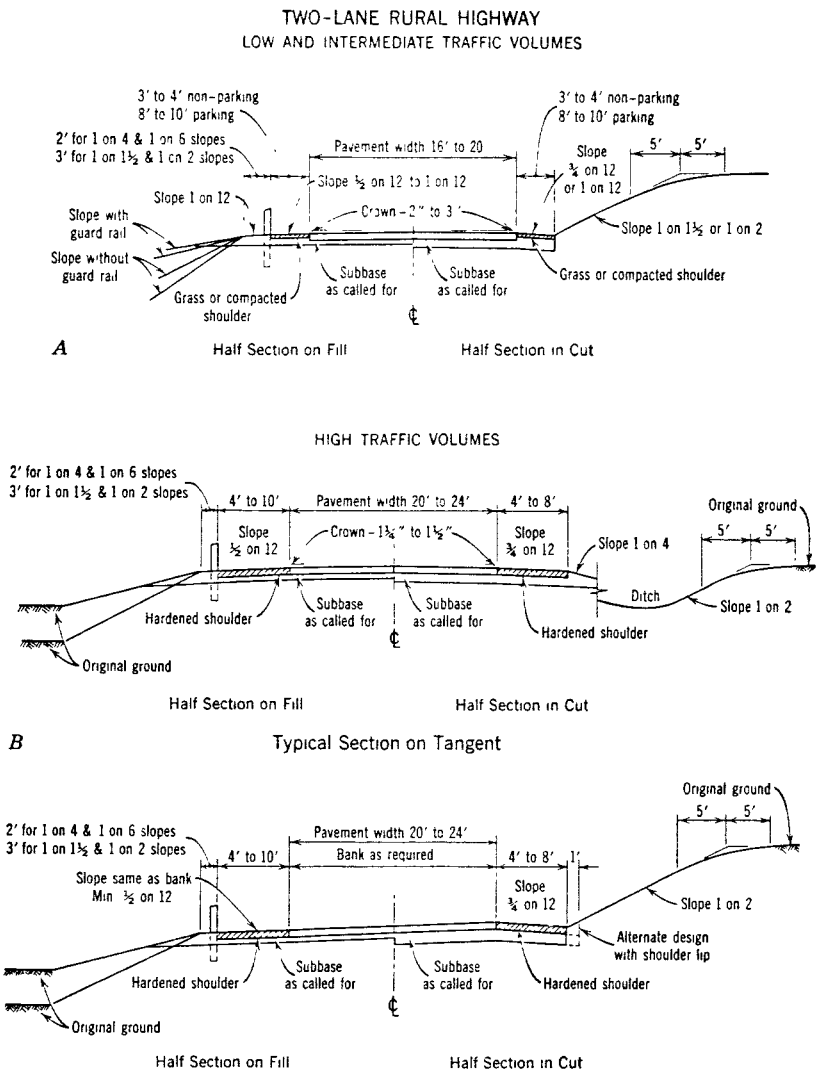


FIG. 5. Typical cross sections for two-lane roads. Underground drainage is not shown, but may be required, see pp. 4-32 to 4-48. For ditch sections see p. 4-29, pavement widths, p. 4-24, shoulder widths, p. 4-24, pavement courses and thicknesses (not shown), Arts. 16, 17, 18, subbase, p. 4-43, required bank, Sect. 3, guard-rail details, Figs. 22 and 23.

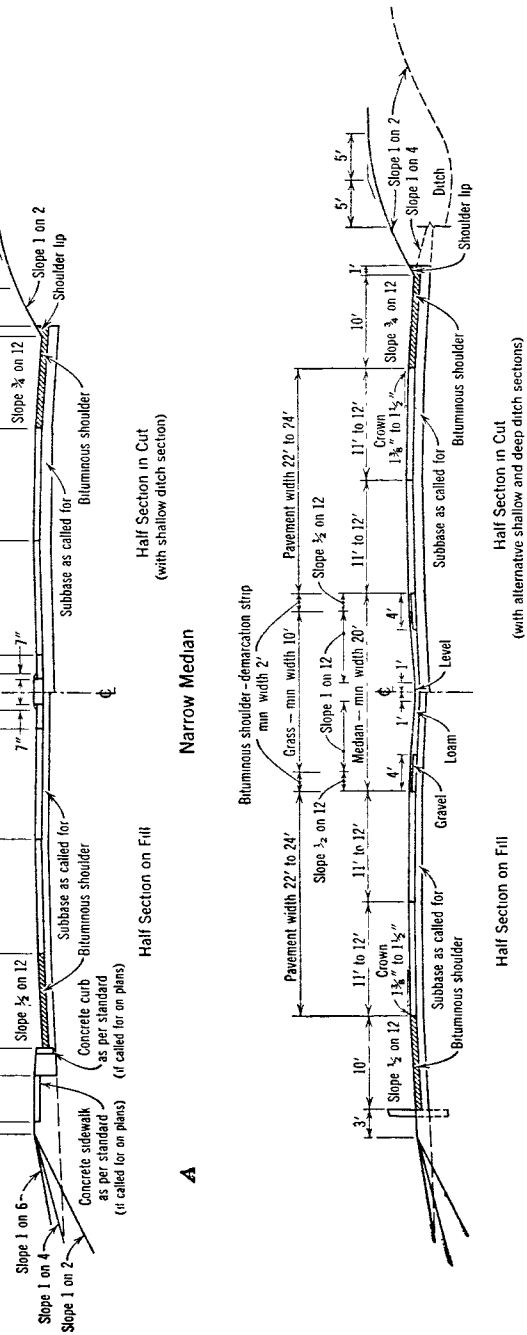
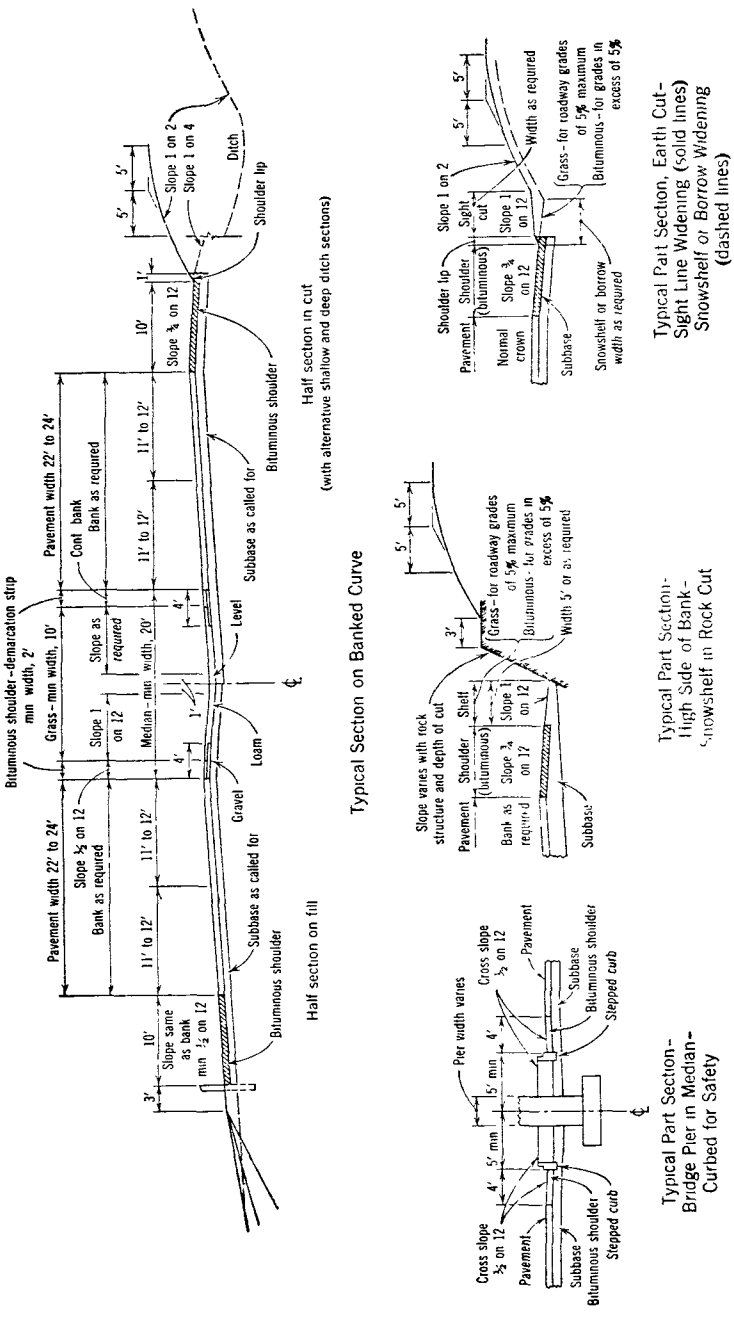


FIG. 6. Typical cross sections for four-lane divided highways; tangent sections, narrow and wide medians. Underground drainage is not shown, but may be required. See pp. 4-32 to 4-48. For ditch sections, see p. 4-29; pavement widths, p. 4-24; medians, p. 4-25, pavement courses and thicknesses (not shown), Arts. 16, 17, 18; subbase, p. 4-43, guard-rail details, Figs. 22 and 23.



PART SECTIONS SHOWING SPECIAL CONDITIONS

FIG. 7. Typical cross sections for four-lane divided highway; banked and other special sections. Underground drainage is not shown, but may be required; see pp. 4-32 to 4-48. For ditch sections, see p. 4-29, pavement widths, p. 4-21, medians, p. 4-25, pavement courses and thicknesses (not shown), Arts. 16, 17, 18, subbase, p. 4-43, required bank, Sect. 3, guard-rail details, Figs. 22 and 23.

Pavement Cross Section[1] On tangents, high-type pavements may have either a straight or a curved cross slope, intermediate- and low-type surfaces are usually curved. Where a curve is used, the difference between a circular and a parabolic arc is usually indistinguishable. The parabolic arc is easier to lay out. The amount of crown should not be more than that necessary to let water run off the road For two-lane concrete or high-type bituminous pavements, a straight cross slope of $^1/_8$ in. per ft is enough except where longitudinal grades are very flat. If a curved cross slope is preferred, an average value of $^1/_8$ in. per ft or a little more, from center line to pavement edge, will ordinarily be satisfactory. On intermediate-type surfaces, $^1/_4$ in. per ft is good. Low-type surfaces may require $^1/_2$ in. per ft or more. Three-lane pavements generally have a curved cross slope, although the two outer lanes may have a straight slope if desired The average cross slope from the center line to the edge of a three-lane pavement should be about $^3/_{16}$ in. per ft. Where curves are banked, it is customary to use a straight cross slope from the outside to the inside edge of the road metal

For secondary and feeder roads, surfacing or pavement widths recommended (1950) by the American Association of State Highway Officials are shown in Table 1.

Table 1

Annual Average Daily Traffic Volume	Width of Surfacing or Pavement, ft	
	Minimum	Desirable
Under 100	12, if any	
100–400	16	20
400–1000	18	20

For two-lane primary roads, the AASHO recommends the widths shown in Table 2.

Table 2

Widths of Pavement for Density and Type of Traffic, ft

Assumed Design Speed, mph	5 to 30 Vehicles per Hr			30 to 100 Vehicles per Hr			100 to 200 Vehicles per Hr			More than 200 Vehicles per Hr		
	P	M	T ‡	P	M	T	P	M	T	P	M	T
30 †	16 *	18	20	18	18	20	20	20	20	22 *	22 *	22 *
40	16	18	20	18	18	20	20	20	22 *	22 *	22 *	22 *
50	18	18	20	20	20	20	20	22	22	22	22	24 *
60	20	20	20	20	20	22	22	22	22	22	22	24 *
70	20	20	20	20	22	22	22	22	24	24 *	24 *	24

* Where adequate treated shoulders are provided, pavements 2 ft less in width may be considered.

† When topographic conditions force a design speed of less than 30 mph, a 14-ft minimum width may be considered.

‡ *Note:* P denotes passenger vehicles exclusively or passenger vehicles and commercial vehicles up to approximately 3 tons gross weight which have little effect on the movement of passenger vehicles and on the essential details of geometric highway design. M denotes mixed traffic consisting of passenger vehicles, buses, and trucks T denotes a large percentage of trucks which impede the smooth flow of other vehicles

Three- and four-lane roads should never have traffic lanes less than 11 ft in width, and 12 ft is preferable. See Sect. 3.

Shoulder Widths and Slopes. Road shoulders serve a number of different purposes, the more important of which are:

1. For the parking of vehicles.
2. As a "safety lane" onto which a vehicle can be driven in emergency to escape a collision.
3. As a pathway for pedestrians and bicyclists.
4. To support the edge of the road metal, and prevent its breaking and raveling.
5. To encourage use of the outer two or three feet of the travel path, since motorists tend to shy away from the pavement edge where the shoulder is defective.
6. As an area for depositing snow from the travel path.

The volume and type of traffic using a road establish the importance of these several shoulder functions and the expense warranted for shoulder construction. The terrain plays a predominant part in fixing the cost of providing a shoulder of any given width.

[1] Pavement thicknesses and other design elements related to pavement strength are discussed later

Joint consideration of need and cost should guide decision as to what should be done. The AASHO specifications require that, on the Interstate Highway System, shoulders shall be at least 10 ft wide in non-mountainous regions and at least 4 ft wide in mountainous regions. For primary roads not on the Interstate System, the AASHO recommendation is as follows.

"*Shoulders* In general, shoulders should be wider on fills than in cuts. Wherever obtainable at reasonable cost the width of shoulders on all highways on which the traffic density classification is more than 200 per hour should be not less than 10 feet on fills on which the slope is steeper than 1 on 4 and not less than 8 feet in cuts or on fills on which the slope is 1 on 4 or flatter. Shoulders should be usable in emergencies. Where the traffic density classification is less than 200 vehicles per hour shoulders should be from 4 to 10 feet in width, the greater widths to be used for the greater traffic densities and higher design speeds. The above shoulder widths do not include widths necessary for guardrail."

The purposes for which road shoulders are provided are largely defeated unless the shoulder is built of material sufficiently stable to support vehicles in all weathers. On lightly traveled roads, the requisite stability may be secured by a good turf mat on a naturally stable soil. As travel increases, shoulders of gravel or other suitable granular material are desirable, and treatment of the shoulder with a bituminous surface coat is increasing in favor. In extreme cases, paved shoulders are warranted.

The cross slope of shoulders should be at least 1/2 in. per ft, and where the shoulder is not treated, or the material is at all questionable, 3/4 in. is better. On the outside of banked curves, shoulders are often given a greater slope (always opposite to the pavement slope) than on tangent sections, to insure getting water away from the pavement. This is especially true when the water may come from melting snow.

Ditch and Gutter Sections. Ditch sections will be discussed as a part of drainage design (p. 4-29). *Curbed* gutters should be avoided alongside rural roads, unless gutter and curb can be located back of the shoulder; otherwise, a hazard results. Gutters may be used along the inner edge of a divided roadway; however, the width of the gutter should not be counted as part of the travel path.

Median Strips. Median strips may be either depressed, or raised and curbed. Choice between the two types is largely dependent on available roadway width. Where this is ample, the depressed cross section is generally preferable. However, it provides less barrier to a car out of control than is afforded by a curbed, raised median. No hard-and-fast rule can be laid down, but, when the distance between inner edges of the two roadway pavements is less than about 20 ft, the raised section should normally have preference. Raised medians are ordinarily crowned to shed water into gutters flanking both sides of the median. Gutters should have a cross slope of 3/4 in. per ft, and are ordinarily made about 2 ft wide, exclusive of curb width. The gutter width should be in addition to the normal pavement width. Unless the raised median is wide enough to allow at least 4 ft of grassed width, it is preferable to surface it with concrete.

When the width between pavement edges is between 20 and 36 ft, the decision as to median cross section should be based on consideration of each individual case. For widths of 36 ft or more, a depressed section is normally desirable. This type median is generally grassed. It should have a bituminous shoulder demarkation strip between the grass and pavement edge, with a minimum width of 2 ft. Widths up to 5 ft or more have been used. With the depressed section, water from the inner half of each pavement, as well as that falling on the shoulder demarkation strips and the grassed area, flows to a shallow ditch located on the center line of the median, from which it is removed through suitable drainage facilities (p. 4-32). If a four-lane road is built with the expectation of its ultimate conversion into a six-lane road, the median should initially have sufficient width to allow the two future lanes to be taken from it.

In northern regions a dual road should be designed to permit ploughing snow to both sides of each roadway. The wide, depressed center median is satisfactory for storage of this snow. On a raised median, stored snow may spill into the inner gutters, blocking them and causing a serious ice hazard from ponded water.

The Interstate System standards specify absolute minimum widths of 15 ft and 4 ft for medians in rural and urban areas, respectively; and specify 40 and 12 ft as the corresponding minimum desirable widths. (See Sect. 3 for additional design features of medians from standpoint of traffic.)

Cut-and-Fill Slopes. Cut-and-fill slopes are often made less steep in modern practice than formerly. This change is due to the relatively lower cost of moving earth now; the greater safety of flatter slopes on fills and better appearance of flatter slopes in cuts; the lower maintenance costs of flatter slopes both in cuts and on fills. No hard-and-fast rules

for slopes can be laid down. In rock cuts, they range from 2 on 1 to about 6 on 1. A slope steeper than 6 on 1 appears to overhang, and a slope as steep as this is unpleasant in a through cut of any depth. Frequently, moreover, rock will not stand so steeply without endangering traffic by slides or falling stones. The slope should be flattened as much as finances permit. When benching into rock on a mountain or steep hillside, where flattening the slope would produce an excessively deep cut, it may be preferable to retain a steeper slope, but to widen the shoulder a few feet so as to set the rock face farther from the travel path.

In earth cuts the slope used will normally be based on study of the soil through which the cut is to be made, and its ability to maintain a stable slope. Water conditions in the cut will affect this ability also The slopes in most general use lie between 1 on 1 1/2 and 1 on 2 Modern practice inclines towards the latter when finances allow. Deep cuts may be benched at vertical intervals of about 30 ft, giving the bench a reverse slope to form a ditch to remove the water running down the face of the slope. The top of a cut slope in earth should be rounded off to a tangent distance of 5 to 8 ft on either side of the P.I. of the slope and ground-surface lines.

On fills, slopes much the same as in cuts are used, except that, on low fills, flat slopes eliminate the need for a guard rail. The AASHO specifications for the Interstate System call for 1 on 2 slopes for earth fills over 10 ft high and 1 on 4 for lesser fills. Sometimes slopes as flat as 1 on 6 are used on fills of intermediate height—say 4 to 8 ft—without a guard rail Few soils will stand on a steeper slope than 1 on 1 1/2. Slopes of this limiting steepness are sometimes used, but with some soils erosion is apt to be severe or slides may occur.

When details of the road-section design have been fixed, a sheet of typical sections is drawn. These may usually be drawn directly onto the 22 x 36-in. tracing cloth that will form part of the set of construction drawings. Normally this sheet will contain a half section on cut and a half section on fill, both on tangent; cut-and-fill sections for a banked curve, together with any other sections that may be useful, such as one showing a cut widened for a sight line, a ramp section if the project includes a traffic interchange, a section in rock cut, etc. The road sections are frequently not drawn to scale, but are fully dimensioned.

Grade Controls. Before a grade line can be added to the profile sheets, careful consideration must be given to the controls to which the grade must be adjusted. Controls are of many sorts, based principally on: (1) requirements of traffic; (2) conditions adjacent to road; (3) maintenance considerations; (4) construction cost.

Few grade-control problems can be resolved by means of formulas or iron-clad rules. They have to be individually recognized and evaluated by the experience and good judgment of the designer. There will often be conflict; as, for example, in making a compromise between property damage and the best grades for traffic or maintenance, in traversing a thickly built-up area. However, the following may be taken as general guides:

1 Stay as far below maximum allowable grade as available funds allow.

2. Maintain continuous non-passing sight distance for the design speed with adequately frequent passing sight distance. (See Sect. 3.)

3. Avoid short "rolling" grades and an excessive amount of rise and fall.

4 Avoid using same P.C. or P.T. for horizontal and vertical curves. The vertical curve should preferably extend on to the horizontal tangent.

5 Avoid leveling off a long descending grade to a flat grade several hundred feet in length as a bridge or grade separation structure is approached.

6. Before designing a highway intersection, determine any existing or probable future plans for reconstructing the intersecting road. Consider possibility of shifting alignment of the intersecting road if desirable to improve conditions.

7. At any intersection at grade where vehicles may have to stop, provide a level or nearly level length of road for them to stop on.

8. Remember that it is usually much easier to secure agreement to raise rather than to lower railroad track.

9 Whenever feasible, hold grade line below sill elevations of nearby buildings.

10. Prefer low fills to shallow cuts. Hold grade above general ground level to facilitate drainage and natural removal of snow.

11 Where a soil profile has been included in the survey, utilize it to take best advantage of good soils and to minimize effects of bad soils that will be encountered.

12. Establish grade line an ample distance above water table to provide a dry roadbed and to prevent frost heave

13. Keep the road high enough not to be subject to flooding at any point. Where drainage lines cross the road, provide ample room for culverts of adequate size.

14. Be sure that no drainage pocket is formed at a low point on the inside of a superelevated curve.

15. Preferably avoid grades of less than 1% in cuts or on embankments where water is confined. In such situations use no grade less than 0.4%. The flatter the grade, the greater is the cost of drainage.

16 In using old railroad or other narrow embankments for fills, consider lowering the grade line to get sufficient width of old fill to support entire width of road metal.

17. Avoid shallow rock cuts. Hold grade line high and use borrow if necessary.

18. Unless there is good reason to the contrary, balance cuts and fills.

Establishment of Grade Line. The establishment of a grade line is essentially a process of trial and error. The profile of the entire project, or of a considerable portion of it, is worked on as a unit. Controls are carefully established from the considerations just outlined, and with attention to recommendations of the survey report. They are plotted on the profile sheet, either as exact elevations or as upper and lower limits. When this has been done, it is convenient to tack the profile to a table and to stretch a light string between two control points an adequate distance apart. By means of pins the string is then tentatively conformed to the intermediate controls, a set of vertical tangents resulting. The trial grade line is then studied to see how well it meets the controls in which there is some leeway. The line is considered from each of the four standpoints listed on p. 4-26, first attention being given to the requirements of traffic, abutting land use, and maintenance. Consideration then is given to construction economy, both from the standpoints of probable types of material to be excavated and quantities of earthwork and lengths of haul. The line is revised as found desirable, by shifting the pins. The ultimate grade will generally be a compromise between conflicting considerations. In considering earthwork quantities and also sight distances, approximate vertical curves will probably have to be sketched in at some places. When the designer is satisfied with his trial grade, tangents are drawn to correspond to the string positions and the pins and string are removed. If the length of profile considered was less than the entire project, this process is repeated. In doing this, a portion of the trial line at the "lead" end should be repinned and reconsidered in the light of the next segment of the job.

Vertical Curves. At every P.V.I. where the break in grade is considerable, a parabolic vertical curve is used. There are two principal reasons for using vertical curves, to secure the necessary sight distance, and to prevent discomfort to road users from too sudden a change in the direction of motion. Where the difference in grades is small, only the last reason is important. For high-type pavements, with correspondingly high design speeds, it is good practice to use vertical curves at all breaks in grade. For intermediate- and low-type surfaces, curves need not be used for differences in grade less than 0.5%.

The length selected for a vertical curve depends on the purpose to be accomplished. In sags and over low crests where adequate sight distance is not a factor, the length that best fits the curve to the ground is ordinarily used. It is important, however, especially in sags, to avoid a curve so long as to create a "flat" length in the grade that will not drain properly. Where this is a factor, curves as short as 50 ft should be used. When a vertical curve is needed for sight distance, Figs 34 and 35, Sect. 3, can be used. Where neither sight distance nor drainage imposes a control, the length of a vertical curve should be as much as can be had without seriously increasing grading quantities. For small differences in grade (say up to 1 $^1/_2$%) the effect of the curve on excavation is small, and amply long curves for appearance can be used; 300 to 400 ft will usually be satisfactory.

As soon as the center-line profile has been fully established, road templets can be added (initially in pencil) to those cross sections that are on tangents and far enough from the P.C.'s and P.T.'s of curves not to be affected by any superelevation or other pavement modification that the curves may necessitate.

Superelevation of Curves: Pavement Widening. Superelevation of curves is employed to counteract the effect of centrifugal force. Section 3 discusses the determination of the proper amount of superelevation for curves of various radii and for various design speeds. Section 3 also develops the transitions, horizontal (when used) and vertical, that join the normal (tangent) pavement section with the fully elevated section. Cross slope of a superelevated section is usually made uniform from edge to edge (Figs. 5 and 7).

The width of pavements is sometimes made greater on curves than on tangents Conditions warranting the increase and a formula used in securing it are outlined in Sect. 3.

Superelevation may be accomplished by rotating the pavement about the center line or about either edge. In rolling terrain, rotation about the center line is quite general. This minimizes the length in which the full amount of bank can be accomplished: however, it requires attention, in establishing the grade line, to avoidance of a drainage "pocket" on the inside of the curve. This is automatically taken care of by rotating

the pavement about its inside edge. However, in flat country, this is apt to create a poor appearance; and to avoid a "hump," the rotation is sometimes made about the outer edge. Where this is done, it is still more necessary to see that the grade line is kept high enough at curves for good drainage.

In working out the superelevation of a curve, it is often desirable to draw a working profile of each pavement edge, deriving these from the previously established center-line grades and from the computed amount of superelevation (and widening, if any) at the section in question. These profiles should commence at the point (on tangent) where superelevation begins and should continue until the normal tangent section is regained. They are studied, especially, from the standpoint of drainage. If they are unsatisfactory, a minor revision may have to be made in the center-line grade to correct the trouble.

When all the details of superelevated sections have been fully and finally worked out, road templets showing what is to be done are added to the appropriate cross sections. The center-line profile is also corrected if necessary. At the proper time, corresponding plan views of the pavement for the length of road in question will be put on the plan sheets.

Drainage

Drainage Design. Good drainage design is fundamental, both for economical maintenance and for comfortable travel. Satisfactory drainage design requires recognition of and attention to a few basic facts concerning water and soils.

Water. Surface water is that which falls upon the roadway and immediately adjacent areas as rain, snow, or sleet, or which enters the highway limits on the surface of the ground, as from streams or the runoff from storms.

Subsurface or *underground* water is that which enters the highway area by movement through the soil, either in the form of springs or subterranean water courses, or by percolation through a pervious soil stratum; plus that which may remain more or less stationary beneath the surface of the water table, or be drawn above it by capillary action.

Subsurface water is of two types: free and capillary. *Free* water is free to move under the influence of gravity—it can be drained off by suitable procedures. *Capillary* water, on the other hand, is held to the soil particles by surface tension and is not free to seek a lower level under gravitational forces. Capillary water has ordinarily been drawn up to the soil stratum where it exists from a saturated stratum beneath (Fig. 8). Saturation of

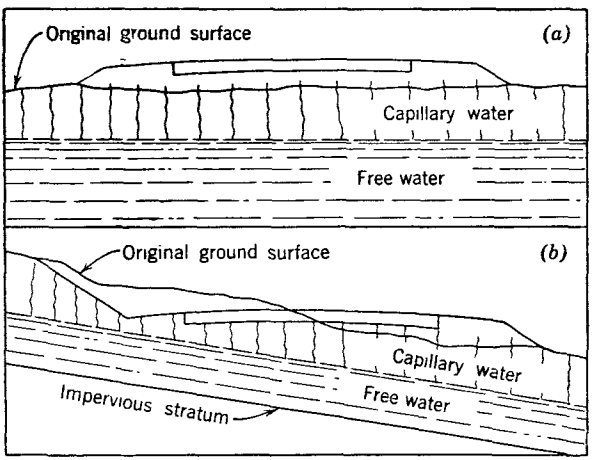


FIG. 8 Free and capillary water (*a*) ground water stationary beneath level surface, (*b*) seam of water flowing through pervious stratum above impervious stratum

the lower stratum is due to one of two causes: either the stratum is below ground-water level; or it overlies an inclined, impervious stratum and has free water percolating through it on top of the impervious stratum. Capillary water cannot be removed directly by drainage; it can be controlled, however, by lowering the ground-water level, above which capillarity can raise the water only a limited distance. Capillary rise can also be controlled, without affecting the free water that is its source, by replacing fine-grained, silty

material of high capillarity with granular material of less capillarity. Each of these remedies is used in highway work.

Soils. The character of the roadbed soil is of great concern to the highway designer. From his standpoint, the most important characteristic of a soil is its *permeability*, or perviousness, which largely determines the ease with which excess water may be removed from it. Permeability is largely dependent on grain size. Soils, simply classified from this standpoint, are *gravel*, *sand*, *silt*, *clay*, and *colloids* (For definitions and detailed treatment, see Sect. 7.) Soils that are to be used for the support of road surfaces usually have more valuable qualities as their prevailing grain size increases; soils that are largely granular in their composition are superior to the silt-clay-colloidal soils The best material for an embankment, for instance, is a well-graded mixture of gravel and sand with a small amount of silt or clay to act as a binder.

Stability of Roadbeds. Stability is the first necessity of a roadbed. No pavement, high or low type, rigid or flexible, will long retain satisfactory usefulness unless the supporting material beneath it remains firmly in place. Hence the roadbed must, above all things, be *stable*—it must neither be heaved by frost in winter nor become rutted in spring. A pavement may sometimes fail because of shortcomings inherent in the pavement itself and not related to what it rests on, but *no* road surface will long remain satisfactory unless the roadbed beneath it is stable.

Lack of stability in a roadbed may sometimes be due to inferior materials, but nearly always it is caused or aggravated by improper conditions of moisture Rarely, there may be too little moisture properly to bond the solid materials together. Much more frequently, lack of stability is due to too much moisture, either permanently or intermittently The moisture is alike injurious whether drawn up into the roadbed from beneath by capillary action, whether it seeps in from water-bearing strata underlying adjoining areas, whether it percolates down through a leaky pavement, or reaches the roadbed in any other manner.

Overcoming Drainage Problems. Drainage installations are classed as "surface" or "subsurface," not as they themselves are on, or are below, the ground surface, but according to the type of water they are primarily intended to remove. In occasional instances, the same pipe may carry both surface and subsurface water. It then forms part of both drainage systems.

The first requisite to satisfactory drainage is a proper grade line. Without it, compensating by means of additional drainage will be costly and not always effective. As indicated in the discussion of grade lines, the subgrade must be an adequate distance above the water table and above capillary water as well; and the grade line must also be high enough to provide adequate waterway for streams and other surface runoff that must be carried from one side of the road to the other.

Surface water, insofar as possible, must be kept from penetrating the roadbed. This means an impervious road surface with proper longitudinal and cross grades to shed water onto shoulders that have been suitably shaped to carry it to the ditches, rather than letting it sink into the roadbed. The situation is better still when the shoulders can be given a waterproof, bituminous surfacing.

Subsurface water, insofar as possible, must be kept out of the roadbed for a sufficient depth below the road surface (1) to prevent frost troubles, and (2) to maintain a layer of stable material thick enough to distribute the pavement pressure successfully to less stable layers beneath.

Surface Drainage: Side Ditches. The flatter the terrain, the more difficult is the removal of water from the right-of-way. Consequently, in rolling or hilly country, side-ditch design tends to be quite different from what it is in flat country.

Figures 5A and *C* show the simple ditch section applicable to rolling terrain where longitudinal road grades are adequate and where precipitation quickly finds its way to natural water courses. The sides of this ditch are formed by the road shoulder and the cut slope. This shallow type of ditch is safest for traffic, and is particularly advantageous where driveways enter the road frequently. Except where heavy snowfall necessitates additional width for snow storage, this ditch section requires the least yardage of excavation for any given depth of center-line cut. The ditch is also very easily maintained. When additional snow storage space is required, it is simply provided (Fig 7) by continuing the shoulder slope as far as needed. The bottom of this type of ditch will usually have a higher elevation than the subgrade opposite it; hence, it will not drain the subgrade where the road profile is flat. On side-hill locations, this is not serious, since any water reaching the subgrade through the pavement can be carried to the outer edge of the road and drained off there In through cuts, however, a deeper ditch section should be used where the profile is flat, unless subsurface drainage is installed. In deep cuts, the saving in excavation will often make subdrainage more economical than the deeper ditch

section. The shallow ditch section is especially satisfactory in long cuts if storm sewers are used, the accumulated flow being dropped into them at intervals of 300 to 500 ft. On side-hill locations, cross culverts should be used at about the same intervals.

Figure 5*B* shows a satisfactory ditch cross section for more level country. Here highway grades are flatter and opportunities to turn water away from the highway are fewer. Hence, the ditch must have a larger cross section. Beyond the normal shoulder line an inside ditch slope of 1 on 4 is considered safe. The back slope should be flat enough to be completely stable; 1 on 2 will ordinarily suffice. The length of the 1 on 4 inside ditch slope is determined by the required depth of ditch. Figure 9 shows the design adopted

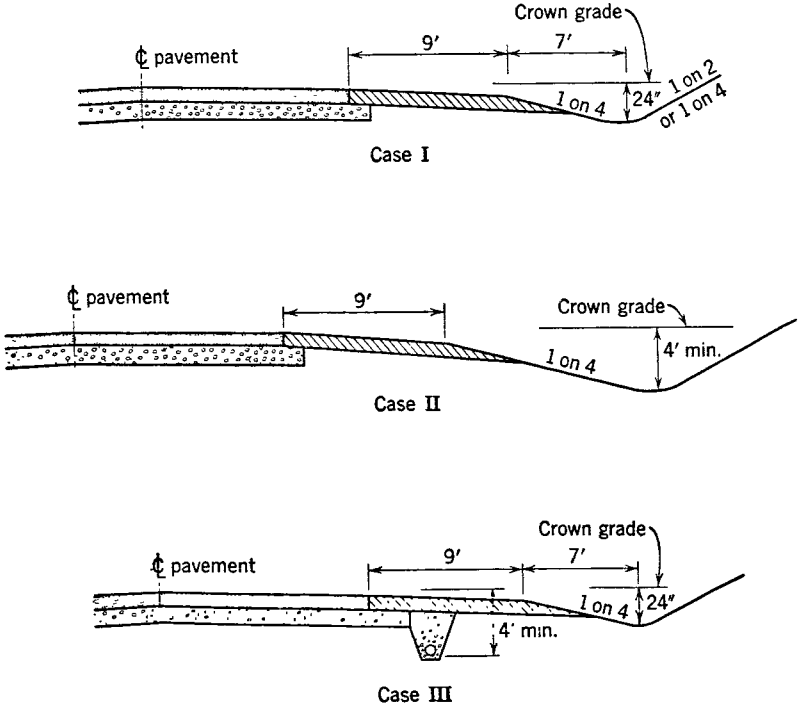


FIG. 9. Variation in ditch section and use of subdrainage to meet varying soil, ground-water, and excavation conditions—New York State Thruway. *Note:* Differentiation between uniform and non-uniform subgrades to be based on soil survey. *Case I:* Subgrade—uniform soil and/or ground-water conditions. *Case II.* Subgrade—non-uniform soil. For shallow cuts (up to 5 ft) and for borrow jobs where increased excavation is economical. *Case III:* Subgrade—non-uniform soil. For deep cuts and for excess excavation jobs.

for sections of the New York State Thruway. Where soil and water conditions allow the ditch bottom to be placed only 24 in. below pavement center-line grade, the ditch center line is only 7 ft from the edge of shoulder (case 1). However, when more complete drainage is required, the 1 on 4 slope is continued until a ditch depth of at least an additional 2 ft is secured (case 2). This widens the cut at least 8 ft on each side of the center line. Where this would cause an uneconomical amount of excavation, subdrainage is used (case 3).

Where the ground is flat and especially where soils are high in capillarity and subbase is expensive, it is desirable to raise the road surface above the general ground level and to maintain a considerable difference in elevation between the pavement crown and the ditch bottom. Both these purposes can be accomplished conveniently by balancing the cut and fill over a cross section. The equation for this (with a small approximation resulting from disregarding the cross slope of the pavement) is:

$$\left(\frac{r_1 + r_2}{2}\right) y^2 + by = \frac{1}{1-k}\left[w_1(h - t) + \frac{w_2}{2}(2h - sw_2) + \frac{r_1}{2}(h - sw_2)^2\right]$$

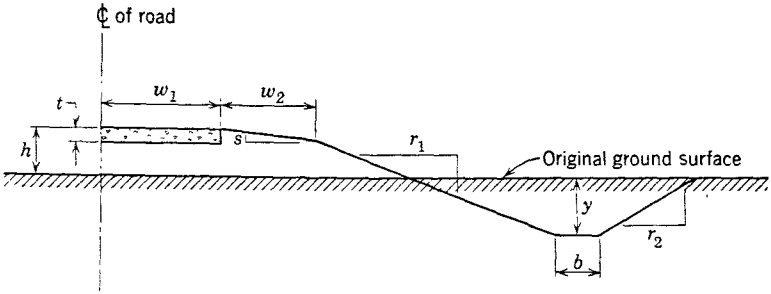


FIG. 10. Balanced grading section.

where (Fig. 10) h = center-line elevation of pavement surface above ground elevation, ft.
t = pavement thickness, ft.
w_1 = $^1/_2$ pavement width, ft
w_2 = shoulder width, ft.
s = cross slope of shoulder, ratio of vertical/horizontal.
r_1 = fore slope of ditch, ratio of horizontal/vertical.
r_2 = back slope of ditch, ratio of horizontal/vertical.
b = bottom width of ditch.
y = depth of ditch below ground surface, ft.
k = shrinkage of cut material, %/100.[2]

If all values except y are assumed, the required ditch depth to balance can be worked out and horizontal dimensions computed. For a 22-ft pavement 8 in. thick, an 8-ft shoulder with a shoulder slope of $^3/_4$ in. per ft, a ditch with a bottom width of 4 ft and with fore and back slopes of 1 on 4 and 1 on 2, respectively, and an assumed value of 0.15 for k, raising the crown 1 $^1/_2$ ft above the ground level requires a ditch about 2 $^1/_4$ ft below ground level, or 3 $^3/_4$ ft below pavement crown. The ditch center line will be approximately 36 ft from the road center line. Where it is desired to minimize the depth of ditch a flatter back slope can be used, or "side borrow" can be taken from cuts, etc.

In the foregoing paragraph it is assumed that the grade of the road will parallel the ground surface. A variation of the procedure, where differences in ground elevation are small, is to hold a straight road grade and to compensate for rise and fall in the ground by varying the height of the fill (and sometimes the depth of the ditch also). In this case, the ditch center line will not parallel the road center line, but will "flare" as the embankment height increases. Whichever of these methods is used, the ditch drainage is discharged into natural watercourses as frequently as possible. It is preferable not to make the ditch grades flatter than 0.5%.

Ditch Erosion. On steep grades, erosion must be prevented. Erosion is due to the velocity of flow, which increases both with the grade and with the quantity of flow. The minimum velocity for serious erosion varies widely with the soil. In sandy soil, erosion may have to be guarded against on grades as flat as 2%; in any ordinary soil, on grades of any substantial length that exceed 4 or 5%. Where a bituminous treated shoulder surface forms the ditch bottom, erosion may be prevented by placing a small berm of bituminous material at the toe of the slope, as shown in Figs. 5, 6, and 7. This is sometimes called a "lip curb." If the shoulder is not treated, the necessary erosion protection may be provided by sodding the ditch; or more elaborately by paving with cobble, bituminous material, or concrete. Alternatively, baffles are sometimes used to retard the flow. Erosion is also restrained by dropping accumulations of water into storm sewers, as aforesaid.

Where the road cross section changes from cut to fill, particular care must be taken to keep ditch drainage from eroding the fill. This can usually be accomplished by construction of a sodded or paved "leak-off" to carry the water along the toe of slope. If the fill is more than 15 or 20 ft high and the ditch carries much water, a 15-in. corrugated metal "downspout" may be preferable. This should commence well within the cut area, for

[2] Fill, properly compacted in an embankment, will occupy from 10 to 20% less volume than that which it had before excavation. This loss of volume is called shrinkage. It is usually expressed as a per cent of the original volume. See Sect. 7.

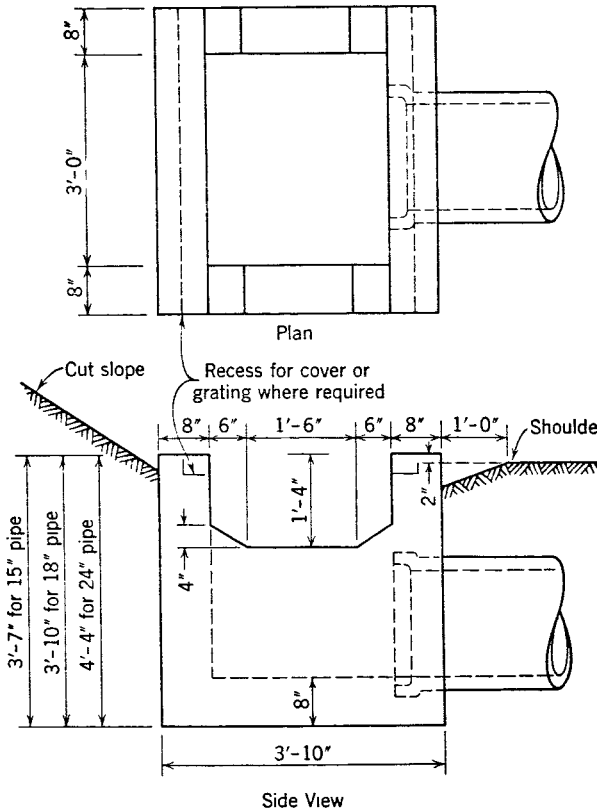


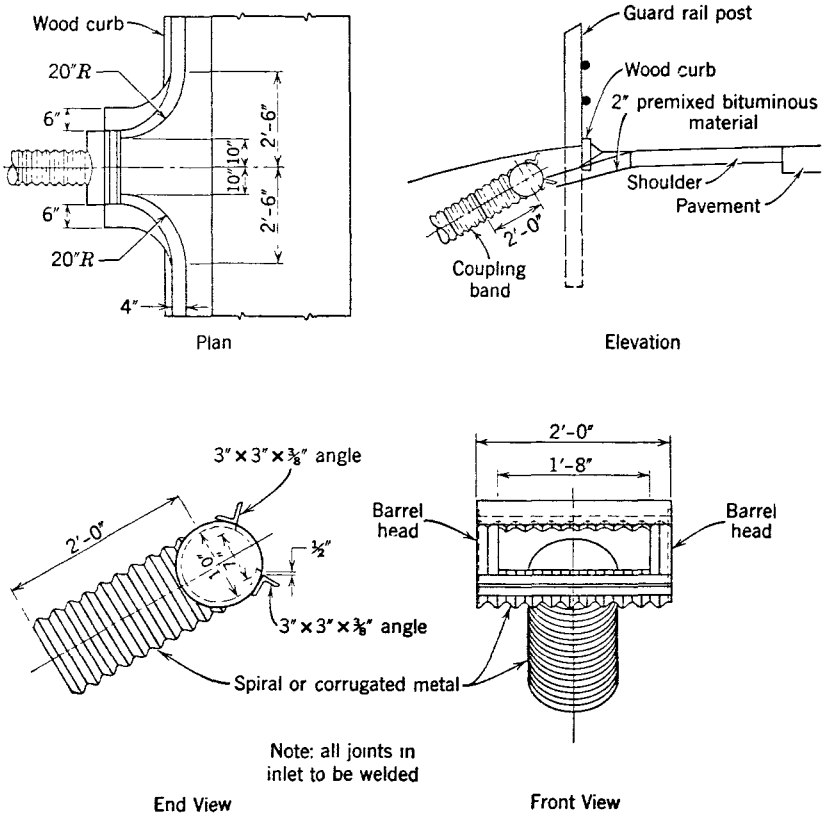
FIG. 11. Drop inlet. Bottom of concrete wall may be of concrete, brick, precast concrete units, or rubble masonry. If rubble masonry, make wall thickness 1' 0".

stability. The inlet end should have a drop inlet (Fig. 11) or end wall (Fig. 16), and the outlet may be provided with a baffle to check the velocity of flow.

Berms on Embankments. Where heavy rainfall occurs it may be desirable to construct a ditch or berm along the top of a high embankment when the embankment is made of easily eroded material Often this berm will need to function only until vegetation is established on the embankment. Such a berm can be simply made by attaching a 2 x 8 in. or 2 x 10 in. board to the inner face of the guard posts, with its lower edge well embedded into the ditch bottom. For more permanence, a sodded gutter or a bituminous gutter and berm may be used. If the fill is long, the gutter must be emptied at intervals by means of inlets from which pipes outlet at the toe of slope (Fig. 12).

Intercepting Ditches. When drainage from areas above the top of slope in cut sections is enough to erode the slope, use should be made of an intercepting ditch. This should parallel the top of the slope and be from 5 to 15 ft back of it. The size of the ditch depends on the area to be drained and its character, and on rainfall characteristics. The size should be generous Such ditches will usually have to be sodded or paved to prevent erosion, at least towards their ends.

Divided Highways. Divided highways present a special problem in the removal of water from the inner half of each pavement and from the median strip. Where freezing occurs, none of this water should be allowed to flow to the outside edge of the pavement, because of the hazard created. When a raised median must be used the inner gutters are drained into drop inlets (Fig 11) or catch basins (Fig. 13). On flat grades this may be at intervals as short as 200 ft; on steeper grades the interval may be as much as 500 ft but in no case so much as to overflow the gutter. When the median is depressed water is carried longitudinally in a surface ditch along its center line. It is dropped into inlets



Detail of Asphalt-Coated Corrugated Metal Inlet

FIG. 12. Inlet and downspout to take surface drainage from embankment with berm.

or catch basins for lateral discharge; or, where points of lateral discharge are far apart, into a longitudinal sewer. The ditch will normally be grassed, but may be paved for a short distance on either side of the inlet, or in other places where conditions warrant.

Pipe Culverts. Pipe culverts are used (1) to remove drainage from highway ditches or to carry it past intersecting crossroads and drives, and (2) to carry stream flows—either continuous or intermittent—from one side of a roadway to the other. It is convenient to treat the two uses separately, although sometimes the same culvert may carry both types of drainage.

Culverts for Ditch Drainage. Both longitudinal and cross culverts are used to remove ditch drainage. For both purposes, reinforced-concrete pipe (AASHO Specification M-41) and corrugated metal pipe (AASHO Specification M-36), coated with asphalt cement, are widely used. Cement asbestos ("Transite") pipe has also come into some use. Vitrified clay pipe is not used as much as formerly because of its tendency to break.

Cross Culverts. Where cross culverts are used to remove ditch drainage, they need rarely be placed under heavy fills, and the choice between concrete and metal pipe can usually be based on comparative cost and availability. Where settlement is feared, metal pipe has the advantage of longer lengths and stronger connections. The top of the pipe should be a distance beneath finished road surface equal to the outside pipe diameter or 24 in., whichever is smaller. Where this amount of cover is difficult or impossible to secure, cast-iron pipe (AASHO Specification M-64) is sometimes used, but it is expensive. Cross culverts should have ample length. The distance between inside faces of culvert headers should never be less than the full width of travel path and shoulder.

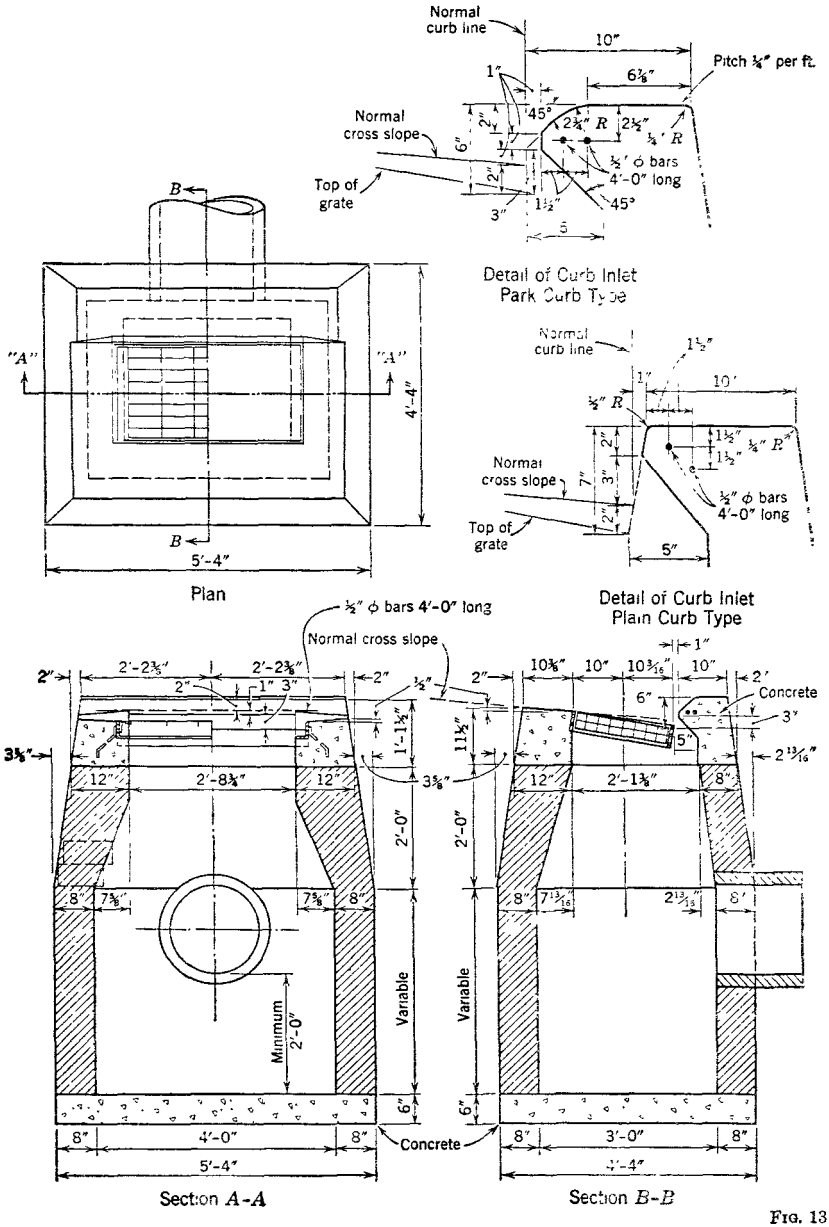
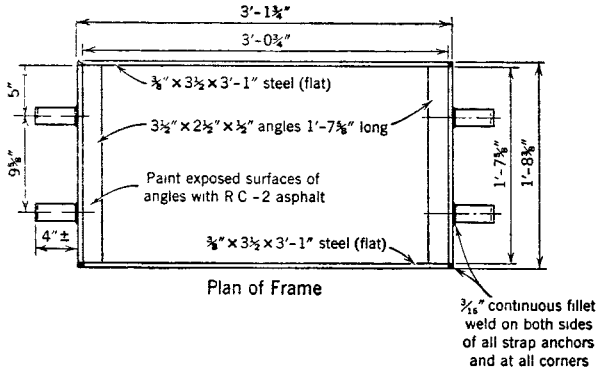
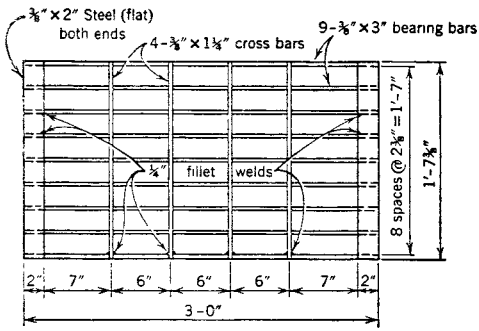

Normal curb line
10"
Pitch ¼" per ft.
6⅞"
1"
45°
2¼" R
2½"
¼" R
Normal cross slope
6"
2"
½" ϕ bars
4'-0" long
Top of grate
3"
1½"
45°
5"
Detail of Curb Inlet
Park Curb Type
B
"A"
"A"
4'-4"
B
5'-4"
Plan
Normal curb line
1½"
1"
10"
½" R
1½"
¼" R
1½"
2"
Normal cross slope
7"
3"
½" ϕ bars
4'-0" long
2"
Top of grate
5"
Detail of Curb Inlet
Plain Curb Type
½" ϕ bars 4'-0" long
Normal cross slope
2"
2'-2⅜"
2'-2⅜"
2"
2"
1" 3"
½"
1'-1½"
3⅝"
12"
2'-8¾"
12"
2'-0"
8"
7⅝"
7⅝"
8"
Minimum 2'-0"
Variable
6"
8"
4'-0"
8"
5'-4"
Concrete
Section A-A
2"
10⅜"
10"
10 3/16"
10"
1"
2'
Concrete
6"
3"
5"
2 13/16"
11½"
12"
2'-1⅛"
8"
2'-0"
8"
7 13/16"
2 13/16"
8"
Variable
6"
8"
3'-0"
8"
4'-4"
Section B-B

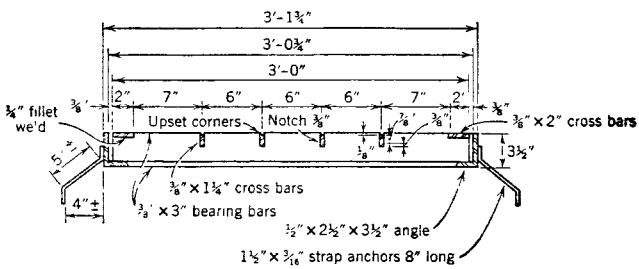
FIG. 13.



Plan of Frame



Plan of Grate



Longitudinal Section of Frame and Grate

Catch basin.

Ditch drainage should ordinarily not be allowed to flow across an intersecting road, but should be put in a culvert, if it must unavoidably be carried from one side of the cross road to the other. The culvert may often be placed a short distance down the intersecting road, past the return. This will substantially shorten it and, where a shallow ditch section is used, may solve the problem of good drainage grades and adequate cover. In contrast to intersecting roads, when driveways are carried across ditch lines it is preferable to avoid using culverts. Since driveway traffic moves at low speeds, a small dip in the driveway profile is not serious; with shallow ditches, no culvert is needed. Where ditches must be deeper, drainage can sometimes be made to flow both ways from a driveway; or perhaps water can be diverted from the right of way near the high side of the driveway. Where culverts must be used, the diameter should never be less than 15 in., for easy cleaning, and the ends should usually be protected with a simple header. The culvert length should be ample.

Longitudinal Culverts. Longitudinal storm sewers in cuts are more accessible in a shoulder area (on multiple-lane roads, they will ordinarily be in the median strip). However, when a culvert is to pick up drainage from both ditches of a two-lane road, it is sometimes put under the pavement, with feeder lines from catch basins discharging into the sewer line at manholes (Fig. 14). The pipe should always be placed in straight runs, with a manhole at each angle point, and also at each grade change. The grade should be at least 2% to prevent deposit of sediment; 1% is the absolute minimum. Pipe diameters less than 15 in. should not be used; with flat grades and especially if drop inlets are used in place of catch basins 18 in. is often preferable.

The minimum sizes that have been recommended for storm sewers carrying road drainage will usually be more than adequate to handle the largest anticipated flows. In an extensive storm-sewer system, however, larger pipe sizes may be needed near the system outlet.

Drop Inlets, Catch Basins, and Manholes. Water is generally removed from shallow ditches by means of drop inlets or catch basins which lead it to a pipe at a lower level than the ditch bottom. Deep ditches may be similarly emptied, or a suitably designed end wall may deliver the water to the culvert.

A drop inlet is essentially a box, outletted through a pipe at the bottom and with water entering it in any one of several ways, according to circumstances. The simplest arrangement is shown in Fig. 11, and consists merely of shaping the sides of the box to the ditch cross section (which may be paved for a short length approaching the box), and leaving the top open. This arrangement is practicable only where the inlet is located so far from the traveled path or is so fully barricaded with posts that it creates no hazard. Otherwise, danger from the open top is obviated by use of a grating or a solid cover. The former has the advantage of providing an alternative opening for the admission of water should the opening in the ditch cross section become obstructed. Where a ditch bottom conforms to the shoulder slope, a grating replaces the ditch openings. For maximum acceptance of water, the grating is often supplemented by a side opening in the cut slope or in a curb, as shown for catch basins (Fig. 13). The outlet may be either a cross culvert, as shown, or a longitudinal culvert. Water may also be delivered to the box through a storm sewer. Drop inlets are appropriate whenever the ditch flow does not carry much solid material which might clog the culvert or be deposited at the culvert outlet.

When the ditch flow carries solids such as the products of slope erosion or sand or cinders used on the road as a protection against ice, it is better to drain the ditch through a catch basin. This is simply a drop inlet with the addition of a sump for the collection and periodical removal of sediment. Sump capacity should be enough for the period between cleanings. Usually a depth of 2 to 3 ft below the outlet pipe invert is appropriate.

Figure 13 shows details of a typical catch basin, including the frame, grating, etc. Walls may be made of brick, stone rubble, or concrete block masonry or of concrete. Wall thickness should be 6 to 8 in. for concrete and concrete block, 8 in. for brick, and 12 in. for rubble. Mortar should be 1:2 cement-sand. The ends of all pipes should be cut flush with the inside walls, and pipes should extend a sufficient distance outside for satisfactory connection to culverts. The design of the inlet to a catch basin is of particular importance, to intercept as much water as possible. A combination curb and gutter inlet, as shown, is desirable. The grate openings should be as large as they can be made without creating a hazard for pedestrians, etc. The size opening shown is about the limit. Steel is a much better material for gratings than cast iron because its strength allows the total area of openings to be substantially larger for any given grate dimensions.

On divided highways with raised medians it is desirable to locate catch basins in cross gaps in the median, arranging gutter grades to deliver water to them from both roadways. The gap should not be wide enough for vehicles to use it as a crossover. Where grades do not allow this location, catch basins are usually placed in the straight gutter line,

with the top of the grating about 2 in. below normal gutter grade to facilitate interception of as much gutter flow as possible. Where there is not much snow, a catch basin may be set behind the curb line, in a pocket formed in the median It is then out of the way of traffic and by proper design of the pocket can be made to intercept all the water that reaches it In cold regions, however, the point of the curb downgrade of the catch basin is very apt to be damaged by snow plows, while deposit of snow on the grating reduces its capacity to receive water.

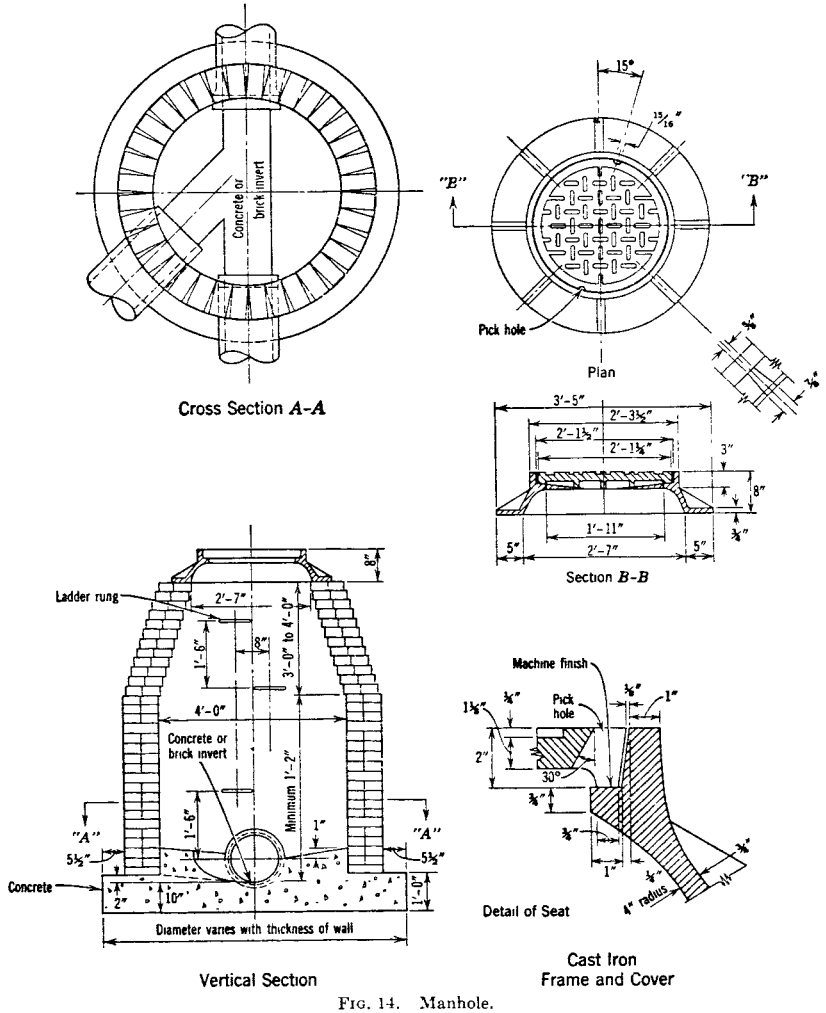


FIG. 14. Manhole.

Figure 14 shows details of a typical manhole and cover. Manholes should be used at all storm-sewer junctions and also at every change in sewer grade or alignment. Even if not otherwise required, they should be placed often enough to eliminate any run of pipe more than about 400 ft long. This is especially important where the sewer is placed beneath a pavement. Careful attention to the location of catch basins will often eliminate need for manholes, some of the catch basins being used for both purposes. Walls of manholes (and of drop inlets) are made of the same materials, thicknesses, etc., as described for catch basins. In a construction contract, payment for all of these minor structures is usually on a unit basis, sometimes with a differential for depth.

Area in Square Feet Required

Acres Drained	Very Flat	Flat	Moderate	Steep	Very Steep
1	.2	4	6	.8	1 1
2	3	6	9	1 4	1 9
3	5	8	1 3	1.8	2 5
4	.6	1 0	1 5	2 2	3 1
5	7	1 2	1 8	2 6	3 6
6	8	1 3	2 1	3 0	4 2
7	9	1 5	2 4	3 4	4 7
8	1 0	1 7	2 6	3 8	5 3
9	1 0	1 8	2 9	4 2	5 7
10	1 1	2 0	3 1	4 5	6 2
11	1 2	2 1	3 3	4 8	6 6
12	1 3	2 2	3 5	5 1	7 0
13	1 4	2 3	3 7	5 4	7 5
14	1 4	2 5	4 0	5 8	7.9
15	1 5	2 7	4 2	6 1	8 4
16	1 6	2 8	4 4	6 4	8 8
17	1 7	2 9	4 6	6.7	9 2
18	1 7	3 0	4 8	7 0	9.6
19	1 8	3 2	5 0	7 3	10.0
20	1 9	3 3	5.2	7 6	10 4
22	2 0	3 6	5 6	8 2	11 2
24	2 2	3 8	5 9	8 6	11.9
26	2 3	4 0	6 3	9 2	12 8
28	2 4	4 3	6 7	9 8	13 4
30	2 6	4 5	7 0	10 2	14 1
32	2 7	4 7	7 4	10 8	14 8
34	2 8	4 9	7 7	11 3	15 5
36	2 9	5.1	8 1	11 3	16
38	3 1	5 4	8 4	12 2	17
40	3 2	5 6	8 7	12 7	18
45	3 6	6 1	9 6	13 9	19
50	3 3	6 6	10 3	15 0	21
55	4 0	7 1	11 1	16	22
60	4.3	7.6	11 9	17	24
70	4.8	8 5	13 3	19	27
80	5 3	9 3	14 7	21	29
90	5 8	10 2	16	23	32
100	6 3	11 1	17	25	35
125	7 4	13 0	20	30	41
150	8 6	15 0	24	34	47
175	9 6	17	26	38	53
200	10 6	19	29	42	58
225	11 6	20	32	46	64
250	12 6	22	35	50	69
275	13 5	24	37	54	74
300	14 4	25	40	58	79
350	16	28	45	65	89
400	18	31	49	71	98
450	20	34	54	78	107
500	21	37	58	85	117
600	24	42	67	97	133
700	27	48	75	109	150
800	30	53	83	120	165
1 000	36	62	98	142	196
1,200	41	71	112	163	224
1,500	48	84	133	193	265
1,800	55	97	152	221	304
2,100	62	109	171	248	341
2,400	69	120	189	274	377
2,700	75	131	206	300	412
3,000	81	142	223	324	446
3,500	91	159	250	364	500
4,000	101	176	277	402	553
4,500	110	192	302	441	604
5,000	119	208	327	476	654
6,000	136	238	375	546	750
7,000	153	268	421	612	841
8,000	169	296	465	677	931
10 000	200	350	550	800	1,100
12,000	229	401	631	918	1,262
15,000	271	474	745	1,084	1,490
18,000	311	544	855	1,243	1,709
21,000	349	610	959	1,395	1,918
26,000	398	696	1,093	1,590	2,187
30,000	456	798	1,254	1,824	2,508
	$C = 0.2$	$C = 0.35$	$C = 0.55$	$C = 0.8$	$C = 1.1$

Pipe 15" Pipe 16" Pipe 24" Pipe 30" Pipe 36" Pipe 42" Pipe 48" Pipe 60" Pipe 72"

FIG. 15. Drainage area table computed from Talbot's formula.

Culverts for Stream Flow. General principles pertaining to other culverts apply also to those carrying streams under a road. These, however, have additional restrictions on their location. Vertically, they should be so placed that their flow line coincides closely with the natural grade of the stream channel, unless this grade is being changed by channel work outside the embankment. The survey should have established the necessary elevations. Horizontally, the culvert should line substantially with the stream flow, producing no sharp bends to cause erosion and loss of culvert capacity. If adherence to the natural channel would cause a serious skew, the channel should be relocated to reduce the skew and secure good flow. When this is done, care must be taken to stabilize the new channel. Sometimes it may be worth while to make rough alternate designs and cost estimates to find the most economical solution. There is a gain in locating a large culvert away from the existing channel in that construction can be carried on without making arrangements for bypassing the stream flow.

The cross-sectional area of a culvert is determined from the drainage area and type of runoff, etc., as recorded in the location survey. Section 2 of Vol. II, Hydrology, develops principles and procedures for this determination. For preliminary studies and in the absence of fundamental hydrologic data, Talbot's formula $a = C\sqrt[4]{A^3}$ may be used for culvert diameters up to 72 in. Here a is the area of waterway (square feet); A is the drainage area (acres); C is a constant to be chosen by the locator or designer. Its values will usually lie between 0.2 and 1.0. For steep, rocky ground, with little vegetation and a valley length several times the width, where floods from melting snow occur, values between 0.7 and 1 may be used. The value increases as vegetative cover decreases and as steepness (including that of the upper valley in comparison with the channel at the culvert site) increases. For rolling agricultural country, with considerable grass cover, a value of about 0.35 is suitable. Any feature of soil, vegetation, or topography that retards runoff lessens the coefficient, and vice versa. Figure 15 gives pipe diameters for different drainage areas and different values of C. However, records of adequacy of other culverts on the same stream form the best criteria for culvert sizes. Such information should be used whenever available.

Pipes are generally used for culverts where the pipe diameter does not exceed 42 in. For diameters between 48 and 72 in., it is well to compare the cost of a pipe with a *box culvert*. Pipes with diameters greater than 72 in. are not usually economical. Sometimes limited head room or the shape of a valley floor makes the use of *multiple pipes* or segmental arch culverts, etc., economical and desirable.

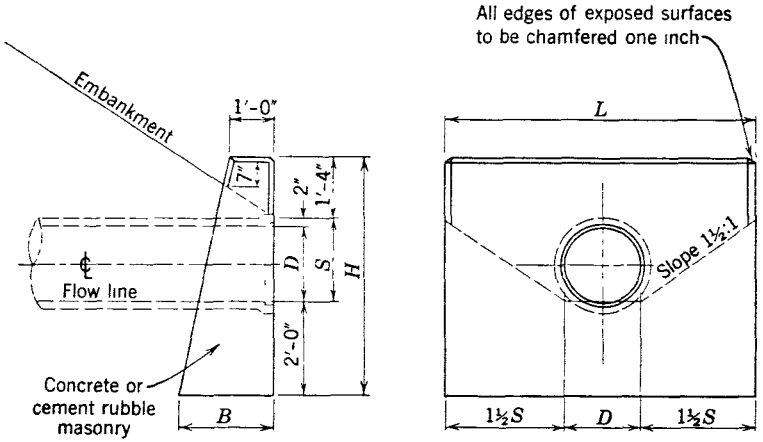
Since the elevation of a culvert for stream flow is largely determined by the channel elevation, these culverts must often be placed beneath heavy fills. Following Connecticut Highway Department practice, either concrete or asphalt-coated corrugated metal (ACCM) pipe may be used for fills up to 25 ft (measured from top of pipe), where the pipe diameter does not exceed 30 in. Deeper fills require metal pipe. For larger pipe diameters, the controlling height of fill is 20 ft. Pipe should conform to the specifications named on p. 4-33.

Culvert End Walls. Culvert end walls must be designed for a variety of situations, and design varies accordingly. Figure 16 shows the simplest type. For the smaller pipe sizes —up to 42 or 48 in.—where there is little or no skew and where the culvert and stream channel are approximately in line, the straight vertical header is generally satisfactory. It should be carried into the ground enough not to be affected by frost. Two feet below the pipe invert will be enough except across the northern tier of the United States and in colder regions. Usually this depth will also provide stability against overturning and undermining.

As pipe sizes increase, wings may be added to the end wall to retain embankment material and, on the inlet end, to minimize scour. The symmetrical design shown (Fig. 17) is appropriate for pipes without skew. For skewed culverts, and where stream flow is deflected at inlet or outlet, modifications of the basic design are made. Wings are set at unequal angles and are adjusted in size to meet whatever condition exists. Sometimes only one wing is used, set obliquely or sometimes parallel to the axis of the pipe. Wing-type end walls merit consideration for pipe sizes from 36 in. upward.

Instead of a wing-type end wall, slope paving (Fig. 18) may be used. When this is done it is essential to support the end of the pipe firmly in a cradle of concrete or masonry. Where suitable stone is abundant, slope paving may provide the most economical treatment.

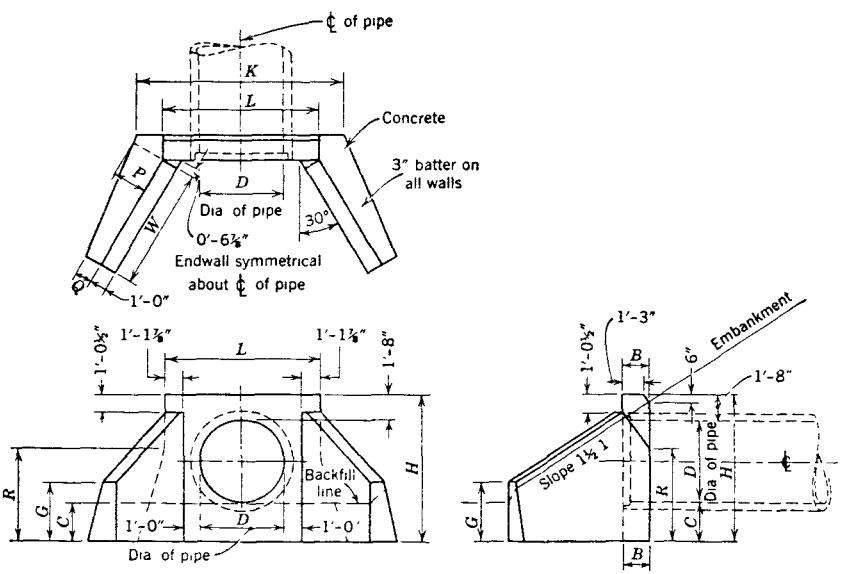
At culvert outlets where the volume of water is substantial and discharge velocity is high, protection from erosion may be necessary. This may take the form of a baffle, a concrete or masonry apron, or simply a layer of loose stone of a size adequate to remain in place.



Dimensions for Straight End Wall Based on $S = D + 2''$

D, in.	S, ft & in.	H, ft & in.	L, ft & in	Batter, in.: ft	B, ft & in.
12	1 2	4 6	4 6	2 1/2	1 11 1/4
15	1 5	4 9	5 6	2 1/2	1 11 7/8
18	1 8	5 0	6 6	2 1/2	2 0 1/2
24	2 2	5 6	8 6	2 1/2	1 3/4
30	2 8	6 0	10 6	2 1/2	2 3
36	3 2	6 6	12 6	3	2 7 1/2
42	3 8	7 0	14 6	3	2 9
48	4 2	7 6	16 6	3	2 10 1/2

FIG. 16. Straight culvert end wall.



Dimensions for Wing-Type End Wall

D	B	C	G	H	K	L	P	Q	R	W
36"	1'-6"	2'-0"	3'-3"	6'-8"	9'- 1 1/2"	7'-3 3/4"	1'-4 1/8"	0'- 9 3/4"	3'- 4 7/8"	4'- 1 3/8"
42"	1'-6"	2'-0"	3'-3"	7'-2"	9'-10 1/2"	7'-9 3/4"	1'-6 3/8"	0'- 9 3/4"	3'-10 1/2"	4'-11 3/4"
48"	1'-7"	2'-6"	3'-9"	8'-2"	10'-10"	8'-3 3/4"	1'-9 3/8"	0'-11 1/4"	4'- 9"	5'-10"
60"	1'-7"	2'-6"	3'-9"	9'-2"	12'- 4 1/2"	9'-3 3/4"	2'-0 3/8"	0'-11 1/4"	5'- 9"	7'- 7"
72"	1'-7"	2'-6"	3'-9"	10'-2"	13'-10 3/4"	10'-3 3/4"	2'-3 3/8"	0'-11 1/4"	6'- 9"	9'- 3 3/4"

FIG. 17. Wing-type culvert end wall.

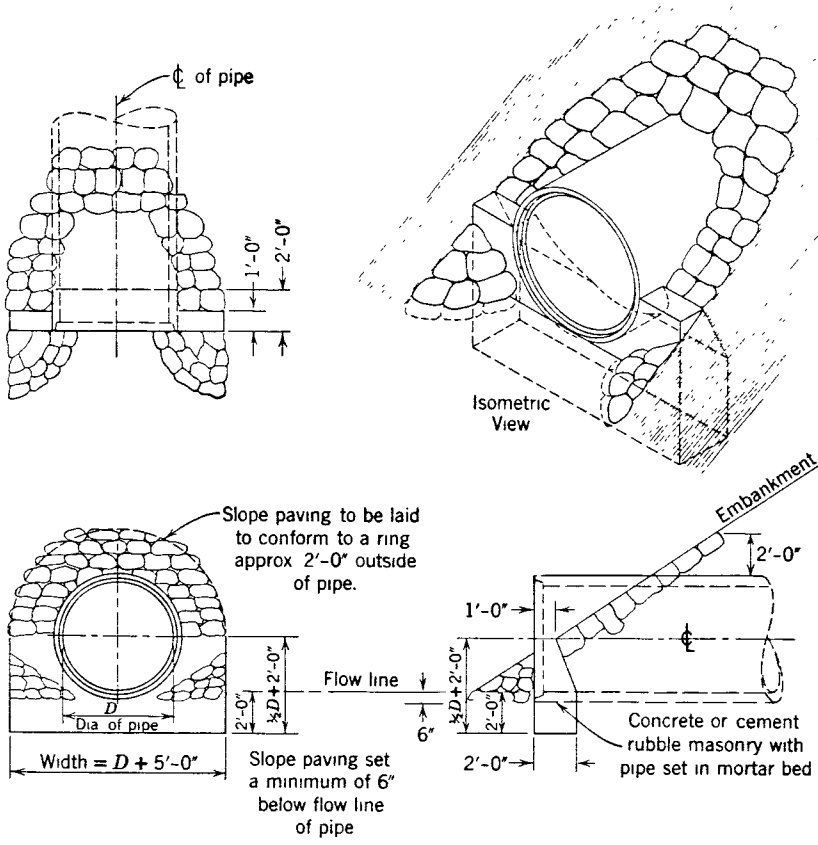


FIG. 18. Footing and slope paving for pipe culvert.

Subsurface Drainage. *Objectives.* Within the roadbed itself, the primary purpose of subsurface drainage is to prevent breakup or deformation of the road surface and shoulders. Under freezing temperatures, certain silty soils of high capillarity develop "frost heaves." These are ice lenses which form, cumulatively, under the road base, pushing up the road surface, sometimes as much as a foot or more. The surface is damaged, either by fracture when it is raised, or by collapse in the spring when the ice melts from under it. Also, in regions or at times of the year when the ground is not frozen, excessive moisture, in parts of the roadbed that are subjected to considerable pressure under vehicular wheel loads, may destroy bearing capacity. This leads to rutting and other forms of deformation of a non-rigid pavement, and to "pumping" and eventual breakup of rigid pavements. In cut slopes, excessive water causes slides, and subsurface drainage should be used to prevent them.

Two principal types of subdrainage are used to remove excessive water One consists of perforated pipe, laid in trenches filled with pervious material, through which water can reach the pipe and be drawn off. The other consists of a pervious fill material, such as crushed stone, gravel, sand, or any sound soil not subject to capillary action. Such material, placed under a road base (p. 4-70) or under the surface layer of the road shoulders, is called *subbase.*

Perforated pipe underdrains or subbase may be used alone; or they may be used together, each to supplement the other.

Underdrain Pipe. The most frequently used materials for perforated underdrain pipe are corrugated metal (ASTM Specification M-136) and concrete. The former possesses some advantage in ease of handling and in strength of joints where there is any uncer-

tainty as to foundation stability. Six-inch pipe is ordinarily adequate in size, though 8-in. is sometimes used. The metal pipe should be thoroughly coated with asphalt cement, inside and out.

Underdrain Trenches. Underdrains are used for two purposes: (1) to intercept a stratum that contains percolating water moving on top of an impervious layer; (2) to lower the ground-water level. Where the purpose is interception, the trench for the pipe is dug to a depth that slightly penetrates the impervious stratum (Fig. 19). The pipe is laid in the bottom of the trench and a backfilling of impervious material is placed and tamped about

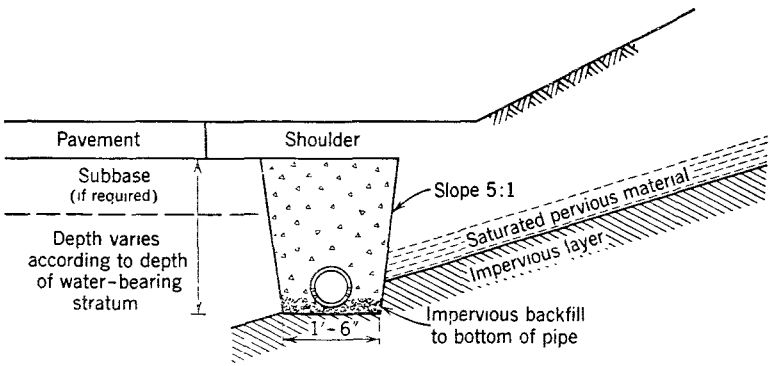


FIG. 19. Underdrain trench. Sand should replace crushed stone if soil conditions warrant.

it to a level just below the lowest rows of holes. The trench is then backfilled with broken stone or gravel of a size just large enough not to pass through the perforations. The following gradations have been used successfully:

5/8"	1/2"	3/8"	#4	#8	and	3/8"	#4	#8	#100
100%	90–100	30–50	0–8	0–3		100%	15–70	0–20	0–1.5

Where the shoulder is not waterproof, the pervious underdrain backfill is not carried to the ground surface. Instead, the trench is sealed against the admission of surface water with a blanket of impervious material. In some silty soils heavily charged with water, there is a tendency for the soil to penetrate and choke the pervious filter. This condition is best met by the substitution of a coarse sand for the broken stone or gravel. Stone or gravel is placed immediately adjacent to the pipe, however, to prevent the sand from passing through the perforations. The filter sand may have the following gradation:

3/8"	#4	#16	#30	#50	#100
100%	95–100	45–80	20–55	10–30	2–10

The underdrain trench is located so *as most effectively to intercept the vein of water.* It will usually be placed in the shoulder area, paralleling the highway center line along the deeper (uphill) side of a cut. However, there may be situations in which the subgrade will intersect a water-bearing stratum transversely or obliquely to the center line of the road and the subdrain will be located accordingly. Frequently an intercepting underdrain must be placed to correct conditions revealed by the excavation. It will, therefore, usually be desirable, in making up the quantity estimate sheet, to provide unassigned underdrain for use as needed.

Sometimes, in deep or moderately deep cuts, a water-bearing stratum is located above the road level, where it saturates the cut slope, creating unstable conditions, and causing sloughing of the slope. It may be possible to correct this condition by locating the intercepting trench above the slope. However, the relative pitches of the ground surface and the impervious soil layer may be such that a prohibitively deep trench would be required. In such cases it may be possible to dry out the surface of the cut slope sufficiently to give it stability by trenching in from the surface of the slope.

In flat areas, underdrainage is used to lower the ground-water level in the roadbed where, for any reason, it is impossible or inexpedient to do this with open ditches and where there is reason for not raising the grade line. Usually two lines of drain, as shown in Fig. 20, are required to accomplish this. A single drain under the center line of the road is sometimes used but is not recommended. In a particularly wet spot, or to remove seepage

of water from a "springy" area, it may be necessary to place lateral branches also in a herringbone pattern under the road surface. The steps taken to accomplish the purposes of underdrainage have to be adapted to whatever conditions are found to exist, either during the soil-survey stage or later, during construction.

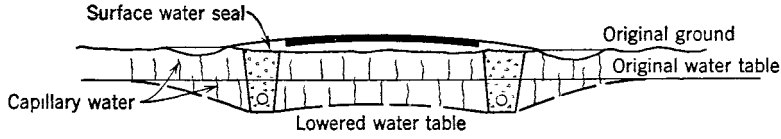


FIG 20 Water table lowered with underdrainage.

In some cases the area requiring underdrainage will not extend to a point where water in the underdrain can be outletted. When this occurs, ordinary culvert pipe can be laid from the margin of the area requiring drainage to the outlet. This length of trench need not be filled with pervious material.

Occasionally the same pipe is utilized for underdrainage and for the removal of surface water. Ordinarily, however, this practice is not recommended.

Subbase. Instead of lowering the free ground-water level with subdrains, it may often be more economical to limit the rise of capillary water above the level by placing a blanket of non-capillary, granular subbase on top of the wet foundation. Blankets of subbase with thicknesses up to 2 ft are used to keep the saturated portion of the roadbed below the frost line and to interpose an adequate thickness of load-distributing, stable material between the pavement base and the softened and weak foundation. AASHO Specification M-147 gives desirable characteristics of subbase.

Subbase is also useful in rock cuts. The irregular bottom of a rock cut can never be perfectly drained. There are certain to be areas where water will be impounded If these areas are too near the road surface, the impounded water will freeze and, in so doing, damage the pavement. Hence the insulating blanket of subbase. Two feet is usually enough. Points in a road where cut changes to fill, and vice versa, are sensitive spots in a roadbed, and are therefore locations where subbase is beneficial.

Unless good subbase is scarce and correspondingly expensive, liberal use of it is an economical insurance against pavement failures.

Whenever subbase is used, care must be taken to see that the subbase itself is adequately drained; otherwise it may simply form a reservoir where water will be impounded immediately beneath the pavement where its effect will be most serious. With adequate grades

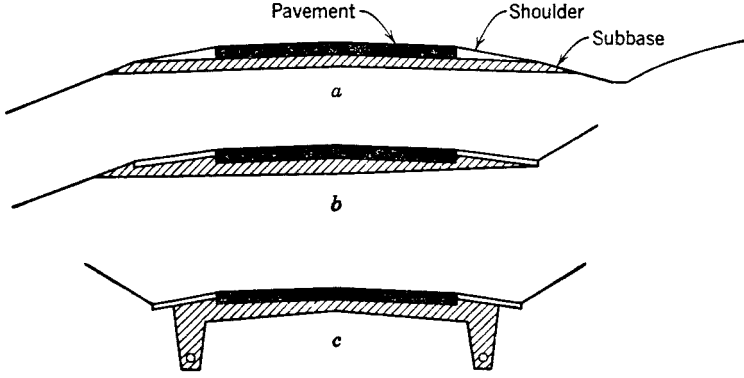
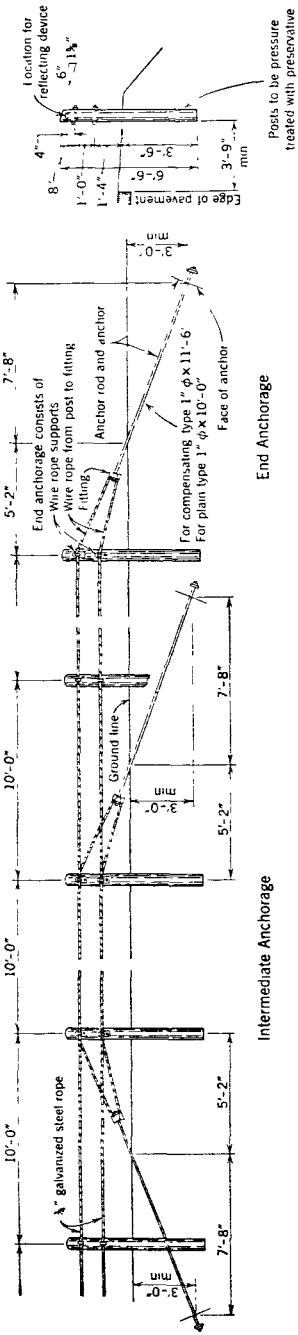


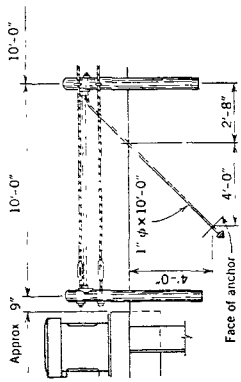
FIG. 21. Drainage of subbase. (*a*) Subbase self-draining to deep ditch or fill slope. (*b*) Shallow ditch section on side hill with subbase self-draining to fill slope. (*c*) Shallow ditch section in cut with subbase draining into longitudinal subdrains.

—say 2% or more—any water reaching the subbase will percolate longitudinally through it, and will drain away provided that leak-offs are available at low spots or provided that diagonal leak-offs are constructed at intermediate points, if necessary, to drain the subbase into the ditches. Where grades are flat, however, more positive drainage must be assured. With the deep ditch section appropriate for level country, the layer of subbase can be carried across the shoulders to the ditch fore slopes (Fig. 21*a*). This will provide

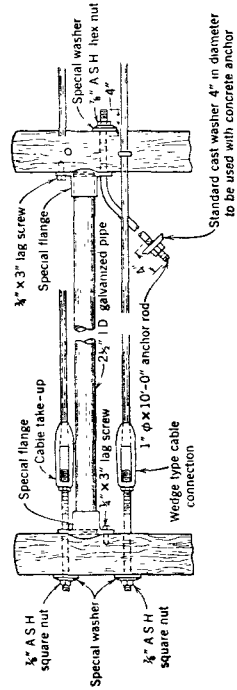


End Anchorage

Intermediate Anchorage



End Anchorage at Bridges



Fittings
End Anchorage at Bridges

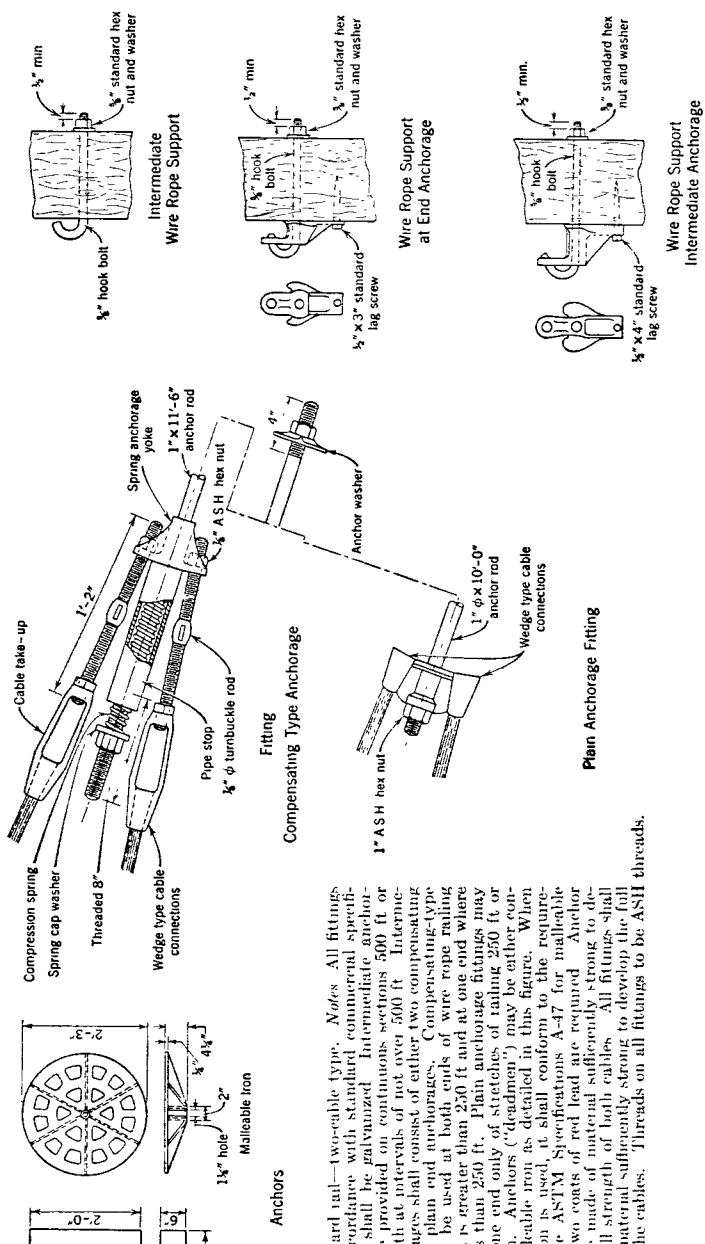
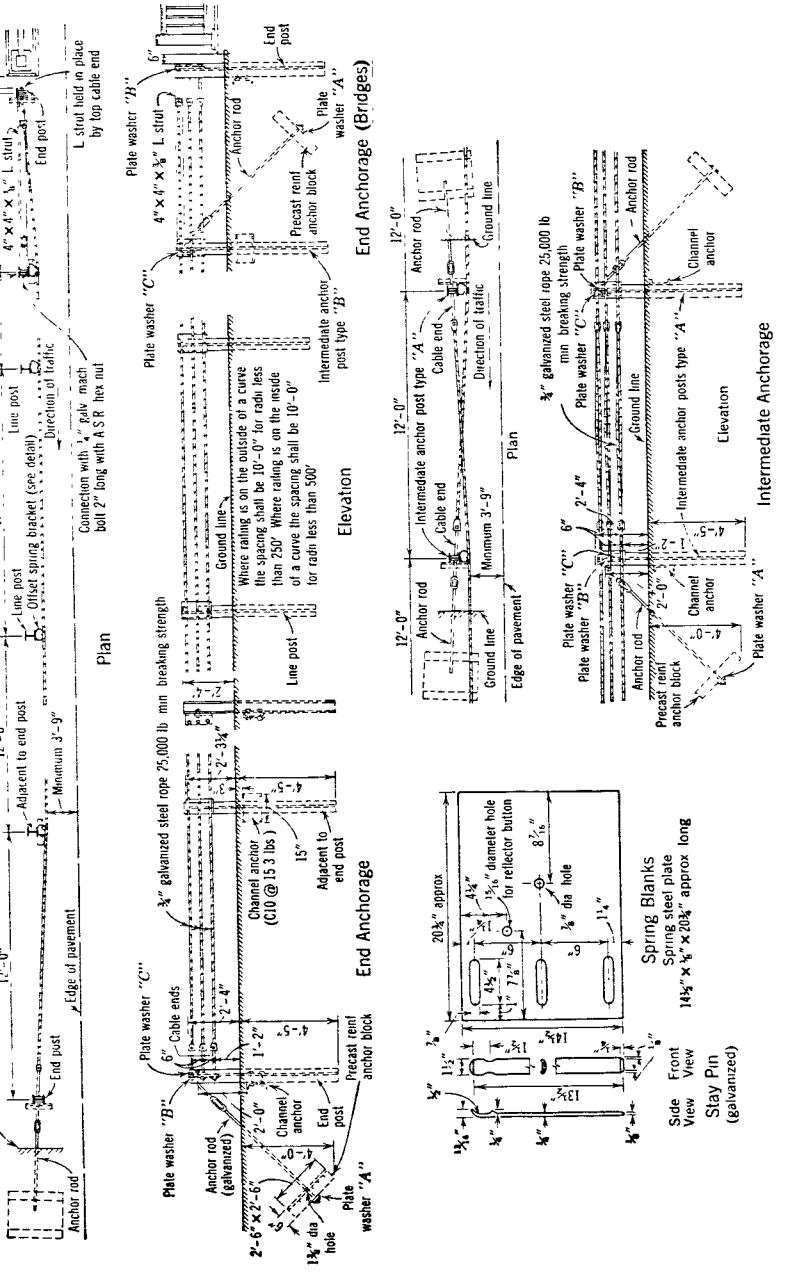


FIG. 22 Guard rail—two-cable type. *Notes.* All fittings to be in accordance with standard commercial specifications and shall be galvanized. Intermediate anchorages shall be provided on continuous sections 500 ft or more in length at intervals of not over 500 ft. Intermediate anchorages shall consist of either two compensating type or two plain end anchorages. Compensating-type fittings shall be used at both ends of wire rope railing where length is greater than 250 ft and at one end where length is less than 250 ft. Plain anchorage fittings may be used at one end only of stretches of railing 250 ft or less in length. Anchors ("deadmen") may be either concrete or malleable iron as detailed in this figure. When malleable iron is used, it shall conform to the requirements of the ASTM Specifications A-47 for malleable castings. Two coats of red lead are required. Anchor rods shall be made of material sufficiently strong to develop the full strength of both cables. All fittings shall be made of material sufficiently strong to develop the full strength of the cables. Threads on all fittings to be ASH threads.


Plan
Elevation
End Anchorage
End Anchorage (Bridges)
Intermediate Anchorage
Spring Blanks
Spring steel plate
14½" × ⅛" × 20¾" approx long
Stay Pin
(galvanized)
Side View
Front View
Ground line
Anchor rod
End post
Edge of pavement
Minimum 3'-9"
Adjacent to end post
Line post
Offset spring bracket (see detail)
Intermediate anchor post type "B"
Direction of traffic
Connection with ¼" galv mach bolt 2" long with A S R hex nut
L strut held in place by top cable end
4" × 4" × ⅛" L strut
Plate washer "B"
Plate washer "C"
Plate washer "A"
Anchor rod (galvanized)
Channel anchor
Precast reinf anchor block
Cable ends
¾" galvanized steel rope 25,000 lb min breaking strength
Channel anchor (C10 @ 15.3 lbs)
Adjacent to end post
Where railing is on the outside of a curve the spacing shall be 10'-0" for radii less than 250' Where railing is on the inside of a curve the spacing shall be 10'-0" for radii less than 500'
Intermediate anchor post type "A"
Cable end
Edge of pavement
Intermediate anchor posts type "A"
20¾" approx
15/16" diameter hole for reflector button
⅞" dia hole

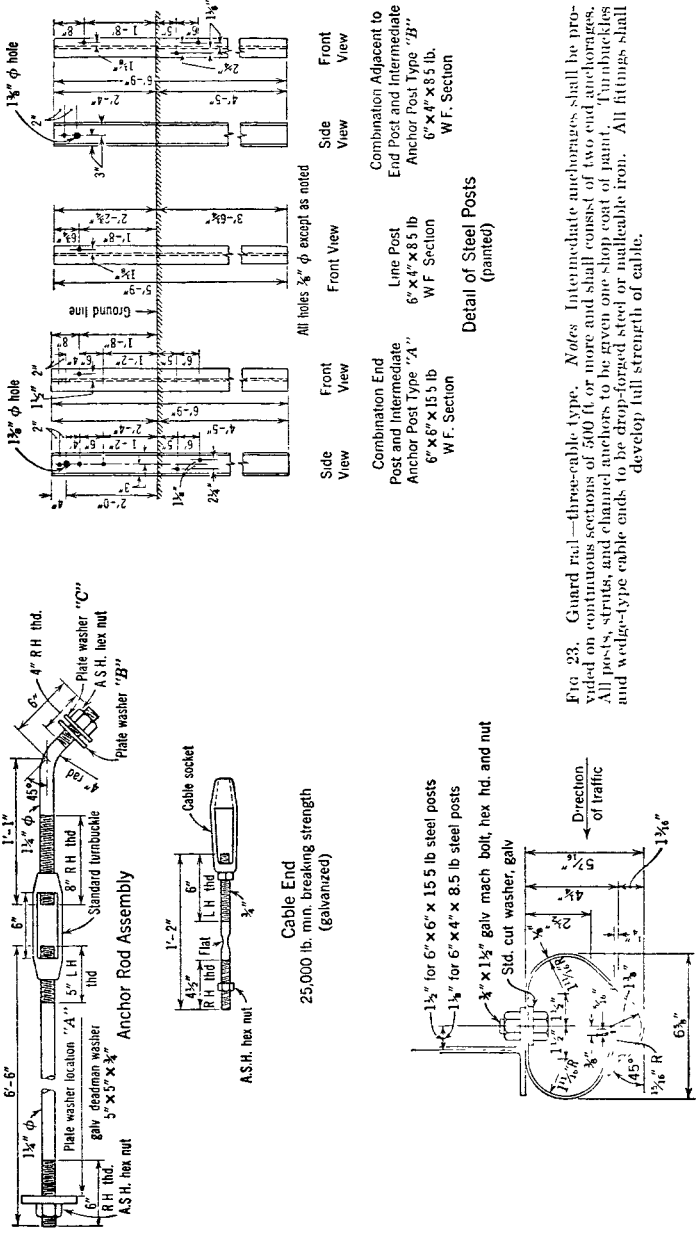


FIG. 23. Guard rail—three-cable type. *Notes* Intermediate anchorages shall be provided on continuous sections of 500 ft or more and shall consist of two end anchorages. All posts, struts, and channel anchors to be given one shop coat of paint. Turnbuckles and wedge-type cable ends to be drop-forged steel or malleable iron. All fittings shall develop full strength of cable.

the necessary drainage if the foundation is properly compacted and smoothed and is given an adequate cross slope (1/2 in. per ft). Even with a shallow ditch section and a flat grade, subbase can be self-drained on a side-hill location by giving the foundation a continuous slope to the downhill side (Fig. 21*b*). Frequently, however, one or more longitudinal subdrains are used in conjunction with subbase (Fig. 21*c* and Fig. 9, case 3).

Guard Rail and Guard Posts. Little uniformity exists as to conditions considered to require protection by guard rail. Practices followed by the different state highway departments reflect the laws of the several states. The Connecticut Department, which is probably at one extreme, uses guard rail at all points where the pavement is more than 8 ft above adjacent ground level, and at all points where the difference in elevation is between 4 ft and 8 ft unless an embankment slope at least as flat as 1 on 6 is used. At all points where the fill is less than 4 ft, a 1 on 4 slope is used instead of guard rail. Michigan, which more nearly represents average practice, specifies railing on fills 14 ft or over, unless the ground slopes away from the road, in which case railing is used on fills of 10 ft and occasionally of as little as 7 ft. Where railing is used, it may well extend closer to the point of transition from cut to fill on the "approach" end of an embankment than at the far end. For guidance in snow plowing and for the safety of traffic, single posts are used to mark the locations of catch basins and culvert inlets and outlets.

Many varieties of posts and rails have been used for guard rail. Posts of both wood and steel are satisfactory and are probably more widely used than any other materials. Wire cable is most widely used for rail.

Figure 22 shows a railing consisting of two strands of wire rope carried on wood posts. The Connecticut Highway Department requires that, for this type of railing, posts must be 6 ft 6 in. long, and not less than 6 in. in diameter at the small end after removal of bark. Oak, pitch pine, Norway pine, and long- and short-leaf yellow pine are considered acceptable woods. Posts are completely cut and drilled and are then pressure creosoted to a final retention of 8 lb/cu ft of wood, or refusal of preservative. This both lengthens the life of posts and eliminates the cost of keeping them painted. For visibility at night, reflector buttons are set in drilled holes, usually in every second or third post. The wire rope is required to conform to AASHO Specification M-30 with a minimum breaking strength of 25,000 lb. No cable should have a length greater than 500 ft. If a section of guard rail is longer than this, intermediate anchorages should be used with all cables. Lengths of cable greater than 250 ft should have compensating-type anchorages at both ends; for shorter lengths, only one compensating type anchorage is required. Dead men of malleable iron or of reinforced concrete may be used. Material used in anchor rods should be strong enough to develop full strength of the cables.

Figure 23 shows a stronger and also more expensive type of wire-rope railing. Here posts are 6 in. steel H sections, 6 ft 9 in. long. The 12-ft spacing should be reduced to 10 ft on curves sharper than 12°. Three wire-rope cables are used, and are held to the posts by means of offset steel-spring brackets. Cable lengths and anchorages should be as specified above.

Besides steel and wood, concrete posts have been widely used. They are apt to break rather than overturn on impact, and have little salvage value. Instead of wire rope, woven-steel wire tape, woven-steel wire fabric, various forms of flexible and semirigid steel strip and plate, and timber are used for rails. AASHO *Specifications for Highway Guards* (1951) gives details.

Right-of-Way Widths. Right-of-way widths specified for the rural portions of the Interstate Highway System are:

	Minimum, ft	Desirable, ft
Two-lane highways	120	220
Divided highways	150	250

Because of the variable conditions encountered, no widths are recommended for urban portions. It is of fundamental importance, however, to secure initially enough width to meet ultimate requirements of the road. Failure to do this has caused a tremendous wastage of highway funds in the past.

The above rights-of-way for two-lane roads are liberal and reflect an expectation that eventually most of the Interstate System will consist of divided highways. Where ultimate planning is for only two lanes, the highway may eventually need a pavement width of 22 or 24 ft, two 8- or 10-ft shoulders, adequate ditches, and slope areas from four to eight times as wide as the depth of cut or height of fill that is consistent with a good grade line. In addition, it is wise to make provision for utility lines and sometimes for roadside shade trees also. Finally, the private development of abutting property should be held a reasonable distance back of the ditch lines. Consequently, 80 ft is widely regarded as the *minimum* acceptable right-of-way width for a state highway or other road of com-

parable importance. Widths of 100 or 120 ft are better, and are usually adequate for roads that are never expected to be developed into divided highways. For expressways, 300 to 350 ft is suitable where frontage roads form part of the design or may be added later; without provision for them, the width may be narrowed 50 to 100 ft. In urban areas, circumstances govern. It will often not be feasible to acquire a right-of-way as wide at all points as would be desirable. For secondary or "feeder" roads AASHO specified right-of-way widths (feet) are as follows:

Annual Average Daily Traffic Volume

Under 100		100 to 400		400 to 1000	
Minimum	Desirable	Minimum	Desirable	Minimum	Desirable
40 *	—	40 *	80	50	80

* Minimum of 40 ft or as required for construction.

In securing rights-of-way, acquiring too narrow a width is a much more serious fault than the reverse.

Establishment of Highway Boundary Lines. Ordinarily, where it is necessary to secure a right-of-way, a minimum width will have been adopted when the other basic features of the proposed road were decided on. The right-of-way, when minimum, is customarily symmetrical about the road center line. Hence the normal position of the boundary or "taking" lines will be one-half the minimum right-of-way width from the center line. It may be necessary, however, to increase this distance, on one or both sides, for some lengths of the road, to bring slope lines (i.e., the top of cut slopes and toe of fill slopes) within the right-of-way. There should also be enough distance between slope line and taking line for any intercepting drainage ditches, etc., that may be required. In rare instances, it may be necessary to narrow the right-of-way below the ordinary minimum, to avoid some disproportionately large payment for property.

When the position of a taking line has been determined, it is plotted on the plan, with ties to the road center line or some other base. Taking lines normally parallel the center line both on tangents and curves, unless there is some contrary reason such as a transition from one right-of-way width to another.

Slope Rights. Occasionally, instead of buying property all the way out to an unusually remote slope line, a "slope right," i.e., the right to construct and maintain a slope, is secured.

Drainage Rights. Frequently drainage rights will also be required. Drainage rights are of two sorts. The most frequent is a right to discharge a concentrated drainage flow from the highway onto private land at a point where no natural watercourse exists. The right may entail no actual construction on the private land, or it may be specified to include the construction and subsequent maintenance of a drainage ditch, slope paving, a culvert, etc. A certain width of property is ordinarily specified within which the drainage may be disposed of. The second type of drainage right is the right to enter private land upstream or downstream of a highway to clear and maintain a stream channel so that it will continue to discharge its water properly into a culvert or ditch within highway limits or to receive water from a culvert. The location and extent of all drainage rights (and slope rights also) should be accurately shown on the plans.

Special Problems in Urban Work

Urban projects can be divided into two definite classes: first, those where the new work is made to conform to existing conditions, with a minimum amount of disturbance to them; second, major projects of expressway type where certain standards are set and adhered to, with secondary regard to the effect on existing conditions.

Design Conformed to Existing Urban Conditions. In this situation consideration will often have to be given to existing underground storm and sanitary sewers, water and gas mains and services, electric conduits, etc. Sidewalks may have been built previously and curb grades established in such relation to adjoining buildings that they cannot be changed appreciably. Frequently these will have been developed without much regard for the roadway between—curbs being at different elevations on opposite sides of the street, etc. The designer should begin by examining all these conditions carefully to decide which of them are susceptible of improvement. These he will modify; beyond this point, however, he will have to adapt his design to what exists.

Sidewalks and Curbs. Sidewalks that exist and are to be retained exercise a control on pavement grades that is subject only to a limited variation in curb heights. The maximum

curb height usually considered acceptable is about 8 in.—anything higher is apt to interfere with the opening of automobile doors. The minimum height may generally be taken as that which will prevent water from overflowing the sidewalk. With sufficiently frequent location of catch basins, this is about 4 in. Hence with existing curbs, a small

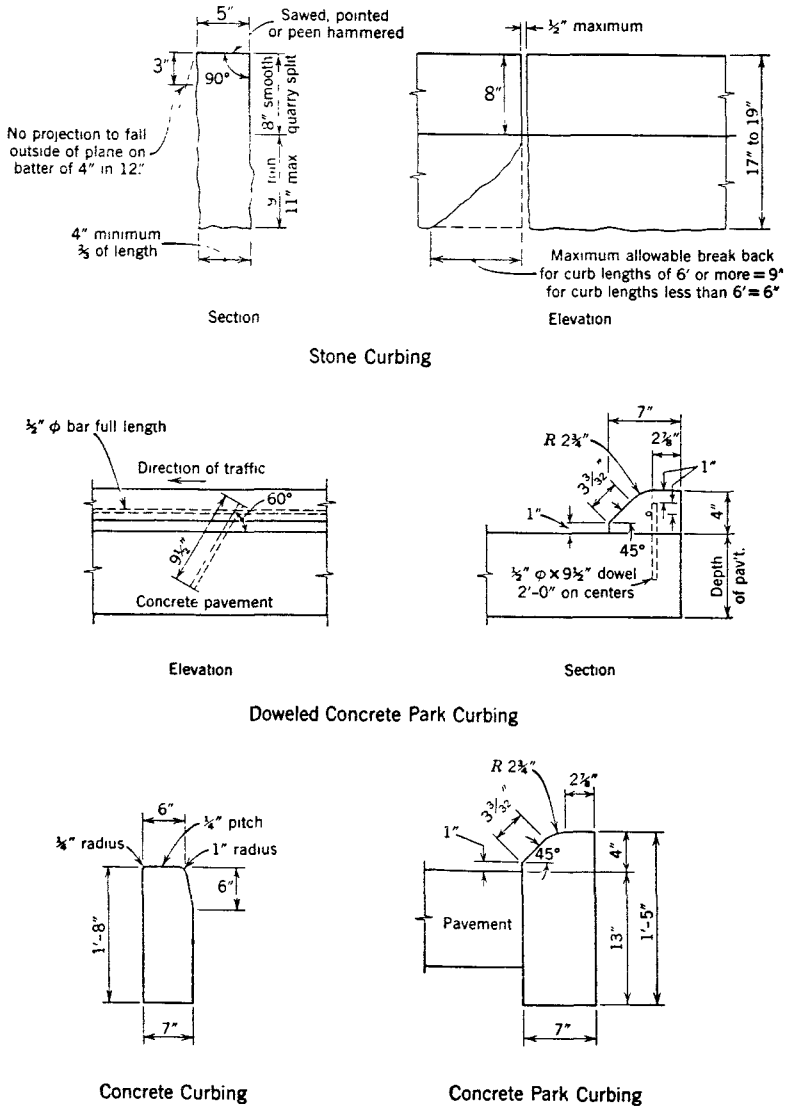


FIG. 24. Stone and concrete curbing.

leeway remains with the pavement designer in establishing gutter grades. Where a sidewalk does not exist but is to be built, or where one is to be replaced, the amount of freedom in establishing gutter grades is increased; but if buildings line the street freedom will be limited by the fact that the sidewalk grade should be at least 2 in. below each sill grade. (It may sometimes be possible to disregard this in the case of a future sidewalk and a building obviously nearing the end of its usefulness.) The sidewalk should have a cross

slope of at least 1/8 in. per ft for drainage; a slope of 1/2 in. per ft should be considered the maximum.

A problem in the design of urban pavements frequently arises when different established curb grades have to be met at opposite sides of the street. If the difference in elevation is small, it can sometimes be taken up by adjusting curb heights. If this is not enough, the pavement cross section must be rotated to throw the road crown nearer the high curb. In doing this, care must be taken to avoid flat pavement areas. A curved pavement section lends itself better to this treatment than a straight cross slope does.

In urban work either a separate curb or a combined curb and gutter may be used. Figure 24 shows typical curb dimensions. Separate curb is generally precast, but may be built in place. In the latter case, the curb is doweled to the pavement, which is built first. Joints in the curb must then correspond with those in the pavement. Precast separate curb is generally made in units 6 to 10 ft long, and should be reinforced.

Combined curb and gutter is built in place after the pavement is built. Dowels should be used to hold curb and gutter to the pavement. Gutter and pavement joints should correspond. The width of the gutter apron is usually made between 12 and 24 in. The cross slope of the apron is often made the same as that of the pavement, or it may be increased to 1/2 or 3/4 in. per ft if required to accommodate the anticipated flow of water.

Sidewalks in residential areas should have a minimum width of 4 ft. Five feet is more satisfactory, and 6 ft is desirable. In business areas, 8 ft may be considered a minimum, but each situation should have individual study. Frequently curb lines have been established by law and should not be changed unless it is quite important to do so.

Minimum Gutter Grades; Drainage. Minimum gutter grades in urban work may be taken as 0.3%, with 0.4% preferable. The flatter the grade, the more frequently catch basins will have to be spaced. Catch-basin spacing is also dependent on the rainfall characteristics of the locality. Decision can best be based on a study of existing installations. Three hundred feet is a maximum distance apart, for average conditions. Water should never be carried across the surface of an intersecting street. Inlets should be ample to receive the full flow of water; combination gutter grating and curb-opening inlets (Fig. 13) are best.

Particular attention must be paid to vertical curves to see that they do not produce gutter grades that are too flat. Short curves—lengths as little as 50 ft—may often have to be used. Storm sewers are constructed as heretofore outlined, with the additional complication that they will have to be adjusted to other sewer and utility lines. Existing storm sewers should be utilized wherever possible. Manholes should be provided as outlined for rural storm sewers (p. 4-37). Concrete pipe and asphalt-coated corrugated metal pipe are used for urban storm sewers on the same basis as in rural work. Subbase and underdrains are used when soil and water conditions indicate. Where soils are impervious, a frequent practice is to use subbase over a foundation shaped with a cross slope (1/2 in. per ft) to drain from both edges to a line of underdrain placed longitudinally as may be convenient near the center line of the pavement.

Designing the Pavement Grades. In designing the pavement grades, profiles should first be established for the top of each curb, and gutter grades should be laid on these and transferred to the cross-section sheets. The pavement cross section is then established on the cross-section sheets, and a corresponding center-line grade is plotted. Revisions are then made as necessary, working back and forth between profile and cross-section sheets, until satisfactory results have been secured. In such urban work little thought can be given to balancing cut-and-fill quantities.

Careful attention must be given to all intersections to avoid a profile that will have unsatisfactory riding qualities on one hand, and to avoid flat spots where water will pond, on the other. Gutter profiles for both roads should be completely worked out before determination of pavement elevations within the intersection is undertaken.

Design of Urban Expressways. Where an expressway is to be carried into a city and high geometric standards for the expressway design have been set, the difficult part of the project is in finding the best location. Before the location can be established, a decision must have been reached as to whether to overpass or underpass each intersecting road or street. This will have established many of the most important grade controls. From that point, the steps in design will be essentially those already discussed for rural roads. The economic necessity of avoiding damage to abutting property assumes much larger importance; and, where the expressway underpasses the existing street system, drainage and sight-line difficulties may develop. The problems will be of the same basic sort, however, as those that have already been discussed, and solutions will be worked out with the same principles.

Completion of Preliminary Plan-Profile and Cross-Section Sheets. The plan-profile and cross-section data of the design should gradually have been added to the appropriate working sheets (or final sheets, in the case of cross sections) while design proceeded. As

soon as all the cross sections have been completed, cut-and-fill quantities are computed from them, and are carefully considered as to reasonableness and desirability. If found unsatisfactory, it may be necessary to revise the grade line and establish new cross sections along some parts of the job. Quantities are recomputed from these, and further adjustments made, if necessary, until a satisfactory design has been produced. This adjustment of profile is worked out almost entirely from profile and cross-section sheets, but any changes made in cross sections will probably necessitate changes in plan sheets as well. Consequently, it is advantageous not to plot the design too completely on plan sheets until grading quantities have been worked out satisfactorily. Pavement and slope lines, etc., may then be transferred to plan sheets from cross sections.

As all three types of sheets, in their preliminary form, near completion, they should be very carefully checked, both for consistency with one another and for completeness.

Plan Sheets. When completed, the preliminary plans should show:

1. The survey data and other information listed on p. 4-19.
2. Pavement lines, for the road itself and such returns as are constructed on cross-roads, also curb and sidewalk lines, where they occur.
3. All culverts, storm sewers, catch basins and inlets, manholes, underdrains, paved gutters, and leak-offs.
4. All guard rail and single posts.
5. Driveways, fences, gates, barways, etc., the construction of which is required.
6. Lines marking the top of cut slopes and the toe of fill slopes.
7. "Taking lines," marking the boundary of the right-of-way to be secured (where this is necessary).
8. Locations of any construction signs, or other temporary or permanent signs to be erected as part of the construction.
9. Construction notes for the guidance of contractors and inspectors.

Profile Sheets. The preliminary profiles should show:

1. Profile data from the survey as listed on p. 4-19.
2. The center-line pavement profile, together with P.C.'s, P.I.'s, and P.T.'s of grade tangents and with notation of per cents of grades and lengths of vertical curves.
3. Profiles, as needed, of any reconstruction of intersecting roads, or construction of driveways, etc.
4. Such supplementary pavement edge, ditch, gutter, or other profiles as may be desirable, including stream-channel profiles.
5. Appropriate views of drainage structures, underground utilities, etc.
6. All appropriate construction notes, including locations where subbase is required. This information is generally entered just above the bottom margin of the individual sheets, opposite the stations where the subbase is to be placed.

Cross-Section Sheets. Cross-section sheets should show:

1. Ground-surface sections from the survey.
2. Finished pavement, shoulder, and ditch or gutter lines.
3. Elevation of pavement at center line (in numerals to feet and hundredths).
4. Bottom surface of pavement; or, if laid in several courses, top and bottom surfaces of each course.
5. Subbase, where used.
6. Pavment lines in rock cuts.
7. All appropriate construction notes.

Preparation of Data for Field Inspection. The sheets that are to be taken to the field for inspection should have written on them all questions which the designer wishes to have answered during the inspection. These questions should be so placed on the field sheets that the answer can be written directly on the print. The question should, whenever possible, be so framed that the answer can be short: "yes" or "no" or a numerical quantity. Notes should also be written on the field inspection sheets, calling specific attention of the inspection party to any design features that the designer particularly wishes to be reviewed and approved or criticized.

8. FIELD INSPECTION

Purpose, Personnel, Procedure. A thorough field inspection is of the utmost importance to a good highway design. It has two principal advantages. First, it reviews the design, while still plastic, through eyes other than the designer's, and preferably from other viewpoints also. Second, it examines the location at a different date, and usually

at a different season than that when the surveys were made. This will frequently allow the terrain to be viewed under a different condition, so far as the amount of water in the soil is concerned.

The inspection should consider the design from four standpoints: traffic adequacy, effect on land use, maintenance of roadway and roadsides, and construction cost. The inspection party should, whenever possible, include someone representative of each viewpoint, and also the engineer who made the surveys, and the designer. Each member of the party should have studied the plans, profiles, and cross sections in advance. In the inspection, the entire project should be walked and thoroughly examined. The aim is thoroughness, not speed

Important Items. The inspection should note particularly the following items:

1. Correctness of the survey map; changes since survey; omissions and visible errors.
2. Adequacy of proposed line and grade and suitability of cross sections as drawn. Particular attention should be given to all intersections and grade separations and to connections at the end of the project; and to the grade line from the standpoints of drainage, snow and ice, sight distance, combinations of horizontal and vertical alignment, avoidance of ledge rock, damage to abutting property, etc. Grade is so important that the field inspection is sometimes called a "grade" inspection. It should verify (*a*) that where drainage lines are crossed the grade line and waterway provided are such as to permit passage of the maximum anticipated flow without flooding or damaging the road or the drainage structure; (*b*) that where a stream is paralleled the grade line is maintained above high-water marks; (*c*) that the grade line utilizes every opportunity to flow water away from the right-of-way; (*d*) that suitable sites for bridges and major drainage structures are used; (*e*) that insofar as possible the grade line maintains a desirable relationship between subgrade and ground-water level.
3. Any soil conditions not reflected in the plans, such as seepages, rank vegetation, and other evidences of wet conditions; evidences of rock or boulders; areas where abnormal grubbing will affect excavation quantities.
4. Correctness and adequacy of proposed surface and subsurface drainage, as regards each item shown on plans and any omissions from plans A check should be made of the condition of any existing drainage structures proposed to be kept, and of all drainage rights that should be acquired, whether they have been noted on plans or not.
5. Probable need of subbase or any special embankment material not called for on plans; location of any available deposits of such material and of loam that should be stock piled and saved for covering slopes.
6. Correctness of all proposed slope-stabilization measures.
7. Trees, from the standpoint of their desirable preservation or removal.
8. Need of additional guard rail or single posts.
9. Possibility of improving project through any changes outside the normal right-of-way, such as drainage of swamps, relocation of stream channels, etc.
10. Adequacy and safety of driveways, including especially their grades, drainage, and sight lines.
11. When necessary, consideration of provision made for the movement of traffic during construction.

The field inspection affords the last opportunity to correct weaknesses in design without issuance of contract change orders There should be no hesitancy in calling for corrections and improvements when their desirability is apparent. Sometimes the inspection may reveal the need of additional subsurface examinations before completion of the design. These should be made.

As the inspection proceeds, all questions on field sheets should be answered and requested information supplied. In addition, an inspection report should be prepared, giving, station by station, all recommendations of the inspection party, with reasons therefor.

9. PREPARATION OF FINAL DRAWINGS

Completion of Final Plan, Profile, and Cross-Section Sheets. All changes in the design resulting from the field inspection are made on the working plan and profile and the cross-section sheets. The plan and profile are then traced onto the final construction sheets, together with any desirable explanatory or directive notes. Where the construction is to be carried out by contract, it is a safeguard to place on each sheet the following statement or its equivalent:

> The information, including estimated quantities of work, shown on these sheets is based on limited investigations and is in no way warranted to indicate the true conditions or actual quantities or distribution of quantities of work which will be required.

ESTIMATE SHEET

STATE	TOWN	COUNTY	ROUTE NO	YEAR	SHEET NO	TOTAL SHEETS
	EXETER	LANCASTER	9A	1952	4	54

FOR THE CONSTRUCTION OF EXETER - LANCASTER ROAD IN THE TOWN OF EXETER COUNTY LANCASTER

FROM STA 165+40 TO STA 337+50 LENGTH 17210 FEET BASE COURSE NONE WIDTH — FEET DEPTH — IN SURFACE COURSE REINFORCED CONCRETE PAVEMENT WIDTH 22 FEET DEPTH 8 IN

EXCAVATION AND BORROW

STA TO STA		EARTH EXCAVATION	ROCK EXCAVATION	FILL	BORROW
		CY	CY	CY	CY
165+40	170+0	1461		798	
170+0	175+0	3068		2387	
175+0	180+0	251		5108	
180+0	185+0	1512	685	1078	
185+0	190+0	3340	2089	742	
190+0	195+0	1315	48	393	
195+0	200+0	2312	1831	312	
325+0	330+0	762		823	
330+0	335+0	945		2340	
335+0	337+50	270		460	
TOTALS FROM SECTS		41987	5362	53365	
MISC		4141	536	5337	
		46128	5898	58702	
BOULDERS		-4564	-4564		
		41012	10962		
LESS SHRINKAGE		2054			
PLUS ROCK SWELL			1475		
AVAILABLE FOR FILL		39018	11937	50955	
NET BORROW				7747	
PLUS SHRINKAGE				387	
TOTAL BORROW					8134
TOTALS		41012	10962		8134

PIPE CULVERTS, CATCH BASINS, ETC.

STA TO STA		18" RCP	24" RCP	36" RCP	MANHOLE	CATCH BASIN	CONCRETE ENDWALL
		LF	LF	LF	EA	EA	CY
168+50		48				1	2.1
180+35		24		32	1	1	5.5
180+35	183+40	48	300		1	2	
183+40	186+60	48	416		1	2	
198+20				60			11.0
209+65			60				6.8
215+15		48				1	2.1
UNASSIGNED		28	24	12			6.7
TOTALS		948	1840	332	11	26	160

WIRE ROPE RAILING

STA TO STA		THREE CABLE RAILING LT	THREE CABLE RAILING RT	ANCHORAGE
		LF	LF	EA
168+0	172+0		400	2
207+20	215+20	800	800	8
220+40	223+60	320		2
225+10	229+20	410		2
232+50	237+50		200	2
265+30	269+50	420	420	4
275+25	279+15	450		2
UNASSIGNED		850		6
TOTALS		10000		68

BRIDGES AND BOX CULVERTS

STA	TYPE AND SPAN

PAVEMENT AND MISCELLANEOUS

STA TO STA		CLEARING	SUBBASE	REINFORCED CONCRETE PAVEMENT	STOCK FENCE	GUARDRAIL
		LS	CY	SY	LF	EA
165+40	337+50	1.5	1692	42000		
UNASSIGNED			1408		15500	25
TOTALS		1.5	2800	42000	15500	25

OTHER CONSTRUCTION ITEMS

Fig. 25 Sample quantity estimate sheet.

Any other notes of a general nature which apply to more than one sheet may conveniently be placed, under the heading "General Notes" or "General Instructions," on the first plan sheet.

Finally, titles, dates, etc., of the sheets are added. When separate sheets must be used for plan and profiles, it is customary, in assembling the set of drawings, to let each profile sheet follow immediately after the sheet showing the plan view of the same length of road.

Typical Road Cross-Section Sheet. This is reviewed for completeness and accuracy, is titled and inked.

Index Plan and Profile Sheets. For convenience of reference, a plan-profile sheet (or separate plan and profile sheets, if necessary) is prepared to show the complete project. The plan scale should be such as to accomplish this without more than one break in the plan. It may perhaps be one-tenth of the scale used on individual plan sheets. The horizontal scale of the profile is made the same as the plan scale; the vertical scale is usually ten times the horizontal. The index plan should show only important features: the center line, stationed at 5- or 10-station intervals; bearings of tangents; complete curve data; locations of only the more important drainage structures; intersecting roads, and the old road in case of a relocation; intersecting and paralleling stream channels of any importance.

The index profile sheet should show the ground surface profile on center line with hatching to indicate swamp or rock outcrop; the grade line; the numerical grades of all tangents, the stations and elevations of points of vertical curvature and tangency; and profile views of major drainage structures. Ten-station intervals are noted along the lower margin of the profile sheet.

Structural Sheets. If the contract is to include any structures requiring special design the structural plans are completed and are titled, dated, etc. In assembling the construction drawings, plans for special structures are frequently placed immediately after the cross-section sheets.

Standards Sheets. Standard designs will normally be used for: drop inlets (Fig. 11); catch basins (Fig. 13); manholes (Fig 14); culvert end walls (Figs. 16, 17, 18); underdrain (Fig. 19); guard rail (Figs. 22, 23). They may also be required for paved ditches, cribbing, retaining walls, box culverts, pavement reinforcing, tree wells, fences and gates, barricades, curbs, gutters, sidewalks, etc. Standards sheets are prepared as needed.

Quantity Estimate Sheet. This sheet (Fig. 25) lists the quantities of all contract items, also showing at or between what stations the quantities will be used, or if they are unassigned. Contract items generally head columns across the sheet; stations or lengths of the project are shown on successive lines; unassigned quantities follow those that are assigned. The quantity of each item is totaled at the bottom of the column. Items are grouped, in several different tabulations as shown, to save space on the sheets.

Every item on which a unit bid price is to be received—except those on a lump-sum basis, such as "Maintenance of Traffic"—should be shown on the estimate sheet.

A list follows of the more important contract items with the unit in which each is ordinarily measured.

Item	Unit
Overhead cost	lump sum
Maintenance and protection of traffic (including signs, lanterns, maintenance of temporary roads and structures or of detours, as per specifications)	lump sum
Clearing and grubbing	lump sum or acres
Removal of existing masonry	cubic yard
Excavation (subclassified as earth, rock, loam, muck, etc., or as unclassified) *	cubic yard
Bridge excavation	cubic yard
Trench excavation (often subclassified according to depth of trench) *	cubic yard
Borrow	cubic yard
Reinforced-concrete pipe *	linear foot
Relaid pipe *	linear foot
Asphalt-coated corrugated metal pipe *	linear foot
Asphalt-coated corrugated metal plate arches *	pound
Underdrain with (type and size pipe) *	linear foot
Outlets for underdrain *	linear foot
Catch basin *	each
Drop inlet *	each
Manhole *	each
Reset catch basin (drop inlet or manhole) *	each
Test piles	each
Furnishing and use of equipment for driving test piles	lump sum
Pile loading test	each
Furnishing timber piles *	linear foot

Item	Unit
Furnishing steel piles *	pound
Furnishing precast concrete piles *	linear foot
Driving piles *	linear foot
Cast-in-place concrete piles *	linear foot
Splicing piles *	each
Steel sheet piles *	pound
Timber sheet piles *	thousand board feet
Concrete, structural	cubic yard
Stone masonry	cubic yard
Brick masonry	cubic yard
Deformed steel bars	pound
Structural steel	pound
Wire mesh	square yard
Gravel fill	cubic yard
Subbase	cubic yard
Formation of subgrade	square yard
Base (stone, gravel, etc.) *	ton or cubic yard
Expansion joint *	linear foot
Mat reinforcement for concrete	square yard
Concrete for pavement	cubic yard or square yard with reinforcement included
Sheet asphalt pavement (separate items for hot or cold, binder or surface course, etc.)	ton
Broken stone for pavement *	ton
Gravel surface (bank-run, screened, etc., and traffic bound, rolled, etc., as needed) *	cubic yard or ton
Bituminous material for bituminous macadam pavement	gallon
Bituminous material for surface treatment	gallon
Sand cover for bituminous surface treatment	square yard
Shaping and cleaning slopes and shoulders	square yard
Gravel shoulders	cubic yard
Curbing *	linear foot
Reset curbing *	linear foot
Sidewalk *	square foot or square yard
Gutter *	square yard
Paved ditch	square yard
Fence *	linear foot
Guard rail *	linear foot
Anchorages *	each
Single post *	each
Reset guard rail *	linear foot
Reset single post	each
Concrete cribbing	cubic or square foot
Riprap	ton
Slope paving	square yard
Signs	each

* Separate item for each type, size, etc.

The quantity estimate sheet may conveniently be placed immediately after the title sheet, in assembling the plans.

Title Sheet. The title sheet will ordinarily include the following items:

Designation of project.
Datum.
Length of project.
Scales used for drawings.
List of drawings.
Location map or maps (frequently a small-scale map of a state or other area to show general location, with a larger scale inset map to show exact location).
Signature block for approvals.

The title sheet will generally be made up after all other sheets of the set have been completed.

10. COMPLETION OF DESIGN

Preparation of Specifications. If the project is of a size or sort to be divided into several contracts, general specifications should be prepared to be applicable to all contracts, with "special provisions" supplementing them for each individual contract. The American

Association of State Highway Officials, the Associated General Contractors, and the American Road Builders Association have jointly prepared a bulletin, "Specifications for General Provisions," which may serve as a guide. In addition, the AASHO has prepared specialized specifications, each covering some particular construction material or type of highway construction.[1] These can be secured from the American Association of State Highway Officials, National Press Building, 14th & F Streets, N.W., Washington, D.C. The American Society for Testing Materials, 1916 Race Street, Philadelphia, Pennsylvania, also has prepared specifications covering many highway materials. All the various state highway departments have standard specifications covering their work. Those of the state in which a project is located, or a nearby state, have the advantage over a more generalized specification of being more closely adapted to climatic and other conditions likely to be encountered on the project.

It is important that either the general specifications or the special provisions should describe each construction item entering into the contract, the materials to be used, the tests they must meet, construction methods, the method of measurement of the item and the basis of payment. Where labor rates or conditions of employment, etc., are prescribed, these must also be covered in specifications or special provisions.

Preparation of Construction Cost Estimate. The construction cost estimate or "engineer's estimate" is made up by listing, on a proposal estimate sheet, all construction quantities from the quantity estimate sheet of the plans, entering a unit price, and extending each. To the total cost of the contract items so obtained, a percentage (often 10%, though sometimes as much as 15%) is added to cover contingencies and construction engineering costs. The cost of any work which it is proposed to do outside the contract is added, to secure the entire estimated project construction cost.

Procedure. In determining the unit prices to be used, consideration must be given both to current prices for comparable work in the same general area, and to factors of the job in question that will influence costs. Such factors include the sizes of the quantities involved, freight charges to the job, lengths of haul from delivery points, probable character of any excavation, depth of swamps and rock cuts, the season when the work is to be done, the time allowed, and many other factors.

Design Report. In any case where a highway design is to be reviewed, for approval or otherwise, by some engineer other than the designer, a design report will facilitate such review. Also if, during construction, question develops as to the desirability of some field change in the plans, the construction engineer will be assisted in his decision by knowing something of the reasons why the design was made as it was.

An outline for a design report follows.

I. *Description of Project.* Beginning and end; length; type, width, thickness, material, etc., of pavement; any other pertinent data, including estimated cost and cost per mile.

II. *Design Requirements.* Design speed, maximum gradient and curvature, required sight distance, and other.

III. *Alignment as Planned.* Maximum curvature, minimum horizontal sight distance, changes made in field location with reasons therefor.

IV. *Grade as Planned.* Maximum and minimum gradients; other grades worthy of comment; minimum vertical sight distance; changes made in recommended field grade line and reasons therefor.

V. *Traffic or Safety Designs or Devices.* Comment on design of all intersections, including traffic data justifying design; comment on any other noteworthy features included in design for safety of public, sidewalks, if included, and reason therefor, etc.

VI. *Grading.* State whether project is well balanced with short hauls, or if it has excessively large percentage of waste or borrow and reason for the excess; explain how quantities of special excavation (rock, muck, etc.) were obtained—from sections, soundings, or borings, or estimated percentages of cuts, etc., amount of available loam and disposition of it.

VII. *Surface Drainage.* Types of pipe used, extraordinary layouts, if any, and description of same; changes made in field recommendations and reasons therefor.

VIII. *Subdrainage.* Subbase: type, extent and reasons for recommendations; unassigned quantity and how estimated, changes in field recommendations and reasons therefor. Underdrain: type, extent, and reasons for recommendations, unassigned quantities and how estimated; changes in field recommendations and reasons therefor. Special comment on any unusual conditions and proposed treatment thereof.

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[1] Many of these are referred to hereafter in the section on highway construction, where the principal road-building materials and methods are outlined.

IX. *Roadway Structures.* List all planned bridges, box culverts, cattle passes, temporary bridges, etc. Comment on important features of design such as use or non-use of piles, etc.

X. *Accessory Structures.* List all planned retaining walls, cribbing, slope paving, riprap, etc., with comment on any alternate designs considered and discarded. List any planned channel changes with any appropriate comment.

XI. *Available Materials.* Give source, type, and quality, so far as known, of available materials such as sand, gravel, borrow, etc.

XII *Public Utilities.* List all utility companies having structures or interests within or near the limits of construction. Note utility lines on private property that are likely to be disturbed by the project. List all transportation lines whose schedules may be affected by the construction. List all notifications given to public-utility owners, with dates and person contacted or notified.

XIII. *Right-of-Way.* List normal and minimum right-of-way width. Explain all reductions below normal. List all property agreements, with dates and names of parties. List all revisions of plans made as result of property agreements.

XIV. *Clearance of Way.* Statement that property is cleared of all right-of-way encumbrances, or a listing of all encumbrances not cleared.

XV *Salvage.* Disposal of salvage, if any; agreements with municipal authorities or other persons or agencies concerning same.

XVI. *Maintenance of Traffic or of Detours.* Give reasons for policy adopted; list detour requirements (traffic volumes, etc.) and recommend routes.

XVII. *Field Inspections.* Note dates of any field inspections and by whom made; list important changes made in design as result of inspection; if important changes were recommended but not made, state why.

XVIII. *Official Contacts.* If municipal or other officials were contacted in regard to design, give dates and persons seen.

XIX. *Recommendations.*

Preparation of Right-of-Way Maps. At any time after taking lines have been determined, right-of-way maps can be prepared. These will normally be to the same scale as the construction plans. Each map should show:

1. The parcel to be acquired, with its dimensions and area, the name of the owners and of abutting owners.
2. The taking lines, accurately located with respect to some permanent base line. If the taking line is a curve, the P.C and P.T. with adequate ties for their locations, and complete curve data.
3. All topography of the area, as shown on the construction map.
4. Date, north point, scale, etc.

HIGHWAY CONSTRUCTION

11. CONSTRUCTION ENGINEERING

Introduction. Construction engineering for a highway project includes: surveying, inspection, and certain miscellaneous activities that may be termed "completion of the project."

Surveying. *Reestablishment of Center Line.* The first surveying activity for construction is the reestablishment, as needed, of the location center line, with any changes that may have been made in it during design. This should be done, whenever possible, before bids are taken. Center-line stakes (or tacks in existing pavements, etc.) should be set at all P.C.'s, P.I.'s, and P.T.'s of curves, at every station and half station and at each plus marked on the plans These center-line stakes have several uses. They form the base from which cross-sectioning of the project is reviewed; from them, offset stakes are located to control grading operations; from them, also, bridge and other special structure stakes are set.

Cross Sections Ground-surface cross sections should be checked as necessary to see whether changes have occurred since the original cross sections were taken and plotted, and to verify the accuracy of sections that may originally have been taken with snow on the ground, etc Corrections are made as necessary. Additional sections (for example, where cut changes to fill, or vice versa) are taken if needed for the accurate computation

of quantities. The plotting of all the cross sections is reviewed to insure that each section was extended far enough to include all required excavation.

Offset Stakes. Offset stakes should be securely set on both sides of the proposed road, at convenient points well outside the area to be disturbed during construction. A pair of stakes should be set opposite each center-line stake—at right angles to tangents and radially on curves. Ordinarily each offset stake should be marked with the station number and plus, the offset distance, and the cut or fill. They may carry additional pertinent information, such as superelevation and pavement widening data. Offset stakes are set as soon as convenient after center-line staking is completed, and early enough not to delay the contractor's grading operations. They need not be set in any regular progression, and it may often be desirable to set them first on the part of the project where the contractor wishes to commence operations.

Structure Stakes. All stakes required for proper construction of bridges, culverts, manholes, and any other structures should be driven on offset lines as agreed with the contractor. This work should be carefully timed so that the contractor can complete the structures as early as possible to permit adequate settlement of backfill. Stakes should be set so as to interfere as little as possible with construction work.

Pavement Stakes. The offset stakes set to control grading and drainage operations will often suffice for construction of road surfacing as well. However, for concrete or other high-type pavements, especially if they are to be curbed, a second set of stakes is usually desirable. These are set after completion of rough grading operations, and before fine grading. On fills, they should be set no longer than necessary before the pavement is placed, so as to allow maximum settlement to take place before the stakes are driven. Stakes should be set with a uniform offset of 2 to 4 ft from the edge of pavement or back of curb. They are set at such intervals as the situation requires—often enough to assure that all forms are set to proper grade and line. Superelevation and pavement widening require close control. All pavement stakes should be tacked. Stakes for curb should be marked to show cut or fill from top of stake to top of curb. Stakes for pavement should show cut or fill from top of stake to top of finished pavement edge.

Additional Surveys. When rock surface has been exposed and before rock excavation is commenced, cross sections of the rock should be carefully taken on the same cross-section lines as the original sections, and on such intermediate lines (usually not more than 10 ft apart) as are necessary to determine accurately the amount of rock to be excavated. Swamps and bogs must be sounded closely enough to delineate hard bottom. If sufficient soundings were not obtained prior to construction, they should be supplemented as necessary during construction. Borrow pits must be sectioned both before and after excavation to determine the quantity of borrow supplied. It may also be necessary to establish line and grade for gutters, walls, fences, and other construction items. Test surveys should be made at frequent intervals to check the accuracy with which construction is being conformed to the plans, especially where the work will be covered by later construction operations, as in the case of bridge and culvert footings.

Recomputation of Earthwork Quantities. As soon as possible after a job has been restaked, excavation quantities for all new or corrected cross sections should be computed and compared with the estimate quantities. Any considerable discrepancy should be investigated before grading operations at that point are begun.

Measurements for Interim Estimates. Measurements of pay quantities for interim estimates must be made accurately for all work which is to be so hidden by later work that it cannot be measured accurately on completion of the job. Otherwise it is permissible to approximate somewhat on interim estimates, since any resulting underruns or overruns will be compensated for on the final estimate.

Final Survey. When the contractor has completed his work, the entire project should be resurveyed (a different survey party is frequently used) to make an accurate determination of final values of all pay quantities. These values will be used in making up the final estimate.

Inspection. The inspector's[1] primary responsibility is to see that construction accords with plans and specifications. This responsibility places on the inspector many related subsidiary duties.

1. The inspector interprets the plans. Through his own efforts or with assistance from a survey party, and through conferences, as necessary, with superiors, he sees that the contractor is supplied with information that will allow the work to proceed. To do this, he must thoroughly familiarize himself with the project, studying general and special provisions of the specifications, the survey report, and the design reports. He should

[1] The chief inspector on a project is often called "project engineer," but for convenience the term "inspector" is used here.

also carefully examine the site of the work, paying particular attention to all critical or difficult spots He must see that enough work is staked out at all times to avoid delays to the contractor and to enable him to carry out his various operations in proper and economical sequence. He should never assume the duties of a foreman for the contractor, but should transmit his instructions through foremen or their superiors. He does not ordinarily have authority to change or to waive any part of the plans or specifications. If agreement cannot be reached between the contractor and him as to any interpretation of plans or specifications, he should refer the matter to his superior, immediately directing the contractor to discontinue work on the disputed feature, and withdrawing inspection from it.

2. The inspector marks or otherwise directs the contractor's attention to all trees and shrubs that are to be saved, and to all existing water and sewer lines, drainage structures, electric conduits, and other utilities, etc., that might be damaged accidentally by the contractor's operations.

3 The inspector should try to correlate the operations of owners of affected public utilities with those of the contractor so as to minimize interference and delays He should give careful attention to details of relocation of utilities that are not explicit in the plans, to see that no conflicts are created.

4. Where existing traffic is affected by the construction, the inspector is usually responsible for seeing that all reasonable precautions are taken for the safety and convenience of traffic. This may include erection and maintenance of warning and direction signs, barricades, lanterns, and flares; the provision of traffic men for maintenance of one-way traffic or similar control; the maintenance of a reasonably smooth and dust-free temporary road surface: provision of temporary bridges, crossovers, etc. The contract should be explicit as to the obligations of the contractor in these matters, and the inspector must see that obligations are met.

5. The inspector is responsible for the accuracy and quality of the contractor's work, and its conformity to requirements. He must be especially careful on all parts of the job that will later be covered; as culverts and drain pipes are laid, the inspector must verify the correctness of their position, the adequacy of foundations, soundness of joints, etc., before he authorizes them to be covered. After forms for pavements have been set, he must check their line and grade from his stakes, and he should also check all forms by eye to see that the top is set to a true line and a smooth grade, with particular attention to transition and superelevated sections.

6. The inspector is responsible for seeing that all property agreements embodied in the construction plans are faithfully carried out by the contractor.

7. The inspector is responsible for the control of materials entering into the project, with regard to both maintenance of quality and verification of delivery to the job of all claimed quantities. Tests of quality are ordinarily accomplished in a variety of ways at the source, on the job, and through samples taken on the job for test elsewhere. On each job, the inspector's responsibility should be clearly defined. A careful routine must be established for sampling, labeling, shipping, testing, and receiving reports of tests. The inspector must see that this routine is meticulously followed. On all materials (broken stone, asphalt, etc.) for which the contractor is to be paid on a per ton, per gallon, or similar unit basis, the inspector should receive a delivery ticket as the material is delivered to the job. The contractor should in addition be required to furnish the inspector, at the close of each day, with a statement of all such materials delivered to the job during the day. The inspector should be required to check his delivery slips against this statement, so that any discrepancies may be immediately resolved.

8. The inspector is responsible for proper preparation of change orders. Specifications usually provide that quantities may be increased or decreased within stated limits at the bid price. When it becomes evident that any quantity is going to deviate from the estimate by more than the limiting amount, it is desirable that a change order covering the variation and any modification of prices be prepared and signed by both parties to the contract before the work in question is done. It is important for the inspector to foresee as fully as possible the need for such change orders in time to prepare them and secure the necessary signatures without delaying the contractor's work. In the event that the change order calls for the work to be done on a cost-plus basis, it is a further responsibility of the inspector to check the contractor's payrolls and receipted invoices before authorizing any payment on the change order. Change orders should give the following information:

a. Description of the proposed change.
b. Location, by stations, of the proposed change.
c Reason(s) for the change.

d. Breakdown of work involved in the change order into unit quantities for payment; or lump sum, cost-plus, or other basis of payment agreed to, in sufficient detail to be completely definite.

e. All other necessary data, such as revision of the time allowed for completion of the contract.

9. The inspector is responsible for job records Records to be kept on a highway construction project will vary according to many different factors, and no hard-and-fast list can be specified. In addition to all other records, however, a running diary or log of the job should always be kept This should form a complete, historical record of the work. It should be kept in a standard engineer's notebook rather than on loose leaves. For each day, it should include the following principal items:

a. Weather conditions—particularly the occurrence of rain or snow, and if in sufficient amount to close down the job The hour of closure, if closed All significant temperatures.

b. Whether or not the day was charged to the contractor as a working day

c. Items of equipment in use during the day, and for how many hours in each case.

d. The labor force employed, divided into classifications.

e. Work in progress during the day—crews working and at what points. Estimate of work done.

f. Any disagreements with or complaints from the contractor.

g. Any complaints made by property owners or by other private citizens.

h. Noteworthy visitors and any noteworthy occurrences not covered by above headings

In its entirety, the log should give a detailed, accurate, over-all picture of the progress of the job. It should form an adequate basis for resolving any disputes and disagreements that may subsequently arise concerning the work or its effect on neighboring properties It should also provide much of the data that will be needed for the construction report that should be prepared after the project has been completed

The log will ordinarily be supplemented by a daily inspection report, furnished to the inspector's superior. This will include such part of the log-book entries as may be required. Figure 26 shows a typical daily inspection report form.

The project records should also include all materials receipts, all requests for and copies of tests of materials, all change orders and any special orders to contractors, and monthly or other interim estimates of work completed, as well as the final estimate.

Estimates. The inspector is responsible for preparing, at the end of each month or other interval specified as the pay period, an estimate of the work done during the period. This estimate should show the work done under each of the contract items, and also must show any additional and extra work done under change orders. A form should be used for tabulating these items. Appropriate column headings for such a form are.

Original Estimated Quantities	Revised Estimated Quantities	Item No.	Item	Unit Price	Work Done in Past Month		Estimate to Date	
					Quantity	Amount ($)	Quantity	Amount ($)

Measurements of quantities are made as described in Sect. 2, Surveying, some approximation being permissible on the interim estimates. Work that is not to be paid for at any contract unit price is listed separately, with the basis of payment (lump sum, cost plus, etc.) and all necessary data. The *Original Estimated Quantities* column shows the quantities on which bids were taken. The *Revised Estimated Quantities* column reflects the best judgment, as of the date of the estimate, of the quantity of each item. The *Estimate to Date* quantities and amounts are obtained by adding the work estimated for the month to the *Estimate to Date* amounts from the previous interim estimate.

Completion of the Project. *Final Inspection and Acceptance.* As the operations of the contractor approach their conclusion, the inspector should carefully review the plans, specifications, contract, and all property agreements and change orders to see that every detail of the contractor's obligations has been met, as well as all obligations to property owners. This review should cover the proper disposal of all waste and salvage materials from the job, the satisfactory completion of all utility work; the removal of all temporary signs, barriers, and barricades and provision for all permanent ones, and the general cleanup of the highway area. When the inspector is satisfied with the completion of all items, the person or persons authorized to accept the contract should make whatever examinations of it they may deem proper. These may include, besides visual inspection of the work, the coring of pavements and any other tests necessary to establish compliance with plans and specifications. Work found lacking or unacceptable is completed or corrected, and the project is then accepted.

INSTRUCTIONS

TO BE PREPARED IN DUPLICATE AT THE CLOSE OF EACH DAY BY EACH INSPECTOR IN CHARGE OF A CONSTRUCTION PROJECT THE ORIGINAL TO BE FORWARDED TO THE RESIDENT ENGINEER IN CHARGE AND THE DUPLICATE RETAINED BY THE INSPECTOR

DAILY INSPECTION REPORT

DATE		NO	
ORGANIZATION UNIT NO		WORK ORDER NO	
CONTRACTOR	WEATHER	TEMPERATURE	
PROJECT	TOWN		ROUTE NO

WORK DONE BY CONTRACTOR

DESCRIPTION OF WORK	FROM STATION	TO STATION	AMOUNT	REMARKS
				AVAILABLE (06 AND 15) —
				CONTRACT TO DATE —
				BAL TO COMPLETE —
				REQUIRED TO COMPLETE —
				FREE BALANCE —

CONTRACTOR'S FORCE AND EQUIPMENT

SUP'T		BULLDOZER		ROLLER	
FOREMEN		COMPRESSOR		SAW-POWER	
SKILLED LAB		CRANE		SCRAPER	
INTERM LAB		GRADER		SHOVEL	
COMMON LAB.		MIXER-CONC		TRUCKS	

REPORT OF MATERIAL TESTS RECEIVED

MATERIAL	SAMPLE NO	PASSED	REJECTED

		MIXER-PAVER			
		PUMP			

ORIG. CALENDAR DAYS- | USED-

WORK DONE BY OTHERS

DEPARTMENT OR COMPANY	DESCRIPTION OF WORK

INSPECTOR'S MILEAGE REPORT

	START	STOP	MILES
HOME-JOB TRAVEL			
ON JOB TRAVEL			
JOB-HOME TRAVEL			
		TOTAL	
HOME-JOB TRAVEL			
ON JOB TRAVEL			
JOB-HOME TRAVEL			
		TOTAL	

RESIDENT ENGINEER OF CONSTRUCTION — ENGINEER OF CONSTRUCTION

SUMMARY OF DELIVERY SLIPS

ITEM	SOURCE	NO LOADS	QUANTITY

CONDITION OF ROAD

ROAD STATUS	STATION	STATION	STATION	STATION

OFFICIAL VISITORS TO JOB

NAME	TITLE

OTHER COMMUNICATIONS SENT WITH THIS REPORT

STATE FURNISHED MATERIAL REPORT		FREIGHT BILLS	
REQUISITIONS FOR MATERIAL		MATERIAL TESTS SENT	
REQUEST FOR CHANGE ORDER		RECEIPTS FOR MAT SOLD OR TRANS	
RECEIVING REPORTS			

INSPECTION FORCE

NAME	HRS	NAME	HRS

INSPECTOR IN CHARGE

FIG. 26. Daily inspection report.

Final Estimate The interim estimates will usually have been made up as the work has progressed, from measurements taken by the inspector, with help of a survey party on some of the items. Upon completion of the work and after final inspection and acceptance of a project, the final estimate is prepared. In contrast to the interim estimates which are built up from month to month, the final estimate should be based on a complete remeasurement (by a new party, when feasible) of the entire job, with the exception of such work as has been finished during course of the construction and has then been covered by more recent work. This independent remeasurement will serve as a check on the correctness of all earlier measurements, and will also act as a deterrent to any collusion in falsifying pay quantities. Quantities included in the *Estimate to Date* columns of the last interim estimate are subtracted from final estimate quantities to establish the payments to be made on the final estimate.

Construction Report. When the project has been completed and the final estimate prepared, a construction report may often be desirable to put into permanent, available form such information concerning the project as is not readily obtainable from the earlier reports that have been enumerated or from other project documents. A suggested outline is.

1. Date.
2. Identification of project: location, designation, termini, length, etc., of road, name of contractor.
3. Changes from design, with reasons therefor, in:
   A. Alignment.
   B. Grade.
   C. Cross section.
   D. Drainage.
   E. Major quantity increases.
   F. Major quantity decreases.
   G. Any other.
4. Difficulties encountered during construction.
5. Equipment used.
6. Performance of contractor, including adequacy of equipment, care of equipment, ability, etc.
7. Materials used.
8. Engineering personnel on project.
9. Signature of author of report.

Revision of Construction Drawings. The final step in completion of the project is the correction of the drawings to show the project "as built." The tracings should be carefully checked against the inspector's notebook and other records, and against the actual construction if necessary. All changes in pipe (sizes, lengths, and locations), in driveways, guard rail, etc., as well as changes, if any, in line, grade, and cross sections should be noted. No erasures should be made on the tracings. The items changed should be crossed or shaded out, in such ways as to leave them still legible. New data should be entered alongside the old. Much of this work should have been done as the job progressed—notably, the cross sections will have been revised as necessary to show true amounts of excavation, subbase, etc., in order that correct payments for the work may have been made. These corrections may have been made on a set of prints, however, rather than on the tracings. They are put on the tracings, and carefully checked for accuracy and completeness.

12. GRADING AND DRAINAGE [1]

Definitions and Purposes. "Grading" and "drainage" together encompass all construction operations required to ready a roadbed for placement of the pavement or other road-surfacing material; they also include shaping shoulders and ditches, and shaping and stabilizing all slopes, in cut or on fill.

The purposes of these operations are:

1. To bring the subgrade, together with shoulders, gutters, and slopes, to the contour called for by the plans

[1] For a more detailed discussion of highway grading and drainage, base courses, and surface courses, and the construction of roads and pavements, see *Manual of Highway Construction Practices and Methods* (1950), American Association of State Highway Officials, National Press Building, 14th and F Streets, N W , Washington 5, D. C.

2. To eliminate unsuitable materials from the subgrade and from other portions of embankments and their foundations, or so to disperse such materials through the earth mass that they will not be harmful.

3. To prevent conditions that will lead to an injurious amount of water in the roadbed and slopes; or, if the water cannot be eliminated, to neutralize any bad effects by the use of appropriate materials, such as rock fills or soils that are stable even when saturated.

Clearing and Grubbing. This initial grading operation includes removal from the construction area of trees, underbrush, stumps, rubbish, abandoned drainage structures, and sometimes buildings. In any unusual situation, notes on plans or paragraphs in special provisions should make clear just what is or is not included. It may be desirable to preserve trees, shrubs, etc., growing within highway limits. Such trees should be carefully marked and, if necessary, protected against accidental damage during construction by putting guards around them, and blanketing exposed or thinly covered roots with a 3-in. to 12-in. layer of gravel and earth. Trees to be left standing within fill slopes should have tree wells of dry masonry built around them.

All brush, stumps, roots with a diameter larger than 3 in., and matted roots, and all trees except those to be left in place, should be removed from the construction area to the extent that no portion remaining will be closer than 18 in. to any subgrade, shoulder, or slope surface, with the further requirement that all stumps be cut off so as not to extend more than 1 ft above the original ground. On cleared areas beyond the road prism slope lines, stumps may be cut flush with the ground in lieu of being removed. Generally, bulldozers and similar equipment are used to remove small trees, underbrush, and surface debris. Larger trees require cutting. Explosives may be used to break up or loosen large stumps. All holes from which stumps, etc., have been removed must be backfilled and compacted, unless they lie within an area to be excavated. Salvageable timber and other material resulting from clearing and grubbing is preserved as may be stipulated in specifications. Waste material, if combustible, is usually disposed of by burning, observing pertinent laws and ordinances. Suitable non-combustible material, such as old masonry, may be used for fill. Clearing and grubbing may be paid for (1) on a lump-sum basis; (2) on an acreage basis; (3) on an acreage basis plus an extra payment for each tree (over a specified minimum diameter), according to its size.

Excavation. Excavation is classified according to its purpose—as roadway, trench, bridge, etc.—or according to the type of materials to be removed, as earth, rock, etc. Bridge excavation is an integral part of foundation construction. Ordinary roadway excavation and trench excavation differ substantially from one another in purposes and problems, and are best considered separately.

Roadway Excavation. Roadway excavation includes removal and satisfactory disposal of all material, removal of which is necessary in constructing the roadbed, ditches, and slopes to dimensions shown on the plans. Most or all of the excavated material is normally used to form embankments, the formation of which is part of the excavation item. Excavation also may be specified to include shaping and cleaning the slopes and shoulders when no such separate items are in the contract.

Roadway excavation is generally classified, for the purpose of payment, as "loam," "rock," or "earth" excavation, or as "unclassified" excavation, as follows.

Loam excavation includes loam or topsoil, removed between limits and to the depth indicated on the plans. Loam excavation is not an item in all contracts, but it has become a widespread practice, where good loam occurs in the right-of-way, to remove and stockpile it, for later spreading on cut and fill slopes.

Rock excavation includes all rock in definite ledge formation, the removal of which requires wedging, barring, or use of explosives. It also usually includes all boulders 1½ cu yd or more in volume.

Earth excavation ordinarily includes all materials other than water or "loam" or "rock" excavation that must be removed.[2]

Unclassified excavation includes all material, other than water, that must be removed for the construction, without any regard whatever to the nature of the materials.

Roadway excavation is carried out as required by the plans. Slopes of earth cuts are made true to the lines given, but are ordinarily not required to be raked perfectly smooth, since they form a better base for loam or vegetation if left a little rough. Ledge rock is excavated at least to the rock-slope lines indicated on the plans, and to a depth below the subgrade elevation as required for subbase. The surfaces of slopes in rock cuts should be made fairly even and uniform. All loose and shattered stone should be scaled off. Where faults or breakage zones are likely to cause slides or fallen stones, unstable rock should be

[2] Occasionally some material such as peat or swamp muck when encountered in a road project, is treated as a separate classification item.

ordered removed and be paid for even though outside normal payment lines Specifications usually make the contractor fully liable for damage due to blasting, and require him to take precautions against it.

All material resulting from excavation that is not required to bring embankments to the specified cross section should be used to flatten slopes, fill low spots in the right-of-way, or widen embankments. Unsuitable material should be disposed of outside of highway limits in unobjectionable ways.

Payment lines for earth excavation are ordinarily specified to coincide with slope shoulder, and subgrade lines as shown on the plans Payment lines for rock excavation are established a short distance—often 12 in.—outside of and parallel to the slope lines. In case of natural faults or fissures which make removal of additional rock necessary for reasons of safety or which produce slides clearly not due to the contractor's methods of operation, payment lines should be fixed to coincide with the natural fissures of the rock. Payment is made for excavation below subgrade to such depth as called for. Payment lines for loam excavation coincide with limits of area and depth specified for the excavation. When excavation is unclassified, payment lines for each material encountered are the same as outlined above for the particular material. Unless the proposal contains a specific item for it or unless excavation is unclassified, the removal of concrete pavement, concrete base, and miscellaneous concrete or cement masonry over 1/2 cu yd in volume may be paid for as rock excavation, based on the actual yardage removed Pavement and road bases other than concrete are treated as earth excavation.

Trench Excavation. Trench excavation is the removal and satisfactory disposal of materials which must be removed in the construction of pipe culverts, end walls, inlets, catch basins, manholes, underdrains, etc. Except for comparatively shallow trenches, unless the soil is exceptionally stable, bracing or sheeting may be required to support the walls. The expense of such support, as well as of any pumping to keep the trench free of water, is covered in the unit price bid for the excavation, as is the cost of any necessary backfilling after a pipe has been placed or a manhole or other structure built. Since the unit cost of trench excavation mounts rapidly as the depth of the trench increases, it is a frequent practice, in preparing a contract, to break down the total amount of trench excavation that must be done into classifications according to depth, 0 to 6 ft, 6 to 10 ft, etc. In roadway cuts, trench excavation commences at the bottom surface of the roadway excavation. Payment is ordinarily made for the actual quantity of material removed in trench excavation, with a safeguarding proviso that the width between maximum payment lines shall not be more than 2 ft greater than the nominal inside diameter of the pipe being installed. This leaves the contractor free to decide on the batter to be used, on the relative economy of greater batter as against the use of sheeting, etc. In the case of end walls, manholes, and similar structures, payment lines are vertical and a specified distance usually 2 ft outside the neat lines of the foundations in each direction, horizontally. In setting up specifications for trench excavation, a widespread practice is to provide that the unit price bid will be applied to the yardage of all material removed except rock; and that the rock yardage taken out will be multiplied by a specified factor representative of the additional cost of removing rock in comparison with earth. When the material in which a trench is being dug does not have enough stability to be self-supporting while the pipe is being laid and the trench backfilled, stay bracing and sheeting are placed to hold it, after the excavation has been roughly completed. Where the ground is so unstable that supports must be driven into the undisturbed ground to hold it in place while the excavating is being done, sheet piling is required.

Borrow. When approval has been given to the contractor's proposed source of borrow, accurate ground-surface measurements are secured before and after the borrow is taken, to determine the yardage of borrow used. No material from within the measured area should be used for anything but borrow. Sometimes a bank will provide material meeting specifications not only for borrow but for some higher type of material, such as subbase, as well Separate areas should then be cross-sectioned for the two uses. Material which the contractor uses for some lump-sum item, such as provision of a temporary road, must not be taken from either cross-sectioned area. When borrow is secured from a commercial pit, a definite portion or portions of the pit should be reserved, and care taken to see that there is no infringement on these portions.

Borrow may be dredged from underwater sources if available in adequate quantities and of satisfactory quality The basis of payment of dredged borrow should be carefully specified. Where there is no current or tidal or wave action to change the contour of the underwater surfaces, measurement of such borrow may be made by computing the volume between the initial and final surfaces, as in ordinary borrow pits. Frequently, however, it is preferable to provide for the payment of dredged borrow on the basis of its yardage as measured in the embankment.

Transporting Excavated Material. Equipment and methods used for transporting excavated material from its original position to its final position in embankments or elsewhere, and the proper organization of the work, are all outlined in Sect. 9. Cost of moving and placing the material is usually included in the price paid for excavation or borrow.

Embankment Foundations. No matter how well an embankment is constructed, it will not give satisfactory service unless it rests on a stable foundation. Important conditions requiring corrective action are:

1. Water in the form of springs or seepages.
2. Unsuitable foundation materials.
3. Steep surface slopes.

Springs and wet areas should be drained, whether or not the condition was discovered in time to be included in the plans Peat, muck, and some types of silt and clay have such an affinity for water and have so little load-carrying capacity that unless they are too deep (15 to 20 ft) it will usually be most satisfactory to remove and replace them with better material. This will especially be true because such materials generally occur in locations that will continue to be saturated after the embankment is built Sometimes the inferior material can be removed with bulldozers, power shovels, or tractors and scrapers. Usually, however, it will be too soft and wet for such equipment. Draglines may then be effective.

Where the natural ground surface in the embankment area is steep (more than about 1 on 3) it should be stripped of topsoil and then be plowed along the contours or benched into steps to provide a better bond with the embankment and to avoid a slippage plane. The harder the ground surface, the more necessary such measures are. Rock slopes require benching.

Construction of Embankments. *Basic Principles.* Equipment and methods employed in building embankments are given in Sect. 9. Listed below are a few principles of particular importance in constructing embankments for highways:

1. Use the most satisfactory materials available, and if inferior materials must be used place them where the effect of their inferiority will be least, as in the slopes beyond the shoulder areas.
2. Compact the material thoroughly.
3. Keep the embankment as free as possible from excessive moisture.
4. In any portion from which excess water cannot be excluded, use materials that will endure saturation.
5. Take particular pains to maintain dry conditions in the part of the embankment immediately below the road base.

Selection and Use of Materials.[3] Normally, most or all material resulting from excavation is used in embankments. However, tree stumps and roots, muck, peat, etc , are discarded, even at the expense of bringing in material to replace them. Where embankment material varies greatly in quality, care should be taken in its disposition. Highly capillary material should be insulated from the source of capillary water by placing an underlying blanket of granular material. If a portion of the fill material is composed of large stones, these should not be placed in nests, but should be distributed and interspersed with spalls, finer rock fragments, or earth to form a dense, compact mass. No large stone should be allowed to extend within 12 in. of the subgrade, and selected material should be reserved for the top layers of the embankment.

Where fill replaces excavated muck or other soft material, the bottom of the fill should be at least as wide as the fill at the ground surface; and where hard bottom is reached only a few feet below ground line, the slope of the embankment above ground should be maintained to the bottom of the fill.

Compaction of Embankments. This is secured by placing the embankment material in relatively thin layers; if necessary, bringing the moisture content of the material to the right amount for most effective compaction; and rolling each layer of the fill with suitable equipment. All of these procedures are set forth in Sect. 7.

Elimination of Excessive Moisture. This is accomplished, in the first place, by good design, especially in establishing the grade line and in providing a suitable embankment cross section and adequate drainage. The important steps to be taken in the construction stage have already been given: use of an insulating blanket of granular material if necessary to prevent capillary water from being drawn into the embankment from below; and thorough compaction to leave as few voids as possible in the body of the embankment.

[3] See also AASHO Specification M-57.

Wet Embankments. The lower portion of an embankment may necessarily be below ground-water level, as, for example, where it crosses a swamp or the arm of a pond. Such a portion must be built of materials that can endure saturation, such as rock, gravel, or sand; and be designed with side slopes that will be stable when saturated. If erosion due to wave action or currents is possible, this must be guarded against by riprapping or slope paving. If the water level fluctuates, the saturation-resistant material must extend to the high-water line. If a coarse rock fill is used for the embankment, the upper portion of this fill must be properly "choked" with granular material, suitably graded so that the normally dry material above, if occasionally saturated, will not flow down into the interstices of the rock

Maintaining Dry Conditions in the Top of an Embankment This is likely to be a problem only in low embankments built of inferior (capillary) material. In such situations, a proper grade line and cross section should normally meet the situation. Any trouble not cured by such means should be controlled by use of subbase and underdrains (p. 4-41).

Installation of Culverts. *Foundation.* Whenever possible, the bottom of a culvert trench should consist of undisturbed soil for its full length and width. If the design has not accomplished this, the best possible compaction of any fill under the culvert is essential. The condition requiring the greatest care occurs when part of the culvert is supported on fill and the remainder on undisturbed ground.

When a culvert is placed beneath a fill in the bottom of a valley or the bed of a watercourse, unstable foundation material should be removed and replaced with satisfactory material so far as practicable. If this cannot be done reasonably, a layer of sand, gravel, or other suitable material should be placed on the foundation and worked into the unsatisfactory material until a stable foundation is formed. If a pipe culvert is to be placed in rock excavation, the rock should be removed for a foot or less (depending on the size of the pipe) below the invert grade and a well-compacted layer of gravel, sand, or other suitable material should then be placed as a cushion for the pipe. The bottom of the excavation for pipe culverts and underdrains should be made to conform as nearly as practicable to the shape of the pipe. When bell-and-spigot pipe is used, holes should be excavated to fit the bells so that the pipe will have uniform bearing along its length.

It is desirable to avoid the installation of drainage structures or systems in embankments because of the possibility of unequal settlement which would be apt to break the line or to produce low spots that will not drain. When there is reason for such an installation, corrugated metal pipe should be used because of its longer sections and stronger joints. Especial pains should be taken to secure compaction of the fill below the pipe. If the structure is one of considerable size—such as a box culvert or large drainage pipe, it is good practice to construct the embankment to a grade several feet above the invert elevation, compacting it very thoroughly, and then to excavate a trench for the pipe in the compacted fill.

In laying culverts, the placing of concrete or other pipe with bell-and-spigot joints should begin at the downstream end. The bell should be laid upgrade. The outside laps of circumferential joints of corrugated metal pipe should point upgrade, and the longitudinal laps should be on the sides.

Corrugated metal pipes of considerable size—48 in. or more in diameter—are ordinarily "strutted" before the fill is placed above them. Temporary sills are placed longitudinally inside the pipe, along top and bottom. Vertical struts are placed between the sills, and jacks are used to increase the vertical diameter of the pipe by 3 to 6% of its normal value. The struts are left in place until the fill has been completed and compacted. When the struts are removed, lateral pressure of the embankment resists lengthening of the horizontal pipe diameter. By the time settlement of the fill has taken place, the pipe is restored to a practically round cross section, instead of having been flattened by the vertical pressure. Headwalls should not be built until the struts have been removed and the culvert has returned or has nearly returned to its permanent cross section.

Backfilling. Backfill adjacent to culverts must be carefully tamped into place so as to be very thoroughly compacted before heavy fill is loaded onto the top of the culvert If not, settlement of the fill alongside the culvert will bring an inordinate load on the top. In the case of a high fill, the pipe may be cracked. Attention must be given to working the backfill under the haunches of the pipe. These especially compacted berms alongside a pipe or box culvert should extend on each side a horizontal distance at least equal to the pipe diameter, or the box culvert height, except insofar as undisturbed material lies within this area. The fill material should be of high quality and should be placed in thin layers and tamped, water being added, if necessary, to bring the material to the best moisture content for compaction. Backfilling should be carried on simultaneously on both sides of the culvert to avoid displacing or unduly stressing it.

A culvert is much more apt to be damaged by loads passing over it during construction than by any loading it will receive after the finished road is in use. Before construction equipment is allowed to be driven over a culvert, a tamped fill equal to at least two diameters of the pipe, or 30 in., whichever is less, should be placed over it. If the culvert grade does not provide space for this much fill to be left permanently, the excess is removed before the pavement is placed.

Backfill for trenches, around catch basins and manholes and in other small areas should be deposited by hand shovels in thin layers and compacted with suitable hand or mechanical tampers For larger structures, heavier machinery may be used, provided the precautions set forth above are taken to prevent damage during construction.

Completion of Subgrade.[4] As formation of the roadbed nears completion, all soft and yielding materials in the top portion of the foundation which will not compact well when rolled or tamped should be removed. All loose rock or boulders over 4 or 5 in. in size should be removed or broken off to a depth of not less than 1 ft below the subgrade. All holes or depressions made by such removal of material should be filled with suitable sound material. If the surface of an existing stone or gravel road is less than 1 ft below subgrade, it should be scarified to permit uniform compaction and bonding between old material and new fill. (Any portions of the old roadway underlying the new road shoulders should be similarly scarified.) The whole subgrade should be thoroughly tamped, rolled, or trucked over for maximum compaction while it is being brought to lines and grades shown in the plans. The embankment or cut surface on which the subbase will rest should be smoothed and given the proper cross slope to drain the subbase, and adequate provision of leak-offs or other means of removing water from every portion of the subbase should be made. In rock cuts, the rock surface of the roadbed should be made reasonably smooth, after which the subbase specified for it should be placed in layers and thoroughly compacted. In earth cuts, the subgrade should be shaped according to the plans, but any plowing or other disturbance of material below subgrade should be avoided, unless necessary for some definite reason.

Formation of Shoulders. The road design specifies the width and cross slope of shoulders and the material of which they are to be made. Whatever the material, the shoulders should be carefully formed, true to the dimensions called for, and thoroughly compacted. If subbase is extended across the shoulders to the ditch line, the surface of the foundation material on which the subbase rests should be smoothed and compacted so that water in the subbase will drain off properly and not sink into the underlying material If the shoulders are to receive a bituminous treatment, this will ordinarily be applied after the pavement is in place.

Constructing Guard Rail. Guard-rail posts are set after the pavement is in place, and either before or after final trimming and finishing of slopes, etc. Where wood posts are used, holes are dug on required lines and centers and to specified depth. The bottom of each hole should be thoroughly rammed so that the post will have a stable foundation. Pressure-treated wood posts should not be cut after being set; hence it is important to dig holes to exact depth so that the post tops will lie at an even grade. If they are irregular in height, a very unpleasing appearance results. Any rock encountered in digging the hole should be removed, to form a hole of adequate size for the post to be firmly held in a vertical position. Posts are set with the large end down, and should be carefully plumbed. Posts should be segregated as to size so that adjacent posts will not have markedly different diameters. Post holes should be carefully backfilled, and thoroughly tamped to hold the posts in true positions.

Steel posts are sometimes set in dug holes, but, where the nature of the soil is suitable, they can usually be more economically set by manual or mechanical driving Precautions should be taken to insure their being correctly placed in line, grade, spacing, and plumbness, and to insure their stability.

Anchorages should be carefully buried, backfilled, and tamped to assure the development of their full resistance. Lengths of cable should be accurately cut to be drawn to a proper tautness by the specified adjusting devices.

Finishing Slopes. *Final Shaping.* If it was not done at the time of excavation, tops of all cut slopes should be rounded to form a smooth, gradual transition to the natural ground surface. Similarly, at the intersection of cuts and fills, slopes should be adjusted to flow into each other or into the natural ground surface. Stump holes and other depressions outside neat slope lines but within the right-of-way should be filled and leveled. Rock slopes should be reexamined for removal of all loosened and hazardous fragments.

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[4] "Subgrade," accurately defined, means the *surface* of that part of the roadbed on which the pavement is placed The term is often used, however, to include the volume immediately under that surface

Ditches should be neatly trimmed to straight lines and smooth surfaces that will present a pleasing appearance and provide for unimpeded flow of water. During excavation of cuts and construction of embankments, every effort should be made to construct and maintain side slopes close to proposed final lines and planes. Nevertheless, some final shaping and trimming will usually have to be done after rough grading is completed.

Placing Loam. Loam removed and stockpiled in grading operations, or occasionally secured from outside sources, should be spread over finished cut-and-fill slopes and over median strips of divided roadways As noted earlier, slopes, when graded, should not have been raked perfectly smooth, but should have been left with a slightly rough surface for better bonding of the loam. Slopes of rock fills should have been choked and covered with an earth blanket to serve as a base for loam. The loam itself should be spread to a thickness of 3 or more inches, depending on the character of underlying material and on the amount of loam available. After the loam has been spread, all heavy clods, large stones, brush, roots or other trash should be raked out of it and removed from the roadway area. Whatever procedure has been specified—seeding, mulching, strip sodding, etc.—should then be carried out, due attention being paid to the fact that seeding can be carried out effectively only at proper seasons.

Final Cleanup. The entire area within highway limits should finally be cleaned of all debris resulting from construction. Areas of right-of-way that were not subject to construction operations should be cleaned of all dead trees, branches, etc., and brought to a condition comparable to that of adjoining areas. In naturally rustic regions, this trimming of the right-of-way should not be carried to the point of producing an artificial appearance.

13. BASE COURSES AND SURFACE COURSES

General Considerations

Base Course. The base course is a layer of material placed between the surface course and the subgrade to provide uniform support for the surface course and to distribute unit pressures from wheel loads, reducing them to an intensity that can be applied to the subgrade (including subbase, where used) without overtaxing its bearing power. This purpose could be accomplished (and often is) by using a sufficiently thick surface course alone. It is often more economical, however, to use a base course which will produce the same load distribution with less expensive materials than those forming the wear-resistant, smooth, and watertight surface course. Base courses ordinarily range from 3 to 12 in. in thickness, the majority being in the range of 6 to 9 in.

Surface Course. Except for the lowest type of road, made simply by grading and draining the natural ground, every road has a *surface course,* made of materials brought to and placed on the roadbed. This is the wearing surface with which the wheels of vehicles are in actual contact. Its principal purpose is to resist destructive abrasion of the road by traffic. Additional purposes (which may be present in varying degrees) are (1) to prevent or minimize penetration of surface water into the roadbed; (2) to provide a suitably smooth and dustless surface for traffic; (3) to assist in distributing wheel loads to the roadbed. Surface courses vary in thickness from 1 to 10 in. or more, according to type. The surface course may be placed on a base course, on a layer of subbase, or directly on the general foundation material. Surface courses are composed of a wide variety of substances, ranging from low-cost, locally available soils to bituminous or portland cement concrete.

Both surface and base courses usually consist of (1) weight-bearing elements; (2) binding elements; and (3) void-filling elements. The weight-bearing elements are granular materials, such as stone or gravel. Binding elements include clay, bituminous materials, cement, chlorides, and other substances. Void fillers may consist of cement, sand, stone dust, silt, or other minerals. Fillers may have binding qualities and functions also.

New Construction, Stage Construction, Reconstruction. In the construction of the wearing surface of a road, three conditions are met. (1) The road may be a piece of new construction which in one continuous series of operations is to be brought to a finished state. (2) It may be a piece of new construction which, however, is to be brought to a temporarily finished state with the expectation that, at a comparatively early future date, it will be further improved. This is called stage construction. In this case, what is initially the surface course will become the eventual base course, which will then receive a new surface. (3) Finally, the construction may entail the resurfacing of an existing road, in which case the old road surface (repaired as necessary) becomes the base for the new surface course.

Selection of Types of Base and Surface Courses. The best types and combination of base and surface courses to use in a particular case will be determined principally by (1) the volume and character of traffic, (2) the availability of funds, (3) the local availability of the various road-building materials, (4) climatic conditions, (5) the nature of subgrade and drainage conditions. Fortunately, there are almost innumerable combinations of different kinds and sizes of aggregates that will give satisfactory results, with various kinds of binders, so that full advantage can usually be taken of locally available materials. The fundamental basis of selection is the traffic to be served. Roads carrying up to 400 vehicles per day may be considered low traffic volume roads; from 400 to 1000 vehicles per day, medium traffic volume; over 1000 vehicles per day, high traffic volume roads. Certain types of surface and base courses are generally considered suitable for each volume classification, as outlined hereafter.

Foundation. No base or surface course can long conceal a defect in the foundation that supports it. In new construction, assuming the roadbed to have been properly built *and properly drained*, with provision of subbase where warranted, it is important that the foundation be accurately brought to the proper cross section, and the subgrade made as smooth and hard as practicable. If considerable time has elapsed between construction of the foundation and commencement of work on the upper courses, it may be necessary to add material to settled areas, to cut down high spots, and to scarify, reshape, and recompact the subgrade. Over swamps or at approaches to bridges, if paving follows closely after formation of the embankment it may be desirable to lay a temporary pavement over the questionable foundation, to be replaced when stability has been attained In surfacing an existing road, it will sometimes be desirable to scarify the old surface. A part of the old material can often be salvaged for use in the new surface course. The remainder is reshaped to form a base on which to place the surface course. In other cases, the old surface is retained as fully as possible, weak spots being carefully dug out and strengthened, holes and depressions being filled and, if necessary, an excessive crown or defective banking being corrected by the provision of wedge courses of base material, before the new surface course is laid.

14. MATERIALS FOR BASE COURSES AND SURFACE COURSES

Load-Bearing Aggregates

These include gravel and crushed stone, which are in practically universal use; and in addition, many materials of local occurrence, which, although often unequal to the requirements of the highest type roads, may serve quite adequately for roads of lesser importance.

Stone. The several varieties of igneous rock commonly called *traprock* are the most satisfactory load-bearing aggregate. Granite, limestone, and dolomite are also often satisfactory, but the quality of different deposits varies widely and they should be carefully tested before acceptance. Sandstones are less satisfactory, especially for wearing courses. Slates and shales are of little value, either for wearing or base courses. The most desirable qualities in stone are hardness and toughness, which give wear resistance. When used with bitumens, it is also important that the stone be not hydrophilic; that is, the stone should not have a greater affinity for water than for bitumen. Aggregate should be angular in fracture for better stability, and should be free from an excess (more than about 5%) of flat, thin, or elongated particles. It should also, of course, be free from vegetable and other deleterious matter (including adherent films of clay on stone that is to be used for bituminous or cement concrete courses). Stone should be tested for resistance to wear and for soundness (resistance to disintegration from effects of weather). Resistance to wear is most often determined by the Los Angeles rattler test, AASHO Method T-96. Soundness is determined by AASHO Method T-104, using sodium or magnesium sulfate solution. The proper gradation of stone and other load-bearing aggregate is very important. Some voids are necessary to provide space for binder, but they should not be too great or they may not be filled. This will allow the entrance and retention of water, with resultant damage; or it may permit further compaction and lateral movement of the aggregate under traffic, impairing the riding qualities of the road and perhaps disturbing the set of the binder and causing raveling or other failures. The most suitable gradings of aggregates vary according to the type of construction. Gradings representative of common good practice are given in discussions of the different road types. AASHO Specification M-43 designates a number of standard "sizes" of stone (and other coarse aggregates) and gives the maximum range of particles within each size (Fig. 27). The specific gravity of stone varies not only with the kind of stone but also

Size Number	Nominal Size Square Openings	Amounts Finer than Each Laboratory Sieve (Square Openings), Per Cent by Weight													
		4 in.	3 1/2 in.	3 in.	2 1/2 in.	2 in.	1 1/2 in	1 in	3/4 in.	1/2 in.	3/8 in.	No. 4	No. 8	No 16	No. 100
1	3 1/2 to 1 1/2 in.	100	90 to 100		25 to 60		0 to 15		0 to 5						
2	2 1/2 to 1 1/2 in.			100	90 to 100	35 to 70	0 to 15		0 to 5						
24	2 1/2 to 3/4 in.			100	90 to 100		25 to 60		0 to 10	0 to 5					
3	2 to 1 in.				100	90 to 100	35 to 70	0 to 5		0 to 5					
357	2 in. to No. 4				100	95 to 100		35 to 70		10 to 30		0 to 5			
4	1 1/2 to 3/4 in.					100	90 to 100	20 to 55	0 to 15		0 to 5				
467	1 1/2 in. to No. 4					100	95 to 100		35 to 70		10 to 30	0 to 5			
56	1 to 3/8 in.						100	90 to 100	40 to 75	15 to 35	0 to 15	0 to 5			
57	1 in to No 4						100	90 to 100		25 to 60		0 to 10	0 to 5		
6	3/4 to 3/8 in.							100	90 to 100	20 to 55	0 to 15	0 to 5			
67	3/4 in. to No 4							100	90 to 100		20 to 55	0 to 10	0 to 5		
68	3/4 in. to No. 8							100	90 to 100		30 to 65	5 to 25	0 to 5		
7	1/2 in. to No. 4								100	90 to 100	40 to 70	0 to 15	0 to 5		
78	1/2 in. to No. 8								100	90 to 100	40 to 75	5 to 25	0 to 5		
8	3/8 in. to No. 8									100	85 to 100	10 to 30	0 to 10		
9	No. 4 to No. 16										100	85 to 100	10 to 40	0 to 10	
10	No. 4 to 0 *										100	85 to 100			10 to 30

* Screenings

FIG. 27. Standard sizes of coarse aggregate, adapted from AASHO specification M-43.

with the particular source of stone of any variety. The weight of broken stone, per cubic yard, varies both with the specific gravity and with the percentage of voids. Hence, the figures in Table 1 are quite general, but they may be used in rough computation of stone tonnages required in road construction.

Table 1. Broken Stone

Pounds per Cubic Yard

Kind	Specific Gravity	Loose Spread 45% Voids	Compacted 30% Voids
Trap	2.8	2590	3300
	2.9	2680	3420
	3.0	2770	3540
	3.1	2870	3650
Granite	2.6	2400	3060
	2.7	2500	3180
	2.8	2590	3300
Limestone	2.6	2400	3060
	2.7	2500	3180
	2.8	2590	3300
Sandstone	2.4	2220	2830
	2.5	2310	2940
	2.6	2400	3060
	2.7	2500	3180

Gravel. Gravel is found both in glacial deposits and in water-formed deposits in the beds of bygone streams and lakes as well as in existing waters. Because of differences both in the original stone from which the gravel was derived and in the way in which the material was deposited, the quality of gravels varies widely. Glacial gravel often has better binding qualities than river gravel, which tends to be loose and sandy. Gravel is used in three forms: bank-run, screened, and crushed. *Bank-run* gravels vary widely in their gradation, according to the deposit from which they are drawn. They may often be used, however, without further processing. *Screening* is resorted to either to get rid of a large proportion of pieces of excessive size (occasional large pieces can be raked out in the pit or on the road without screening) or to eliminate an excessive proportion of sand. *Crushed gravel* is obtained by running the pit material through a stone crusher and screening the product into graded sizes. Crushing gravel has the advantage of producing angular in place of rounded fragments. These produce a better bond and greater stability. In addition, crushing permits the use of oversize pieces and will generally be economical where oversize pieces constitute more than 20% of the pit output. Gravel should have good degrees of wear resistance and soundness, and is ordinarily subjected to the same tests as stone (AASHO T-96 and T-104). The desirable gradation differs according to the purpose for which the gravel is used, and will be considered in connection with different road surfaces.

Slag. Slag should be blast-furnace slag, air-cooled, with a weight of 60 (preferably 70) lb per cu ft, or more. It should be reasonably uniform in density and quality, and contain few glassy fragments. It is tested for wear resistance and soundness, using the methods heretofore given for stone and gravel, and is graded according to the particular purpose for which it is used, as described later.

Selected Soil Materials. Low-cost roads must be made of materials available nearby and capable of being used without expensive processing. Fortunately, most areas have materials which, alone or in combination, will adequately carry low traffic volumes. Such materials include (besides stone and gravel) sand, clay, cinders (including products of spontaneous combustion in coal-mine dumps), caliche, marl and other soft lime deposits, scotia, shell, iron ore, etc. Road construction with all these materials (see p. 4-77) is of a very simple sort. Quarrying, crushing, and screening are not ordinarily resorted to. The material is simply dug from the pit, hauled, spread, shaped, and compacted, sometimes with the addition of binder material and water. These materials are more satisfactory for bases, since all lack hardness and wear resistance, and are subject to drying out and being blown away as dust unless the moisture content is controlled. When used as surface courses, unless traffic is very light, they should be surface treated with chlorides or light road oils.

Binders

Binders exert their stabilizing effects in various ways. Sometimes, as in the case of granite sand or traprock screenings, the action is largely mechanical. In limestone screenings, chemical action is also present. The chlorides, calcium and sodium, act principally by fostering moisture conditions conducive to stability. Calcium chloride is more widely used, and in most climates is the more effective However, the binding value of the most useful binders—bituminous materials and portland cement—is due to cohesion and adhesion.

Bituminous Materials. Bituminous materials include asphalts and tars. All **asphalts** are products of the distillation of petroleum crude oil. The distillation may have been natural or artificial. In the United States, asphalts distilled artificially are commonly used, although *natural asphalts* chiefly from Trinidad and Venezuela, are sometimes employed

Rock Asphalt (from deposits of limestone or sandstone impregnated with natural asphalt) has a limited use, principally near the areas of its occurrence.

Artificial asphalt may have either an asphaltic or a paraffin base. The former is preferable, because of its superior stability and adhesiveness. For road construction, asphalt is used in various forms, as follows.

Asphalt Cements These are semisolid residues remaining after the volatile components of petroleum oils have been removed by distillation. They are mixtures of hard asphalt and non-volatile oils. They are brought to a usable consistency by heating, without being softened with a fluxing or emulsifying agent. They are designated by their "penetration"—by which is meant the distance to which the cement is penetrated by a needle in a standard test (AASHO T-49).

Slow-Curing Liquid Asphalts or Road Oils are produced by fluxing an asphalt cement with an asphaltic residual oil of low viscosity or with some other light *non-volatile* oil. Seven grades, designated SC-0 to SC-6, are made. Asphalt content and viscosity increase as the designating number increases. SC-0 is a highly penetrating road oil—a "dust-layer" with very little binding quality, which can be used at normal atmospheric temperatures. SC-6 has strong binding properties and must be heated above 300°F to be usable.

Cutback Asphalts are asphalt cements that have been liquefied by fluxing them with a light *volatile* petroleum substance that was removed in the distillation process. There are two series of cutbacks, designated as medium curing (MC) and rapid curing (RC). The respective fluxes are kerosene and gasoline or naphtha. Each series has six grades, numbered 0 to 5, the asphalt content and viscosity increasing with the grade number. After placement, the flux evaporates from the cutbacks, leaving the semisolid asphalt cement as the binding agent.

Emulsified Asphalts are produced by combining heated asphalt with water by intense agitation in the presence of an emulsifying agent. Asphalt particles of colloidal size are dispersed in the water. Emulsions are brown liquids, turning black when applied as the asphalt coagulates on the aggregate particles while the water flows away or evaporates. Emulsions are divided into three classes, according to their stability. These are (1) the penetration or quick-breaking type, used where no manipulation of the aggregates is involved; (2) the coarse-aggregate mixing or medium-setting type, used for mixing with aggregate practically all of which is retained on a No. 10 sieve; (3) the fine-aggregate mixing or slow-setting type, used in densely graded mixes with considerable filler. The emulsified asphalts are applied without heating, and can be worked on the road in cool and damp (not cold and wet) weather. They have low viscosity and will penetrate deeply.

The American Association of State Highway Officials has prepared a specification for each of the several types of asphalts, and has established a number of standard tests for the determination of the specified characteristics.[1] The American Society for Testing Materials has also established standard tests.[2] In general the tests prescribed by the two agencies are identical.

Types and grades of asphaltic materials suitable for the several types of surface and base courses are given hereafter, in the presentation of the different road types.

Tars are produced from coal (coal and coke-oven tar) or oil vapors (water-gas tar). In highway work tars have their principal use as prime coats or tack coats (p. 4-76) or for surface treatments (p 4-100) with light cover, where they are favored because they produce

[1] See "Standard Specifications for Highway Materials and Methods of Sampling and Testing," American Association of State Highway Officials, National Press Building, 14th & F Streets, N W., Washington 5, D C.

[2] Compiled in a Book of ASTM Standards Those relating to road materials are currently in Part 3 of the book (obtainable separately)

a less slippery surface than asphalt. They are considerably less used in road construction than are asphalts. Tars are prepared in 14 grades, designated RT-1 to RT-12, and RTCB-5 and RTCB-6 (see AASHO Specification M-52). Grades RT-1 to RT-12 are residual products; the CB grades, cutbacks The viscosity increases with the grade number. The low-numbered tars are very light, penetrating oils, and can be applied cold. At the other end of the scale are heavy materials, requiring heat.

Recommended bituminous materials for each type of base and surface course are given hereafter. Table 2 gives recommendations of the AASHO for application temperatures

TABLE 2

	Temperature, °F			
	Spraying		Mixing	
Asphaltic Material	Min.	Max	Min.	Max
A.C. 200–300 Pen.	275	350	200	275
A C all others	275	350	275	325
RC-0	50	120	50	120
RC-1	80	150	80	125
RC-2	100	175	80	150
RC-3	150	200	125	175
RC-4	175	250	150	200
RC-5	200	275	175	225
MC-0	50	120	50	120
MC-1	80	150	80	150
MC-2	100	200	100	200
MC-3	175	250	150	200
MC-4	200	275	175	225
MC-5	225	275	200	250
SC-0	50	120	50	120
SC-1	80	200	80	200
SC-2	150	200	150	200
SC-3	175	250	175	250
SC-4	175	250	175	250
SC-5	200	275	200	275
SC-6	300	400	300	400
Emulsified asphalts	50	140	50	140
Tars				
RT-1	60	125		
RT-2	60	125		
RT-3	100	150		
RT-4	100	150		
RT-5	125	150		
RT-6	125	150		
RT-7	170	225		
RT-8	170	225		
RT-9	170	225		
RT-10	200	250		
RT-11	210	250		
RT-12	225	260		

of asphalts and tars. The *maximum* temperature is the more important figure since, if it is exceeded, the material is apt to suffer severe damage. Each material, generally speaking, should be used at the lowest temperature at which it can be satisfactorily worked

Bituminous materials vary in specific gravity and consequently in volume as their temperature changes They are ordinarily purchased at a price per gallon based on a temperature of 60°F They will usually be delivered in tank cars or tank trucks at a temperature other than 60° The volume at the delivery temperature is corrected to 60° in order to determine the gallonage to be paid for. Convenient correction tables are available.[3]

Portland Cement. Portland cement for use in concrete pavements should comply with AASHO Specification M-85. A number of types of cement with different specialized qualities are included in this specification Types I and II, I-A and II-A are the most widely used in highway pavements In regions subject to alternations of freezing and

[3] "Policy on Maintenance of Roadway Surfaces" (AASHO 1948).

thawing weather, and especially where chlorides are used on pavements to facilitate travel under winter conditions, scaling of concrete pavements has been a serious problem. To control it and add to the durability of concrete, materials (AASHO Specification M-149) have been developed that cause the entrainment of air in concrete while it is being mixed. The most widely used of these are sold under the names of Vinsol resin and Darex AEA, respectively. Cements of types I-A and II-A have such materials ground with them during manufacture, and consequently are called "air-entraining" cements (AASHO Specification M-134). Alternatively, air-entraining concrete can be secured by using Type I or Type II cement and adding the air-entraining agent at the mixer. Increase in concrete volume due to air entrainment should be between 3 and 6%. A smaller increase than 3% will not provide the desired additional durability; a larger increase than 6% will reduce the strength of the concrete without an offsetting increase in durability. Intergrinding the air-entraining agent with the cement in its manufacture provides the simplest means of using air entrainment and is now the prevailing method. It may not always, however, produce the right amount of entrainment, since this is affected somewhat by conditions apart from the cement used. In such cases, an adjustment must be made at the mixer, either by adding more of the air-entraining agent to increase the amount of entrained air, or by replacing a part of the air-entraining cement with other cement to lessen entrainment.

The proper gradation and proportioning of aggregates and cement, and the amount of mixing water to be used in making concrete for pavements are treated later in this section, in the discussion of cement concrete pavements.

Reinforcing Steel. Reinforcing steel may be billet-steel bars conforming to AASHO Specification M-31, rail-steel bars conforming to Specification M-42, axle-steel bars conforming to Specification M-53, fabricated bar or rod reinforcement conforming to Specification M-54, or welded wire fabric reinforcement conforming to Specification M-55.

Joint-Filling Materials. For filling expansion joints (p. 4-87) in concrete pavements, either premolded or poured types of fillers may be used. Premolded fillers are made of bituminous materials, cork, rubber, redwood, and other materials and combinations of materials. AASHO Specification M-33 applies to bituminous types and Specification M-153 to non-extruding and resilient types. Poured fillers usually consist of asphalt (AASHO Specification M-18), or of asphalt with a mineral filler (AASHO Specification M-89).

15. PRIME COATS, TACK COATS, AND SEAL COATS

Prime Coat. A prime coat does not constitute either a base course or a surface course, but may be used as a preparation for the construction of either one. Its purpose is to penetrate the foundation or base course on which some sort of bituminous mat is to be placed later. It should fill the pores, bond together any loose material, and in general produce a firm impervious layer to serve as a foundation for the material to be superimposed, and to bond with it. Since the purpose is to penetrate and seal a porous surface, prime coats are not used on bases of portland cement concrete or of tight bituminous mixtures that cannot be readily penetrated, and on which a *tack coat* will be used instead. The following materials are commonly used for prime coats: SC-1 and SC-2, MC-0, MC-1 and MC-2; RT-1, RT-2 and RT-3.

Choice between asphalt and tar depends largely on relative cost. The more porous the base and the cooler the weather, the lower should be the viscosity. Conversely, if too much material is being absorbed, a more viscous grade should be used. The amount applied should ordinarily be absorbed in 24 hr. It may vary from $^1/_{10}$ to $^1/_2$ gal per sq yd. The larger amounts will usually have to be put on in two applications to give time for absorption before any material runs off the road surface. Traffic should not be allowed on the primed surface for at least 24 hr. When curing is complete, any spots showing surplus bituminous material should be blotted with sand, to any spots still showing porosity, a little more material may be applied.

Tack Coat. Whereas the purpose of a prime coat is to reduce the porosity and so to stabilize and improve a foundation or base course, a *tack coat* is designed to establish a bond between a highly stable and impervious foundation, such as an existing pavement or a new concrete base, and the superimposed mat which must be made to adhere to it. Thus while both prime and tack coats are films of bituminous material, the purpose and resulting character of the two films are quite different. Materials commonly used for tack coats include: RC-2, RC-3, and RC-4; emulsified asphalt; asphalt cement of 150 to 200 penetration; RT-4 to RT-9, inclusive.

The amount of bituminous material is usually from $^1/_{10}$ to $^1/_4$ gal per sq yd. The tack coat should precede placing of the superimposed mat only long enough to allow it to become "tacky." Care must be taken not to cover so large an area at one time that the tack

coat will cool and harden before the mat is placed on it. Care must also be taken to guard against applying so much material per square yard that it will unduly enrich the superimposed course and lessen its stability.

Seal Coat. A seal coat is a bituminous application, usually with a small stone or sand cover (although, rarely, with no cover at all) placed on a bituminous surface course for the primary purpose of sealing it against the intrusion of water. Frequently, subsidiary purposes, such as provision of a better riding surface, may also be accomplished. Since many types of bituminous surface courses require sealing, application of a seal coat is a frequent final step in the construction of a black-top road. Seal coats are also used extensively in maintenance to forestall the disintegration of a road surface or deterioration of a road because of percolation of water through the pavement. In this use, they merge into surface treatments (p. 4-100).

In applying seal coats, suitable relationships must be maintained between the type and viscosity of bituminous material, the amount of bituminous material, and the grading and amount of mineral cover. The materials for a suitable seal coat, and details of their application, are discussed under the particular road surface to be sealed.

16. LOW TRAFFIC VOLUME ROADS

Natural-Soil Roads. For the lowest traffic count road, the natural soil from the sides of the road is bladed to the center to form a crown with a ditch along each side to which water sheds. This is the lowest form of road surface used, and is the first stage in the development of a road which is to be further developed as increasing traffic requires. The surface obtained is generally dusty and ruts are quickly formed, destroying the crown so that water pockets are developed on the surface.

Natural Soil, Treated. A natural-soil, bladed road can be improved considerably by an oil surface treatment, as follows: Immediately after the road has been brought to a good grade and cross section, 1/2 gal per sq yd of bituminous material such as SC-0, SC-1, or MC-0 (p. 4-74) is applied by a pressure distributor directly to the surface and allowed to penetrate without any cover. As soon as this application has soaked in, a second application of the same material in the amount of 1/4 to 1/3 gal per sq yd is applied and is also left to be absorbed without cover. This will take more time to absorb than did the first application. The surface should be watched and additional applications of 1/8 to 1/4 gal per sq yd placed on the road from time to time as needed. This is all that is done, except to see that the surface is maintained by patching, as necessary. Patching is done with a mix of the natural road material with the same kind of bituminous material that was used on the road. This mix is made up and stockpiled, to be used as required.

Instead of completing the construction with a bituminous application, calcium chloride can be used. About 1 lb per sq yd is applied initially, with the treatment repeated as found necessary.

Roads of Other Local Materials. Where suitable and easily processed local materials (such as the soft limestones *coquina*, *caliche*, and *marl* found in some of the southern and southwestern states, *scoria* in Montana and North Dakota, *cinder* from mine dumps in bituminous coal regions, *shell* along the Gulf Coast) are available, a layer of such material 6 in. to 12 in. thick is often spread on the natural soil of the roadbed, compacted either by rolling or by traffic, and shaped with graders to a suitable cross slope (1/2 in. to 3/4 in. per ft). The material is placed on the full width of the roadbed, feathering out on the shoulders. Sometimes such material is used alternately with the natural soil surface, as it occurs in the roadbed. The natural surface is used where soil and drainage conditions are good. When sections of poorer soil are encountered, and where drainage is poor, the better material is hauled in and spread. All these materials are low in wear resistance, become dusty and rut under traffic. They may be much improved by the light oil or chloride treatment outlined above. They are also sometimes covered with a heavier treatment of a harder crushed stone or gravel to add to their wear resistance.

Sand-Clay Roads. Although neither sand nor clay alone forms a stable road surface, a mixture of the two materials, in proper proportions, is much more satisfactory. From 10 to 20% of silt, with 5 to 10% of clay mixed with 70 to 85% of sand is a satisfactory mixture, the exact proportions being determined by experiment during the construction (see AASHO Specification M-147). The use of a disc, spring-toothed, or spike harrow is satisfactory for mixing the sand and clay on the road. A blade scraper can be used to form a cross slope of 1/2 to 3/4 in. per ft. In dry weather, the use of water from a sprinkler cart will aid in "bringing down" and compacting the mixture. The use of calcium chloride at the rate of 1 lb per sq yd will help in keeping the surface in proper condition. Additional applications of chloride at the rate of 1/2 lb per sq yd may be needed at intervals.

Gravel Roads. Gravel roads are built for permanent use as such. They are also frequently built as the first stage in a stage construction program that will eventually see the gravel road converted into a base for some higher type of surface course.

Bank-Run Gravel Roads. A subgrade is prepared on the road with a cross slope of $^1/_2$ to $^3/_4$ in. per ft. This is done with a grading machine. Soft foundation material is removed and replaced with suitable material. Raised shoulders of the natural side-slope soil may be placed on each side of the subgrade to hold the gravel, if desired, or the gravel surface material can be feathered out at the edge. Material from the gravel pit should not contain more than 50% passing a $^1/_4$-in. screen, and should be free from deleterious material such as an excess of clay, loam, or vegetable matter. A uniform layer of gravel 6 to 8 in. deep is placed, and is well harrowed with a spring or spike harrow. Any stone over 4 in. in size appearing on the surface is hooked out with a potato hook and the surface is then dragged with a road drag or is smoothed with a 4-wheel grader. As traffic compacts the road, the grader or drag is used to keep ruts from forming until the surface is uniformly compacted and smoothed. It is a good practice to start the spreading of the gravel at the end of the road nearest the source of supply so that the material is trucked over as the work progresses. The use of a road roller and a watering cart is desirable. When the first course has the proper crown and is compacted and smooth, a second course of the same material from 4 to 6 in. deep is placed on the first course and is finished in the same manner. Calcium chloride or bituminous material should then be applied to the surface to prevent its raveling.

Screened-Gravel Roads. For screened-gravel roads, gravel from a pit is screened to obtain three classes of material: "sand," passing through a $^1/_2$-in. square; "gravel aggregate" retained on the $^1/_2$-in. screen but passing through a 2 $^1/_2$-in. screen; "tailings" retained on the 2 $^1/_2$-in. screen. The tailings are either discarded or used as foundation material. The subgrade is prepared to the desired cross section, with shoulders made of satisfactory side-cut material. The subgrade is compacted with a 10- to 15-ton power roller, and soft spots are dug out and replaced. The gravel aggregate is then spread uniformly to a depth of 5 to 6 in., preferably from a box stone spreader. The gravel is then rolled, while sand from the screening plant is spread dry over the surface until the voids in the gravel are filled and the surface is well compacted. During the rolling, water is flushed over the surface in sufficient quantity to harden and set up the surface, but not to the point that the subgrade will be softened. As soon as a smooth, well-compacted surface is obtained, a second uniform layer of 5 to 6 in. of loose gravel is spread over the bottom course, filled with sand, rolled, watered, and compacted as before to obtain the required depth of pavement. As wind and sun will dry out the surface of such a road very quickly, and as traffic will soon tear the surface apart, it must be protected immediately after being built by the application of calcium chloride as mentioned on p. 4-77 for sand-clay roads, or by an application of bituminous material, such as MC-1, MC-2, RT-2, or RT-3. The bituminous material is applied during dry weather at the uniform rate of $^1/_2$ gal per sq yd, using a modern bituminous material spreader. This application should be allowed to penetrate with a minimum amount of sand cover applied. The surface should then be dragged and honed to mix the bituminous material on the surface with any loose sand of the road surface. After the mixture is thoroughly set up and is in a satisfactory condition the surface is sealed, using an application of about $^1/_3$ gal per sq yd of the same material used for the first coat, with a somewhat heavier sand cover. This is then dragged and honed and left to set up.

Crushed-Gravel Roads. For a crushed-gravel surface, material from a gravel pit is run through a stone crusher and then screened in the same manner as for a screened-gravel road. The advantage of the crushed-gravel road over the screened-gravel road is that all the stone in the gravel pit is utilized, and, as there are more angular pieces in the resultant aggregate, there is less difficulty in compacting the road under the roller. The interlocking of the angular aggregate produces a surface more resistant to breaking up under traffic. The construction details are the same as for a screened-gravel road.

Both screened- and crushed-gravel roads are frequently used in stage construction programs. The gravel road, following application of bituminous material, is used until the growth of traffic, with resulting higher maintenance costs, warrants its conversion into a base for a higher type of bituminous surface course.

Slag Roads. In regions where blast-furnace slag is readily available, roads for low traffic volumes may be built of it using the methods outlined for crushed-gravel roads. To be satisfactory, slag should have a per cent of wear not greater than 50 at 500 revolutions (AASHO Method T-96). A typical gradation of the coarse aggregate would require 100% to pass a 2 $^1/_2$-in. square mesh screen, 90–100% to pass a 2-in., 35–70% to pass a 1 $^1/_2$-in., and 0–15% to pass a 1-in. screen. The fine aggregate usually consists of crushed slag also, 100% passing a $^3/_4$-in. screen, 90–100% passing a $^1/_2$-in. and 0–30% passing a No. 100

sieve. Immediately following completion of the slag road, it should receive two applications of bituminous material, as described for screened-gravel roads.

As in the case of gravel, slag roads may be built either for permanent use as completed roads, or with the intention of later converting them to road bases.

Waterbound Macadam Roads.[1] Waterbound macadam roads are generally built as part of a stage construction program—to be given a bituminous application and used to carry low-count traffic for a number of years; then to be used as the base for a higher type surface when traffic of medium volume calls for a better pavement.

The road generally consists of two courses of waterbound macadam constructed on a well-prepared subgrade. The coarse agreegate should be crushed stone of uniform quality, free from deleterious material. It should not break excessively under rolling. Traprock, hard limestone, dolomite, and granite are suitable. AASHO Method T-96, using the Los Angeles rattler test, should not show a loss over 50%, if possible, and a smaller loss (40% or less) is desirable. A good grading for the coarse aggregate is 100% passing a 2 1/4 in. square screen, 90–100% passing 2-in., 30–55% passing 1 1/2-in., 0–15% passing 1 1/4-in., 0–5% passing 1-in.[2] The filler can either be the crushed stone dust product of the crusher or a clean sand. The stone dust is preferable. All the filler should pass through a 5/8-in. square screen and at least 70% be retained on a No. 50 sieve. The method of construction is as outlined for screened-gravel roads except that, because of the cohesiveness or stickiness of stone dust, this material must be worked into the voids of the coarse aggregate by use of hand brooms, broom drags, or brooms attached to the roller. Water is used until the voids in the aggregate are filled, and a slight wave of stone-and-water grout is pushed ahead of the roller. The second course is placed and handled in the same manner as the first course. As soon as the second course is thoroughly dry, the road surface should be broomed free of excess stone dust. An application of 1/2 gal per sq yd of MC-1, MC-2, RT-2, or RT-3 is made as noted for screened-gravel roads and is allowed to stand so that it will penetrate the voids in the filler and around the stone. A light sand cover is then applied either by hand from trucks or with a sand-distributing machine. The operation must be such that the truck wheels will at no time run over the freshly applied bituminous material. The surface then should be dragged so as to distribute, mix, and smooth the combination of bituminous material and sand. This is followed in fair weather, in about a week, by a second application of 1/3 gal of the same bituminous material. The surface is sanded sufficiently to blot up the bituminous material, and then is dragged.

17. PAVEMENTS FOR INTERMEDIATE TRAFFIC VOLUMES

Pavement Types. It is generally considered that a volume of traffic exceeding about 400 vehicles a day requires better pavement surfaces than those previously discussed. In the intermediate traffic range (400 to 1000) a bituminous pavement, of which there are many types and classifications, is generally employed. The pavement may be of the road-mixed, the plant-mixed, or the bituminous macadam (penetration) type. Also, the road-mixed and plant-mixed pavements may be constructed of dense-graded aggregate or of open-graded aggregate, with some resulting variation in the bituminous materials used, and with marked differences in construction methods.

Selection of Pavement Type. Bituminous macadam is not often used for resurfacing existing roads, either in reconstruction or stage construction programs. Its use is in new construction, especially in regions where a clean aggregate of superior wear characteristics is available. With such aggregate, and when carefully constructed, a very durable and satisfactory road can be produced. Plant mixes and road mixes are used both in new construction and in stage construction and reconstruction. They enjoy a preference over bituminous macadam when a surface course is to be laid over an existing base of ample strength, in that they can be made as thin as 1 1/2 or 2 in. Plant-mixed and open-graded road-mixed surfaces are made of all-new aggregates. Dense-graded road-mixed surfaces, however, frequently are made of aggregate secured by scarifying the base. The dense-graded road mix has particular applicability, therefore, to the reconstruction of an old road which contains an adequate amount of aggregate both for base and surface courses, but which was originally constructed without a bituminous surface or which has a bituminous surface that is worn to the point of requiring renewal. This advantage is of espe-

[1] Sometimes a crushed-stone road is made using a "dry" rather than the waterbound process described here. The same materials are used and the same general methods are followed, except that the materials are *sprinkled* rather than *flushed* with water during construction Better compaction is secured, however, by the flushing process

[2] Other satisfactory gradings are given in AASHO Specification M-77

cial importance in regions where good aggregate is scarce and expensive. The road-mix process is also appropriate when the work to be done is not enough to warrant moving in and setting up a mixing plant in an area where the road net is not dense enough to warrant the existence of permanent central mixing plants.

The *plant*-mix process, however, permits more accurate control both of aggregates and bituminous materials, and therefore normally results in the production of a superior job. It has other advantages in that: (1) It prolongs the construction season, since both the aggregate and bituminous materials can be heated. (2) More viscous grades of bituminous materials can be used. (3) Wet weather, with attendant delays and with danger of inferior results, is a much less serious problem. (4) Traffic is much less inconvenienced. Because of these various considerations, there is a trend towards the plant-mix process, except where aggregate is to be reused or where some other special condition obtains.

A dense-graded mix is generally preferable to an open-graded mix, both in road-mix and plant-mix construction.

Road-Mixed Pavements. The road-mixed pavement is a surface course of 1½ to 3 in. compacted thickness, which is placed on a base course of gravel, slag, dry stone, or water-bound macadam. The base may have served previously as a road, or it may be newly built. In the latter case, it should have a thickness of 6 to 12 in., depending on the quality of subbase and foundation and on the anticipated traffic, as well as on the thickness decided on for the surface course. The base is built using materials and methods as heretofore outlined for low traffic roads.[1] In every case where the base is not newly constructed, preliminary work on the base may be necessary before the surface is applied. If the base is too narrow, it is widened with stone or gravel placed to an adequate thickness and thoroughly compacted. Weak spots in the subgrade are strengthened. If the old base is stable and solid and of reasonably good cross section, it may simply be repaired where needed, and the cross section and profile trued up to the extent necessary. Often, however, it is desirable to scarify the old base, either because of its poor condition, or to secure from it part or all of the aggregate needed for the new surface.

Dense-Graded Type. For this type, aggregate secured from the old surface will normally be used, though it may be supplemented with new aggregate or occasionally all new aggregate may be used. The gradings shown in Table 1 are often used for new aggregate. It is desirable, where possible, that the old aggregate or a mixture of old and new aggregate should conform approximately to them also.

Table 1

Sieve Designation	Percentage by Weight Passing Square Mesh Sieves				
	Grading *A*	Grading *B*	Grading *C*	Grading *D*	Grading *E*
1 in.	100	100	100	100	100
3/4 in.	75–100	75–100	85–100	85–100	85–100
No. 4	35–45	40–60	45–65	50–70	60–95
No. 10	20–35	25–45	30–50	35–55	45–80
No. 200	2–7	3–8	5–10	5–12	5–15

The aggregate should be graded so that at least 10% of the total will pass a No. 4 sieve and be retained on a No. 10 sieve. Per cent of wear for the portion retained on the No. 4 sieve should not exceed 50. Plasticity index of the portion passing the No. 40 sieve should not exceed 6.

The bituminous material used may be one of the following:

Cutback asphalts MC-2, MC-3, or MC-4.
Emulsified asphalt, slow-setting type.
Tars RT-6, RT-7, or RT-8.

The bitumen content should be between 3 ½ and 7% of the weight of the dry aggregate. Both the type and amount of bituminous material must be adjusted to the aggregate used on the basis of preliminary laboratory and field tests. The greater the proportion of fines, the less viscous should be the bituminous material and the greater will be the amount required. The material should be applied at temperatures shown on p. 4-75.

The old road surface is lightly scarified and is then bladed to the correct profile and cross section. It is then scarified again to the depth required. If some new aggregate is to be supplied, it is spread over the loosened material from the road and thoroughly mixed with it. All the loose material is then windrowed along the side of the road, leaving exposed an undisturbed foundation roughly parallel to the proposed finished surface. The foun-

[1] See also AASHO Specification M-75.

dation is then watered, bladed, and rolled until satisfactory, after which it is primed (p. 4-76). When no aggregate is to be secured from the existing road, the surface is repaired, shaped, and compacted as necessary, and then given a prime coat if necessary. The aggregate is measured in the windrow, the volume is computed and the necessary amount of bituminous material is determined. Moisture content of the material should be tested, and if it is as much as 2% the material should be dried and again windrowed.

Mixing is preferably done by a mixing machine. This picks up the material in the windrow, mixes it, and redeposits it in a windrow ready for spreading—a much faster process than blade mixing. When machine mixing is used, a tack coat (p. 4-76) may be desirable if a prime has not been used.

For blade mixing, the aggregate is spread on the foundation and the bituminous material is sprayed on in three approximately equal applications. The material is thoroughly bladed and harrowed after each application until the mix is uniform. Mixing must be continued until the mix is just workable enough for satisfactory spreading. The material is then windrowed. Following either method of mixing, the mix is bladed from the windrow and spread in two operations. The first half is rolled once and then planed with a grader to remove high spots before the second half is spread. The material is then compacted thoroughly, with such blading or dragging as may be needed to give a good surface. No more mixed material should be spread from the windrow than can be compacted during the day. Similarly, all incompletely mixed material should be windrowed before any interruption of the work by night or weather. If the surface, after compaction, is so open as to make sealing desirable, it should be given an application of 1/6 to 1/3 gal/sq yd of RT-10 or RT-11, or of asphalt cement with a penetration of 120–150 or 150–200. This may then be covered with sand, as described for screened gravel roads (p. 4-78); or 10 to 20 lb/sq yd of 1/4-in. stone can be used for cover. The surfacing should not be attempted when atmospheric temperature is less than 50°F, nor unless the weather is clear. A complete specification for this type of road surface is contained in "Specifications for Construction—Bituminous Surfacing," AASHO (1949).

Open-Graded Type. The open-graded type of road mix is much less used than the dense-graded type. Particulars concerning the open-graded type can be found in the AASHO *Manual of Highway Construction Practices and Methods* (1950), and in the specifications named just above.

Plant-Mixed Pavements. *Dense Graded.* When this type of pavement is used in new construction, considerable latitude exists as to the types of bases and the relative thickness of surface and base courses. For best results, no single layer of the plant-mixed material should be more than 2 in. thick, though thicknesses as large as 2 1/2 in. are used. Sometimes a base of water-bound macadam, or of broken stone, slag, or gravel, is constructed with sufficient thickness to support the weight of traffic; the surface course of plant mix then needs simply to be thick enough to resist wear. It may be anything from 1 to 2 in. On the other hand, two or even three courses of plant mix can be laid down, on a correspondingly lighter foundation. Whatever the base under the first course of plant mix, it should be bonded to the plant-mix course. If the base is non-bituminous, it should be well primed with one of the penetrating materials listed on p. 4-76. If the plant mix is laid down on a bituminous base, however, care must be taken to avoid overpriming; otherwise the primer may work up into, and seriously soften, the plant-mix surface. Tar, as a primer, will be less apt to do this than asphalt. In any case, a prime coat should merely be lightly "fogged" onto a bituminous base. Where the base calls for a tack coat rather than a prime coat, it should also be very sparingly applied.

In reconstruction, the thickness of plant mix to be used and the number of courses should depend on the character of the existing base. If the base has required much repair, a leveling course of plant mix under the surface course (each course being about 1 1/2 in. thick) will normally result in a smoother road. A single course, however, is often used. It should ordinarily be at least 2 in. thick.

Many different gradings of aggregates give satisfactory results in dense-graded plant-mix pavements. Table 2 is typical. The gradings shown therein are primarily for surface courses. For base courses, the maximum size may be increased.

The aggregate for all the gradings in Table 2 except *E*, *E*-1, and *E*-2 should consist of coarse crushed gravel or stone, with a filler of finely crushed stone or sand. For gradings *E*, *E*-1, and *E*-2, the aggregate should consist of fine gravel and sand, disintegrated granite, etc. At least 10% of the total should be retained on a No. 10 sieve. A per cent of wear not greater than 50 is desirable. Any of the following bituminous materials may be used with any of the above gradings:

Rapid-curing cutback asphalt RC-3, RC-4, or RC-5.
Medium-curing cutback asphalt MC-4, MC-5.

Slow-curing asphalt SC-6.
Asphalt cement 85–100, 100–120, 120–150, 150–200.

The bituminous material should be from 3 1/2 to 7% of the aggregate, by weight, the exact amount being determined by laboratory and field tests.

Table 2. Requirements for Grading of Aggregate

1-In. Maximum Size

Sieve Designation	Percentage by Weight Passing Square Mesh Sieves				
	Grading *A*	Grading *B*	Grading *C*	Grading *D*	Grading *E*
1 in.	100	100	100	100	100
3/4 in.	75–100	75–100	85–100	85–100	85–100
No. 4	30–45	40–60	45–65	50–70	60–95
No. 10	20–35	25–45	30–50	35–55	45–80
No. 200	2–7	3–8	5–10	5–12	5–15

3/4-In. Maximum Size

Sieve Designation	Percentage by Weight Passing Square Mesh Sieves				
	Grading *A*-1	Grading *B*-1	Grading *C*-1	Grading *D*-1	Grading *E*-1
3/4 in.	100	100	100	100	100
No. 4	35–50	45–65	50–70	55–75	60–95
No. 10	25–45	30–50	35–55	40–60	45–80
No. 200	2–7	3–8	5–10	5–12	5–15

1/2-In. Maximum Size

Sieve Designation	Percentage by Weight Passing Square Mesh Sieves				
	Grading *A*-2	Grading *B*-2	Grading *C*-2	Grading *D*-2	Grading *E*-2
1/2 in.	100	100	100	100	100
No. 4	40–55	50–70	55–75	60–80	65–100
No. 10	30–45	35–55	40–60	45–65	50–85
No. 200	2–7	3–8	5–10	5–12	5–15

The roadbed is repaired as necessary and is swept clean of all foreign materials. A prime or tack coat, as the base may call for, is used as noted above. The mineral aggregate is passed through a rotary drier to reduce the moisture content to 1% or less, and to heat the aggregate to the mixing temperature, which should not exceed the proper temperature for the bituminous material (p. 4-75) by more than 25°. The aggregate is then screened into two or more sizes for proper proportioning, and is binned. It is weighed from the bins into the mixer, where it is combined with the predetermined amount of bituminous material and mixed thoroughly for at least 1/2 min, or more if needed. Either a rotary-drum batch-type mixer, a pug-mill batch-type mixer, or a continuous flowing pug-mill-type mixer may be used. The mixed material is hauled to the job in clean trucks the metal bodies of which have been sprayed with suitable materials to prevent adhesion. The trucks should have insulated bottoms if the length of haul requires, and the mix should be covered for protection from the weather The material should be spread on the road when the atmospheric and ground temperatures are not less than 40°. A combination spreading and finishing machine is desirable; otherwise, blade graders are used for spreading and shaping. When the desired cross section has been obtained, compaction is secured by rolling with rollers of 7 to 10 tons weight. During rolling, the surface is dragged or bladed to secure requisite smoothness.

Where sand of the desired quality is abundant, as in the Cape Cod area of Massachusetts, and in some parts of Delaware and North Carolina, a sand-asphalt base course of 2 1/2 in. thickness and a surface course of the same thickness have been very successfully used. This is simply one particular type of a dense-graded plant mix. The same materials can be used for a road-mixed pavement. Detailed information can be found in the AASHO *Manual of Highway Construction Practices and Methods* (1950).

Open-Graded Type. What has been said with respect to bases and wearing courses of dense-graded plant mixes applies to open-graded types also. A typical grading of aggregates for this type of pavement is shown in Table 3.

Table 3

Sieve Designation	Percentage by Weight Passing Square Mesh Sieves: Primary Course Plant-Mix Aggregate	Choker Aggregate	Cover Aggregate
1 in.	100		
3/4 in.	90–100		
3/8 in.	20–55		100
No 4	0–10	100	75–100
No 8	0–5	85–100	0–10
No. 200	0–2		0–2

Any of the following bituminous materials may be used. The same type is used for the entire sequence of construction.

Rapid-curing asphalt RC-3, RC-4, RC-5.
Slow-curing asphalt SC-6.
Asphalt cement 150–200
Emulsified asphalt, medium-setting type.
Tar RT-7, RT-8, RT-9.

The bituminous material should be from 3 1/2 to 7% of the amount, by weight, of the plant-mixed aggregate, the exact amount to be determined by trial The approximate amount of the other materials, per square yard (in sequence of application), is as follows:

Plant-mixed aggregate, 150–200 lb
Choker aggregate, 5 lb.
Bituminous material for choker, 0.10 gal.
Choker aggregate, 5–10 lb.
Bituminous material for seal, 0.16 to 0.33 gal.
Cover aggregate, 10–20 lb.

The materials for the mixed course are processed, transported, spread, and compacted as in the case of the dense-graded mix. The first choker aggregate is then applied uniformly before the mixture has set, and is broomed into the surface voids. The bituminous material is next applied, and the remaining choker aggregate is spread on it immediately. The road surface is then alternately broomed and rolled until the mixed course and the choker material are thoroughly bonded together. Traffic should be allowed to use the road for at least 2 weeks before the seal is applied.

Plant-mix pavements and bases, either open or dense-graded, can also be laid cold if suitable types and grades of bituminous material are used. They are described in the AASHO *Manual of Highway Construction Practices and Methods* Cold mixes have the advantage that they can be mixed and shipped long distances, and also that they can be stockpiled and used as needed Under other circumstances than these, however, hot-laid mixes are generally preferable.

Bituminous Macadam Pavement. This pavement consists of a course of large aggregate of a fairly uniform size, first penetrated with a bituminous binder, then keyed with a smaller size stone, and finally given a seal coat with a small stone cover Total thickness, after compaction, is at least 2 1/2 in and may be as much as 4 in The pavement is built on a base of crushed gravel or waterbound macadam, as referred to previously (p. 4-78).[2] As in the case of plant-mixed pavements, there is a good deal of latitude in the relative thickness of surface course and base. Aggregate for the pavement may be broken stone, slag, or gravel, and should consist of clean, hard, tough material, free from dirt, crusher dust, and other deleterious substances Tested with magnesium sulfate for soundness (AASHO Test Method T-104), loss should preferably not exceed 5% after 5 cycles, although material with a loss as high as 12% has been successfully used The abrasion loss in the Los Angeles rattler test (AASHO Test Method T-96) should preferably not exceed 25%, although stone showing up to 40% loss has been used. The grading should be such that the larger stone sizes are fairly uniform, while the key or filler stone should fill in and wedge between the large pieces to obtain a mechanical locking action to aid the bond. A successfully used grading of stone is shown in Table 4.[3]

[2] Or see AASHO Specification M-75

[3] For gradings adapted to softer aggregates, see AASHO Specification M-78

Table 4. Total Percentage Passing Square Opening Sieves

Sieve Sizes	Coarse	Key or Choke	Cover
2 1/4 in.	100		
2 in	90–100		
1 1/2 in.	30–55		
1 1/4 in.	0–15		
1 in.	0–5	100	
3/4 in.		90–100	
5/8 in.			100
1/2 in		10–40	90–100
3/8 in		0–20	30–50
No. 4		0–5	0–8
No 8			0–3

The bituminous binders generally used are asphalt cements of 85–100, 100–120, and 120–150 penetration, and tars RT-11 and RT-12 In special cases, an emulsified asphalt may be used Asphalt cement of 85–100 penetration is used in warm climates, the higher penetrations in cooler or colder climates Tars are not as widely used as the asphalts. Asphalt emulsion is useful when it is difficult to keep the aggregate free from moisture, as in fall weather

The subgrade is swept clear of undesirable material The large aggregate is then spread in a uniform layer preferably from stone spreader boxes between boxed out shoulders or forms It is spread to such a depth that after compaction it will have the required thickness A loose layer 3 in. thick should compact to 2 1/2 in. or a little more Immediately after this stone is spread, it is rolled with a power roller weighing 10 tons or more to consolidate, smooth and lock the stone, reducing the voids Rolling should be longitudinal and should commence at the outer edges of the pavement, progressing towards the center in successive overlapping passes to preserve the crown Care must be taken not to continue rolling to the point where the coarse aggregate is cracked to an appreciable extent The bituminous material is then uniformly applied from a pressure distributor at a rate depending on the amount of voids For a 2 1/2-in. pavement, the amount is usually between 1 1/2 and 2 gal/sq yd, for ordinary conditions, 1 3/4 gal/sq yd should be suitable. The bituminous material should not be applied when the temperature is less than 50°, or when the aggregate is wet If asphalt is used for the binder, it should not have been heated above 350° and it should be applied at 300° to 350° If tar is used, it should not have been heated above 250° and it should be applied at 200° to 250°

Dry, clean 3/4-in key stone is immediately spread longitudinally over the bituminous material The stone trucks should be backed, so that the tires do not touch the uncovered asphalt. Spreader boxes are desirable, although the stone may be spread by hand from the truck Approximately 40 lb/sq yd of key stone is ordinarily required. The road is then well rolled a second time More key stone is added and brushed into the voids with hand brooms or broom drags until all voids are well filled and the surface presents a uniform appearance and no further compaction is obtainable

Before application of the seal coat, the surface should again be cleaned of all foreign material, such as the leaves from trees alongside the road The seal coat of 1/3 to 3/4 gal/sq yd is then applied the amount depending on the voids in the keyed surface. The seal coat is immediately covered with a sufficient amount of clean, dry 1/2-in. stone to cover the asphalt Usually about 20 lb/sq yd will be sufficient. The cover stone is thoroughly rolled and broomed until all voids are well sealed and no further compaction is obtainable. The slight excess of 1/2-in. stone left on the road will be swept to the sides of the road by traffic, and may then be salvaged An even tighter and smoother surface may be obtained by making three applications of bituminous material of 1 1/2, 1/2 and 1/4 gal/sq yd The first application is covered with 3/4-in stone, the second and third applications with 1/2-in stone. In this method, care must be taken not to overseal the voids in the second application of bitumen. Otherwise there may be flushing of the asphalt after the road is in use

18. PAVEMENTS FOR HIGH TRAFFIC VOLUMES

Pavement Materials and Types. The materials most used for high traffic volume roads are portland cement concrete and hot-laid asphaltic concrete, with sheet asphalt also in wide use for surfacing city streets. Asphaltic concrete is a combination of an appropriate grade of asphalt cement with a completely graded aggregate, proportioned (as in the case of a cement concrete) so that the fine materials fill the voids in the coarse material and the

bitumen fills the voids in the fine material, creating a very dense mixture. Asphaltic concrete differs from the dense-graded plant mixes heretofore described in the care with which it is proportioned, mixed and shaped—and consequently in the uniformity, quality and cost of the finished product. The aggregates, both coarse and fine, are more closely graded by the use of additional screens. More attention is paid to securing close temperature control in mixing, transporting, and placing the mixture. Frequently, more elaborate equipment is used throughout the entire process, and especially in bringing the finished pavement surface to close tolerances of smoothness. Consistent with the higher cost of the surfacing material, more attention is also paid to the adequacy of the base. Sheet asphalt is akin to asphaltic concrete, but differs in that no coarse aggregate (retained on a ¼-in. screen) is used in it.

In new construction, cement concrete pavement is generally placed directly on the subgrade, without a base course. Asphaltic concrete is customarily laid on a base course, which may be built up of several thicknesses of more than one type, as will be discussed. Sheet asphalt is usually laid on a cement concrete base, with a binder course of asphaltic concrete.

Selection of Pavements. For city streets, both in new construction and resurfacing of main arteries, an asphaltic surface on a cement concrete base is usually satisfactory. Sheet asphalt is widely used. It has a very smooth surface which is quiet and pleasant to ride on. When properly constructed it is durable and requires little maintenance, and it is easy to clean. Although its smooth surface tends to be slippery when wet, at usual city street speeds this is not a serious hazard. Sheet asphalt, however, is more expensive than asphaltic concrete. A pavement of the latter material has most of the good qualities of sheet asphalt in equal or nearly equal measure, and because of this, together with its low cost, its use in urban work is growing. In industrial and other heavy trucking areas of cities, cement concrete pavements are widely used because of confidence in their ability to withstand the impact of heavy traffic without destruction or surface deformation. The stone block pavements formerly used in such areas have declined in popularity because of their unsatisfactory traffic characteristics, and because of a realization that smoother types of pavements can be built to endure hard treatment.

For rural roads carrying large volumes of heavy traffic, concrete is also extensively used for new construction, with satisfactory results. It provides a durable pavement, which does not corrugate, pot-hole, or ravel under severe usage. Its light color gives it good visibility in night driving, and it affords good skid resistance. When the surface has deteriorated to the point of requiring renewal, the concrete slab makes an excellent base for a comparatively inexpensive new surface coat. On the other hand, the joints in a concrete pavement have always been a disadvantageous feature. They require constantly recurring maintenance and unless the subgrade has been very well constructed are apt to lead to uneven settlement of the slabs, impairing the riding qualities of the road and frequently causing cracking of the pavement as well. Where alternations of freezing and thawing weather are frequent, a serious problem of surface scaling may also occur. Asphaltic concrete surfaces eliminate some of the above problems. In recent years, their ability, when placed on a suitable base, to stand up under heavy traffic has been demonstrated and they have gained rapidly in use for roads of the highest quality. Among notable recent roads, the Austin (Texas) Expressway, the Valley Freeway in Colorado, the Maine Turnpike, the New Jersey Turnpike, and the Oklahoma Turnpike have asphalt surfaces; the Pennsylvania Turnpike, the Wilbur Cross Parkway in Connecticut, the Hollywood Freeway in California, the Central Expressway, Dallas, Texas, the Edsel Ford Expressway in Detroit, and the New York State Thruway are built of cement concrete. Either type of pavement can be constructed to give good service under heavy traffic. Choice between types may often rest on comparative cost. Sometimes this has been determined by taking alternate bids on pavements designed of cement and bituminous concrete, respectively.

For reconstruction of pavements, asphaltic concrete has an advantage over cement concrete in that it can be laid in much thinner courses. Frequently all a worn pavement needs in the way of a new surface is enough new material to restore good riding qualities and to protect the original surface against further deterioration. This can often be accomplished with a thickness of as little as 2 in. of bituminous surface course, whereas a cement concrete resurfacing course is apt to crack unless it is at least 5 in. thick. Both hot- and cold-laid asphaltic materials are used for resurfacing. For jobs of any size, hot asphalt is most often used. Cold material is especially useful for repairs and for the resurfacing of comparatively small areas, in that it can be stockpiled and drawn upon as convenient over a period of weeks or even several months.

Portland Cement Concrete Pavements. Portland cement concrete for pavements has the general characteristics and qualities possessed by the material when used in structures.

Its use in pavements, however, imposes some specialized limitations. The *cement* should be as described on p. 4-75. Air-entraining cement should generally be specified for concrete that will be used under conditions conducive to surface scaling. The cement should, whenever feasible, be tested at the mill so that 7- and 28-day tests can be completed before the cement is used. The *fine aggregate* should preferably consist of a natural sand, free from organic material (AASHO Method T-21), and containing not more than 3% of inorganic silt and clay by dry weight using AASHO Method T-11. Mortar should show 100% of the tensile or compressive strength of a comparable Ottawa sand mortar. AASHO Specification M-6 calls for the following gradation:

Passing a 3/8-in. sieve	100%
Passing a No. 4 sieve	95–100%
Passing a No. 16 sieve	45–80%
Passing a No. 50 sieve	10 (5) *–30%
Passing a No. 100 sieve	2 (0) *–10%

* May be reduced to these minimum values at option of engineer.

Other gradations may be necessary or desirable to meet local conditions. The *coarse aggregate* consists of broken stone or gravel or occasionally slag (see pp. 4-71, 4-72, and 4-73). It should conform to AASHO Specification M-80. Gradation of the coarse aggregate is very important in securing an economical and superior concrete. It is generally specified that the stone be furnished in two separate sizes, usually from 2 in. to 1 in. and from 1 in. to No. 4, or from 1 1/2 in. to 3/4 in., and from 3/4 in. to No. 4. These gradations are as shown in Table 1.

Table 1

Sieve Sizes	Total Per Cent Passing Square Opening Sieves			
	2 in. to 1 in.	1 in. to No. 4	1 1/2 in. to 3/4 in.	3/4 in. to No. 4
2 1/2 in.	100			
2 in.	95–100		100	
1 1/2 in.	35–70	100	90–100	
1 in.	0–15	95–100	20–55	100
3/4 in.			0–15	95–100
1/2 in.	0–5	25–60		
3/8 in.			0–5	20–55
No. 4		0–10		0–10
No. 8		0–5		0–5

The two coarse aggregate sizes are combined in such proportions that the resulting mixture has the smallest percentage of voids. This will usually require about twice as much of the large size as of the small size. The exact proportions to produce the densest mixture should be determined experimentally when the source of the aggregates has been decided on.

Water for mixing concrete should be clean and clear, free from salt, oil, acid, injurious alkali, and vegetable matter. If doubt exists as to the quality of the proposed supply, it should be tested by AASHO Method T-26. Where the water must be taken from a shallow source, an intake should be constructed that will be free from silt, mud, grass, etc., and the intake pipe should be at least 2 ft above the bottom.

Proportioning Concrete. Concrete for pavements is usually proportioned in one of two ways. In the first, a minimum flexural strength (modulus of rupture) is specified. For the sources of materials that will be used, a job gradation schedule is made up. It includes the coarse and fine aggregates, the cement, and mixing water. This schedule must produce a concrete of the specified strength. The schedule may be prepared by the contractor and approved by the engineer; or sometimes it is made up by the engineer and given to the contractor. As the job progresses, daily tests are made, both of the accuracy of the mix and the strength of the concrete. A new source of any material component, or failure of the concrete to meet strength requirements requires a revision of the mix formula. Where adequate facilities for test control are available, this is the desirable procedure. Alternatively, proportions are based on a constant cement factor, regardless of the source or characteristics of the aggregates selected by the contractor, so long as they meet the gradation and other tests required of the aggregates. This method is much simpler than the first, but is apt to be wasteful of cement.[1] In quite general terms, concrete for pavements should have a water-cement ratio of 5 to 6 gal of water per bag of cement (including free moisture in the aggregates); a slump between 2 and 3 in. when the concrete is not to

[1] A specification for the use of either of these methods is given in "Specifications for Concrete Pavement Construction," AASHO.

be vibrated, and 1 to 1 1/2 in. when it is; and 5 3/4 to 6 1/4 bags of cement per cu yd of concrete.

Thickness of Concrete Pavement Slabs. Over the years, cross sections of concrete pavements have been given a variety of forms involving thickening of the edges of slabs in various ways and degrees (Fig. 28). However, there has been a decided trend towards a uniform slab thickness. This is now used in all but a few of the state highway departments. More than one-third of the states have standardized on a single thickness, 8 in. in nearly all cases. About half of the remaining states use two thicknesses, usually 8 and 9 in. A few states build slabs less than 8 in. thick, and a few use thicknesses of 8, 9, and 10 in. Several of the states which use uniform thicknesses use thickened-edge sections also, and three or four states use thickened-edge sections altogether.

Formulas originally derived by H. M. Westergaard and since modified by other investigators may be used for the semi-empirical determination of slab thicknesses. These formulas give primary consideration to the anticipated wheel loads and to the proposed strength of the concrete to be used. They also include consideration of the area of contact of the tire with the pavement and the resistance offered by the particular type of subgrade (as experimentally determined). The formulas and their application are discussed in standard treatises on highway engineering.[2] Use of such formulas is restricted by the impracticability of varying the thickness of a pavement slab along the length of a contract to take into account variations in subgrade conditions. Moreover, forms for highway pavements are standardized in inch widths. Finally, the increment in the over-all cost of a highway project that results from an added fraction of an inch of pavement thickness is minor. Hence, as stated, a large measure of standardization has been incorporated into highway practice.

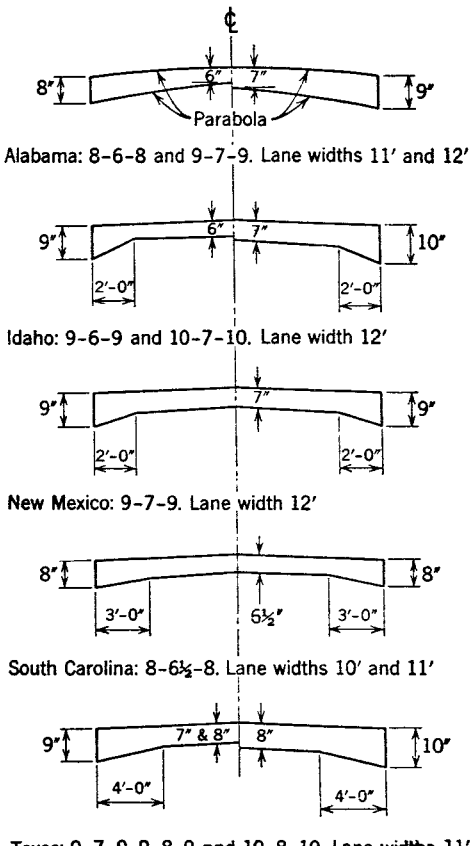

FIG. 28. Concrete pavement slabs of non-uniform thickness.

Other Elements of Slab Design. Aside from thickness, there are four elements of slab design that are closely interrelated. These are (1) the use and spacing of expansion joints, (2) the use and spacing of contraction joints, (3) the use of dowels or other load transfer devices at joints, and (4) the use of steel reinforcement, or distributed steel.

Expansion joints, placed transversely, permit increase in the length of a slab as its temperature rises. They are made by embedding in the concrete a vertical filler strip of non-extruding, compressible material (p. 4-76), usually 3/4 in. or 1 in. thick. The filler is

[2] See also H. M. Westergaard, "Stresses in Concrete Pavements Computed by Theoretical Analysis," *Public Roads*, Vol 7, No 2, April 1926, and "Analytical Tools for Judging Results of Structural Tests of Concrete Pavements," *Public Roads*, Vol. 14, No. 10, December 1933 See also "Concrete Pavement Design," The Portland Cement Association, 33 W. Grand Ave., Chicago 10, Ill.

held about 1 1/2 in. below the top surface of the slab, leaving a slot which is later sealed with a plastic material. Alternatively, a removable bulkhead may be used in place of the filler strip. When the pavement has hardened sufficiently, the bulkhead is withdrawn and the joint is poured, usually with an asphalt cement. The filler strip or the bulkhead must be held in exact position as the concrete is placed. This can be accomplished in conjunction with the load transfer device.

Contraction joints are usually "planes of weakness" provided at intervals to localize and control transverse cracking of the slab during curing or with a fall in temperature. They are most often of the "dummy" joint type. A temporary strip of wood or metal is embedded in the fresh concrete to form a groove 1/4 in. to 3/8 in. wide and extending from the top surface about 1/4 or 1/3 the depth of the slab. After the slab has been consolidated and finished, the strip is removed and the edges of the slot are finished and rounded. Before the pavement is put in use, the joint is filled with asphalt cement of a penetration that, in the locality where used, will neither "bleed" in summer nor crack in winter. When the slab contracts, cracks will normally form at these weakened planes.

Load transfer devices of one sort or another are usually used at expansion joints and sometimes at contraction joints, to prevent the full effect of a wheel load from coming onto the end of a slab as the load approaches or leaves the joint, and also to maintain vertical alignment of the adjacent slabs at the joint. These devices most often take the form of dowels (3/4 to 1 in. diameter, 12 to 14 in. long, and spaced 12 to 15 in. on centers) arranged, at least in theory, to slip back and forth, longitudinally, in one of the slabs as expansion and contraction occur. Many different types have been marketed. Most of them consist, essentially, of a framework to hold the dowels in correct position, parallel with one another and perpendicular to the joint, together with the dowels and dowel caps. An additional essential element is some means (usually a coat of grease or bituminous material) to coat one-half of each dowel and break the bond between it and the concrete. Most of these devices are expensive in themselves and in their installation or are lacking in durability. Frequently, the dowel appears to bind in the concrete socket and become immobilized. Consequently, load transfer devices are an unsatisfactory part of a concrete pavement, and this has stimulated efforts to eliminate their use.

Reinforcing steel is composed of sheets of welded steel-wire fabric or of mats of steel bars. These sheets are placed about 2 in. below the top surface of the slab. The principal function of this reinforcement is to localize cracks in the slab, to prevent the opening of such cracks as may occur, and to prevent relative vertical movement of slabs on opposite sides of cracks. As a rule, the amount of steel used is not enough to increase materially the flexural strength of the slab, although sometimes a substantial additional amount of reinforcement is placed in the end portions of slabs adjacent to expansion joints to increase resistance to cracking. Reinforcement is not ordinarily carried across contraction joints, since to do so would largely defeat the purpose of the joint. However, reinforcement is sometimes made continuous between closely spaced expansion joints. In this case, any joints between the expansion joints are more accurately termed "warping joints."

Spacing of Joints. Recent highway practice reveals two distinct schools of thought in respect to joint spacing and the use of load transfer devices and reinforcement. At one extreme—generally characteristic of practice in the northeastern part of the country—expansion joints are located at comparatively frequent intervals, usually from 30 to 100 ft (besides being used at all structures, around manholes, etc.). Contraction joints are either not used at all, or are used sparingly between expansion joints. Load transfer devices are used both at contraction and expansion joints. Slabs are reinforced quite heavily between joints to prevent the formation of transverse cracks. At the other extreme—generally typifying practice west of the Mississippi River, though not limited to that area—expansion joints are omitted except at bridges and other structures; contraction joints are spaced at 15- or 20-ft intervals; reinforcing is usually limited to areas near bridge approaches, etc.; and load transfer devices are used only at expansion joints and closely adjacent contraction joints. The Portland Cement Association lays down four conditions that should be met to justify the omission of expansion joints and steel reinforcement, as follows: (1) that the pavement be constructed of materials with normal expansion characteristics; (2) that the pavement be constructed during periods of the year when normal construction temperatures prevail; (3) that the pavement be divided into relatively short panels by contraction joints so spaced as to prevent the formation of intermediate cracks; and (4) that the contraction joints be properly maintained to prevent the infiltration of relatively incompressible materials.

Between these extreme positions there are a few states whose practice is not clearly consistent with either line of reasoning. It may be noted that in general the states adhering to the older, more conservative practice make a relatively greater use of concrete pavement than do most of the states adhering to the newer line of thought. Nearly one-half

the states prescribe the use of reinforcement generally throughout slabs, a somewhat smaller number use it over culverts, in bridge-approach slabs, etc.; and about the same number use reinforcement very sparingly or not at all.[3] To a rather limited extent and somewhat experimentally, long slabs (a mile or more) have been built without any joints and with heavy continuous reinforcing to prevent cracking.

Design of Reinforcement. Reinforcement for concrete pavements is most often of the bar-mat type or the welded-wire-fabric type. Both types are extensively used and are made up in sheets 4 to 6 in narrower than the width of one lane, and with a length determined by convenience in handling and shipping and by the spacing of transverse joints. Lengths between 13 and 16 ft are common. Steel for reinforcement should comply with the specifications given on p. 4-76. The amount of reinforcing used varies widely, but is most often between 40 and 75 lb per 100 sq ft. The requisite amount of longitudinal steel can be computed from the formula $A_s = LfW/2S$, where A_s is the amount of steel (square inches) per foot width of slab, L is the distance in feet between transverse joints, f is the coefficient of friction between slab and subgrade, W is the weight in pounds of 1 sq ft of slab, and S is the allowable stress (lb per sq in.) in the steel. f may range from 1 to 2; it is usually taken as 1.5. S may be taken as one-half the yield point of the steel. An analogous formula can be used to compute the required amount of transverse steel. Bar-mat reinforcing is often made up on the job, the bars being wired together at each crossing point. The most commonly used bar sizes are 1/2, 3/8, and 1/4 in. Spacing of bars across the width of the slab may be uniform, or more steel may be concentrated near the edges. Longitudinal bars are frequently spaced about 12 in. on centers. Transverse bars are often the same size as the longitudinal bars, but are spaced about twice as far apart. Except where continuity is deliberately broken at a joint, adjacent mats should be lapped 6 to 12 in. and securely fastened together.

Welded wire fabric usually has longitudinal strands about 6 in. apart and transverse strands about 12 in. apart. Wires may be spaced uniformly across the width or concentrated somewhat near the sides. The cold-drawn wire used in the fabric has higher tensile strength than the bars in mats, which is an advantage. Also the welded joints of the fabric are more secure than the wired joints of the mats, and the larger number of small wires more closely spaced than the bars is an advantage. On the other hand, the cost, per pound, of fabric generally exceeds the cost of mats.

Longitudinal Joints. To prevent the formation of unsightly, irregular longitudinal cracks, longitudinal joints are constructed on the center lines of two-lane pavements and between the lanes of wider pavements. Two-lane pavements are constructed one lane at a time, or the full width of the road may be laid in one operation. In the first case, the form for the inner edge of the slab is made with a triangular key about 1 in. deep. The key is stopped about 6 in. short of each expansion joint. The two slabs are tied together with tie-bars. These may be 1/2-in. diameter deformed bars, 2 ft 6 in. long and spaced 2 ft 6 in. on centers, although larger bars, longer and spaced somewhat farther apart are sometimes used. When the entire width of road is laid in one operation, the longitudinal joint is often formed by installing a metal parting strip, to be left in place with its top 1/4 or 1/2 in. below pavement surface. A convenient way to make the installation is to cap the strip with an installing shield. This is removed after the pavement is laid. A groove then extends from the top of the strip to the pavement surface, and is filled with a plastic seal. A longitudinal "dummy" joint or plane of weakness is also frequently used. In either case, particular pains should be taken to secure a straight joint, or a poor appearance will result. The construction joint automatically forms the longitudinal joint when the pavement is laid one lane at a time.

Construction Methods. Bulk cement is generally used. It should be so stored that none will remain too long in storage. Cement stored longer than 90 days should be resampled and tested before use. Cement should not have a temperature in excess of 160°F at time of delivery to the paver, to avoid danger of a "flash" set. There should usually be six stockpiles of aggregates, three in use after completion of tests, three being built up. Each set of three will include large stone, small stone, and sand piles. Piles should be far enough apart not to run into one another, or, alternatively, should be separated from one another by adequate partitions. Each pile should be of size sufficient to supply material for a day's run. In order to prevent segregation in building the piles, the materials should not be "coned" but should be spread in layers not more than 3 ft thick, commencing with an area large enough for the finished pile. If suitable hard, well-drained ground is not available, the ground should be covered with 2-in. plank, or a concrete slab. It is essential that

[3] For the practices of the individual states in respect to all characteristics of concrete pavements, see the "Summary of Concrete Road Pavement Specifications Used by State Highway Departments," issued by The Portland Cement Association, 33 W. Grand Ave., Chicago 10, Ill.

conditions be such that stockpiled materials have a uniform moisture content. Washed aggregates should be held in the stockpile 24 hr to drain Sometimes a sprinkling system is used to maintain uniform moisture conditions in both stone and sand.

The best and generally used method of mixing concrete for pavements is through use of a paver which mixes the concrete and places it directly in the forms[4] A batching plant is set up at the materials delivery point Cement and aggregates from the stockpiles are binned so that they can be accurately measured, by weight, separately before being loaded for transportation to the paver. The cement should be put into the truck on top of the coarse aggregate, and the fine aggregate deposited on it, the truck being covered with a tarpaulin to prevent loss. Alternatively, cement may be transported in a separate, waterproof container. Usually a truck carries enough material for two or three batches of concrete. The materials for each batch should be in a separate compartment

Pavers in general use are of 27 or 34 cu ft nominal capacity (known as *E*-27 and *E*-34). The latter is of the "dual-drum" type, which permits two batches of material to be mixed simultaneously, passing from the skip to the first drum to the second drum to the bucket. The cement and dry aggregates are dumped from the truck into the skip and then are charged into the drum of the paver and mixed for at least 1 min and preferably for 1 1/4 min, at 14 to 20 rpm. In the dual-drum paver, mixing should continue for at least 30 and preferably 40 sec in each drum. Water is admitted at a uniform rate through an accurate water-measuring device during the first 15 sec of mixing. In setting the amount of water, allowance should be made for any free moisture in the aggregates. Mixing and placing of concrete is generally allowed when the air temperature is 40°F and rising, and is stopped when a falling temperature reaches 40°. When it is anticipated that the temperature may drop below 40°, the aggregates should be heated before being placed in the mixer, or a flame-thrower may be used to heat the concrete while it is being mixed. Normally the concrete should have a temperature between 60° and 90° when placed in the forms.

Before any concrete is placed, care should be taken to see that the subgrade is thoroughly compacted. Steel forms are then carefully set along one side of the first lane of pavement to be poured. These are set from tacked stakes set 50 ft apart (or closer on curves, as needed), accurately on line and marked for grade. The line of forms should be checked by measurement and by eye. The second line of forms is set from the first line, giving close attention to proper crown and superelevation. Width of the lane or pavement should be accurate within 1/4 in. The earth under the forms should have been very thoroughly compacted, with mechanical tampers or otherwise, since the pavement can be no smoother than the forms on which the finishing machine operates. It is usually required that forms shall be kept set for at least 500 ft ahead of the paver. After forms are set, a templet is drawn along them to check the accuracy of the subgrade, every point of which is normally required to be within 1/2 in. of its correct position. Low spots are filled and must be very thoroughly compacted, high spots are planed down. Sometimes a subgrading machine, riding on the forms, is used to bring the subgrade to exact elevation. The finished subgrade is sprinkled, if necessary, to wet it sufficiently to prevent absorption of water from the concrete It should be thoroughly moist, but not muddy. Just before placing concrete, the forms should be oiled to prevent adhesion. The mixer should deposit the concrete near its final position so as to require as little rehandling as possible The concrete is then spread with mechanical spreaders or shovels to the required depth and cross section (usually to the level at which the reinforcement is to be placed). In doing this, care must be taken not to displace tie-rods, dowels, etc. No material should be deposited by the mixer within 3 ft of an expansion joint where a load transfer device is used. Concrete should be deposited on each side of the joint and shoveled simultaneously against both sides of the filler. Concrete should be thoroughly spaded along the forms and along the sides of transverse joints to prevent honeycombing Mechanical vibrators are effective along side forms and joints, particularly where an elaborate load transfer device is used Vibration should not be overdone, however. The placing of concrete should be continuous from transverse joint to transverse joint Construction joints are normally made to coincide with expansion joints When an equipment breakdown or other unavoidable interruption occurs closer to an expansion joint than the length of one mat of the reinforcement being used, the short length of pavement that has been placed should be removed and the pavement relaid from the joint. Preferably no construction joint should be located within 25 ft of an expansion joint.

Reinforcing mats or *mesh* are generally located from 2 to 3 in. below the surface of the pavement. Concrete is placed to that depth The reinforcing is laid on the concrete and

[4] Plant or transit-mixed concrete should generally not be allowed in concrete pavements, although it is usually allowed for culvert headers, manholes, etc

is secured as required. The remaining concrete to complete the full slab thickness is then placed. When *expansion joints* include a load transfer device, this may consist of a wire frame (to be left in place) which supports the joint filler and dowels. Alternatively, the device may consist of some type of removable bulkhead. When concrete has been brought to the level of the top of the permanent filler strip, the bulkhead is removed and the space it occupied is filled with concrete. Frequently a joint is designed so that the top of the premolded filler is depressed about 1 1/2 in After any removable parts have been taken away, a steel strip of the proper depth and as thick as the filler strip is clipped to the top of the filler strip to form the slot and to serve as a guide for finishing the joint. When the concrete has set sufficiently to retain its shape, this strip is removed and the joint is repaired, edged, and later filled with a poured filler. If the entire expansion joint is of the poured type, a temporary bulkhead occupies the place of the premolded filler. When *contraction joints* with dowels are used, the dowel assembly is placed as for an expansion joint, the concrete is shoveled around it, and the supports are removed when the concrete has reached the proper depth A strip to form the slot for the dummy joint is then placed and is later removed and the joint finished and sealed with a poured filler. Alternatively a slot is cut in the green concrete to the required depth and width. Contraction joints without dowels are made by forming or cutting a slot of the proper cross section, finishing its edges, and sealing it with suitable poured filler. Where adjacent pavement lanes are constructed one at a time, all transverse joints should line up across both slabs.

Consolidation and finishing of a concrete pavement is best done with a power-driven finishing machine. The machine usually has two strike-off screeds, the leading screed sometimes being a vibrating tamper. Concrete is placed to a depth a little above the top of the forms. The leading screed is set a little high, but pushes most of the excess material ahead of it while allowing a slight amount to pass beneath it. This is picked up and spread by the second screed, which shapes the pavement surface to the correct height and crown. Any low or porous areas of the screeded surface should be filled at once with fresh concrete and leveled off. If the finishing machine has only one screed, this is raised off the forms slightly for the first pass, and is brought to true elevation in successive passes. Intersections, driveways, and other irregular areas are screeded by hand. As soon as possible after the screeding operation is completed, the concrete should be further smoothed and finished with a longitudinal float. This may be machine or hand operated, preferably the former. It is simultaneously moved forward along the pavement and sidewise across the pavement with a sawing motion It carries ahead of it any thin mortar or laitance on the pavement surface This is pushed to, and wasted over, the side forms. The surface is then tested for smoothness with a 10-ft straightedge. This is held in successive positions parallel to the center line, from one side of the slab to the other. All variations of more than 1/8 in. from the true surface contour should be corrected by filling or planing, as necessary, and the resolved areas refinished. When this has been done, and just before the concrete surface becomes non-plastic, any float marks or other surface blemishes are removed by belting or dragging the surface. The *belt* is a strip of canvas or burlap about 6 in. wide and 2 ft longer than the slab width. It is operated by a man on either side of the slab, with short strokes perpendicular to the center line, accompanied by an advance along the center line. A *drag* consists of a piece of burlap about 3 ft wide and 2 ft longer than the slab width, with one long edge of the material fastened to a pole. The drag is rolled on the pole and is thoroughly wetted. When dripping has stopped, the drag is held above and transverse to the new slab, and is gradually unrolled and lowered until 18 to 24 in of burlap are in contact with the concrete Maintaining this contact area, the drag is then drawn slowly along the surface so as to leave an even texture behind it This is sometimes followed by brooming the pavement to produce shallow (1/16-in.) transverse grooves on the surface. The edges of the slab are then finished with an edging tool to round them to a radius of about 1/4 in Edges of transverse joints are carefully finished from a bridge which does not touch the concrete. Proper *curing* of the finished slab is very important. In hot weather, loss of moisture from the slab must be prevented. Three methods of doing this are in general use. In the first, wet mats of quilted cotton or burlap (AASHO Specification M-73), 5 ft 9 in. long and 2 ft 6 in. wider than the slab, are unrolled on the pavement from a supported roll so as not to mar the surface. They are kept saturated for a minimum of 72 hr, and it is sometimes specified that, when the wet mats are removed, paper covers shall replace them for a total curing period of 2 weeks. In the second method, the wet mats are dispensed with and waterproof paper covers are used throughout the curing period. They are lapped at least 12 in. and are weighted down around all edges to prevent air getting through to the concrete. It is important that the covers be placed before hair-checking has an opportunity to commence. The third curing method consists in spraying the surface of the freshly laid slab with a film of material (AASHO Specification M-148) which will harden into a waterproof membrane. Curing

with wet hay or straw, or with wet earth, is no longer generally practiced. In winter, or at any season when *freezing* of the new concrete is a possibility, this must be guarded against A layer of the waterproof curing paper just referred to will be adequate for temperatures just under freezing. For greater protection, the paper should be covered with 6 to 12 in. of straw or hay, and then another layer of paper *Forms* are not removed until concrete has set for at least 12 hr. After forms are removed, the ends of all joints are cleaned full width, and all honeycombed areas in the sides of the slab are pointed up Earth is then shoveled against the sides of the slab to top-of-slab level.

During placing of the slab, test beams (AASHO Method T-23 and T-97) are made. These are cured in the same manner as the slab, and are tested for flexural strength as soon as it is believed they will show a modulus of rupture of at least 550 lb per sq in Traffic is not ordinarily allowed on the pavement until this strength has been attained. Before opening the road to traffic, all joints should be carefully cleaned and then sealed with bituminous material, as specified.

To be fully acceptable, concrete pavements should have average thickness (as shown by test cores taken at intervals of approximately 500 ft in each lane) within 1/8 in. of the specified value. When average thickness is less than the proper amount by more than 1/8 in. and less than 1/2 in., the unit price to be paid for the pavement is adjusted downward in the ratio of the square of the actual thickness to the square of the specified thickness. No additional payment is made for excess thickness. If average thickness is deficient by 1/2 in. or more, the engineer may require the defective pavement to be removed and replaced, or he may permit it to be left in place, but without any payment therefor.[5]

Curbs. These may be built integrally with the pavement slab or be doweled to the top of the slab; or precast concrete or stone curb may be set against the side of the completed slab with joint filler between. When an integral curb is to be used, the uncurbed slab is consolidated and finished as heretofore described. A set of curb forms designed to give the curb the specified cross section is then immediately fastened to the pavement side forms and is filled with concrete. This is thoroughly spaded or mechanically vibrated against the forms, and a spade is worked into the slab concrete far enough to insure that the curb will be well bonded to the pavement When the curb has hardened sufficiently, the forms are removed and finishing of the curb is completed. Joints must be formed in the curb wherever they occur in the slab, and must be of the same width. The curb must be carried ahead as the pavement is laid, so that its construction is always accomplished before set occurs in the slab. When curb is to be *doweled* to the top of the slab, dowels are set into the pavement slab while the slab concrete is still plastic The curb area is left rather rough for better bond A 1/2-in. reinforcement bar, extending from joint to joint of the pavement, is wired to each dowel (Fig. 24) Forms are then set for the curb, and a piece of premolded joint filler, of the same thickness as the pavement joints and cut to the cross section of the curb, is placed exactly above each transverse pavement joint. The forms are then filled with concrete The concrete should be well spaded (but not vibrated) to prevent honeycomb Construction of the curb should follow the pavement construction as soon as practicable. As soon as the concrete has set (generally the following day) curb forms are stripped and the curb surfaces are finished.

Concrete Bases. Cement concrete is often used as a base course with a sheet asphalt or asphaltic concrete surfacing. What has been said about cement concrete pavements applies also, in general, to bases, with the following exceptions and additions.

Cement. Type I or Type II cement may ordinarily be used without the addition of air-entraining materials.

Reinforcement. Reinforcement is often dispensed with but modern practice tends to make use of a lightweight wire mesh, especially where heavy traffic is anticipated. When reinforcement is provided, it is distributed as discussed under concrete pavements.

Joints. Frequently bases are built without joints. Where joints are used, they are usually dummy joints to control contraction cracks There is little uniformity in practice in the spacing of transverse joints, but they need not be as closely spaced as in pavements. When the full width of the base is placed in one operation, a longitudinal dummy joint should be provided on the center line.

Thicknesses. Slabs are very generally of uniform thickness, from 6 in. to 8 in. or more depending on foundation and traffic. Most of the supporting value of the combined base and surface courses is provided by the base.

Curbs. A header curb is generally used, with a height equal to the designed thickness of the asphaltic surfacing and a width of 6 in. This curb is cast integrally as explained for concrete pavements.

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[5] "Specifications for Concrete Pavement Construction," AASHO, paragraph 35 (1950).

Construction Methods. Except for the omission of expansion joints, construction methods are substantially as outlined for concrete pavements. The only difference of importance is in the surface texture to be obtained. Instead of belting and light brooming to secure a smooth-riding surface, as in the case of a concrete pavement, the surface of the base should be heavily scored, transversely, with a stable broom. This scoring should roughen the base to a degree that will prevent sliding of the asphaltic surface on the base. At the same time, it should leave the crown of the base substantially true to that of the finished road, so that the superimposed asphaltic material will have a uniform thickness.

Hot Asphaltic Concrete Pavements. There are two basic types, one with aggregate graded down from 1 1/2 in. or 1 in. (called coarse-aggregate type) and one in which the largest aggregate size is 3/4 in. or 1/2 in. (called fine-aggregate type). Both types consist of a densely graded combination of broken stone, gravel, or slag, with sand, mineral filler, and asphalt cement. Sometimes the two types are used in combination, a wearing course of fine-aggregate type being used on a binder or base course of coarse-aggregate type.

Alternatively, either the fine- or the coarse-aggregate type may be used independently. The material adapts itself to wide variations in use, dependent, very largely, on the judgment of the engineer. Coarse aggregate adds stability to a layer of considerable thickness. Conversely, the fine-aggregate type can be placed in thinner layers, and facilitates securing a tight and smooth surface. In each type, attention must be given both to the grade and the quantity of asphalt cement used. Too hard a cement will crack; too soft a grade will soften and "shove," producing a "washboard" surface. The proper grade depends primarily on the temperature range, secondarily on the traffic. In the United States the following grades should prove satisfactory:

	Light or Moderate Traffic	Heavy Traffic
Warm climates	60–70	50–60
Moderate climates	70–85, 85–100	60–70, 70–85
Cold climates	100–120	85–100

From the standpoint of quantity of asphalt cement, too lean a mixture will lead to raveling; too rich a mixture, to surface deformation. The asphalt cement should conform to AASHO Specification M-20. The broken stone or gravel aggregate (retained on No. 10 sieve) should meet the quality requirements stipulated by AASHO Specification M-79. Fine aggregate (passing No. 10 sieve) should consist of sand or stone screenings and should conform to the quality requirements of AASHO Specification M-29. Mineral filler should consist of portland cement, ground limestone, fly ash, traprock float or like material conforming to AASHO Specification M-17.

In new construction for mixed traffic the pavement may consist of a comparatively large thickness (6 in.) of asphaltic concrete laid in three courses directly on a waterbound macadam or broken-stone base of approximately the same thickness. Alternatively, a lesser thickness of asphaltic concrete may be used on a heavier base. Figure 29 shows the

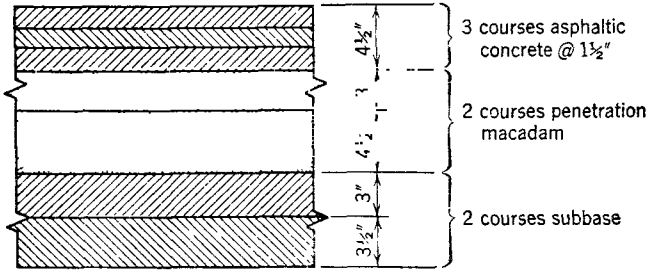

FIG. 29. Arrangement of subbase, base, and surface courses, New Jersey Turnpike.

design of the New Jersey Turnpike, in which 6 1/2 in. (compacted thickness) of selected subbase material is laid on the subgrade in two courses, and is followed by two courses of penetration macadam on which are placed two leveling courses and a wearing course of asphaltic concrete. Another possible arrangement is to use about 7 in. of waterbound macadam, in two courses; then a single course (approximately 3 in.) of penetration macadam; and finally a wearing course (1 1/2 to 2 in.) of asphaltic concrete.

In resurfacing work, the asphaltic concrete is most often laid either on an old cement concrete pavement or on some type of bituminous pavement. If badly broken, the old

road should be thoroughly repaired and patched and all weak spots in the foundation corrected before the new pavement material is applied. This should then be placed in two layers: a binder or leveling course and a wearing course. The leveling course should have a minimum thickness of 1 in., and should be brought to a true profile and cross section so that the wearing course will have a uniform thickness. The size of aggregate to be used in the leveling course may depend on the thickness of the course, which should never exceed 2 in. If this thickness will not be sufficient to correct irregularities that exist, separate repair operations should fill the deeper depressions, and separate wedge courses should be used to eliminate excessive crown or produce necessary superelevation before the leveling course is applied. A small-sized aggregate should be used in this work. If the old road requires widening before the new surface is applied, this must be very carefully done or longitudinal cracks along the old pavement edges are likely to form. The base widening material may be penetration macadam, cement concrete, or asphaltic concrete. It must be thoroughly compacted on a completely stable subgrade. It is desirable, when possible, to widen the old pavement and allow traffic to use it for a year before the new surface is added.

The wearing or surface course is ordinarily between 1 and 2 in. thick. If the old road that is to be surfaced has an adequately true profile and cross section (as when the surface is applied because of the "scaling" of a portland cement concrete road) the leveling course may be omitted and the entire new surface applied in a single course 2 in. thick. Also when an asphaltic concrete wearing course for a city street is to be laid on a new cement concrete base, the entire thickness may be applied in a single operation, provided that the base has been constructed to close tolerances of smoothness. In both cases, a light tack coat, preferably of tar, should be used. Under asphaltic concrete, asphaltic tack coats must be applied very sparingly or they will result in an excess of asphalt which, working upward through the concrete, will produce an unstable mixture, or a slippery surface, or both.

Proportioning Asphaltic Concrete. In using asphaltic concrete, the usual practice is to specify a single grading that will include all sizes of mineral constituents, and will give allowable minimum and maximum percentages passing each size screen. The engineer then establishes or approves a *job-mix formula* for the mixture that will be supplied for the project. This formula establishes a single definite percentage of aggregate passing each screen size, a single definite percentage of bitumen to be added to the aggregate, and a single definite temperature at which the mixture is to be delivered. This formula should produce a mixture the voids in which will comprise from 3 to 6% of the volume, after rolling of the pavement is completed. This amount of space should be available for the asphalt to occupy as the pavement gradually compacts further under traffic.[6] Tolerances, as given hereafter, limit the allowed departures from the job-mix formula. Samples of the mix are checked against the formula several times a day as the work progresses.

Table 2. Percentage by Weight Passing Square Mesh Sieves

Sieve Designation	Dense-Graded Coarse-Aggregate Type		Dense-Graded Fine-Aggregate Type	
	Grading *A*	Grading *B*	Grading *C*	Grading *D*
2 in.	100			
1 1/2 in.	95–100	100		
1 in.	75–100	95–100		
3/4 in.	60–90	80–100	100	
1/2 in.	. .	. . .	85–100	100
3/8 in.	35–65	45–80	75–100	90–100
No. 4	25–50	28–60	50–85	70–100
No. 10	20–40	20–45	30–75	60–90
No. 40	10–30	10–32	15–40	30–70
No. 80	5–20	8–20	8–30	10–40
No. 200	1–8	3–8	5–10	5–12
Bitumen (sol. in CS2) %	4.0–7.5	4.5–8.0	4.0–10.0	4.5–11.0

Source: AASHO "Specifications for Construction, Bituminous Surfacing" (1949).

Typical gradations for coarse-aggregate and fine-aggregate types of asphaltic concrete are given in Table 2. In each case, two gradings are given, one somewhat coarser than the other. The table shows the amount of asphalt cement to be used with each grading.

[6] Machines are available in which a sample cut from the finished pavement can be tested to foretell the characteristics the pavement will develop under traffic In the light of such tests, the mix formula can be adjusted during progress of the paving as may be found desirable.

The fraction retained between any two consecutive screens must not be less than 4% of the total weight.

Allowable tolerances for use with Table 2 are shown in Table 3.

Table 3

Passing No. 4 and larger sieves	±7%
Passing No. 4 to No. 40 sieves, inclusive	±5%
Passing No. 80 sieve	±3%
Passing No. 200 sieve	±2%
Bitumen	±0.3%
Temperature of mixture on delivery	±20°F

Equipment.[7] Hot asphaltic concrete mixing plants are of three general types: permanent, railway, and portable. The permanent plant is most reliable, but such plants may not be within practical hauling distance of the job. Railway plants are permanently mounted on railway trucks and are operated from them on sidings. Portable plants may be dismantled and set up where needed. A permanent plant should have a capacity of at least 25 tons per hr. The aggregate and mineral filler are dried, heated to the proper temperature, screened, and stored in an adequate number of separate, heated compartments to assure maintenance of the proper gradation. Proper proportions of aggregate are then carefully weighed and thoroughly mixed with the hot asphalt, which is weighed or metered in the proper amount. The mixing may be done either in a batch-type or a continuous-mix-type plant. Care must be taken not to overheat the asphalt, either directly or by contact with overheated aggregate, during the mixing process. The completed mixture is discharged into trucks at the specified temperature. Truck bodies should be tight, well-cleaned, wetted with a soluble oil solution to prevent sticking, and, when necessary, covered with canvas for weather protection. A lining of 1/4-in. plywood provides cheap and effective insulation.

Placing. Machines for spreading, leveling, and compacting asphaltic concrete are self-propelled, on caterpillar treads, and utilize long runners to equalize minor subgrade irregularities and produce a smooth and even surface. Placing the material in several courses is advantageous because thin courses facilitate better compaction and also because each course provides a progressively smoother surface for the machine to travel over in spreading and screeding the next course. The surface produced should be true to designed profile and cross section, free from all hollows and irregularities, and of uniform compaction throughout. The paver is adjusted to spread a depth of material such that, after proper compaction, the course will have the specified thickness. It should also confine the edges of the mixture to true lines. Pavers are generally designed to be able to spread and finish a lane width of anything between 8 and 12 ft. The material should be laid only on a dry surface when the air temperature is above 40°F and the weather is not foggy or rainy. In any area where mechanical spreading is impossible, hand spreading and finishing are used. In this case, the material should never be dumped from the truck directly onto the subgrade, but onto a steel dump sheet, from which it is distributed with shovels and spread with rakes or other suitable hand tools to the proper depth.

Compaction. After each course has been laid and screeded, it is rolled while still hot until it is thoroughly compacted. Ten to twelve-ton tandem rollers are used; or it may be specified that 3-wheel finishing rollers be used also. To maintain the crown, rolling should

Table 4

	Rollers Required	
Tons Laid per Hour	Leveling Course	Surface Course
Less than 125	3	3
125–175	3	4
175–225	4	5

begin at the sides and progress gradually to the center; or, on superelevated curves, it should begin at the lower edge and progress to the higher. Rolling should commence as soon after the material is laid as is possible without causing undue displacement. New Jersey Turnpike specifications required that rolling should commence when the sum of the temperatures of the air and of the mixture was between 300 and 325°F; and also required that the number of rollers used be not less than those shown in Table 4.

[7] For greater detail in requirements for mixing plants, see "Specifications for Construction, Bituminous Surfaces," AASHO.

AAHSO specifications (1950) require one roller for each 25 tons of material spread, or each 400 sq yd covered, whichever is less. If pavement width permits, the pavement should be rolled diagonally, in both directions, after having been rolled longitudinally. Areas inaccessible to rolling must be thoroughly tamped until full compaction is secured.

Joints should not be formed more often than necessary. When unavoidable, as at the end of a day's run, they are formed by cutting back on the previous run so as to expose the full depth of the course. The exposed edge is painted with a thin coat of hot asphalt cement just before new material is placed against it. Upon completion of rolling, the surface should be checked for smoothness—with a crown templet and a 10-ft straightedge applied parallel to the center line. Any surface irregularity exceeding 1/4 in. should be corrected by cutting down and recompacting, or by loosening the surface mixture already laid, adding more material, and recompacting. After final rolling and checking of the surface, it is sometimes required that a light coat of mineral filler be dusted over the surface. This practice seems to offer no real advantage, however, and is often omitted. Traffic should not be allowed on the pavement until it has entirely cooled, which will ordinarily require from 6 to 12 hr.

Cold Asphaltic Concrete Pavement. This differs from hot asphaltic concrete pavement in that the asphalt is in the form of a cutback or an emulsion, instead of an asphalt cement. This enables the mix to be placed and manipulated cold. When a cutback is used, the mix can be made up in a central plant and shipped long distances by truck or rail. This may be an important advantage. However, for sizable jobs where suitable aggregates can be found close to the site of the work, freight charges on the cold mix may more than offset the cost of setting up a hot-mix plant at the job location. The cold-laid asphaltic concretes have their greatest usefulness for small jobs and repair operations. A number of cold-laid processes have been patented in past years, most of the patents having now expired. Cold-laid pavements are of two general types: the liquid-bituminous type using a cutback or emulsified asphalt (or sometimes a tar) and the liquefier type, which employs an asphalt cement that is fluxed with a naphtha or equivalent liquefier. Specifications for both types are included in, "Specifications for Construction, Bituminous Surfaces," AASHO.

Sheet Asphalt Pavements. As most often used, this term implies the combination of a sheet asphalt wearing course laid on a "binder" course of fine-aggregate type asphaltic concrete. Each course is ordinarily from 1 to 1 1/2 in. thick. The binder course is an economy measure, since it requires only about half the asphalt content of the sheet asphalt surface. Sometimes the binder course is given an "open" rather than a dense texture by omitting part of the finer gradations of aggregate. The trend appears to be, however, towards using a dense gradation. The binder course is made, laid, finished, and compacted as described for hot asphaltic concrete pavements. The wearing course, or sheet asphalt proper, consists of sand, mineral filler (portland cement, ground limestone, or similar substances), and asphalt cement. The sand should be graded as shown in Table 5.

Table 5

Sieves	Min.	Max.
Passing No. 4—retained on No. 10	0%	5%
Passing No. 10—retained on No. 40	12%	40%
Passing No. 40—retained on No. 80	25%	60%
Passing No. 80—retained on No. 200	18%	45%
Passing No. 200	0%	6%

The mineral filler should be graded as shown in Table 6.

Table 6

Passing No. 200 sieve	Not less than 75%
Total passing No. 80 sieve	Not less than 95%
Total passing No. 30 sieve	Not less than 100%

The sand and mineral filler should be combined in such proportions that the resulting mixture will have the ranges of gradation and bitumen content shown in Table 7.

Table 7

	Passing Sieve	Retained on Sieve	Per Cent
Aggregate		No. 10	0–5
	No. 10	No. 40	10–32
	No. 40	No. 80	20–48
	No. 80	No. 200	16–36
	No. 200		10–18
Bitumen			9.5–12

As with a hot asphaltic concrete pavement, a job-mix formula conforming to the above ranges is established Variations from the job-mix formula should not exceed the values shown in Table 8.

Table 8

Passing No. 4 to No. 40 sieves, inclusive	±5%
Passing No. 80 sieve	±3%
Passing No. 200 sieve	±2%
Bitumen	±0.3%
Temperature of mixture on delivery	±20°F

The asphaltic cement should conform to AASHO Specification M-20. The same penetration grades are used as for hot asphaltic concrete (p. 4-93). The sheet asphalt is proportioned, mixed, transported, placed, compacted, and surface tested as described for hot asphaltic concrete pavements. For mixing, a pug mill is used, rather than a continuous-mix plant

Brick and Block Pavements. Because of their high labor cost, brick pavements are not as frequently used now as in the past Paving bricks are manufactured in sizes to give a pavement thickness of 2 1/2, 3, 3 1/4, and 4 in. They have a width on top of 3 1/2 and 4 in., and are standardized at a length of 8 1/2 in. They may have lugs to provide space for filler, or may be lugless As a rule, brick pavements are built on a 6-in. concrete base with a 6-in.-wide curb on each side the same height as the combined thickness of the brick and the bedding material used. Bedding is frequently an inch of sand, though sometimes sand and cement are used to form a mortar. In other cases a mastic of bituminous material and sand has been found satisfactory. After the bedding material is screeded to a true crown, the bricks are laid by hand by masons from piles previously distributed along the sides of the road and brought to the masons by helpers or by mechanical conveyers. The rows are laid in a straight line with joints broken in the adjoining rows The tops of the bricks are kept true to grade. The surface is then rolled with a 3- to 5-ton tandem roller, and the joints are filled with a bituminous filler. A cement grout filler is sometimes used, in which case allowance must also be made for expansion and contraction at the curbs and around manholes and catch basins. The filler is squeegeed into the joints, care being taken not to overfill or to leave a surplus on the surface. A light coating of sand or screenings is sometimes used to cover the pavement surface as a finish to blot up any surplus of joint material.

Stone, wood, and asphalt block pavements are practically obsolete at the present time In the rare cases when they are used, they are laid in practically the same manner as brick pavement

Shoulders for High Traffic Volume Roads: These should meet several diverse and important requirements

1. They should maintain such stability throughout all seasons that no hazard will result from a vehicle running on them in emergency, or from vehicle wheels running accidentally from the pavement to the shoulder.
2. They should so obviously be able to support any vehicle at any time that they will be no deterrent to driving a disabled vehicle off the pavement for repairs.
3. They should adequately support the edge of the pavement and protect it against failure
4. They should minimize the entrance of water into the subgrade.
5. In some locations, they should provide an acceptable path for pedestrians.

These requirements will very generally preclude the use of grass or natural earth shoulders and will call for some type of stabilization This may most simply take the form of an untreated shoulder of bank-run gravel or other selected local soil that can be well compacted and that will remain hard and firm in wet weather. At somewhat greater expense, crushed stone, gravel, or slag can be used to provide a still firmer and more stable surface. Any of the foregoing types should be built much as described for a low traffic volume road of the same material, and should have about the same thickness. Also any of the foregoing types can be given added strength, stability, and watertightness by applying a bituminous surface treatment as described for screened-gravel roads (p. 4-78). If still greater strength and durability are desired, they can be had through use of waterbound or bituminous macadam, or of a mixed-in-place or plant-mixed bituminous shoulder on a suitable base. Care should always be taken to provide a very obviously different shoulder surface from that of the pavement, otherwise the shoulder may be used as part of the travel path. For example, where a bituminous macadam shoulder is to be adjacent to a bituminous concrete pavement, the shoulder should generally be given a rough surface texture to differentiate it from the pavement and to discourage its use for vehicular travel. However, where the shoulder should also serve as a pathway for pedestrians, if the texture is

made unpleasant for walking, pedestrians will walk on the pavement despite the hazard of doing so. Sometimes a surface of light-colored gravel is applied to the shoulder adjacent to a black-top road. This delineates the pavement edges during dark hours. Paint lines may also be used.

HIGHWAY MAINTENANCE

Maintenance of highways falls into five classifications: (1) road surfaces; (2) shoulders, (3) drainage; (4) roadsides, (5) structures. The first four of these are treated in this section. Allied to maintenance of highways is maintenance of aids to traffic (signals, signs, and pavement markings) and facilitation of traffic through removal of snow and sanding of ice.

19. MAINTENANCE OF ROAD SURFACES

Regardless of the type of road surface, maintenance, for economy, should be *preventive*. Defects should be corrected before they become serious, keeping the surface in good repair rather than allowing it to fall into disrepair and then correcting it. When a rut, pothole, or other defect appears, it progresses very rapidly. When a surface has been treated, either with chloride or with a bituminous material, timely renewal of the treatment is much more economical than allowing the road surface to disintegrate. In a time of prolonged wet weather, it may be worth while to fill small holes, temporarily, removing the temporary repair material later when a more careful, permanent repair is made. A rich, gummy mixture of coarse sand with a medium-grade slow-curing residual asphalt (SC-2 or SC-3) or a medium grade of tar (RT-6) can be made up and stockpiled for such use. Generally, a road constructed with asphalt should be patched with asphalt, and one constructed with tar should be patched with tar.

Low Traffic Volume Roads. Since these roads include many types, from natural soil to macadam, the forms of deterioration to which they are subject, and the proper maintenance procedures, are widely varied. They include. (1) dragging and blading, (2) patching and repairing local breaks, (3) scarifying, reshaping, and restoring lost material, (4) applying dust palliatives, and (5) stabilization with bituminous surface treatments. The most critical time in the life of low-type roads is at the spring thaw If heavy traffic can be kept off the road at this time, it will be helpful. If a road shows distress at this season, a sand blanket will dry and help to stabilize it over the critical period.

Dragging and Blading. Natural or selected soil roads, and in general all roads lacking the stability afforded by an adequate proportion of hard, coarse aggregate, typically fail by rutting under traffic. This type of deterioration is combated by dragging and blading, which are the principal maintenance operations on the lowest types of roads The purpose of both these operations is to plane down high areas and to move the resulting loose material into low areas. Blading, however, also implies reshaping and reestablishing a proper crown on the cross section The two operations differ principally in the equipment used. A drag consists of scraping members, normally fixed in position. The simplest form is shown in Fig 30. It is hauled behind a truck or tractor, and is weighted as needed to enable it to "hone" or plane down the material of the road surface. Many more complex types have been built, frequently of structural steel members, set diagonally within a rectangular frame, so that, as they are drawn along the road, they move loose material transversely as well as longitudinally.

Blading is usually accomplished with an adjustable steel blade, carried on a frame which may be self-propelled, or which may be dragged after a truck or tractor. The power-driven grader is one of the most valuable pieces of maintenance equipment. Blading and grading are best done after a rain when the surface materials are sufficiently moist to compact well. In blading, material is generally first drawn from edges to center to establish a crown. The loosened material is then drawn back again and deposited in depressed areas. When a satisfactory surface has been restored, it should be given an application of suitable bituminous material or of calcium chloride.

Patching and Repairing Local Breaks. In contrast to the road types just discussed, surfaces constructed with a suitable proportion of a good coarse aggregate are likely to "pothole" rather than to form ruts. If inspection reveals no foundation trouble, a localized poor gradation of surface materials is usually the cause. In any case, careful inspection should be made to determine the extent of the area of potential weakness, and the area should be excavated and enlarged until sound materials are encountered The walls of the hole should then be made vertical to provide a solid shoulder, and the hole filled with

carefully graded material of the sorts used in the original road surface, and thoroughly tamped. Water may be used to secure better compaction. If the patch is 4 in. or more in depth, the material should be placed in layers 2 to 3 in. thick, thoroughly tamping each layer. Frequently, examination will show that the surface break was due to a foundation failure, often induced by poor drainage. In any such case, surface repair without first correcting the underlying condition is a waste of money. This may require removal of inferior material and replacement with a granular subbase, or the improvement of surface

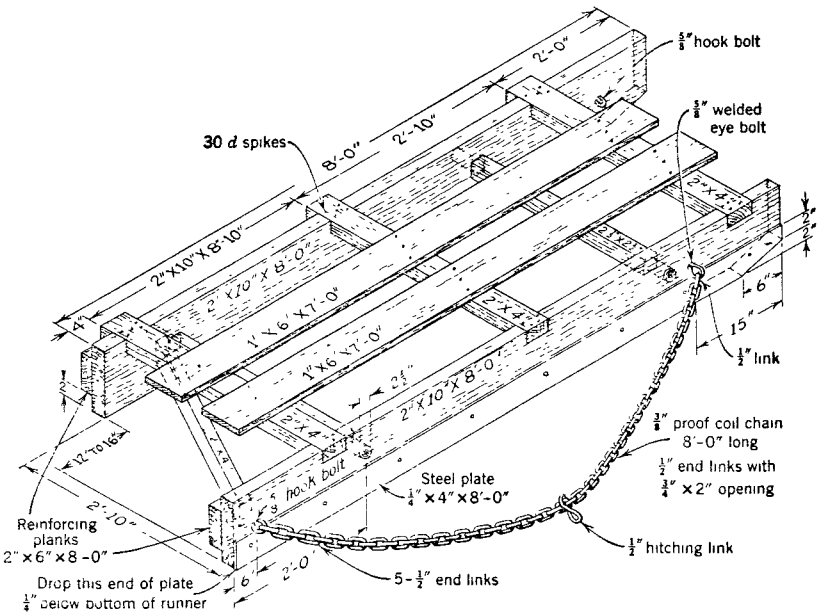

FIG. 30. Simple type of road drag. (U. S. Bureau of Public Roads.) All wood to be of good quality seasoned white or burr oak, or other timber equally as satisfactory. Cutting plate to be of medium steel. The drag to be securely made in a workmanlike manner.

Metal

No.	Item	Size
1	Steel plate with holes as shown	1/4" × 4" × 8'-0"
8	Bolts with nuts and washers (countersunk)	3/8" × 3"
1	Welded eye bolt with nuts and washers	5/8" × 5"
2	Hook bolts with nuts and washers	5/8" × 5"
1/3 lb	Wire nails	8d
3 lb	Wire spikes	30d
1	Hitching link	1/2"
1	Proof coil chain	3/8" links 8'-0" 6 1/2" links

Lumber

No.	Size
3	2" × 4" × 3'-2"
2	2 1/2" × 2 1/2" × 3'-2"
1	2" × 4" × 3'-0"
2	1" × 6" × 7'-0"
2	2" × 6" × 8'-0"
2	2" × 10" × 8'-0"

drainage facilities, or the installation of suitable underdrainage. Only careful examination of each location will indicate the necessary corrective treatment, which is then carried out as described under road construction.

Scarifying, Reshaping and Restoring Surface Materials. When areas of local failure become general, either through neglect or as an indication of a generally inadequate foundation or improperly constructed surface, complete scarification and reconstruction of the road may be required. The old surface should be loosened by scarifying to the full depth of the surface material. New material is then added as necessary. It should be of such nature that, when well mixed with the old material, a suitable gradation will result. It is mixed on the roadbed with blade graders or harrows. After mixing is complete, the road is bladed to a proper cross section and is then recompacted. This may be done by traffic, with occasional blading to maintain proper cross section; or it may be more quickly and better accomplished with rollers. If deterioration of the road surface was due to inadequate drainage or other foundation conditions, these must be corrected before the surface

is reshaped and compacted. Gravel and some types of soil roads sometimes corrugate or "washboard" under traffic. This usually indicates a faulty composition and gradation of surface materials. The surface should be removed and replaced with suitable material of the same type; or, if traffic is heavy, the unsatisfactory surface may better be replaced with one of a higher type. When a road of gravel or crushed slag or stone aggregate has been scarified and reconstructed, it should generally receive a bituminous treatment

Dust Palliatives The lower-type road surfaces are subject to very rapid deterioration from the blowing away of surface materials unless calcium chloride or light, penetrating road oils are used to prevent this. Besides preserving the surface, dust palliatives make roads much pleasanter to use. Calcium chloride is applied either in granular or flake form, using a mechanical spreader of the gravity or spinner type It is best applied after rain Otherwise, water may be added to the surface, if feasible, although this is not necessary unless the surface is very dry. The first treatment in the spring may be $^3/_4$ to 1 lb per sq yd, followed by lighter treatments as needed. The season's requirements will ordinarily be less than 2 $^1/_2$ lb per sq yd. Bituminous dust layers include the lightest two or three grades of tars or of slow-curing and medium-curing asphalt cutbacks Tars are suitable for granular surfaces only. The particular type and grade of bitumen to be used, as well as rate of application, will depend on nature and condition of road surface. The normal quantity for a treatment (on a road that was initially treated as the final step in construction) varies from 0 10 to 0.25 gal per sq yd. Before the material is applied, the road should be bladed as necessary to restore proper crown and contour. Traffic should be kept off the treated surface until the bituminous material has been absorbed This is usually accomplished by treating one-half the width of the road at a time It may sometimes be necessary to "blot up" with a light sand cover spots where the material did not fully penetrate, before opening the road to traffic.

Bituminous Surface Treatment The action of the dust palliatives is to penetrate and impregnate the top layer of the road material, giving it enough cohesiveness to enable it to resist being blown from the road surface or drawn from the surface by tire suction This is about all that can be accomplished by treating natural soil roads with bituminous binders. In the case of more granular materials, however, such as gravel, crushed slag, and dry or waterbound macadam, a somewhat heavier grade of bitumen can be used with sand or other light mineral aggregate to form a protective bituminous mat on the road surface. This is called a surface treatment. The term is a broad one, and covers the application of varying amounts of a number of different bituminous materials, with widely varying amounts and types of cover, and occasionally no cover at all. Surface treatments, as a maintenance measure, are applied to medium and high traffic volume roads, as well as to low traffic volume types. There is no well-established terminology covering the various kinds. Classified primarily according to the heaviness of the treatment, they may be called (1) sand-mat types, (2) armor coats, and (3) retreads

Sand-Mat Treatment The *sand-mat* types employ a natural sand cover, or stone screenings, or other aggregate passing a $^1/_4$- to $^5/_8$-in. screen These types have particular applicability to gravel, crushed slag, and dry crushed stone or waterbound macadam surfaces The road surface should have had (as part of or immediately following construction) a penetrating bituminous treatment as described on p. 4-78 This should be allowed to cure for a period of 3 to 6 weeks The surface is then cleaned of all loose material and repaired, if necessary. An application of $^1/_8$ to $^1/_3$ gal per sq yd of bituminous material is then made with a pressure distributor, preferably in hot, dry weather. The best bituminous material is one of the medium grades of tar (RT-4 to RT-9), the heavier grades being used with the more open surfaces. The most stable mat is secured by using the heaviest grade that the surface will accept Cover material is next applied, in amounts of 12 to 18 lb per sq yd The road is then "honed" with a heavy drag to plane off high spots and move the displaced material, together with the sand mulch, into depressions. The dragging or honing is continued until the new material is well distributed and until a smooth surface has been produced. Building up too thick a mat (more than $^1/_2$ in.) in one application should be avoided. Any excess of mulch can advantageously be bladed to the shoulder, which will be strengthened by it. A great advantage of the sand-mat treatment is that it can be applied not only to seal and preserve intact a smooth road surface but also to restore a true contour to a road that has been put badly out of shape by corrugation, frost heaves, etc. This type of treatment is used, therefore, on roads that do not possess sufficient stability to retain a true surface, until opportunity has been afforded to assure that underlying causes of lack of stability have been corrected. Several repetitions of the treatment, moreover, on a low-grade type of road lacking surface stability will build up a bituminous mat that (assuming sound foundation conditions) can then hold a heavier and more permanent type of surface treatment. The sand-mat treatment will usually have

to be renewed annually, or at least every second year, and, with some types of traffic and surface course conditions, may have to be applied oftener than once a year.[1]

Armor Coat Treatment. The armor coat treatment (another somewhat loosely used term) consists of applying a suitable grade of bitumen to the road surface and then covering it with a heavier (3/8 in. to 3/4 in.) stone cover. This is a single treatment. A double treatment may also be used, in which case there are two applications of bitumen and two applications of cover stone. Occasionally triple treatments are used, although, generally speaking, other methods of adding so much material will give better and more economical results. The distinguishing characteristic of armor coat treatments is that there is a minimum of manipulation of the stone cover. Each individual stone preferably remains where it first encounters the bitumen. In this way, if proper balance is maintained between size of stone and amount of bitumen, an uncoated surface of stone comes in contact with tires, affording high antiskid properties, while the stone is sufficiently embedded in bitumen to be securely anchored. This type of treatment is much more durable than the sand-mat treatment, and should normally have a life of 4 to 8 years, depending on size of stone and number of applications. It should be noted that the process produces a wearing surface of substantially uniform thickness—one stone, in the case of a single treatment. Hence it will not correct irregularities in the surface contour. Where such exist, they should first be corrected with a honing (sand-mat) treatment, applied a year ahead of the armor coat; or by a retread or heavier "mix" treatment the thickness of which can be varied to eliminate irregularities. It should also be noted that an armor-coated road cannot be honed back to a true cross section if it loses its correct shape because of any foundation weakness. Hence this type of treatment should be applied only to roads of established stability. If the roadbed is generally stable, except for a few weak spots, these should be very carefully repaired before the treatment is given. For roads of the low traffic volume type, the process is applicable only after the surface has been stabilized by a sand-mat type of bituminous treatment, and it will normally not be applied until that treatment requires some renewal.

Application of a single armor coat treatment is made as follows: The road surface is first cleaned of any loose or foreign material. If inspection shows that in places the treated surface is more porous than in other places, from 1/8 to 1/4 gal per sq yd of a medium tar (say, RT-5) is applied, lightly sanded and honed. This process may have to be repeated one or two weeks later, care being taken to hone any surplus mulch off the road surface. (This preparation is unnecessary if the road surface is uniform in texture.) A week or more after the second honing operation, an application of RC-5 or of asphalt emulsion RS-2 is made and is immediately covered with the stone, which may be either 1/2 in. (No. 7, p. 4-72) or 3/4 in. (No. 6). Quantities of stone and bitumen should be as given in Table 1.

Table 1

Type of Bituminous Material	1/2-in. Bitumen, gal/sq yd	Treatment Stone, lb/sq yd	3/4-in. Bitumen, gal/sq yd	Treatment Stone, lb/sq yd
RC-5	0.22–0.27	25–30	0.32–0.36	35–40
RS-2	0.28–0.35	25–30	0.33–0.42	35–40

The lesser amounts of bitumen are used on a tight surface, the greater on a more porous surface. Care must be taken not to use too much bitumen, or "fat" spots will result, the stone being submerged. On the other hand, if too little bitumen is used, the stone will be inadequately anchored. Determination of the proper amount of bitumen for the aggregate being used and, thereafter, close control of the amount decided on are essential to success. The application temperature for the bitumen should be as given on p. 4-75. One-half-inch stone produces a smoother and pleasanter riding surface; 3/4-in. stone produces a rather rough, noisy surface, but one which is more durable and which does not require quite such close control in application. The stone should be spread with mechanical spreaders of the roll-feed type, and applied as evenly as possible. Hand brooms or a light broom drag are used to correct unevennesses. Since it is essential that the road surface be completely covered, some hand spotting of stone on thinly covered areas may be necessary, though the use of roll-feed spreaders will eliminate most of this. The road should be lightly rolled, sufficiently to set the stone without crushing it. Traffic may then be allowed to use the road, but should be held to 20 mph for several days until the stone

[1] On light traffic roads where *little honing* is required, an MC-2 or MC-3 asphalt with a coarse cover can be substituted for the tar, with satisfactory results. It will usually be more durable than the tar treatment. Relative original costs will depend on relative prices of tar and asphalt. A coarse cover material should be used to avoid creation of a slippery surface.

is well set. Weather conditions have a great effect on the satisfactoriness of armor coat treatments. Ideal weather has a minimum temperature of 75°, with low humidity. The work should be deferred or discontinued when there is any likelihood of rain. Should rain fall on a newly applied armor coat, the road should be closed to traffic until it has dried. Asphalt emulsion has the advantage over the cutback that moderate dampness of aggregate and road surface is not harmful This allows the work to be carried on under a wider range of weather conditions.[2]

In a double armor coat treatment, an application of bitumen and $^3/_4$-in. cover stone is made as described above, broomed, spotted, and lightly rolled. A second application of the same bitumen is then made, covered with $^1/_2$-in. stone, broomed, spotted, and rolled. The double treatment is obviously more expensive than the single treatment, but has a longer life and adds some structural strength to the road.

"Retread" or Drag Treatment. This is essentially a light mixed-in-place surface course (p. 4-80), utilizing all new aggregate. It results in a new wearing course of variable thickness, which makes it suitable for trueing up a road surface that has developed some, but not too great, irregularities. When employed as a maintenance operation, the average thickness is generally not greater than 1 in.

Intermediate Traffic Volume Roads. The higher the road type, the greater the importance of preventive maintenance. If the road has proper drainage, an adequate base, and a well-constructed surface course, maintenance should consist principally of periodically resealing the surface as the bituminous material ages, dries out, and begins to show signs of cracking or raveling. The nature of the seal coat will depend on the condition of the road surface and the nature of the available aggregates. If the surface condition is generally good and the contour is satisfactory, an armor coat treatment may be applied. The single treatment will effectively seal the surface; a double treatment may be used if it is desired to strengthen the pavement also. If the surface has been allowed to become porous, or if it has already raveled somewhat or has developed minor irregularities, it will often be desirable to use a sand-mat treatment to smooth and tightly seal the surface, following it a year later with an armor coat.

If the road needs repair, rather than merely resealing, the trouble will be due to (1) faulty drainage, (2) faulty foundation material, or (3) a poor surfacing job. Repair will be dependent on the cause of the trouble, and the extent to which the damage has progressed. If the pavement has raveled and broken in localized spots, the basic condition should be corrected and the spots patched. In doing this, the disturbed material should be removed until satisfactory material is reached, both on the sides and bottom of the excavation. The sides are cut vertically. The base course is replaced if and as needed and is tamped until true and firm. The hole is then filled with premixed patching material This should normally consist of a dense-graded mix of aggregate having a $^3/_4$-in. or $^1/_2$-in maximum size with an asphalt cutback, say MC-3 or MC-4.[3] Before placing the patching material, the sides of the hole should be painted with the same bitumen used in the patch The patch should be thoroughly compacted in layers about 2 in. thick. The surface of the finished patch may be slightly above the existing road surface to allow for settlement, but care must be taken not to cause a "bump." Alternatively, the patch may be made by a penetration method, placing dry, coarse aggregate in the hole, compacting it, and then penetrating it with just enough asphalt cement (100–120 penetration for average climatic conditions) to cover the aggregate, after which it is keyed with small stone. Generally, patches should be *sealed* with 0.1 to 0.25 gal per sq yd of a suitable bituminous material and covered with an aggregate (10 to 25 lb per sq yd) like that forming the road surface. Where road shoulders are inadequate, a frequent location of raveled areas is along the edges of the road metal. These are patched much as outlined above, care being taken to cut back the damaged surface sufficiently to establish a solid foundation for the patch, and to compact the new material solidly against the side of the hole. Along the free edge, a stepped back or "pyramid" construction should be used, the lowest layer of the patch extending into the shoulder area with each succeeding layer drawn in 6 in. or more, so that the top layer will line with the established pavement edge.

Where deterioration of the road has progressed beyond an occasional spot requiring patching, a retread treatment may be in order; or it may be desirable to use the existing surface (repaired as necessary) as a base for a plant-mix or penetration surface course;

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[2] Tar has been found less satisfactory than asphalt for armor coats If it is used, it is of especial importance that both the road surface and the aggregate be dry until the operation has been completed. Stripping of the bitumen (either asphalt or tar) from wet aggregate is lessened by the use of various chemical additives that have been developed for the purpose. There is, however, no completely satisfactory substitute for dry weather conditions

[3] If the road surface was originally constructed with tar, RT-4, RT-5 or RT-6 should be substituted for the cutback asphalt

or it may be found necessary to scarify and rebuild the existing road, using construction methods heretofore outlined. If the road surface has become dangerously smooth, it can be "skid-proofed" with an armor coat treatment.

High Traffic Volume Roads. *Bituminous Pavements.* Maintenance of asphaltic concrete and sheet asphalt pavements involves somewhat the same problems as those discussed for intermediate traffic roads. Sheet asphalt does not ordinarily require surface treatments of any sort, and, if the base is adequate, should not give trouble except from the occasional formation of cracks These should be filled with asphalt cement, after being carefully cleaned with a broom or with compressed air. The crack and adjacent area should then be sealed with SS-1 asphalt emulsion, covered with 3/8-in. stone. More extensive failures are usually traceable to foundation defects The failed material is removed, the foundation repaired, and the patch (consisting of sheet asphalt surface on a binder course) placed as described in the preceding paragraph. In urban streets, pavement openings for sewer lines and other utilities often constitute a problem. The cut in the pavement should be made wider than the trench to be dug, in order that the replaced concrete base may have areas of solid ground to support it. The backfilling of the trench should be thoroughly compacted. Even so, there will be some settlement and a layer of reinforcement, transverse to the trench, should be placed about 2 in. above the bottom of the base slab to enable it to bridge the trench. The cut faces of the base should be vertical, but will provide better bond if left rough. The new concrete should be mixed dry—not more than 4 gal of water per sack of cement being used. Just before the new concrete is placed, the sides of the cut should be scrubbed with wire brushes, then wetted thoroughly and well painted with a mortar composed of equal parts of cement and sand. The old sheet asphalt is cut to a vertical surface and is painted with asphalt cement (85–100 penetration for average climatic conditions) before the new binder and wearing course patches are placed. Eventually, with age. sheet asphalt pavements lose their "life" and crack to such an extent that it is no longer practical to repair the cracks. They must then be removed and a new pavement laid. Asphaltic concrete pavements are maintained much as are intermediate traffic volume pavements.

Cement Concrete Pavements. These are subject to several quite different types of deterioration, each of which calls for its own remedial measures. These types are: (1) joint filler deterioration, (2) cracking, (3) pumping, (4) scaling and other surface defects, and (5) loss of true cross section because of foundation settlement.

Joint Pouring. Renewing the joint filler is routine, the purpose being to waterproof the joints and at the same time to prevent dirt, pebbles, etc., from getting into them. The work is done when needed, but preferably as late in the fall as weather will permit. A considerable contraction of the slab should have taken place. At the same time, the slab *must* be dry and above freezing to secure adhesion. The refill material most often used is an asphalt cement, usually of a comparatively low penetration (50–60 and 60–70, although in cold climates penetrations as high as 100 to 120 are used). Sometimes from 15 to 25% by weight of mineral filler (rock flour, volcanic ash, etc.) is mixed with the asphalt (AASHO Specification M-89).[4] The joint is first cleaned of dirt and of old, oxidized filler by use of an appropriate router or with compressed air. Thorough cleaning of the ends of joints is important to prevent corner spalling. The filler is heated as hot as possible, without burning (Table 2, p. 4-75). It is applied, usually from hand pouring pots, although machines have been developed to apply the material through nozzles, from a tank maintained at the proper temperature. Joints are sometimes poured in the spring as well as in the fall. All types of joints should be sealed.

Repair of Cracks. It is now the general practice to do nothing about hairline cracks that have not spalled at the edges, or opened to a width that will permit satisfactory introduction of the filler. Wide and badly spalled cracks are cleaned and filled like joints.

Pumping of Joints. This is a serious problem where the slab is laid on a subgrade containing an excess of silty material Water, permeating a joint, softens this material and forms a mud. Wheel loads passing over the joint suddenly depress the slab ends, forcing the mud upward, through, and out of the joint. Many repetitions of this process create a cavity under the ends or corners of the slab and cause a lack of support that eventually leads to cracking of the ends or corners. Sealing the joints retards this destructive process and would prevent it, were the joint perfectly sealed. Perfection in this is hard to accomplish, however. When a cavity has formed under the slab, it can be filled with a water-resisting mixture, as will be described under mudjacking. This is an expensive but usually effective treatment.

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[4] Rubber joint fillers are also available and give satisfactory results when the joint has been thoroughly cleaned and the pouring temperature is closely controlled through use of a thermostatically regulated oil-bath heating kettle.

Surface Deterioration. This includes spalling at joints and also a progressive disintegration of the surface, commonly called scaling. Where the scaling is not deep (less than 1 in.) the usual corrective practice is to clean the scaled surface thoroughly, brushing loose all of the deteriorated concrete that can be moved. The surface is then sealed with one or more applications of MC-2, MC-3 or MC-4, RC-2 or RC-3, or a light to medium grade of tar, each application being covered with coarse sand, pea gravel, or ½-in. or smaller stone. Tar and sand will be satisfactory for thin patches, asphalt with stone cover when the deterioration has progressed farther. In the event the depth of the scaling exceeds 1 in., it should be patched with premixed material, using a graded stone. The maximum stone size will depend on the depth of the scaling. After the surface has been thoroughly cleaned, it should have a light (not more than 0.1 gal per sq yd) prime coat of RC-1 or RC-2, to bond the mixed material to the concrete. The bituminous mix is then placed and well compacted. It may be feathered over the borders of the scaled area to improve riding qualities. Care must be taken to avoid using too much bitumen, in the prime coat or the mix itself, lest an unstable or slippery surface be created. A seal coat may be used, if the texture of the patch requires it. Scaling, begun, is difficult to check. It is likely to progress beneath the patch, the patch ultimately loosening and breaking up. The patching process can be repeated, but eventually it will usually be desirable to transform the concrete pavement into a base for an adequately heavy surface course of asphaltic concrete

Repairing Settlements; Mudjacking. Unstable subgrades beneath concrete slabs, especially under heavy traffic, sometimes yield, leading to settlement of the slabs. This usually does not occur uniformly. As a result, the correct contour of the pavement surface is destroyed, relative vertical displacements at joints may occur and produce rough riding conditions, and the pavement slabs may be cracked. Pumping of joints may also produce settlement and cracking at the ends and corners of slabs. Temporary relief from the discomforts of a rough pavement may be secured by adjusting the surface with bituminous patches or wedges, but mudjacking the slabs back to their correct position, and filling any voids that may have been created beneath them, is the more permanent cure. The mudjacking process utilizes a machine consisting of a slurry mixing tank, and a high-pressure pump, together with air compressor, hose, drilling equipment, etc. The process consists in drilling holes through the slab and forcing the slurry through them until the slab is raised and "floated" back to its correct elevation. If, in the process of settlement, horizontal as well as vertical displacement has occurred, tackle and a winch are used to move the slab horizontally while it is supported on the semifluid slurry. The slurry may contain any type of earth (loam, clay, silt, etc.) that will retain water under a pressure of 500 lb per sq in. No more than 5% of sand should be present. Usually about 2 bags of cement should be used with each cubic yard of earth. The amount of water to be used is largely a matter of experience—large areas imply a thinner mix than small ones. Holes are ordinarily 2½ in. in diameter, and are placed on 5-ft centers in rows about 3 ft from each edge of the slab. Holes in the two rows are staggered. Pumping progresses from hole to hole, in such sequence as to relieve rather than increase the stresses that were caused by settlement of the slab. Only enough material is forced through a hole to produce a small lift at any one time. The hose is then moved to another hole, also chosen so as to relieve rather than increase stresses. The process continues from hole to hole until the proper movement has been accomplished. This may involve pumping several times through the same hole. An earth dam is maintained around the free edge of the slab to prevent leakage of the slurry. Drilling may be done well in advance of the actual jacking. Joint material should be removed to prevent binding between adjacent slabs. About 2 weeks after the slabs are raised, the holes are filled with concrete plugs the full thickness of the slab, with a cement grout to hold them in place. Traffic should be kept off the raised slabs for 24 hr.

Cuts in Concrete Pavements. When a cut must be made in a concrete pavement, it should be made with straight, parallel edges. The concrete that is to be removed should be broken up to expose the reinforcement for a length of a foot or more, and the bars or mesh should be cut at that distance from the edge of the remaining slab. As for a sheet asphalt pavement, the slab opening should be made somewhat larger than the excavation that is to be made in the subgrade, in order to provide a shoulder of solid material for the patch to rest on. The excavation is backfilled and thoroughly compacted. It is desirable then to remove the subgrade material under the edge of the old slab, for 4 to 6 in. back from the edge and to a depth of 4 to 6 in. The new concrete is worked into this space. The reinforcement of the old slab is pulled out again to its original position to bond with the new concrete. New reinforcement should be placed in the patch, about 2 in. from the bottom, and transversely to the trench. The patch is made of dry concrete, sometimes using high early strength cement or an accelerator.

20. MAINTENANCE OF SHOULDERS

The most important functions of shoulders are: (1) to provide space for vehicles leaving the travel path, in emergency, by accident, or in order to park; (2) to conduct surface water from the pavement area to the ditches, (3) to support the edge of the pavement or road surface. Maintenance should preserve these functions, regardless of the type of shoulders. Hence shoulders should be kept stable, smooth, and with the proper elevation and cross slope. The most frequent defect of shoulders is erosion at the pavement edge. This defeats all three functions; it creates a traffic hazard, it collects and holds water; and it deprives the edge of the road surface of any support.

Turf Shoulders. These require maintenance of a firm, tough mat of sod, and require that the grass be mowed often enough to make the solid character of the shoulder evident to the driver, as well as to maintain the sod in healthy condition and to lessen fire hazard in dry seasons. Turf shoulders erode readily under the wheels of traffic running accidentally off the pavement edge Especially in the spring, rutting and tearing of the sod may occur under traffic. Such ruts should immediately be filled with suitable soil, and grass reestablished by sowing seed as early in the growing season as possible. Sodding may sometimes be desirable Fertilizer should be applied as needed. When the growth is heavy, cuttings should be raked from the shoulder; light cuttings, unless they create a fire hazard, may advantageously be left to supply humus to the soil. Power-driven mowing machines especially designed for shoulder and slope work are available. Shoulders should never be burned off. Turf shoulders have a tendency to build up, gradually, through natural growth processes aided by the deposit of foreign matter and by frost action. They must periodically be brought back to the proper elevation. Sometimes rolling will correct the trouble, if done while the ground is saturated Otherwise the shoulder must be planed down. This should be done early in a grass-growing season—spring or fall. Any tall grass or weed growth should first be mowed. The shoulder is then disked or harrowed so that large chunks of sod will not be planed off. The harrowed material, to a depth that will include much of the grass roots, is then bladed to a windrow along the outer edge of the shoulder. Enough of the underlying soil is then planed off so that when the sod fragments have been replaced and have reestablished themselves, the shoulder elevation will be satisfactory. A drop-off of approximately $^1/_2$ in. from the pavement edge to the turf shoulder is permissible, in order to increase the time before the shoulder may have to be planed down again The new shoulder subgrade is lightly scarified. The windrowed material is then spread uniformly over it, and is disked and bladed to produce a smooth surface. Fertilizer, loam, and seed are applied if and as necessary to create conditions requisite to the reestablishment of turf, after which the shoulder may be lightly rolled. Turf shoulders cannot be maintained satisfactorily on a heavily traveled road; hence, as traffic increases it may be necessary to replace them with a more appropriate type. Also, where turf shoulders are generally maintained, it may be desirable to use a shoulder of a firmer type opposite mailboxes and at other points where there is a considerable passage over the shoulder. This will especially be true on the inside of curves Turf shoulders are sometimes used in combination with a bituminous shoulder strip adjacent to the pavement.

Stabilized-Soil and Untreated Gravel or Stone Shoulders. The most frequent and most serious fault of such shoulders is erosion at the pavement edge. This is corrected by dragging the shoulder to redistribute loose material, drawing it higher on the shoulder. From time to time, the addition of new material may be required This should be of the best quality obtainable so that the stability of the shoulder is gradually improved by the additions rather than lessened. New material may be applied by hand or with mechanical spreaders. A box-type drag is sometimes used. Material, especially that placed adjacent to the pavement edge, should be compacted to give it stability. A correct cross slope ($^3/_4$ in. to 1 in. per ft for this type of shoulder) should be maintained. Besides dragging to redistribute loose material, it may sometimes be necessary to blade or plane the shoulder to the proper contour. In shaping the shoulder, no loose material should be left on the pavement surface. Care must also be taken not to create a false ditch between the pavement edge and the true ditch.

Bituminous-Treated Shoulders. Maintenance of such shoulders consists of patching, surface treating, and occasionally scarifying and rebuilding. Each of these operations is carried out as described for road surfaces of comparable materials. Prevention is more economical than repair; and prompt repair is cheaper than delayed and consequently more extensive repair Especially, holes developing near the pavement edge should have quick attention, not only for economy but to avoid a serious traffic hazard The higher the type of shoulder, the greater is the responsibility to keep it in good repair, since the

higher are the speeds at which cars are likely to be driven onto the shoulder. Not only should surface breaks be promptly repaired, but obstructions such as fallen branches and rocks should be removed as quickly as possible.

Paved Shoulders. Paved shoulders are made by penetration or by road or plant-mix methods, and their maintenance is generally as described for road surfaces of those types. Defects are the same as for the surface-treated shoulders just described. Since paved shoulders are generally found alongside important, high-speed roads, it is especially important that the shoulder area, particularly that part adjacent to the pavement edge, be suitable for high-speed vehicle operation.

Combination Shoulders. It is sometimes found economical and satisfactory to construct a shoulder of two types, the higher type adjacent to the pavement The usual combination is a bituminous or paved strip with turf beyond. This will resist erosion from traffic, support the edge of the pavement, and prevent the entrance of water near the pavement edge. Furthermore the problem of planing down the turf will be less serious. The bituminous strip should never be less than 3 ft wide, and 4 ft is preferable.

21. MAINTENANCE OF DRAINAGE

Proper drainage maintenance requires that ditches and stream channels be kept open, that catch basins, manholes, storm sewers, and culverts be cleaned as often as needed, that erosion of ditches, of stream channels, and at the inlets and outlets of culverts be promptly repaired, that failed sewers or culverts be replaced, and a constant lookout be kept for all evidences of inadequate or faulty drainage and proper corrective steps taken. Faulty drainage can nullify almost any other maintenance effort.

Ditches and Stream Channels. The shallow ditch section recommended for rolling country lends itself to simple and economical maintenance, since the normal shoulder slope is continued to the outside of the ditch and a proper ditch bottom contour is maintained by the normal shoulder blading and grading operations. Great care must be taken, however, *not to undercut the back slope of the ditch*, creating a steeper surface than will be stable If this happens, the slope will be undermined and a long process of slope disintegration is likely, unless counter measures are taken. When the back ditch slope is undercut, it will often be cheapest, in the long run, immediately to install a berm of bituminous mixed material (nothing else is likely to be sufficiently stable to resist erosion) to replace the material that was cut away.

If the road has a deeper (say 1 on 4 slope) ditch section, this can ordinarily be kept open with grading equipment. If the ditch sides and bottom are sodded, the grass should be kept well mowed so as not to impede the flow of the water and lead to silting up of the ditch. If the ditch design was inadequate to prevent erosion, it is important that this be checked as soon as discovered. Sodding the ditch may be effective, or paving it with cobbles, bituminous material or cement concrete may be required. Whatever the ditch problem, delay in recognizing and solving it will be costly Stream channels which receive highway drainage must be kept clear and open. This may at times require corrective work on private property outside highway limits Channels subject to scour should be closely watched to forestall erosion which might be very costly to the pavement, road shoulders, or drainage structures.

Catch Basins, Manholes, Culverts, Storm Sewers, and Underdrains. All such elements of a drainage system should be inspected frequently enough to assure their proper functioning. When sand is used on roads in winter, catch basins often become filled rapidly They can be shoveled out by hand, but, if there are many, use of a catch-basin *eductor* will be economical. This machine consists of a pump which puts water into the catch basin with enough velocity to stir up the sand or other deposit and which, through another hose, withdraws the water and suspended solids Storm sewer or culvert pipes which have partially silted up should be flushed with a fire hose, if water and pumping facilities are available. A pipe culvert which is completely plugged can usually be opened with a hose, operated from the downstream end. The outlets of all perforated drains should be periodically inspected, at times when they should normally be discharging water, to see that they are continuing to function properly. If not, the likelihood is that the filtering material in the trench has become clogged with silt. The trench will have to be excavated and refilled with clean material. This should be done before the road has suffered from failure of the drainage system to dewater the roadbed.

22. MAINTENANCE OF ROADSIDES

Roadside areas, from a highway maintenance standpoint, may conveniently be defined as all areas within the highway right-of-way designed for vegetative treatment. This will include medians, unless paved. All cut-and-fill slopes are also considered part of the roadside, although, in the case of rock cuts, vegetative treatment may be lacking. The principal roadside maintenance activities are:

Mowing and otherwise caring for grassed areas.
Weed control and eradication.
Care of trees and shrubs.
Maintenance of slopes.
Maintenance of picnic, recreational, or rest areas.

Some of this work is more or less continuous in its requirements, some is seasonal, and some may be done at any season, enabling it to be fitted into a continuing program, as the pressure of more urgent work permits. Hence, for economy, the work should be carefully scheduled.

Mowing and Other Care of Grassed Areas. This should be in keeping with the surroundings. In more or less formal, urban areas, close mowing may be desirable and warranted; in rural areas, more attractive as well as more economical results will usually be secured if mowing is restricted to that necessary for (1) maintenance of shoulder usability and of traffic sight lines, (2) maintenance of the health of the grass (3) control of fire hazards. The grass should not be too closely cropped. Frequency of mowing varies with rainfall, etc., but it usually will not have to be repeated more than two to four times a growing season Turf shoulders will require more frequent mowing than slopes and other areas farther from the traveled way. Ground cover within highway areas need not necessarily be grass. The most pleasing effect will often be secured through use of the plant materials that are native to similar areas in the region. On new slopes, this cover may often be secured by mulching the area with seed-bearing roadside cuttings. The Connecticut Highway Department utilizes the following practices for the establishment of roadside ground cover to meet different conditions:

"(*a*) Application of mulch—consisting of roadside mowings, poor grade hay, litter from the floor of the forest, et cetera—is a method which in some instances may be used satisfactorily. In many areas, particularly adjacent to existing stands of desirable plants, volunteer growth quickly becomes established in the mulch without further treatment.

"(*b*) Application of mulch with seed sowed on top or underneath in some locations gives satisfactory results Seed shall be sowed either in the fall or spring For late seeding a light cover of mulch over the seed serves to furnish the new grass protection against the excessive cold of winter or the heat of summer. Mulch underneath serves to furnish the moisture and humus necessary to germination and subsequent growth.

"(*c*) Where the application of mulch is a potential fire hazard, it must be covered lightly with topsoil and seeded. However, even in such instances, the use of mulch reduces the depth of topsoil which would otherwise be required. The mulch serves to stop surface wash, adds humus to the soils, and conserves moisture, encouraging the influx of volunteer growth.

"(*d*) In the interest of economy, machine methods have been used experimentally with excellent results for spraying seed, fertilizer, and topsoil over mulch. Beyond question, judging from results obtained through experimentation in machine spraying, this method will become one of the foremost standard practices for the establishment of turf ground cover.

"(*e*) The application of topsoil to a given depth and seeding, is the standard method for median strips, grass shoulders, traffic control areas, and in front of houses where the turf must be maintained as a lawn or frequently mowed

"(*f*) Occasionally, little or no topsoil is required where soil tests reveal that the quality of the existent subsoil is such that it can be economically built up by proper amendment, such as the addition of fertilizers, lime, or sowing cover crops.

"(*g*) The planting of shrubs or vines may be used in conjunction with mulch, confining the selection largely to native species, preferably indigenous to the locality, which will readily colonize or adapt themselves. In conjunction with this method no pockets are prepared, the plant material being installed directly into the existing soil. Where soil acidity or any other necessary soil constituent is lacking, the proper fertilizer or soil correcting agent is usually added.

"(*h*) Sodding may be done with either turf or low-growing compact plant material, such as Lowbush Blueberry, Bearberry, certain ferns, et cetera. Sodding is done primarily for protection on steep slopes which cannot otherwise be established; in interception drainage ditches and 'leakoffs'; and around structures where erosion is likely to occur. Solid sodding of slopes is not the usual practice in this state due to the fact that the establishment of the ground cover through this method is not economical."

Weed Control. Allied to the maintenance of suitable cover on roadside areas is the elimination of noxious weeds. Weeds may be undesirable because of appearance, hazard, or annoyance to the public from such growths as poison ivy or ragweed, or because of the infection they carry to weed-free agricultural areas. Adequate mowing is the foremost form of weed control, but weed-killing chemicals may often be economically and effectively used to supplement it In rural areas, plant growth of non-noxious types may desirably be left in fence corners, etc., as a shelter for wild life

Care of Trees and Shrubs. This includes, primarily, the elimination of diseased and dead wood and, in some regions, spraying of certain species of trees for the control of insect pests or insect-borne diseases. When spraying is done, care should be taken to keep spray materials off passing automobiles, and off adjacent homes. The planting of young trees and such supplementary care as fertilizing, watering, pruning, and guying may also be part of roadside maintenance, as is the emergency removal of fallen trees and branches after storms. Much tree work is seasonal, and fortunately, it is naturally well distributed through the year. The trimming of large trees is hazardous work that should be entrusted only to competent personnel adequately provided with the proper tools, ropes, and other safety equipment. Especial care must be taken where any utility wires may be encountered. The trimming of trees to clear such wires should generally be a responsibility of the owning utility, with supervision from the highway authority to see that unnecessary damage is not done to roadside growth. In trimming trees, care must also be taken not to endanger the traveling public from falling branches, etc. Besides eliminating dead and diseased wood, selective trimming and pruning of roadside tree and shrub growth may be required to maintain sight lines at horizontal and vertical curves, intersections, etc ; to preserve the visibility of warning and direction signs; to prevent interference with the prescribed standard vertical vehicle clearance, and to eliminate certain types of vegetation that serve as hosts to insect pests. The waste products of roadside trimming may be disposed of by burning, care being taken to observe laws relating thereto. Noxious weeds may also advantageously be burned, to insure destruction of their seeds.

Maintenance of Cut-and-Fill Slopes. Both cut and fill slopes are best stabilized with vegetable growth, where this is feasible. In some cut slopes, it may be necessary to remove excessive water by installing drains. Erosion should be closely watched, on both cut and fill slopes, and checked, if possible, before it causes serious damage This may require the construction of berms and downspouts on fills, and of intercepting ditches on the top of slopes. Rock slopes in cuts should be watched for loose rocks and scaled as necessary.

Picnic and Rest Areas. These should be carefully planned from the standpoint of pleasantness and sanitation of use, and lack of any hazard in entering or leaving. This latter implies adequate advance notice of the area (preferably about 1000 ft), as well as suitable parking facilities with well-designed entrances and exits, where necessary. The area may vary from a simple table with attached benches and a trash receptacle, under a tree beside the road, to areas of substantial size with parking facilities, an extensive layout of footpaths, tables, and fireplaces, drinking water, toilet facilities, etc. In every case, convenient receptacles for the deposit of trash and litter must be provided, and, even so, frequent attention must be given to cleaning the area.

SECTION 5

AIRPORT ENGINEERING

BY

PHILLIPS MOORE

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GENERAL DATA

This section covers the essential planning and design considerations involved in the development of airports to be used primarily for civilian operations. Although civilian and commercial requirements are stressed, it should be understood that the entire section is applicable to military airports However, since operational needs at military airports may differ greatly from those at civil airports, each military agency should be consulted relative to the planning and design details peculiar to its branch of service.

1. PRELIMINARY PLANNING

In planning the development of an airport, the principal problem is to determine what type of airport will adequately serve the future as well as the immediate aviation needs of the community, with due consideration being given to the relationship that the proposed airport will bear to existing and other proposed airports in the area. Information relative to other airports in the area can be obtained from local, state, and federal aviation agencies. Coordination with these agencies during the preliminary planning stages is extremely important. It will insure the future of the airport by the fact that it will be part of an approved airport system.

Preliminary planning for any airport should take into consideration (1) population of the city and its trading area; (2) distance between adjoining cities or airports and the area in square miles to be served by the airport; (3) local topography as it affects the cost of construction; (4) local flying, including the present and future requirements of flying schools, the number of privately owned aircraft located in the community, the number owned by commercial firms, and charter operations, both local and itinerant; (5) scheduled air-line requirements both intrastate and interstate, present and future; (6) aerial tourist travel, including private planes, charter operations, and seasonal fluctuations of air-line traffic due to tourist travel; (7) postal receipts of the city and its trading area to evaluate air-mail and express possibilities; (8) retail sales of the city and trading area which will give some indications of the potential aviation business in the locality; (9) national defense; (10) intermediate point on an established airway for emergency, refueling, or convenience; and (11) possibility of diversity and volume of air traffic necessitating the location of several airports of different classes around a municipality.

2. NATIONAL AIRPORT PLAN

A National Airport Plan is prepared by the Civil Aeronautics Administration in accordance with the requirements of the Federal Airport Act (Public Law 377, 79th Congress, as amended) To assist in the development of airport facilities required for locations included in the National Airport Plan, the Administrator is authorized to grant federal aid to appropriate sponsoring agencies

3. FEDERAL AID

The Federal Airport Act was passed by the 79th Congress (2nd Session, Public Law 377) to provide for the development or improvement of public airports within the continental United States, Alaska, Hawaii, and Puerto Rico. On April 17, 1948, the act was amended by the 80th Congress (2nd Session), to include the Virgin Islands in the program. Under the act, as further amended, annual appropriations amounting in the aggregate to $520,000,000 are authorized to be made to the Administrator over a period of twelve fiscal years. The maximum appropriation allowed for any fiscal year shall not exceed $100,000,000 and shall remain available until June 30, 1958, unless sooner expended.

Within the limits of fiscal-year appropriations and contract authority, projects are programmed to the extent possible from the National Airport Plan in the following manner: (1) submission of requests for federal aid by eligible sponsors, (2) evaluation of the proposed projects for the preparation of a tentative allocation, (3) issuance of tentative allocations and funds to sponsors.

Throughout each fiscal year the program is constantly under review. It is analyzed in regard to progress on the specific projects included and in relation to general aeronautical trends. Every effort is exerted toward creating a comprehensive national network of airports to encourage the development of air commerce in the United States, and to increase the national security.

In general, the United States' share of costs of an approved project for the development of an airport is 50% of allowable project costs (other than costs of land acquisition or of installation of high-intensity lighting on designated instrument-landing runways).

Exceptions to this rule are (*a*) the states containing unappropriated and unreserved public lands and non-taxable Indian lands in excess of 5% of the total area of all land within the state, and (*b*) the Territory of Alaska and the Virgin Islands where the U. S. share is 75%.

Of the federal-aid funds appropriated by Congress, 75% is apportioned among the states, one-half in the proportion which the population of each state bears to the total population of all the states, and one-half in the proportion which the area of each state bears to the total area of all the states.

The remaining 25% goes into a discretionary fund which the Administrator may use for allowable project costs on approved projects regardless of the states in which they are located.

Engineers should check with local CAA [1] district offices relative to the current changes in the U. S. share, which may result from amendments to the Federal Airport Act.

4. TRAFFIC AND ECONOMIC STUDIES

In order to formulate and revise the National Airport Plan, it is necessary to (1) determine the amount and kinds of existing and potential needs of both air commerce and private flying for airports on a nationwide basis, (2) examine and evaluate the existing airport system; (3) utilize to the best advantage existing facilities; (4) anticipate additional facilities needed; and (5) determine the relative urgency of necessary airport development for inclusion in the National Airport Plan so that the right kind of airports may be provided in the right places—and at the right time.

Fundamental to these determinations are general economic studies aimed at establishing criteria and standards of community aeronautic need, measuring the aeronautic potential of the communities of the country and translating this potential into airport requirements. Such general studies are carried on by the CAA, and the results are available to airport planners and designers. These studies deal with air-transportation and air-cargo potentials, traffic volume and peak movements; space requirements and utilization at terminal airports are issued as airport planning publications. As of June 1950, the following reports have been issued. (1) Economic Character of Communities, (2) Airline Passengers, (3) Airmail Potentials, (4) Domestic Air Cargo, (5) Airport Activities, (6) Personal Aircraft, (7) Peak Air-Carrier Movements at Airports, (8) Airline Travel Distances, (9) Airport Facilities, (10) Community Airport Requirements, and (11) Effective Community Air-Traffic Potential.

The information contained in these reports is extremely useful in estimating the airport needs of a community, and they should be consulted in the preparation of an airport plan.

5. AIRCRAFT TYPES

Dimensions, weights, and other characteristics of typical aircraft are given in Table 1; a key to the dimensions is shown in Fig. 1. Conventional landing gear refers to aircraft having a tail wheel as compared with planes having tricycle landing gear. Transport planes having tricycle landing gear are equipped with dual wheels on each landing strut; all others have single wheels.

[1] In the following pages the symbol CAA will be used when referring to the Civil Aeronautics Administration of the U. S. Department of Commerce.

Table 1. Aircraft Types

Company	Model Designation	Dimensions, ft and in: A Wing Span	B Length	C Height	D Tail Width	E Tread	Gross Weight, lb	Number of Engines	Total Hp
Small planes, conventional landing gear									
Aeronca	7AC-Champion	35 0	21 6	7 0	10 2	6 10	1,220	1	65
Aeronca	11CC-Super Chief	36 1	20 9	6 7	10 2	6 0	1,350	1	85
Bellanca	14-13-2 Cruisair	34 2	21 3	6 3	10 10	8 11	2,100	1	150
Cessna	140	32 10	21 6	6 3	8 10	6 5	1,450	1	85
Cessna	195	36 2	27 4	7 2	10 7	7 11	3,350	1	300
Fairchild	F-24	36 4	25 10	7 8	12 2	9 3	2,562	1	165
Luscombe	8E	35 0	20 0	5 11	10 10	6 4	1,400	1	85
Luscombe	11-A	31 0	23 0	5 2	11 10	8 4	2,280	1	165
Meyers	MAC-125C	30 0	20 10	8 6			1,675	1	125
Piper	PA-11 (Trainer)	35 3	22 5	6 8	9 6	6 1	1,220	1	65
Piper	PA-12 (Cruiser)	35 6	23 0	6 10	9 6	6 3	1,750	1	100
Stinson	150 (Voyager)	34 0	24 6	6 10	11 2	7 1	2,150	1	150
Taylorcraft	BC12-D	36 0	22 0	6 10	10 0	6 0	1,200	1	65
Texas	GG-1A (Swift)	29 4	19 7	6 1	9 9	9 9	1,710	1	125
Small planes, tricycle landing gear									
Beech	35-Bonanza	32 10	25 2	6 7	10 6	9 7	2,550	1	165
Erco	415-D Ercoupe	30 0	20 9	6 0			1,400	1	75
Ryan	Nav-4 Navion	33 5	27 3	8 9		8 3	2,750	1	185
Transport planes, conventional landing gear									
Beech	D-18-S	47 7	34 0	9 3	13 6	12 11	8,750	2	900
Boeing	247D	74 0	51 4	15 5	25 6	17 3	14,000	2	1,100
Boeing	307B-Stratoliner	107 3	74 4	20 9	44 0	24 7	54,000	4	4,400
Douglas	DC-3	95 0	64 5	16 11	26 8	18 6	25,200	2	2,400
Lockheed	18-08 Lodestar	65 6	49 10	17 10	25 10	15 4	18,500	2	2,400
Northrup	N-23 Pioneer	87 0	66 6	16 10	32 6	23 3	25,700	3	2,400
Transport planes, tricycle landing gear									
Boeing	377-Stratocruiser	141 3	110 4	38 3	43 0	28 6	140,000	4	14,000
Consolidated Vultee	Convair-Liner	91 9	74 8	26 11	36 6	25 0	39,500	2	4,800
Douglas	DC-4	117 6	93 5	27 7	39 6	24 8	73,000	4	5,800
Douglas	DC-6	117 6	101 6	28 5	46 5	24 8	93,200	4	8,400
Lockheed	749-Constellation	123 0	95 1	23 10	50 0	28 0	102,000	4	10,000
Martin	202	93 4	71 4	28 5	36 6	25 0	39,900	2	4,800

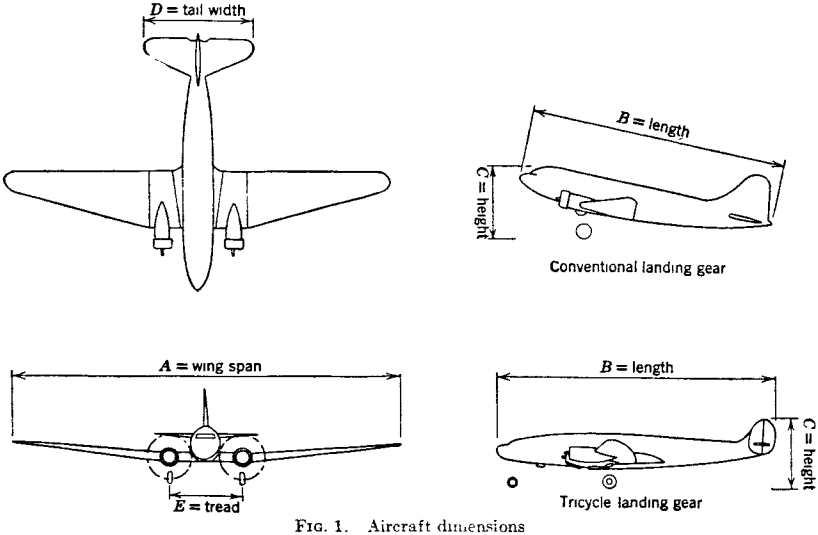


FIG. 1. Aircraft dimensions

6. DESIGN STANDARDS

Runways and Taxiways. Dimensional standards, strength requirements, gradients, and clearances for runways and taxiways, as recommended by the CAA, are given in Tables 2 and 3. The airport service types are defined as follows:

Personal: Airports to handle light (up to 3000 lb) aircraft for small communities or urban areas.

Secondary: Airports for larger (2000–15,000 lb) aircraft in non-scheduled flying activities.

Feeder: Airports to serve certificated feeder airlines.

Trunk line: Airports to serve smaller cities on airline trunk routes.

Express: Airports at important cities or junction points on trunk routes.

Continental: Airports serving aircraft making long non-stop domestic flights.

Intercontinental: Airports terminating long international flights.

Intercontinental express: Airports serving the highest type of transoceanic flights.

Table 2. Airport Design Standards

Type of Service	Runway Length, ft	Minimum Widths, ft: Landing Strip	Minimum Widths, ft: Runway	Minimum Widths, ft: Taxiway	Maximum Grades, %: Effective *	Maximum Grades, %: Longitudinal	Maximum Grades, %: Transverse	Pavement Loading per Wheel in 1000 lb: Single Wheel	Pavement Loading per Wheel in 1000 lb: Dual Wheel
Personal	1500–2300	200	50 †	20 †	2	3	3	‡	‡
Secondary	2301–3000	250	75 †	30 †	1 1/2	2	2	‡	‡
Feeder	3001–3500	300	100	40	1	1 1/2	1 1/2	15	20
Trunk line	3501–4200	400	150	50	1	1 1/2	1 1/2	30	40
Express	4201–5000	500	150	60	1	1 1/2	1 1/2	45	60
Continental	5001–5900	500	150	75	1	1 1/2	1 1/2	60	80
Intercontinental	5901–7000	500	200	75	1	1 1/2	1 1/2	75	100
Intercontinental express	7001–8400	500	200	100	1	1 1/2	1 1/2	100	125

* Maximum effective gradient is obtained by dividing the maximum difference in runway center-line elevation by the total length of the runway.

† Pavement not required

‡ When pavement is used it shall be designed in accordance with Part VII of this section of handbook

Table 3. Minimum Clearance Standards on Airports

Type of Service	Runway Center Line to Taxiway Center Line	Center Line of Parallel Runways, Contact Operations *	Center Line of Parallel Taxiways	Center Line of Taxiways to Aircraft-Parking Area	Center Line of Taxiway to Obstruction	Center Line of Runway to Building Line (Instrument)	Center Line of Runway to Building Line (Non-Instrument)
Personal	75	150	60	50	50	. .	150
Secondary	150	300	125	110	75		225
Feeder	200	500	150	130	100	750	300
Trunk line	250	500	200	175	100	750	350
Express	300	500	250	220	125	750	425
Continental	350	500	275	240	150	750	500
Intercontinental	400	700	300	260	175	750	575
Intercontinental express	450	700	325	280	200	750	650

* For instrument operations, a minimum of 3000 feet has been suggested but no firm standard has been established. The matter is still under study at this writing.

Runway lengths shown in Table 2 are for sea-level elevation, standard sea-level temperature of 59°F, and zero per cent effective gradient.

These lengths should be increased for airport elevation at the rate of 7% for each 1000 ft of elevation above sea level. This corrected length should be further increased at the rate of one-half of 1% for each degree that the mean temperature of the hottest month of the year, averaged over a period of years, exceeds the standard temperature.

The runway length should also be increased to correct for runway gradient at the rate of 20% of the length corrected for altitude and temperature for each 1% of effective runway gradient. The effective runway gradient is determined by dividing the maximum difference in runway center-line elevation by the total length of the runway.

The mean (average) monthly temperature of the hottest month may be obtained from the local U. S. Weather Bureau station, or from the Weather Bureau's publications Bulletin W and Climatological Data When the mean temperature is available for July only, this may be used It can be obtained from the 1941 Yearbook of Agriculture *Climate and Man*, publication of the U. S. Department of Agriculture. The standard temperature of a site can be obtained by reducing the standard sea-level temperature of 59°F at the rate of 3.566° per thousand feet of elevation

Example of Runway-Length Correction The following example shows the corrective procedure:

Determine the length of runways required when the runway length under standard conditions is 4200 ft and the airport has an elevation of 3400 ft, with 70°F as the average temperature of the hottest month and an effective gradient of 0.25%.

1 Altitude correction: Runway length to be increased 7% per 1000 ft of elevation above sea level, or

$$(0.07 \times 3.400)4200 + 4200 = 5200 \text{ ft}$$

2. Temperature correction: Runway length corrected for altitude to be further increased 0.5% for each degree which the temperature of the hottest month exceeds the standard temperature of the site, or

$$59 - (3.566 \times 3.400) = 46.9°\text{F}$$

$$0.005(70 - 46.9)5200 + 5200 = 5800 \text{ ft}$$

3. Effective gradient correction: Runway length corrected for altitude and temperature to be further increased 20% for each 1% of effective gradient, or

$$(0.20 \times 0.25)5800 + 5800 = 6090 \text{ ft}$$

Using a length to the nearest 100 ft, the corrected runway length for temperature, elevation, and gradient, therefore, is 6100 feet.

Obstructions to Air Navigation. In order to provide uniform criteria, the CAA, in conjunction with the Federal Communications Commission, the Department of Defense, and appropriate branches of the aviation and broadcasting industries, has developed a standard method for determining obstructions to air navigation. This standard has been issued as Technical Standard Order N18.[2] It establishes the allowable heights of obstructions in the approach ways and turning zones.

Clearance over Highways and Railroads. Figure 2 shows the recommended minimum clearances for (1) the horizontal distance between the end of the landing strip and the railroad or highways, and (2) the vertical distance between the railroad or highway and the obstruction clearance line. It may be desirable to increase these minimum clearances where the extended center line of the runway intersects the highway or railway at an acute

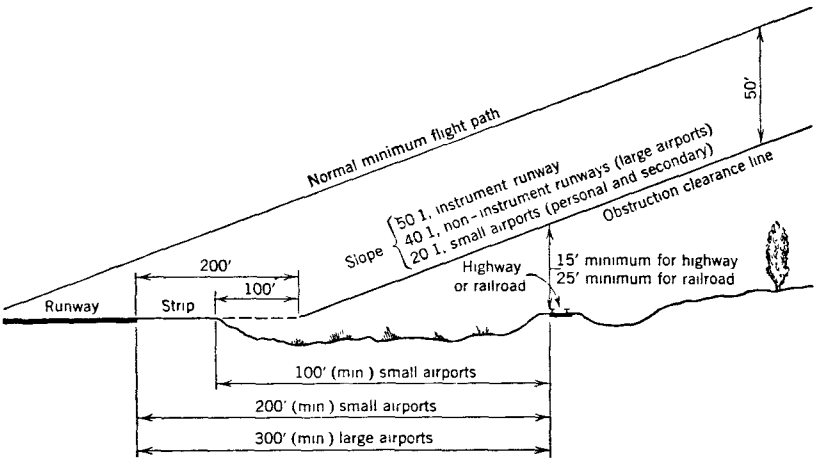


FIG 2 Clearance over highways and railroads

[2] TSO-N18, Criteria for Determining Obstructions to Air Navigation, April 26, 1950, available from Aviation Information Office, Civil Aeronautics Administration, Washington 25, D. C.

angle or where the portion of the highway or railway adjacent to the intersection has an appreciable gradient.

Vertical Curves and Sight Distances. For safety during take-off and landing operations, abrupt grade changes on runways should be avoided. For this reason, longitudinal grade changes should not exceed 3% for personal and secondary airports or 1 1/2% for feeder or larger airports. Longitudinal intersecting grades should be connected with vertical curves when the grade change is greater than 0.40% The length of vertical curves should be 300 ft for each 1% change in grade for personal or secondary airports, and 500 ft for each 1% change in grade for feeder or larger airports

Changes in grades should be located so that the distance in feet between two successive points of intersection will not be less than 25,000 times the sum of the grade changes (expressed as a decimal) at these successive points of intersection for personal or secondary airports, and 50,000 times the sum of the grade changes for feeder or larger airports. These requirements for grades are illustrated in Fig 3

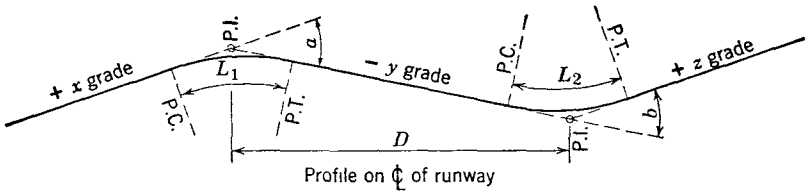


FIG. 3. Vertical curves required when grade changes such as a or b exceed 0.40%.

	Personal or Secondary	Feeder or Larger Airport
Maximum grade change such as a or b not to exceed	3%	1 1/2%
Length of vertical curve (L_1 or L_2)	300′ for each 1% grade change	500′ for each 1% grade change
Distance in feet between points of intersections for vertical curves, or D	25,000 $(a + b)$	50,000 $(a + b)$

Example Assume $x = +1.0\%$, $y = -0.5\%$, and $z = +0.4\%$. Then a (algebraic difference between x and y) = 1.5% and b = 0.9%.

For personal or secondary airports.

L_1 (length of vertical curve) = 300 × 1.5 = 450′ and L_2 = 300 × 0.9 = 270′
D (distance between points of intersections) = 25,000 (0.015 + 0.009) = 600′

For feeder or larger airports:

L_1 = 500 × 1.5 = 750′ and L_2 = 500 × 0.9 = 450′
D = 50,000 (0.015 + 0.009) = 1,200′

As a minimum requirement, runway grade changes should be such that there will be an unobstructed line of sight from any point 5 ft above the runway to any other point also 5 ft above the runway within a distance of at least 500 ft plus one-half of the length of the runway. The length of runway required for ultimate development of an airport should be considered when determining these sight distances.

7. ICAO DESIGN STANDARDS

The International Civil Aviation Organization (ICAO) has recommended certain standards and practices which differ somewhat from those in Tables 2 and 3. The ICAO recommendations are summarized in Tables 4, 5, 6, 7, and 8. These tables give the physical characteristics of the various classes of airports. The classification is based on the two principal characteristics: a dimensional characteristic (the basic length for standard conditions at sea level of the main runway) and a strength characteristic (the load-bearing capacity of that runway). The class of the airport is denoted by a capital letter indicating the basic length followed by a numerical index indicating its load-bearing capacity. Thus, a Class A1 airport would be defined as one whose main runway had a basic length of not less than 8400 ft or 2550 meters (see Table 4), and a strength (see Table 7) capable of withstanding the maximum expected frequency of traffic of aircraft imposing single isolated wheel loads of 100,000 lb (45,000 kg) at a tire pressure of 120 lb sq in. (8.5 kg sq cm).

In Table 5, all strips consist of a basic strip plus lateral supplementary rectangular areas as required. The length of the basic strip should be such that it extends 200 ft (60 meters) beyond each end of the runway, and the supplementary rectangular areas should be the same length as the basic strip.

All basic strips should have a width of 500 ft (150 meters). The overall widths in Table 5 include the added widths of the supplementary areas. For example, in the case of an instrument runway, the overall strip should be 1000 ft (300 meters) wide which is equal to the basic strip width of 500 ft (150 meters) plus supplementary rectangular areas 250 ft (75 meters) wide adjoining each side of the basic strip

In the case of non-instrument runways at airports of Classes A, B, and C, the total strip width should be 700 ft (210 meters), which equals the basic strip of 500 ft (150 meters) plus supplementary rectangular areas 100 ft (30 meters) wide adjoining each side of the basic strip. At airports of Classes D, E, F, and G, the strip width should be the basic strip width of 500 ft (150 meters).

Table 4. ICAO Recommended Standards: Runways

Airport Class	Basic Length of Main Runway, ft (meters)	Length of Other Required Runways	Minimum Width of All Required Runways, ft (meters)	Longitudinal Slopes of Runways: Not to Exceed, Overall	Longitudinal Slopes of Runways: Not to Exceed, Partial	Longitudinal Slopes of Runways: Rate of Change of Slope Not to Exceed	Transverse Slopes of Runways Not to Exceed
A	Not less than 8400 (2550)	Generally not less than 85% and never less than 70% of basic length of main runway	Not less than 200 (60)	1%	1.25%	0.3% per 100 ft (30 m)	1.5%
B	Less than 8400 (2550) and not less than 7000 (2150)	Same	Not less than 200 (60)	1%	1.25%	Same	1.5%
C	Less than 7000 (2150) and not less than 5900 (1800)	Same	Not less than 150 (45)	1%	1.5%	Same	1.5%
D	Less than 5900 (1800) and not less than 5000 (1500)	Same	Not less than 150 (45)	1%	1.5%	Same	1.5%
E	Less than 5000 (1500) and not less than 4200 (1280)	Same	Not less than 150 (45)	1%	1.5%	Same	1.5%
F	Less than 4200 (1280) and not less than 3500 (1080)	Same	Not less than 100 (30)	1%	1.5%	Same	1.5%
G	Less than 3500 (1080) and not less than 3000 (900)	Same	Not less than 100 (30)	1%	1.5%	Same	1.5%

Table 5. ICAO Recommended Standards: Strips

Airport Class	Length of Strips (applies to both basic strip and supplementary rectangular areas)	Widths of Strips: Instrument, Basic, ft (meters)	Widths of Strips: Runway, Overall, ft (meters)	Widths of Strips: Other Runways, Basic, ft (meters)	Widths of Strips: Other Runways, Overall, ft (meters)	Slopes of Strips *: Longitudinal, Overall (basic strip)	Slopes of Strips *: Longitudinal, Partial (basic strip)	Slopes of Strips *: Transverse, Basic Strip	Slopes of Strips *: Transverse, Supplementary Strips
A	Full length of runway plus 200 ft (60 m) beyond at each end	500 (150)	1000 (300)	500 (150)	700 (210)	Essentially the same as runway	Not to exceed 1.75%	Not to exceed 2.5%	Not to exceed 5%
B	Same	500 (150)	1000 (300)	500 (150)	700 (210)	Same	Same	Same	Same
C	Same	500 (150)	1000 (300)	500 (150)	700 (210)	Same	Not to exceed 2%	Same	Same
D	Same	500 (150)	1000 (300)	500 (150)	500 (150)	Same	Same	Same	Same
E	Same	500 (150)	1000 (300)	500 (150)	500 (150)	Same	Same	Same	Same
F	Same	500 (150)	1000 (300)	500 (150)	500 (150)	Same	Same	Same	Same
G	Same	500 (150)	1000 (300)	500 (150)	500 (150)	Same	Same	Same	Same

* Slope changes on basic strip may be greater than on runways but should be as gradual as practicable, and abrupt changes or reversals of slopes should be avoided

Table 6. ICAO Recommended Standards: Taxiways, Approaches, Clearances

Airport Class	Minimum Width of Taxiways ft (meters)	Slopes of Taxiways: Longitudinal, Not to Exceed	Slopes of Taxiways: Transverse, Not to Exceed	Slopes of Taxiways: Rate of Change of Longitudinal Slopes Not to Exceed	Approach Surface Slopes: Instrument Strip	Approach Surface Slopes: Non-Instrument Strip	Between Center Line of Taxiway and Runway: Instrument Runway, ft (meters)	Between Center Line of Taxiway and Runway: Non-Instrument Runway, ft (meters)	Between Center Lines of Taxiways, ft (meters)	Between Center Lines of Taxiways and Fixed Obstructions, ft (meters)
A	100 (30)	3%	1.5%	1% per 100 feet (30 m)	1 in 50	1 in 40	700 (210)	550 (165)	325 (100)	180 (55)
B	75 (23)	3%	1.5%	Same	1 in 50	1 in 40	650 (200)	500 (150)	300 (90)	165 (50)
C	75 (23)	3%	1.5%	Same	1 in 50	1 in 40	600 (180)	450 (135)	250 (75)	140 (43)
D	60 (18)	3%	1.5%	Same	1 in 50	1 in 40	600 (180)	350 (105)	250 (75)	125 (38)
E	50 (15)	3%	1.5%	Same	1 in 50	1 in 30	600 (180)	350 (105)	200 (60)	100 (30)
F	40 (12.5)	3%	1.5%	Same	1 in 50	1 in 30	550 (165)	300 (90)	150 (45)	100 (30)
G	40 (12.5)	3%	1.5%	Same	1 in 50	1 in 25	550 (165)	300 (90)	125 (38)	80 (24)

Table 7. ICAO Recommended Standards: Strengths of Runways, Taxiways, and Aprons

Airport Class	Pavement to Be Capable of Withstanding the Maximum Expected Frequency of Traffic of Aircraft Imposing Single Isolated Wheel Loads of the Weight Shown Below: Lb	Kg	Tire Pressure to Be Associated with Loads for Purpose of Design: Lb/sq in.	Kg/sq cm
A	100,000	45 000	120	8.5
B	75 000	35,000	100	7 0
C	60,000	27,000	100	7 0
D	45,000	20,000	100	7.0
E	30,000	13 000	85	6.0
F	15,000	7.000	70	5.0
G	5,000	2 000	35	2.5

Table 8. ICAO Recommended Standards: Minimum Clearances

Airport Class	Between Center Lines of Taxiways and Runways: Instrument Runways, ft (meters)	Between Center Lines of Taxiways and Runways: Non-Instrument Runways, ft (meters)	Between Center Lines of Taxiways, ft (meters)	Between Center Line of Taxiway and Face of Fixed Obstruction, ft (meters)
A	700 (210)	550 (165)	325 (100)	180 (55)
B	650 (200)	500 (150)	300 (90)	165 (50)
C	600 (180)	450 (135)	250 (75)	140 (43)
D	600 (180)	350 (105)	250 (75)	125 (38)
E	600 (180)	350 (105)	200 (60)	100 (30)
F	550 (165)	300 (90)	150 (45)	100 (30)
G	550 (165)	300 (90)	125 (38)	80 (24)

A taxiway should generally leave the end of the runway it serves at right angles for at least 100 ft (30 meters).

In all cases, the minimum specified clearance between center lines of taxiways and the runways they serve should be attained within a distance not exceeding the specified clearance, such distance being measured along the center line of the runway (extended if required) from the junction of the center lines of the runway and the taxiway.

When parallel runways are provided for simultaneous use under visual conditions, the minimum distance between their center lines should be 700 ft (210 meters) for classes A, B, and C, and 500 ft (150 meters) for classes D, E, F, and G.

With respect to strip slopes (Table 5), the longitudinal slopes of the basic strips should be essentially the same as those of the runways they enclose. The longitudinal slope of any portion of a basic strip should not exceed 1.75% for Classes A and B and 2% for Classes C, D, E, F, and G.

In connection with pavement-strength requirements given in Table 7, it is recommended that a particular strength class should be capable of bearing the specified loading for a frequency of operations corresponding to the expected traffic, but not less than 10 operations (landing or take-off) per day.

SITE SELECTION

8. SAFETY FACTORS

Freedom from Obstructions. An airport site should be so selected that it may be kept free from what would constitute a hazard to aircraft in the turning and approach zones. The removal of an existing obstruction, whether natural or man-made, should be physically and economically possible to warrant consideration of the site as an airport. Future growth or erection of obstructions should be guarded against by adequate zoning laws, and property interests and avigation easements, if necessary.

Visibility. Good visibility is more essential for air transportation than for any other type. This requirement becomes most acute in the vicinity of an airport and in the approach zones. Often, a condition of poor visibility exists only in relatively small areas and results either from local features of terrain (swamps and bodies of water) or from nearby smoke-producing industrial plants. The direction of industrial development within an area should be determined and the airport so located as not to be in the direction of a potential smoke hazard arising from future industrial development. The properly chosen site should be the one least affected by poor visibility, thus reducing hazards and interruptions to operations.

Direction of Prevailing Winds. An important consideration relative to safety in operations is the direction of the prevailing winds with respect to the landing areas provided by an airport. Landings and take-offs that can be made into the wind are the most desirable. Hence, an airport site should be selected which permits the economical construction of a landing strip oriented into the prevailing winds.

9. ECONOMIC FACTORS

In most cases, the choice between possible airport sites will be greatly influenced by various economic factors. These factors must be recognized and carefully considered in light of the present and future activities at the airport This can best be accomplished by using engineering studies which develop the relative values of the various sites considered and lead to the most logical selection Two salient factors to be analyzed are cost of development of airport and cost of operation.

Cost of Development. Extremes of cost are possible in the development of an airport of any particular class. Some of the factors which contribute to the variation in cost are the conditions peculiar to the site selected, the character and quality of the essential facilities installed, and the extent to which the design is modified for future expansion. Development costs include land, construction, and utilities.

Cost of Operation. The cost of operating the airport comprises management, maintenance, and replacement charges. Although many of the items included in the cost of operation will be identical for the different sites under consideration for a particular airport project, it is well to estimate completely what these charges will be. Any study of cost of operations should also include an appraisal of the revenue to the airport. Frequently, the location of the airport will have a great influence on the activities which can be conducted at the airport and the revenue which can be obtained from these activities. Analysis of the cost of providing various services and the income to be derived therefrom will also indicate the provisions which should be made for the various activities and will, therefore, influence the development proposed at each site. Some of the services to be rendered at an airport may not be a source of direct revenue, but are required to supplement other activities which provide income.

10. AIRSPACE REQUIREMENTS

The site chosen for an airport should be that site which, when developed, will have a traffic pattern that causes the least interference with traffic of nearby airports. If such interference cannot be avoided, unconventional flight patterns are required, these, though they tend to minimize the hazard, may cause confusion The minimum distance between airports for contact operations should be equal to the sum of the radii of their traffic patterns. The radii of airport traffic patterns for contact operations are as follows: Personal

and secondary, 1 mile; feeder, 2 miles; trunk line and express, 3 miles; continental and larger, 4 miles.

Greater separation, depending on terminal air space needs, may be required if instrument operations are to be conducted. A study of current or potential instrument traffic control at airports in the area is essential to insure sufficient separation between airports in order to permit maximum usage.

Most cross-country flying is done within the limits of well-defined airways which are established, regulated, and equipped with navigation aids by the Federal Government. Viewing the amount of traffic expected to use these routes in the future, it is important that airports be so located as to contribute the least possible unnecessary congestion to these air spaces, while providing accessible landing facilities for planes using the airways.

11. INTEGRATION WITH CITY PLAN

Since airports are an integral part of the community, whether municipally or privately owned, they are subject to the same community planning principles and policies as other elements in the master plan and must be coordinated with existing as well as planned features. Especially important is the provision of adequate water supply, sewerage, electric light and power, fire and police protection. Attention must be given to the possible effects of the airport on land use in the contiguous areas, and vice versa. The relatively vast area of an airport, as in the case of a large park or a golf course, may provide a barrier to the extension of the arterial-highway system as well as to the street pattern. An attempt should be made to select sites which will not conflict in this regard.

The provision and protection of adequate approach zones will necessitate more or less restrictive height zoning near the airport and in line with the landing strips, in a few instances restrictive-use zoning may also be needed. In general, however, the territory between the approach zones will lend itself to more intensive community development, frequently to a complete neighborhood development. The more restricted area in the approach zones, especially the open areas near the airport boundary, may be found useful as playground and recreational areas.

12. ACCESS HIGHWAYS

In the selection of an airport site, and particularly in weighing the relative merits of several locations, the facilities for ground transportation to and from the area in question assume considerable importance. To assure uninterrupted transportation service, the selection of a site which is conveniently located with respect to the existing highway system and other means of transportation is an important factor in enabling a city to obtain the maximum benefits from air travel. The frequency, speed and cost of existing or proposed means of transportation should be considered more than the distance in miles of the proposed site from the city.

LAYOUT PLAN

13. GENERAL

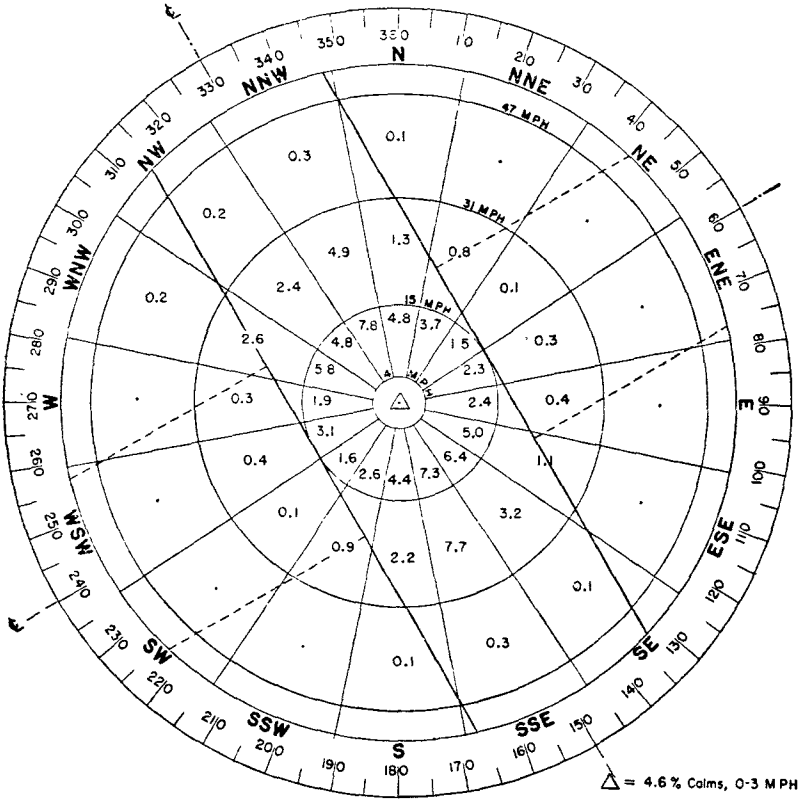
Each airport presents an individual design problem, the solution of which depends on the proper consideration of several factors. The principal determinations to be made are number and orientation of runways, amount and location of space to be devoted to aircraft servicing and storage, and the circulation and parking of ground vehicles.

The landing area, being the least flexible, should be studied first. From an analysis of the expected traffic and wind data, the number and orientation of runways can be determined. Runways should be oriented so that aircraft may be landed at least 95% of the time with cross-wind components not exceeding 15 mi/hr because this is the most desirable condition for conventional aircraft, other than transports which can be landed without hazard or discomfort to the occupants in winds with cross components of 20 to 30 mi/hr.

Usually, 95% coverage with cross-wind components not exceeding 15 mi/hr will provide 98–100% coverage with less than 30 mi/hr cross-wind components. Because nearly all air-carrier airports will also serve light aircraft, the wind data should be analyzed on a basis of the 15-mile cross-wind component in addition to any other studies which may be made.

14. WIND ROSE

The most desirable direction of runways for wind coverage can be selected by examining wind-data characteristics of the area, which has been plotted on a wind-rose base such as that shown in Fig. 4. On this wind rose, the numerals (shown in the segments bounded



HURON, S.D.

AIRPORT LOCATION

Surface Wind Velocity over a period of 4 years and 0 months. U.S. Weather Bureau (AIRPORT) (CITY) Station
Check Which

FIG 4. Wind rose.

by the arc of the circles representing wind velocity and the radial lines representing compass direction) show the percentage of time winds of the indicated direction and velocity can be expected to occur at the designated location.

Use of the wind rose in determining desirable runway directions requires a strip of transparent material on which three parallel and equally spaced lines have been plotted. The middle line represents the runway center line, and the distance between it and each of the outside lines represents the allowable cross-wind component. The distance between the outside lines equals twice the allowable cross-wind component and must be plotted to the same scale as the wind-velocity scale on the wind rose with which it is to be used.

Procedure for establishing runway orientation is as follows: First, place the transparent strip over the wind rose in such a manner that the line representing the runway center line passes over the center of the wind rose. Then, using the center of the wind rose as a

pivot point, rotate the transparent strip until the sum of the percentages appearing between the outside lines on the strip becomes a maximum. The percentage shown in each segment is assumed to be equally distributed over the area of the segment. When one of the outside lines on the transparent strip crosses a segment, a fractional part of the percentage appearing in that segment is to be used in the summation of the percentages. This fractional part is equal to the fractional part of the segment appearing between the two outside lines on the transparent strip. Determining these fractional areas visually and tabulating numbers to the nearest tenth is adequate and consistent with the accuracy of wind data.

The next step is to read the bearing for the runway on the outer scale of the wind rose where the center line on the transparent strip crosses the angular scale. Because true north is used for published wind data, this bearing usually will be different from that used in designating runways for numbering and marking, which is based on the magnetic bearing. Referring to Fig. 4, it will be noted that a runway oriented 150–330° (S. 30° E., true) will permit operations 95% of the time with the cross-wind components not exceeding 15 mi/hr.

15. WIND DATA AND RUNWAY ORIENTATION

Winds for a particular location frequently have not been recorded. Thus, the data from the two or more nearest wind-recording stations must be used to establish wind characteristics for the site. When the recording stations are located around the airport site and the intervening terrain is level or slightly rolling, a composite wind rose made from those of the surrounding stations is usually sufficient to give the desired information. If the intervening terrain is mountainous, allowance for its effect on wind can be made by weighted averages. In determining the values to be applied to the records of each individual station, a topographic map of the area is needed on which the location of the wind-recording station has been plotted.

The direction of runways which give the most wind coverage may have to be shifted because of obstructions in the approach zones. Runways should be oriented in the direction of the best approaches meeting the standards or exceeding them where possible. At least one runway should be provided for instrument operations for airports which ultimately will be developed for express-type service or larger. Such runways should be oriented so that the instrument approaches will not conflict with instrument-approach patterns of other airports in the area. Runways with approaches free of obstruction are more desirable than those with maximum wind coverage but limited approaches.

16. RUNWAY CONFIGURATION

Traffic volume is the major item to consider when determining the number of runways required. At one time it was thought that satisfactory coverage could only be obtained

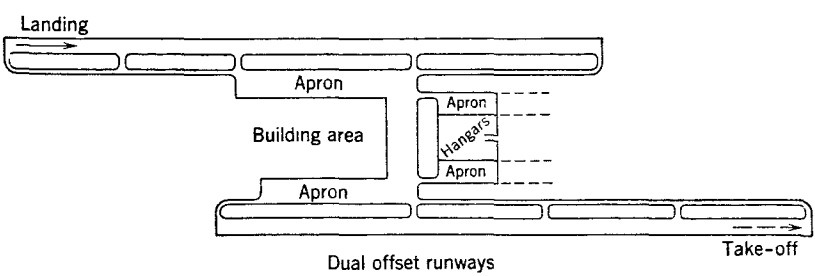


FIG. 5. One-runway direction layout

with not less than four runways, all of them oriented in different directions. With the increase in construction costs, the development of caster-type landing gear to permit landings in cross winds of relatively high velocity, and the increased use of larger and faster aircraft that are not so sensitive to cross winds, there is a trend toward a reduction in the number of runways considered necessary on airports.

It has been found that in many instances, one runway, properly oriented, would give adequate wind coverage. This has led to the elimination of many runways, even on the

largest airports, and to the policy that more than one runway is justified only to meet the requirements of traffic. This policy is qualified, however, by the fact that wind conditions in some locations are such that a single runway is not practical. Therefore, proper evaluation depends on a thorough investigation of local conditions and traffic requirements.

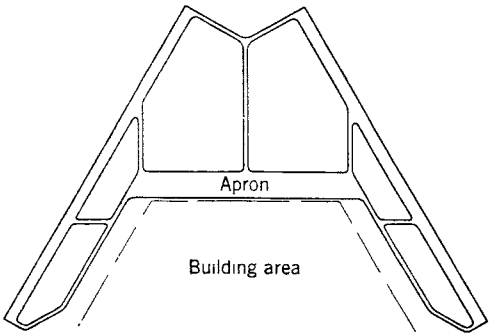


FIG 6. Two-runway, non-intersecting layout

If it should be determined that more than one runway is required, they should be oriented so that simultaneous use may be effected without conflict of traffic patterns. This will greatly increase the capacity and permit more direct approaches and departures, saving aircraft time and reducing traffic congestion. Typical runway layouts are shown on Figs 5, 6, and 7.

In general, the need for parallel runways should be based on a single runway handling 40 to 60 landings and take-offs per peak hour in visual flight conditions and 25 to 35 landings and take-offs per peak hour under instrument flight conditions In this connection, it should be mentioned that the development of electronic devices for navigation, air-traffic control, and ground-traffic control will permit, in the near future, an appreciable increase in the number of operations per runway, provided the runway and taxiway layouts are properly arranged.

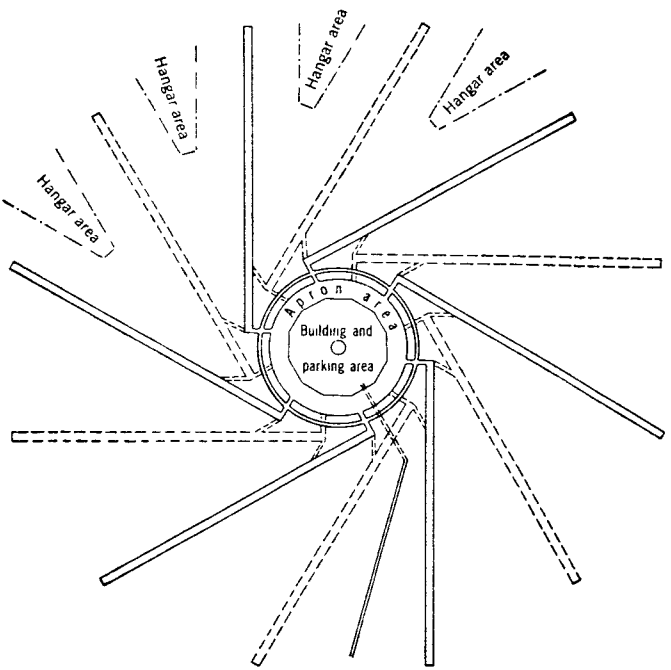


FIG 7. Tangential layout

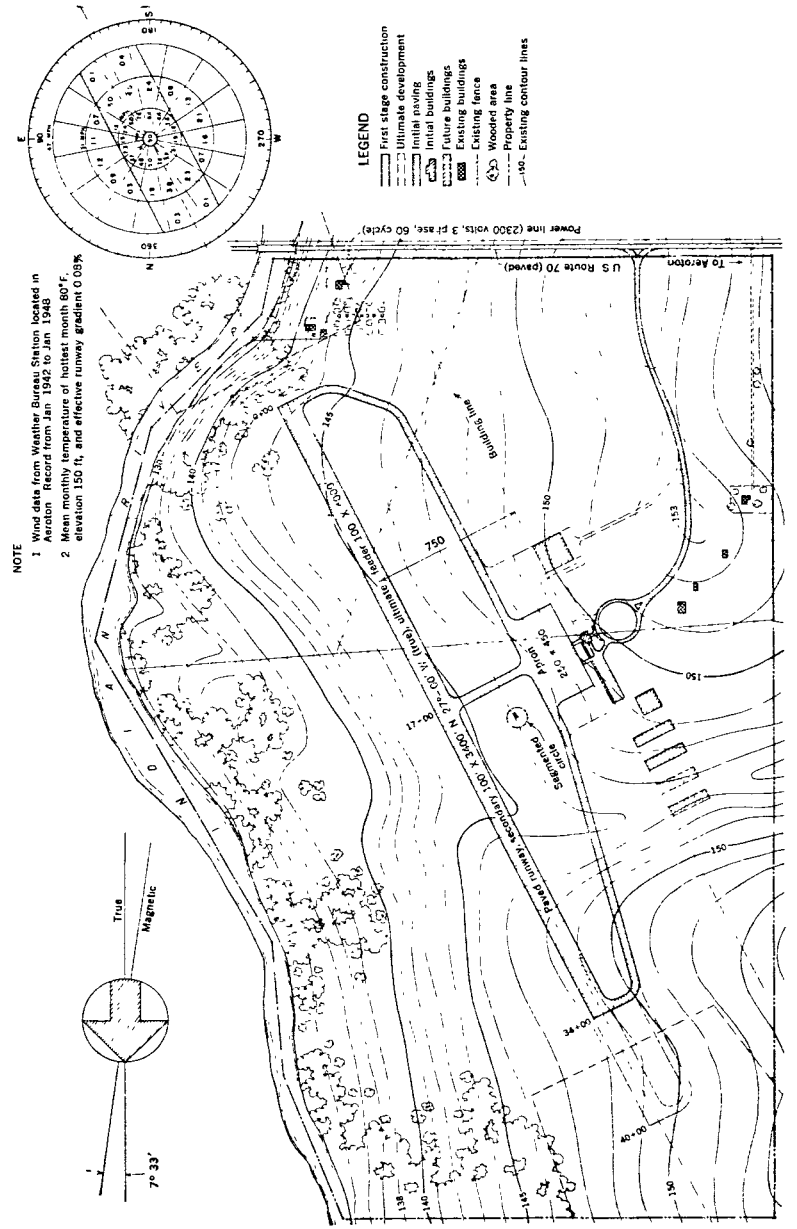


Fig 8 Master plan, feeder-type airport

17. TAXIWAY CONFIGURATION

Taxiways should be located at various points along the runway (see Figs. 5, 6, and 7) to permit landing aircraft to turn off the runways as quickly as possible to clear it for other aircraft and to provide minimum taxi distances to the service or ramp area on completion of the landing roll. Present practice, in general, connects both ends and the two-thirds points of runways with taxiways on the theory that (*a*) access must be available to the ends of the runway for take-offs, (*b*) all landings should be accomplished within the first two-thirds of the runway, and (*c*) aircraft that make landing runs longer than the first two-thirds of the runway may continue to and turn off at the far end This layout is completely satisfactory for the majority of airports.

The turnoff from runway to taxiway may be expedited by designing the taxi exits so that an aircraft will not have to come to a virtual halt before making the turn. Results of configuration studies suggest that taxi exits should permit an aircraft to turn off with a maximum of 45° initial turn This can be accomplished by providing curved turnoffs to taxiways (in both directions) although the taxiway proper may be perpendicular to the runway. For aircraft turning off the runway at speeds of 35 mi/hr, a minimum radius of 250 ft is desirable.

Another method of expediting the movement of traffic is to mark the taxi exits so that the pilot may anticipate a turnoff at night or in poor visibility.

On busy airports it is desirable to provide a holding or run-up apron connecting the taxiway and the end of the runway. The apron should be so designed that from three to five aircraft can simultaneously and independently perform cockpit checks and engine run-ups, and that either can pull out of the run-up position on to the runway without interference from other holding aircraft. This apron should be of such a size that the wheels of the aircraft using it will clear the edge of the pavement by 10 ft and the aircraft parked thereon will have a 15-ft wing clearance with aircraft taxiing by.

18. MASTER PLAN

Having determined the number and orientation of runways and taxiways and the space requirements for the other facilities, the entire layout should be shown on a master plan. The master plan, showing the site as it exists and also as it is planned ultimately, should be prepared and approved before any construction work is undertaken on a newly acquired site or before any additional improvements are made on an existing airport This should cover all existing facilities, the improvements planned under the current project, and future expansion. Included in these facilities are runways, taxiways, aprons, drainage, lighting, administration buildings, service buildings, hangars, etc. The purpose of this master plan is to achieve the orderly development of a serviceable airport which can be expanded, when necessary, with the least disturbance to the operation of the airport. A typical master-plan layout for a secondary airport to be ultimately developed into a feeder-type airport is illustrated in Fig. 8.

SURVEYS AND INVESTIGATIONS

19. TOPOGRAPHIC SURVEY

A topographic map is the basic work sheet for the design and development of an airport For this reason it must be accurate and must show all the surface features of the site. These features include contours of an appropriate interval to show the elevations and slopes of the terrain and all drainage courses; locations and top elevations of towers, pole lines, trees, and other obstructions; location, dimensions, and descriptions of structures such as existing buildings, pavements, and drainage structures, location and description of all public utilities, highways, railroads, and waterways.

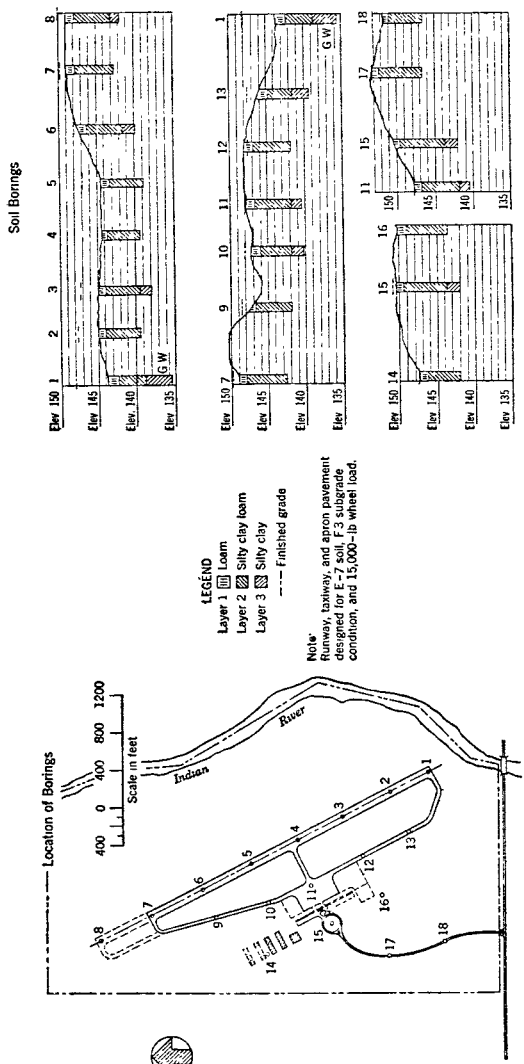
In addition the map should show the topography of lands adjacent to the site including all natural water courses, the areas contributing runoff onto the site, and possible outfalls and ditches.

20. SOILS AND MATERIALS SURVEY

Investigations of soil conditions at airport sites include:

1. A soil survey to determine the character and arrangement of the different layers of the soil profile, the ground water level, and drainability of the individual layers and the soil formation as a whole.
2. Sampling of the different layers of soil.
3. Testing the samples to determine the physical properties of the various soil materials with respect to stability and subgrade support.
4. A survey to determine the availability of materials for use in the construction of subbases, base courses, and surfaces.

Soil-survey information is generally obtained by means of borings made with a soil auger. The spacing and depth of borings cannot be definitely specified since they depend



Elevation water table 136 ft. On May 16, 1949, maximum frost penetration 8 in. Drainage condition good

Soil Analysis and Classification

Soil Layer	Boring Number	% Retained on #10 Sieve	Mechanical Analysis: Material Finer Than #10 Sieve: Coarse Sand, % pass #10 ret'd #40	Fine Sand, % pass #60 ret'd #270	Combined Silt and Clay, % pass #270	Liquid Limit	Plasticity Index	CAA Class
1	1	0	1	44	55	33	8	E-6
1	b	0	2	37	61	37	9	E-6
1	11	0	0	37	63	38	10	E-6
2	1	0	1	25	74	43	15	E-7
2	6	0	1	24	75	46	17	E-7
3	1	0	0	13	87	51	20	E-8
3	6	0	0	10	90	53	22	E-8

FIG. 9. Soil-survey sheet

on the topography and the uniformity of the soils. The number and depth of borings should be sufficient to adequately map all important variations. This survey should not be confined to soils encountered in grading nor to the boundaries of the airport site. Possible sources of material that can be used in the paving operations should be investigated on the airport site as well as off the site within reasonable and economical hauling distance.

Samples, representative of the different layers of the various soil types and available construction materials, should be obtained and tested in the laboratory in order to determine their physical characteristics and engineering properties. Since the results of the tests are only as good as the sampling, it is of utmost importance that a sample represent a certain type of material and not a careless mixture of several materials.

A standard method of surveying and sampling soils is outlined under Method T86-42 of the American Association of State Highway Officials.

This method of survey is based entirely on the soil profile. The different layers of the soil profile are identified in the field by color, texture, structure, consistency, compactness, cementation, and, to some extent, chemical composition. The method of field identification is essentially the same as that used by the U. S. Department of Agriculture for making soil surveys in connection with the preparation of county and other areal soil maps. When these maps are available, they should be carefully studied. Intelligent use of published soil maps will be invaluable in a study of the soil on the airport site, as well as indicate the availability of construction materials.

An example of a soil-survey sheet showing soil profiles and results of tests is presented in Fig. 9.

In some cases the only available site for an airport may be in a low, wet area consisting of soft organic silts or clays which will not support the weight of the necessary fill material unless special precautions and remedial measures are taken. Such locations will require an exhaustive subsurface investigation to determine the depth of the soft material, its moisture content, density, permeability, and shear and consolidation characteristics. Special drilling and sampling equipment is necessary to obtain samples of the materials in an undisturbed condition. The drilling, sampling, and testing procedures for this type of investigation are described in Sect. 7 (Soil Mechanics and Site Examination).

21. SOIL TESTS

The essential laboratory tests utilized in the CAA classification of soils for airport construction consist of (1) the mechanical analysis which determines the percentages of sand, silt, and clay in the soil, and (2) the liquid and plastic limits and plasticity index which indicates the stability of the soil in relation to changes in moisture content. In some instances it will be desirable to perform additional tests, such as shrinkage-factor and field-moisture-equivalent tests, to provide information for a better estimate of the performance of a soil. Standard methods of performing these tests, as published by the American Association of State Highway Officials, should be followed in all cases. The test designations and brief definitions are given as follows.

Mechanical analysis (AASHO Method T88-42) is a method for determining, quantitatively, the distribution of particle sizes in soils.

Liquid limit (AASHO Method T89-42) refers to the water content at which the soil passes from a plastic to a liquid state. The liquid state is defined as the condition in which the shear resistance of the soil is so slight that a small force will cause it to flow.

Plastic limit (AASHO Method T90-42) is defined as the minimum moisture content at which the soil becomes plastic. At moisture content above the plastic limit there is a sharp drop in the stability of a soil.

Plasticity index (AASHO Method T91-42) is the numerical difference between the liquid limit and the plastic limit. It indicates the increase in moisture content required to change a soil from a plastic to a liquid condition.

California Bearing Ratio. In connection with the design of flexible pavements the Corps of Engineers, Department of the Army, makes use of the California Bearing Ratio (CBR) which indicates the relative shearing resistance of a soil as compared to a standard value for crushed stone and is expressed as a percentage of this standard value. (See Sect. 7.)

22. SOIL AND SUBGRADE CLASSIFICATION

Although the results of the individual tests indicate certain physical properties of the soils, their principal value lies in the fact that through their interrelationship it is possible to draw up an engineering classification of soils related to their field behavior. Such a

classification is presented in Table 9. For other methods of soil and subgrade classifications see Sect. 7.

For each soil group there are corresponding subgrade classes based on the performance of the particular soil as a subgrade for rigid or flexible pavements under different conditions of drainage and frost. The subgrade class is determined from the results of soil tests, the information being obtained by means of the soil survey and a study of climatological data. The subgrade classes and their relationship to the soil groups are shown in Table 9. R refers to rigid pavements and F to flexible pavements. The thickness of subbase required in addition to the minimum pavement thickness for a given aircraft load is determined according to the subgrade class.

Table 9. Classification of Soils for Airport Construction

Soil Group	Mechanical Analysis				Liquid Limit	Plasticity Index	Subgrade Class			
		Material Finer Than No. 10 Sieve					Good Drainage		Poor Drainage	
	Retained on No. 10 Sieve,* %	Coarse Sand, Pass No. 10, Retained No. 60, %	Fine Sand, Pass No. 60, Retained No. 270, %	Combined Silt and Clay, Pass No. 270, %			No Frost	Severe Frost	No Frost	Severe Frost
E-1	0–45	40+	60−	15−	25−	6−	Fa R1a	Fa R1a	Fa R1a	Fa R1a
E-2	0–45	15+	85−	25−	25−	6−	Fa R1a	Fa R1a	F1 R1a	F2 R1a
E-3	0–45	. . .	.	25−	25−	6−	F1 R1a	F1 R1a	F2 R1a	F2 R1a
E-4	0–45	. .		35−	35−	10−	F1 R1a	F1 R1a	F2 R1a	F3 R2a
E-5	0–45		.	45−	40−	15−	F1 R1a	F2 R1b	F3 R1b	F4 R2b
E-6	0–55	. .	.	45+	40−	10−	F2 R1a	F3 R2b	F4 R2b	F6 R2b
E-7	0–55	. .	. .	45+	50−	10–20	F3 R1b	F4 R2b	F6 R2b	F7 R2c
E-8	0–55		.	45+	60−	15–40	F4 R1b	F6 R2c	F7 R2c	F8 R2d
E-9	0–55	.	.	45+	40+	30−	F5 R2b	F7 R2c	F7 R2c	F9 R2d
E-10	0–55	.		45+	70−	20–50	F5 R2b	F7 R2c	F8 R2c	F9 R2d
E-11	0–55		.	45+	80−	30+	F5 R2c	F8 R2d	F9 R2d	F10 R2e
E-12	0–55			45+	80+		F8 R2d	F9 R2e	F10 R2e	F10 R2e
E-13	Muck and peat, field examination						Not suitable for subgrade			

* Classification is based on sieve analysis of the portion of the sample passing the No. 10 sieve. When a sample contains material coarser than the No. 10 sieve in amounts equal to or greater than the maximum limit shown in the table, a raise in classification may be allowed provided the coarse material is reasonably sound and fairly well graded.

With respect to rigid pavements, subgrades are divided into two broad classes, R1 and R2, and each is subdivided into five classes, a, b, c, d, and e. Subgrades are placed in the R1 class where the combination of soil, drainage, and climatic conditions are favorable; otherwise they are classed as R2. Subbase requirements are the same for R1 and R2 subgrades but the R2 condition requires a concrete pavement 1 in. thicker than the R1. For certain conditions an additional inch of rigid pavement is considered more desirable than a proportionate increase in subbase thickness.

In the case of flexible pavements, subgrades classed as Fa furnish adequate subgrade support without the addition of subbase material. Soils in group E-1 fall in the Fa subgrade class under all conditions of drainage and frost while E-2 soils are classed as Fa subgrades only where the drainage is good. The other subgrade classes are designated F1 to F10.

To distinguish between good and poor drainage requires a knowledge of the topography of the site, the properties and arrangement of the different layers of the soil profile, and the elevation of the water table. Poor drainage is defined as a condition where the subgrade may be rendered unstable owing to (1) inadequate internal drainage caused by the character of the soil profile, (2) capillary rise from a high water table, (3) topographic features such as a flat terrain at elevations only slightly above sea level, or (4) any other cause that may result in instability or produce saturation of the subgrade.

Good drainage may be defined as a condition where (1) the internal drainage characteristics are such that there will be no accumulations of water that would develop spongy areas in the subgrade, (2) the water table is at such an elevation that the soil will not become waterlogged either by percolation from above or capillarity from below, and (3) the topography is such that surface water will be removed rapidly. These conditions can be met by any of the soil groups with the exception of E-13.

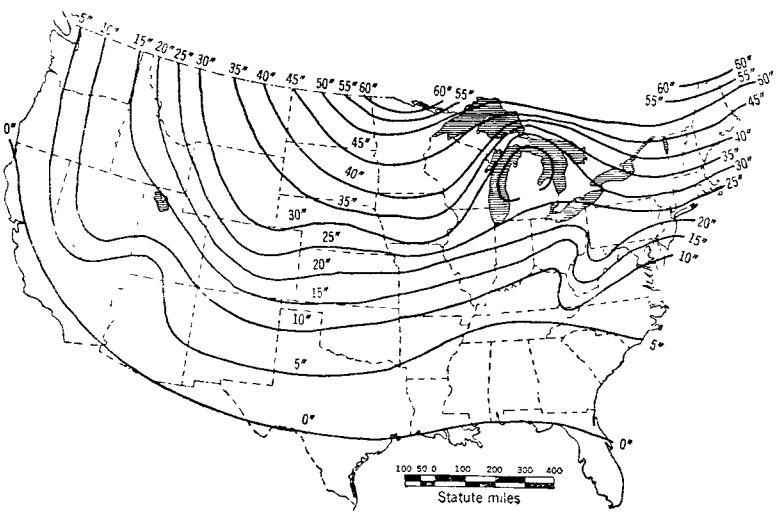


FIG. 10. Average annual frost penetration

Good drainage cannot be distinguished from poor drainage on the basis of soil-test results alone. This criterion has been used in the past and has led to the erroneous conception that a good drainage condition cannot exist when the soils consist of clays and silty clays of the E-7 to E-12 groups. There is no logical reason for such an assumption, although it is recognized that soils of these groups are expansive and subject to detrimental volume change with variations in moisture content. The degree to which these detrimental properties may develop will be shown by studying the topography and the entire soil profile and then applying the definitions of good drainage and poor drainage outlined above. It is important to remember that, although the soil group may be determined by means of laboratory test results, the drainage conditions must be established by the surface and subsurface characteristics of the site.

With respect to frost action, a "severe frost" condition exists if the depth of frost penetration for the particular site is greater than the anticipated thickness of surfacing, base, and subbase as determined for "no frost" and the drainage condition as defined above. Otherwise the condition of "no frost" prevails.

Some explanation may be necessary relative to a combination such as "good drainage" and "severe frost." It is obvious that a good drainage condition may exist in a location where the average depth of frost penetration exceeds the required thickness of surface, base, and subbase for a "no frost" condition. In this case, the subgrade class would be determined on the basis of "severe frost." The importance of "severe frost" in a location having good drainage has been questioned. From the standpoint of detrimental frost heave, it may not be important but it is well known that even under the best drainage conditions, the penetration of frost below the non-heaving subbase material, together with alternate freezing and thawing, produces a softening of the subgrade soil immediately un-

der the subbase. The only remedy for this situation is additional subbase thickness to overcome the loss in stability.

Figure 10 shows the average annual frost penetration throughout the United States.

Frost heaving is due to the formation of ice lenses within the soil. These lenses will continue to grow in thickness as long as freezing temperatures remain, and provided that the water upon freezing, can be replenished by water from a source such as ground water. This water may rise to the lenses by means of capillary action within the soil. The formation of lenses is therefore dependent upon (*a*) an adequate water supply and (*b*) a means of moving the water from the water source to the freezing zone. It is possible, therefore, to have a high water table but no frost heave if the material above the water table is coarse enough to prevent capillary action. Conversely it is possible to have material above the water table that will allow capillary action and yet not produce frost heave; this will occur if the water table is at such a depth that the height of capillary rise in the soil above the water table does not extend into the freezing zone.

In general, sands will not produce frost-heaving conditions since they cannot sustain capillary action. Clay, although it sustains capillary action, is not susceptible to frost heaving, since the rate at which water can flow through the soil is not great enough to permit growth of the ice lenses. Silts on the other hand are highly susceptible to frost heave. Uniform soils, containing 10% or more particles less than 0.02 mm in diameter, and well-graded soils, containing 3% or more particles less than 0 02 mm in diameter, may usually be considered susceptible to frost heave.

Bad surface drainage may be the cause of frost heave during alternate periods of freezing and thawing due to water from thawed snow being trapped beneath the pavement and thereby forming the water supply required for the growth of ice lenses.

Although the curves in Fig. 10 show average frost penetration, it may be necessary to design against greater depths of penetration, depending upon the frequency of deeper penetration and the cumulative number and intensity of degree days for the area under consideration.

PREPARATION OF SITE

23. CLEARING LANDING AREAS AND APPROACHES

Clearing the landing area is a preliminary to the grading and other construction work included in the airport development. It covers the removal of trees, poles, fences, structures, etc , within the usable landing and service areas. This must be accomplished prior to construction of runways, taxiways, aprons, and buildings, and before the installation of a drainage system The areas to be cleared should be shown on the plans and the details of the work covered in the specifications. Clearing may involve individual structures and trees or groups of trees by the acre. A typical clearing plan is illustrated in Fig. 11.

The amount of clearing in the landing area and approaches will be governed by the clearance and approach standards shown in Table 3. Clearing of approaches outside the airport property lines will require easements from the property holders.

24. EXCAVATION AND EMBANKMENTS

Excavation and construction of embankments on airports are carried out in the same manner as grading work on highways, parks, and other developments. The very heavy loads and aeronautical considerations, however, may require special attention to gradients, slopes, and compaction.

When a landing strip is located in a cut, the cut area should be of sufficient width to provide for any necessary drainage ditches at the edges of the landing strip in addition to the width of the landing strip shown in Table 2. If a taxiway is planned on the cut side of the runway, the back slope of the cut may be an obstruction to the taxiway in that case, the cut should be widened or the slope flattened to provide the distances between taxiway and obstruction shown in Table 3 In addition to the above requirements no part of the cut slope should project into the approach or transition slope zone except transversely 250 ft from the instrument-landing strips, where the height above the runway grade should not exceed 15 ft unless the obstruction is lighted.

The edges of graded areas should be so shaped that abrupt grade changes will not exist between graded and ungraded areas. By rounding the conventional angular shape at the top and bottom of cut or fill slopes, these areas are rendered less hazardous to aircraft and less expensive to maintain

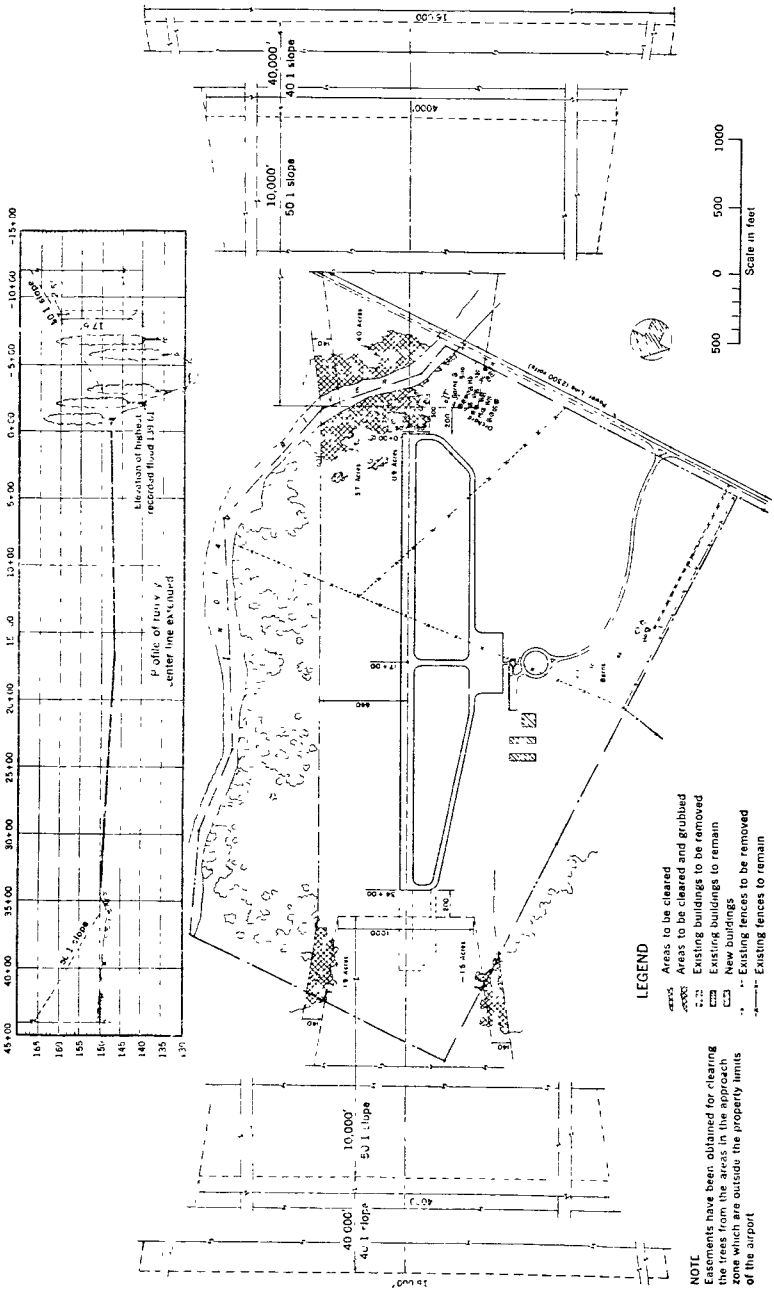


FIG 11. Typical clearing plan

Grading of the areas at the end of landing strips should result in a surface which will cause the least damage to aircraft inadvertently getting into this area because of an unsuccessful take-off or because of overshooting or undershooting the strip when landing.

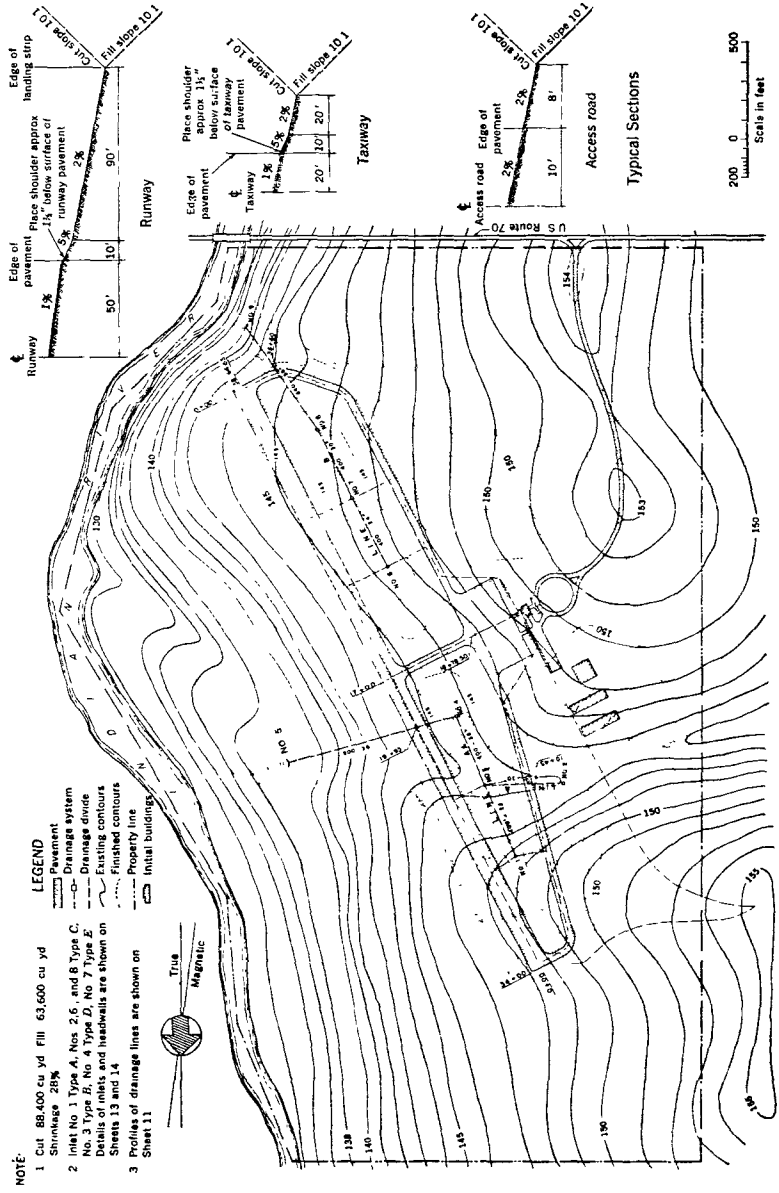


FIG 12 Grading and drainage plan.

Where practicable, vertical curves should be used to connect the grade of the landing strip with the grade of the terrain existing beyond the limits of the grading operations. Reverse vertical curves will be required in most cases. The point of intersection of the vertical curve nearest the landing strip should be placed not less than 100 ft from the end of the runway pavement or at the end of the landing strip. A curve having a vertical

distance from the point of intersection to the curve of not more than 3 ft should be used to connect the two grade tangents.

Grading usually can be so planned that ditches which are potential hazards to aircraft in distress are unnecessary around the ends of landing strips. When this is not practicable, either drainage pipe or drainage swales with gently sloping sides should be used to make this area less hazardous The work of grading should be closely coordinated with the drainage requirements.

A typical grading and drainage plan is shown on Fig. 12.

25. COMPACTION REQUIREMENTS

All fills should be properly compacted in accordance with the standard recognized practices after local soil conditions are analyzed. Particular care must be used in securing compaction around drainage and similar structures not only to avoid future settlement but to provide bearing pressures for the structures. Generally, it is desirable to compact cut sections when such a section occurs under areas to be paved.

CAA specifications require that embankments be compacted to not less than 90% of the maximum density at optimum moisture. This is increased to 95% for the upper 9 in. in fills and the upper 6 in. in cuts under all areas to be paved.

Maximum density is defined as the maximum dry weight in pounds per cubic foot obtained when a material is mixed with different percentages of water and compacted in a standard manner The percentage of water at which maximum density is secured represents the moisture content necessary to give maximum compaction and is called the optimum moisture content.

For further details regarding compaction, excavation, embankments, and construction of fills on unstable foundation soils, reference should be made to Sects. 7, 8, and 9.

DRAINAGE

26. FUNCTIONS AND CHARACTERISTICS OF AIRPORT DRAINAGE

The purpose of airport drainage, briefly stated, is to reduce the amount of water entering the soil and to maintain a firm, stable, reasonably dry surface, within economic limitations, for the safe use by the aircraft for which the airport was designed. The drainage system should provide for the interception and removal of surface and underground water flowing toward the airport from areas adjacent to the airport, the collection and removal of surface runoff from the actual airport area, the removal of excess underground water, the lowering of the water table, and the protection of slopes

In very few cases of airport construction is the natural drainage on the site sufficient to provide all of the requirements necessary for a properly drained area. Therefore, artificial drainage facilities need to be installed to provide for present requirements and also for future enlargement of the system when needed. This may mean the installation of a portion of a drainage system to supplement the natural drainage on the site or a complete system to drain the entire airport area.

27. SURFACE DRAINAGE

The principal problem in the design of a surface drainage system is to estimate the maximum runoff flow which must be accommodated by each part of the system. Once the runoff flow has been estimated the computation of pipe sizes and dimensions of drainage structures can then be made.

Runoff by Rational Method. There are in use at present several procedures for determining runoff. The most common of these and the one recommended by CAA is the *rational method*. The fundamentals of this method are covered in Sect. 8 of Vol. II, Sewerage and Sewage Disposal. Its application to airport drainage is discussed here.

In the rational method formula, $Q = CIA$, where

Q = maximum runoff rate in cubic feet per second or in acre-inches per hour.
C = a coefficient representing the ratio of runoff to rainfall.
I = average rainfall rate in inches per hour during the time of concentration.
A = drainage area in acres.

The time of concentration (T.O.C.) is the time required for overland flow to reach the drainage inlet or other point of concentration from the farthest extremity of the drainage area. It is usually considered as being composed of two components referred to as "inlet time" and "time of flow." The "inlet time" is the period of time consumed by the flow of water from the most distant point in the drainage area to the inlet. The "time of flow" is the period of time during which water flows through the drainage system from the inlet to any point being investigated below the inlet.

To determine the rainfall intensity I a design storm based on a 5-year frequency is considered adequate for airports. Intensity-duration curves should be plotted for storms of a 5-year frequency, and the graph used for runoff supply quantities. If complete Weather Bureau data are not available locally, the data in *Miscellaneous Publication* 204, U. S. Department of Agriculture, Rainfall Intensity-Frequency Data by David L. Yarnell, should be used for plotting the intensity-duration curve. Other curves such as 15-year, 20-year, etc., should also be plotted for checking excess storms. In many instances, storms appreciably more severe than the design storm may cause very little damage or inconvenience, whereas in other cases flooding of important areas may result. It is generally advisable to investigate the probable consequences of storms more severe and less frequent than the design storm before making final decisions regarding the adequacy of proposed drain-inlet capacities.

The runoff coefficient (C) is the percentage of the rainfall on a given area that flows off as free surface water. This percentage will seldom reach 100%, even when the entire surface is composed of pavement or other hard impervious material with steep slopes, because a certain amount of evaporation takes place even during the storm. The impervious surfaces absorb some moisture, and small depressions and irregularities hold back additional amounts.

Information for estimating the runoff from different types of airport surfaces in proportion to the rainfall intensity is presented in Table 10.

Table 10. Runoff Coefficients

Type of Surface	Factor C
For all watertight roof surfaces	0.75 to 0.95
For asphalt runway pavements	0.80 to 0.95
For concrete runway pavements	0.70 to 0.90
For gravel or macadam pavements	0.35 to 0.70
For impervious soils (heavy) *	0.40 to 0.65
For impervious soils, with turf *	0.30 to 0.55
For slightly pervious soils *	0.15 to 0.40
For slightly pervious soils, with turf *	0.10 to 0.30
For moderately pervious soils *	0.05 to 0.20
For moderately pervious soils, with turf *	0.00 to 0.10

* For slopes from 1–2%.

If the drainage area contributing to a certain inlet is composed of several surfaces for which different coefficients from Table 10 must be assumed, then the coefficient used in the formula should be a weighted average in accordance with the respective areas.

The formula $Q = CIA$ gives the maximum instantaneous runoff rate to be expected for a given design storm. If no temporary flooding can be permitted, the capacity of the drain lines, inlets, and other components of the drainage system must be large enough to handle this peak flow. Because of the short duration of maximum flow conditions, however, it is often possible to achieve substantial economies in design if even a small amount of temporary storage can be provided. This permits use of smaller pipe sizes with a subsequent flattening of the peak of the outflow hydrograph. Where sufficient storage capacity is available, it is recommended that ponding be utilized in airport drainage design.

Runoff by Overland Flow Method. Another method for determining runoff from airport surfaces is based on the hydraulics of *overland flow*. A number of investigators[3] have developed formulas and hydrographs of runoff which takes place as overland flow and which refers to runoff, resembling sheet flow, before it has reached a defined channel or ponding basin. This method, based on studies by Robert E. Horton, has been adopted by the Corps of Engineers, Department of the Army, for the design of airport drainage

[3] The Interpretation and Application of Runoff Plat Experiments with Reference to Soil Erosion Problems, by Robert E. Horton, *Proc. Soil Sci. Soc. Am.*, Vol. 3, pp. 340–349 (1938), Surface Runoff Determination from Rainfall without Using Coefficients, by W. W. Horner and S. W. Jens, *Trans. ASCE*, Vol. 107, pp. 1039–1117 (1942), A Method of Computing Urban Runoff, by W. I. Hicks, *Trans. ASCE*, Vol. 109, pp. 1217–1268 (1944); Hydraulics of Runoff from Developed Surfaces, by C. F. Izzard, *Proc. Highway Research Board*, pp. 129–150, December 1946.

systems. A complete explanation of the method is contained in Design of Drainage Facilities for Military Airfields by Gail A. Hathaway, *Proc. ASCE*, January 1944, pp. 55–88, and in *Engineering Manual*, Corps of Engineers, Part XIII, Chapter 1.

Collection and Disposal of Runoff. The next step in the design of airport drainage, following determination of rainfall and runoff requirements for each drainage area, is the collection and disposal of the runoff by means of a connected system of inlets, manholes, and pipes. The proper location of the various items of the drainage system will depend on the character of the terrain as disclosed by the topographic map. This map should be extensive enough to indicate the areas adjacent to the airport boundaries; it should show the necessary contours (2-ft interval) and all natural water courses, swales, draws, ditches, slopes, ridges, and configurations. It should also show all improvements that might have a bearing upon the runoff and drainage of the immediate area, such as railroads, highways, canals, and irrigation and drainage installations.

An additional layout plan is necessary to show the layout of the runways, taxiways, aprons, and building area with the finished contours drawn to a 1-ft interval or less. This layout should be the "drainage working drawing" (see Fig. 12). The entire system should be sketched upon it to show the outline and identification of each subarea contributing to each inlet or channel; all main and lateral storm pipe drains, pipe sizes, direction of flow, gradients, catch basins and inlets, structures, gutters when required, surface channels, peripheral and outfall ditches, and other essential features.

Inlets should be placed at all low points in the surface drainage created by grading. In the case of a long run of surface drainage where the fall is all in one direction, the inlets should be spaced so that the runoff will not have to travel excessive distances in reaching the system. Normally, the inlets should not be spaced less than 200 ft nor more than 400 ft apart with a general spacing of about 300 ft. They should not be located less than 75 ft from the edges of the pavement, unless a gutter section in the pavement is used; in that case, the inlets will be in the line of the gutter.

Special treatment for the location of inlets usually will be necessary near runway intersections, and the inlets should be as far away from the pavement as practicable. The inlets must be designed so as to present a satisfactory contour for use by planes that may traverse that area

Manholes, or combination manholes and inlets, should be provided where necessary; their spacing also should vary from 200 ft to 400 ft, with the most desirable interval at approximately 300 ft

Flow in Conduits. After the locations of the inlets, manholes, pipe runs, and outfalls have been determined and the design runoff for all subareas has been computed to each of the inlets, the next step in the design will be the computation (by appropriate hydraulic formulas) of the size and gradient of the pipe drains. Also, the flow time in the pipes from the various inlets can be computed according to the hydraulic characteristics of the pipe.

Several formulas are used to determine the flow characteristics in pipes; many of them give practically the same results. However, the Manning formula is the most widely used and has largely replaced the more complicated Kutter formula.

Charts have been compiled for the solution of the Manning formula and are included in most engineering handbooks. They usually are used instead of the formula to determine the size of pipe required.

Figure 13 shows the design data and computation for the drainage system illustrated in Fig. 12.

The conduits in the drainage system may be constructed of poured-in-place concrete, reinforced concrete pipe, concrete pipe, vitrified clay pipe, corrugated metal pipe, or any combination of these. The pipes should be of conventional standard sizes and provided with either bell-and-spigot or tongue-and-groove joints in the precast pipes and adequate metal bands for the corrugated metal pipe.

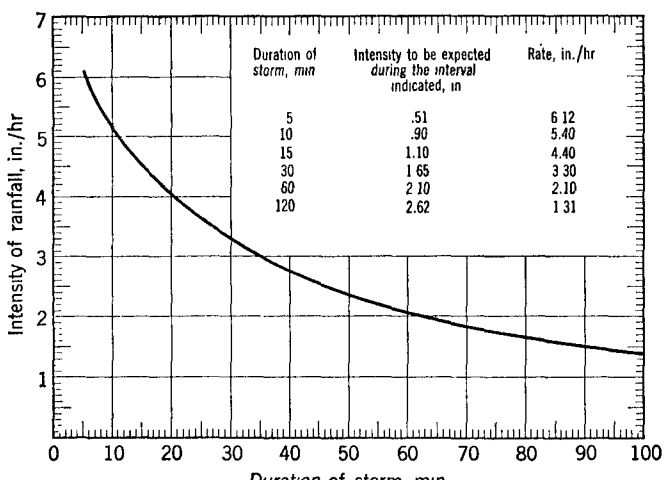
Structures. The structures most generally used are field and pavement inlets, manholes, combination manholes and inlets, occasional catch basins, lamp holes, and head walls on outlet ends. Structures located in the usable areas on airports must be designed so that they will not extend above the ground level. The tops of such structures should be one- or two-tenths of a foot below the ground line to allow for settlement around the structure, to permit unobstructed use of the area by aircraft, and to facilitate the entrance of surface water.

Manholes are more or less standardized as to type and can be of round, oval, square, or rectangular design. They are usually constructed of concrete, brick, concrete and brick, or corrugated metal, depending upon the stresses to which they will be subjected.

Many practical considerations must be recognized in the design of the spacing of grate openings. The openings must be large enough to serve the purposes for which they were intended, but small enough to prevent tail wheels of small planes or maintenance equipment

Drainage Design Computations
(Airport Site: Aeroton)

Number of Structures From	Number of Structures To	Length of Drain, ft	Subarea, acres: Increment A	Subarea, acres: Total A	Concentration Time, minutes: Inlet Time	Concentration Time, minutes: Flow Time	Concentration Time, minutes: Total Time to Inlet Below	Runoff Coefficient C	Subarea Rainfall Intensity I, in./hr	Maximum Runoff, $Q = CIA$ cu ft/sec: Inc'm't	Maximum Runoff, $Q = CIA$ cu ft/sec: Acc'm'd	Slope of Drain, Dec. Ft per Ft	Velocity Drain, ft/sec	Diameter Pipe, in.	Pipe Capacity, cu ft/sec	Hydraulic Gradient	Elevations: Top Inlet Grate	Elevations: Upper End Invert	Elevations: Lower End Invert	Notes
Lines A and AA																				
1	3	400	3 20	3 20	36	2	38	0 47	3.00	4.51	4 51	0.0015	2.8	24	8.5		145.20	139.80	139.20	
2	3	400	14 81	14.81	45	2	47	0 34	2.60	13 09	13 09	0.0015	3.2	30	16 0		145 80	139 30	138.70	
3	4	400	3.19	21 20	31	2	33	0.42	3.20	4 29	21.89	0 0015	3.6	36	26 0		143.80	138 20	137.60	
4	5	900	9 61	30 81	36	3	39	0.48	3.00	13.84	35.73	0.0025	4.6	36	33.0		142 60	137 60	135.35	
Line B																				
6	7	400	7.42	7 42	41	2	43	0 51	2.70	10 22	10.22	0 002	3.2	24	11.0		143 60	139.60	138.80	
7	8	400	4 20	11 62	36	2	38	0 39	3 00	4 91	15.13	0 0015	3.2	30	16 0		142.70	138 80	138.20	
8	9	640	7.44	19 06	36	3	39	0 41	3 00	9.15	24.28	0.0015	3.6	36	26 0		142 70	137.70	136.74	



5-year design storm

Drainage design criteria:

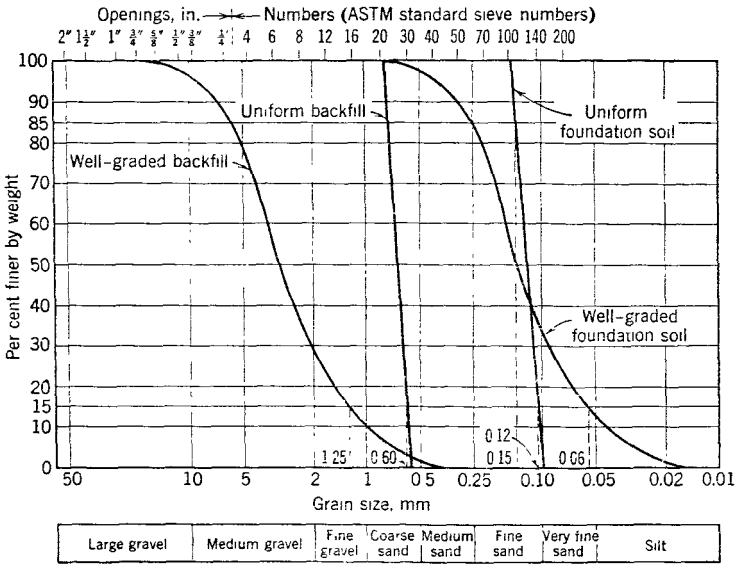
1. 5-year designed storm based on Yarnell's rainfall data
2. Coefficients of runoff: 0 3 for turf and 0 9 for pavement
3. Quantity of runoff estimated by the rational formula, $Q = CIA$.
4. Required inlet grate opening determined by formula, $A = \frac{Q}{0\ 7\sqrt{2gh}}$.
5. Discharge through pipes based on Manning's formula and $N = 0\ 013$.

FIG. 13. Drainage-design data and computation sheet.

from being caught in them. Small grate openings often present a difficult maintenance problem because they clog readily with grass cuttings, leaves, and sticks. Therefore, the actual effectiveness of such grates is less than the design capacity. It is suggested that the grate openings have a factor of safety of approximately 150% applied to their design to offset this ineffectiveness. The maximum width of a slot opening of the grate should be 1⅝ in. if rectangular; and, for openings approaching a square, the maximum dimensions should be 3 in. To curtail bypassing, the inlets should be installed to position the largest overall dimension across the direction of surface flow with the small grate openings parallel to that flow. It is important to check, by computations, the area of the grate opening to make sure there will be sufficient area to admit all of the runoff estimated for each inlet.

28. SUBSURFACE DRAINAGE

Subsurface drainage on airports consists, in general, of intercepting lines, diverting subterranean flows, draining wet masses or areas, controlling moisture in base or subbase of pavement, or any combination of these. Draining large field areas by subsurface drainage



Piping ratio

$\frac{\text{15\% size uniform backfill}}{\text{85\% size uniform foundation}}$, not greater than 4.0

$\frac{\text{15\% size well-graded backfill}}{\text{85\% size well-graded foundation}}$, not greater than 5.0

Permeability ratio.

$\frac{\text{15\% size uniform backfill}}{\text{15\% size uniform foundation}}$, not greater than 5.0

$\frac{\text{15\% size well-graded or uniform backfill}}{\text{15\% size well-graded foundation}}$, not greater than 25

FIG. 14. Limiting gradation for backfill surrounding drain pipe

is not usually necessary on airports as this can be accomplished more efficiently by proper grading and the installation of surface drainage.

The presence of a high water table on an airport site calls for a thorough soil survey and a determination of the cause of such underground water. It might cover the entire area of the airport or be located in one or more isolated portions of the site. The relative permeability and arrangement of the different soil layers will determine the need for and the location of underdrains. Localities in which the water table fluctuates with the seasonal rainfall should be carefully observed when making the soil survey.

Under normal conditions, a 4-in. to 10-in. pipe will suffice to handle subsurface flow. Pipe for subdrains may consist of vitrified clay, concrete, or corrugated metal, and may be perforated or plain, designed with open joints, bell-and-spigot joints, or cradle inverts. Great care must be exercised during construction and provision made for adequate cover-

Table 11. Recommended Minimum Depth of Cover in Feet for Conduits

	Wheel Load, 15,000 lb												Wheel Load, 30,000 lb											
	Nominal Diameter of Pipe, in.												Nominal Diameter of Pipe, in.											
Kind of Pipe	6	8	10	12	15	18	24	30	36	42	48	60	6	8	10	12	15	18	24	30	36	42	48	60
Clay sewer pipe	1.5	2.0	2.5	3.0	3.0	3.0	3.0	3.5	3.5				2.5	2.5	3.5									
Clay culvert pipe	1.5	1.5	1.5	1.5	1.5	1.5	1.5	2.0	2.0				1.5	2.0	2.0	2.5	2.5	3.0	3.0	3.0	3.0			
Concrete sewer pipe	1.5	1.5	2.0	2.5	2.5	3.0	3.0						2.5	2.5	3.0	3.5								
Reinforced-concrete sewer pipe				2.0	2.5	3.0	3.0	3.5	3.5	4.0	4.0	4.0				3.0	3.0	3.5						
Reinforced-concrete culvert pipe				1.5	1.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0				2.0	2.0	2.5	2.5	3.0	3.0	3.0	3.0	3.0
Reinforced-concrete culvert pipe (extra strength)							1.0	1.0	1.0	1.0	1.0	1.0							2.0	2.0	2.0	2.0	2.0	2.0
Corrugated metal pipe																								
Gage 18	1.0	1.0	1.0	1.5									1.0	1.5	1.5	2.0								
16	1.0	1.0	1.0	1.0	1.0	1.5	1.5						1.0	1.0	1.0	1.5	1.5	1.5	2.0					
14				1.0	1.0	1.0	1.0	1.5	2.0							1.0	1.0	1.5	2.0	2.5	2.5			
12								1.0	1.0	1.5	2.0								1.5	2.0	2.5	3.0		
10											1.0	1.5								1.5	2.0	2.0	2.5	
8												1.0								1.0	1.5	1.5	2.0	

	Wheel Load, 45,000 lb												Wheel Load, 60,000 lb											
	Nominal Diameter of Pipe, in.												Nominal Diameter of Pipe, in.											
Kind of Pipe	6	8	10	12	15	18	24	30	36	42	48	60	6	8	10	12	15	18	24	30	36	42	48	60
Clay sewer pipe	3.0	3.5	4.0										4.0	4.5	5.0									
Clay culvert pipe	2.0	2.5	3.0	3.0	3.0	3.5	4.0	4.0	4.0				2.5	3.0	3.5	3.5	3.5	4.0	4.5	4.5	4.5			
Concrete sewer pipe	3.0	3.5	4.0	4.0									4.0	4.0	4.5									
Reinforced-concrete sewer pipe				3.5																				
Reinforced-concrete culvert pipe				2.5	2.5	3.0	3.0	3.5	3.5	3.5	4.0	4.0				3.0	3.5	4.0	4.0	4.5	4.5			
Reinforced-concrete culvert pipe (extra strength)							2.5	2.5	2.5	2.5	3.0	3.0							3.0	3.0	3.5	3.5	3.5	3.5
Corrugated metal pipe																								
Gage 18	1.5	1.5	2.0	2.0									2.0											
16	1.0	1.0	1.5	2.0	2.5	2.5	3.0						1.5	1.5	2.0	2.5	3.0	3.5	4.0					
14				1.5	1.5	2.0	2.5	3.0	3.5							2.0	2.0	2.5	3.0	3.5	4.0			
12							2.0	2.0	2.5	3.0	3.5					1.5	1.5	2.0	2.5	2.5	3.5	4.0	4.0	
10									2.0	2.5	3.0	3.5							2.0	2.5	3.0	3.0	3.5	4.0
8											2.5	2.5									2.5	3.0	3.0	3.5

	Wheel Load, 75,000 lb												Wheel Load, 100,000 lb											
	Nominal Diameter of Pipe, in.												Nominal Diameter of Pipe, in.											
Kind of Pipe	6	8	10	12	15	18	24	30	36	42	48	60	6	8	10	12	15	18	24	30	36	42	48	60
Clay sewer pipe	4.5	4.5	5.0										5.5	5.5	6.0									
Clay culvert pipe	3.0	3.0	3.5										4.0	4.0	4.0									
Concrete sewer pipe	4.5	4.5	5.0										5.0	5.0	5.5									
Reinforced-concrete culvert pipe				3.0	3.5	4.0	4.5	4.5	5.0							3.5	4.0	4.5	5.0	5.0	5.5			
Reinforced-concrete culvert pipe (extra strength)							3.0	3.5	4.0	4.0	4.0	4.0							4.0	4.0	4.5	4.5	4.5	5.0
Corrugated metal pipe																								
Gage 18	2.0												2.5											
16	1.5	2.0	2.5	2.5	3.5	4.0	4.5						2.0	2.5	2.5	3.0	3.5	4.5	5.0					
14				2.0	2.5	3.0	3.5	4.0	4.5							2.5	3.0	3.5	4.0	5.0	5.5			
12							3.0	3.5	4.0	4.5	4.5								3.5	4.5	5.0	5.5	5.5	
10							2.5	3.0	3.5	4.0	4.0	4.5							3.0	3.5	4.5	5.0	5.0	5.5
8									3.0	3.5	3.5	4.0									4.0	4.5	4.5	5.0

Note 1. Cover for pipe in airfield areas not used by aircraft traffic shall be designed for a single wheel load of 15,000 lb.

Note 2. Cover for pipes within runway or taxiway strips and other aircraft traffic areas shall be provided in accordance with the above table except as provided for rigid pavements in Note 3 below

Note 3. Pipe placed under concrete pavements on airfields shall have a minimum cover measured below the slab of 1 ft for single wheel loads of 60,000 lb and 2 ft for single wheel loads of 150,000 lb.

age by proper specifications to assure that the trench will be backfilled with well-compacted, properly graded material. Grades should not be less than 0.1 ft in 100 ft. In order to prevent the infiltration of the surrounding natural soil into the interstices of the porous backfill material, it is recommended that the relationship in grading between the natural foundation soil and the backfill material shown in Fig. 14 be followed.

29. EROSION CONTROL

In airport construction, cut and fill slopes are usually made as flat as possible. Cut slopes more than 8 or 10 ft deep, with higher ground above them, should be provided with a cutoff surface ditch constructed several feet back from the top of the cut and running parallel to the top-of-cut line to intercept the surface water flowing down from the higher ground before it reaches the cut bank. To protect the cut slopes, it may be necessary to riprap, sod, sprig, or seed with rapid-growing grass or vegetation. It is good practice also to construct a surface ditch at the base of the cut slope to intercept the flow of runoff before it reaches the airport area.

All fill slopes of more than 3 ft in height should be protected against surface-water erosion by the construction of berms or gutters along the top of the slope to intercept the surface water and to prevent it from spilling down the slope. The surface water, thus intercepted, may then be disposed of by properly constructed rubble or concrete spillways, pipe spillways, vertical drop inlets, or by other suitable means of conducting the water down the slopes to proper outfall ditches.

Spillways should be provided with an adequate cutoff wall beneath the entrance apron to the spillway. This cutoff is most important as it prevents water from seeping down under and along the spillways and causing failure from lack of support.

Where open-trough-type spillways are constructed, their cross-sectional area should be larger than that required for the design storm, and provision should be made in the design for ample free board.

30. COVER REQUIREMENTS FOR CONDUITS

Conduits installed on airfields are normally subjected to either a dead load (loads due to the weight of soil or other overburden placed over the conduit) or a combination of a dead load and live loads (stationary and moving airplanes). The relative effect of each type of loading may vary considerably. In some instances, the dead load may comprise the total load. This is true of conduits installed in areas away from the landing strips, runways, and taxiways. Usually, the conduits will normally be subjected to both a dead load and a live load. On the smaller type airports, the effect of the live, or surface loads, will generally be minor in relation to the load due to the weight of the soil or other overburden The problem in this case is somewhat analogous to the problem usually encountered in the design of conduits installed on highway projects. On the larger-type airports, the problem is different. In many instances, the surface loads may comprise the major percentage of the total load on the conduit.

Investigations have shown that proper installation has considerable effect on the ability of a conduit to support loads. Provision should, therefore, be made to insure that proper construction procedures are followed for installing conduits. Table 11 shows recommended minimum depth of cover required for conduits installed with good foundations and properly backfilled. Figure 14 shows the gradation characteristics for backfill.

PAVING

31. GENERAL CONSIDERATIONS

Airport pavements are subjected to impact, dynamic, and static loads When an airplane is standing with the motors dead, the load on the pavement is static and equal to the gross weight of the aircraft. This is the heaviest load imposed on the pavement. Tests and observations have disclosed that the gross load transmitted to the pavement is reduced when the plane is standing and the propellers operating at high speeds as well as when the plane is taxiing. This may be explained by the aerodynamical forces acting

on the wings producing an air-borne condition to a certain degree. Because the aircraft is at least partially air-borne when taxiing and largely air-borne when landing and taking off, the dynamic (moving) and impact loads are less critical than the static load. For this reason, pavements designed for static loads will be adequate for all conditions of loading.

For design purposes it is the usual practice to consider the gross weight of the aircraft as distributed between the two main wheels or group of wheels, disregarding the portion of the load that may be carried by the tail or nose wheel. The static wheel loads may, therefore, be taken as one-half the gross weight of the plane.

To determine the pavement thickness required on a given airport, it is necessary to know (1) the weights and number of aircraft to be accommodated, (2) the engineering properties of the soil on which the pavement will rest, (3) the general character of the drainage, and (4) the local climatic conditions, especially with respect to depth of frost penetration.

Service records have shown definitely that taxiways, aprons, turnarounds, and warm-up pads at ends of runways are the most critical areas from a standpoint of aircraft use. Pavement failures have occurred on these areas when pavement of the same type and thickness on the runway proper showed no signs of distress. This difference in condition results from the numerous repetitions of loads due to channelization of traffic in these critical areas as compared to the wide distribution of traffic on the runway. This is important in pavement design because of its influence on pavement-thickness requirements.

For safe and efficient aircraft operations, pavement gradients and vertical curves should comply with the standards set forth in Art. 7 of this section. Particular care must be taken to provide easy transitions at intersections, not only with respect to grades but in providing convenient turning radii. Other features of aeronautical importance follow.

1. Rough wearing surfaces should be avoided because of the excessive wear caused to tires of landing aircraft. Bituminous surfaces may be finished with a sand or fine-crushed aggregate (1/4-in. maximum size) seal. Concrete surfaces should have an even, fine sandy or gritty texture. A smooth trowel finish should never be permitted.

2. The wearing course should be dense and well bonded to prevent loose surface aggregates and consequent damage to propellers.

3. Pavements at loading and warming-up aprons and turnarounds at the ends of the runways should be of sufficient stability to withstand, without damage, the heavy shearing stresses occasioned by short-radius brake turns, the vibration of engine run-up, and drippings of gas and oil.

4. Crowns and transverse gradients should be sufficient to expedite surface water runoff.

A description of pavement types and requirements as to quality of materials and workmanship are covered in Sect. 4, Highway Engineering. Standard specifications published by federal, state, and other public agencies should be consulted regarding details relative to quality of materials and mixes, control tests, and methods of construction.

Design procedures vary in accordance with the agency responsible for the development of standards which they consider applicable to their particular needs. In general, pavements on civilian airports are designed in accordance with the methods described in Airport Paving, published by the CAA. For military airports pavement design follows either the method of the Corps of Engineers as established in Part XII of the *Engineering Manual for War Department Construction* or the procedure presented in the *Design Manual* of the Bureau of Yards and Docks of the Navy Department.

All the design procedures specify thicker pavements for aprons, taxiways, and runway ends than those required on the major portion of the runway. Special turnaround areas and warm-up pads, when required, should have the same thickness as taxiways.

32. RIGID PAVEMENTS

Rigid pavements for airports consist of Portland cement concrete pavements, with or without reinforcement, and with or without additions of natural cements or other admixtures. In any case, regardless of aircraft loading, concrete pavements should have a minimum thickness of 6 in.

Most rigid-pavement design is based upon the principles of elastic deflections of slabs as developed by H. M. Westergaard. This analytical method has been widely accepted. Both the CAA and the Corps of Engineers have published a series of design curves based on the Westergaard principles but modified in accordance with the observed behavior of pavements under service conditions.

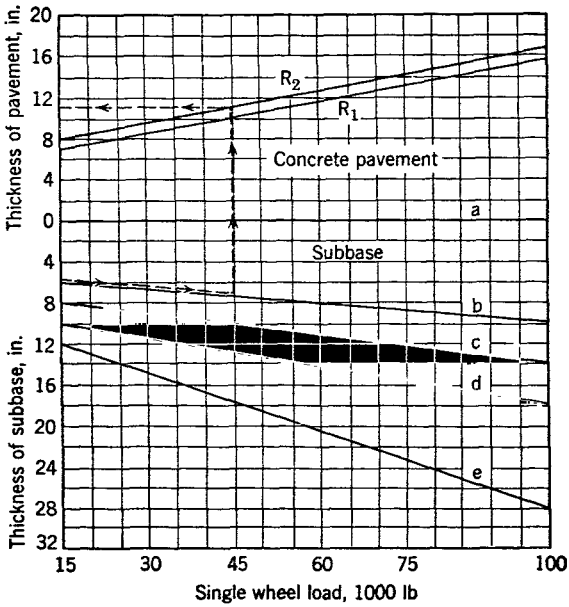


FIG. 15. Concrete pavement: taxiways, aprons, and runway ends.

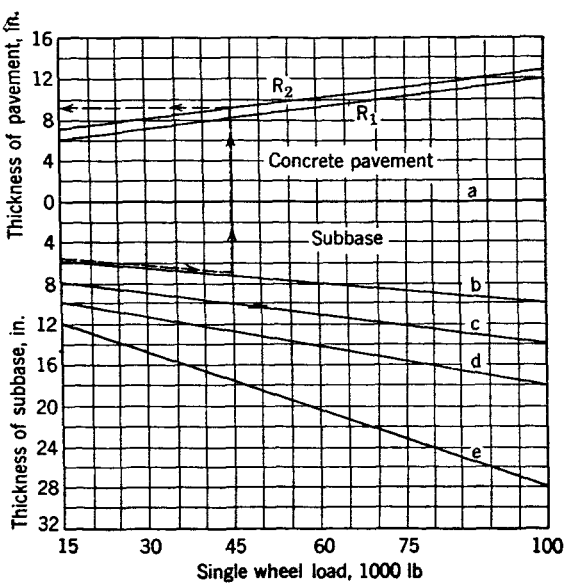


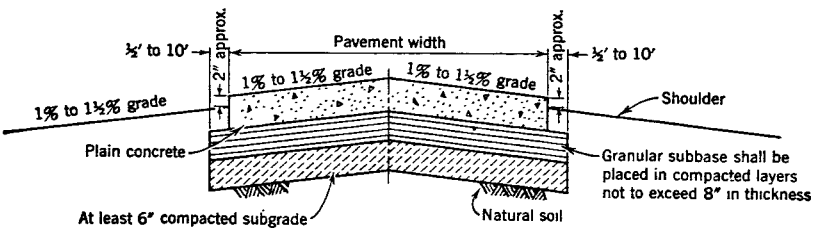
FIG. 16. Concrete pavement: runways (CAA method).

CAA Design Procedure. Thickness requirements for single wheel loads[4] from 15,000 to 100,000 lb are shown graphically on Figs. 15 and 16. The designations R1, R2, and a, b, c, d, and e refer to subgrade classes discussed in Art. 22 of this section.

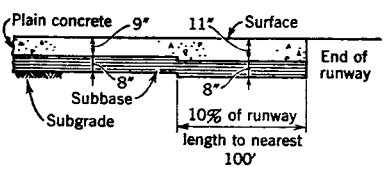
Referring to Figs. 15 and 16, line *a* represents the subgrade class that requires no subbase. Subbase requirements are the same for R1 and R2 subgrades, but the R2 condition requires a concrete pavement 1 in. thicker than the R1. For certain conditions an additional inch of rigid pavement is considered more desirable than a proportionate increase in subbase thickness. In general, subgrades are placed in the R1 class where the combination of soil, drainage, and climatic conditions are favorable; otherwise, they are classed as R2.

Figure 15 shows the required concrete pavement thickness for runways: the requirements for taxiways, aprons, and runway ends are shown in Fig. 16. Their use may be illustrated by an example as follows:

Assume a single wheel load of 45,000 lb, an E-6 soil, good drainage, and severe frost. The corresponding subgrade class for these conditions is R2b which will require a concrete pavement 9 in. thick on the runways and 11 in. thick on taxiways, aprons, and runway ends. The subbase requirement is 8 in. in both cases. Thicknesses are taken to the nearest inch.



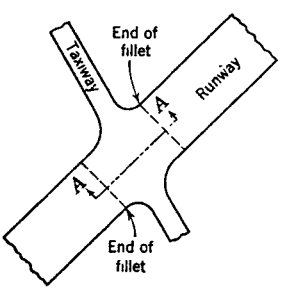
Cross Section of Runway



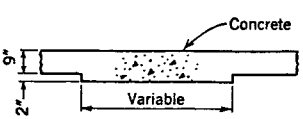
Runway End Section

This longitudinal section illustrated runway end design for a single wheel load of 45,000 lb and a R_2b subgrade. Pavement thickness increased for 10% of runway length to nearest 100'.

Reinforced concrete:
When pavements are reinforced concrete, the pavement may be reduced 1" in thickness but in no case shall the thickness be less than 6".



Plan



Section *A-A*

Increase in depth of concrete runway at intersection of a taxiway and a runway. Length of section variable depending on angle of intersection. Length shall be equal to maximum distance between fillet ends.

FIG. 17. Typical sections, concrete pavements.

Cross-section details for the thicknesses given in the above examples are shown on Fig. 17; Figs. 18, 19, 20, and 21 and Table 12 give jointing, reinforcing, dowel, and tie-bar details.

[4] For relationship between single wheel loadings and dual wheel loadings, see Table 2.

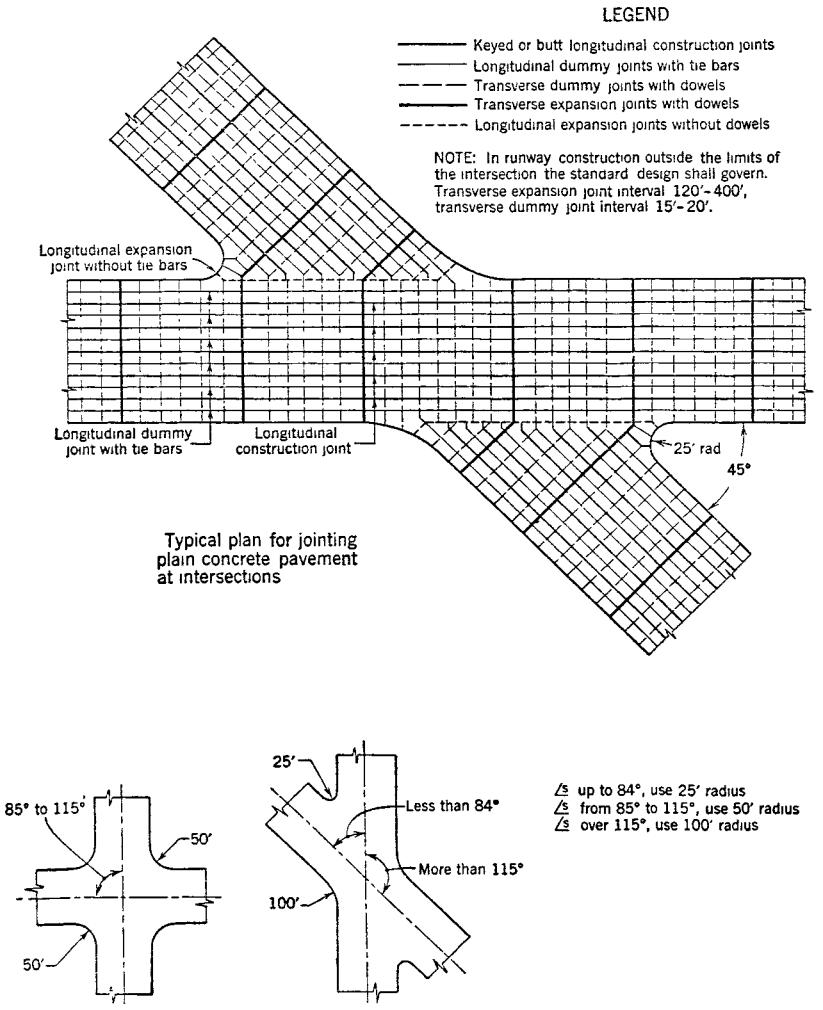
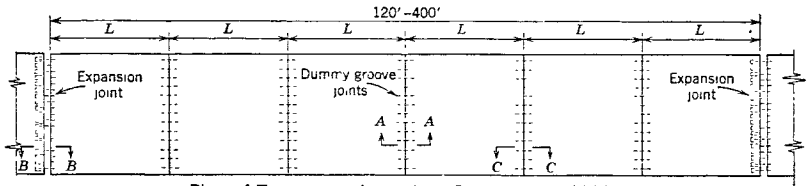
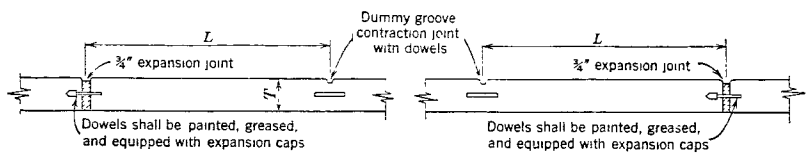


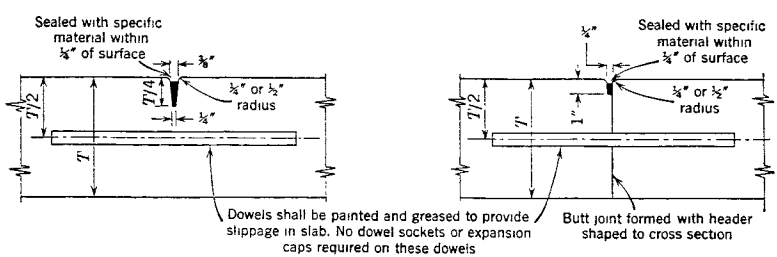
FIG. 18 Details of pavement intersections.



Plan of Transverse Joints in a Construction Width



Longitudinal Section

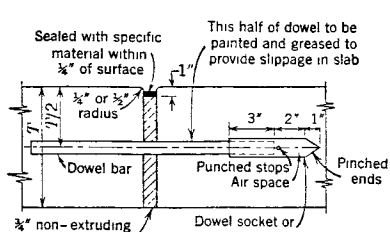


Section *A–A*

Section *C–C*

Dummy Groove Construction Joint With Dowels

Construction Joint When Needed



Section *B–B*

Expansion Joint

1. *T*, cross section thickness.
2. *L*, according to type of coarse aggregate may vary from 15′ to 20′.
3. Dowel bars shall be used in dummy groove type of joints on aprons, taxiways, and critical thickened end sections of runways, and on all runways designed for wheel loads of 60,000 lb or more. They may be omitted in the main portions of runways between the critical thickened end sections when the wheel loads are less than 60,000 lb.
4. Dowels shall be supported and held in place by an approved assembly within 1/8″ per foot of being parallel with the plane of the slab surface and within 1/8″ per foot of being parallel to center line of the pavement. Dowel size, length, and spacing as required.
5. All joints shall be edged.
6. Spacing of transverse expansion joints may vary from 120′ to 400′.

FIG. 19. Transverse joint details plain concrete.

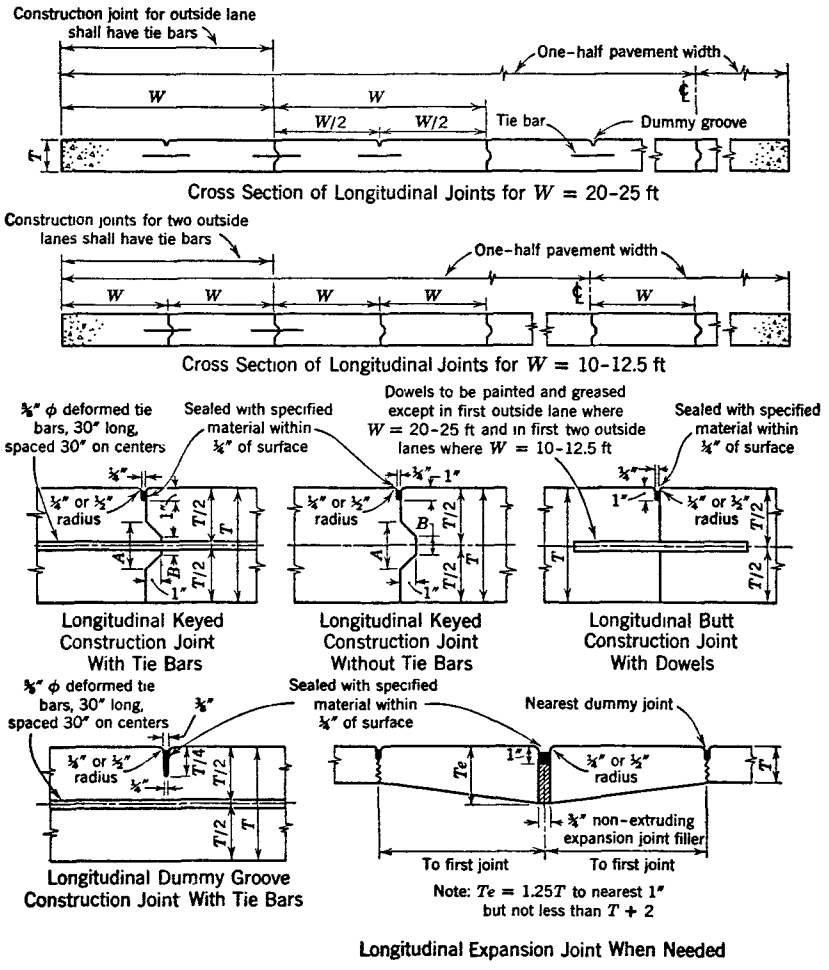
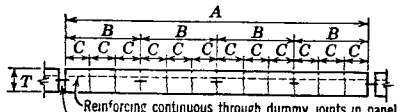
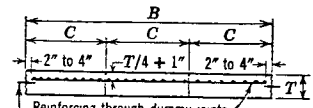


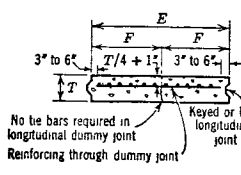
FIG. 20. Longitudinal joint details, plain concrete.



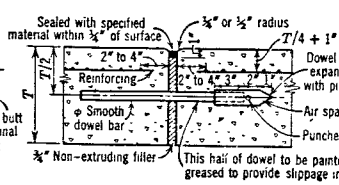
Longitudinal section



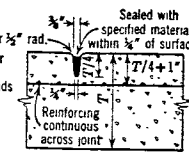
Panel section



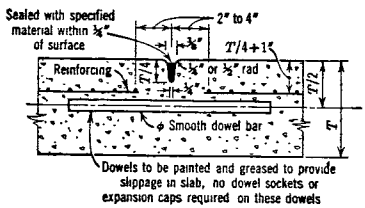
Transverse section of construction lane



Transverse expansion joint



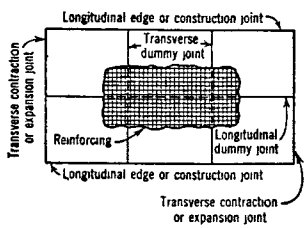
Transverse dummy joint without dowels



Transverse dummy groove contraction joint with dowels

Dimensional Limitations

Slab Thickness T	A		B		C	
	Min	Max	Min.	Max	Min	Max.
6"–8"	120'	400'	40'	60'	15'	20'
9"–12"	120'	400'	40'	60'	15'	25'
13"–20"	120'	400'	40'	60'	20'	25'



Cutaway top view of typical panel

Amount of Reinforcement

The amount of sectional area provided by the reinforcing steel shall be at least equal to that determined by the following formulas:

For longitudinal members $\left\{ A_s = \dfrac{3.7B\sqrt{BT}}{f_s} \right.$

For transverse members $\left\{ A_s = \dfrac{3.7E\sqrt{ET}}{f_s} \right.$

in which

A_s = square inches per foot of length or width
B = length of panel in feet
E = width of panel in feet
T = thickness of slab in inches
f_s = unit stress in steel in lbs per sq in.

Allowable unit stresses $\left\{ \begin{array}{l} f_s = 28{,}000 \text{ for cold-drawn wire} \\ f_s = 23{,}000 \text{ for bars} \end{array} \right.$

Legend

A = Spacing of transverse expansion joints
B = Spacing of transverse contraction joints
C = Spacing of transverse dummy joints
E = Spacing of longitudinal construction joints
F = $1/2\,E$ (locates longitudinal dummy joints)
T = Thickness of slab

General Requirements

Reinforcing may be either welded wire fabric or bar mats, furnished in flat sheets or mats, and installed with end and side laps so as to provide continuous reinforcing throughout each slab panel

Longitudinal members shall be spaced not less than 4" nor more than 12"; transverse members not less than 6" nor more than 18"

The term "slab panel" refers to a unit of pavement having B length and E width. Each panel shall have one center longitudinal dummy joint; and at least one but not more than three equally spaced transverse dummy joints. This layout is designed for construction width E of either 20' or 25'.

Runways exceeding 250' in width shall have one center longitudinal expansion joint.

Aprons and other large paved areas shall have longitudinal expansion joints spaced 120' to 400' apart

Grade of material, fabrication, lapping, etc., to meet requirements prescribed in the general specifications.

FIG. 21. Reinforcement details for concrete pavements.

Table 12. Dowel-Bar Details for Longitudinal Butt Joints and Transverse Expansion, Construction, and Contraction Joints

Slab Depth *T*, in.	Bar Diameter, in.	Bar Length, in.	Bar Spacing, in.
6	3/4	16	12
7	3/4	16	12
8	1	16	12
9	1	16	12
10	1	16	12
11	1 1/4	20	15
12	1 1/4	20	15
13	1 1/4	20	15
14	1 1/4	20	15
15	1 1/4	20	15
16	1 1/2	24	15
17	1 1/2	24	15
18	1 1/2	24	15
19	1 1/2	24	15
20	1 1/2	24	15

33. FLEXIBLE PAVEMENTS

Flexible pavements for airports consist of a bituminous wearing surface resting on a non-rigid base and a subbase course when required by subgrade conditions. Bituminous material may also be used in the base course or subbase.

Just as in the case of rigid pavements, procedures for designing flexible pavements vary according to the agency responsible for the establishment of design criteria.

CAA Design Procedure. Thickness requirements for single wheel loads from 15,000 to 100,000 lb are shown graphically on Figs 22–25 inclusive. The designations Fa to F10 refer to subgrade classes discussed under Art 22 of this section. The line Fa represents the subgrade class that requires no subbase. When the subgrade falls into Classes F1 to F10, subbases of varying thicknesses are required.

To illustrate the use of the design curves, assume a single wheel load of 45,000 lb, an E-6 soil, good drainage, and severe frost. The corresponding subgrade class for these conditions is F3. For a non-bituminous base, the pavement requirements are:

Runways (Fig. 22)

Bituminous surface	2 1/2 in.
Base	8 in.
Subbase	6 in.
Total	16 1/2 in.

Taxiways, Aprons, Runway Ends (Fig. 23)

Bituminous surface	2 1/2 in.
Base	10 in.
Subbase	8 in.
Total	20 1/2 in.

Base course and subbase requirements are taken to the nearest full inch of thickness.

When a bituminous base course is used, thickness requirements are determined in the same manner from Figs. 24 and 25; Figs. 26 and 27 show the required thicknesses for a bituminous-coated aggregate base.

Cross-sectional and other design details for flexible pavements are shown on Fig. 28.

With respect to the design curves, Figs. 22–27, *non-bituminous base* refers to base courses consisting of dry-bound macadam, water-bound macadam, crushed stone, gravel, caliche, lime rock, shell, etc., whereas *bituminous base* is limited to a central-plant hot mix of bituminous material and aggregates of approximately the same design as the bituminous surface course. *Bituminous-coated aggregate base* is defined as a mixture, produced in a central plant or travel plant, of bituminous material and aggregates which are inferior in gradation and quality to the aggregates meeting the requirements for non-bituminous base courses.

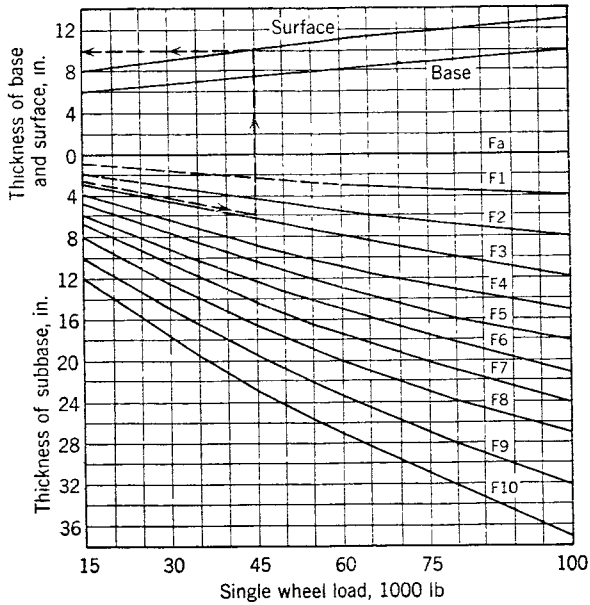


FIG. 22 Flexible pavement, non-bituminous base runways (CAA method).

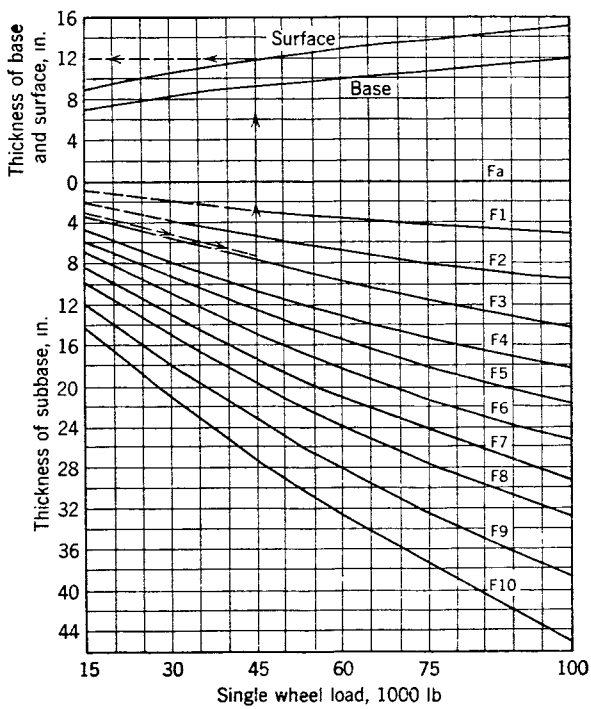


FIG 23. Flexible pavement, non-bituminous base. taxiways, aprons, and runway ends (CAA method).

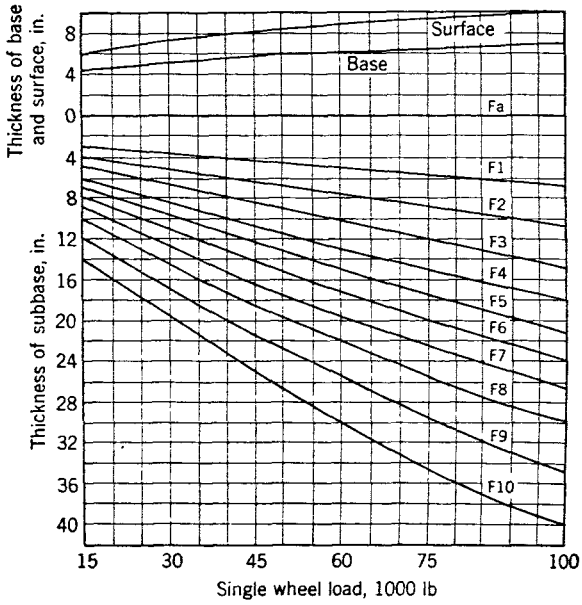


FIG. 24. Flexible pavement, bituminous base runway (CAA method).

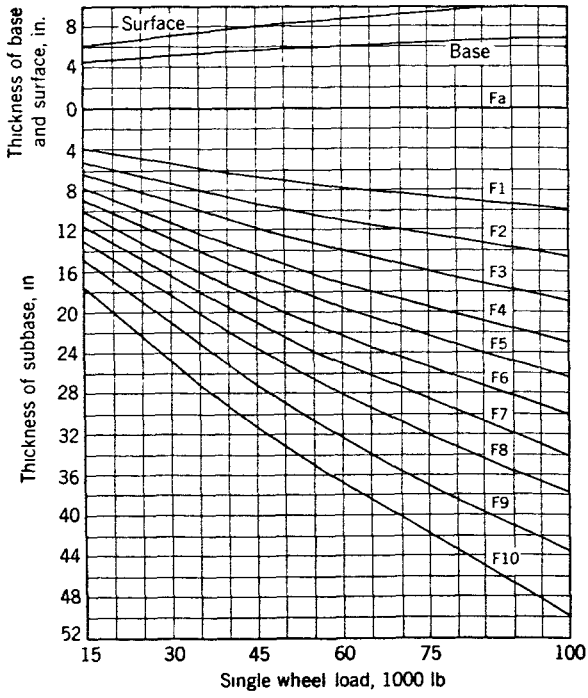


FIG. 25. Flexible pavement, bituminous base: taxiways, aprons, and runway ends (CAA method).

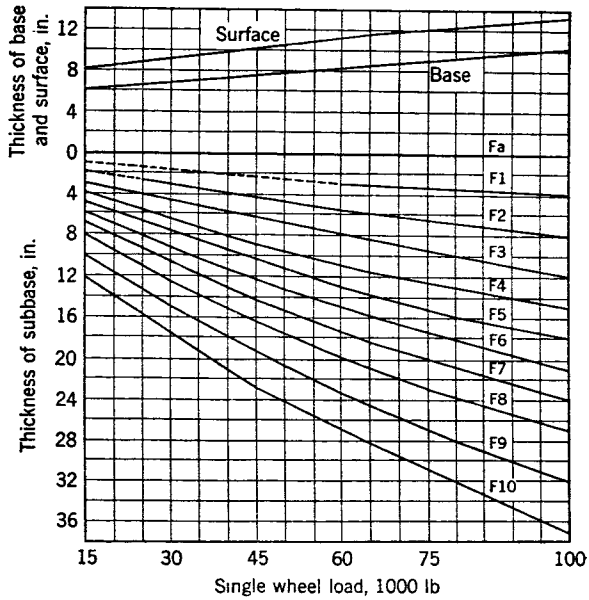


Fig. 26. Flexible pavement, bituminous-coated aggregate base: runways (CAA method).

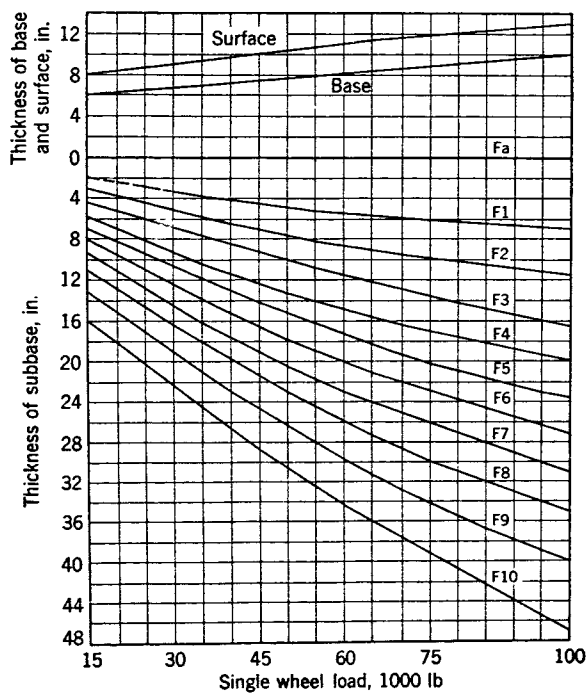
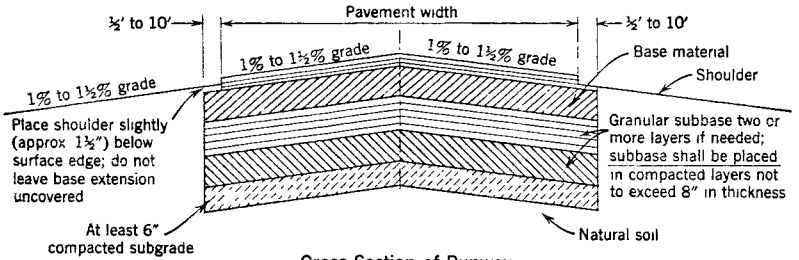
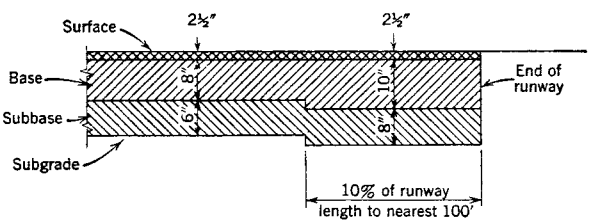


Fig. 27. Flexible pavement, bituminous-coated aggregate base: taxiways, aprons, and runway ends (CAA method)

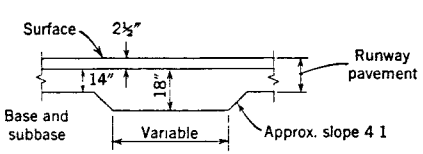


Cross Section of Runway



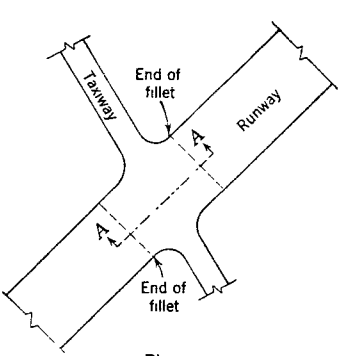
End Section With Subbase

This longitudinal section illustrates runway end design for a single wheel load of 45,000 lb and a F3 subgrade. Pavement thickness increased for 10% of runway length to nearest 100'.



Section *A-A*

Increase in depth of base and subbase at intersection of a taxiway and runway. Length of section variable depending on angle of intersection Length shall be equal to maximum distance between fillet ends.



Plan

FIG 28 Typical sections, flexible pavements.

Selective grading during construction is recommended so that the better types of soil found on the site will be placed in the upper part of the subgrade. This can result in a reduction in total costs, since the required thickness of pavement is so closely associated with the type and condition of the subgrade soil.

Corps of Engineers Design Procedure. Combined thicknesses of pavement (surface, base, and subbase) required for different wheel loadings, tire pressures, and landing gear assemblies are presented on Figs. 29–32. In these curves the pavement design depends on the California Bearing Ratio (CBR) of the subgrade soil, a test specially developed for

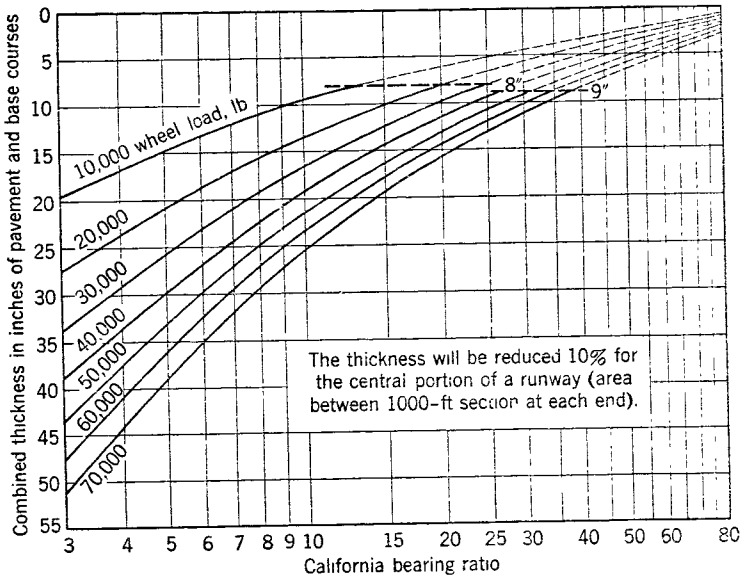


FIG. 29. Flexible-pavement design curves for taxiways, etc., single wheels, 100 lb/sq in. tire pressure. (U. S. Army Corps of Engineers, revised 5-21-51.)

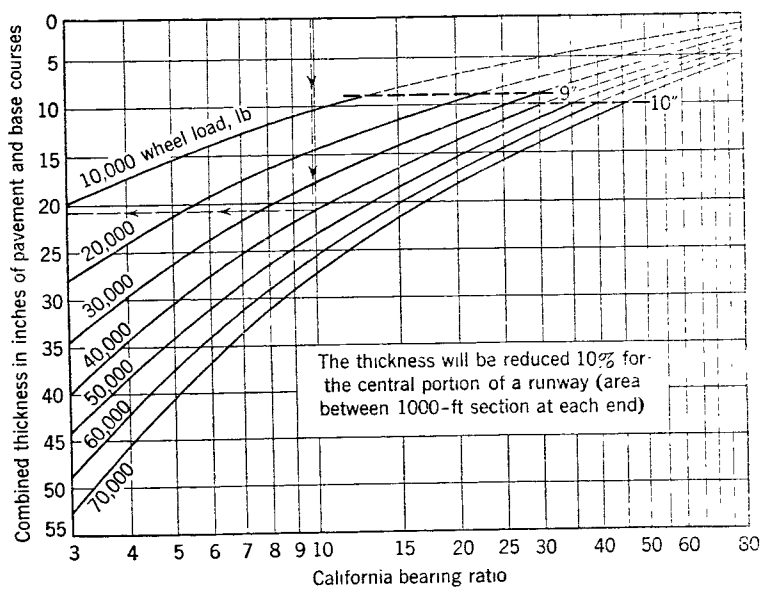


FIG. 30. Flexible-pavement design curves for taxiways, etc., single wheels, 200 lb/sq in. tire pressure. (U. S. Army Corps of Engineers, revised 5-21-51.)

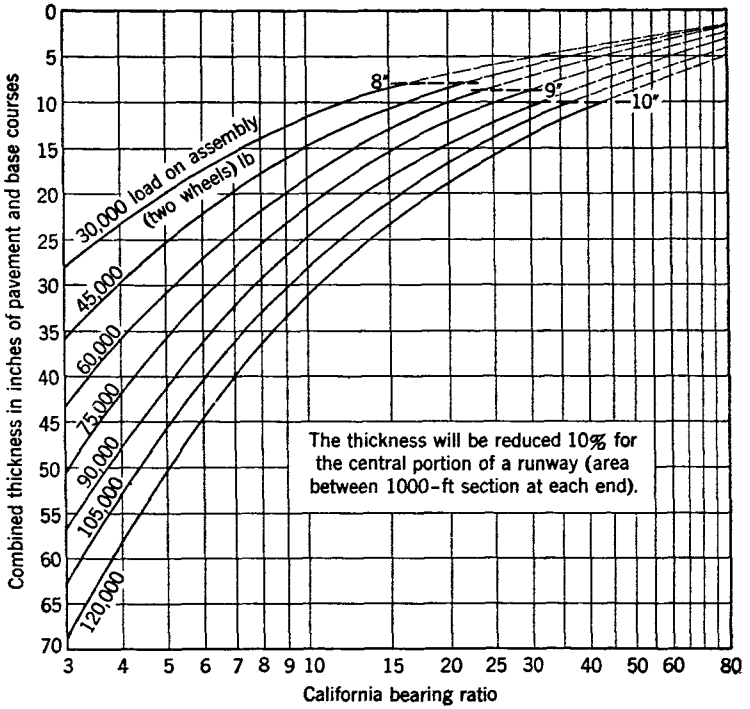


FIG. 31. (Tentative) Flexible-pavement design curves for taxiways, etc., dual wheels 37 5" c.c., 267 sq in. contact area each wheel. (U. S. Army Corps of Engineers.)

use in flexible pavement design. This test and its interpretation are described in Sect. 7. Table 13 gives CBR values for representative soils.

To illustrate the use of the design curves, assume a single wheel load of 40,000 lb, tire pressure of 200 lb/sq in., CBR = 10. Figure 30 shows the requirements for aircraft equipped with single wheel landing gear and with tires inflated to 200 lb/sq in. The chart is entered at the top for a CBR of 10 and the line is dropped vertically to intersect the curve for a single wheel load of 40,000 lb. It is then carried horizontally to the left-hand margin where the combined thickness in inches of pavement and base is seen to be 21 in.

This method of design permits the construction of a pavement by means of layers of different materials, each having an increasing CBR value as the surface is approached. This may result in more economical construction at sites where high-grade materials may be difficult to obtain.

Bureau of Yards and Docks Design Procedure. Results of plate loading tests performed on specially constructed trial sections are the basis for flexible-pavement design by the U. S. Navy. As a first step in the design an estimate is made of the required pavement thickness. Then trial sections, approximately 20 ft by 20 ft, are constructed at selected locations prepared and graded to subgrade elevation in (1) cut area, (2) fill area, and (3) transition area where there is no cut or fill except for the removal of plant growth. Three trial sections having thicknesses equal to, two-thirds of, and 1.5 times the estimated required thickness are constructed in each of the test areas (cut, fill, transition). In each case the surface course is replaced by an equal thickness of base course material so that it is not necessary to lay bituminous surfaces on the trial sections. Loading tests are made with circular plates on each of the test sections, and the unit pressures and deflections are measured. As an example of this method, it is desired to determine the pavement thickness required for a single wheel load of 50,000 lb and a tire pressure of 150 lb/sq in. From loading tests on the subgrade soil and a theoretical analysis of the deflections, it is estimated that this particular loading will require a pavement thickness of about 15 in. Therefore, the trial pavement sections are constructed with compacted base course thicknesses of 15 in., 10 in., and 22.5 in.

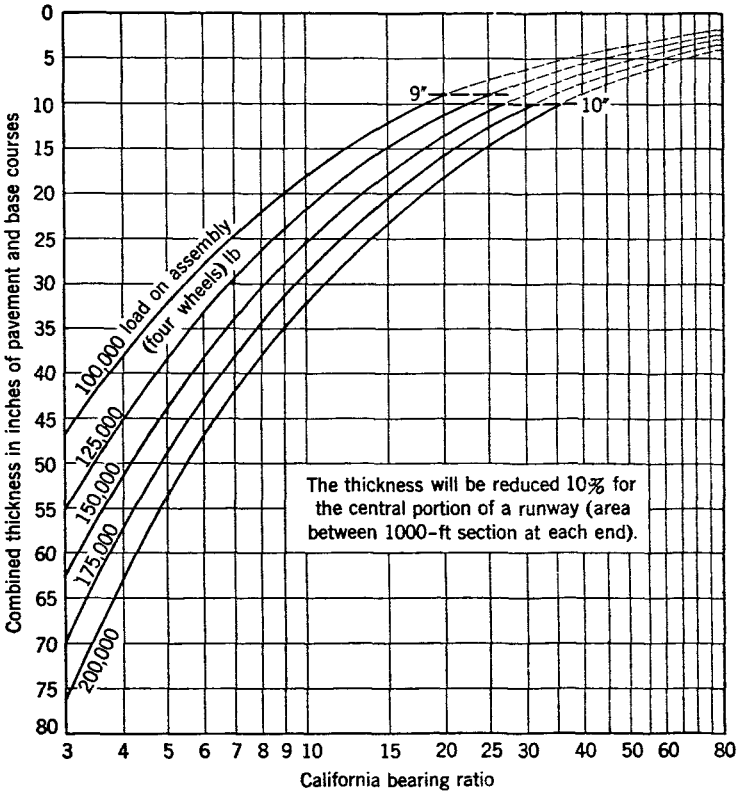


FIG 32. (Tentative.) Flexible-pavement design curves for taxiways, etc., twin tandem assembly 31" × 60" c.c., 267 sq in. contact area each wheel. (U S Army Corps of Engineers, revised 5-21-51)

Since the tire contact area is equal to 50,000/150 or 333 sq in., the diameter of the equivalent circular area is 21 in. to the nearest inch. Consequently, the pavement tests are made with a 21-in. diameter plate. Tests on the subgrade are made with a plate 30 in. in diameter. Deflections or settlements of the circular plate on each of the test pavements in each of the test areas are measured and plotted against the pavement thickness as shown on Fig. 33. A smooth curve is drawn through the points corresponding to the greatest settlement. The required pavement thickness is taken from the point on the curve corresponding to a settlement of 0.20 in.

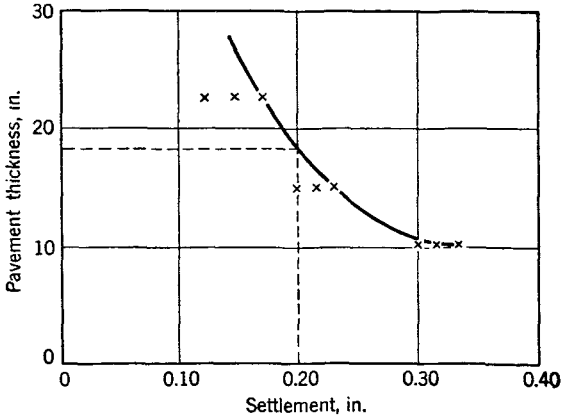


FIG. 33. Flexible-pavement design (Bureau of Yards and Docks).

If conditions at the site indicate that it is desir-

Table 13. Characteristics Pertinent to Road and Runway Foundations

(July 1951)

Major Divisions (1)	(2)	Letter (3)	Name (4)	Value as Foundation When Not Subject to Frost Action (5)	Value as Base Directly under Wearing Surface (6)	Potential Frost Action (7)	Compressibility and Expansion (8)	Drainage Characteristics (9)	Compaction Equipment (10)	Unit Dry Weight, lb/cu ft (11)	Field CBR (12)
Coarse-grained soils	Gravel and gravelly soils	GW	Gravel or sandy gravel, well-graded	Excellent	Good	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tired equipment, steel-wheeled roller	125-140	60-80
		GP	Gravel or sandy gravel, poorly graded	Good to excellent	Poor to fair	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tired equipment, steel-wheeled roller	120-130	35-60
		GU	Gravel or sandy gravel, uniformly graded	Good	Poor	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tired equipment	115-125	25-50
		GM	Silty gravel or silty sandy gravel	Good to excellent	Fair to good	Slight to medium	Very slight	Fair to poor	Rubber-tired equipment, sheep's-foot roller, close control of moisture	130-145	40-80
		GC	Clayey gravel or clayey sandy gravel	Good	Poor	Slight to medium	Slight	Poor to practically impervious	Rubber-tired equipment, sheep's-foot roller	120-140	20-40
	Sand and sandy soils	SW	Sand or gravelly sand, well-graded	Good	Poor	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tired equipment	110-130	20-40
		SP	Sand or gravelly sand, poorly graded	Fair to good	Poor to not suitable	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tired equipment	105-120	15-25
		SU	Sand or gravelly sand, uniformly graded	Fair to good	Not suitable	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tired equipment	100-115	10-20
		SM	Silty sand or silty gravelly sand	Good	Poor	Slight to high	Very slight	Fair to poor	Rubber-tired equipment, sheep's-foot roller, close control of moisture	120-135	20-40

		SC	Clayey sand or clayey gravelly sand	Fair to good	Not suitable	Slight to high	Slight to medium	Poor to practically impervious	Rubber-tired equipment, sheep's-foot roller	105-130	10-20
Fine-grained soils	Low compressibility, $LL < 50$	ML	Silts, sandy silts, gravelly silts, or diatomaceous soils	Fair to poor	Not suitable	Medium to very high	Slight to medium	Fair to poor	Rubber-tired equipment, sheep's-foot roller, close control of moisture	100-125	5-15
		CL	Lean clays, sandy clays, or gravelly clays	Fair to poor	Not suitable	Medium to high	Medium	Practically impervious	Rubber-tired equipment, sheep's-foot roller	100-125	5-15
		OL	Organic silts or lean organic clays	Poor	Not suitable	Medium to high	Medium to high	Poor	Rubber-tired equipment, sheep's-foot roller	90-105	4-8
	High compressibility, $LL > 50$	MH	Micaceous clays or diatomaceous soils	Poor	Not suitable	Medium to very high	High	Fair to poor	Rubber-tired equipment, sheep's-foot roller	80-100	4-8
		CH	Fat clays	Poor to very poor	Not suitable	Medium	High	Practically impervious	Rubber-tired equipment, sheep's-foot roller	90-110	3-5
		OH	Fat organic clays	Poor to very poor	Not suitable	Medium	High	Practically impervious	Rubber-tired equipment, sheep's-foot roller	80-105	3-5
Peat and other fibrous organic soils		Pt	Peat, humus, and other organic soils	Not suitable	Not suitable	Slight	Very high	Fair to poor	Compaction not practical	...	...

Note 1 In column 5, values are for subgrades and base courses except for base course directly under wearing surface.

Note 2 In column 6, the term "excellent" has been reserved for base materials consisting of high-quality processed crushed stone.

Note 3 In column 10, the equipment listed will usually produce the required densities with a reasonable number of passes when moisture conditions and thickness of lift are properly controlled. In some instances, several types of equipment are listed, because variable soil characteristics within a given soil group may require different equipment. In some instances, a combination of two types may be necessary. *a. Processed base materials and other angular materials.* Steel-wheeled rollers are recommended for hard angular materials with limited fines or screenings. Rubber-tired equipment is recommended for softer materials subject to degradation. *b. Finishing.* Rubber-tired equipment is recommended for rolling during final shaping operations for most soils and processed materials. *c. Equipment size.* The following sizes of equipment are necessary to assure the high densities required for airfield construction. Crawler-type tractor, total weight in excess of 30,000 lb. Rubber-tired equipment, wheel load in excess of 15,000 lb. Wheel loads as high as 40,000 lb may be necessary to obtain the required densities for some materials (based on contact pressure of approximately 65 to 150 lb/sq in.) Sheep's-foot roller, unit pressure (on 6- to 12-sq-in. foot) to be in excess of 250 lb/sq in. Unit pressures as high as 650 lb/sq in. may be necessary to obtain the required densities for some materials. The area of the feet should be at least 5% of the total peripheral area of the drum, using the diameter measured to the faces of the feet.

Note 4 In column 11, unit dry weights are for compacted soil at optimum moisture content for modified AASHO compactive effort.

able to compensate for anticipated increase in moisture content, laboratory compression tests are made on 2 by 4 in. cylinders cored from the subgrade. These are tested at optimum moisture content and at a moisture content of 2% above optimum. From the results of these tests a correction factor is applied to the settlements determined by means of the loading tests.

Accelerated Traffic Tests. Where time permits and different types of base course materials are available, it may prove economical, on certain projects, to construct a test track consisting of short sections of pavements of variable thicknesses and using various combinations of materials and construction techniques. The test track is then subjected to accelerated traffic of a specified wheel loading, and the relative performance of each type and thickness of pavement is determined. Information from these tests will aid in the establishment of a pavement design for the materials and conditions peculiar to the site. It may also result in monetary savings far in excess of the costs of the tests.

34. STRENGTHENING EXISTING PAVEMENTS

Because of increasing loads imposed by new types of aircraft, it is frequently necessary to strengthen existing pavements. In the case of flexible pavements, this can be accomplished by adding a bituminous concrete overlay to the existing surface and the thickness can be determined by one of the design methods given above. In connection with concrete pavements, however, the problem is more complex since the addition of concrete slabs less than 6 in. in thickness is not satisfactory.

With respect to concrete overlays, the Portland Cement Association suggests the following formula as a guide:

$$h_c = \sqrt{h^{1.87} - Ch_e^2}$$

where

h_c = thickness of overlay slab.
h = thickness required of an equivalent single slab placed directly on the subgrade with a working stress equal to that of the overlay slab.
h_e = thickness of existing slab.
C = coefficient, depending on condition of existing pavement.

For example:

C = 1 when existing pavement is in good condition.
C = 0.75 when existing pavement has initial corner cracks due to loading but no progressive cracks.
C = 0.35 when existing pavement is badly cracked or crushed.

When conditions are such that satisfactory bond cannot be obtained between the overlay and existing slabs, the overlay may be determined by the following formula:

$$h_c = \sqrt{h^2 - Ch_e^2}$$

The design of bituminous overlays for concrete pavements is still under investigation. For this reason, no design curves or formula are presented herein.

35. PAVEMENTS FOR JET-POWERED AIRCRAFT

Although pavements to be used by jet aircraft are designed, structurally, in the same manner as pavements accommodating aircraft powered by reciprocating engines, the operation of jet aircraft has raised many questions concerning the effects of fuel spillage and high exhaust temperatures on airport pavements. These factors are still under observation, but it is generally agreed that spillage is not a serious problem except in those areas where refueling is performed. This problem exists with all types of aircraft. It has been observed that dense-graded, asphaltic concrete offers excellent resistance under normal operating conditions. Also, the use of tar in bituminous mixtures and in seal coats makes them even more resistant. Bituminized rubber compounds seem to give the best service as joint sealers in Portland cement concrete pavement.

The effect of jet blast, however, is severe in certain areas. These areas are small in extent and are almost entirely confined to locations where engines are tested. In these locations, the pavement is subjected to sustained blasts of 5–10 minutes' duration, or longer. This has caused serious erosion of bituminous pavements and has blasted out the sealing material in joints of rigid pavements.

There is no damage to either concrete or bituminous pavements as long as the jet aircraft is in motion.

Some minor surface deterioration of bituminous pavements has been observed at ends of runways and on short sections of taxiways where they join the end of the runway. To overcome these conditions certain agencies recommended the construction of concrete surfaces for a distance of 300–500 ft at the ends of runways and on the adjoining taxiways. Also, concrete is recommended in engine test areas.

It is important to note that the effects of jet aircraft on pavements will depend on the design of the aircraft with respect to velocity and temperature of the blast and the angle of the exhaust. The more the angle of the blast approaches the horizontal, the less importance it has as far as pavements are concerned. Finally, it should be remembered that any conclusions reached regarding the performance of pavements under the operation of jet aircraft must be qualified as tentative and subject to change as new models of aircraft are developed.

36. PERSONAL AND SECONDARY AIRPORTS

The foregoing design requirements cover air-carrier-service types of airports. For personal and secondary airports, similar types of pavements will be used but requirements, both as to quality of material and thickness, may be relaxed so that the best possible use may be made of locally available materials. Bituminous surfaces may consist of surface treatments and seal coats rather than plant-mixed bituminous concrete. In most cases, pavements, including base courses, need not be thicker than 6 to 8 in. except under severe conditions of drainage and frost, when it may be necessary to include subbases ranging from 3 in. to 12 in. in depth.

In many instances, a turf surface will be satisfactory for personal and secondary types of airports. If the soils are such that they become unstable in wet weather, it is recommended that the upper 4–10 in. be stabilized with granular materials before establishment of a turf surface. Stabilization requirements are given in Table 14. Article 41 of this section gives general information on the development of a turf. For additional information at a particular site local agricultural authorities may be consulted.

Table 14. Aggregate-Turf Pavement, Wheel Load Up to 5000 Lb

	Good Drainage		Poor Drainage	
Soil Group	No Frost	Severe Frost	No Frost	Severe Frost
E-1	Stabilize or condition existing soil to a depth of 4–6 in.			
E-2				
E-3				
E-4				
E-5				
E-6	Stabilize to obtain 55% retained on No. 200 sieve for 6-in. depth			
E-7				
E-8	Stabilize to obtain 65% retained on No. 200 sieve			
E-9				
E-10	For 6-in. depth	For 8-in. depth	For 8-in. depth	For 10-in. depth
E-11				
E-12				
E-13	Unsatisfactory	Unsatisfactory	Unsatisfactory	Unsatisfactory

If coarse aggregate is available efforts should be made to include such material in stabilization.
The above requirements are minimum and should be increased where conditions warrant.

37. BITUMINOUS SURFACES

Bituminous surfaces on airports must (1) provide resistance to the penetration of surface water, (2) protect the base from the raveling and disintegrating effects of traffic, (3) provide a smooth, well-bonded surface free from loose particles that might cause damage to planes, (4) resist without damage the shearing stresses occasioned by short-radius turns, and (5) furnish a non-skid surface while avoiding a rough surface that would cause excessive tire wear. In order to accomplish this, the bituminous materials and aggregates must be combined in such a manner as to produce mixtures which, when properly compacted,

will furnish a uniform surface of suitable texture and provide the maximum in stability and durability.

These objectives can best be achieved by the use of a dense-graded, hot-laid bituminous mixture produced in a central mixing plant where accurate control of materials and proportions can be assured. When properly designed and compacted, they are practically impervious to water and possess high structural strength.

CAA thickness requirements for the bituminous surface range from 2 in. for a single wheel load of 15,000 lb to 3 in. for a 100,000-lb wheel load when the base courses consist of non-bituminous materials or bituminous-coated aggregate. When a bituminous base is surfaced, the range in thickness varies from 1 $^1/_2$–3 in The Corps of Engineers specify minimum thicknesses of bituminous surface ranging from 2–4 in. depending on wheel loading and tire pressure.

38. BASE COURSES

Base courses, together with a subbase when required, serve to distribute the wheel load pressures to the underlying subgrade, and, as such, should be of adequate stability and thickness (1) to prevent failure of the subgrade, (2) to withstand the stresses produced in the base, (3) to resist vertical pressures that tend to produce consolidation with resultant distortion of the bituminous surface, and (4) to resist volume changes caused by fluctuations in moisture.

Stability of the base course is controlled by specifications which cover grading, physical properties, and compaction of the mixtures as well as quality of the individual materials comprising the mixtures. An important requirement, common to all gravel, stone, or slag base courses, is the limitation that the liquid limit shall not exceed 25 and the plasticity index shall not exceed 6.

39. SUBBASES

Subbases form an integral part of the flexible pavement structure on all but the very best of subgrades where the base course alone is considered to be adequate in thickness. Subbase materials are generally of inferior quality compared to base course materials but superior compared to the subgrade soil on which they are placed. They usually consist of locally available materials that can be obtained at a much lower cost than the higher-quality base course. Their use results in great savings in construction costs since the required amount of the more expensive base course material can be kept to a minimum.

In addition to increasing the load support furnished by the entire pavement structure, properly designed subbases serve the following purposes.

1. Provide a drainage course.
2. Eliminate frost heave.
3. Minimize the effects of volume change, elasticity, and capillarity.

A subbase under concrete pavements serves several purposes:

1. It improves the support furnished by the foundation soil.
2. It provides a more uniform bearing for the pavement.
3. It prevents pumping action, which may be defined as the displacement and ejection of water carrying soil particles in suspension caused by the deflection of slab ends at joints and cracks under heavy loads.
4. It reduces the detrimental effects produced on pavements by volume change in soils.
5. It prevents the detrimental effects produced on pavements as a result of frost action in soils, provided the subbase is of sufficient thickness.

In connection with frost action, there may be instances where the combined thickness of pavement and subbase may not be adequate to prevent detrimental frost heave with the resulting breakage of the pavement. In this case, the thickness of subbase should be increased over that shown on the design curves. The actual combined thickness required must be determined from a study of local conditions.

40. SPECIAL COMPACTION REQUIREMENTS

In order to avoid the development of pavement settlement and rough surface conditions as a result of base and subgrade compaction under the action of heavy aircraft, the following compaction requirements have been established by the CAA for all areas to be paved:

Base courses, 95% of maximum density.

Subbases, 95% of maximum density.

Subgrade (fills), 95% of maximum density in upper 9 in.
Subgrade (cuts), 95% of maximum density in upper 6 in.
Compaction requirements specified by the Corps of Engineers are given in Table 15.

Table 15. Compaction Requirements, Military Airfields

Wheel Load, lb	Depth in Inches below Pavement Surface to Which Indicated Per Cent of Modified AASHO Density Should Extend			
	All Subgrades except Cohesionless Sands		Cohesionless Sands	
	100%	95%	100%	95%
5,000	.		...	12
15,000	.	12	12	24
40,000	12	18	24	36
70,000	18	30	30	48
150,000	30	54	48	78

The depths shown are for cases where the indicated compaction extends into the subgrade. In no case should the compaction in the top 6 in. (after compaction) of the subgrade be less than 95% for all subgrades except cohesionless sands, and for cohesionless sands compaction should be 100%. All fills will be compacted to 90% of Modified AASHO density below the depths to which greater compaction is indicated.

Compaction requirements are constantly under investigation, and requirements are subject to change as more information becomes available.

TURFING

41. FUNCTIONS AND CLASSIFICATION

Turf is used on airports as a wearing surface for traffic areas; to control dust and erosion; and to improve the appearance of the airport, especially the grounds around buildings, drives, and parking areas. The primary advantage of turf over other types of cover is economy, both in initial development and subsequent maintenance.

Turfed areas of an airport differ in purpose and vary in usage. The kind of turf and the cultural practices needed to develop and maintain it will vary somewhat, depending on the specific area involved.

The use of frequent-traffic areas necessitates the development of the best type of turf that it is practical to produce under the soil and climatic conditions prevailing on the site. This turf should be long-lived, dense, durable, low-growing, and sod-forming, and have maximum bearing capacity, ability to recover rapidly from dormancy or abuse, and a short establishment period. On occasional-traffic areas, a thinner turf may be satisfactory and a longer establishment period allowed; thus smaller quantities of seed and fertilizer are required. Non-traffic areas are varied in character and each group may require different treatment. A typical turfing plan is shown on Fig. 34.

42. SELECTION OF SPECIES

The selection of grass species for any airport planting is dependent on many variable factors, including climatic conditions, physical and chemical properties of the soil, the intended use of the areas, the speed of coverage that is desired, the availability of seed or vegetative planting material, and the kind of maintenance that is likely to be given to the area Each species has its limitations as to the areas where it will thrive; some are adaptable to extensive areas, whereas others are restricted to areas limited to only a few states. It must be borne in mind that for any specific location, the local soil and climatic conditions, as well as other variables, must be considered before selecting the grass species to be used.

The use of legumes on landing strips or other frequent-traffic areas on airports is not encouraged because they do not have the ability to withstand intensive plane traffic.

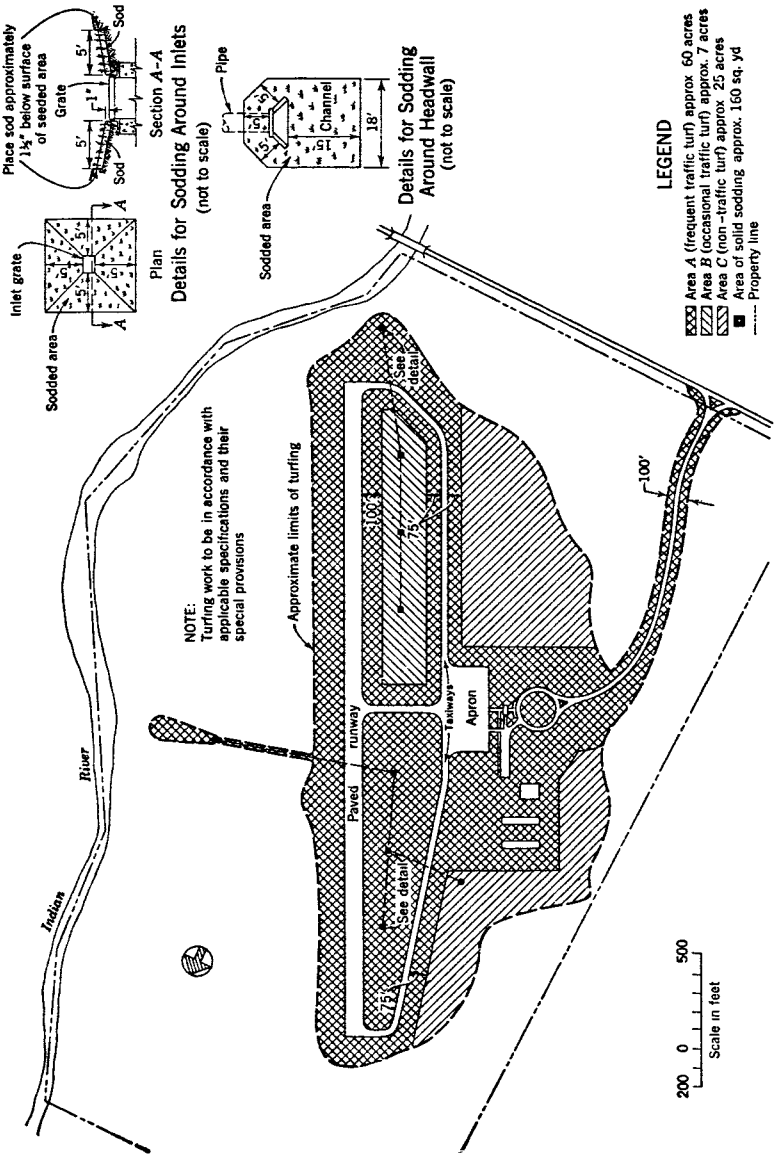


FIG. 34. Typical turfing plan.

When used as a mixture with turf species, they tend to crowd out the grasses in localized spots rather than to grow uniformly throughout the area. On non-traffic areas, however, they sometimes may be used to advantage, particularly on infertile soils that are subject to erosion.

43. PLANTING METHODS AND PROCEDURES

Turf grasses may be established either by seeding or by vegetative planting. In most cases, the establishment of turf by seeding is much less expensive than by sprigging or sodding. On the other hand, the latter methods usually provide a surer means, especially under extremely adverse conditions.

Important factors to be considered in seeding are time and rate of seeding, even distribution of seed, depth of planting, and compaction. Best results are obtained when seed is sown at the season when conditions for germination are most favorable and while there is still time for a good growing period.

Important considerations in sprigging are that the material be viable, dense, and thickly matted, that there is sufficient moisture in the soil to insure rooting and establishment, and that the sprigs are placed deeply enough to prevent drying out. During the operations of harvesting, transporting, and planting, the sprigs should not be permitted to dry out.

Another procedure often used in developing turf is mulching, the purpose of which is to provide an immediate cover to protect bare soils against water and wind erosion, to conserve moisture, to control temperature, and to aid in the establishment of permanent vegetation. The most common forms of mulch materials are hay and straw. Anchoring is a very important phase of the mulching operation. It is greatly influenced by previous tillage operations, soil conditions, and distribution of mulch material, but above all, it is contingent upon the use of suitable anchoring equipment.

Seeded, sprigged, topsoil-planted, hay-mulch seeded, or sodded areas should be compacted immediately following planting operations. Compaction firms the soil around the seed, sprigs, or sods, eliminates air pockets, and encourages the movement of capillary moisture through the surface soil. An ordinary water-ballast roller, Culti-Paker, or similar equipment is sufficient for compaction. The use of heavy road rollers is undesirable.

When turfing work is done at the right season, watering is ordinarily not necessary. The high cost of application of water to large areas in quantities sufficient to be of any value usually prohibits its use, especially in seeding operations. Light surface watering, such as is done with sprinkler trucks, is rarely effective. There are instances, however, where watering may be necessary and justified. This is more often the case with sprigging or sodding operations than with seeding.

44. LIME AND FERTILIZER

In many sections of the country, the airport soil is too acid even for the more acid-tolerant species of grass. Lime is valuable as a neutralizer of excessive soil acidity and also assists in making certain elements available for plant use.

The need for lime should be determined by one of the approved methods of soil testing, either in the field or in a laboratory. Ground limestone or agricultural lime generally is the preferred form of lime to use on airports. When lime is needed, $^1/_2$–2 tons per acre are adequate except in extreme cases.

No other controllable factor is more essential to the development of an adequate and serviceable stand of airport turf than the addition of fertilizer. The function of fertilizer is to supplement the nutrient-supplying power of the soil so that an adequate quantity of each element required for plant growth is available to the grass.

The decision as to the grade and quantity of fertilizer required for turf at any particular airport site is dependent upon several local factors, including mineral deficiencies of the soil, requirements of the species to be grown, the intended use of the area, the speed of development required, type of soil, rainfall, and local sources of fertilizer materials.

Nitrogen is usually deficient in the soil, and its application may be required for several years after the initial planting.

Under most climatic and soil conditions where all three of the common fertilizer ingredients are needed, mixtures which contain approximately half as much phosphoric acid as nitrogen and still less potash have been found to give the best results at the minimum cost. For establishing new turf, fertilizer should be applied immediately before or during planting operations.

45. SEEDBED PREPARATION

The best turf will be obtained on a correctly prepared seedbed. Improper ground preparation usually results in turf stands which are weedy, spotty, and rough. In the humid regions, a well-pulverized, firm surface is preferred to encourage the rapid growth of the small-seeded grasses that are generally used for airports. In the arid and semiarid regions, the surface may be left firm but roughened to check wind erosion and loss of seed.

The ideal soil for turf development is one which is relatively stable when wet or dry, readily permeable to water, inherently fertile, and capable of supporting a dense turf. In general, fertile, porous sandy loams or sandy clay loams will meet these requirements.

46. COVER AND NURSE CROPS

In many cases, grading, paving, or other construction operations on an airport are completed at a season when permanent grasses cannot successfully be established. In such instances, it is often desirable to plant annual or temporary cover crops to control dust and erosion until the permanent species can be developed. Rapid-growing cereal grains or grasses are the commonly used temporary covers, although in some cases legumes may be grown to advantage.

In some sections of the country, temporary grasses are used in combination with the permanent species and are referred to as nurse grasses. The most commonly used species of permanent grass develop slowly in the seedling stage and, consequently, provide little protection to the soil for several weeks, or much longer in the case of dry-land native grasses. The quick-developing temporary species are included in the seed mixture to afford this initial protection. However, if the nurse-crop seeding is too heavy, or if the nurse crop is allowed to grow too high before mowing, it may fail in the objective of aiding the permanent species and may actually smother them. The seeding rate for the temporary species as compared to that for the permanent species should therefore be kept low.

47. ESTABLISHMENT

The most critical period in the development of turf is the several weeks or months during which the new growth is being established. During this period, turfed areas usually require much more work on them than is needed after the desired species are well established. It is most important that any excess growth of weeds or temporary species be mowed in time to avoid too much competition for the permanent grasses. Areas where the stand of grass is thin or weak should be reseeded or refertilized, or both, as needed to produce a uniform good cover

Neglect of turfed areas during the period of establishment may result in loss of much of the turf and lead to extensive and costly renovating or repair work.

LANDSCAPE PLANTING

48. PLANTING DESIGN AND LAYOUT

Planting design of an airport involves the consideration of the entire area of the airport, including the entrance, access, and other roads and walks, buildings, parking lots, outlying areas, and, in some cases, even areas outside the property boundaries. It should be planned not only from the point of view of the observer on the ground, but also in the air. Aerial observations of an airport give a comprehensive impression of the whole area, and the lack of proper planning or appropriate planting is quite conspicuous

By planning well in advance of airport construction, desirable existing vegetation can often be protected and saved in place, or removed to other locations on the site much more economically than it could be provided by subsequent operations.

One of the first items to consider in planting design is the purpose for which the planting is intended. The choice of material, its location and arrangement, and other considerations are dependent upon the purpose or function of the planting.

Simplicity of layout is another important objective in airport planting design. Straightforward planting schemes which take broad simple forms create an orderly appearance

and are in keeping with other elements of the airport design in which form follows function. Plantings which are too elaborate or follow complicated layouts are not conducive to economy, either in original cost or in subsequent maintenance.

Informality in planting design has certain advantages over formality for most areas in the airport because of its greater flexibility, softer appearance, less exacting requirements, and lower maintenance costs. Only in some cases may formality, implying a more or less symmetrical treatment about an axis, be justified and desirable for limited areas (see Fig. 35).

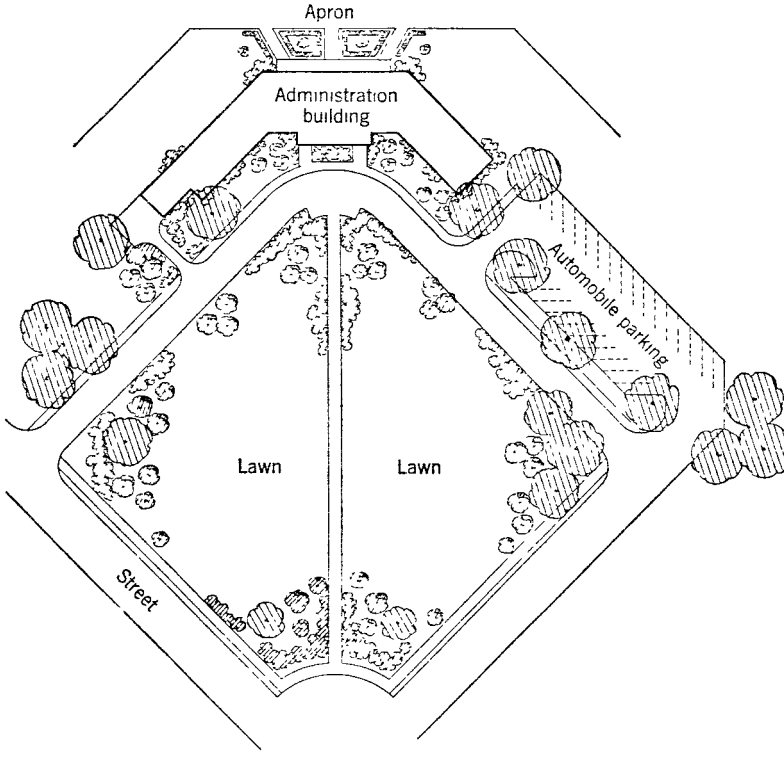


FIG 35 In this simple plant arrangement, shrubs and trees are used effectively and, once established, require little maintenance

Plantings should in no instance be located so as to make the use of the airport hazardous for flying. Trees or shrubs should not be planted in the approach zones or other critical areas where their ultimate height will be above the obstruction clearance lines.

49. SELECTION OF PLANTS

Selection of plant materials to be used on airports is influenced by many factors. Fundamentally, plants are chosen for their ability to produce a given desired effect; that is, for the natural growth characteristics of the plants themselves. In one situation the most pleasing appearance may be obtained from using evergreen shrubs; in another, deciduous flowering shrubs may be most appropriate; in a third, tall upright or low spreading forms may be desirable. The many varying characteristics of plants, including size, shape, texture, and character of foliage, color and time of blooming, fruiting display, etc., offer many different effects from which to choose. Then, of course, the plants selected must be hardy and suitable to the local conditions of soil, climate, altitude, exposure, and other environmental features influencing plant growth.

In determining whether trees, shrubs, or combinations should be used for a certain location, it should be borne in mind that trees furnish shade which is often desirable and frequently lacking on airports. In the original planting, trees are more difficult to handle, and with their wide spacing do not give an early effect as do shrubs. However, trees are generally longer-lived and less expensive to maintain than shrubs.

50. PLANTING OPERATIONS

The success or failure of airport plantings is often determined by the sequence and detailed execution of planting operations. Even under the most favorable conditions, the moving of trees, shrubs, or vines from one location to another results in a severe shock to them. But by proper planting and care thereafter, recovery can be hastened and the plants will become adjusted to their new environment.

Airport building or grading operations are not usually completed at the most opportune time for planting operations. In such cases it is advisable to delay planting operations until a favorable planting season. The best time varies with local conditions. Generally, the most favorable period for the planting of most species is in the spring or fall.

Mulching is of value in all planting work. It adds a surface cover which provides shade to the ground beneath and reduces the loss of water through evaporation. In cool climates it protects the plants from the injurious effects of alternate freezing and thawing. The more stable mulch materials tend to reduce both wind and water erosion.

LIGHTING

51. PLANNING THE SYSTEM

Planning an airport lighting system must take into consideration, (*a*) the size and shape of the airport, (*b*) the amount of traffic, (*c*) type of night operations planned, (*d*) the type of aircraft to be used, (*e*) type of landing surface provided (paved or unpaved), (*f*) ultimate development of the airport, (*g*) funds available, (*h*) power service available, and (*i*) climatic conditions.

General overall standards and practices apply to all airports, but each airport lighting installation must, to some extent, be separately designed to suit local conditions. Essential basic elements of all airport lighting systems are: (*a*) beacons, (*b*) runway, strip, or boundary lights, (*c*) taxiway lights, (*d*) obstruction lights (if required), and (*e*) illuminated wind indicators. The beacon tells the pilot the location of the lighted airport; the runway, strip, or boundary lights outline the area the pilot must use in setting the aircraft on the ground; the obstruction lights warn the pilot of hazards to avoid in making his landing: the illuminated wind indicator gives him necessary wind direction information which he needs in making his final approach and touchdown.

To assist the airport operator or designer to choose the type best suited for his airport, the CAA has prepared various lighting bulletins and equipment specifications which are available on request. There are also well-qualified lighting engineers in each regional office of the CAA who can furnish assistance in the selection of units and general design of lighting systems.

52. AIRPORT BEACONS

For identifying the airport at night there are three types of airport rotating beacons approved by the CAA. The best-known type is the large 36-in. rotating beacon conforming to Specification CAA-291. This beacon consists of a single large drum with a 1000-watt lamp in the center and a lens at each end of the drum providing two beams of light 180° apart. For indicating a lighted airport a green color screen is placed between the lamp and the lens at one end which results in the pilot's seeing an alternate green and white flash as the beacon rotates.

Another type is the 24-in. beacon which consists essentially of two 24-in. drums placed back to back on a turntable with a green lens in one drum and a clear lens in the other. Each drum has its own lamp of 500 or 1000 watts and each drum is essentially a separate projector. The two drums mounted back to back on the turntable are rotated so as to produce alternate green and white flashes for the pilot.

The third type of beacon used by the CAA is the small airport beacon made in conformance to CAA Specification L-801. This beacon is used only at small fields where there are no regularly scheduled commercial flights. It produces much less candlepower in its main beams than the other two types mentioned above.

53. RUNWAY, STRIP, AND BOUNDARY LIGHTS

Runway lights are divided into two general categories: medium and high intensity. Medium-intensity lights are used for runways on which landings are made normally only during contact conditions. They do not provide enough candlepower for landings under low visibility conditions; i.e., instrument conditions.

Medium-intensity lights are of the elevated type and are designed to conform to CAA Specification L-802. They are a considerable improvement over the old-style flush marker lights, many of which are still in use.

High-intensity runway lights should always be installed on designated instrument runways where landings and take-offs are to be made under conditions of restricted visibility. These lights provide much more candlepower in their main beams than do the medium-intensity lights. Three different types of high-intensity lights are available from which the airport operator or designer may choose. The three CAA specifications covering the high-intensity runway lights are listed below:

1. L-818. Specification for Controllable Beam, Bi-Directional High Intensity Runway Light, dated May 1, 1948.
2. L-819. Specification for Fixed Focus Bi-Directional High Intensity Runway Light, dated January 1, 1950.
3. L-820. Specification for Fixed Focus, Uni-Directional High Intensity Runway Light, dated May 1, 1948.

A standard configuration and layout for runway lights, both medium and high intensity, provides that there shall be clear lights along the sides of paved runways at intervals of as near 200 ft apart as is practicable. The lights shall be located in straight rows not more than 10 ft out from the runway edge.

Across each end of the runway green threshold lights are placed in a row at right angles to the runway center line. The row of green threshold lights should be not more than 50 ft from the end of the runway. For runways not over 200 ft wide six threshold lights are used, three on each side with an 80-ft clearance gap in the center. For runways over 200 ft, eight threshold lights are used at each end maintaining the same clearance gap of 80 ft in the center.

Strip lights are identical with medium-intensity runway lights. The configuration used for strip lights is the same as that for runway lights with the exception of the green threshold lights. On an unpaged strip the corner lights at the end of the strip are clear white lights, and between these corner lights are placed a minimum of four green threshold lights equally spaced, 50 ft apart, and grouped symmetrically about the center line of the strip.

Boundary lights are seldom used today. However, where it is desired to light a small "all-way" field which contains no paved runways and no definite "strips," clear white boundary lights are placed around the perimeter of the field at intervals of 300 ft. These boundary lights are of the elevated type quite similar to the medium-intensity runway lights except that the light distribution is symmetrical; i.e., they do not produce definite beams 180° apart as is the case with runway lights.

54. TAXIWAY LIGHTS

Taxiway lights are quite similar in physical size and shape to the elevated medium-intensity runway light. The principal differences are that taxiway lights are always blue in color and the light distribution normally used is symmetrical. Taxiway lights are placed on both sides of the taxiway at a distance of not more than 10 ft from the edge of the paving. On straight sections of the taxiway the lights are spaced as near 200 ft apart as practicable, but closer spacing is used on curves to help guide aircraft around the curved portions at night. Closer spacing is also used where the taxiways join a runway or an apron. At these intersecting points the taxiway paving is joined to the runway apron by means of comparatively large curved "fillets." These fillets are treated the same as curved portions of the taxiways and additional taxiway lights are used to mark these entrances and exits to the taxiway.

55. OBSTRUCTION LIGHTS

Obstruction lights are small lighting fixtures with red globes that are designed for marking obstructions in the vicinity of an airport (or on the airport itself). These units can be operated from either series or multiple circuits and the red glass globes have prisms designed to concentrate most of the light output in a slight elevation from the horizontal. Looking at the unit from above the horizontal light, output is symmetrical. It is designed to warn pilots of obstructions in the approach ways and in the turning zones of an airport The obstruction lights may be included as part of the airport lighting system or may be operated from separate sources of power when located a remote distance from the airport The determination of which obstructions in the vicinity of an airport should be lighted is made by following the criteria established in the previously mentioned TSO-N18.

56. ILLUMINATED WIND INDICATORS

Some type of illuminated wind-direction indicator is necessary at all airports where night operations are planned. It is required that all airports, regardless of size, should have an illuminated wind cone. Larger airports may also find it desirable to have an illuminated wind tee in addition to the wind cone.

The illuminated wind cone consists essentially of a long fabric sleeve or "sock." The wind sock or cone is held open at the throat by a metal ring or cylinder, and the metal part is in turn attached to a vertical supporting pole by means of brackets fastened to the pole on bearings. The lubricated bearings permit the wind cone to rotate freely about the vertical supporting pole as the wind direction changes. Lamps with inverted reflectors above the wind cone furnish sufficient illumination to enable pilots to see the wind cone at night from a considerable distance out from the airport. An obstruction light is normally mounted on top of the supporting pole.

The lighted wind tee, when viewed from above, has the appearance of the letter T lying flat on the ground. The length of the stroke of the tee is approximately 18 ft, and the length of the head is approximately 12 ft Actually, the wind tee does not lie flat on the ground but is supported some distance off the ground on a vertical shaft. The wind tee is balanced on top of the shaft, and the shaft is free to rotate in any horizontal direction. At the base of the stroke of the tee there is a large vertical fin or tail which causes the tee to be actuated by whatever wind is blowing. It is designed to show true wind direction within 5° when the wind is approximately 4 miles per hour or more. Lamps with green globes are placed along the top edge of the stroke and head of the tee at intervals of 12 in. to provide a good lighted signal to aircraft at night.

Lighted wind cones and wind tees are normally placed inside a segmented circle marker, which is described in Airport Marking.

57. TRANSFORMER VAULT AND MISCELLANEOUS EQUIPMENT

There are many items of power-supply equipment needed in conjunction with the airport lighting at large airports. This equipment must be housed in a vault conforming to all of the requirements of the local code having jurisdiction, or to the requirements of the National Electrical Code. This transformer vault may consist of a separate concrete or brick building or it may be a suitable fireproof room inside one of the other airport buildings. For the small airport which has only one or two short runways to light, together with a small beacon, a small wind cone, and a few obstruction lights, the specially prepared vault might not be necessary These small airports may use any one of a number of low-voltage cabinets available which contain all the power-supply equipment needed, as well as control switches mounted directly on top of the cabinet.

As for miscellaneous airport lighting equipment, the most commonly used devices are apron floodlights for the loading and unloading zones. These apron floodlights are produced commercially in many different sizes. Where it is desired to purchase floodlights to light any particular area, the various manufacturers will furnish the necessary illumination data as a guide for obtaining proper light in the area.

In addition to the apron floodlights, there are a number of other items of miscellaneous airport lighting equipment that can be added as desired. Such items would include a tower traffic control signal light, a ceiling projector, a control-tower searchlight, etc.

In the design of an airport lighting system, the engineer must include complete wiring diagrams, runway layouts, and installation details (see Fig 36). These will be needed before any part of the lighting installation can be started

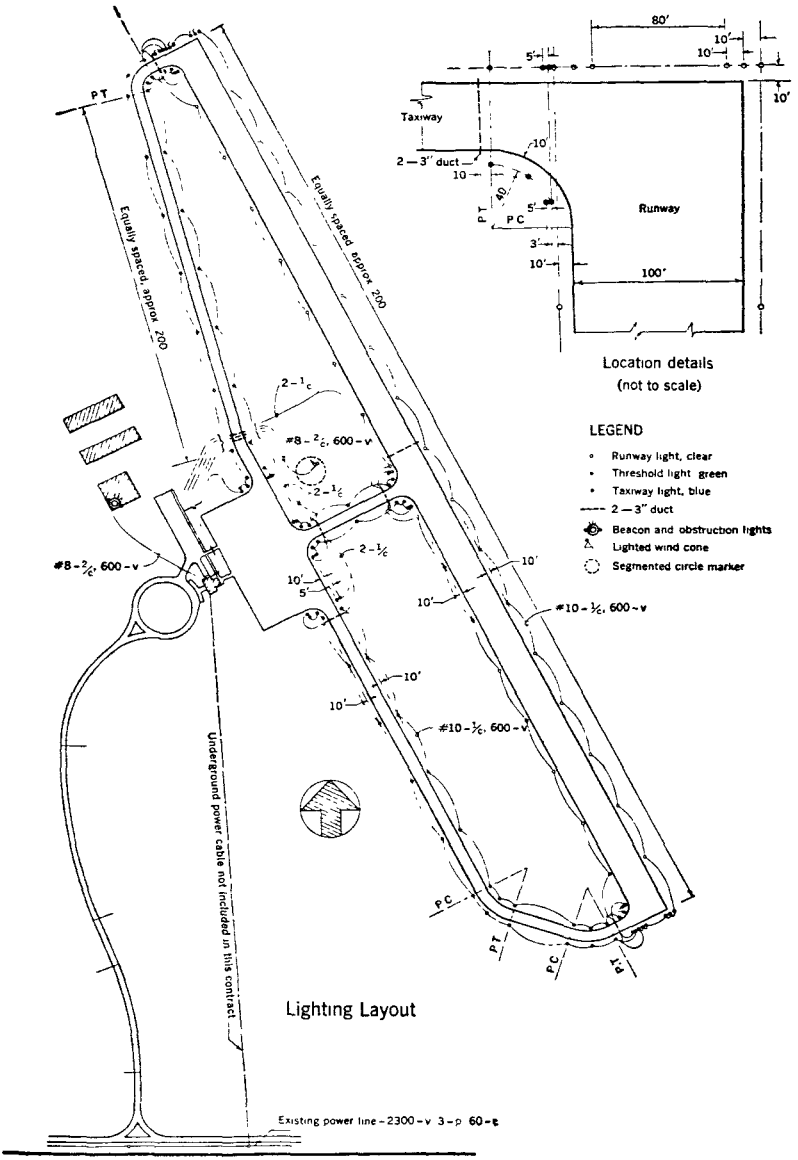


FIG. 36. Lighting layout.

AIRPORT MARKING

58. RUNWAY MARKING

In the interest of safety, representatives of the U. S. Air Force, U. S. Navy, and the CAA have formulated a uniform system for the marking of airfield runways and taxiways.[5]

Runway markings consist of numbers and letters, longitudinal stripes, length symbols, and distance markers. Numbers and letters assigned are determined from the approach direction. The number is the whole number nearest one-tenth the magnetic azimuth of the center line of the runway measured clockwise from magnetic north.

Symbols are used on the runway ends to indicate the runway length in feet, whereas distance markers are used to mark points 1500 ft from the ends of the runway.

All markings on runways are white and a background of black may be provided if considered necessary. All paints, except for numbers, letters, and length symbols, are required to be reflective.

59. TAXIWAY MARKING

Taxiway markings, consisting of longitudinal stripes and holding point markers, are yellow in color and should be reflective. The longitudinal stripe is a single line (1) terminating 10 ft from the nearest edge of the runway symbols at intersections with runway ends, (2) terminating at the nearest edge of the runway stripe at other intersections with runways, and (3) intersecting at taxiway intersections.

Taxiway intersections and points of junction of taxiways and runways are indicated by a holding point marker placed across the taxiway at right angles to the taxiway stripe. The nearest end of the holding stripe is not less than 100 ft from the nearest edge of the intersected runway or taxiway measured on a line perpendicular to the intersected center line.

60. BOUNDARY MARKERS

On small airports, especially those that are unpaved and unlighted, boundary markers are installed so that a pilot may identify the landing area. These should clearly indicate the area utilized for take-offs and landings. The placement and suggested construction of boundary markers are shown on Fig. 37.

61. SEGMENTED CIRCLE

The segmented circle airport marker system provides for a minimum installation consisting of a segmented circle located off the traffic area with a conventional wind cone located at its center. To this minimum installation other pilot aids and traffic-control devices are added as may be required to meet the conditions existing at a particular airport.

62. WIND-DIRECTION INDICATORS

Three types of wind-direction indicators are in common use: they are the wind cone, wind tee, and tetrahedron. They are manufactured in accordance with prescribed standards and need no further description here. It is important, however, that they be located where they can be readily seen from the air, and arrangements should be made for lighting them at night.

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[5] TSO-N10: Air Force-Navy-Civil Uniform Requirements for the Numbering and Marking of Airfield Runways and Taxiways, April 8, 1948, available from Aviation Information Office, Civil Aeronautics Administration, Washington 25, D. C.

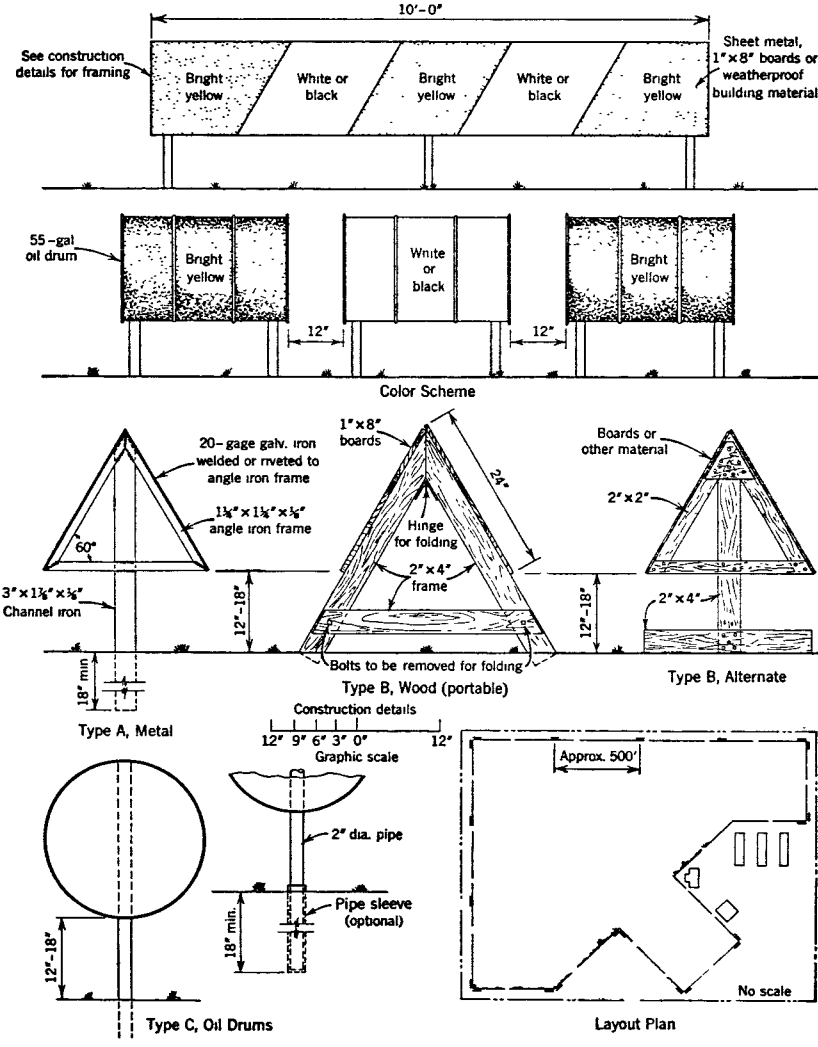


FIG. 37. Boundary markers.

AIRPORT BUILDINGS

Because speed is the prime advantage of air transportation, airport buildings should be located and planned so that rapid exchange will be achieved between air and ground transportation. To fulfill this requirement, buildings should be readily accessible to the people using them; also, the floor plan should be so arranged that activities performed in the buildings will be expedited. The highly specialized nature of this problem makes it essential, especially in large airport projects, that competent architectural service be used to coordinate the design and development of all buildings with other airport facilities.

Monumental treatment of airport buildings should be avoided. They should be functional and of an architectural style which is suited to the particular area. The design should

be such that the structure can be economically enlarged or altered. Also, the designer must comply with local building codes covering method of construction, acceptable materials, and fire and health requirements.

Usually, the airport is developed in stages of construction to meet the current aeronautical activities and demands. The master plan should depict the various stages from the initial to the ultimate development. The master plan of the building area must allow for expansion of any facilities in addition to being inherently flexible to permit accommodation of the continually changing pattern of airport activities.

63. BUILDING AREA

The building area provides for the transition of persons and property between surface and air-borne equipment. This area includes access roads, walkways, auto parking lots, loading aprons, and various buildings, all unified through appropriate landscape planting and architectural treatment.

The building area should be so located (1) that it will be accessible from the community with a minimum of travel time and distance, and (2) that it will provide direct access to the ends of all runways to minimize costly and time-consuming taxiing of aircraft, to simplify and reduce the amount of taxiway construction, and to ease the problem of ground control of aircraft.

Topographical features of the airport site must be considered in locating the building area in order to keep grading and drainage costs as low as possible and to provide the maximum in visibility from the control tower that is usually located on the terminal building. Regardless of whether the control tower is located on the terminal building or some other building, it is of utmost importance that unobstructed sight lines be provided to all aprons, taxiways, runways and their ends and approaches for the control of aircraft on the ground and in the air.

64. ADMINISTRATION OR TERMINAL BUILDING

The terminal building generally is the center of airport interest and activity. Therefore, it should be given first consideration with respect to location in the building area. In general, all managerial, operational, and service functions are performed in this building

It serves as a processing center for passengers and property between ground and air transportation, in addition to providing concessions for the comfort and convenience of passengers. Also, it serves as an operating base for scheduled air carriers and in some instances, for fixed-base operators and private flyers The terminal building may function as an Air Traffic Control Center and U S. Weather Bureau station, and, where international traffic is accommodated, it will house Customs, Immigration, U. S. Public Health, and Department of Agriculture inspection facilities.

The principal problems of the terminal building area layout are adequate circulation of passenger and vehicular traffic on the access road side of the building, and of passenger and cargo planes on the landing field side and their relationships to auto parking and buildings housing other activities.

Access Roads and Parking. Direct connections from the access roads to the terminal building and automobile parking areas must be provided to make possible rapid transfer of passengers and property from vehicles to the terminal building. Direct access must be provided to the service area adjoining the terminal building for use of commercial and service vehicles. Automobile parking can be divided into several parking lots to provide limited periods of parking close to the building for the use of those visitors who will require parking of only a few minutes' duration. Long-time parking areas can be provided at a greater distance from the building. Cross circulation between passengers and vehicles should be avoided wherever possible in the interest of safety and the rapid movement of traffic. The service court should be adjacent to the terminal building to provide for the loading and unloading of express, mail, food, and cargo (when handled from the terminal building) in addition to providing vehicular unloading of items of a building service nature.

Separation from Hangars and Other Buildings. In order for the terminal building to serve its primary purpose as a control center to expedite the movement of passengers, property, and aircraft on the ground and in the air, it must be separated from the buildings housing other airport activities. Each activity has its own peculiar problems and requirements which can best be satisfied by providing separate building areas, although the overall plan must be tied together for administrative control. Each area is indirectly

related to the terminal and administrative area to provide for the interchange of passengers, property, and information and is directly related to the runway and taxiway systems.

Separation will be governed by the nature and extent of the activities. In any case the buildings and building areas should be arranged in such a manner that orderly expansion will be possible, when necessary.

Size and Layout. Among the basic considerations affecting the size and layout of the terminal building are the types of airline service offered, e.g., whether the location under consideration is an originating, terminating, through, or transfer point. Generally speaking the airline's operating-space requirements are larger at originating and terminating points, since more airline personnel are likely to be required for ticketing and ramp-service activities, and the public space requirements, such as waiting room, eating, and rest-room facilities, are greater at transfer points, since transfer passengers have more time to patronize the various concessions and other public spaces in the building.

At large airports serious consideration should be given to a building with two-level operation (Fig. 38) in order to simplify the problem of passenger and baggage circulation.

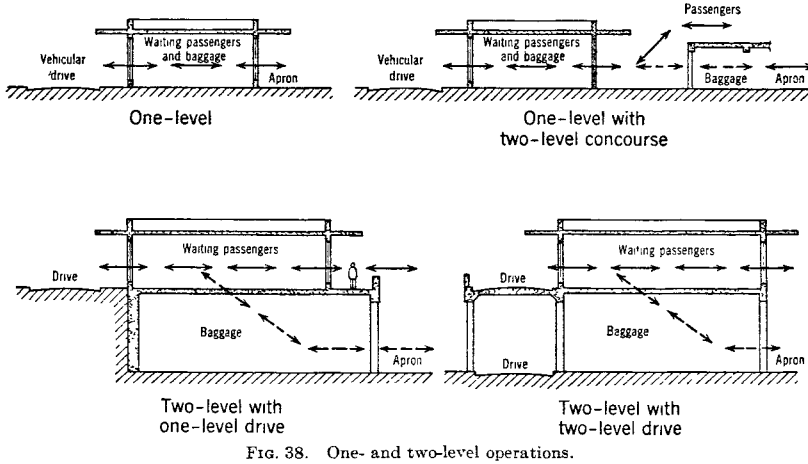


FIG. 38. One- and two-level operations.

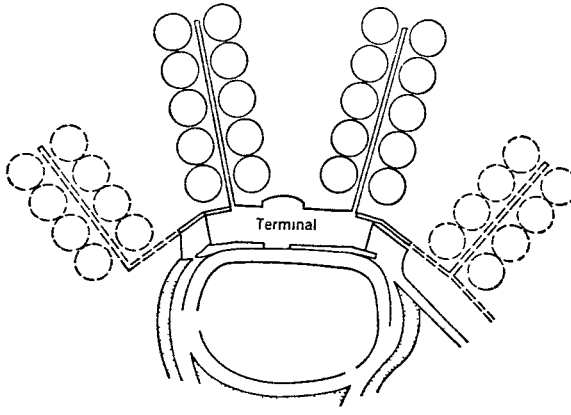
In this type of operation, baggage and cargo handling is confined to the apron level and all passenger facilities to the level above. Vehicle driveways to both levels are provided on the access-road side of the building with the passenger-vehicle driveway elevated to the main entrance.

The break point in passenger volume at which a two-level operation becomes feasible is believed to be about 500,000 annual passengers, that is, 250,000 enplaned and 250,000 deplaned, or approximately 1500 total average daily passengers.

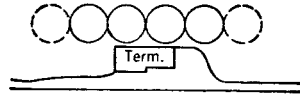
In a two-level operation, outgoing baggage is received at the ticket counter and by means of chutes or conveyors is sent to the lower level and transferred on carts to the plane. Incoming baggage is transferred from the plane to the lower level and thence directly to the baggage-claim room either on the lower or upper level. The upper level is preferred in order to keep passenger vehicles and loading on the upper driveway, reserving the lower driveway for service and mail trucking. A ramp from the lower level to the baggage-claim room above is considered more desirable than elevators or a conveyor system because (1) the baggage can be delivered quicker and with less handling, (2) fewer service personnel are required, and (3) even though the ramp may take up more space the problem of maintenance of mechanical equipment for conveyors or elevators make these systems more costly in the long run.

Traffic volume, as measured by the number of planes to be loaded or unloaded during peak hours, is an important factor to be considered when planning the building layout or configuration. At medium or small airports, plane loading positions can be placed parallel with the face of the building and the frontal scheme (Fig. 39) will be satisfactory. At large airports, the finger scheme has been evolved to reduce the walking distance between plane and terminal building (Fig. 39). It consists of one or more covered passageways projecting from the face of the building with space for baggage carts and airline ramp service on the apron level, and passenger concourse on the level above.

International Traffic. Facilities for international traffic will be required in relatively few locations. It is desirable that these functions be separated from, yet adjacent to, the other public spaces in the terminal building. Few of the conveniences of the terminal building proper will be duplicated there. However, the problem of passenger and baggage circulation is considerably more complex where international flights are accommodated and will require special study in order to achieve a unified plan layout and at the same time keep separate the lines of communication of domestic and international traffic.



Finger scheme



Frontal scheme

FIG 39. Loading position layouts.

Loading Apron and Gate Positions. The apron is the area of transition between the landing field and the terminal building. Its design must be closely integrated with the design and layout of the landing field as well as the terminal and other buildings. The loading, unloading, and servicing of aircraft is just as important to the efficient operation of an airport as is the layout of the runway system. Sufficient loading positions and maneuver area should be provided so as to prevent delays and aggravation to the users of the airport.

Size of the loading apron and size and number of loading or gate positions depend on the volume and concentration of traffic, the number of participating airlines, and the various types of aircraft. It is generally accepted that airline operations can be satisfied by an average spacing of 160 ft between plane parking positions.

The number of loading or gate positions required at a given airport is determined by the number of planes to be berthed at one time. For planning purposes it may be assumed that the average time each plane will occupy a berth will be 30 minutes. On this basis, each position can handle two planes per hour. Since each landing and each take-off is considered a separate operation, each plane accounts for two operations per hour. Therefore, the number of positions can be estimated as one-fourth the number of operations the runways can handle per hour. This formula applies where the traffic is such that the runways are being utilized to capacity. Thus, a capacity of 40 to 60 operations per hour on a single runway under non-instrument conditions will require from 10 to 15 positions; a parallel runway handling 120 operations would have to berth 30 planes.

A reduction in the number of gate positions may be made when it is obvious that the capacity of the runways will not be utilized. Also, an effort is being made to improve loading, unloading, and servicing techniques so as to reduce the amount of time a plane

must occupy a loading position. Layover parking areas should be provided close to the apron so aircraft can be moved to this area from the gate position during extended layover.

In addition to the apron area required for the loading and servicing of aircraft, adequate space must be provided for the circulation of planes to and from the taxiways. The recommended standards for distance between center line of runways and airport buildings are given in Table 3. These are minimum clearances for safe operation of aircraft on the field and have nothing to do with the space requirements for maneuvering and berthing planes. The minimum depth of apron (from edge of landing area to edge of building area) at an airport having scheduled airline service should be 300 ft. This allows for a line of berths and one-way traffic.

At major air terminals, a minimum depth of 500 ft and expansion to 1000 ft are recommended. This depth can be provided by either a solid apron or apron plus taxiway. The latter is more economical in first construction and the space between may be paved later, if required. Also, it may be necessary to provide for two-way taxiing at terminals where apron activity demands it.

Apron Services. It may be necessary at some airports to install such apron facilities as sewerage lines, underground water and power supplies, air-conditioning lines, and aircraft waste-disposal units. At large airports, consideration should be given to the installation of passenger- and cargo-loading equipment or special devices for docking aircraft.

Means for fueling aircraft should be studied, and these facilities should be planned so as to cause the least fire hazard and interference with airport activities. Gasoline should be stored in a location away from buildings or parked aircraft.

65. HANGARS AND OTHER BUILDINGS

Hangars, buildings to house aviation schools, commercial activities, maintenance, and fire-fighting equipment, warehouses, and related facilities are other buildings to be considered for construction. The number, type, and size can be determined from the data collected from the potential users.

Hangars are generally considered as being in two categories, repair or maintenance and storage. Repair or maintenance hangars will vary in size according to the intended use. It is recommended that they be built with doors at each end to permit ready expansion and additional flexibility for the removal or storage of aircraft. T hangars in individual units or multiples of the T are very adaptable for storage of personal planes.

Commercial Fixed Base Activities. Local commercial aviation enterprises such as sales services, repair, storage, charter, flight instruction, crop dusting, etc., normally require hangars, office and salesroom, classroom, and aprons for repair, storage, and tiedown. This area must provide automobile parking for employees and visitors together with service drives and entrances for commercial vehicles.

Access from the commercial and passenger vehicle drives must be provided to the hangar area with automobile parking adjacent both to the passenger and service roads and to the respective administrative areas of the base operators. Generally, the hangar activities of repair and maintenance, and sometimes storage, are connected directly to the administrative activities to provide direct management control of the hangars. Aprons for the purpose of repair are usually situated in close proximity to the openings of the hangars. Tiedown and other storage areas or aprons are usually located within easy access of the hangars.

Airline Maintenance Activities. Maintenance and or overhaul of aircraft and components used in scheduled service usually calls for hangars, shops, repair aprons, aircraft parking, commercial vehicle unloading areas, administrative offices, and automobile parking. The operational requirements depend upon the nature and extent of the work to be accomplished by the occupying airlines in their respective facilities.

The relationships between various activities within the area are governed by the operating procedures of the occupants. However, the aprons adjacent to hangars and repair positions should provide for easy and direct access to other airline operational or maintenance buildings adjoining them. Each airline should be consulted for its operational requirements when a layout is being prepared for this area.

Cargo Area. Space for cargo handling may be provided in the terminal building in the vicinity of the mail and express quarters. In cases where the volume is great enough, separate buildings may be required for these activities. The trucking court should be adjacent to the service drive and to the cargo transfer warehouse which adjoins the cargo apron and aircraft loading positions.

These buildings should be located as close as possible to the terminal building to provide for rapid interchange of cargo to and from combination cargo and passenger aircraft that

are normally loaded from the terminal building In the layout of these buildings, proper allowances must be made for their orderly expansion and for the expansion of the terminal building as well.

Airport Maintenance. Buildings for airport maintenance should be adequate to accommodate offices, personnel, and grounds maintenance equipment. Depending on the type of airport, there may be space demands for (*a*) offices for the superintendent and his staff, (*b*) drafting room, (*c*) photo and radio laboratories, (*d*) offices for police and fire departments, (*e*) cafeteria and lunch rooms, (*f*) recreation and rest rooms, (*g*) first aid, (*h*) garages for ambulance, fire, and crash equipment, service trucks, official cars, and ground maintenance equipment, (*i*) workshops for carpenters, electricians, plumbers, metal workers, and painters, and (*j*) storage room for supplies.

At small airports these quarters may be part of the terminal building, but at large developments, where separate buildings are necessary, they should be located in an area convenient to the landing facilities and the drive road.

Fire trucks, crash trucks, and other emergency equipment should be stationed at such a location so they will have ready and rapid access to all areas of the airport.

HELIPORTS

66. GENERAL

Obvious advantages of the helicopter in certain types of civil operations is becoming increasingly apparent. A number of substantial helicopter operations have been inaugurated in various parts of the United States with different degrees of success.

The helicopter has been found to be an aircraft which is ideally suited to such services as passenger shuttle, mail and cargo, commuting and taxi, property survey and mapping, geological survey, forestry patrol, crop seeding, spraying, orchard frost and moisture prevention, pipe- and power-line patrol, border patrol, emergency police control, air rescue, fish spotting, cattle roundup, ship-to-shore mail, and emergency communications. Other uses for this versatile aircraft will develop as its inherent advantages become better known. Among the first and most important services to be offered by the helicopter are the inauguration of mail and cargo networks in metropolitan areas. The inauguration of passenger service over such a network should logically follow.

67. SITE SELECTION

In selecting a heliport site the area it is designated to serve as well as the type of activity will be of primary interest. For commercial operation to midtown locations, roof tops may serve more advantageously, whereas in suburban areas ordinary ground sites are readily available. In any event clear channels of approach will be the main requisite. These may be waterways, railroads, etc., that lead to the most suitable central landing site for the activity intended.

Where a heliport is developed on a major airport, it should preferably be located where landings and take-offs may be made with no taxiing. A roof site on the terminal building proper of the airport post office may be satisfactory and should be considered, although a ground site is preferred. The only advantage of a roof site on a terminal building is the time factor to be considered in the handling of mail and cargo. A service and maintenance area as close as possible to the operations area, but separated from fixed-wing aircraft operations, service, and maintenance, should be provided.

68. SITE DEVELOPMENT

With equipment being used at present, the heliport should have approaches to permit landings and take-offs at angles from the outer limits of the touchdown pad of 10:1 in the prevailing wind direction and 5:1 in other directions. These angles will permit safe and economical operations under all conditions. (The larger helicopter now being designed will require flatter angles than shown above.)

Size. Touchdown pads should be a minimum of twice the diameter of the rotor where not more than one helicopter is landing at one time. The safety area to be improved around the touchdown pad will vary depending upon obstructions and dust conditions. This area

should extend a minimum of 50 ft from the outer edge of the touchdown pad. Where more than one helicopter is using the site simultaneously touchdown pads should be arranged in appropriate multiples having a minimum edge to edge distance of 100 ft between safety zones to eliminate dangerous air currents between aircraft. However, the combined touchdown pad areas may be paved as an integral unit provided they are satisfactorily marked.

Protection. The ground heliport should be enclosed by a substantial fence having a minimum height of 4 ft to prevent trespassing and should be provided with suitable gates for passenger and vehicle entrance. Heliports on roofs not having the protection of a parapet of adequate height should be provided with a guard rail at least 4 ft high

Design. For ground-surface sites a touchdown pad may be either square or circular. The pad should be surrounded by either a square or circular clearance and safety area in a contrasting color, e g., (1) a white concrete touchdown pad surrounded by a bituminous surface safety area, or (2) a white concrete or dark bituminous touchdown pad surrounded by a turfed safety area For a roof heliport a bituminous touchdown pad may be surrounded by a tar and gravel safety area, a painted wood platform touchdown pad may also be set on a tar and gravel surfaced roof. The safety area in each case should be free of dust or loose particles which may damage the craft's rotors. The color combinations may vary depending upon materials available and the location of the site. However, as sharp a contrast as possible between the touchdown pad and the surrounding safety area should be obtained so that the heliport may be readily identified under all conditions.

When contrasting colors between the touchdown pad and the safety zone are not feasible, the heliport may be marked by a circle with an H in the center. This marking may be formed by embedding a contrasting material on the touchdown pad or by painting. To aid in precision operation it may be desirable to omit the circle and paint on the entire touchdown pad crisscross lines approximately 4 ft apart.

Lighting. Heliports to be used for night operation will need a flashing or rotating beacon showing distinctive light signals, which will serve to identify the landing areas as a heliport.

In addition to the beacon, it will be necessary to outline the perimeter of the helicopter landing area with suitable boundary or perimeter lights.

The surface of the heliport may be floodlighted by suitable floodlights designed to show no light above the horizontal and such that they will provide sufficient illumination on the landing surface.

In addition to the above, obstruction lights should be installed on all structures and objects which rise above the predetermined allowable approach surfaces.

Touchdown Pad Loading. Where heliports are built on roofs the strength of the contact surface should be greater than the helicopter alighting gear by a comfortable margin. Helicopter alighting gear structures are customarily designed to withstand from two and one-half to four times the static load on each wheel without taking a permanent set, and to enjoy a factor of safety of 50% in addition to this load without failure. Since no helicopter can impose its maximum load upon landing until at least three wheels are in ground contact, the maximum strength of either wheel gear structure is approximately two times the gross weight of the helicopter. Drop tests have indicated that this load can be built up in a period of from $^1/_{20}$ to $^1/_{125}$ of a second and is relieved back to static load in from $^1/_8$ to $^1/_5$ of a second. It is recognized that the strain on a roof surface induced by a hard landing will be appreciably greater than that of a steadily applied load. It is believed, therefore, that a roof structure would have sufficient strength if no part was stressed beyond the stresses induced by a concentrated load equivalent to three-quarters the gross weight of the helicopter on any one square foot.

SEAPLANE FACILITIES

Increased interest in flying water-based planes and the use of waterways by water-borne aircraft have placed added emphasis upon the necessity for planning and developing seaplane facilities. The essential elements of each development will vary considerably depending upon the location and type of operation.

In order to develop a satisfactory and efficient seaplane base, it is necessary to make an estimate of the number and types of planes expected to use the facility and a survey of the water area, bordering shore line, terrain, land development, and other factors relating thereto. Items such as length, width, and depth of the water operating area; current velocity and water level differential; shore line and bottom characteristics; prevailing winds and their velocities; sheltered areas and mooring areas; approach conditions and obstructions will have to be considered Due regard should also be given to local boating, nearby property owners, and onshore activities.

The proper location of floats, docks, ramps tiedown aprons, hangars, and other buildings can be determined from the estimate of the expected activities at the base. These component parts should be so placed as to be convenient to planes and the public using the seaplane facility. Routes which planes should follow in landing and take-off operations can be established from an analysis of the expected volume and type of traffic. These routes will aid in selecting the site most suitable for each of the several facilities. The accessibility to local transportation and public utilities should not be overlooked.

An investigation should be made to determine if any existing federal, state, county, or local regulations might restrict or prevent the construction, operation, or development of the seaplane facility.

MAINTENANCE

69. PAVEMENTS

Flexible-type pavements may require maintenance for the following reasons: failure of base or subbase; raveling, drying, or cracking of surfacing material; shrinkage of soil due to extensive periods of drought, frost heave in subgrade, inadequate drainage; inferior materials; and improper construction in initial work. The cause of any pavement deficiency or incipient failure should be determined and complete corrective measures applied to prevent a recurrence of the trouble A base failure will usually require replacement of the base course after correction of the subgrade and drainage, if necessary, and replacement of surface course

Raveling or breaks in the surface course should be repaired by removing the defective material, squaring up the edges, and filling with new surfacing material similar to that used in the pavement. The material should be thoroughly compacted and sealed. At intervals of 3–6 years, surface treatments similar to what is known by many as a seal coat, should be applied to seal cracks, restore life to the bitumen, and correct the surface texture. Rolling the pavement with pneumatic tire rollers in warm weather will extend the life of the surface course and lengthen the period between surface treatments. Cracks wider than $\frac{1}{8}$ in. should be cleaned out and sealed with bituminous material as soon as discovered.

The maintenance of rigid-type pavements generally consists of sealing joints and cracks, replacement of broken areas, adjustment of pavement elevation to correct settlement or errors in construction, and the repair of spalls. Expansion-joint material will compress or extrude as the concrete expands, and cracks will open as contraction occurs. The maintenance of joints usually involves replenishing or replacing the seal material and trimming off of excessive filler material which has extruded Contraction joints, construction joints, cracks, and hinge joints should be kept filled and sealed Replacement of broken areas should be repaired with concrete in such manner that the patch will be equal in strength and riding quality to the remainder of the pavement Patching should be done at the earliest opportunity and by accepted engineering practices. Correction in elevation of concrete pavement necessitated by settlement can be accomplished by the use of mud jacks which force a mixture of cement and mud through holes drilled into the pavement to the subgrade. Bituminous materials are also used for this purpose. Spalling of joints is usually caused by improper construction. When repairs are made to spalled areas, all unsound material should be cut away to neat lines and the area replaced with new concrete.

Both flexible- and rigid-type pavements may require resurfacing because of disintegration or excessive loading of the pavement The pavement should be resurfaced with bituminous concrete, and two courses used if extreme irregularities in the surface contour are to be corrected. Resurfacing should not be delayed until the deterioration has progressed to such an extent that the pavement cannot be salvaged as a base course. Badly broken areas should be repaired before resurfacing

70. DRAINAGE SYSTEM

Inspection and maintenance of drainage facilities are required at regular intervals, and in addition, during or following every heavy storm. Most drainage systems are integrated systems and generally a deficiency in one portion of the system will cause undue stress and resultant damage to other parts of the system

Frequency of regular inspections varies with amount and intensity of rainfall and with adequacy and condition of drainage facilities. As a minimum program, a complete inspection is made in the fall in preparation for the winter season, and another in the spring, to

determine extent of repairs required. Maintenance includes cleaning inlets, culverts, storm drains, ditches, and underdrains; correction of washing, undermining, and settlement of structures; stabilizing ditches and slopes; removal of high shoulders and other obstructions; enlarging, replacing, or providing additional waterways and structures, as necessary. Proper inspection and maintenance require familiarity with design, capacity, and location of drainage facilities in place.

Drainage facilities should, if possible, be patrolled during storms to relieve stoppages at critical points such as inlets, culverts, and other structures to prevent serious damage and unnecessary flooding. Inspection should immediately follow a storm to observe needed corrections in the drainage system, and to survey extent of damage to be repaired.

71. LIGHTING

Proper maintenance consists of establishing a regular routine of testing, cleaning, adjusting, repairing, and, when necessary, replacing worn-out or damaged parts. Lamps must be replaced; glassware and reflectors must be cleaned; moving elements in the beacon, wind tee, and wind cone must be lubricated; broken glassware must be replaced; and wiring and cable insulation must be maintained.

To effect quick repair and replacement, an adequate supply of cable, lamps, glassware, film cutouts, fuses, gaskets, and spare parts for field lighting units should be kept on hand where they are readily available when the need arises. A suitable workshop and storeroom are also of prime importance, and the maintenance electrician should have available a small panel or pick-up truck.

It will be necessary, of course, for the maintenance electrician to make periodic inspections of the lighting equipment. In addition to that, a daily check or test of the equipment operations should be made at about one-half hour before susnet. Any reliable and competent person stationed at the airport during the evening hours can accomplish this test. This should consist of turning on all airport lighting facilities and seeing that each is functioning normally. If any failure is noted, the maintenance electrician should be notified immediately.

All equipment must be maintained free of dust, sand, surplus grease and oil, and other foreign matter. Experience has proved that dirty equipment contributes greatly to operational failures.

Maintenance instruction books and descriptive parts lists for each piece of lighting equipment at the airport should be procured from the manufacturer. A complete inventory should be made of the equipment installed and of the replacement articles carried in stock. A record of the style number, rating, model number, etc., of each piece of apparatus will facilitate obtaining replacement parts.

72. TURF AND LANDSCAPE PLANTING

Mowing is one of the most important factors in the establishment and maintenance of turf. It not only encourages the rapid coverage of turf species and promotes density and vigor, but it also serves to check the growth of weeds and undesirable grasses that tend to smother or compete seriously with the desired species.

Mowing to a height of 3 in. is generally advisable; below 2 in. is neither practical nor desirable for airport turf. It should be done when the grass needs cutting rather than on a fixed schedule. On well-established stands, this is usually when it reaches a height of 6–8 in. If mowing is neglected too long, the clippings must be raked up and removed, which adds greatly to the cost of mowing.

It is necessary to apply fertilizer occasionally to established turf in order to maintain a heavy stand. On frequent-traffic areas, or other places where vigorous thick turf is essential, nitrogen is of utmost importance. Results of soil tests should be considered in estimating quantities of the nutrients needed. Time of application varies with the growth characteristics of the species to be fertilized, but in general, fertilizers should be applied just prior to the time of most rapid growth of the species involved.

In the event that existing turf has deteriorated to the point that it is very thin and weedy or numerous bare spots occur, a general renovation may become necessary. A careful analysis of the soil, turf, and other factors involved should be made in order to determine the cause for the turf failure and to provide an effective program for the reestablishment of a smooth turf surface.

Spring rolling of turfed areas is desirable in regions where the winter's freezing and thawing heave the grass plants out of the soil and leave an irregular surface. It should be done

as early in the spring as feasible. Rollers of the type used on farms or on golf courses should be used.

The control of weeds is generally a minor problem on properly managed turf areas. Timely mowing, adequate turf fertilizing, and other regular maintenance practices ordinarily keep weeds in check without any special treatment. Under certain conditions, however, immediate control is required. Several chemicals or herbicides which may be used to advantage in controlling weeds are now commercially available.

The degree of maintenance required for landscape plantings depends to a large extent on the planting design. Formal plantings or extensive hedges require more constant care than simple informal plantings and, generally, trees require less attention than shrubs or vines.

Under certain conditions artificial watering of trees and shrubs at airports becomes necessary. Lack of water immediately after planting is extremely dangerous especially during hot, dry, and windy weather. After the plants have become well established, watering is not usually necessary in the humid regions except occasionally during periods of extreme drought.

Cultivating and mulching are beneficial during the establishment of plants in that they help to keep down the weeds, conserve soil moisture, and improve the textures of the soil. Also the regular mulching of most broad-leafed evergreens or other shallow-rooted shrubs is helpful and worth while.

The need for refertilizing trees, shrubs, or vines is influenced by such factors as the fertility of the soil in which they are growing, the kinds of plants, and the kind and quantity of fertilizer used at the time of planting. Obviously no formula as to the proper kind and quantity of fertilizer to use can be made to cover all conditions. It must be made on the basis of local requirements. However, under normal conditions, trees and shrubs respond well to a fertilizer high in nitrogen.

As a general rule, the best time to refertilize is at the beginning of the natural growing season. Plants should not be forced into lush growth by fertilizing just before freezing weather.

With most trees and shrubs a certain amount of pruning from year to year is desirable. The purposes of periodic maintenance pruning are to remove dead or diseased wood, to shape the plants, to prevent bad crotches in trees, to remove low branches, to promote flowering or fruiting, and to trim hedge plants into a regular shape.

SECTION 6

RAILROAD ENGINEERING

BY

WILLIAM W. HAY

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RAILROAD ENGINEERING

ROADWAY

1. RIGHT OF WAY

Width of right of way for a single track line usually varies from 4 rods (66 ft) to 100 ft, but widths of 200 and 400 ft are not uncommon. Within station limits and yards, the width is increased to accommodate necessary tracks, structures, and facilities. Depths of cuts and fills control minimum width. On level ground, the minimum width is $w = b + 2sd$, where b is the width of roadbed, s the rate of sideslope, and d the depth of cut or fill. Sufficient additional width must be provided for side and berm or intercepting ditches, for fences, and to keep slides or sloughed-off material from encroaching on the property of others. Needs for future double tracking, siding construction or extension, and other expansion should be considered at the outset since the construction of a railroad often enhances land values and makes later purchase more expensive. When additional width for more than one track is required, it is based on the spacing between track centers, usually 13–14 ft. The center line of right of way has usually coincided at one time with the center line of one of the tracks but the track is frequently shifted from its original position in the processes of maintenance and line revision. Property lines should be marked by monuments. Right-of-way monuments are not standardized. A satisfactory marker consists of sections of steel rails or of concrete posts about 8 in. square, 3–4 ft in length, and with at least 2 ft imbedded in the earth. The exact line is scratched on the top.

Areas of land for different widths of right of way are given in Table 1.

Table 1. Area of Right of Way

Width, ft	Acres per Mile	Acres per 100 Ft	Width, ft	Acres per Mile	Acres per 100 Ft	Width, ft	Acres per Mile	Acres per 100 Ft
16.5 1 rod	2.00	0 038	27	3.27	0 062	40	4.85	0.092
			28	3.39	0.064	41	4.97	0.094
17	2.06	0.039	29	3.52	0.067	41.25 2.5 rods	5.00	0.095
18	2.18	0 041	30	3.64	0.069			
19	2.30	0 044	31	3.76	0.071	42	5.09	0.096
20	2.42	0 046	32	3.88	0.073	43	5.21	0.099
21	2.55	0.048	33 2 rods	4.00	0.076	44	5.33	0.101
22	2 67	0 051				45	5.45	0.103
23	2.79	0.053	34	4.12	0.078	46	5.58	0.106
24	2.91	0.055	35	4.24	0.080	47	5.70	0.108
24.75 1.5 rods	3.00	0.057	36	4.36	0.083	48	5.82	0.110
			37	4.48	0.085	49	5.94	0.112
25	3 03	0.057	38	4.61	0.087	49.5 3 rods	6.00	0.114
26	3.15	0.060	39	4.73	0.090			

For greater widths use sums or multiples; thus, for 87 ft take the sum of the acres for 43 and 44 ft.

2. FENCES, SNOWSHEDS, AND SAND PROTECTION

Legal requirements for fencing vary from state to state. It is often economical to fence land regardless of legal requirements, especially in densely populated areas, to reduce the incidence of trespassing. If the right of way constitutes the fourth side of a field, the law usually requires the railroad to fence that fourth side when and if the owner has fenced the other three sides.

Wire fences, with wood, metal, or concrete posts, of the following design are recommended by the American Railway Engineering Association (AREA) for right-of-way fences.[1] Height is generally about 4 ft 6 in., but it and all other features must conform to

[1] *AREA Manual*, 1953 Ed., pp. 1-6-1 to 1-6-18, American Railway Engineering Association, Chicago, Illinois.

statutory requirements. Four classes are provided by the AREA with the longitudinal wires spaced as follows:

Class A: 5 in. above ground, 4, 4 1/2, 5, 5 1/2, 6, 7, 8, and 9 in. To make a Class A fence "hog-tight," an additional wire is stretched midway between the ground and the first wire listed above.

Class B: 7 in. above ground, 6 1/2, 7, 7 1/2, 8, 8 1/2, and 9 in.

Class C: 2 in. above ground, 3, 3 1/2, 4, 4 1/2, 5, 5 1/2, for woven wire (25 1/2 in. high) with 3 strands of barbed wire above spaced 4 1/2, 10, and 12 in.

Class D: 10 in. above ground, 10, 10, 12, and 12 in.

Wire. Smooth, round wire of No. 9 gage is recommended for Classes A and B, and the woven part of C, except that for the top and bottom strands of a Class A fence No. 7 wire is required. Ribbon, smooth, round, or barbed wire may be used for Class D fences. Vertical wire stays of No. 9 gage are recommended for Classes A, B, and C, spaced 12 in. apart. Woven-wire fencing giving the same combinations of spacing is commonly used. Longitudinal wires for Classes A, B, and C should be provided with tension curves to take up expansion and contraction. Wires are attached to the side of post away from the track with No. 9 wire staples, 1 in. long for hardwoods and 1 1/2 in. for softwoods. Staples are set diagonally with the grain of the wood and driven home. Wire for fencing and staples should be galvanized with an even coating of zinc and have a minimum ultimate tensile strength as follows:

	Open-Hearth Steel	Wrought Iron
Line wire, No. 7 gage	2200 lb	1800 lb
Line wire, No. 9 gage	1500 lb	1200 lb
Stay wire, No. 9 gage	1100 lb	1000 lb

Electrically welded fence should be galvanized after fabrication.

Wooden posts should be straight and free from splits, rot, large knots, and other defects. Black locust, cypress, catalpa, cedar, chestnut, Douglas fir, black walnut, heart yellow pine, larch, mulberry, redwood, and white oak may be used without preservative treatment. Other woods should be treated. End, corner, anchor, and gate posts should be at least 8 ft long, 8 in. in diameter at the small end, and 40 in. in the ground; intermediate posts should be at least 7 ft long, 4 in. in diameter at the small end, and set 28 in. in the ground. Dimensions of sawed or split posts should be at least equal to those of round posts. Posts are set with the large end down and spaced not more than 20 ft. The first line post from any corner, anchor, or gate post is spaced 10 ft center to center with it. Holes are dug to full depth. To avoid blasting in solid rock, intermediate posts may be set on 6 by 6 in. sills 4 ft long braced on both sides by 2 by 6 in. braces 3 ft long. Not more than two such posts should be placed consecutively. After a fence is erected, the tops should be sawed off on a one-quarter pitch with the high side on the same side as the wire and about 2 in. above it. Intermediate posts in hollows or sags are anchored down by gaining and spiking two cleats near the bottom. End and corner posts are anchored by cleats set one near the bottom and the other just below the ground surface and also braced by cleats to adjacent posts. Cleats are of 2 by 6 in. common lumber 3 ft long. Braces are of 4 by 4 in. common lumber. Wire braces made of a double strand of No. 9 wire are used where bracing by tension is needed.

Concrete posts are recommended as practical, economical, and a suitable substitute for wood. The cross section is preferably a square or a circle, the latter having the more extended use, whereas square or nearly square sections are slightly more efficient in resisting the forces that ordinarily cause failure. Greater resistance to deterioration and better methods of manufacture of round posts may offset the greater efficiency of the square. Corners should be rounded off to a radius of not less than 5/8 in., and posts should be tapered from base to top. End, corner, and gate posts should be at least 8 ft long, 8 in. at the base, 6 in. at the top, set 40 in. in the ground, and reinforced with four 3/8-in. rods. Intermediate posts should be at least 7 ft long, 5 in. at the base and 3 1/2 in. at top for round posts, and 4 1/2 in. at the base and 3 in. at top for square ones.

Steel posts are widely used. Higher first cost is more than offset by reduced labor costs since these may be driven either by hand or by a one-man post driver. Metal fence posts may be made of steel, wrought iron, or alloys of iron of any kind which meet the following conditions: minimum tensile strength of 40,000 lb/sq in., and, with a finished post, clamped rigidly, and a load of 300 lb applied at a point 3 ft from the face of the clamp, a maximum deflection of 6 in. and a permanent set of 2 1/2 in. Posts should be provided with some convenient way of attaching the wire in at least twelve positions in a height of 5 ft above ground. One-third of the post length is set below ground with a minimum depth of 2 1/2 ft.

Farm gates should be of all-metal construction and 12 ft wide or wider depending upon the size of farm machinery passing through it or statutes of the state. Height should

be at least 4 ft 6 in.. Gates are hung so as to open away from the track, and, if hinged, as is desirable, to shut by gravity and overlap the post a sufficient distance to prevent opening by side pressure

Costs. The first cost of fencing depends on prices of labor and materials and efficiency of the former. It should be estimated in each case from quotations on material and estimated output of labor. The annual cost involves also the estimated life and interest rate and should be used in making comparisons.

Quantities of Materials. For wire fences, the 16 1/2-ft spacing requires 320 posts per mile Barbed wire varies in weight with the type, but may be estimated roughly at 1 lb per rod or 320 lb per mile, whereas No 7 and No. 9 wire weigh 439 and 306 lb per mile, respectively.

Staples are put up in 100-lb kegs. If of No. 9 wire, the number per keg is approximately 1 in , 10,800; 1 1/4 in., 8,700; 1 1/2 in , 7,200; and 2 in., 5,800. For posts 20 ft apart, 83% of the weights given in Table 2 will be required. Locks are of various types and may be estimated at 25 lb per 1,000.

Table 2. Materials, except Posts, Required for Wire Fences, AREA Standards

Per Mile of Fence

	Wire					Staples		
	Longitudinals			Verticals				
Class	No. 7 Lb	No. 9 Lb	Barbed Lb	No 9 Lb	Total Lb	1-in. Lb	1 1/4-in. Lb	Locks No
A	878	2142		1300	4320	31.1	46.4	47 500
B		2142		1236	3378	24.9	37.1	37 000
C		2142	960	700	2842	34.4	30.8	37 000
D		..	1600	.	1600	18 0	26.8	.

One inch extra was allowed at each end of vertical or stay wires and 5% was allowed for loss of staples. Nothing was allowed for loss of locks or ties, as, if fabricated in the field, some of the stay wires would be omitted at posts.

Table 3. Labor in Man-Days per Mile of Fence

	Wooden Posts		Concrete Posts		Steel Posts
Class	Auger	Bar and Shovel	Auger	Bar and Shovel	Driven
A	42	48	44	50	31
B	35	41	37	43	28
C	40	46	42	48	30
D	26	28	28	34	21

Snow Fences. A snow fence is a structure erected to accumulate drifting snow. Snow is carried by the wind close to the surface of the ground and is deposited in railway cuts because of the eddies and diminution of wind velocities caused by the cuts The drifting of snow in shallow cuts may be prevented or alleviated by raising tracks and eliminating the cut or by reducing slopes on both sides to a 4:1 or flatter Widening cuts provides room for snow to accumulate free of the tracks. A snow fence is so placed that artificial eddies will form on the windward side of a cut at sufficient distance to cause the snow to be deposited between the snow fence and the cut. Thus the fence is set on the windward side of the track and at right angles to prevailing winds. The fence may be set in a crescent form to give partial protection against variations in wind direction.

A tight fence of sufficient height causes snow to accumulate on the windward side. An open fence accumulates snow principally on the leeward side. Frequently the best type of fence and its proper location may be determined only by experiment; therefore portable snow fences are used. When practicable, a permanent fence located on the right-of-way line may prove the most economical. Where this line is 50 ft or less from the center of the track, the boards should be close For greater distances, space is left between the boards. When the distance is 100 ft, 50% of the fence should be open space. The height of fence should not exceed 10 ft (Hedges of California privet, Armour barberry, evergreens, and locust have been planted in recent years for this purpose. Their economic efficiency is still doubtful)

A specification, quoted from Orrock's *Railroad Structures and Estimates*, calls for cedar posts 8 in. in diameter, 12 ft long, set 3 ft 6 in. in the ground, and spaced 8 ft on centers. The boards are 7/8 in by 6 in by 16 ft, and are nailed on with 6-in. clear spaces. This requires eight boards, four breaking joints at each post. A *portable fence* is made in panels

16 ft long. Six boards 1 in. by 6 in. by 16 ft, spaced 6 in. clear, are fastened onto three pieces 2 in. by 6 in. by 9 ft, spaced 7 ft apart. Two pieces 1 in. by 6 in. by 10 ft act as stiffeners. Three other pieces, also 2 in. by 6 in. by 9 ft, are bolted, using 1/2 in. by 4 3/4-in. carriage bolts, with plate washers, scissorwise to the other three similar pieces. Two more boards are fastened to the tops of the second set of pieces. These sections may be folded up flat for storage. When in use, the scissors are opened until the lower ends are spread about 6 ft 6 in. apart and are fastened to stakes, 3 in. by 3 in. by 2 ft, driven into the ground. On hard, rocky ground a tie-board, 2 in. by 6 in. by 7 ft, may be nailed to the lower ends of the scissors and weighted down with old ties, stones, etc. This construction is estimated to cost $40 per panel of 16 ft.

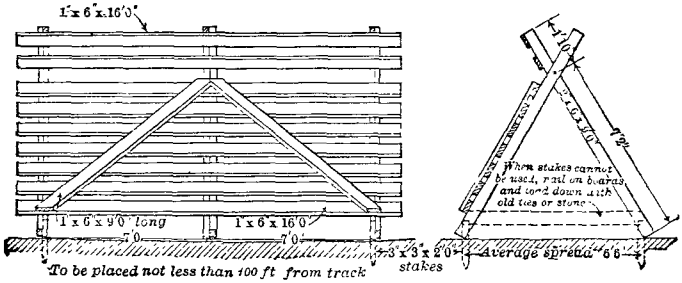


FIG. 1 Portable snow fence.

Another type of portable snow fence is made by weaving wooden laths 4–6 ft long with five to seven double strands of smooth galvanized wire, making a sort of picket fence. By varying the widths of the lath and the number of turns of wire between them considerable difference in the percentage of area shut off may be obtained. This fence is made in rolls up to 100 ft long and is set up by fastening it at intervals to posts set or driven into the ground. By keeping the laths well painted and using wire heavily galvanized the fence may be made quite durable; and it is easier to handle than the other type.

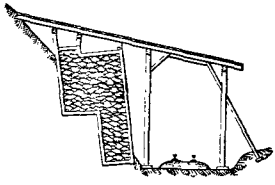


FIG 2 Side Fall, Northern Pacific Ry.

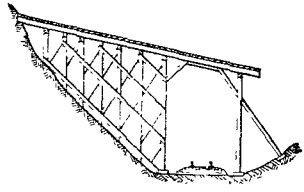


FIG. 3 Side Fall, Northern Pacific Ry

Snowsheds are expensive to build and to maintain. Their probable necessity and the possibility of avoiding them should influence the decision on the location of the road. There are two types of construction: (1) those to protect the track against slides, the track being on a side hill or at the base of a hill, and (2) those to prevent the filling of cuts or to protect the track from an excessive level fall. Although the detail varies somewhat with nearly every location, typical designs for timber sheds are shown in Figs. 2 to 5.

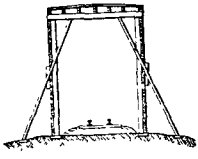


FIG 4 Level Fall, Southern Pacific Ry

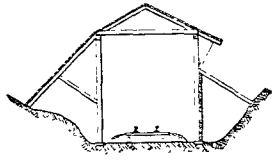


FIG 5. Lower Fall, Canadian Pacific Ry.

Sheds cut off the view considerably and are disagreeable to operate; hence a "summer track" is sometimes laid outside. To partially open up level fall sheds and to localize fires, movable sections 100 ft long are provided which are mounted on wheels and telescope into enlarged adjacent sections. The cost of maintaining the extremely heavy avalanche sheds,

particularly those for double track, and the great fire risk have led some roads to build combination concrete-and-timber sheds and reinforced concrete sheds. See *Railway Engineering and Maintenance* pp. 54 and 58, 1930.

Sand. Protection against drifting sand begins with initial construction. Where possible the roadbed should be constructed above the natural ground. Cuts are widened and have flat slopes as in snow protection. Cut slopes may be oiled or paved with clay to keep sand moving. If sand is drifting in the track, the track may be half-skeletonized permitting the wind to blow the sand clear. It is sometimes possible to remove the original source of the sand. Sand dunes can sometimes be dissipated by scattering gravel or small stones on the windward slope. Where undercutting occurs from sand blowing away, a layer of clay placed on top of the sand may achieve stability. Possibly the best method of sand control is to plant vegetation on the source to form a root matting which will hold it in place.

3. SUBGRADE

Soils serve both as a foundation for the subgrade and as a material from which the subgrade is constructed. A close adherence to the principles and good practices of soil engineering as found in Sect. 7 is necessary to obtain the requisite stability of the structure.

Subgrade design involves the selection of three critical dimensions: width, height (or depth), and slope. Each of these may be influenced by the type of soils used in construction.

Width of roadbed is determined by the width of base of ballast section (see p. 6-20) and the supporting shoulders which extend beyond. The AREA recommends a minimum of 1 1/2 ft as the width for each shoulder, but a width of 2 1/2 ft (5 ft for both) is preferable to cover normal erosion and sloughing.[2] When soil materials are unstable, a greater width is helpful as determined by the principles of Sect. 7. Estimated settlement requires a wider shoulder on the basis of 3 additional feet of subgrade width per foot of settlement (or whatever a slope ratio other than a 1 1/2:1 requires). Roadbeds under multiple tracks must be widened by the distances between track centers, usually 13–14 ft. Customary widths of single track subgrades on U. S. railroads as reported by an AREA survey are as follows:

Main lines: 18–24 ft; weighted average, 20 ft 6 in.
Secondary lines: 16–22 ft; weighted average, 19 ft 3 in.
Third-grade or branch lines: 16–22 ft; weighted average, 17 ft 6 in.

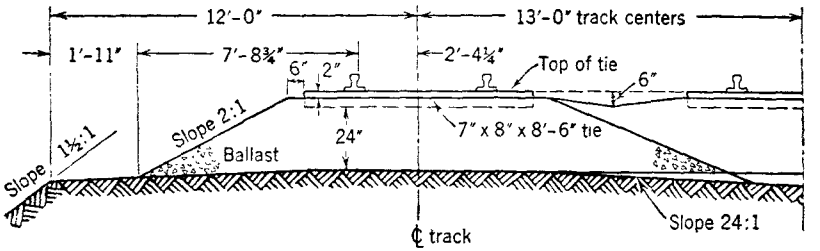


FIG. 6. Typical roadbed and ballast section, prepared materials of stone, slag, and gravel having over 20% crushed particles. (*AREA Manual*, 1953 Ed., p. 1-2-7.)

Some roads widen the roadbed for high fills but the practice is non-uniform. The Santa Fe widens as follows: 22-ft width for fills under 20 ft, 24 ft for fills more than 20 ft but less than 50 ft, and 26 ft for fills of 50 ft or higher. Cut widths of 30 and 40 ft are used.[3]

In cuts additional provision must be made for drainage ditches (see p. 6-13). Good practice today recommends that ditches be made wide enough or that low berms be provided to permit use of off-track equipment for ditch cleaning, snow accumulation and removal, and other maintenance operations.

Height of fill (or depth of cut) is principally determined by the established grade line. The practice of adding additional height to finished grade to compensate for estimated settlements is not recommended. In addition to the conventional factors which govern the design of a grade line, it is sometimes desirable to give it added height to bring it above high-water lines or above drifting sand or snow.

[2] *AREA Manual*, 1953 Ed., pp. 1-1-1 to 1-1-7.
[3] Grading Specifications, Sept. 12, 1940, Atchison, Topeka, and Santa Fe Railway System.

Sideslopes must be designed for stability and should not be less than the angle of repose of the soil materials. Most soils are stable with a 1 1/2:1 slope. Rock fills will stand on a 1:1 slope. Some rock cuts stand on a 1/2:1 or even a 1/4:1 slope. Unstable materials having a low shearing strength require a flatter slope, 2:1, 3:1, or less. Stability and proper slope design are based on the principles of Sects. 7 and 4. Submerged slopes should be flat, 4:1 to 6:1 unless otherwise protected, see p. 6-08. Cuts subject to snow drifting should also have flat slopes, 4:1 to 6:1. With these exceptions there is no advantage in a flatter slope than required by needs of stability. There is a disadvantage in the greater area exposed to rain wash and erosion on the flatter slopes.

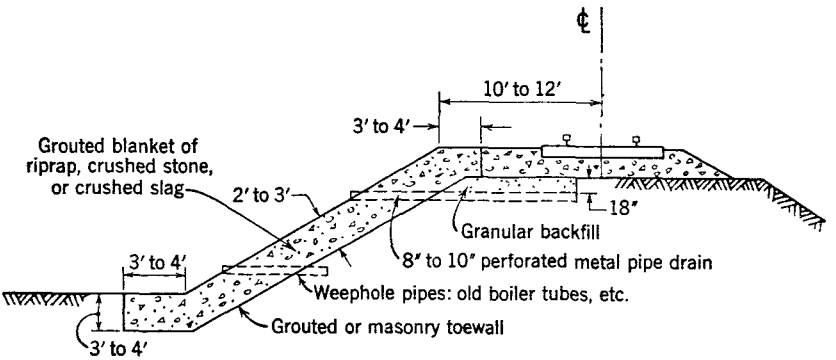


FIG. 7. Grouted blanket to protect roadbed slopes from erosion

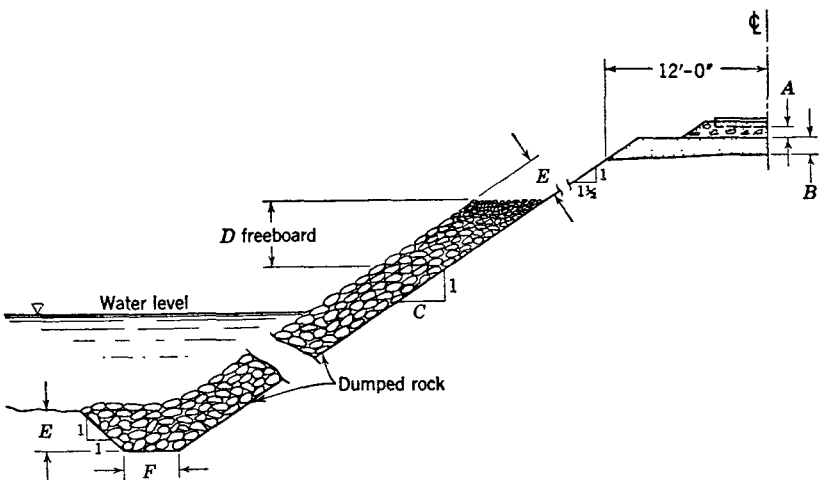


FIG 8 Protection for water-immersed embankments *A* and *B* conform to ballast section, *C* varies with stability of slope completely saturated, 2 1/2–3 1/2, *D*, 4–10 ft depending on height of wave; *E*, 2–4 ft depending on wave height, depth of water, etc ; *F*, 4–10 ft depending on depth of water, etc. (After *Proceedings of the AREA*, Vol 50, p. 684, 1949)

High slopes may be further stabilized by use of offsets or terraces. These should be well drained by a paved gutter at the toe of the upper slope to prevent accumulation of moisture and saturation of fill materials. Terracing practices are non-uniform. One road-terracing practice is as follows:

6-ft terraces for fills 25–50 ft in height.
12-ft terraces for fills over 50 ft in height.

Again reference is made to Sect. 7 for further details on the application of soil engineering to this problem.

Bank protection is essential to reduce erosion and promote subgrade stability. Quick protection and a pleasing appearance are obtained by sodding, but the method is expensive.

Planting with grasses or shrubs, especially those native to the vicinity, is recommended. Successful plantings are had with kudzu, Bermuda grass, the clovers, perennial rye, oats, woodbine, honeysuckle, roses, sumac, locust, and evergreens. Russian wild rye is good for sandbank stabilization. Submerged slopes, especially those subject to swift currents, should be protected with riprap. One-man stone dumped in place is sufficient for many situations. Severe erosion conditions may require two-man or even derrick stone. An alternate method covers the slope to a depth of 2–3 ft with one-man stone, crushed stone, slag, or other heavy granular material. A cement grout in proportions of one part of cement to two parts of sand is flowed over and through the porous cover. Asphaltic compounds may be used instead of the grout. The blanket material should be made heavier and thicker at the base to form a toe wall. Where swift currents or wave action cause severe erosion, hand-placed two-man stone or even derrick stone should be used in constructing the toe wall. Drainage back of the blanket must be provided. Weep holes are made of 4-in. cast-iron pipe or old boiler tubes set in the blanket face and extending through the grouted depth of cover. Larger seepage drains may also be provided of 6–8 in. corrugated-metal pipe, perforated and extending under the near rail of the track. The pipe, laid on 100-ft centers, is backfilled with filter sand Blanket material or riprap should be carried high enough up the slope to provide a freeboard space protecting against wave action and high water.

4. SUBGRADE CONSTRUCTION

Principles and methods of subgrade construction as presented in Sect. 7, Soil Mechanics and Site Examination, and Sect. 4, Highway Engineering, should be followed in the construction of railroad subgrades These procedures include a soil-exploration program in keeping with the importance of the road under construction, adequate subsoil preparation by clearing, grubbing, and stripping; proper soil selection and mixing; adequate compaction by rolling, vibrating, and control of the moisture content; and finishing by placing porous, free-draining materials in the top layers and rolling and blading to a smooth, uniform surface. Small depressions, tire and tractor tread marks, and other undulations are forerunners of potential water and ballast pockets and must be removed. The AREA recommends sloping the subgrade 1 in 12 from a point under the outside end of crossties of the outside tracks to promote drainage. Although good techniques should always be followed, there may be some departure from the more severe requirements of highway construction because irregularities which develop in the track surface due to subgrade conditions can be taken up by relatively slight surfacing in the ballast section. The highway surface cannot be so readily adjusted and must conform to stricter requirements. For high-speed main lines going into immediate service there should be no relaxing of standards, and requirements comparable to those for highways are maintained.

5. SPECIAL ROADWAY PROBLEMS

Unstable ground accounts for the sinking and sideslips of railroad subgrades through foundation failures and may lead to complete failure of the fill, sometimes under traffic. Instability occurs from soft underlying clay layers, the presence of muck and peat bogs, or along planes of sliding on a sidehill embankment. The first two are often concealed under a stiff crust which eventually fails under the imposed loads During construction, early stability is had by excavating the soft material and backfilling with good soils, by overloading the fill to hasten settlement, by drilling blasting holes transverse to the overloaded subgrade and shooting the muck from underneath the fill, or by drainage, using surface ditches or vertical sand drains beneath the fill, to lower the water table. Timber mats or corduroys may be placed under the fill to give floating support. In severe cases, trestles are constructed across the unstable area or the line is relocated on firm ground Where slow subsidence occurs, the track will have to be raised to proper surface and elevation and the ballast replenished at frequent intervals. The ballast, therefore, should be of a cheap material cinders, pit-run gravel, etc A more complete analysis and solution to foundation soil instability requires the application of principles of soil engineering as presented in Sect. 7

Slides as considered here include both individual and mass movements of soil and rock occurring in natural or in graded slopes. Section 7 should be studied for a more detailed analysis. Slides in general are caused by overstressing of slopes by load increases or by a decrease in the strength of slope with a constant load. Lack of shearing strength in a slope may result from too steep a slope, especially in homogeneous soft clays; from moisture penetration in joint cracks of stiff clays; from transfer of intergranular to pore water pres-

sures in a saturated sand, detritus, or talus slope; or as a result of the rapid drawdown of a flooded stream. Clays of high sensitivity may flow like a liquid due to structural breakdown caused by remolding processes of the initial slide. Loess slopes which stand vertically when dry break off in small portions along vertical root holes and slump like a soft clay on becoming saturated.

Slide prevention includes avoidance of troublesome ground during location, grading of stable slopes, and adequate drainage. Safe slopes are determined by methods described in Sect. 7, but all slopes should be checked during excavation to observe if there is any need to revise the original design for greater stability. Tunnels and vertical drains supplement surface drainage to remove excess moisture. Since any slide is almost always preceded by noticeable movement, all cuts and slopes, especially those suspected of slide capabilities, are observed for movement. In removing slides, excavation should begin at the top to lighten the load and impart greater stability to the sliding mass.

Rock slides result from moisture alternately freezing and thawing in joints and cracks, from erosion or excavation of underlying supporting materials, or from shattering through overshooting in excavation. Loose rocks should be removed each spring from rock cuts. Other unstable masses are brought down under controlled conditions. Good location, careful excavation and sloping, adequate drainage, sealing of joints with grout, and continual inspection and vigilance are measures necessary to prevent rock slides.

Slide fences may supplant cut watchmen to provide protection against slides and rock falls where these are a recurring source of danger. Wire-mesh fences are strung at the toe of slopes and along cut slopes and connected to the signal system so that a break or strain in the fence will interrupt a detector circuit and set the wayside or locomotive cab signal at the "stop" or "stop-proceed" indication against oncoming trains.

6. ROADBED STABILIZATION

Unstable roadbeds may arise from inadequate compaction during construction. Frozen soil lumps, arching around large boulders and stumps, or the decay of logs, stumps, and timbers leave voids in the fill which collect moisture and develop instability. The "working" of plastic clays and silts under the ballast as traffic moves forms a soft slurry-like material into which the ballast sinks. The process is repetitious and cumulative, until large moisture-filled ballast pockets up to 10 ft or more in depth are formed. These conditions are reflected in sinking track, muddy ballast, pumping joints, worn ties, battered rails, slow orders, increased maintenance costs, and sometimes complete failure of the embankment. Instability is often accompanied by a characteristic heaving of the roadbed shoulder or push-up at the toe of the fill or at points even more remote. Slides within the roadbed limits are a form of instability. They arise from slope failures resulting from a failure of the soil in shear, sliding along a lubricated surface as in the case of a sidehill fill, and from foundation failures in the underlying subsoil.

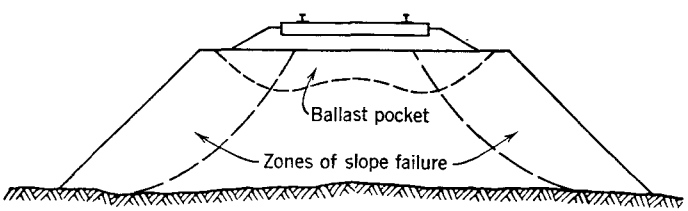


FIG 9 Ballast pockets and slides in fills

Roadbed instability is best controlled at the time of construction by proper soil engineering including selection of stable soils, avoidance of compressible silts, plastic clays, and swelling clays and shales, providing adequate compaction and moisture control, and finishing of the top to a smooth surface free of undulations, ruts, and depressions. Clay materials if used should be kept well below the ballast section. For roadbeds which are unstable after construction, a variety of remedies are available.

Grouting

Roadbed grouting is one of the most useful and recent of the stabilization procedures. The following methods have been developed by the AREA with the assisting research of

Dr. Ralph Peck of the University of Illinois Engineering Experiment Station.[4] Grouting is most effective when applied to ballast pockets but has been successfully applied to high fills. A cement grout slurry is forced into the fill either by pneumatic or by hydraulic injection. A hydraulic displacement pump for hydraulic injection forces the grout through flexible rubber hose, usually 1 1/4 in. in diameter with standard train line connections. Pipe of the same size may be substituted for the hose. The grout is mixed in an agitator tank or tub. Holes for injection points are first driven with a spud bar if the soil or ballast material is hard. Injection points usually consist of a section of 1 1/4-in. pipe. A loose plug or bolt in the driving end prevents the pipe from filling with soil during the driving. It is forced out by a long rod after the driving is completed. Commercial points combine a hardened steel tip with an inner steel core which can be withdrawn after the point is in place. Spuds are driven with 70-lb air hammers, and points with 52-lb hammers. The mixing tub, injection points, and air hose are the same for all systems of grouting.

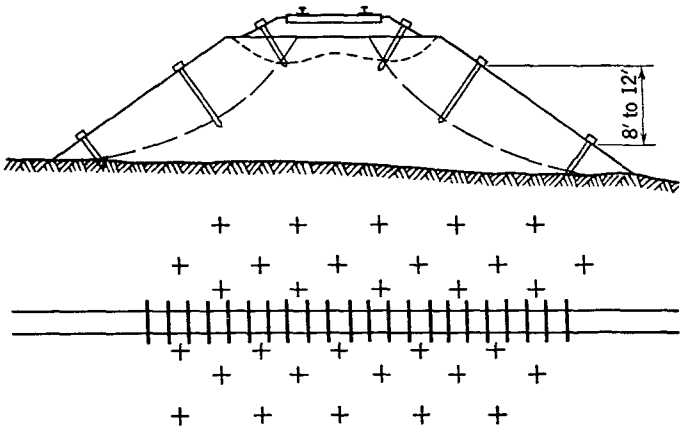


FIG. 10. Location of grouting points.

Pneumatic injection is accomplished with an air compressor delivering 65–210 cfm at 90 lb/sq in. pressure.

Cement-Sand Ratios. Rich-mix grouting uses a cement-sand ratio of 1:1 to 1:6. The average is about 1:4. Rich-mix grouting is usually applied to "spot" grouting whereby small individual soft spots are grouted. The portability of the pneumatic injection equipment commends it to spot grouting. Lean-mix slurries vary from 1:6 to 1:32; 1:12 is the most commonly used. Because of its low cost, the lean-mix is used for out-of-face grouting in areas of extended instability. The less portable hydraulic injection equipment is usually associated with lean-mix grouting.

Standard Portland cement is used in the slurries. Air-entraining cement promotes flowability in the line. Gradations commonly encountered have been:

Passing a 100-mesh sieve (0.149 minimum grain size) 99.5%.
Passing a 200-mesh sieve (0.074 minimum grain size) 96.0%.
Passing a No. 325 sieve (0.44 minimum grain size) 90.0%.

Fly ash, a product of combustion of powdered coal under forced draft in power plants, is sometimes used as a filler for sand. It lacks cementing qualities but promotes flowability producing the same flow characteristics with less cement.

Asphalt emulsions in quantities of 0.1 gal of emulsion per cubic foot of sand lubricate the flow and aid in suspending solids in the slurry, preventing the separation of sand from cement during injection.

Sand should be flat and angular to promote stability but for flowability a rounded wind-blown or beach sand is preferable. For flowability the sand should pass a No. 20 screen 100% with a large percentage passing a No. 40 screen. Sand with organic content, from upper coniferous areas, will not set readily; it should be corrected by adding about 1% of calcium chloride.

Water content has varied from 3 to 6 gal per cu ft of dry material. Experience indicates 4–5 gal of water per cu ft of material to be desirable.

[4] Dr. R. B. Peck, First Progress Report of the Joint Investigation of Methods of Roadbed Stabilization, *Proceedings of the AREA*, Vol 47, 1946, pp. 324–353.

Grout acceptance varies from a few cubic feet per track foot (one 94-lb bag of Portland cement being assumed as 1 cu ft) in shallow fills and pockets to 35 and 100 cu ft per track foot in high fills. Grouting continues until the grout breaks out or the track begins to rise. When acceptance is high as in high fills, it may be desirable to do the grouting in stages allowing the first 20 or so cubic feet per track foot to set up before going on with the grouting

Costs depend on the mixture used and the acceptance per track foot. Actual costs have ranged from $0.28 to $1.04 per cubic foot of grout and $1.12 to $22.50 per track foot. Most installations have shown savings in maintenance costs sufficient to cover the costs of grouting in 3–4 years. In only a few cases has it been necessary to repeat the grouting procedures, although this might become necessary over longer periods of time than have thus far elapsed.

Other Methods

Other stabilization methods have been successfully developed and used in certain situations.

Piling 10–14 in in diameter and driven in the roadbed shoulder has proved helpful in some types of instability. To be effective at least 50–65% of the pile length should be in

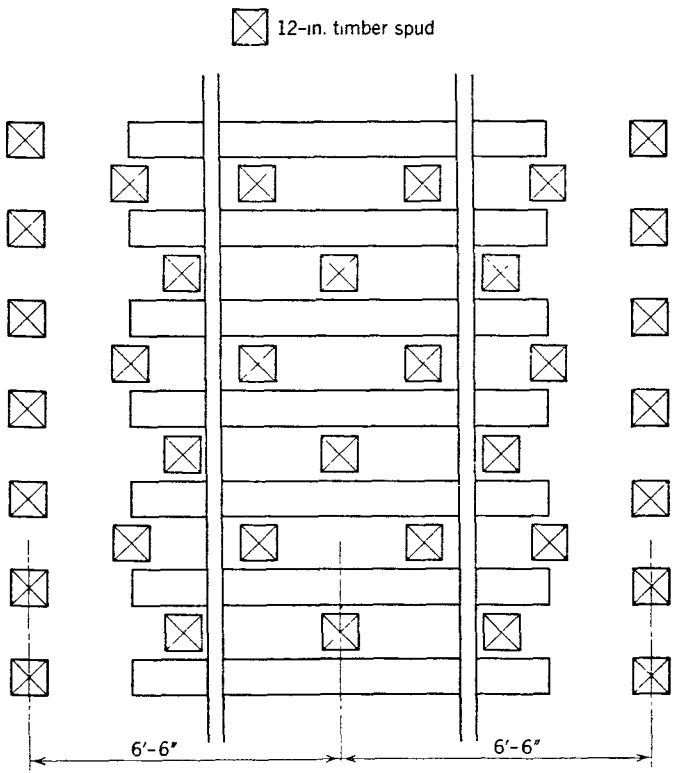


FIG 11 Driving pattern for sand piles (*Proceedings of the AREA*, Vol 51, p 741, 1950)

firm subsoil. Piles are driven with conventional-type pile drivers on 3- to 6-ft centers, tied through the subgrade to a corresponding row on the opposite shoulder with tie rods or wire cables, and braced longitudinally with scrap rails or 8 in by 16 in. timber walers. In addition to mechanical stability, the driving process tends to consolidate the fill material and provide vertical drainage, especially when an underlying impervious stratum is completely penetrated. Costs depend on length of piles, difficulty of driving, and extent of interruptions from and to revenue traffic.

Short poles 6–10 ft in length and 4–12 in. in diameter driven at the ends of every other tie have been successful in low fills and in shallow swampy areas. The stabilizing action is similar to that of piles in deeper fills. Again 50–65% penetration in firm material is essential. Old ties have been successfully used. Light commercial on-track and caterpillar-mounted off-track pile drivers have been developed for this work. Care should be observed to avoid shoving off part of the fill. Costs were about $1.50 per track foot for one job on a southwestern railroad.

Sand piles involve the driving of series of holes into the subgrade with a steel or timber spud inside and outside the running rails. The holes are then filled with a clean, dry engine sand. About 20% of the subgrade top surface should be covered with the holes. (Fig. 11.) The spud is about 12 in. by 12 in. and, for ease in driving and withdrawal, should be about 10–12 ft in length. A light on-track or caterpillar-mounted off-track pile driver sets and draws the spud. The sand pile provides consolidation (while driving) and vertical drainage, and the sand particles add shearing resistance to the soil. The spud should always puncture the bottom of all water pockets. Costs have run about $1.60 per track foot.

Sand-filled blast holes are a variation of the sand pile. Holes are drilled with a 2- or 3-in. auger and sprung at the bottom with 1 1/2 sticks of 40% dynamite. The holes are then filled with clean, dry sand blown in and compacted by an air compressor. Costs are about $1.68 per track foot.

Drainage must always be adequate to promote stability. Relief from water and ballast pockets in fills may be had by locating the low spots in the pockets and placing a corrugated, perforated metal pipe or open-jointed clay-tile drain (metal pipe is preferred) 12–18 in. below the bottom of the pocket, transverse to the track, and extending to the far rail of the near track. Pipes are laid with perforations downward. The ditch in which the lateral stub is laid is then backfilled with a sand filter material (see Sect. 7) and allowed to drain down the slope (Fig. 12). Where drainage is excessive, a paved gutter is required. A fall of about 1/2 in. per ft is sufficient for the stubs. A screen of heavy wire mesh is placed over the pipe openings to keep out rodents.

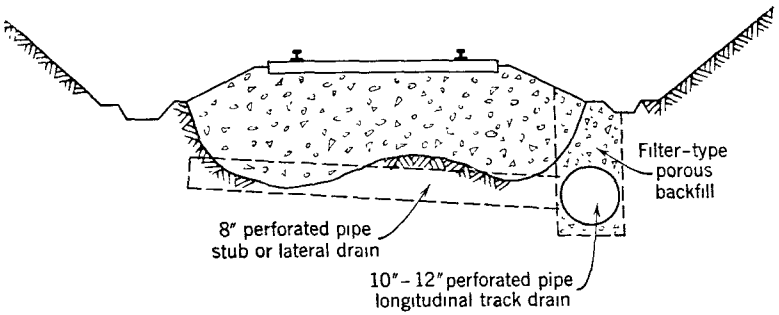


FIG. 12. Subdrainage for ballast pockets.

In cuts, the lateral drain stubs connect to a longitudinal drain located beneath the roadbed shoulder (Fig. 12). The drain should be at least 2 in. greater in diameter than the transverse stubs and of perforated corrugated metal or of drain tile laid with open joints. Again a backfill of filter material is used. Laterals are placed at low spots in the ballast pockets as indicated by a subsurface survey made with a hand or power auger. A minimum grade of 0.30% is held on the longitudinal. The longitudinal drain must be led away from the track at its exit to insure discharge in a direction away from the roadbed, preferably into an existing surface flow channel. The exit may be protected by a light headwall and a wire screen.

With *all types of stabilization*, the track is likely to be disturbed and should be given a light surfacing and lining.

7. DRAINAGE

Drainage is the most important single item in railroad construction and maintenance. Roadbed stability and low maintenance costs can only exist when adequate drainage is provided. For additional information on drainage reference is made to Vol. II, Sect. 12, Hydraulics and Pumping, Sect. 13, Hydrology, and Sect. 17, Irrigation and Land Drainage.

Ditches

Longitudinal drains include (*a*) intercepting or berm ditches which protect the slopes of cuts and intercepting or toe ditches which similarly protect embankments on saturated soils, (*b*) side or track ditches in cuts and subsurface drains of 6- to 10-in tile, corrugated and perforated metal pipe, or concrete pipe laid about 2 ft or more underground and backfilled with porous filter material as described in the preceding paragraphs on subdrainage.

Ditch gradients may conform to the natural slope of the ground for ditches listed under (*a*) and of the roadbed for those listed under (*b*). The AREA recommends a minimum gradient of 0.30% to make the ditches self-flushing and eliminate standing water. Too-steep slopes giving flow velocities of 5–10 fps lead to scouring and, if used, must be protected by paving, by check dams, or by drops.[5] Long ditches may have a crest in the middle and drain both ways. Since pipes will carry water on a flatter slope than mere earth ditches, drains listed under (*b*) are particularly necessary when the grade of the road is level and especially if the soil is retentive and impermeable. The AREA further recommends maximum slope channel cross sections of 1:1 with a minimum base width of 3 ft for intercepting ditches and 1 ft for track ditches. Intercepting ditches must be 15 ft or preferably more from the berms of slopes or toes of fills so that the water content will not saturate the slope and cause sliding. They should be deep enough to lower the adjacent water table. Borrow pits should be included in the system of toe ditches to prevent slope and subsoil saturation. All drains and ditches must have a free and unobstructed outfall

Precise values for cross-sectional areas of ditches are obtained by applying the methods described below under Computing Sizes of Openings, and by referring to the previously mentioned Sects. 12, 13, and 17, all in Vol. II.

Ditch maintenance includes restoration of scoured slopes and bottom and removal of sediment and debris. Paving, sodding, or timbering restores slopes and prevents scour Riffle or check dams also reduce scour as well as velocities. Clamshell buckets in work trains are used today for ditch cleaning only when on-track ditching blades or off-track scraping and loading equipment is not available or cannot be used because of restricted space.

Cross Drains and Culverts

Cross drains and culverts are used to conduct water from one side of the roadbed to the other. Culverts are used to drain side ditches; pass side streams; drain pockets, coulees, and small valleys; serve as equalizers through pond and swamp areas; and form cattle and pedestrian underpasses.

Computing Sizes of Openings. The **cross-sectional area** of an opening is $A = Q_c/v$ where A is the area of opening in square feet, Q_c the quantity of flow through the opening in cfs, and v is the velocity of flow in second-feet. Manning's formula gives $v = \frac{1.486}{n} r^{2/3} s^{1/2}$, where s is the energy slope in feet of drop per foot of length, r is the hydraulic radius equal to A divided by the wetted perimeter and n is a roughness factor or coefficient of frictional flow varying from 0.012 for finished concrete, 0.015 for smooth, tiled channels, 0.021 for corrugated metal pipe, 0.025 for bare earth and rubble, to 0.06 for weed-grown ditches. It may be taken as 0.04 for average stream channels when other data is lacking. If Q_r is the rate of runoff from the drainage area, $Q_c = Q_r$, since the opening must handle the runoff from the area. By assuming a value for A and determining corresponding values for r, s, and v, the pipe capacity, Q_c, is obtained and compared with Q_r. If the first comparison is not in close enough agreement, a second value for A is assumed, the choice being guided by the error in the first assumption.

The **runoff,** Q_r, is obtained from the expression

$$Q_r = A_D i R \text{ cfs}$$

where A_D = area of the watershed or drainage area in acres.

i = rainfall intensity in inches per hour for a selected storm of given frequency and duration, the maximum design storm.

R = the runoff factor, a difficult value to determine accurately since it depends on topography, vegetation, and permeability and condition of the soil. The factor will vary from 0.10 to 0.15 for flat, vegetated, or gently rolling country; from 0.3 to 0.5 for built-up sections, from 0.8 to 0.9 for completely built-up sections or rocky, hilly, or mountainous areas; and will be 1.00 (or more when snow is melting) for frozen ground

[5] *AREA Manual*, 1953 Ed., pp. 1-1-45 to 1-1-46.

Saturation by long-continued rains makes most soils more or less impervious. Frozen soil is also impervious. Vegetation retards flow and holds back some of the rainfall. Shape and slope of the watershed affect the time required for water to reach the culvert from different parts of the area. Runoff factors have been determined with great care in some areas by state and federal agencies and may be used when thus available.

Good judgment and knowledge of an area are essential in making an informed selection of values for runoff factors. Ample use should be made of field observations and of the historical evidence of high-water marks and other flow characteristics. Section 13, Vol. II, should be consulted for this solution as well as for the use of the unit hydrograph, a more accurate method for determining runoff where stream flow is involved.

The **size of opening** of a culvert or bridge should be sufficient to care for the flow resulting from all ordinary floods. When loss of life or damages from interruption to traffic are likely to result from roadway damage, it is necessary to provide for phenomenal flood conditions that occur at 50- to 75-year intervals. For other locations, it is more economical to repair the occasional damage which occurs than to tie up additional capital in construction for long periods. However, cost does not increase in proportion to capacity, this fact together with the difficulty of accurately determining required sizes and the advantages of standards in estimating and constructing have led most roads to adopt a few standard sizes for small openings rather than try to fit each case accurately.

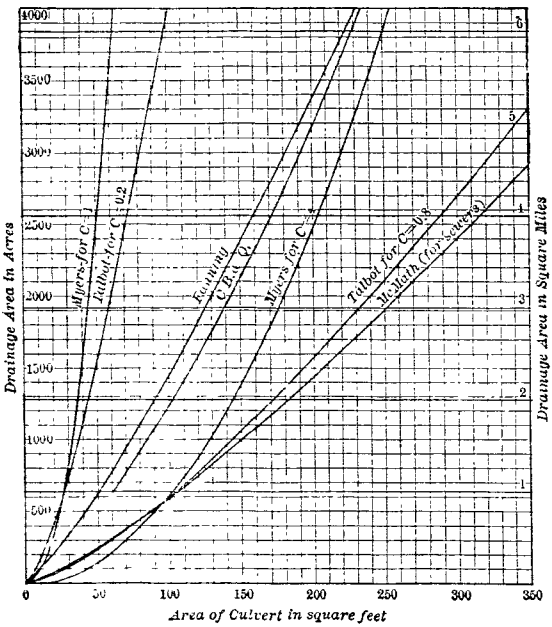


FIG. 13. Required culvert areas by different formulas.

Empirical formulas, of which there are many, are useful as guides where only general information is available or where long usage has shown the results to be reliable. Empirical formulas are derived for the conditions of a particular watershed and cannot safely be transferred to any other. In the several following examples, A = drainage area in acres, a = area of waterway in square feet, and C = a coefficient depending upon topography, vegetation, and soil conditions:

E. T. D. Myer's formula, $a = C\sqrt{A}$, in which C is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground.

A. N. Talbot's formula, $a = C\sqrt[4]{A^3}$. "For steep and rocky ground C varies from $^2/_3$ to 1. For rolling agricultural country, subject to floods at times of melting snow, and with the length of the valley three or four times its width, C is about $^1/_3$; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, $^1/_5$ or $^1/_6$ or even less may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert."

J. T. Fanning's formula, $a = 0.23\sqrt[6]{A^5}$, in which no allowance is made for variations in conditions which affect the flow.

R. McMath's formula, $a = 0.5908\sqrt[5]{A^4}$, which was designed primarily for application to sewers. Like Fanning's, it makes no allowance for variations in conditions.

C. B. & Q. formula, $a = 0.46875\,A/(3 + 0.079\sqrt{A})$.

Fanning's formula and the C. B. & Q. formula agree very closely, and also agree with Talbot's for a mean value of C. But Talbot's has the advantage of flexibility. Curves for these formulas are plotted in Fig. 13 from which values of a may be readily found.

Table 4. Area of Full Semicircular Arches with Vertical Abutment Walls

Height of Abutment Walls, ft	Span of Arch, ft											
	5	6	7	8	9	10	11	12	13	14	15	16
1	14.8	20.1	26.2	33.1	40.8	49.3	58.5	68.5	...	...	...	...
2	19.8	26.1	33.2	41.1	49.8	59.3	69.5	80.5	92.4	...	...	...
3	24.8	32.1	40.2	49.1	58.8	69.3	80.5	92.5	105.4	119.0	...	...
4	29.8	38.1	47.2	57.1	67.8	79.3	91.5	104.5	118.4	133.0	148.4	...
5	34.8	44.1	54.2	65.1	76.8	89.3	102.5	116.5	131.4	147.0	163.4	180.5
6	...	50.1	61.2	73.1	85.8	99.3	113.5	128.5	144.4	161.0	178.4	196.5
7	...	...	68.2	81.1	94.8	109.3	124.5	140.5	157.4	175.0	193.4	212.5
8	...	...	...	89.1	103.8	119.3	135.5	152.5	170.4	189.0	208.4	228.5
9	...	...	...	...	112.8	129.3	146.5	164.5	183.4	203.0	223.4	244.5
10	...	...	...	...	...	139.3	157.5	176.5	196.4	217.0	238.4	260.5

Table 5. Diameters and Areas of Small Openings

Pipe Diameter, in.	Area, sq ft	Pipe Diameter, in.	Area, sq ft
8	0.351	30	4.91
10	0.55	36	7.07
12	0.79	42	9.62
15	1.23	48	12.57
18	1.77	60	19.64
21	2.41	72	28.29
24	3.14		

Unless local conditions specifically dictate otherwise, no pipe smaller than 18 in. in diameter should be used as a culvert or cross drain.

Culvert types in railroad use include

(*a*) corrugated metal pipe which combines lightness in weight with strength, durability, low cost, and ease of installation, especially in old embankments. Conventional sizes range from 12 to 48 in. For diameters over 48 in. multiple plates prebent to circular shape, and field-assembled, replace the tubular sections. When subject to scouring or corrosion, the entire pipe may be asphalt-coated and the invert paved with asphalt to a depth well above the depth of corrugations. The *AREA Manual* establishes recommended specifications for corrugated metal culverts on pages 1-4-6 to 1-4-16 and 1-4-25 to 1-4-32, 1953 Ed.

(*b*) Precast concrete pipes, which are usually cheaper than metallic sections, are more fragile, heavier, and not so easy to install. Concrete sections are non-flexible, in contrast to corrugated metal pipe. Large sizes of concrete pipe, 60–72 in. in maximum cross-sectional dimension, may be elliptical or rectangular for use as underpasses. Pipes come in lengths of 24, 48, 72, or 144 in. The large sections may have an eyebolt set in the top to which a hook, cable, or chain can be attached for ease in handling with crane or block and tackle.

(*c*) Concrete or stone masonry box or arch culverts are used for larger-sized openings and for initial construction. The box type is rectangular in cross section. Both types are laid or cast in place. For design of these culverts, reference is made to Sect. 23, Vol. III, Masonry and Plain Concrete Structures, as well as to the following general notes.

Design of Culverts. For other than pipe culverts, the following general design features should be noted:

The **length** of the **barrel** of a culvert between the inside faces of the walls is $L = 2sd + b$, in which s = the slope ratio of the embankment (horizontal to vertical), d = depth to top of culvert, b = width of roadbed. This length may also be used for pipe culverts with head walls.

End or Wing Walls. These act as retaining walls for the lower part of the embankment and are of three types (see Fig. 15).

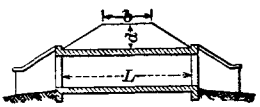


FIG. 14. Length of culvert.

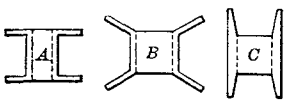


FIG. 15. Types of end walls.

Type A is the simplest form, but the volume of masonry is usually greater than that required by the other types except in narrow gorges. It is generally used for pipe and other low and small culverts.

Type B requires less masonry and is more commonly used for large culverts.

Type C is liable to the objection that the water may work its way behind the wing walls, scour them out, and endanger the embankment and culvert. When this can be unquestionably prevented, the type has the advantage of simplicity and economy of construction.

The **length** of an end wall of type A should be a little more than sufficient to keep the earth of the embankment spilling around its ends from reaching the opening or more than twice the slope ratio times the height under coping plus the width of opening minus twice the thickness of the wall. Type B is tapered down to a height of from 2 to 4 ft and type C to practically nothing. The types are often combined for culverts on a skew in order better to guide the stream into the culvert.

Standard Designs. Many railroads have prepared sets of standard plans for culverts and other structures, track work, signs, etc. These usually give quantities and are convenient for both estimating and construction as the men become accustomed to their use.

Stone Box Culverts. The thickness of the roof slabs for stone culverts usually varies from 10 in. for a 2-ft span to 15 in. for a 4-ft span. Figure 16 represents what may be considered a typical design. Such a culvert requires 7.0 cu yd of masonry for each of the end walls and 1.41 cu yd of masonry for each foot of length between the inside faces of the coping stones. This estimate does not include the volume of the paving. Dry masonry culverts should not be used on soft soil because of the danger of their being undermined; for these conditions concrete is practically always used at the present time.

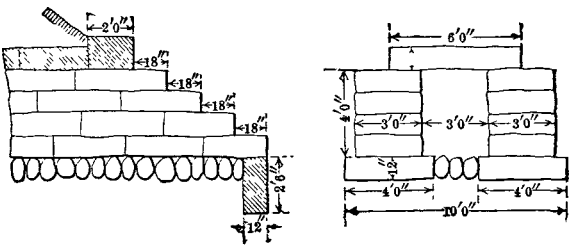


FIG. 16. Typical stone box culvert

Wooden box culverts, similar in design to the stone box, are used to save time and expense during construction, where timber is cheap. They are made large enough to allow a pipe culvert to be put through them before the timber gives out. The sides are of 12 by 12 in. timbers, laid up solid, and the top varies in thickness from 8 in. for 3-ft span to 12 in. for 5 ft. The bottom should be planked and the ends protected from undercutting.

Correct culvert location is important for successful operation. Culvert alignment insofar as practicable should coincide with that of the stream. Where the stream is at an

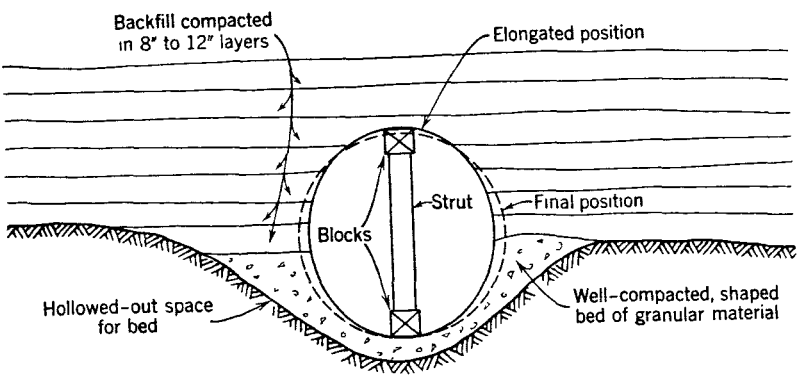


FIG. 17. Strutting and placement of flexible metal pipe culvert.

angle to the roadbed, a skewed culvert should be installed. Culverts may be given the same slope as the stream but should be held to a minimum gradient of 0.50% to prevent sedimentation. The pipe should be set about 2 in. above stream bed. Where the pipe

discharges down a slope, a paved spillway should be provided. When installed under a high fill, a slight camber, obtained by a break in grade of the pipe bed, is placed in the pipe under the midpoint of the fill to compensate for any settlement of the fill onto the pipe. Head walls, provided where necessary with wings and aprons, hold back the fill, permit a shorter length of culvert barrel, and prevent wash and erosion around the entrance and exit. Head walls also resist longitudinal forces tending to separate the pipe sections.

Flexible culverts are laid in a well-tamped bed of porous material with a contour of the lower third of the pipe hollowed out. All backfill must be thoroughly tamped to develop approximately equal earth pressures around the pipe and thus in turn develop the maximum resistance of the flexible barrel to those pressures. Large-diameter pipes should be elongated vertically about 5% by jacking and strutting the pipe out of round. Struts are removed about two months after installation. Strutting has the effect of developing full earth pressures around the pipe. Corrugated metal pipes are conveniently installed in old roadbeds by jacking through the fill (Fig. 18).

Rigid culverts (concrete and cast-iron pipes) are also set on a prepared bed and carefully backfilled. The prepared bed should include a space for the bell end laid upstream. Joint cracks are filled with mortar or asphalt. The latter has flexibility and will not crack under slight movements of the pipe sections. Smooth-barreled sections of the male and female type may also be jacked into place. Culverts should be set at least below the ballast section. Clay tile when used is set a minimum of 3 ft below top of cover because of its low resistance to crushing. No stones should be in contact with it.

Culvert maintenance includes the following:

(*a*) Inspection by local forces should be made at least once a week and after every storm.

(*b*) Cleaning is performed in the spring and fall and whenever else inspection discloses a need for it.

(*c*) Sags and settlements which impede the flow of water or cause icing and sedimentation are corrected by digging out and resetting shallow pipe. When pipe is deeply buried, new pipe must be installed.

(*d*) Crushed sections near the end may be replaced. Temporary relief is had by installing struts under the weakened portions. Cracks are filled with asphalt or mortar. A new flexible metal pipe may be threaded through the old pipe.

(*e*) Separated joints are jacked together and the joints filled with asphalt or mortar. Longitudinal steel tie rods may be used to hold joints together. When jacking is difficult, metal segmented collars are installed by excavating a slight amount around the joint from inside the pipe. Installation or repairs to head walls may also be required.

(*f*) Scour at the entrance and exit is reduced by resetting the pipe on a flatter slope or by constructing a stone or concrete apron or spillway at each end.

(*g*) Concrete and structural failures in large box and arch culverts call for repairs common to all major structures of that type of construction.

An adequate program of inspection, usually semiannual, is required to preserve the safety and life of these structures. Reference should be made to Sect. 23, Volume 3, Masonry and Plain Concrete Construction.

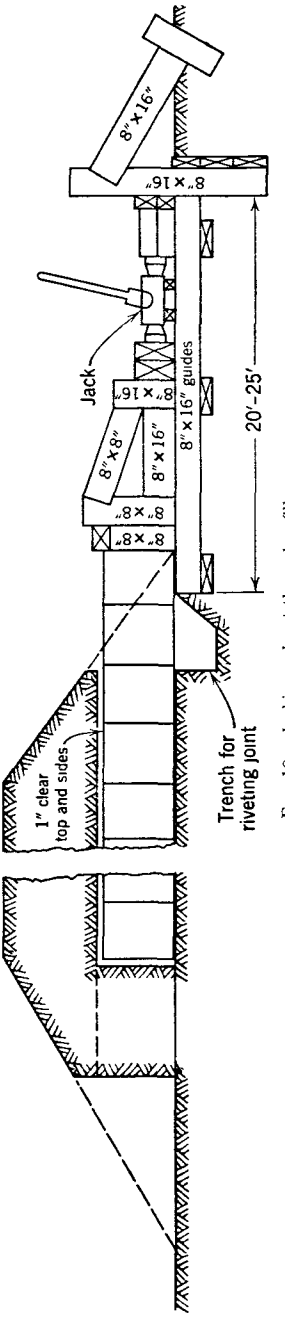


FIG. 18 Jacking culvert through a fill.

TRACK

8. CLASSIFICATION OF TRACK

The following classification of railway track, based on annual tonnage and on maximum speed of freight and passenger trains, is recommended by the AREA as the basis for construction and maintenance of roadbed and track. In the *AREA Manual*, 1953 Ed., pp. 16-4-1 to 16-4-2, all track is grouped as follows:

Class A. Railway main track carrying 12.5 million or more gross tons annually, or on which freight trains normally operate at average speeds of 40 mph, or passenger trains at average speeds of 60 mph or more between stops.

Class B. Railway main track carrying more than 5 million but less than 12.5 million gross tons annually, or on which freight-train speeds are more than 25 but less than 40 mph, or passenger-train speeds are more than 40 but less than 60 mph average between stops.

Class C. Railway main track carrying not more than 5 million gross tons annually, or over which freight trains normally operate at speeds less than 25 mph, or passenger trains at speeds less than 40 mph average between stops.

9. BALLAST

Ballast is a granular, non-cohesive material placed on the subgrade to hold the track in line and surface, to provide immediate drainage, to transmit track loads to the subgrade with diminished unit pressure, to facilitate maintenance operations, and to discourage vegetation.

Characteristics of a good ballast are:

(*a*) Strength and toughness to resist static and dynamic crushing loads.

(*b*) Durability, including resistance to abrasion and weathering.

(*c*) Stability or resistance to flow or movement when in place, i.e., high internal friction.

(*d*) Drainability to provide rapid movement of moisture from the track especially from around the ties.

(*e*) Cleanability.

(*f*) Workability.

(*g*) Availability.

(*h*) Low overall economic cost (rather than low first cost).

Types of ballast in general use today and in order of desirability include:

Crushed Stone. Basalt, granite, limestone, or other hard rock material artifically broken into sizes from 3/4 in. to 3 1/2 in. The larger sizes are used on roads with very heavy axle loads. Stone possesses strength, durability, stability, and is readily cleaned.

Table 6. Grading Requirements for Ballast Prepared from Crushed Stone and Crushed Air-Cooled Blast-Furnace Slag *

Size No.	Nominal Size Square Opening	Amounts Finer Than Each Sieve (Square Opening Laboratory Sieve), % by weight									
		3″	2 1/2″	2″	1 1/2″	1″	3/4″	1/2″	3/8″	No. 4	No. 8
24	2 1/2″–3/4″	100	90–100	.	25–60		0–10	0–5	...	...	...
3	2″ –1″	.	100	90–100	35–70	0–15	.	0–5	.	...	...
4	1 1/2″–3/4″	.	..	100	90–100	20–55	0–15		0–5	...	...
5	1″ –3/8″		..	...	100	90–100	40–75	15–35	0–15	0–5	
57	1″ –No. 4	...	...	..	100	90–100	...	25–60	.	0–10	0–5

* *AREA Manual*, 1953 Ed., p. 1-2-3.

Crushed slag, the waste product of iron and steel production. Crushed to the same sizes as stone, it possesses most of its characteristics but is less expensive in first cost.

Prepared gravel which has had excessively fine and deleterious materials removed and large particles crushed to proper size. Gravel lacks the strength and durability of stone or slag and cannot be cleaned without undue loss of the smaller particles.

Table 7. Grading Requirements for Prepared Ballast Gravel, Depending upon the Percentage of Crushed Particles *

Size No.	Crushed Particles, %	Amounts Finer Than Each Sieve (Square Opening), % by weight							
		1 1/2"	1"	1/2"	No. 4	No. 8	No. 16	No. 50	No. 100
G-1	0–20	100	80–100	50–85	20–40	15–35	5–25	0–10	0–2
G-2	21–40	100	65–100	35–75	10–35	0–10	0–5		. .
G-3	41–100	100	60–95	25–50	0–15	0–5		. .	

* *AREA Manual*, 1953 Ed., p. 1-2-3.

Pit-run gravel, which may occur sufficiently free of deleterious materials and graded in nature to permit immediate application to the track.

Table 8. Pit-Run Gravel Ballast *

Sieve Size (Square Openings)	Amounts Finer Than Each Size, % by weight	
	Grade A	Grade B
2 1/2"	97–100	97–100
No. 4	20–55	20–65
No. 200	0–2	0–3

* *AREA Manual*, 1953 Ed., p. 1-2-5.

Chat is the tailings or refuse from zinc, lead, and silver mines, and consists of small rock particles combined with bits of ore. Chat is easily worked and possesses reasonable strength, durability, and stability.

Cherts are compact, flint-like siliceous rocks or hornstone occurring in natural deposits and possessing characteristics similar to chat.

Cinders lack compressive strength and will pulverize and harden. Cinders are suitable for branch lines, yards, sidings, subballast, and for full ballast application on unstable and settling portions of roadbed.

Sand as used for ballast is any hard, granular, finely divided rock material of grain size passing a No. 10 screen but retained on a No. 50 screen. It lacks the required stability if the particles are rounded or possess excess moisture. It will blow and drift.

Other Materials. *Burnt clay* is formed by burning clay in beehive mounds and then crushing to 3/4-in. to 1 1/2-in. size. It lacks crushing strength, absorbs moisture, and is generally obsolete today. *Caliche, burned tailings of coal mines, seashells*, and even *soil* may be used for light, temporary lines. *Volcanic ash*, which occurs in certain parts of the Southwest, has been used with success. It can be crushed to the same sizes as stone, and is considerably cheaper if it is located close to the tracks of the using carrier.

Ballast tests as recommended by the AREA on p. 1-2-4 of the 1953 Manual are based on ASTM and AASHO recommendations.

(*a*) *Size and Gradation Tests.* ASTM Test C-136, Method of Test for Sieve Analysis for Fine and Coarse Aggregates.

(*b*) *Tests for Deleterious Particles.*

1. *Material finer than No. 200 sieve* shall not exceed 1% in stone, slag, and washed gravel ballasts.
2. *Soft and friable pieces* must not exceed 5% in prepared ballasts, in accordance with the Method of Test, ASTM C-235-49T.
3. *Clay lumps* must not exceed 0.5% as determined by the current Method of Test for Clay Lumps in Aggregates, ASTM C-142.

(*c*) *Resistance to abrasion* is determined in accordance with the current Method of Test for Abrasion of Coarse Aggregates by Use of the Los Angeles Machine, ASTM C-131. The difference in weight before and after rotating and screening expressed as a per cent of initial weight should not exceed 40%.

(*d*) *Soundness test* is made in accordance with Test for Soundness of Aggregates by Use of Sodium Sulfate or Magnesium Sulfate, ASTM C-88. For use in freezing temperatures, the ballast should not show a weighted average loss exceeding 10%.

(*e*) *Weight per cubic foot* is given by the ASTM Test C-29, Method for Testing Unit Weight of Aggregates.

(*f*) *Experience* is deemed a satisfactory test by the AREA even though a ballast may not meet all formal test requirements.

Selection of ballast will depend on wheel loadings, volume and speed of traffic, stability of roadbed, weather conditions, desired ease in surfacing (2-in. surfacing lifts are hard to make with 3-in. stone), availability, and overall economic cost.

Shrinkage of ballast is the decrease in volume from that loaded at the pit and measured dumped loosely in the car to that measured in the track after ballast has been tamped and compressed by trains. The AREA has recommended in the past an "allowance for shrinkage to be added to measurements in cars at the loading point for calculated quantities" as follows [6]

Stone, slag, and prepared gravel ballasts	12–15%
Pit-run gravel ballast	8–20%

The **ballast section** as shown in Fig. 6 has a horizontal shoulder extending at least 6 in. beyond the end of tie and then sloping to the subgrade or subballast section on a 2:1 slope. The subballast section holds the same slope as the subgrade. Quantities of ballast required per mile are a function of the depth of ballast beneath the tie. This in turn depends on traffic volume, weight and speed, type of ballast, weather, and condition of subgrade. An unyielding subgrade might require only 6 in. of ballast beneath the tie. No practical subgrades are unyielding. The minimum depth for A should be at least equal to the tie spacing but may be divided between the subballast and ballast up to 50% of each Class A track may require 16 in. of ballast and 14 in. of subballast, or 30 in. of ballast without a subballast section. The preferred depth is that which gives a minimum economic cost of maintenance. Quantities of ballast per mile of track for various combinations of ballast and subballast depths and for curves are found on pp. 1-2-7 to 1-2-11 of the *AREA Manual*, 1953 Ed These are based on 3200 ties per mile with a 15% allowance for shrinkage.

Unloading of ballast is best performed with special-type on-track ballast cars of 70 tons capacity with side and center doors, ratchet controlled, which permit distribution of ballast in the places and quantities desired. Unloaded ballast should be plowed down to rail level or lower by spreader blade or flanger, or with a reinforced timber skid placed in front of the rear truck of the car being unloaded. Other car types, i e., bottom-dump coal cars may be used by chaining the doors to a predetermined opening. Unloading costs include the wages of the train and engine crew, the foreman and at least four to six laborers, and the rental charges on the cars, spreader, and locomotive. The ratio of the per cent of track-usage time to the total time on the road and the length of haul also affects costs.

Cleaning Ballast

Ballast becomes dirty and "dead" in from 2 to 7 years. Engine sparks and cinders, dirt and dust blown or dropped onto the track, abrasion of ballast particles, and clay and silt working upward from the subgrade are principal causes of dirty ballast. Loss of draining qualities, pumping joints, and general track deterioration quickly follow. Low-grade ballasts (chat, cherts, cinders, sand, etc.) must be dug out and replaced or, more commonly, the track is raised on the old ballast and filled with a spread of new ballast. Stone, slag, and some prepared gravels can be cleaned.

Frequency of cleaning depends on local conditions of ballast, subgrade, weather, locomotive types, and traffic. Some roads clean on a 2- or 3-year cycle. The best rule is to clean ballast when it needs cleaning, i.e., when it begins to lose its "life" and drainability.

Cleaning Procedures. The oldest method, and still a common one, is hand cleaning. Ballast is forked out of the shoulders, intertrack, and cribs by hand. It is cleaned by shaking on the fork or by throwing it on an inclined screen before returning it to the track. A mechanized alternative is to use clamshell buckets to dig the ballast and dump it on inclined screens mounted in railroad cars. The dirt falls through the screens into the cars; the ballast falls back onto the roadbed.

A variety of commercially developed cleaning machines are available for purchase or lease including both on- and off-track designs. Essentially all mechanized cleaners consist of a combined plow and digging head which carries the ballast from shoulder or intertrack onto a conveyor or elevator, over a series of vibrating screens, then back to a conveyor for deposit on the roadbed. Dirt is wasted from another conveyor or chute onto the right of way or, in the case of on-track equipment, into cars back of the conveyor. Cribbing machines, recently developed, force the crib ballast onto the shoulder or intertrack for pick-up and cleaning with conventional equipment.

Two or more cleaners and cribbers are efficiently operated in groups. Cleaning rates vary from 2 to 60 ft per minute. Shoulders should be cleaned to subgrade, and intertrack to tie bottom and preferably to 12 in. below the bottom. Cribs should be cleaned 2 to 3 in. below the bottom of tie. The ballast bed under the ties partially cleans itself by leaching

[6] *AREA Manual*, 1948 Ed., p. 2-15.

out into the cleaned portions of the section. Ballast fouled with clay may require two passes of the cleaner About 0.1 cu yd of dirt is obtained per track foot, averaging about 470 tons per mile. Dirt wasted close to the track is likely to be drawn back into the ballast by the suction of passing trains Usually some new ballast will have to be added after cleaning to replace the "fines" lost through the cleaning screens.

10. TIES

Railroad ties, set transversely to the rails, hold the rails to correct gage, bear the rail and traffic loads, and transmit those loads with diminished unit pressure to the ballast Ties are classified by species of wood, by method of dressing (sawed four square or hewed), by use (plain, switch timber, bridge tie, or third-rail tie , or by the type of preservative treatment (usually zinc chloride or creosote-oil impregnation).

Tie lengths vary from eight feet, suitable for branch lines and yards and for street and interurban lines, to the present in-track standard of 8 ft 6 in. and the 9-ft tie now recommended by the AREA for new construction and maintenance renewals.[7] Narrow-gage ties are proportionately shorter. Other recommendations based on AREA standards for size of cross section are found in Table 9

Table 9. AREA Recommended Tie Grading *

Size	Dimensions: Sawed or Hewed 4 Sides	Dimensions: Sawed or Hewed Top and Bottom Only	Usual Assignments
0	5″ depth × 5″ width	5″ depth × 5″ width	Not accepted for standard gage
1	6″ depth × 6″ width	6″ depth × 6″ width	Yard and side tracks only
2	6″ depth × 7″ width	6″ depth × 7″ width	Yard and side tracks only
3	6″ depth × 8″ width	6″ depth × 8″ width 7″ depth × 7″ width	Sidings and secondary main lines
4	7″ depth × 8″ width	7″ depth × 8″ width	Primary main lines
5	7″ depth × 9″ width	7″ depth × 9″ width	Primary main lines (especially on curves)
6	7″ depth × 10″ width	7″ depth × 10″ width	Primary main lines

* *AREA Manual*, 1953 Ed., p. 3-1-2.

Tie woods should be free of splits, shakes, and knots, should be hard, tough, and resistant to splitting, abrasion, and decay, and should preferably be grown local to the using railroad. White and red oak are preferred materials; Southern pine, Douglas fir, and gum are in widespread use. AREA specifications are found in *AREA Manual*, 1953 Ed., pp. 3-1-1 to 3-1-8.

Spacing. The usual maximum spacing is 2 ft on centers. Class A track will require 19.5–22 in. spacing. An open space of 10 in. between ties is sufficient for tamping. A maximum bearing area on ballast is obtained by use of wider and longer ties. The number of ties per mile for various spacings follows:

Number of ties per 39-ft rail	26	25	24	23	22	21	20	19 5	19	18	17	16
Average spacing, in.	18 0	18.7	19.5	20.3	21 3	22.3	23.4	24.0	24 6	26 0	27.5	29.2
Number of ties per mile	3520	3385	3250	3114	2980	2843	2708	2640	2573	2437	2302	2166

Tie renewals are normally carried out by "spotting" in ties to replace those no longer serviceable The year's program is usually based on a fall inspection, and later by a spring inspection to locate any additional ties that did not last through the winter. Out-of-face renewals call for the 100% replacement of all ties in a location without regard to individual condition. Such renewals reduce overall tie life and, since all the ties wear out at approximately the same time, may cause unsafe track conditions. Where the cost of opening the track exceeds any losses otherwise incurred, out-of-face renewals are necessary, including paved crossings and station platforms, in tunnels, on bridges, and through retarders.

Tie life for an untreated tie varies from 3 to 15 years (with an average of 7 years) depending on type of wood, traffic, and roadbed and climatic conditions. Ties normally are removed from track because of natural decay and mechanical failure Ties can be given protection against insects and decay for as long as 40–50 years by chemical treatment. The older, somewhat injurious, zinc chloride treatment has been largely replaced by im-

[7] *AREA Manual*, 1953 Ed., p. 3-1-6.

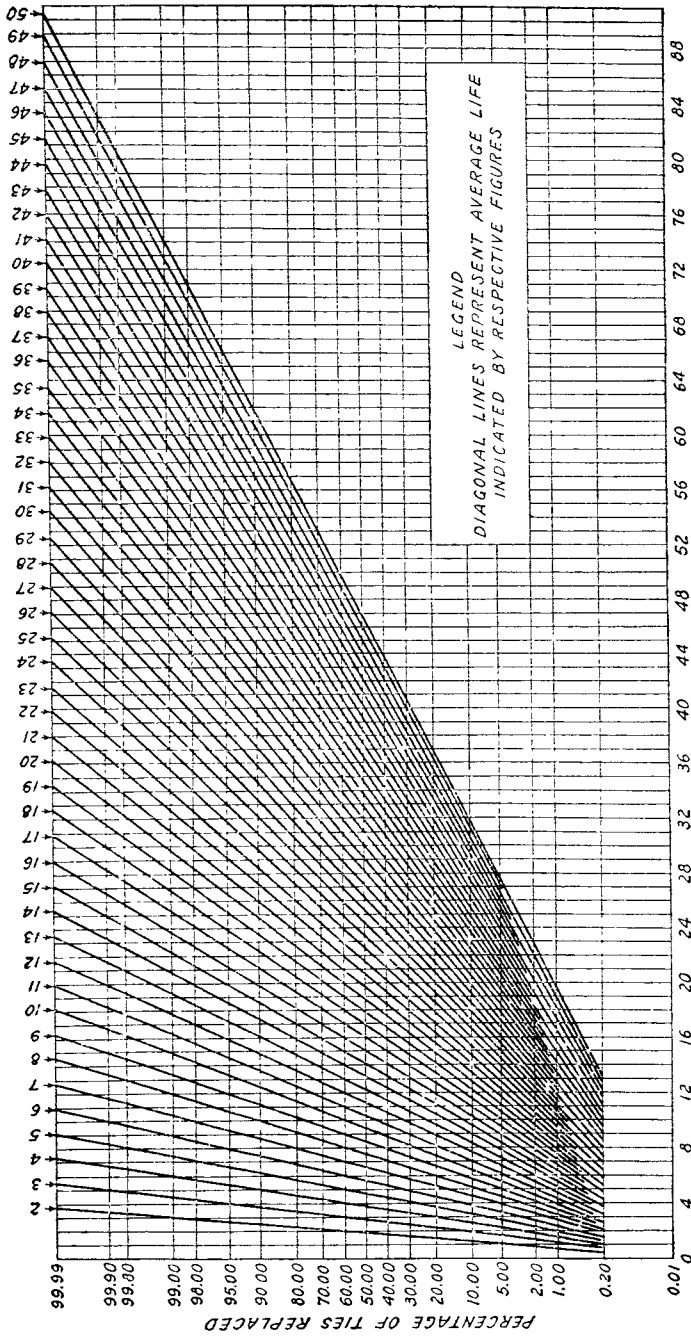


FIG. 19 Annual replacements for ties of various average life (Forest Products Laboratory, Madison, Wis.)

pregnation under pressure in a vat or cylinder with a creosote oil after the tie has been thoroughly seasoned or dried.

Mechanical failure arises from splitting during the drying process, from abrasion of rail or tie plates, from the crushing effect of wheel loads, and from rough handling, excessive respiking, and derailments. Antisplitting irons driven in tie ends reduce splitting. A considerable reduction in splitting has resulted from a quick drying process which can be combined with preservative treatment in the same chamber. Drying is accomplished in about 14 hours instead of the 6 months required for natural seasoning. Tie plates spread the load over a greater tie area and absorb abrasion from the rails. The plate itself may cause abrasion if not secured tightly to the tie by plate-holding spikes Screw spikes are frequently used for plate holding. Rubber or fiber pads between plate and tie reduce plate cutting and keep away moisture. Moisture greatly intensifies abrasion of the wood fibers. Cementing compounds are being developed to hold plate, pad, and tie in a watertight union. Additional and helpful waterproofing has resulted from spraying the tie with an asphaltic compound of the type used to waterproof ballast sections.

In 1953 the 5-year average renewal rate for the United States was 99 ties per mile or 3.0% of the total ties in track. This gives an average tie life in track of 33.3 years but includes light traffic lines. In Class A track, the life may be no more than 12–20 years. Ties cost $4–$6 in track. Figure 19 gives the annual replacements for ties of various average life.

11. RAILS

Manufacture. Rails are made from the steel of open-hearth furnaces and rolled to the desired cross section. The rolling process gives fine grain, strength, and toughness to the metal. Rails are hot-sawed to proper length and, for main-line use, are control-cooled to allow hydrogen gases to escape, thus preventing the formation of shatter cracks from which the dangerous transverse fissure defect develops. Rails from the hot saw are placed in cooling boxes at a temperature of 725–1000°F and kept there for at least 10 hours and until the temperature of the top layer has fallen to 300°F or less. Rail ends are now frequently end-hardened by heat-treating to reduce wear and batter. Some rails are heat-treated throughout for use in frogs, turnouts, and crossings. After cooling, rails are straightened, ends and edges ground free of burs from the rail saw, ends slotted or beveled in some cases, bolt holes drilled, and the rails are inspected, classified, and loaded for shipment.

Rail Designations. Rails are usually designated by weight, section, and length. The weight is expressed in pounds per yard of rail length. The section refers to a particular design of cross section and is usually identified by the initials of the agency creating the design.

Example: 132 lb RE rails weigh 132 lb per yard and were designed by the AREA.

Weights and sections are given in Table 10. Rails are further identified by the mill stamp or brand placed on the web in rolling. On one side is stamped the weight, section, rolling mill, country (U.S.A.), year, and month rolled. On the reverse is stamped the heat number, a letter designating the portion of ingot used for that rail, and the ingot number. Control-cooled rails bear the letters CC preceding the heat number, CH if control-cooled and end-hardened, B if Brunorized.

Specifications

Most rail is rolled to recommended specifications of the AREA and must pass the tests thereby established.[8]

Inspection. Inspection and tests are made at the works of the manufacturer before shipment. The works management must afford all reasonable facilities for determining the satisfactory quality of the rails accepted.

Chemical Composition. The chemical composition of the steel, determined as prescribed in the AREA specifications, shall be within the following limits.

	Weight, lb/yd		
Constituents, %	70–90	91–120	121 and Over
Carbon	0.55–0.77	0.67–0.80	0.69–0.82
Manganese	0.60–0.90	0.70–1.00	0.70–1.00
Phosphorus, not to exceed	0.04	0.04	0.04
Silicon	0.10–0.23	0.10–0.23	0.10–0.23

[8] *AREA Manual*, 1953 Ed., pp. 4-2-1 to 4-2-8.

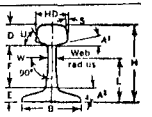
Table 10. Standard Tee Rail Sections

(Courtesy of Ramapo Ajax Division, Chicago, Illinois, American Brake Shoe Co.)

H—height
B—base
HD—head
W—web at thinnest point

D—depth of head
F—fishing
E—depth of base
A¹—head angle

A²—base angle
S—slope of head
L—see sketch
U—head-free rail head undercut
Dimensions in inches



Type	Wt per Yd	[illegible]	Beth. Steel Co.	Colo. F. & I Co	Inland Steel Co	H	B	HD	W	D	F	E	A¹	A²	S	L
A.R.A.-A.	100	[illegible]	100-RA	1003*	10020*	6	5½	2¾	9/16	1 9/16	3⅜	1 1/16	1 to 4	1 to 4	1/16 to 1	21 5/16
"	90	[illegible]	90-RA	902	9020	5⅝	5⅛	2 9/16	9/16	1 15/32	3 5/32	1	"	"	"	29½
"	80	[illegible]	80-RA			5⅛	4⅝	2½	33/64	1 7/16	2 23/32	31/32	"	"	"	29 1/16
A.R.A.-B.	100	[illegible]	100-RB	1002	10030	54 1/64	59/64	2 21/32	9/16	1 45/64	2 55/64	1 9/64	13°	13°	3°	26 5/128
"	90	[illegible]	90-RB	905*		51 7/64	44 9/64	2 9/16	9/16	1 39/64	2⅝	1 1/32	"	"	"	21 1/32
"	80	[illegible]*	80-RB*			41 5/16	47/16	2 7/16	35/64	1 15/32	2 15/32	1	"	"	"	21 5/64
A.R.E.A.	133	[illegible]	133-RE	1330	13331	7 1/16	6	3	1 1/16	1 15/16	3 15/16	1 3/16	1 to 3	1 to 4 011	4°	33/4
"	132	[illegible]	132-RE	1321	13225	7⅛	6	3	2 1/32	1¾	4 3/16	1 3/16	1 to 4	1 to 4	1 to 40	37/8
"	131	[illegible]	131-RE	1311	13128*	7⅛	6	3	2 1/32	1¾	4 3/16	1 3/16	1 to 4	1 to 4	1 to 40	4¼
"	130	*	130-RE	1300*	13025*	6¾	6	2 15/16	2 1/32	1 27/32	3 11/16	1 7/32	"	"	1/16 to 1	3⅜
"	115	[illegible]	115-RE	1150	11525	6⅝	5½	2 23/32	⅝	1 11/16	3 13/16	1⅛	1 to 4	1 to 4	1 to 40	3¼
"	112	[illegible]	112-RE	1121	11228*	6⅝	5½	2 23/32	19/32	1 11/16	3 13/16	1⅛	"	"	1 to 40	3¾
"	110	[illegible]	110-RE	1100	11025*	6¼	5½	2 25/32	19/32	1 23/32	3 13/32	1⅛	"	"	1/16 to 1	3⅛
"	100	[illegible]	100-RE	10025	10025	6	5⅜	2 11/16	9/16	1 21/32	3 9/32	1 1/16	"	"	"	23 1/32
A.S.C.E.	100	[illegible]	100-AS			5¾	5¾	2¾	9/16	1 45/64	3 5/64	31/32	13°	13°	Str.	26 5/128
"	90	[illegible]	90-AS		9040	5⅜	5⅜	2⅝	9/16	1 39/32	2 55/64	59/64	"	"	"	24 5/128
"	85	[illegible]	85-AS	851	8540	5 3/16	5 3/16	2 9/16	9/16	1 35/64	2¾	57/64	"	"	"	21 7/64
"	80	[illegible]	80-AS	800	8040	5	5	2½	35/64	1½	2⅝	⅞	"	"	"	23/16
"	75	[illegible]	75-AS	753		4 31/16	4 13/16	2 15/32	17/32	1 27/64	2 35/64	27/32	"	"	"	21 5/128
"	70	[illegible]	70-AS	701*		4⅝	4⅝	2 7/16	33/64	1 11/32	2 15/32	1 3/16	"	"	"	23/64
"	65	[illegible]	65-AS	653		4 7/16	4 7/16	2 13/32	½	1 9/32	2¾	25/32	"	"	"	13 1/32
"	60	[illegible]	60-AS*	603		4¼	4¼	2⅜	31/64	1 7/32	2 17/64	49/64	"	"	"	1 115/128
A. T. & S. Fe.	90	9021*	90-SF*	903*	9021*	5⅝	5 3/16	2 9/16	9/16	1 15/32	3 5/32	1	1 to 4	1 to 4	1/16 to 1	22 9/32
Can. Pac.	85	8524*	85-CP		8524*	5⅛	5	2½	9/16	1 7/16	2 11/16	1	"	"	1/32 to 1	21 1/32
C. & N.-W.	100	10035	100-OM		10035*	54 5/64	59/64	2 9/16	9/16	1 39/64	2 61/64	1 9/64	13°	13°	3°	27 9/128
"	90	9035	90-OM			51 7/32	5 3/32	2½	½	1 17/32	2 31/32	1 1/32	"	"	3/64 to ¾	23 3/64
"	72	7250*				4¾	4¾	2⅜	9/16	1 13/32	2½	27/32	14°	14°	Str.	23/32
C. B. & Q.	85	8543*	85-CB*	852*		5 3/16	5 3/16	2 21/32	9/16	1 35/64	2¾	57/64	13°	13°	5°	21 7/64
D. L. & W.	105	10533*	105-DL			6	5⅜	2¾	⅝	1 23/32	3¼	1 1/32	13°	13°	4°	—
"	101	10133*	101-DL*			5 7/16	5¾	2¾	⅝	1 23/32	2 11/16	1 1/32	"	"	"	2¾
D. & R. G.	90			906*		5½	5⅛	2 9/16	9/16	1⅝	2⅞	1	14°	14°	4°	—
"	85			850*		5¼	5¼	2½	9/16	1¾	2⅝	⅞	13°	13°	4°	Str.
Grt. Nor.	110	11036*	110-GN*		11036*	6½	5½	2¾	19/32	1⅝	3¾	1⅛	1 to 4	1 to 4	1/16 to 1	3¼
"	100	10036*	100-GN*			5¾	5	2¾	9/16	1⅝	3	1⅛	"	"	"	21 3/16
"	90	9024*	90-GN*		9024*	5⅜	5	2⅝	9/16	1 15/32	2⅞	1 1/32	13°	13°	"	25 3/64

Interb'gh	100	10005*	100-RT*			5¾	5¾	2⅞	9/16	14 5/64	3 5/64	3 1/32	13°	13°	8°	26 5/128
"	90	9050*	90-RT			5	5	2⅞	1 1/16	12 5/32	2 11/32	⅞	"	"	"	2 1/16
Lehigh Val.	136	13633*	136-LV			7	6½	2 15/16	2 1/32	1⅞	3⅞	1¼	"	"	4°	3½
"	110	11033*	110-LV*			6	5½	2⅞	1 9/32	1⅞	3 1/16	1 1/16	1 to 4	1 to 4	4°	2 15/16
Mo Pac.	85	8550*				5 7/32	5¼	2 15/32	7 5/128	1¾	2 39/64	5 5/64	13°	13°	2°-30′	Str.
"	75	7550*	75-MP*			4¾	4¾	2 9/16	9/16	1 7/16	2 15/32	2 7/32	"	"	Str	2 5/64
Dudley	127	12722	127-DY		12722	7	6¼	3	2 1/32	1 11/16	4 5/32	1 5/32	1 to 4	1 to 4	1/16 to 1	3⅞
"	115	11522				6½	5½	3	⅝	1 11/16	3¾	1 1/16	"	"	"	3⅜
		11523*														
"	105	10524	105-DY		10524	6	5½	3	⅝	1⅝	3 13/32	31/32	"	"	"	3⅛
"	90		90-DY*			5½	5	2 21/32	9/16	1½	3 1/32	31/32	"	"	"	2 31/64
"	80	8022*	80-DY*			5⅛	5	2 21/32	7/32	1½	2¼	⅞	"	"	"	2⅝
N. Y., N. H & H.	107	10734*	107-NH			6⅛	5½	2¾	1 9/32	1 23/32	3 11/32	1 1/16	13°	13°	4°	2 47/64
"	100	10034*	100-NH			6	5½	2¾	1 9/32	1 23/32	3 11/32	1 1/16	"	"	"	2 39/64
Pa	155	15531	155-PS			8	6¾	3	¾	2 1/16	4 21/32	1 9/32	18°-26′ 10″	14°	4°	4 1 1/16
"	152	15222*	152-PS			8	6¾	3	1 1/16	1 27/32	4⅞	1 9/32	14°	14°	1 to 40	5
"	140	14031	140-PS			7 5/16	6	3	¾	2 1/16	4 1/16	1 1/16	18° 26′-10″	14°	4°	4
"	130	13031	130-PS		13031*	6⅝	5½	3	1 1/16	2	3 13/32	1 7/32	18°	14°	Str	3¼
"	100	10031*	100-PS		10031*	5 11/16	5	2 43/64	9/16	1 13/16	2 25/32	1 3/32	15°	13°	3/32 to 1	2 31/64
"	100	10033*				5½	5½	2 13/16	⅝	1⅞	2 11/16	1 5/16	13°	13°	4°	2 9/32
"	85	8531*				5⅛	4⅝	2½	1 7/32	1 21/32	2 13/32	1	15°	13°	Str.	2 15/64
P. & R.	100	10032*	100-RG			5⅝	5⅜	2 21/32	9/16	1 45/64	2 55/64	1 1/16	13°	13°	3/64 to 1	2 63/128
Soo Line	85	8520*				5⅜	4⅞	2½	9/16	1 15/32	2 29/32	1	14°-02′ 11″	14° 02′ 11″	3°-34′-35″	2 23/32
U Pac.	90	9023*		901*		5¾	5⅜	2¾	1 7/32	1½	3⅜	⅞	13°	13°	Str	—
"	75	7524*	75-SP*	757*		4 15/16	4 7/16	2 7/16	3 3/64	1⅜	2⅝	15/16	1 to 4	1 to 4	1/16 to 1	2½
Misc	67	6733*				4½	4½	2 13/32	½	1⅝	2⅛	¾	13°	13°	1/16 to ¾	1 13/16
"	60	6051*				4¼	4 1/16	2 5/16	½	1 7/16	2⅛	1 1/16	14°	12°-50′	1/16 to 1	1¾
"	56	5651*				4¼	3 31/32	2 7/32	1 3/32	1 7/16	2⅛	1 1/16	14°	12° 50′	1/16 to 1	1¾
Head Free So Pac	132			1320		7 5/16	6	2 31/32	2 1/32	1 15/16	4 3/16	1 3/16	1 to 4 U = 60° 30′	1 to 4	1 to 40	4¼
" " R. E.	131	13127*				7¼	6	2 21/32	2 1/32	1⅞	4 3/16	1 3/16	1 to 4 U = 60°-30′	1 to 4	1 to 40	4¼
" " P S	130		130-PS T*			6 13/16	5½	3	2 1/32	2 3/16	3⅜	1 7/32	18° U = 58° 30′	14°	Str	3¼
" " R E.	130	13027*	130-RE-T		13027*	6 15/16	6	2 27/32	2 1/32	2 1/32	3 11/16	1 7/32	1 to 4 U = 61°	1 to 4	1/16 to 1	3⅜
" " So Pac.	113			1130		6 13/16	5½	2 11/16	1 9/32	1⅞	3 13/16	1⅛	1 to 4 U = 58°	1 to 4	1 to 40	3¾
" " R E	112	11227*	112 RE T*		11227HF*	6¾	5½	2 23/32	1 9/32	1 13/16	3 13/16	1⅛	1 to 4 U = 58°	1 to 4	1 to 40	3¾
" " R E	100					6 1/16	5⅜	2 39/64	9/16	1 23/32	3 9/32	1 1/16	1 to 4 U = 57°	1 to 4	1/16 to 1	2 31/32
" " R A	100		100 RA T			6 5/32	5½	2 11/16	9/16	1 23/32	3⅜	1 1/16	1 to 4 U = 49°	1 to 4	1/16 to 1	2 15/16
" " R A	90	9027*	90-RA T	TC 1013*		5 25/32	5⅛	2 31/64	9/16	1⅝	3 5/32	1	1 to 4 U = 54°	1 to 4	1/16 to 1	2 29/32
T. R	129	12929			12929	7 5/16	6	2⅝	2 1/32	1 27/32	4 9/32	1 3/16	1 to 4	1 to 4	1 to 40	3½
"	112	11229		1122		6¾	5½	2½	⅝	1¾	3⅞	1⅛	1 to 4 U = 77° 45′	1 to 4	1 to 40	3⅛
C T A	110	11050				7	6	2¾	9/16	1⅞	4 5/16	1 1/16	14°	9°	3° 34′-30″	—

* Not rolled at present.

Rail Classification and Assignments. The customary and AREA recommended rail classifications and assignments are as follows.

No. 1 Rail. Rails free from injurious defects and flaws of all kinds; for use in main tracks preferably on tangents The ends are unpainted.

No. 2 Rail. Rails with only minor surface imperfections not in sufficient number to render them unfit for recognized use are classified No. 2 and accepted up to 8% of the total order. Also classed as No. 2 rails are those reaching the straightening press with kinks or camber greater than 6 in. in 39 ft: also those that are not hot-stamped. These rails may be laid on tangents or on curves not exceeding 3°, preferably in freight tracks where speed does not exceed 50 mph. The ends are painted white and a figure 2 is stamped on both ends.

Hi-Carbon Rails. All heats with the carbon content in the upper five points of the mean carbon percentage are classed as Hi-Carbon rails and are painted blue on the ends. Assignments are on curves, through turnouts, or through hazardous locations or places difficult to renew on bridges or in tunnels.

A Rails. Those rails from the A position in the ingot are painted yellow on both ends and should not be laid on curves or in hazardous locations including tunnels and bridges or where inspections cannot be made.

X-Rayls. Rails which on fracture of a test specimen exhibit seams, laminations, cavities, interposed foreign matter, or a distinctly bright or fine-grained structure are classified as X-Rayls and painted brown on the ends. X-Rayls may only be laid in locations approved for No. 2 rails and in yards, sidings, and other safe, slow-speed locations.

Short Rails. Rails are rolled in 39-ft lengths. Shorter lengths down to 25 ft may constitute 11% of the entire order. The ends are painted green. Some 66-ft rails have been rolled for use in turnouts and crossings. The rolling of 78-ft rails is not uncommon and is being considered for recommended practice.

Rails of each classification must be loaded in separate cars for shipment from the mill.

Rail Design

The rail sections in common use today originated in the tee rail designed by Col. Robert T. Stevens, Chief Engineer of the Camden and Amboy Railroad in 1830. It was reinvented in England in 1836 by Charles Vignoles and is used in Europe and England as the Vignoles rail. In England, the old double-headed or bullhead rail which required chairs for its support on the tie was officially replaced by the tee rail in 1948.

Although some railroads design their own rails, most railroads have adopted AREA designs as standard. The first series of RE sections were issued in 1915. The latest revisions are those of 1946. The tendency is to increase the resistance to bending by using a heavier rail section (the moment of inertia varies as the square of the weight) and a higher section with a moment of inertia increasing as the cube of the height. Heavier, stiffer rail sections bring about a marked decrease in maintenance on the entire track structure.

Street Railways. Interurban and some city railways use the ordinary tee rail in weights varying from 60 to 110 lb per yard as recommended by the ASCE, ARA, AREA, and the American Transit Engineering Association. On paved streets, the rail head must not interfere with traffic. A special high web tee rail may be used in conjunction with a special paving block, providing a 2-in. flangeway or a groove may be formed in the concrete pavement on the gage side of the rail. A better system is the use of a grooved girder rail having a flangeway rolled into the rail head. Paving can then be set tightly on both sides of the rail.

Rail Life. Excluding defects, the service life of rails is a function of tonnage, axle loading, speed, curvature, gradient, and degree of maintenance. The limit of rail wear is usually taken as 25% of the head area and may be produced by 300–500 million gross tons of traffic. For curves over 7°, curve wear is the limiting factor and rails may carry only 100 million gross tons. On tangents and curves up to 4° direct wear or abrasion has no practical effect but joint and surface wear set in as early as the passage of 200 million tons. On the average, heavy traffic lines will have to renew rail every 10 years and light traffic lines every 25 years giving an overall average of 15 years. Rail removed from main-line track often finds additional service life in secondary lines, yards, and sidings.

Laying Rail

In laying rail for new construction, the track is usually built directly on the finished subgrade or subballast section. Ties are placed in approximate alignment, rails are spiked to the ties and to correct gage, ballast is unloaded and the track raised, surfaced, and lined. Rails, fastenings, and ties should be laid out in advance by truck or flatbed trailer.

In **old track,** rail renewals are made out of face. The track must be in good line and surface especially where heavy lining on curves is required. The re-lay should be followed at least by a light surfacing raise to protect the rail. Heavy tie renewals and ballasting may also follow the re-lay. Turnouts, railroad crossings, and highway crossings may be renewed separately from the re-lay, preferably beforehand. Rail and other materials should be laid out at the approximate spot to be used a few hours in advance of the re-lay. One side of the track is laid at a time. The actual re-lay procedures involve: (*a*) distributing materials; (*b*) unspiking, unjointing, and setting out old rail; (*c*) preparing the track by plugging spike holes, adzing the plate-bearing surface, swabbing it with creosote oil, and setting tie plates; (*d*) setting in the new rail (once performed by hand, now with a light on-track crane); (*e*) fastening the new rail, which involves applying joints, gaging every third or fourth tie, driving intermediate spikes, and applying rail anchors and signal-bond wires; (*f*) spotting up old joint locations; (*g*) cleaning up all old material. A new device permits gaging the plates as placed (step *c*), thus eliminating the gaging from step *e*.

It is customary to insert temporary expansion shims in the joints according to the temperature of the rail as indicated by a rail thermometer and as required by the AREA[9] (Table 11). Metal shims should be removed every 12 rails, and reused. Other types are left in track.

Table 11. Re-Lay Shims for Temperature Expansion

Rail Temperature, degrees F	Size of Shim, 135 joints per mile for 39-ft rails
0–25	1/4 in.
26–50	3/16 in.
51–75	1/8 in.
76–100	1/16 in.
Over 100	None

Continuous Welded Rail

Continuous welded rail is formed by welding standard-length 39-ft rails in continuous length. Three processes are used on railroads.

The **Thermit process** makes use of the production of molten Thermit metal from the chemical reaction between iron oxide and metallic aluminum: $FaO + Al_2 \rightarrow AlO + 2Fe$. The revised Thermit process makes a full fusion weld (the older process combined a fusion weld of web and base with a pressure weld of the head area). Rail ends, aligned by clamps and held 3/4 in. apart and inclosed in a two-part mold, are preheated by a kerosene jet. A crucible set above the joint in the mold contains Thermit powder which, upon ignition, proceeds with the foregoing reaction at a temperature of 5000°F. The crucible is then tapped, and molten steel flows into the mold. After 10 minutes of cooling, the mold is removed and the rail ground to a smooth contour. A foreman and 20–25 trackmen can produce 30–40 welds in an 8-hour day. This process has had extensive use in street-railway work since early in the century. The process is simple and reliable, and the equipment is portable.

Electric flash welding depends on electrical heating for fusion and welding of rail ends. The rails to be joined are gripped in current contact pads and brought to fusing temperature by intermittent short-circuiting contact. The ends are then brought into continuous short-circuiting contact; the metal becomes molten and flashes out, removing rough spots and metal from the rail faces. Rail ends are squeezed together under pressure, fusing the ends in an upset arc which must be ground to proper contour. Stresses from high heat concentrations are relieved by heating in an oil-fired oven and cooling at atmospheric conditions. The process is reliable and produces 17–20 welds per hour. Equipment is cumbersome and expensive.

Oxyacetylene process forms a weld below the melting point of the steel by diffusion and recrystallization under pressure. Held in gripper blocks, the rail ends are forced together cold under a pressure of 2500 lb/sq in. while subject to flame from four blocks of oxyacetylene tips. Under heat and pressure, the metal upsets and shortens each rail 3/8 in., pressure is released, flames are turned off, and the rail cooled for 2 minutes before normalizing with welding tip flames and grinding to correct contour. The process produces 20–30 high-quality welds per day with a 12-man crew. Most rail welding today is being done with this system.

Other Processes. An earlier electric weld known as the Jacobs process was used by the Larkin Steel Co. on street railways. The bar-weld process also used by the Larkin Steel Co. for street railways welds two plates 18 in. long, 3 in. wide, and 1 in. thick to the

[9] *AREA Manual*, 1953 Ed., p. 5-5-4.

webs of the rails. Raised bossed surfaces on the plates are pressed against the web with pressures up to 35 tons while a current of 25,000 amperes taken from the trolley wire is run through the joint.

The only **limitation** on **length** of continuous welded rail is that imposed by the location of insulated joints and of crossings and turnouts. Continuous welded rail is still a subject of test and study and has not yet been recommended by the AREA for use in high-speed main-line territory.

Restraint of Continuous Welded Rail.[10] To overcome the natural expansion and contraction in welded rail due to temperature changes, the rail must be restrained by rail anchors, spring clips, tie-plate shoulders, and joints at the ends of the section. Using the stress-strain relation and a modulus of elasticity for rail steel of 30,000,000 lb/sq in., an internal stress or restraining force of 195 lb/sq in. must be established per square inch of cross-sectional area per degree change in temperature. The amount of rail movement that will occur at the ends of the long rail before a full restraining force has been established by tie restraint alone is

$$M = NTL/2AE$$

where N = number of ties required to produce full restraint.
T = average tie resistance in pounds per tie per rail (tie spacing and restraints considered uniform) and equal to 500–800 lb per tie spaced 22 in. in stone ballast (and twice as much when the ballast is frozen).
A = cross-sectional area of tie in square inches.
E = modulus of elasticity of rail steel = 30,000,000.
L = length of rail showing movement = Ns where s is the center-to-center tie spacing in inches.

If joint restraint is considered, tie restraint can be less and

$$M = \frac{(F - J)^2 s}{2TAE}$$

where J = joint restraint varying from 5000 lb for loose joints to 100,000 lb for a tight 6-hole bar. Because it is non-uniform and difficult to determine, joint restraint is usually omitted and used only as a factor of safety.

Economics of Continuous Welded Rail. Since rail joints consume as much as 45% of track labor, a saving is accomplished by their elimination. Joint assemblies cost $5 to $8 and are usually renewed twice during the life of rail. A welded joint costs $10 to $12 in track but lasts as long as the rail. Labor savings of 30% have been reported on actual installations.

12. TRACK FASTENINGS

Rail Anchors

The longitudinal running or creepage of rails is the result of rail flexure under a moving wheel load, of expansion and contraction due to temperature changes, and of other causes not fully understood. Rail anchors, of great variety, usually depend on attaching the

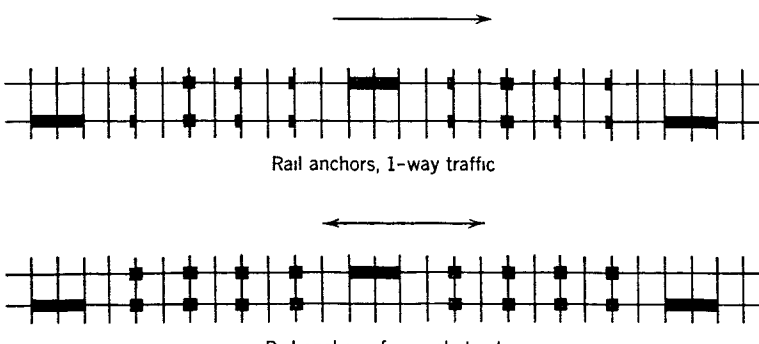


FIG. 20 Rail anchor placement. (*AREA Manual*, 1953 Ed., p. 5-5-5.)

[10] *Proceedings of the AREA*, 1937, Vol. 38, pp. 493–498.

anchor to the base of rail by spring force and friction and securing the assembly by a bearing between the anchor and tie. The AREA recommends that a minimum of 8 anchors spaced uniformly per 39-ft rail be used for one direction of traffic with two additional back-up anchors near the quarter points for occasional movements against traffic, Fig. 20. With traffic in both directions, 16 per 39-ft rail, 8 in each direction, are recommended. If traffic is lighter in one direction, the number of anchors may be changed accordingly.

Rail Joints

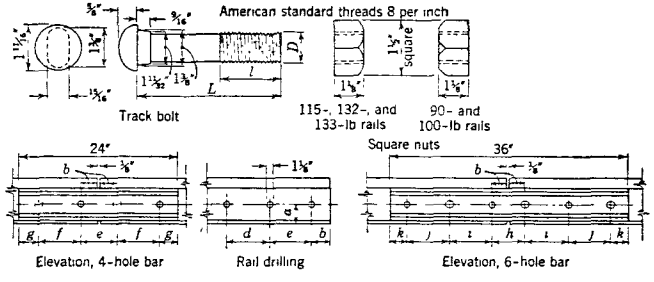
Rail joints are made necessary by the discontinuity in rail length. The ideal joint bar must preserve insofar as possible the continuity of the rail by providing the same strength, stiffness, and flexibility as the rail itself. Rails must be held to the same horizontal and vertical alignment forming through the joint a continuous girder distributing the load applied at the joint over several ties.

A variety of joint-bar types have been developed. Of these the **angle-bar design** was successful and is still used even with modern rail sections. It offers a greater moment of inertia than the strap type of bar for small rails with limited space between the base and head. Good results are being had from angle bars with short toes.

Recent preference is shown to bars with almost 100% symmetry. Bending and deflection are both restricted giving greater resistance to flexure and lateral bending and with lower bolt tensions. The conventional contact area between bar and rail head is the inclined bearing space under the head. The **head-free bar** is loaded through the fillet curve under the head increasing the area of contact and permitting a heavier load on the bar.

Length. Early joint bars drilled for 4 bolts were 18–24 in. long. Bars up to 40 in. with 6 holes have been used. Short bars have been found adequate to develop resisting moments but tests indicate a substantial increase in service life of 36-in. bars as well as a smoother joint surface and prolonged life of rail.

Table 12. Recommended Rail Drillings, Bar Punchings, and Track Bolts *



	Dimensions, in.				
	90 lb, RA-A	100 lb, RE	115 lb, RE	132 lb, RE	133 lb, RE
Rail drillings					
a	2 37/64	2 45/64	2 7/8	3 3/32	3
b	2 11/16	2 11/16	3 1/2	3 1/2	3 1/2
c	5 1/2	5 1/2	6	6	6
d	5 1/2	5 1/2	6	6	6
Bar punchings:					
24-in. 4-hole bar					
e	5 1/2	5 1/2	7 1/8	7 1/8	7 1/8
f	5 1/2	5 1/2	6	6	6
g	3 3/4	3 3/4	2 7/16	2 7/16	2 7/16
36-in. 6-hole bar					
h	5 1/2	5 1/2	7 1/8	7 1/8	7 1/8
i	5 1/2	5 1/2	6	6	6
j	5 1/2	5 1/2	6	6	6
k	4 1/4	4 1/4	2 7/16	2 7/16	2 7/16
Track bolt					
D	1	1	1	1	1
L	5 1/4	5 1/2	5 1/2	5 3/4	5 3/4
l	2	2	2 1/4	2 1/4	2 1/4

* After *AREA Manual*, 1953 Ed., pp. 4-1-14 through 4-1-19.

The AREA recommended specifications, for high-carbon steel joint bars, are found in the *AREA Manual*, pp. 4-2-9 to 4-2-11; also, for quenched carbon steel joint bars, pp. 4-2-12 to 4-2-14. Both types are for general use in standard railroad tracks.

The breakage resulting from stress concentrations at the corners of **spiking slots** has led to the discontinuance of those slots on many roads. Rough spots on rail ends due to hot sawing gouge the bars and develop cracks therein. Any unevenness in the fishing area leads to stress concentrations and cracks. Better millwork and grinding of rail ends and fishing spaces is necessary to reduce joint failures.

Wear. The joint bar acts as a beam distributing applied load over several ties. The unit intensity of pressure will be greatest under the rail-head ends where the load is applied and maximum wear on both rail and joint will occur there. Less severe wear also occurs at the bottom ends of the bars. Initial wear is taken up by bolt tightening. Excess wear causes cracked joints and leads to rough track. Excessive wear of $^1/_{64}$ in. to $^1/_{16}$ in. can be improved by inserting metal shims between the rail and bar. The worn space on the bar may be rebuilt by welding and grinding or by reforming the bar after heating. The welding or reforming should include a small amount of overfill to compensate for wear of the bearing surface under the rail head.

Track bolts for rail joints use the U.S. Standard coarse thread and are made from heat-treated carbon steel. Some roads specify alloy steels. Recommended specifications are given in pp. 4-2-15 to 4-2-21 of the *AREA Manual*. Ultimate tensile strength must be within the limits of 100,000 to 115,000 lb/sq in. Diameters in use are: $^3/_4$ in. for 60- to 70-lb rail, 1 in. for 70- to 100-lb rail, and 1 in. or 1 $^1/_8$ in. for heavier than 100-lb rail. Smaller sizes are available for very light rail. Bolts are shipped in 200-lb kegs, the size of bolt determining the number per keg.

Bolt tension must be maintained to draw the bars into the fishing spaces and affect a close fit. Initial bolt tension is lost under traffic. The AREA recommends an initial bolt tension of 20,000–30,000 lb/sq in. to overcome the roughness of fishing spaces and provide a proper seating.[11] Bolts should then be retightened 1–3 months after application and at 1-year intervals thereafter with 15,000 to 20,000 lb/sq in. tension per bolt.

Joint lubrication is necessary to prevent scale, corrosion, and dirt from causing bars to "freeze" to the rails preventing movement as the rails expand and contract. Rail ends should be cleaned with a flame or by grinding and painted or sprayed with a lubricating compound. On the fishing spaces and web, heavy oil or graphite grease give temporary lubrication. Preformed joint packings are available.

Offset or compromise bars compensate for differences in height, base, head width, etc., when joining rails of dissimilar weights or sections. Weight and section of both rails must be specified when ordering as well as whether a right- or left-hand joint is required. When facing an offset bar from inside the track, the side on which the higher or heavier rail occurs defines its right- or left-hand designation.

Insulated joints are used where adjacent track circuits flowing through the rails must be isolated from each other. Fiber inserts impregnated with an insulating compound are placed between the rails and the bars. Bolts are inserted through fiber bushings and a fiber plate is placed back of metal plate washers. A fiber end post is placed between the rail ends.

Nutlocks maintain track-bolt tension between tightenings. Recommended specifications for a single spring washer are found in pp. 4-2-19 to 4-2-21 of the *AREA Manual*. Many roads find nutlocks to be ineffective and do not use them. The AREA has not made a recommendation as to the use of nutlocks.

Spikes

It has been demonstrated that of all the various forms of driven spikes which have been proposed, the plain spike of uniform section has the greatest holding power. The holding power is increased by a cutting edge at the lower end which cuts rather than crushes the fibers, and by a beveled wedge whose length is about twice the width of the spike rather than a short blunt point. The sizes of spikes recommended for different weights of rail are given in Table 13.

The specifications of the AREA require the steel to be made by the Bessemer or open-hearth process. The required properties are:

Ultimate strength, not less than 55,000 lb/sq in.
Elastic limit, not less than 50% of ultimate strength.
Elongation, not less than 25% in 2 in.

[11] *AREA Manual*, 1953 Ed., p. 5-5-6.

Table 13. Railroad Spikes

Size Measured under Head, in.	Average Number, per keg of 200 lb	Ties 24 in. between Centers, 4 Spikes per Tie, quantity per mile		Suitable Rail, lb per yd
		Pounds	Kegs	
6 1/2 × 5/8	230	11,360	56.8 *	110 and heavier
6 × 5/8	250	10,880	54 4	120 to 150
5 1/2 × 5/8	275	7,680	38 40	85 to 110
5 1/2 × 9/16	340	6,220	31.10	45 to 90
5 × 9/16	360	5,840	29.40	40 to 56
5 × 1/2	495	4,260	21.30	40
4 1/2 × 1/2	540	3,920	19.60	35
4 × 1/2	600	3,520	17.60	30
4 1/2 × 7/16	655	3,220	16.10	25 to 30

* Based on 19.4 in. tie spacing center to center.

The driving of spikes should be by blows in line with the spike. Eccentric or glancing blows tend to enlarge the spike hole and reduce the holding power of the spike. The spikes should not be driven directly opposite each other, but should be staggered. The staggering should be reversed for the two rails on one tie; that is, the spikes on the inner side of the two rails should be near the same side of the tie to prevent slewing of the tie.

Screw Spikes. The advantages of screw spikes are: (1) The increase in the life of the tie by the avoidance of spike killing; (2) The decreased cost of maintenance (although this is denied), and (3) their greater holding power. It is said, as an objection to their use, that when they are rusted or the head broken off by accident, it is difficult or impossible to extract the stump. One strong objection is the entire loss of holding power when the wood fiber between the threads becomes worn as it does under the action of the extremely heavy traffic. The screws cost considerably more than common spikes, and require more work to put them in place. The screwing must be done with a hand or power-driven wrench. The auger hole should have the same diameter as that of the screw at the base of the thread, or perhaps 1/16 in. larger. Screw spikes find use today as plate-holding spikes. They are extensively used in Europe for rail-holding purposes. The AREA specifications require them to be made of open-hearth steel, having not more than 0.05% of either phosphorus or sulfur. The minimum allowable figures for the finished spike are ultimate strength 60,000 lb, elastic limit 50% of ultimate strength, elongation 22% in 2 in., reduction of area 40%. The material must be capable of being bent 180° and hammered down flat and the finished spike bent 90° without sign of fracture. Various forms of spring clips and other hold-down devices are available but are not in general use.

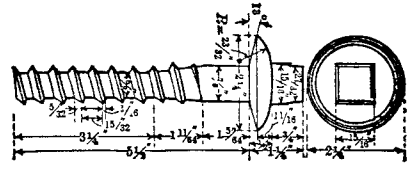


FIG 21 Screw spike

Tie plugs of plain or preferably creosoted wood are used to fill holes from which spikes have been withdrawn. The plug keeps out moisture and insures a better fit of the spike if redriven in or near the same hole.

Tie Plates

Tie plates absorb rail abrasion and reduce the unit pressure of the rail on the tie thus reducing cutting the tie. Better track gage is maintained. Double shoulders reduce skewing tendencies of the plate and promote better line and surface. Corrugations on plate bottoms aid in gripping the ties but the cutting of wood fibers overcomes the other advantages and flat-bottomed plates are preferred. Plate edges should be beveled to avoid cutting the base of rail. The AREA recommends a cant of 1:40 on the bearing surface to tilt the rail inward and more nearly in full contact with the coned treads of worn wheels. To offset the strong outward thrusts which accompany vertical loadings and cause plate cutting of ties, the outer end of the plate is made longer than the inner. One-half of the excess length, as measured from the center line of the bearing surface, is known as the eccentricity. The AREA recommends an eccentricity of 3/8 in. for use with 115 and 132- to 133-lb rails.

Based on a crushing load of 400 lb/sq in. for hardwood ties and 250 lb/sq in. for softwoods and dynamic loadings of 40,000 lb on the inner rails of curves and 25,000 lb on tangents, 100 sq in. of plate area are needed to protect hardwoods on curves and softwoods on

Table 14. Tie-Plate Dimensions, in.*

AREA Plan No.	Weight and Section	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*k*	*l*	*m*	Service	Weight, lb
13	14 3/4″ plate—RE 6″ base/rail 130, 131, 132, 133#	4 13/16	6 1/16	3 7/8	6	3/8	25/32	15/16	7/16	3 9/16	14 3/4	2 5/8	Extraheavy	23 32
12	14″ plate—RE 6″ base/rail 130, 131, 132, 133#	4 3/8	6 1/16	3 7/8	6	7/16	23/32	7/8	7/16	2 15/16	14	2 15/16	Heavy	21.47
11	13″ plate—RE 6″ base/rail 130, 131, 132, 133#	3 7/8	6 1/16	3 1/16	6	3/8	21/32	13/16	7/16	2 5/8	13	2 1/16	Medium	18.41
10	13″ plate—RE 6″ base/rail 130, 131, 132, 133#	Flat ends 3 3/4	6 1/16	3 3/16	6	1/2	5/8	25/32	7/16	2 5/8	13	2 1/16	Medium	17.87
	12″ plate—RE 6″ base/rail 130, 131, 132, 133#	Note 7 holes. Inside plate-holding spike hole centered. 3 3/8	6 1/16	2 9/16	6	5/16	9/16	23/32	3/8	2 3/8	12	1 9/16	Light	14.88
	14″ plate—RE 5 1/2″ base/rail 110, 112, 115#	4 5/8	5 9/16	3 13/16	5 1/2	7/16	13/16	15/16	7/16	3 3/8	14	2 9/16	Extraheavy	22.90
	13″ plate—RE 5 1/2″ base/rail 110, 112, 115#	4 1/8	5 9/16	3 5/16	5 1/2	7/16	23/32	27/32	7/16	3	13	2 3/16	Heavy	19.60
6	12″ plate—RE 5 1/2″ base/rail 110, 112, 115#	3 5/8	5 9/16	2 13/16	5 1/2	3/8	5/8	3/4	3/8	2 3/8	12	1 13/16	Medium	15.94
5	12″ plate—RE 5 1/2″ base/rail 110, 112, 115#	Flat ends 3 1/2	5 9/16	2 15/16	5 1/2	1/2	5/8	3/4	7/16	2 3/8	12	1 13/16	Medium	16.25
4	11″ plate—RE 5 1/2″ base/rail 110, 112, 115#	Note 7 holes. Inside plate-holding spike hole centered. 3 1/8	5 9/16	2 5/16	5 1/2	5/16	9/16	11/16	3/8	2 1/4	11	1 7/16	Light	13.39
3	12″ plate—RE 5 3/8″ base/rail 100#	3 1/2	5 7/16	3 1/16	5 3/8	3/8	5/8	3/4	3/8	2 3/8	12	1 15/16	Medium	15.86

* As recommended by the *AREA Manual*, 1953 Ed., pp. 5-1-7 to 5-1-22.

tangents, and 63 sq in. to protect hardwoods on tangents. Plate width is limited to slightly less than width of tie. The more important AREA plate designs and recommended assignments appear in Fig. 22 and Table 14.

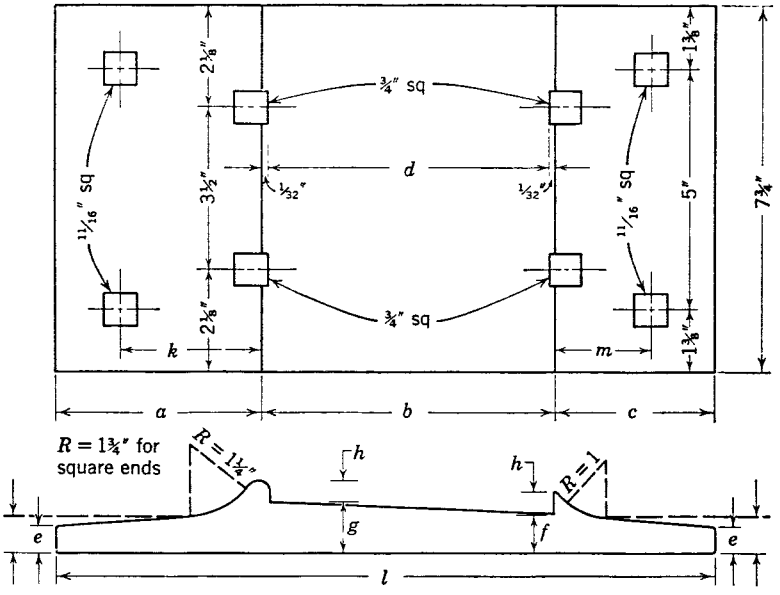


FIG 22 Tie plate dimensions *Note*· Round holes may be specified for use with screw spikes for plate-holding purposes. May also specify 4- or 6-hole punching.

Rail Bonds

Rail bonds are used to insure the continuity of signal and traction current return circuits through rail joints. The older-style signal bond was one or two cables, single wire or stranded, with a tapered soft metal plug cast or pressed into each end. The plugs were inserted in holes about 3 in. beyond each end of the joint bars and driven home. To reduce theft and tripping hazards, some roads placed the wires between the joint bars and web of rail. Recent designs are small twisted copper cables about 6 in. long Holes are drilled in the heads of the rails about 2 in. from the ends and the small plugs attached to the cables are driven home.

Heavier bonds are usually required for **traction bonding.** These may be attached by mechanical pressure to the head or web, or by soldering, or brazing. Traction bonds consisting of two strands of heavy braided copper cable inserted in welding heads are brazed or welded to the rail head. The rail-head surface is first prepared by grinding it smooth and clean.

Other Track Devices

Derails protect traffic on one track from interfering with movements on another and derail trains in preference to running into open movable span bridges. Railroad crossings use derails only when there is a definite preponderance of movement over one line Sidings have been so protected but the practice is falling into distavor. Storage tracks and industrial spurs leading from main or running tracks are accepted locations. Derails may be of the split switch, the lift, or of the frog type. In all cases the derail must be set so as to throw the car or train to be derailed away from the protected track.

Bumping Blocks and Car Stops. Car stops are used to prevent cars from rolling off the ends of stub tracks Bumping posts engage the coupler or buffer of rolling equipment and are designed to stop movement without regard to car damage Mounds of earth or massive concrete blocks have been used Other types are made a part of the track structure and transmit the shock to the ground through timber blocks or steel frames. The height of block which receives the impact is set 2 ft 9 in above top of rail (2 ft 11 in. where pas-

senger equipment only is to be used). For freight cars, the block should be set at the center of impact which is slightly left of center going away from the end of track.

Wheel stops engage the car wheels rather than the coupler or buffer. The rigid type is bolted or clamped to the rail and bears against the tie. A friction type allows the wheel tread to slide along on the rail. Car stops are for much lighter service than bumper blocks. Neither is a substitute for careful switching procedure.

13. TRACK

Gage

Standard gage for tangents and curves of less than 8° is 4 ft 8 $^1/_2$ in., as measured $^5/_8$ in. below the top of rail. At least one road has adopted 4 ft 8 $^3/_8$ in. in high-speed tangent territory to reduce nosing action (lateral motion). For curves over 8°, gage is widened when necessary to prevent locomotive wheels from binding in going around the curve. The practice varies. One road widens $^1/_8$ in. per additional degree over 6° to a maximum of 4 ft 9 $^1/_8$ in. Gage widening due to wear should never exceed 4 ft 9 $^1/_4$ in. Standard gage is in general use in the United States, Canada, Mexico, Britain, Europe, and parts of Asia and Africa. Narrow gage (36-in., meter, 42-in. and 22-in. gage) is used in many foreign countries and on a few common carrier and mining and logging roads in the United States. Broad gages of 5 ft, 5 ft 3 in., and 6 ft are used in India, Russia, Portugal, and Spain.

Curvature

A railroad route is composed of a series of curves connected by straight track or tangents (to the curves). Computation of horizontal curves for alignment and vertical curves for the profile will be found in Sect. 2, Surveying, and will not be discussed further here.

Railroad Spirals. The general principles of spiral or easement curves will also be found in Sect. 2. These curves have the dual purpose of providing an easy riding approach to full curvature and of introducing superelevation in the outside rail at a rate corresponding to the rate of increase in curvature from zero at the point of spiral to full elevation at the point of full curvature. The theoretical value for superelevation is

$$e = \frac{lv^2}{gR}$$

where l = horizontal projection of distance between bearing points of wheels on rails = 4.7′ for standard gage.
v = speed in fps.
R = radius of curvature = $5730/D$ approximately.
g = acceleration due to gravity.

In railroad units, $e = 0.00066DV^2$, where e is in inches of superelevation, D = degree of curve, and V = speed in mph.[12] Excessive superelevation causes rapid wear and crushing on the low rail. Too little superelevation causes excessive wear on the gage corner of the high rail and may in extreme cases cause a derailment where the overturning speed $= \frac{170.7}{D}$ for equipment with a center of gravity 7 ft above the rails. Experience indicates an unbalanced superelevation of 3 in. as both safe and comfortable. Thus a track may be superelevated to provide equilibrium conditions for a slow-moving freight train with passenger trains operating at a speed corresponding to an equilibrium elevation 3 in. higher. When this alone does not suffice for both fast and slow speed, the track will be given excessive superelevation for slow trains or the fast trains will have speed restrictions imposed on the curves depending on which service predominates. The AREA recommends a maximum superelevation of 6–7 in. but many roads consider this excessive and set an even lower maximum.

The comfort and smoothness of spiral operation depend upon the rate at which it runs in. The AREA recommends nothing faster than 1 $^1/_4$ in. per second. Some roads have used 1 $^1/_6$ in. per second in high-speed territory, and several for 85–100 mph use 1 $^1/_8$ in. per sec. The length of spiral in feet thus determined is $L_s = CeV$, where e is the superelevation in inches, V is the speed in mph, and C is a coefficient based on the rate of run-in

[12] *AREA Manual*, 1953 Ed., pp. 5-3-1 to 5-3-9. A recommendation is being made to the AREA that a value of l = 5.0′ more closely represents the distance between bearing points of wheels on the rails. The equation for superelevation thus becomes $e = 0.00070DV^2$.

Table 15. Superelevation of Outer Rail on Curves

e = elevation of outer rail in inches

	1/2		1		1 1/2		2		2 1/2		3		3 1/2		4		4 1/2		5		5 1/2		6	
	Speed in mph to nearest 5 mph																							
D	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc	Ve	Vc
0° 30′	40	70−	55	110	65	95−	75	105	85		95		100		110									
1° 00′	30−	60−	40−	70	45	80−	55	85	60	90	65	95	70	100−	75	105−	80	105+	85	110	90	110	95	115−
1° 30′	25	50+	30+	60+	40−	65+	45	70	50	75	55	80	60	80	65+	85−	65+	85+	70	90	75	95−	80−	95
2° 00′	20	50−	25	55+	35−	60−	40+	60+	45−	65	50−	65+	50+	70	55	75−	60−	75	60+	80−	65	80	65	85−
2° 30′	20−	45	25	55	30	55	40	55	40	60	45	60	45	60+	55−	65	55	65+	55	70	60	70	60	70+
3° 00′	15+	40+	20+	45	30−	50−	30+	50	35	55	40−	55	40+	55	45	60−	50−	60+	50	65−	55−	65+	55	65
3° 30′	15	40−	20	40+	25	45−	30−	45+	35−	50−	35	50	40−	55	40+	55	45−	55+	45+	60−	50−	60+	50+	60+
4° 00′	15−	35+	20	40−	25−	40+	30−	45	30	45+	35−	50−	35+	50	40−	50+	40+	55−	45−	55	45+	55+	50−	60−
4° 30′	15−	35−	20−	35	20+	40−	25+	40+	30−	45	30+	45	35−	45+	35+	50−	40−	50	40+	50+	45−	55−	45	55
5° 00′	10+	35−	15+	35	20+	35+	25	40−	30−	40+	30	45−	35−	45−	35	45+	35+	50−	40−	50−	40+	50+	45−	50−
5° 30′	10+	30+	15+	35−	20	35+	25−	35+	25+	40−	30−	40+	30+	40+	35−	45−	35	45	35+	45+	40−	45	40+	50
6° 00′	10+	30	15+	30+	20−	35	20+	35+	25	35+	30−	40−	30	40	30+	40+	35−	45−	35	45	35+	45−	40−	50−
7° 00′	10	30−	15	30−	20−	30+	20	35−	25−	35	25	35+	30−	40−	30−	40−	30+	40	35−	40+	35	45−	35+	45−
8° 00′	10	25+	15−	30−	15+	30−	20−	30+	20+	30+	25−	35−	25+	35	30−	35+	30−	40−	30+	40−	30+	40	35−	40+
9° 00′	10−	25−	10+	25+	15+	30−	20−	30−	20	30	20+	30+	25−	35−	25+	35−	30−	35+	30−	35+	30+	40−	30+	40−
10° 00′	10−	25−	10+	10+	15+	25+	15+	30−	20−	30−	20+	30	25−	30+	25	35−	25+	35−	30−	35	30−	35+	30−	35+
11° 00′	10−	20+	10	10+	15	25	15+	25+	20−	30−	20	30−	20+	30	25−	30+	25	30+	25	35−	30−	35	25+	35
12° 00′	10−	20−	10−	10	15−	25−	15−	25	15+	25	20	25	15+	20	20	25	20+	30−	20+	30+	25+	35−	25−	35−

Ve = equilibrium speed where $e = 0.00066DVe^2$.
Vc = comfortable speed where $e = 0.00066DVc^2 - 3$.
± = Rounded off from an exact value one or two mph higher (+) or lower (−).

having values of 1.17 for 1 1/4 in per sec, 1.26 for 1 1/6 in. per sec, and 1.301 for 1 1/8 in. per sec

Vertical curves, used to connect all changes in gradient, are also explained in Sect. 2. The AREA recommends (p. 5-3-13 of the 1953 manual) that, for high-speed main tracks, the rate of change in gradient along the curve should not be more than 0.05 ft per 100 ft in sags and not more than 0.10 ft per 100 ft over summits On secondary lines, the rates may be twice those for primary lines. Tracks of lesser importance may have even greater rates of change but the rate should be kept to a minimum.

Limiting effects of curvature are most pronounced in the limitations imposed on train speeds as indicated by the superelevation tables and restrictions on its maximum values.

(*a*) *Length of rigid wheelbase* of car or locomotive is given by the AREA formula

$$R = \frac{6ab}{p}$$

where R = radius in feet of maximum curve that can be negotiated safely.
a = distance in feet from center pin of lead truck to center of lead driver in feet.
b = distance in feet from center of lead flanged driver to center of rear flanged driver.
p = side play in inches assumed as 5/8 in.

When using a swing motion truck

$$R = \frac{6ab}{p + \frac{sb}{a + b}}$$

where s = side motion in inches, usually 1 1/2 in. See p. 6-34 for a discussion on widening the gage on curves.

(*b*) *Coupling difficulties* arise from poor alignment of couplers, especially of passenger equipment, on curves of 6° and over.

(*c*) *Clearance.* Additional clearance, both vertical and horizontal, must be provided on curves in the vicinity of bridges, tunnels, buildings, platforms, and adjacent tracks. The AREA recommends that clearance for overhang on curves be based on a car 85 ft long with 60-ft truck centers. For vertical clearance, allowance must be made for superelevation, a car height of 14 ft, and rail height of 8 in. The car plan and cross section may be plotted on coordinate paper (with the cross section oriented with respect to the superelevated plane of the rails) and necessary allowances—actual plus 6 in. for tolerance—read from the coordinate scale.

Track Elasticity

Since ties are closely spaced in modern track and elastically supported, the rail may be assumed without undue error to be a continuously and elastically supported beam with applied loads distributed over two or more ties, depending on the flexibility of the beam and its supports. The ratio of applied load to track depression is related through a factor u termed the modulus of elasticity of rail support, or simply the track modulus. It is the

Table 16. Values of Track Modulus u *

Rail	Ties, spaced 22″ c.c.	Track and Ballast	u
1. 90 lb	None used	Concrete roadbed	8000
2. 110 lb RE	7″ × 9″ × 8′ 0″ GEO fastenings	Limestone ballast, wide, stable roadbed	5100
3. 110 lb RE	7″ × 9″ × 8′ 0″ GEO fastenings	Flint gravel ballast, wide, stable roadbed	2900
4. 130 lb RE	7″ × 9″ × 8′ 6″	24″ gravel ballast plus 8″ of heavy limestone on well-compacted roadbed	2800
5. 85 lb RE	7″ × 9″ × 8′ 6″	24″ crushed limestone on loam and clay	1200
6. 85 lb	6″ × 8″ × 8′ 0″	6″ crushed limestone on loam and clay roadbed after tamping	1080
7. 85 lb	6″ × 8″ × 8′ 0″	6″ crushed limestone on loam and clay roadbed before tamping	970
8. 85 lb	7″ × 9″ × 8′ 6″	6″ cinder ballast on loam and clay subgrade	750
9 85 lb	7″ × 9″ × 8′ 6″	6″ fine cinder ballast, poor condition, loam and clay subgrade	530

* Dr. A. N. Talbot, Chairman Special Committee to Report on Stresses in Railroad Track, First Progress Report, *Proceedings of the AREA*, 1918, Vol. 19.

pressure per unit length of each running rail (the wheel load) required to depress the track one unit; or it is the load on one rail required to depress a tie one unit divided by the wheel spacing. The modulus, a measure of elasticity or stiffness of support, is independent of the rail stiffness but varies with spacing and stiffness of ties, type, depth, and compaction of ballast, quality of surface and tamping, and stability of the subgrade.

Track depressions and upward pressures result from the applied wheel load P so that $p = uy$. On the master diagram of Fig. 23 and the equations shown thereon, p_0 is the upward pressure per unit length at the point of wheel load where $X = 0$, y is the rail depression at any point x, and y_0 is the rail depression at the point of wheel loading. X_1 is the distance from wheel load to point of zero upward bending and $3X_1$ the distance from the wheel load to a point of zero upward pressure. The values of bending moment M, depression y, and reaction p developed at any point by a single wheel load may be determined from maximum values from the master diagram of Fig. 23. Where wheels are closely

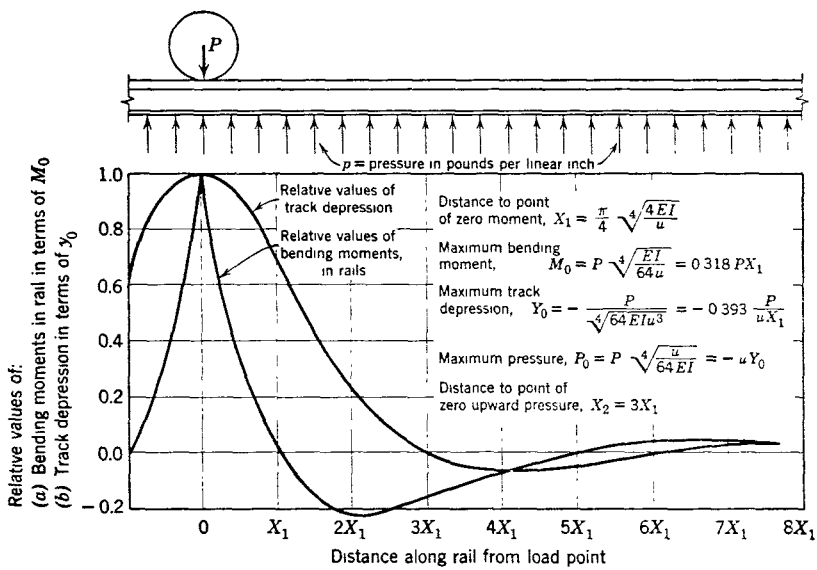


FIG. 23 Master diagram. I = moment of inertia of rail, E = modulus of elasticity of rail steel = 30,000,000, u = track modulus

spaced as in a car or engine truck, the combined effect is obtained through the principles of superposition by taking the algebraic sum of the effects of individual loads. Reasonably good agreement is had between computed and observed values of rail deflection and bending moments.

Track Maintenance

In addition to maintaining individual units of the track structure, close attention must be given to securing and maintaining correct gage, good line, and smooth surface.

Gage. The railroad usually specifies a gage rail (the north or the west rail, for example), and the other rail is gaged to it regardless of effect on alignment. The track can later be properly aligned. Wide gage is likely to occur where decayed ties provide insufficient holding power for spikes. In regaging the cut surface of the tie must be smoothed by hand or machine adzing to provide an even bearing for the plate.

Line is detailed adherence to the established alignment. The railroad designates one rail or the other as the line rail. On curves, the high rail is always the line rail. Local irregularities are removed according to eye, but stakes should be set by transit every 200–400 ft on tangents and every 31–50 ft by transit or by stringline on curves. Even when stringlining, spirals should be established by transit. The transit, set well back from the point of tangency, is "wiggled in" on a true tangent line. Initial throws are recorded to place the existing "tangent" on true tangent. Tangent offsets are then recorded for the curve, those inside the line of sight being negative, and those outside the

line of sight positive. Tangent offsets are then reduced to midordinates for stringline computation by the formula [13]

$$M_n = O_{n-1} + O_{n+1} - O_n$$

where M_n = computed midordinate at any station.
O_n = tangent offset at same station.
O_{n-1} = tangent offset at preceding station.
O_{n+1} = tangent offset at next station in advance.

Good surface requires adherence to the established grade and uniformity of cross level in the plane of the rail heads as well as to established superelevation on curves. Nonuniform tamping, soft and decayed ties, poor or dirty ballast and subgrade conditions, lack of recent tamping, center-bound track, and depressed ballast under joints cause poor surface. Resurfacing, track raising, and reballasting are required for track in poor surface. All track defects are cumulative and contribute to an accelerating rate of track deterioration unless corrected.

14. TURNOUTS AND CROSSINGS

Turnouts are used to divert trains from one track to another. A conventional turnout consists of a switch to inaugurate the diversion, a frog to carry wheel flanges over opposing rails, guard rails to protect the frog points, and the closure rails between the frog and switch.

The stub switch of Fig. 24 is now obsolete on all main line railroads and on most of the others as well. It has some use as a temporary expedient for light contractors' and mining railways and emergencies but should otherwise be rejected. In the stub switch, both of the main rails are cut at H and K. The rails from S' to H and from S to K are not fastened to the ties but are tied to each other by four or more tie rods. The gaps at H and K are always sufficient to cause a considerable jar to the rolling stock. In the winter, the gaps become excessively wide. In summer, expansion of the steel rails may close the gaps completely, sticking the switch. When operated as a trailing switch, a derailment is inevitable if the switch is incorrectly set.

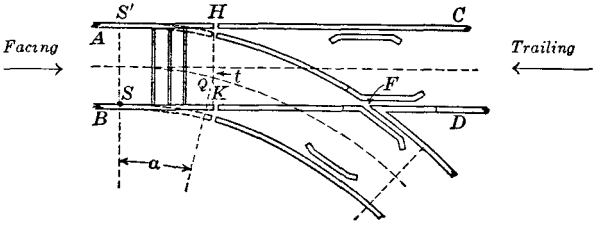


FIG. 24. Stub switch.

The standard switch in use today is the **split switch** of Fig. 25. The switch rails or points which initiate the diversion taper from a full section at one end, the heel, to a $^1/_4$-in. point at the other. The point is further beveled and planed to a $^1/_8$-in. thickness at the gage line. The switch angle between the gage lines of point and stock rails varies from 0° 45′ to 2° 30′. For good riding, the ratio between frog and switch angle should be approximately 4:1. At the heel, the point rail is fastened to the adjoining lead rail by bent splice bars with cast-steel heel blocks inserted for rigidity between the point and lead rails and the stock rail. Thimbles in the heel-block bolt holes permit the point to hinge or pivot at the heel. The point rails rest and slide on switch or riser plates which have planed bearing surfaces of varying thickness. These raise the point $^1/_8$ to $^1/_4$ in. above the stock rails to prevent derailments from cupped wheel treads. The points are bound together by switch rods. The throw rod is coupled at one end to the points or switch rod and at the other to a power-switch operating mechanism, a ground-throw lever, or a switch stand. The switch stand has a lamp and target to indicate the position of the switch. Yard switches may have a banner only. Braces, fixed or adjustable, attached to the switch plates, bear against the outside base and underside of the head of the stock rail to resist

[13] H. B. Christiansen, Asst. Chief Engineer, C. M. St. P. & P. Ry., Milwaukee Method of Straightening Curves, 1940, pp. 4–7.

side or lateral thrust on point and stock rails. There is an abrupt bend in the stock rail on the turnout side equal to the switch angle which houses the point. A pocket milled in the main-track stock rail prevents wear on the point. Spring switches hold the points in place by a stiff spring which yields to permit wheels to trail through on reverse moves.

Tongue Switch. For railway work in paved streets, the tongue switch is used, Fig. 26. A steel wedge-shaped tongue, pivoted at the heel, fits into a groove in the main running track rail when aligned for a turnout move. Its mate at the opposite rail may be a similar tongue when used in line-haul railroads in which case the operation is like that of a split switch. For street railways, the tongue mate will be a fixed frog point in the opposite rail. Both point and fixed mate are set in steel casings to permit placement of street paving materials. The double tongue is operated by a ground-throw lever in a covered box often set between the rails.

Frogs permit wheel flanges to cross opposing rails in turnouts and crossings. Frog types include:

(*a*) *Bolted rigid frogs* of standard rail sections bolted together.

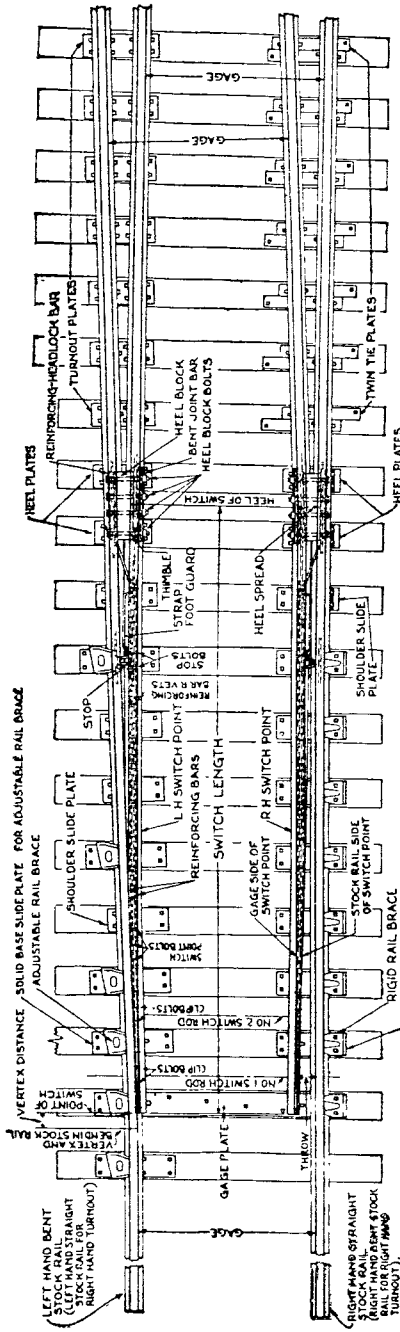
(*b*) *Hard-center frogs* having a replaceable manganese steel center or insert secured within standard rail sections.

(*c*) *Spring-rail frogs* which hinge the wing rail on a spring at the joint on the turnout side. For main track moves, the wing is held in line, by a spring, with the running rail to provide a solid support for the wheel. In a turnout move, the wheel pushes the wing outward against the spring to provide an open flangeway. Solid support is provided for the side having the prevailing volume of movement; hence spring frogs are designated as right or left hand depending on which side the spring rail is located when facing the point.

Frogs are designated by the **frog angle** F but more commonly by the **frog number** N which is the ratio between a unit spread and the axial distance from the theoretical point to the place where unit spread is measured. In Fig. 27, the relation between the frog angle F and the frog number N is $N = \frac{1}{2} \cot F/2$. If the point were brought to a knife edge, it would soon batter and chip away. It is therefore ground back or cut off to the "actual" point where it is approximately $\frac{1}{2}$ in. in width. The "frog bluntness" or distance from the theoretical to the actual point in feet is $NP/12$ where P is the measured width in inches.

Guard rails are set opposite the frog point to hold wheels away from the point, thus preventing point wear and possible derailment. The midpoint of the guard rail is set a few inches ahead of the point. The distance from the gage of the frog point to the gage of the running-rail side of the guard rail must be within 4 ft $6\frac{1}{2}$ in. to 4 ft $6\frac{7}{8}$ in. based on AAR specifications for wheel mounting. This provides a minimum flangeway of $1\frac{7}{8}$ in. Where gage is widened on curves, the point to guard rail distance must be held and the flangeway widened. Guard rails may be made from ordinary rail sections, sheared head and base on one side, to fit close to the running rail, with flared ends, and held in place with spikes, rail braces, filler blocks, bolts, and clamps. Modern designs include manganese cast-steel guard rails with plates, rail braces, and foot guards integrally cast or attached to form a complete unit. On self-guarded frogs, vertical risers on the outside of the wing rails bear against the outside treads of the wheels, forcing them away from the point.

Turnout design is shown in Fig. 28. The length of point, l, the point width, usually taken as $\frac{1}{8}$ in., and the heel spread, h, taken as $6\frac{1}{4}$ in., are known. The sine of the switch angle, S, equals $(h - \frac{1}{8})/l$. The lead distance from actual point of frog to actual point of switch is $L = l + b + d + NP/12$. Note that the actual or "$\frac{1}{2}$-in." point of frog is shown in Fig. 27 and is used in all field layouts as a point of reference. In triangle 1, angle F and side f are known. Solve for sides d and e. In triangle 2, side $Q = g - (h + e)$ and the angle $(F + S)/2$ are known. The triangle is solved for b and C. C, the long chord of the lead curve equals $2(R + g/2) \sin (F - S)/2$. From this, R is obtained. Turnout dimensions depend on the length and thickness of switch rails, the heel spread, frog angle, and length of wing rails of the frog. These may all vary within narrow limits depending on the standards of the various railroads or the manufacturer. The 4:1 ratio between frog and switch angles is maintained only approximately so that a few standard combinations of switch length and frog angle can be had. The AREA recommended turnout dimensions are given in Fig. 35.



PLAN VIEW INDICATING (FOR ILLUSTRATION ONLY) VARIOUS PARTS OF SWITCH LAYOUTS
DRAWN FOR LEFT HAND TURNOUT

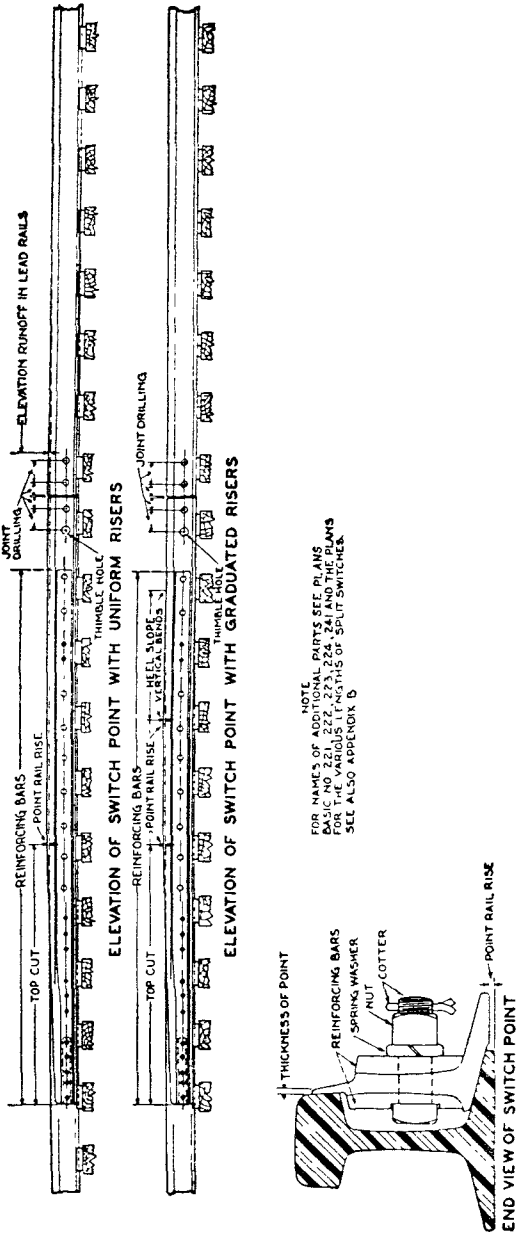


FIG. 25 Preferred names of parts for split switches. (Trackwork Plan 190-42 of the AREA, Chicago.)

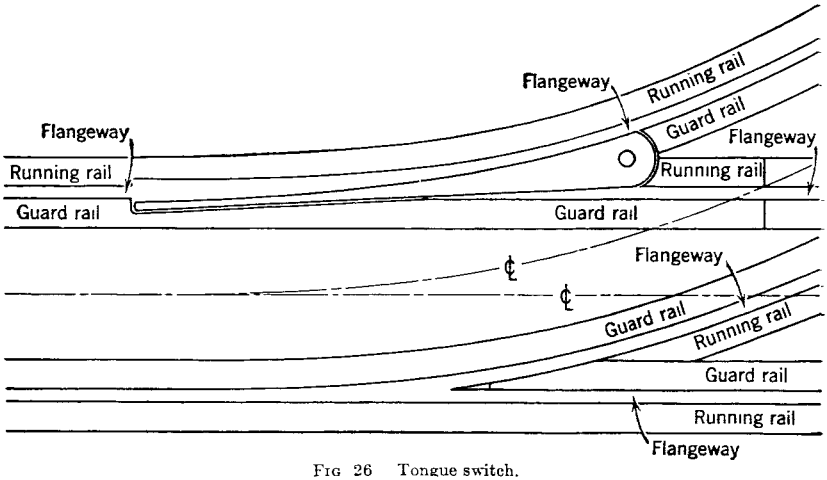


FIG 26 Tongue switch.

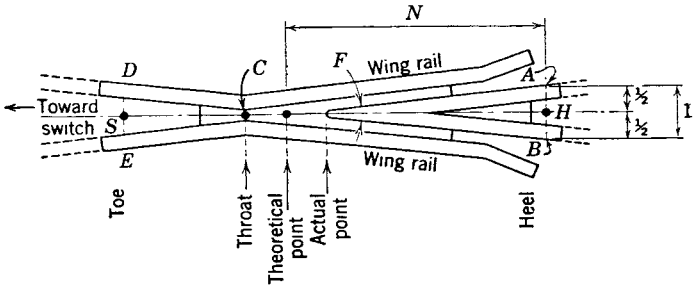


FIG. 27 Diagrammatic design of frogs (rigid)

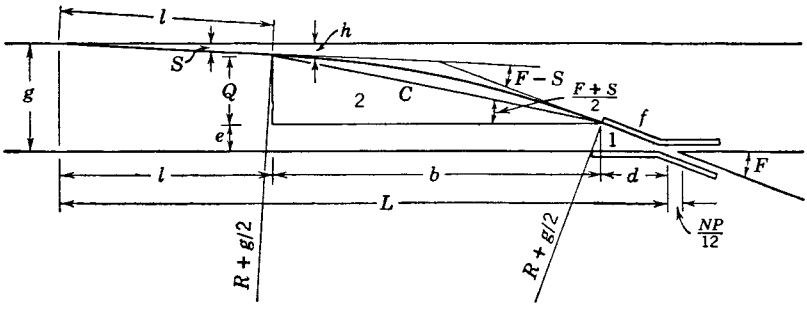


FIG. 28. Turnout design. (After G W. Pickels and C. C. Wiley, *Route Surveying*, John Wiley & Sons, 1949)

Turnout Installation

Basic problems involving the use of turnouts include connections to diverging tracks or routes, siding connections, crossovers, and ladder tracks.

The diverging route, Fig. 29, may be a main line, branch line, utility or industrial spur, or siding. Usually the engineer needs only to locate the point of frog and the P.C. and P.T. of the connecting curve if any. The foreman installing the turnout will be provided with all other necessary information on a standard plan giving frog and switch dimensions, length of lead and closure rails, and spacing of switch timbers.

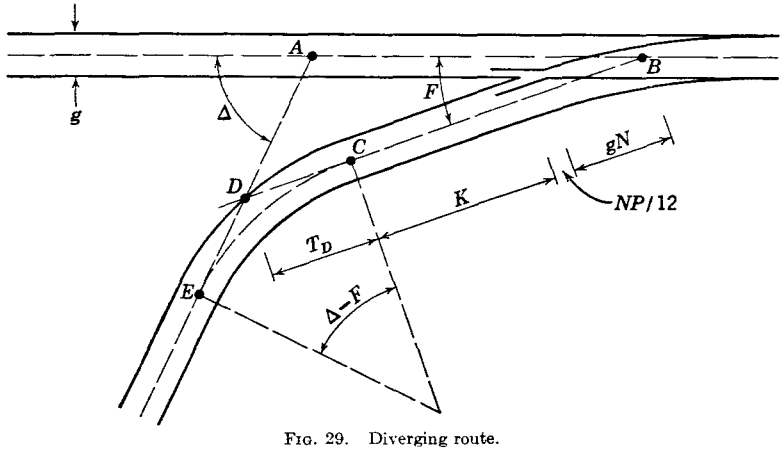


FIG. 29. Diverging route.

The intersection angle Δ between the main track and the proposed diversion is measured, Fig. 29. All angles in triangle ABD are known. The side DB is known where $gN + NP/12$ is distance from the intersection of center lines to the actual point of frog, K is the heel distance of the frog plus any additional desired distance, preferably at least a full rail length, and T_D is the tangent distance of the connecting curve, where one is used, having a degree equal to or less than the degree of turnout curve and an intersection angle, $\Delta - F$. Triangle ABD is solved and AB laid off from point A, the frog angle, F, is turned off from B, and the frog point located a distance $gN + NP/12$ from B. Point C is laid off from point B, and E is located from point A and checked by running in the connecting curve. To avoid cutting a rail, it may be necessary to shift the position thus determined by as much as half a rail length. The triangle ABD must be reused in the ratio of side AB to the shift in point of frog. The problem is more complicated when the route diverges from a curve. Turnouts should not be located on curves for safety reasons and because of difficulties in maintaining superelevation, maintaining line and surface, and keeping the switch in proper adjustment.

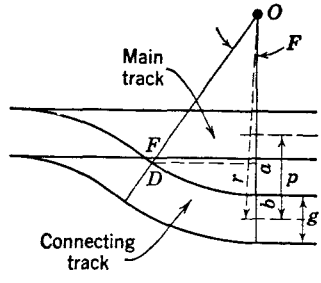


FIG. 30. Connection to parallel tracks, straight

Connecting Curves from Turnouts. The following solutions for connecting curves and crossovers between straight parallel tracks are based on the use of straight frog rails beyond the frog point. Let p = distance between track centers, g = gage, F = frog angle, w' = length of wing rail back of theoretical frog point DF in the figures, R = radius of main curved track, r = radius of connecting curved track, N = frog number corresponding to frog angle F, ψ = central angle, measured at center C of main curved track, required for connecting curve, ϕ = central angle, required for connecting curve, measured at center O of connecting curve.

For a connecting curve from a straight track, see Fig. 30.

$$(r - {}^1/_2\, g) = \frac{p - g - w' \sin F}{\text{vers } F}$$

Although the distance from F to the "fouling point" (which is on the line ab) may be shortened somewhat by using a sharper curve and using a short tangent at each end, the method indicated in Fig. 30 is preferable. Under such conditions, using the standard dimensions given in Fig. 35, the distance from F to the "fouling point" measured along the main track, is a definite quantity for any frog number, and is as given in Table 17 for $p = 13$, $g = 4.708$.

Table 17. Connecting Curve from Turnout, Straight Main Track

Frog Number	Distance, ft	Radius, ft	Degree of Curve ° ′ ″	Frog Number	Distance, ft	Radius, ft	Degree of Curve ° ′ ″
5	85.06	424.93	13 29 04	12	186.93	2102.35	2 43 32
6	93.41	525.63	10 54 16	14	217.91	2855.30	2 00 24
7	108.87	713.19	8 02 25	15	234.04	3292.44	1 44 25
8	124.68	935.56	6 07 37	16	248.45	3723.27	1 32 20
9	139.38	1168.59	4 54 16	18	281.30	4749.75	1 12 23
10	156.55	1464.51	3 54 47	20	312.23	5863.15	0 58 38
11	171.38	1768.46	3 14 25				

For a connecting curve on the outside of a main curved track, see Fig. 31.

$$\tan {}^1/_2\,\psi = \frac{2N(p - g - w' \sin F)}{2R + p + w' \sin F}$$

$$(r - {}^1/_2\,g) = (R + {}^1/_2\,g + w' \sin F)\,\frac{\sin \psi}{\sin (F + \psi)}$$

$$DS = 2(r - {}^1/_2\,g) \sin {}^1/_2\,(F + \psi)$$

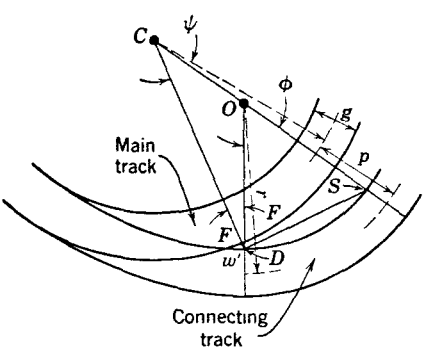


FIG 31. Connection to parallel tracks, curved

For a connecting curve on the inside of a main curved track, see Fig. 32.

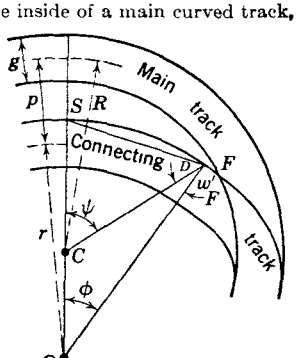


FIG 32 Connection to parallel tracks.

$$\tan {}^1/_2\,\psi = \frac{2N(p - g - w' \sin F)}{2R - d - w' \sin F}$$

$$(r + {}^1/_2\,g) = (R - {}^1/_2\,g - w' \sin F)\,\frac{\sin \psi}{\sin (\psi - F)}$$

In case the frog angle F is larger than ψ, the center of curvature of the connecting curve rails will be on the other side of the main track as shown in Fig. 33. Then

$$- {}^1/_2\,g) = (R - {}^1/_2\,g - w' \sin F)\,\frac{\sin \psi}{\sin (F - \psi)}$$

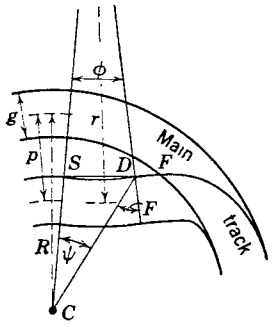


FIG. 33. Connection to parallel tracks.

In case ψ should exactly equal F, the connecting rails would be straight and r = infinity. This is true when

$$2R - p - w' \sin F = 4N^2\,(p - g - w' \sin F)$$

The solution of this equation will show the value of R which makes this case possible for any given values of N, p, g and w'.

A crossover, Fig. 34, permits movement from one track to a parallel track. Using frogs of the same angle, F, and track centers p ft apart, the solution is based on assuming one frog point location and locating the other by measuring the distance from a point on the parallel track opposite the first point of frog. $BC = AD = Ng + \frac{Np}{12}$ in Fig. 34; $d = AE \cot F - 2\left(Ng + \frac{Np}{12}\right)$ where D and B represent the actual or half-inch points of frog. When frogs of unequal number are used, a short connecting curve of about the same degree as the shorter turnout is necessary between the two frog tangents.

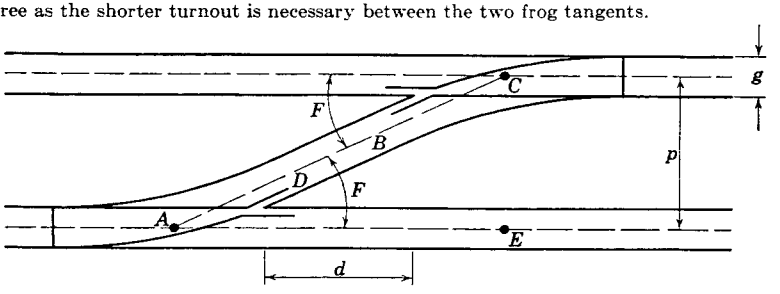
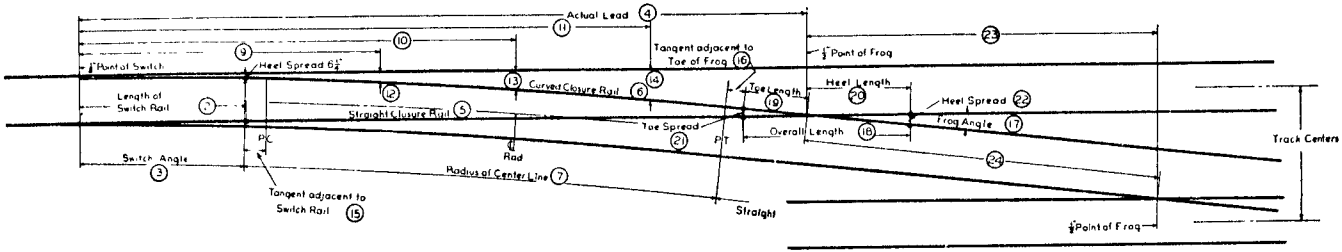


FIG 34 Crossover; equal frogs

For crossovers between parallel curved tracks and using equal frog numbers and a simple connecting curve tangent to the heels of the frogs, the distance along the main track between frog points is practically the same as for parallel tangent crossovers. Diamond crossovers connect parallel tracks in both directions at the same location and make use of crossing frogs designed for the track centers involved.

Ladders connect the body tracks of a yard with the approach or switching lead. The simple case in Fig. 36 is that of parallel tracks equally spaced with the same frog angle throughout. Knowing p and frog angle F, triangle ABE is solved for AB and AE. The



TURNOUT AND CROSSOVER DATA

	Properties of Switches			Closure Distance		Lead Curve		Gage Line Offsets					
Col (1)	Col. (2)	Col. (3)	Col. (4)	Col. (5)	Col. (6)	Col. (7)	Col (8)	Col. (9)	Col. (10)	Col. (11)	Col (12)	Col (13)	Col (14)
Frog Number	Length of Switch Rail	Switch Angle	Actual Lead	Straight Closure Rail	Curved Closure Rail	Radius of Center Line	Degree of Curve						
	Ft In.	Deg Min.	In	Ft In.	Ft. In.		Deg Min. Sec.	Ft In	Ft In	Ft In	Inches		Ft In
5	11-0	2-39-	2-6 1/4	28-0	28-4		32-39-56	18-0	25-0	32-0	11 13/16		-8 7/8
6	11-0	2-39-	1-6	32-9	33-0		22-17-58	19-2 1/4	27-4 1/2	35-6 3/4	12 3/8		-10
7	16-6	1-46-	2-1	40-10 1/2	41-1 1/4		15-43-16	26-2 1/4	35-10 1/2	45-6 3/4	11 3/8		-6 7/8
8	16-6	1-46-	3-0	46-5	46-7 1/2		11-46-44	27-7 1/2	38-8 1/2	49-9 3/4	11 7/8		-8 5/16
9	16-6	1-46-	3-3 1/4	49-5	49-7 1/4		9-19-30	28-10 1/4	41-2 1/2	53-6 3/4	12 5/16		-9 7/16
10	16-6	1-46-	3-9	55-10	56-0		7-21-24	29-11 3/4	43-5 1/2	56-11 1/4	12 1/4		-8 5/8
11	22-0	1-19-	-10 1/4	62-10 1/4	63-0		6-10-56	37-8 1/2	53-5	69-1 1/2	12 1/4		-9 3/4
12	22-0	1-19-	5-8	66-10 1/2	67-0		5-11-20	38-8 1/2	55-5	72-1 1/2	12 7/16		-9 7/8
14	22-0	1-19-	7-0 3/4	76-5 1/4	76-6 3/4		3-37-28	41-1 1/4	60-2 1/2	79-3 3/4	12 7/8		-10 1/2
15	30-0	0-58-	5-4 1/2	86-11 1/2	87-0 3/4		3-19-48	51-9	73-6	95-3	12 7/8		-9 3/4
16	30-0	0-58-	-4	91-11	92-0		2-51-18	53-0	76-0	99-0	12 7/16		-10 3/16
18	30-0	0-58-	0-11 1/2	99-11	100-0		2-13-20	55-0	80-0	105-0	12 3/4		-10 7/16
20	30-0	0-58-	-11 1/2	110-11	111-0		1-44-32	57-9	85-6	113-3	13 1/16		-11 3/16

Notes:

1 Turnouts and crossovers recommended: For main-line high-speed movements, No. 16 or No 20 For main-line slow-speed movements, No. 12 or No. 10. For yards and sidings, to meet general conditions, No 8

2 The data shown are computed for turnouts out of straight standard 4′ 8 1/2″ gage track If the wheel base of the equipment used requires wider gage for the switch alignment or curvature shown, the lead and alignment of curved closure rail shall be maintained, and inside stock and curved rails shall be moved out the necessary amount. The gage of straight track through switch will then be increased and the straight closure rail shall be bent to true alignment in advance of toe of frog.

3 Frog designs: For short spring-rail-type frogs, the straight closure, and for solid manganese frogs, the straight and curved closures shall be lengthened to conform.

4 Modification of alignment: The alignment specified may be used without appreciable wear for switch points made in accordance either with Detail 4000 (thickness of point—1/4″), or with Detail 5000 (thickness of point—0″).

		Properties of Frogs							Data for Crossovers			
									13′-0″ Track Centers		For change of 1′-0″ in Track Centers	
Col (15)	Col (16)	Col (1)	Col. (17)	Col. (18)	Col. (19)	Col. (20)	Col (21)	Col (22)	Col. (23)	Col. (24)		
Tangent adjacent to Switch Rail	Tangent adjacent to toe of Frog	Frog Number	Frog Angle	Overall Length	Toe Length	Heel Length	Toe Spread	Heel Spread	Straight Track	Crossover Track	Straight Track	Crossover Track
Feet	Feet		Deg.	Ft In	Ft In	Ft In	Inches	Inches	Ft In	Ft In	Ft In	Ft In
0.00	0.78	5	11-	9-0	3-6 1/2	5-5 1/2	7 15/16	13 9/16	16-10 5/16	18-1 7/8	4-11 7/16	5-0 5/8
0.00	1.75	6	9-	10-0	3-9	6-3	7	13	20-5 1/2	21-6 1/2	5-11 1/2	6-0 1/2
0.01	0.00	7	8-	12-0	4-8 1/2	7-3 1/2	7 9/16	13	24-0 3/8	24-11 5/8	6-11 9/16	7-0 7/16
0.64	0.00	8	7-	13-0	5-1	7-11	7 1/8	12 3/8	27-7 1/8	28-4 7/8	7-11 3/8	8-0 3/8
0.00	0.17	9	6-	16-0	6-4 1/2	9-7 1/2	8	13 5/16	31-1 5/8	31-10 3/8	8-11 11/16	9-0 5/16
2.08	0.00	10	5-	16-6	6-5	10-1	7 3/16	12 5/8	34-8 1/8	35-3 7/8	9-11 11/16	10-0 5/16
0.00	0.13	11	5	18-8 1/2	7-0	11-8 1/2	7 1/8	13 1/4	38-2 1/2	38-9 1/2	10-11 3/4	11-0 1/4
0.00	0.50	12	4-	20-4	7-9 1/2	12-6 1/2	7 5/16	13 1/16	41-8 3/4	42-3 1/4	11-11 3/4	12-0 1/4
0.24	0.00	14	4-	23-7	8-7 1/2	14-11 1/2	6 7/8	13 5/16	48-9 1/4	49-2 13/16	13-11 13/16	14-0 1/4
1.56	0.00	15	3	24-4 1/2	9-5	14-11 1/2	7	12 7/16	52-3 7/16	52-8 5/8	14-11 15/16	15-0 3/16
0.66	0.00	16	3-	26-0	9-5	16-7	6 9/16	12 15/16	55-9 5/8	56-2 1/2	15-11 13/16	16-0 3/16
0.57	0.00	18	3-	29-3	11-0 1/2	18-2 1/2	6 7/8	12 3/8	62-9 7/8	63-2 3/16	17-11 15/16	18-0 3/16
2.47	0.00	20	2-	30-10 1/2	11-0 1/2	19-10	6 1/8	12 3/4	69-10	70-2	19-11 7/8	20-0 1/8

FIG. 35. Turnout and crossover data. (Plan 910-41 of the Trackwork Plans of the AREA, Chicago.)

initial point of frog is then located by measuring $AK = Ng + \frac{Np}{12}$ and $AB = BC = CD$. The outside track is connected to the ladder with a curve of intersection angle F and a radius approximately equal to that of the turnout curves in use, Fig 36. Variations of this problem include ladders and body tracks on curves, ladders with frog angles greater or less than the one in the lead and layouts in which every other track comes out of a body track instead of from the ladder.

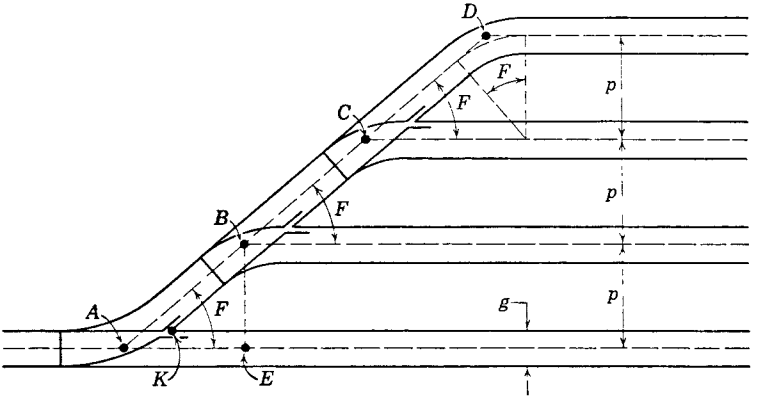


FIG 36 Simple ladder track

Slip Switches

Slip switches are used with small angle crossings to affect movement between the two tracks and are termed "double" or "single" depending on whether connections exist on one or both sides of the crossings, Fig. 37. Standard frogs serve as end frogs. Reference points in design and construction are the end frogs and the longitudinal and transverse axes. Movable points are often used as center frogs as well as in plain crossings (see next paragraph) to provide a solid bearing for the wheels and avoid the long, open flangeways

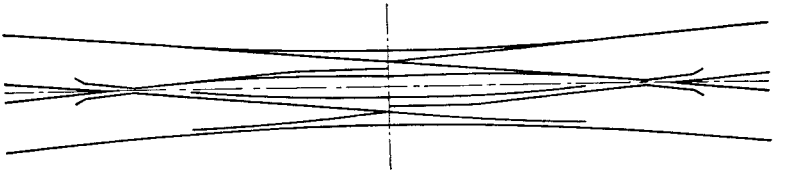


FIG. 37 Slip switch with movable point frogs

characteristic of small angle frogs; see Fig. 38. Slip switches are seldom installed under traffic but are built in one piece or as units and set in place with a crane. The slip should be secured under a light girder or truss combination to give stiffness and prevent sagging during the lifting operations.

Notes on Fig. 38

1 The graph shows minimum angles between tangents at intersection of center lines of track for which rigid center frogs may be used as referred to on Plan Basic No 700

2 For curved track the center line of track having the greater degree of curvature within the limits of the diamond of the crossing shall apply

3 For crossings in the Area C between the lower solid graph line and the dash graph line raised guards are required to provide effective guarding For crossings in the Area B between the dash graph line and the upper solid graph line, level guards will provide sufficient guarding, but raised guards are recommended to protect the frog points from flange wear For crossings in the Area A above the upper solid graph line, level guards are recommended

4 Where raised guards are specified, the inside guarding face shall be 1″ higher than the top surface of the frog

5 Dotted graph applies only for installations in yards and terminals and requires that the gage and guard lines of the points be extended straight (with no side chamfer) to a point where they are 1 2″ apart

6 Locomotives with blind drivers should only be operated at restricted speeds over center frogs having the 1″ high guard

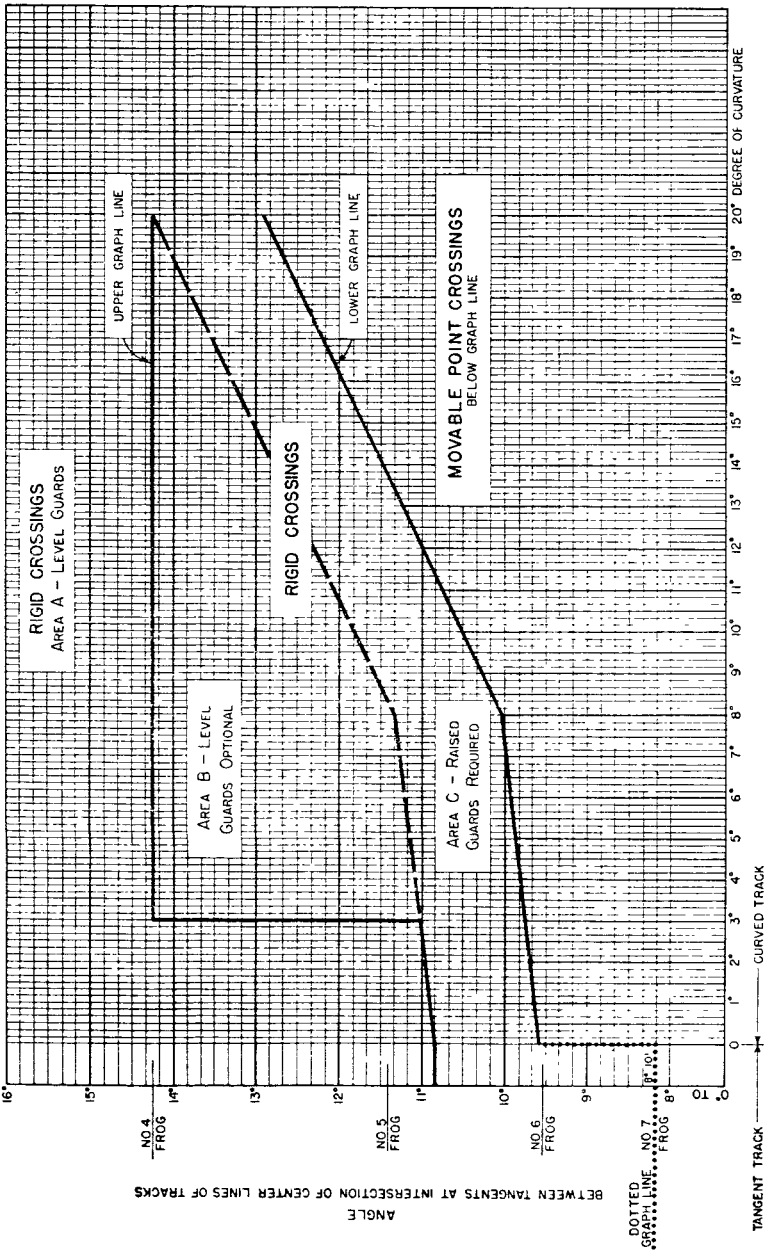


FIG. 38. Graph of crossing angles requiring movable point frogs (Plan 820-30 of the AREA) *(See notes at foot of facing page.)*

Crossings. For *two straight tracks* the frog angles are the angle (and its supplement) made by the two alignments. For *one straight and one curved track*, Fig. 39 applies with these equations:

$$\cos F_1 = \frac{R \cos M + {}^1/_2\, g}{R - {}^1/_2\, g}, \qquad \cos F_2 = \frac{R \cos M + {}^1/_2\, g}{R + {}^1/_2\, g}$$

$$\cos F_3 = \frac{R \cos M - {}^1/_2\, g}{R + {}^1/_2\, g}, \qquad \cos F_4 = \frac{R \cos M - {}^1/_2\, g}{R - {}^1/_2\, g}$$

$$F_3F_4 = (R + {}^1/_2\, g) \sin F_3 - (R - {}^1/_2\, g) \sin F_4$$

$$HF_4 = (R - {}^1/_2\, g)(\sin F_4 - \sin F_1)$$

$$F_1F_2 = (R + {}^1/_2\, g) \sin F_2 - (R - {}^1/_2\, g) \sin F_1$$

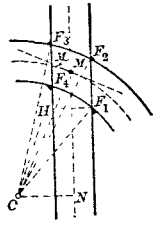


FIG 39. Crossings.

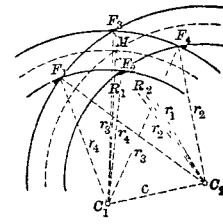


FIG 40. Crossing.

For *two curved tracks* all four frogs are unequal, see Fig. 40. The radii of the two tracks are of course known and also the angle of intersection of their tangents (M). Then r_1, r_2, r_3, and r_4 become known by adding (or subtracting) $^1/_2\, g$ to the known track center radii. In the triangle C_1MC_2,

$$^1/_2\,(C_1 + C_2) = 90° - {}^1/_2\, M$$

$$\tan {}^1/_2\,(C_1 - C_2) = \cot {}^1/_2\, M \frac{R_2 - R_1}{R_2 + R_1}$$

From these equations C_1 and C_2 become known.

Then
$$c = C_1C_2 = R_2 \frac{\sin M}{\sin C_1}$$

For abbreviation $s_1 = {}^1/_2\,(c + r_1 + r_4)$; $s_2 = {}^1/_2\,(c + r_2 + r_4)$; $s_3 = {}^1/_2\,(c + r_1 + r_3)$ and $s_4 = {}^1/_2\,(c + r_2 + r_3)$.

Then
$$\text{vers } F_1 = \frac{2(s_1 - r_1)(s_1 - r_4)}{r_1 r_4} \qquad \text{vers } F_2 = \frac{2(s_2 - r_2)(s_2 - r_4)}{r_2 r_4}$$

$$\text{vers } F_3 = \frac{2(s_3 - r_1)(s_3 - r_3)}{r_1 r_3} \qquad \text{vers } F_4 = \frac{2(s_4 - r_2)(s_4 - r_3)}{r_2 r_3}$$

$$\sin C_1C_2F_4 = \sin F_4 \frac{r_3}{c}; \quad \sin C_1C_2F_2 = \sin F_2 \frac{r_4}{c}; \quad F_2C_2F_4 = C_1C_2F_4 - C_1C_2F_2$$

$$\sin F_1C_1C_2 = \sin F_1 \frac{r_1}{c}; \quad \sin F_2C_1C_2 = \sin F_2 \frac{r_2}{c}; \quad F_1C_1F_3 = F_1C_1C_2 - F_2C_1C_2$$

From these equations the chords F_1F_2 and F_2F_4 are readily computed.

For crossings greater than 25°, the various pieces, cut to fit, are united by bolting through filler blocks and heavy straps. A much-used recent design contains a replaceable manganese insert at each frog point. Except for 90° crossings, each installation is a special job. In *Route Surveying*, Pickels and Wiley give the following data as necessary in ordering crossings from manufacturers:[14]

(*a*) Crossing angle.
(*b*) Gage of each track.
(*c*) Curvature by degrees or radii.
(*d*) Direction of curvature.

[14] G. W. Pickels and C. C. Wiley, *Route Surveying*, 3rd Ed., 1949, p. 223.

(*e*) Length along each gage line from one gage-line intersection (theoretical point of frog) to the nearest rail joint.
(*f*) Length overall along each gage line.
(*g*) Height, weight, and section of rail from which the crossing is to be made.
(*h*) The spacing and size of holes for splice bars.
(*i*) The type of crossings, etc., unless covered by general specifications.
(*j*) A small sketch based on field measurements correct to the nearest $^1/_8$ in. (0.01 ft).

15. MISCELLANEOUS ROADWAY STRUCTURES

Track tanks, or pans, are used to give a supply of water to a steam locomotive while the train is in motion. A scoop, lowered from under the tender, is attached to a pipe leading to the tender tank. The rapid motion forces the water up the pipe from the long shallow pan between the rails. The height of the top of the tender tank above the track pan is usually about 9 ft, and this means that water will not flow into the tank unless the speed of the engine is more than 16 miles per hour. Even at 20 miles an hour more water is wasted by slopping over the sides than reaches the tender. The minimum wastage is about 12 $^1/_2$% and occurs at a speed of 45–50 miles per hour, although with a properly designed pan the wastage is fairly constant at speeds from 25 to 60 miles per hour. Adequate drainage is an important feature of the installation.

Track pans are made of $^3/_{16}$-in. plate 19 in. wide, 6 in. deep, and 1200–2400 ft long. They must be on absolutely level track and on tangent track or flat curves. To prevent freezing in winter, steam pipes having $^1/_8$-in. nozzles are inserted every 40–50 ft of tank length and jets of live steam forced through from the pump-house boiler. Another system circulates the water, feeding steam into it at the inlets.

Section Tool House. Class A houses, 14 ft by 20 ft, and class B, 12 ft by 18 ft, AREA recommendation, are set with the long side parallel to the track with an 8-ft sliding door at the extreme end for motor-car storage. Class C houses, 10 ft by 14 ft, place the short side nearest the track with double swinging doors on the track end. Temporary locations shall be on wooden posts, others on brick or concrete piers. Wood, galvanized iron, and corrugated metal have had common use.

Highway Grade Crossings. Unimportant country roads and farm crossings may be made of 3-in. planks spiked to blocks on the ties on either side of each rail, the outer planks tight against the heads of the rails. As with all types of crossings, a flangeway of 2 $^1/_2$ in. (2 $^3/_4$ in. on curves in excess of 8°) is allowed. Space between the planks is filled flush with ballast. Rails laid on side, head against web of running rail or standing with separators between it and the running rail, may be used. The ends are bent inward to give a flangeway entrance opening of at least 4 in. As a safety precaution, crossings should be at least 4 ft wider than the adjacent approach of pavement or roadway. Surface of highway shall be in the same plane as tops of rails for at least 2 ft outside and, where superelevation does not require otherwise, shall be no lower than 9 in. or higher than 3 in. above tops of rail at points 30 ft beyond on the highway. Prevailing highway grades should not be exceeded beyond the 30-ft point. Adequate drainage must be provided. Nine-foot ties are preferred. Rail joints should be kept at least 6 ft from the crossing, using longer than standard rails if necessary. In addition to the foregoing, AREA recommendations for bituminous, wood plank, prefabricated sectional treated timber, precast concrete slab, and monolithic concrete crossings will be found in the *AREA Manual*, pp. 9-1-1 to 9-1-16.

Stock Guards. A stock guard consists essentially of wing fences extending from the right-of-way fences to **aprons** which are short sections of fence, usually flaring, set parallel with the track at the track end of the wing fences; and a rough surface, on which stock will not walk, which extends the entire width of the track between the outer ends of the ties.

Recommended Practice. The AREA recommends surface guards in preference to pit guards, chiefly on account of the disastrous consequences of derailment at the guard or a failure of the pit structure. A surface guard should be so designed that it will not catch dragging brake chains or other rigging; it should not endanger employees who must walk over it; it should not rattle during the passage of trains; it should be reasonable in first cost, durable and easily applied or removed for track repairs: finally, it should be effective to deter all kinds of livestock from attempting to cross and yet should not catch and hold such as should make the attempt.

Surface Guards. Home-made wooden guards consist of slats about 8 ft long, 2 $^1/_2$ in. wide, 4 in. high, spaced 4 $^1/_2$ in. c.c., using 2-in. fillers and tied together with three $^3/_8$-in. round rods running through slats and fillers. The slats are chamfered on the upper corners. Metal guards are usually patented and consist of rods (replacing the slats), or some one of a variety of designs in stamped sheet metal which, by bending out lugs, make a rough

surface, or of rollers. A surface may also be made by a combination of interlocking tiles about 4 in. wide, 15 ft long, and with their upper surface forming ridges. This type is free from rust or decay but is more liable to breakage.

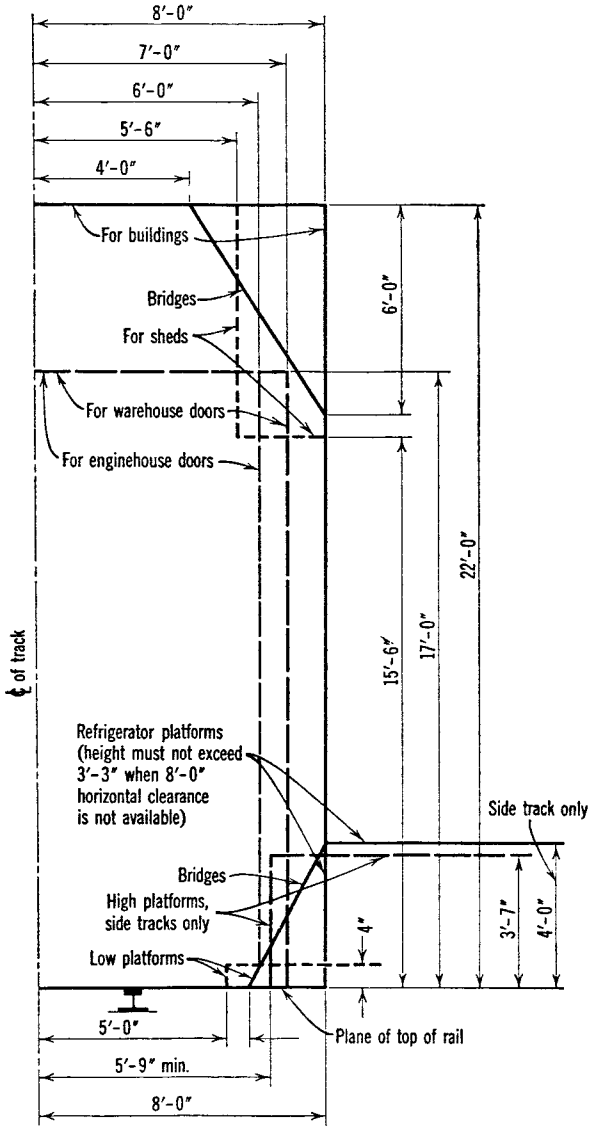


FIG. 41. Clearance diagrams. State laws govern any conflicts. (*AREA Manual*, 1953 Ed., pp 28-2-1 to 28-2-7)

Clearances. Side clearances are measured with references to the center line of track, vertical clearance from the top of rail. All structures should clear a man on the side of a car and failing this should be protected with close clearance signs. Overhead clearances are protected with tell tales, rope ends suspended above the tracks. Most states establish

clearance requirements which must be observed. The AREA, in their manual, has recommended a series of clearance diagrams for various locations—tunnels, bridges, station platforms. Figure 41 shows an all-purpose diagram which will meet practically all situations. Clearances may be obtained by direct measurement or by use of a clearance car mounting a frame of adjustable, graduated measuring rods which can be set to the individual cross section of track and structure under study.

YARDS AND TERMINALS

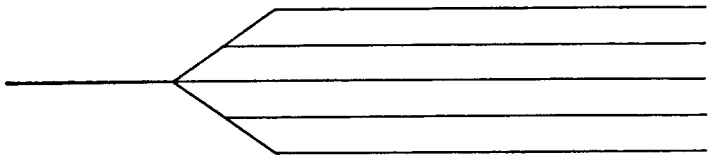
16. YARDS

(Condensed from Definitions and Recommended Practices of the AREA)

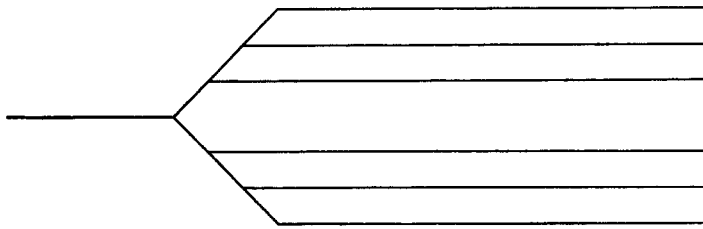
A yard is a system of tracks within defined limits for receiving, separating, and making up trains, storing cars, and other purposes. A large yard will have some or all of the following features: A receiving yard where the incoming train may wait, and the locomotive and caboose can be removed, until the cars can be separated; a holding yard where cars can be moved if not ready for classification; a classification yard in which cars are classified or grouped in accordance with requirements preliminary to forwarding in trains; a storage yard where some cars await further disposition; a departure or forwarding yard where cars are assembled into trains. The distribution of cars onto various tracks is often accomplished by pushing them over a summit beyond which they run by gravity. Such a yard is called a summit or hump yard. In rare cases natural grades give a true gravity yard. A yard in which movement of cars is accomplished entirely by locomotives is called a flat switching yard. Saucer yards are flat-switched but have a slight descending grade at one end to aid in car movements and a slight ascending grade at the other to keep cars from rolling past the clearance point.

Tracks

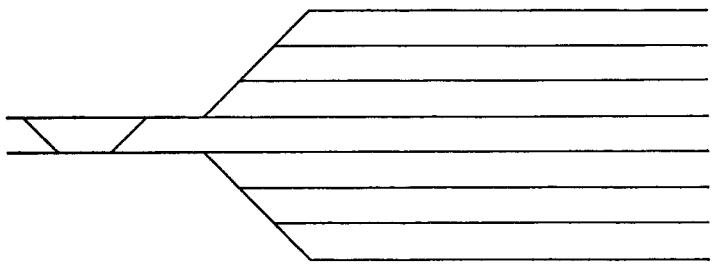
Tracks in a group or a set of parallel tracks all used for some one purpose are called **body tracks.** They should be spaced from 13 to 14 ft c.c. and 15 ft from a main or a running track. A group of such tracks all lead into a **ladder track** which should be not less than 15 ft c.c. from any parallel track and 18 ft if parallel to another ladder. Number 8 is the minimum frog number which should be used on such tracks The track connecting either end of the yard with the main line is called a **lead track** which should be interlocked with the main line Running tracks and open tracks are provided so as to permit the free movement of cars and switching engines from any portion of the yard to any other. When riders are used to control cars on classification tracks a rider track may be used for the gasoline car which takes them back to the hump or they may be required to walk back. Many large yards are now equipped with car retarders which control the speed of cars by pressing against the sides of the wheels. These, together with switches, are operated from towers by one man in each tower, thus eliminating riders and switch tenders. Caboose tracks should be located between the receiving and forwarding yards, and are preferably so constructed that a caboose may be readily pushed thereon from a receiving track and then dropped by gravity to the train departing in the direction from which the caboose has arrived. Scale tracks should be located between the receiving and separating yards or on the descending hump lead. Fueling, ashpit, sand, and engine tracks should be located on the route to and from the engine house; they should be so arranged that water, fuel, and sand may be taken and ashes disposed of in convenient rotation and also provide that switching engines may clean fires, take fuel, water, and sand and pass around the waiting engines. Bad-order tracks should be convenient to the classification yard, so that bad-order cars may be set off and easily run to the repair tracks. The repair tracks should have a maximum capacity of about 15 cars each and should be spaced alternately 18 and 24 ft apart c.c. Fifty-five linear feet of track should be estimated in rating the capacity of freight-car repair tracks, in order to provide working room about each car. Parts of the yards should be provided with air and water pipes with outlets 55 ft apart for testing cars. A materials supply track should be placed in the space between each pair of repair tracks. Heavy freight-car repairs should be made under cover in a shop provided with overhead traveling cranes to facilitate heavy lifting. Icing tracks should be located between the receiving and separating yards so that the cars to be iced may readily be moved from the receiving to the icing tracks and thence to the separating yard. A coach-cleaning yard should be located for ready and quick access to and from the station. The tracks should be long



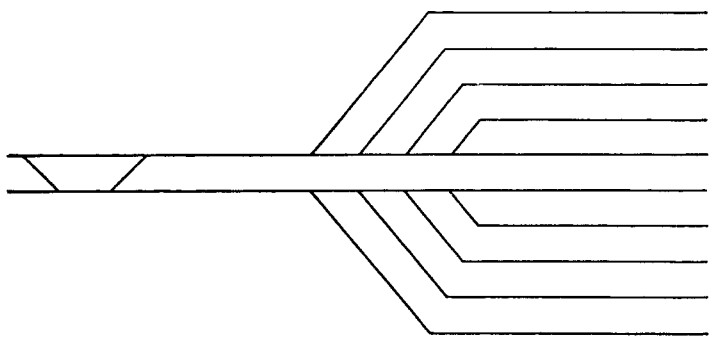
(a) *V*-type



(b) *V*-type



(c) Double lead *V*-type



(d) Central lead

FIG. 42. Yard leads and ladder types.

enough to accommodate trains without cutting, and should preferably be stub-ended, with a car-cleaners' repair supply building located at right angles at their ends. Yard track capacity is estimated by allowing 45 linear feet of track to each freight car, 70 ft for passenger cars. Truck delivery yards should be located convenient to the freight house so that the receipt and shipment of freight may be easily under the control of the freight agent's force. The tracks should be stub tracks in parallel pairs, the tracks of each pair 12 ft between centers and the pairs 52 ft between centers. For convenience of shifting, the tracks should have a capacity of about 20 cars each. A crane for handling heavy freight should be provided. If possible, ingress and egress for trucks should be provided for at each end of each teamway. Truck scales should be provided near the truck entrance of the yard, and track scales should be provided and located for convenient switching.

Ladders and body track may be laid out to form a simple gridiron as in Figs. 42*a* through *d*. Figure 42*d* shows a type of central ladder having all switches in line, clearly visible from the engine, and accessible to the switchman. The double central ladder arrangement provides space for tools and material in carrying on maintenance work and to deposit snow cleaned from the switches.

The main tracks should preferably go around both sides of the yards to eliminate any interference between through trains and those moving in and out of the yard. A second-best arrangement (and the only desirable one for a single track line) is placing the yard entirely on one side of the main tracks despite the traffic interference which must follow. The worst possible arrangement is locating the main tracks through the yard.

Assisting Gradients. The operation of flat-switched yards may be improved by the use of assisting gradients. The following were proposed to the AREA in 1951:[15]

Drill Tracks. Level or with a slightly assisting gradient except for 300 ft of 0.20% gradient preceding the first switch.

Ladder Tracks. 0.1–0.3% depending on length of ladder and possible use of an accelerating gradient.

Body Tracks. 0.2–0.25% for 300 ft following the switch, gradually decreasing with about 400 ft of level track at the end. Some roads place a slightly ascending grade beyond the level section to stop cars from rolling out of the yard.

Drainage is very important to good yard design and operation. Terrain conducive to drainage should be selected. The midportion of the yard should be kept higher than the outside. Flat areas which are not free draining should be drained by a system of subdrains consisting of one or more trunk drains with lateral stubs or feeders made of 6- to 10-in. vitrified tile laid with open joints or perforated metal pipes, 2–3 ft below the surface, and backfilled with filter sand materials.

The Hump Yard

A hump yard should have receiving, classification, and departure tracks. Trains may be handled through it faster and at less cost than through any other form of yard. The receiving tracks and lead from the main track should be of sufficient length to hold a maximum train, and should be sufficient in number to hold as many trains, arriving in quick succession, as the character of the traffic renders probable. If possible the grades of the receiving tracks should be such that one engine can push the maximum train over the hump. The length and number of the classification tracks depend on the number of classifications required, number of cars in each classification, whether or not a sorting yard is used, etc. Classification of through traffic may often be shifted from one yard to another with advantage. The departure tracks should be of full train length and sufficient in number to provide ample standing room for trains being tested for air and waiting for engines. An air hose and an air-brake testing plant should be provided in conjunction with the departure tracks.

Hump Design. Trains should be brought to the highest elevation possible in the receiving yard without exceeding 80% of the division's ruling grade. The hump should be as close to the receiving tracks as possible consistent with the necessary hump elevation and a gradient which can be overcome by one locomotive. The hump master's office, the controls, and the pin puller's position should be on the right-hand side of the hump so the coupler knuckles will be opened from that side and thus stand open for coupling on impact in the yard. Height of hump is based on two principal situations: (1) the empty, hard-rolling car in winter with a head wind and (2) the loaded, easy-rolling car in summer with an assisting wind. The hard-rolling car must be given sufficient elevation and velocity head so that it will roll well beyond the clearance point. The easy-rolling car, once under way, must not accelerate unduly and, in retarder operation, must not accelerate after leaving the last retarder. Grades for manually operated hump yards are shown in Fig. 43.

[15] *Bulletin of the AREA*, Vol. 51, No. 490, November 1950, p. 258.

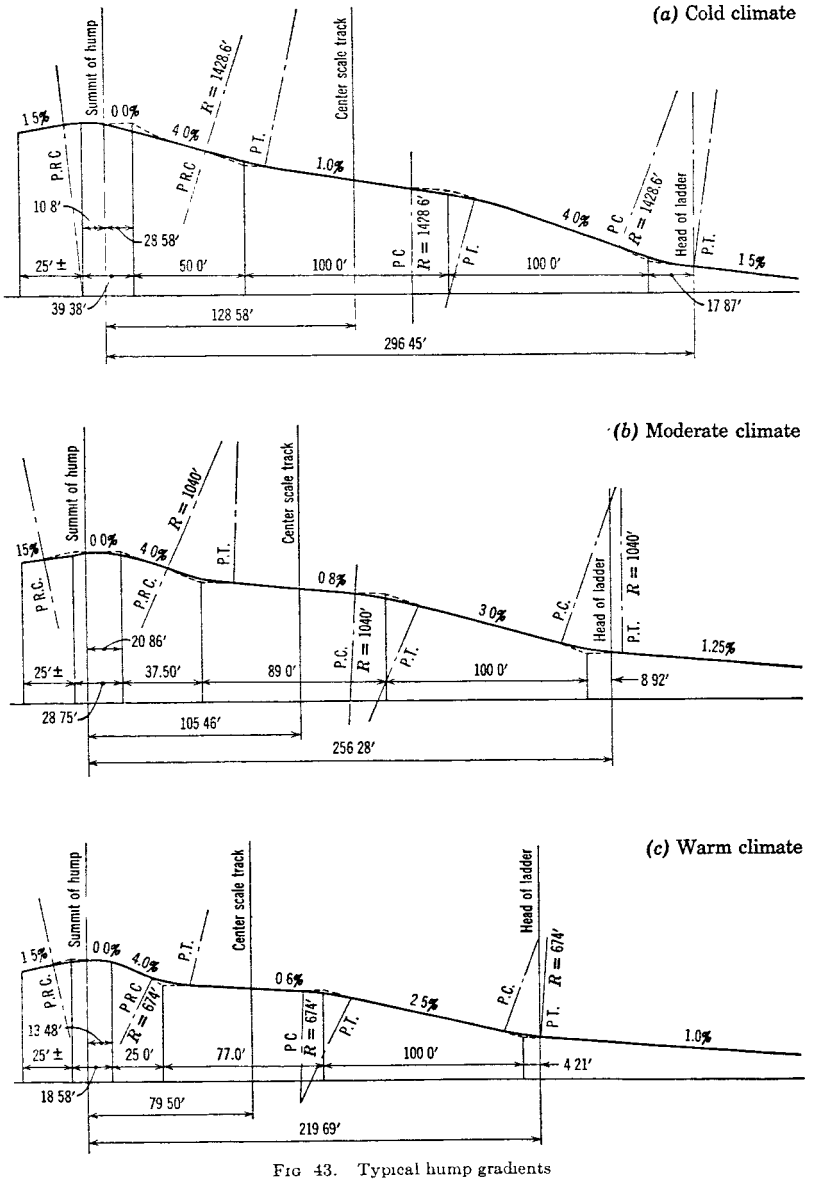


FIG 43. Typical hump gradients

Design for Retarder Operation. In order to make the most efficient use of retarders, the ladders and yard leads should follow a pattern similar to that of Fig. 44, dividing the classification tracks into groups with group- or subleads. This layout is facilitated by use of the lap or three-way switch shown in Fig. 45. The drop from summit to the clearance point A in Fig. 46 must be sufficient to roll the hard-rolling car well into the yard. Height B from last retarder to the clearance point must provide a gradient sufficient to start a car stopped in the last retarder but not unduly accelerate an easy-rolling car. The drop, $1 - B$, from the summit to the last retarder should be apportioned so as to separate

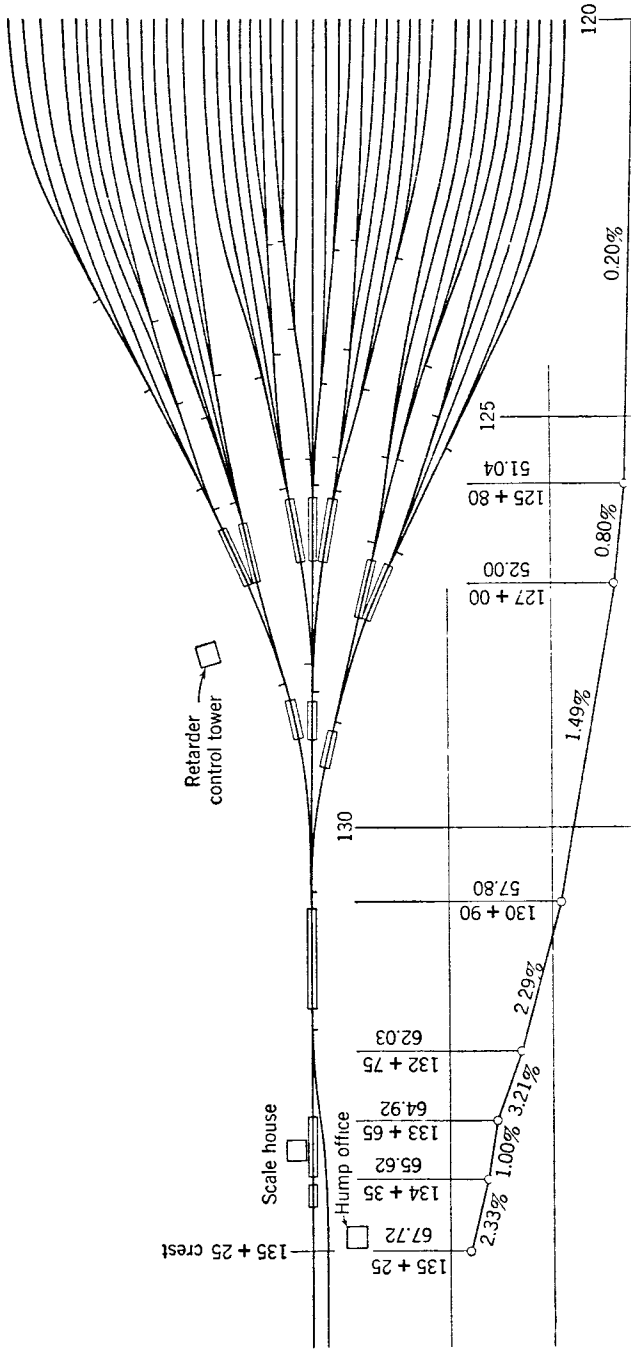


FIG. 11 Retarder leads and hump profile. Track groupings with 3-way lap turnouts for single tower retarder control (Southbound hump, Markham Yard, I C R R

quickly the cars or cuts and give adequate spacing for operation of switches. Increased height is necessary when scales or a double hump lead with diamond crossover is placed on the descending slope of the hump. In Fig. 46, $A = Dz + D_1z_1 + \Delta c$ where z and z_1 are the per cent gradients equivalent to the resistance of hard-rolling cars in the distances D and D_1 and equal respectively to 1.4 and 0 9 per cent gradients. Δ is the total degrees of central angle in distance $D + D_1$ and c is the equivalent curve resistance of a hard-rolling car expressed as feet of drop per degree of central angle It is usually taken as 0.045 ft.

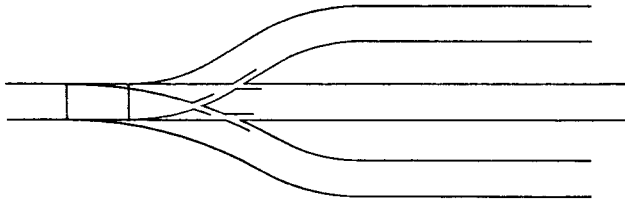


FIG 45. Lap or 3-way turnout with switch points separated

$B = D_1Y + C_1\Delta_1 + a + b$, where Y is per cent gradient equivalent of the easy-rolling car = 0.22%, and C_1, the curve resistance of easy-rolling cars = 0.025 ft per degree of central angle for the curvature, Δ_1, in distance D_1. Switch resistance in equivalent feet per switch in distance D_1 is a = 0.06 ft for single switches, 0 12 ft for lap switches. (Those factors are often omitted when compensated grades are used) An allowance in feet of drop, b, is sometimes made for accelerative increment over and above all other resistances in distance D_1. Beyond the last retarder non-accelerating grades of 0.20–0.15% are used depending on whether the hard- or easy-rolling car traffic predominates.

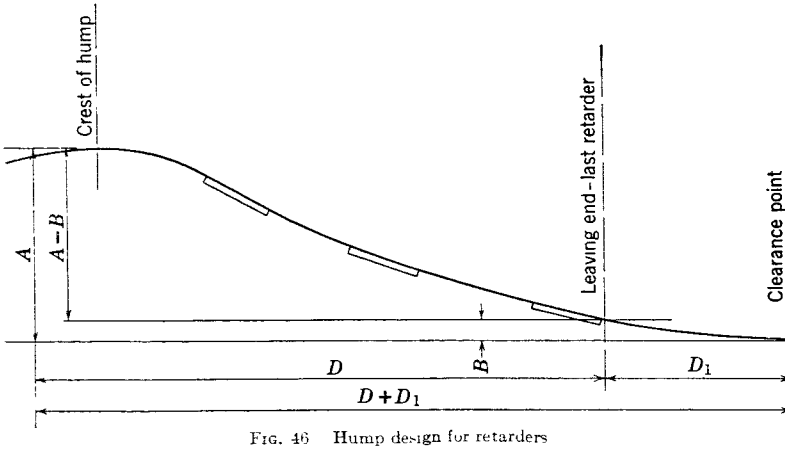


FIG. 46 Hump design for retarders

Inspection pits are sometimes placed under the ascending hump lead, permitting inspection through a glass window of the underside of the cars. Pits must provide for adequate drainage, lighting, a comfortable inclined seat for the inspector, a wiper to keep the window clear in bad weather, a means of spraying an identifying paint or whitewash on the trucks of bad-order cars and a means of communication with the hump master and retarder-switch operators so the cars can be diverted to the bad-order track.

Lighting must be provided for night operation. Floodlight towers are placed at each end of the yard facing into it. A row of towers with banks of lights facing into both the receiving and classifying yards is installed on or near the hump. Horizontal flood lights should be used to illuminate the crest of the hump

Communications are carried on by telephone, teletype, pneumatic-tube system, loudspeaker and talk-back system, inductive telephone, and radio. Adequate communications are vital to modern yard operation and most of the foregoing are usually required to maintain efficient contact between yard units, the general yard office and outlying points, be-

tween yard supervision and yard engine crews, between the yard and dispatcher, and between the yard and switch and transfer runs.

Track scales location depends on volume of weighed to unweighed traffic. The scale usually should not be placed on a drill track or hump lead when the volume of traffic to be weighed is less than 60% of the total since that location involves increased maintenance, inspection, and testing costs and is a hazard to trainmen. When weighing in motion, as on a hump lead, a minimum of 4 sec of weighing time per car is required and the car must be uncoupled and alone on the scale during the operation. Dead rails should be provided on all scales not designed to support the heaviest locomotives which might run over the weigh rails. Where only spot weighing is done a runaround track is desirable. Much additional detail is provided in the *AREA Manual*, 1953 Ed., pp. 14-5-1 to 14-5-92.

Enginehouses

Locomotive terminals are located to provide the minimum of traffic interference, delay, and running time in moving locomotives to and from their trains. The design should be coordinated with all other facilities and provide for future expansion. Servicing facilities are laid out in the order in which the servicing will be performed on inbound and outbound locomotives. The track layout should provide for an orderly movement of locomotives without reverse moves and with at least one runaround track to provide flexibility. Where turntables are used, approach and departure tracks should line across the table to the house, with enough tangent to permit all engine trucks to be on straight track before moving onto the table, and crossovers should permit engines not requiring turning to be serviced without use of the turntable.

Enginehouses. A circular house is preferred for steam locomotives but for diesel and electric locomotives or where a wye is used for turning, a rectangular house is better. Length of stall for steam locomotives should exceed overall length of engine and tender by 20 ft to provide trucking and walking space and room to detach the tender. Stall angles should permit tracks across a half circle to line up across the turntable. Inspection and repair pits, 4 ft wide and 4–4 ½ ft deep below top of rail, should be paved and well drained. Details of **drop pit** design for removing driving wheels are found in the *AREA Manual*, 1953 Ed., pp. 6-21-4 to 6-21-8. In unpaved houses, a concrete apron on each side of the pit extending about 24 in. beyond the rails provides a firm base or jacking pad on which to set jacks. **Smoke jacks** set over locomotive stacks, or exhausts draw off smoke, steam, or diesel exhaust gases. In a circular roundhouse the locomotive normally has its tender toward the table. Fixed-type smoke jacks of non-combustible, non-corrodible material should have a bottom opening at least 42 in. by 12 ft and should be set as low as clearance permits, usually 16 ft 6 in., at the ends and 15 ft 6 in. at the sides above top of rail. The upward slope should be uniform to a flue of 7 sq ft minimum cross section with an annular space of 2 ft around the flue for enginehouse ventilation.

Doors should have a minimum clear entrance of 13 ft in width and 17 ft in height and should be of non-corrodible materials. Roof design may be of the monitor or of the ridge type. Windows have advantages over skylights in providing adequate lighting. Heating, concentrated at the pits, should maintain a temperature not less than 50°F with openings closed. Piping must provide for air, steam, and water supply, and for boiler blowoff.

Rectangular House. The size of a rectangular house is set by the number and lengths of units to be simultaneously housed. Running repair tracks should be through tracks; heavy repair tracks may be stub-ended. To allow for a 12-ft width of working platform, minimum track centers should be 23 ft. Inspection and repair pits should be 4 ft wide, 4–4 ½ ft deep (from top of rail) and long enough to serve the maximum assembly of units. The floor between tracks should be 30 in. below top of rail to facilitate working on sides of locomotive mechanisms. Elevated platforms with side clearance of 5 ft 6 in. from track center line and 4 ft 10 in. above top of pit rail, as long as the longest diesel and with minimum width of 12 ft for two-track service is necessary for work at locomotive-floor level. A 25 or 30 ton overhead crane with a 5-ton auxiliary hoist is necessary for the handling of engines, generators, motors, trucks, and other heavy parts.

Turning Facilities. Most diesels and electrics need not be turned. Wye, loop, or balloon tracks may be used for turning both locomotives and trains. Turntables provide conventional and rapid roundhouse turning. The three-point bearing type is preferable to the balanced. Modern locomotives require table length of 80–135 ft. The design of the table and determination of loads will follow the principles of steel-structure design in Sect. 28, Vol. III. A deck-type table is more economical but a through table for a balanced type may be cheaper and facilitate drainage. Electric power or a compressed-air motor is used for power. The circle rail supporting the end or ends of the table should be on a concrete base, and the circle wall should be of concrete masonry with a steel or timber coping.

The pit floor should be paved and well drained. Easy access to turntable mechanism is desirable for inspection, lubrication, and repair. Proper spacing must be maintained between ends of rails of radial tracks and the table tracks, and a mechanical means must be provided to lock the table in line with radial tracks.

17. WATER STATIONS

Chemical treatment is necessary when the water contains objectionable amounts of incrusting or corrosive matter or of alkali salts and is desirable for practically all natural waters, especially those containing even a fraction of a grain of calcium or magnesium sulfate When water is boiled at a pressure of over 60 lb, the carbonates of lime and magnesia will precipitate and form mud or soft scale whose presence in the boiler is objectionable It may be blown off, but this wastes water and more or less heat. The sulfates of lime and magnesia when boiled form a hard scale which adheres to the tubes and is only removed with difficulty. Treatment with lime in a tank will precipitate the carbonates, which are thus easily removed The cost of this treatment is very moderate. The use of sodium carbonate (or soda ash) in a water containing sulfate of lime will produce carbonate of lime, which precipitates forming soft scale and also sulfate of soda which is objectionable because of its effect on "foaming." **Foaming** is primarily due to suspended matter in the water but is aggravated by the alkali salts, sulfate, carbonate or chloride of sodium, and is objectionable because it is difficult to maintain the proper amount of water in the boiler and keep wet steam out of the cylinders. If the sulfate hardness is very great the water is practically worthless, since too much alkali salts would be developed by treatment.

The quantities of pure reagents required to remove 1 lb of various scaling and corroding substances are given in Table 18. These should be increased to give equivalent quantities

Table 18. Reagents Required for Water Softening, per Pound of Substance

American Railway Engineering Association

Substance	Reagent and Amount	Foaming Matter Increased
Sulfuric acid	0 76 lb lime and 1.08 lb soda ash	1.45 lb
Free carbon dioxide gas	1.68 lb lime	None
Calcium carbonate	0 56 lb lime	None
Calcium sulfate	0.78 lb soda ash	1 04 lb
Calcium chloride	0.96 lb soda ash	1.05 lb
Calcium nitrate	0.65 lb soda ash	1.04 lb
Magnesium carbonate	1.33 lb lime	None
Magnesium sulfate	0.47 lb lime and 0.88 lb soda ash	1.18 lb
Magnesium chloride	0.59 lb lime and 1.11 lb soda ash	1.22 lb
Magnesium nitrate	0 38 lb lime and 0 72 lb soda ash	1.15 lb
Calcium carbonate	3.15 lb barium hydrate	None
Magnesium carbonate	3 76 lb barium hydrate	None
Magnesium sulfate	2.62 lb barium hydrate	None
* Calcium sulfate	2.32 lb barium hydrate	None

* In precipitating the calcium sulfate, there would also be precipitated 0 74 lb of calcium carbonate or 0.31 lb of magnesium carbonate, the 2 32 lb of barium hydrate performing the work of 0 41 lb of lime and 0 78 lb of soda ash, or for reacting on either magnesium or calcium sulfate, 1 lb of barium hydrate performs the work of 0 18 lb of lime and 0 34 lb of soda ash, and the lime treatment can be correspondingly reduced.

of pure reagents in case commercial products are used. The pounds of matter per 1000 gal may be found by dividing parts per 100,000 by 12 or grains per gal by 7. The total amount of lime must be sufficient to take care of the free carbon dioxide gas as well as the solids.

Concentration of foaming solids reaches the critical point between 100 and 200 grains per gal, depending on the character of the alkali salts and the amount of suspended matter in the water. Systematic and frequent blowing off and occasional complete blowing down and washing out at terminals are the best means for keeping below this limit.

The cost equals the cost of pumping, treating, and heating to boiler temperature the amount of water blown off, plus cost of washing out boiler. With the limit 100 grains per gallon the percentage of wastage equals the number of grains per gallon in the water supplied; for other limits the per cent varies inversely as the limit.

The **equipment** for a lime soda treating plant includes the tanks in which the chemical reactions and sedimentation are carried out, a smaller tank for mixing and storing the

chemical charge, and chemical pumps and feeder lines to deliver the required amount of chemicals.

The **zeolite process** uses sodium aluminum silicate, which first exchanges its sodium for the calcium and magnesium in hard water, and then exchanges these hardness ingredients for sodium when the zeolite bed is regenerated with sodium chloride. The process is simple, can readily be installed in existing structures, and is cheaper for water that is low in hardness; hardness removal is practically 100%, and waste products can be disposed of in the sewer. The process may cause foaming where the water is already high in soluble salts, and those with iron and manganese contents exceeding 1/4 gram per gallon require a prior treatment. Water for zeolite treatment should be clear enough not to exceed 20 on the Jackson scale. The plant includes a zeolite bed tank through which the hard water is passed and a tank, plus equipment, for the preparation of the regenerative brine solution and its introduction into the zeolite bed.

Supply. The AREA recommends the purchase of water where it can be obtained in sufficient quantity and of suitable quality at a reasonable price. Ordinarily the quantity should be sufficient, if economically possible, to give a 24-hr supply in 7 hr at terminal stations and 4 hr at intermediate stations, except for large stations when one may figure on 10-hr or even 20-hr pumping service.

Pumping. The AREA recommends the use of Table 19 for determining the size of pumping plants. See Sect. 12, Vol. II, Hydraulics and Pumping, for details of pumping.

Table 19

Quantity per 24 Hr, gal	Terminal Stations: Time Pump to Run in 24 Hr, Hours	Terminal Stations: Gallons per Minute	Intermediate Stations: Time Pump to Run in 24 Hr, Hours	Intermediate Stations: Gallons per Minute
2 000 000	20	1 666	20	1 666
1 750 000	20	1 458	20	1 458
1 500 000	20	1 250	20	1 250
1 250 000	20	1 042	20	1 042
1 000 000	20	833	20	833
900 000	20	733	20	733
800 000	20	666	20	666
700 000	20	583	20	583
600 000	20	500	10	1 000
500 000	7	1 189	10	833
450 000	7	1 071	10	750
400 000	7	928	10	666
350 000	7	838	10	583
300 000	7	714	10	500
250 000	7	595	4	1 041
200 000	7	476	4	833
150 000	7	357	4	625
100 000	7	238	4	416
50 000	7	119	4	208
25 000	7	60	4	104

Tanks of ordinary size are sometimes built of wood: redwood, cypress, fir, white pine, or other approved species. Steel is used especially for the larger sizes and permanent installations, and a few reinforced-concrete tanks have been constructed with varying re-

Table 20. Capacity, in U.S. Gallons, of Tanks of Various Inside Dimensions

Height, ft	Diameter, ft	Gallons	Height, ft	Diameter, ft	Gallons	Height, ft	Diameter, ft	Gallons
10	12	8 460	14	16	21 057	18	22	51 185
	13	9 929		18	26 650		24	60 914
	14	11 515		20	32 901		26	71 489
	15	13 219		22	39 810		28	82 910
12	14	13 817	16	18	30 457	20	24	67 682
	15	15 863		20	37 601		26	79 432
	16	18 049		22	45 498		28	92 123
	18	22 843		24	54 146		30	105 752

sults. The construction of wooden and steel tanks is now a specialized business which may be left to the builders under general specifications. (See the *AREA Manual*.) Their capacity usually varies from 10,000 to 80,000 gal. Two or even three smaller tanks are preferable to one excessively large tank.

The total cost of wooden tanks per 1,000-gal capacity may be roughly estimated at from $60–$70 for 100,000-gal tanks, $90–$100 for 30,000-gal tanks, $140–$150 for 15,000-gal tanks, and $260–$280 for 5,000-gal tanks. Steel tanks will cost from 25% to 50% more.

Water columns serve as hydrants to deliver water into the locomotive tender from the storage tanks or lines. The rate of flow depends upon the head and the length and diameter of the line as shown in the *AREA Manual*, pp. 13-2-10 to 13-2-14.

Track pans to supply water to the locomotive while it is in motion have been discussed in Art. 15, Miscellaneous Roadway Structures, p. 6-51.

18. OTHER LOCOMOTIVE FACILITIES

Coaling stations are placed both in terminals and at roadside locations as determined by the needs of operation. For light traffic, branch lines, and temporary locations, hand-shoveling or locomotive- or crawler-type crane operation may prove economical. To provide for the quick release of coal cars, a storage bin or platform should be erected about 5 ft higher than the top of car. Small portable loading conveyors also may be used.

Mechanical stations are designed to deliver coal to one, two, three, four, or more tracks. The station includes an elevating conveyor which raises the coal from a hopper underneath the receiving track into a storage and delivery bin from which it is dumped by gravity through a hopper bottom and delivery spout operated by chain hoist or a counterbalance. The receiving hopper should be at least 20 ft in length, about 16 ft in depth, and equipped with a car haul to aid in spotting cars. Approaching and leaving the hopper, the track should have a slightly descending grade to assist car movements. Storage bins should have a capacity equal to at least 30-hr operation. Hoisting equipment should permit raising of a 24-hr supply in one 8-hr shift. Sectionalized bins permit storing of different grades of coal and maintaining some operation while one section is undergoing repairs. Stations are built of wood, steel, or concrete. Units, prefabricated to a standardized design, are available commercially.

Sanding facilities should be adjacent to the fueling station for steam locomotives, so that both fuel and sand can be taken on at the same time. For diesels, facilities should be separate to prevent sand mixing with the fuel oil. Facilities must provide for unloading, storing, and drying the green sand, and for elevating, storing, and delivering the dry sand. Mechanical conveyors or compressed air conveys the dry sand into an elevated storage tank from which it flows by gravity through a pipe into the locomotives.

Fuel oil facilities, whether for steam or diesel use, must include provision to unload the oil from cars, hold it in storage, and deliver it to the locomotive. Oil may be unloaded from the bottom by gravity, or by pumping or air pressure. For steam locomotives, the oil may be discharged by gravity into steel or concrete boxes depressed between the rails and thence carried to a sump for pumping into a storage tank. The size of storage tank depends not only on daily consumption but on availability and market conditions. Storage is in cylindrical steel tanks on leveled earth foundations with steel or fireproofed-wood covers and surrounded with an earth dike with a volume 1 1/2 times the tank capacity. Ventilation must provide for the escape of gases evaporating from the oil. Delivery to steam locomotives is made through columns which can be swung into position and the valve opened from the tender and equipped with a drip bucket or other antidrip device. For diesel locomotives a section of hose with a pressure-type fuel nozzle and quick-acting coupling is connected to the fuel line on a fueling crane or in a box. Where low temperatures are anticipated, steam coils are placed in the tank to improve flowability. Provision must be included for metering, fire protection, and draining of water from lines and tanks.

Ashpits. The simplest form of ashpit is made by laying the rails on 12-in. by 14-in. wooden stringers, which rest on crossties, leaving a net space 4 ft wide. The stringers and ties are covered with old boiler plate to protect them from the hot ashes. Water service with hose to quench burning cinders and suitable drainage are essential. Although such a pit is justifiable for a road doing small business, mechanical means of handling ashes are recommended for pits serving 25 or more engines per 24 hours. The next step is to construct a pit with concrete walls and bottom and with a clear depth of about 3 ft, or a wider pit with one rail on one side wall and the other rail on a series of cast-iron or concrete columns. This affords greater freedom in the removal of the ashes. For large terminals the pits are often filled with water and the ashes removed by a grab bucket operated from a gantry, traveling, or locomotive crane.

Inspection pits are usually located on the inbound track near the terminal entrance, of a depth for convenient inspection, about 3 1/2 to 4 1/2 ft below top of rail, at least as long as the longest locomotive and well drained. Stairway access, ample lighting, and telephonic and pneumatic tube connections to the enginehouse are also recommended.

Wash racks for steam locomotives are preferably located between the cinder pit and turntable. Platforms level with the top of rail and equipped with drains and sloping surfaces to facilitate drainage provide a place for the cleaners to work and also facilitate drainage. The platforms should extend a minimum of 7 1/2 ft from the center of track and be about 10 ft longer than the longest locomotive and tender. If oil is used in cleaning, a means must be provided to keep it from entering the drainage system. Diesel and electric locomotives as well as cars may be washed on the foregoing racks or by passing them between large vertical cylindrical brushes. The brushes rotate and cleaning sprays are turned on automatically when the locomotive shunts the rails at the washer.

Oil Houses. The essential features are a fireproof place of economical construction for the storage of oil, combined with conveniences for its distribution. The oil should be stored in tanks located in the basement, the basement having masonry walls and the floor above being made preferably of reinforced concrete A trap door of fireproof materials should be the only opening into the vault beside the pipes. No illumination except electric lights should be permitted. The vault is surmounted by a house which may be of wood, and which is used for pumping the oil from the tanks into small cans for distribution. Since the oil is put in and removed entirely by pumping, the vault need not be opened except for occasional inspection and repairs.

19. OTHER TERMINAL FACILITIES

Icing Facilities. Icing platforms are usually built in conjunction with a storage house for natural ice, a plant making artificial ice, or a combination of both. The handling of ice may be by gravity or mechanical conveyor system. The former is cheaper to construct but slower to operate. The design should be such as to avoid back handling and duplicate handling as well as collison of ice cakes in transit to avoid breakage loss. Icing platforms are single or double decked. The AREA recommends single platforms be 13–16 1/2 ft above top of rail; double-deck platforms 13–14 ft for the lower, 20–23 ft for the upper. Both decks are 12 ft wide, the upper deck 6 ft from center line of track, the lower 7 ft. Crushed ice is handled in two-wheel carts on the upper, cake ice on the lower. Single platforms should be wider than 12 ft. to permit handling both crushed and caked ice, and set 7 ft from center line of track. (These close clearances may require permission from state commissions.) Where traffic is heavy icing tracks are placed on both sides of the platform to work two trains at once An endless chain platform conveyor may be used to move the ice. Provision for salt is required The platform must be well lighted for night operation. Roller mounted and pivoted spouts permit delivery of crushed ice to car bunkers.

Livestock must be loaded, unloaded, fed, rested, and watered. Facilities include stock pens and sheds, loading and unloading chutes both for single- and double-decked cars, watering troughs, and drenching hydrants. Construction is of heavy timber, simple but firm and substantial.

Produce terminals design will be governed in part by the type of produce marketing operation. Usually the structure must provide a large display and auction hall or perhaps a series of display rooms. Refrigerating facilities protect extremely perishable produce. Adequate provision must be made to dispose of large quantities of garbage. Platforms providing easy unloading of cars to the display floor should be supplemented by team tracks when purchasers unload directly from the cars.

Freight houses should be built of fire-resistant materials and in every way conform to requirements of the National Board of Fire Underwriters. Separate inbound and outbound houses or platforms are used only where volume of freight justifies the investment. For hand trucking, a minimum platform width of 30 ft for outbound and 40–60 ft for inbound houses is required. For tractor trucking, the outbound width is 50–60 ft and preferably free of columns. Minimum length of platform is determined by tailboard room requirements for trucks. The AREA recommends 1.12 tons of freight per foot of length. Maximum length depends on an economic combination of platform length and number of tracks required to handle a given tonnage per day. Platforms outside the house on the track side protected by a canopy shed eliminate the need for precise door spotting of the cars. These should be no less than 8 ft in width for hand trucking and 12 ft for tractor trucking. The street side should provide a canopy over the trucks or even provide bays inside the building. The street-side platform should be 3–4 ft above the street or driveway grade depending on the type of truck used but low enough to permit opening end doors over it.

Doors may be of the counterbalanced type, rolling shutters, or horizontal sliding. A maximum amount of opening is desirable on the trucking side. With no outside track platform (as in cold-weather country) continuous doors should be used. All doors should be at least 8 ft high. The platform should be approximately car-floor level, namely 3 ft 7 in. above top of rail or slightly higher. All clearances must conform to state requirements or the clearance diagram of Fig. 41.

Passenger stations generally have only one waiting room with rest rooms, ticket offices, baggage, express, and other facilities grouped around it. Ramps, surface-treated with non-slip material, are preferable to stairways and steps up to grades of 12%, but should be less than 10%. Long rises should use moving stairways. High, car-floor level platforms are recommended only for tracks used exclusively for passenger traffic, and where the volume of traffic is heavy and speed in moving passengers to and from cars is a factor as in handling commuting traffic. Umbrella-type sheds (with gutters under each side of platform canopy) are preferable for heavy-snow country, butterfly sheds (with a single gutter along the center of the canopy) for localities with little or no snow. Adequate parking facilities should be free of streets and station approaches.

20. SIGNALS AND INTERLOCKINGS

Signals have a twofold purpose of providing for the safe operation of trains and for their rapid movement with a minimum of delay time. To accomplish this, block signals are used to space trains and to provide speed control when approaching restrictive indications. Speed control is also provided by interlocking signals for crossover and diverging route movements. Train-order signals, usually of semaphore type, tell a train to reduce speed or stop to pick up train orders. Signal types include semaphore blade and color lights, color lights, position lights, and color-position lights. A common feature possessed by all is the showing of the most restrictive indication whenever anything goes wrong with the signal.

Signal Location and Spacing. Signal systems must provide advance warning of the most restrictive indications to permit the proper control of the train speed. Thus distance signals are set in advance of home signals in manual block and interlocking territory. In automatic-signal territory, each signal in turn serves as an advance signal for the signal ahead. All signals acting as distance signals must be set at least maximum braking distance apart for the authorized maximum speed involved. The signal mast is usually placed to the right of and adjoining the track to which it refers. On a signal bridge the signal is set over the right-hand rail (engineman's side) of the track governed. At turnouts, the signal is set back of the fouling point to protect converging and opposing train movements. Signals should be set for maximum visibility with due regard for background, curvature, sight distance, and prevailing weather conditions.

Track circuits are fundamental to all modern systems of signaling.

Simple closed track circuits are used in automatic block territory so that a train, by its presence on the track, will provide its own protection as it moves down the track. A simple track circuit, Fig. 47, is comprised of a source of current, usually a dry- or wet-cell

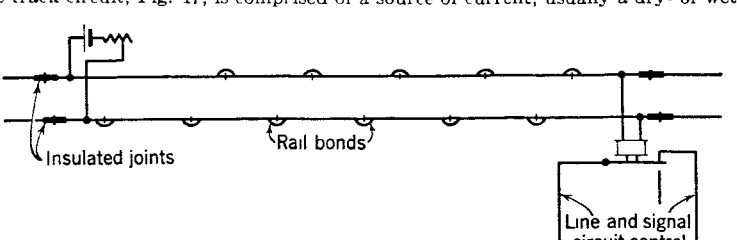


FIG 47. Simple closed track circuit.

battery of 0.5–1.5 volts delivering 0.115–0.300 amp, or a transformer winding, the rails forming the principal path for the current, and a track relay which is de-energized and permits the armature to fall away from the magnet when the track is shunted by the wheels and axles of a car or the current interrupted by a broken rail. Insulated joints isolate one circuit from another, bonds provide a continuous electrical path around joints (see p. 6-33), and a resistor in series with the battery prevents rapid discharge when a train short-circuits the battery. The track relay is used to control one or more line circuits which

actuate the signal mechanism to display proper indications. Traction direct currents are often combined with a-c signal circuits by using an a-c relay with a shielding impedance coil across the terminals which shunts out the direct current. Traction return currents are carried around insulated joints through impedance coils which permit the flow of direct current or of alternating current of another frequency but offer too much impedance to permit the flow of alternating current of undesired frequency

Dry insulation must be maintained in the insulated joints and ballast kept at least an inch below the base of rail Excessive leakage under conditions of wet or dirty track requires the signal maintainer to increase the voltage through the circuit to prevent false stop-proceed indications.

Coded track circuits make use of conventional track circuit elements. Instead of a steady current flow, periodic energy impulses are sent through the circuit from a continuously operating coding device or transmitter A code following track relay picks up during each "on" period of the transmitter and releases during the "off" period. A track circuit detector relay is held up continuously while the code-following relay follows the code, but is released when the code-following relay is de-energized by the shunting action of a train on the circuit. This in turn controls the signal indication The particular code being sent by the transmitter depends on the code which is picked up by a block approach relay depending in turn on the condition of the block ahead. Coded circuits offer the advantages of improved sensitivity to train shunting, less susceptibility to stray currents, a greater variety of signal indications with few line wires, and a greater length of track circuit without cut sections. The coded circuit can also be picked up inductively, electronically amplified and used to operate cab signals and automatic train control. Coded circuits (in line wires) form the basis for modern CTC operation.

Interlockings are defined as an assemblage of switches, signals, locks, and levers so interlocked that the movement of each can only be made in a predetermined order Crossover and diverging route layouts, junction switches, railroad crossings at grade, and movable span bridges may have their controls and signals assembled in an interlocking plant. An interlocking must prevent setting up conflicting signals or conflicting or incomplete routes. Mechanical, electromechanical, and electropneumatic plants make use of mechanical interference in the control lever racks to provide the lock. Recently designed **all-relay** plants provide interlocking by a series of intercoordinated circuits and relays in the control circuits. On the ground outside the plant further protection is obtained by the use of track circuits which provide:

(*a*) *Section locking* to prevent throwing a switch under a train.

(*b*) *Route locking* whereby a route once established and accepted by a train cannot be changed while a train is in the route.

(*c*) *Sectional route locking* which releases each section of a route for other use as soon as it has been cleared by a train.

(*d*) *Approach or time locking* which prevents the taking away of a signal which may have been accepted by a train until a mechanical time release has run down.

(*e*) *Switch indication circuits* whereby the switch points must move completely into a tight fit with the stock rail in order to release an electric lock on the lever in the plant.

(*f*) *Traffic locking* giving signal protection against opposing moves within the plant.

MOTIVE POWER AND OPERATION

21. TRAIN RESISTANCE

Definition. Train resistance is that resistance to motion encountered by a train moving at constant speed on level track in still air. It is expressed in pounds per ton An empirical formula for train resistance takes the general form $A + BV + CV^2$ in which A represents a coefficient varying with the gross axle load and comprising mainly journal or axle bearing friction, rolling friction, and the resistance which results from track depression under load The second term includes that portion of journal friction which varies with speed and also the effects of flange resistance and of swaying and concussion. The third term represents air resistance almost entirely.

Davis Formula. Mr W. J. Davis, Jr., combined the results of several sets of field tests into an empirical formula similar to the foregoing with the form $R = 1.3 + \frac{29}{w} + bV + \frac{CAV^2}{wn}$ where R is the unit resistance in pounds per ton, w is the weight per axle, n

the number of axles, V is the velocity in mph, A the cross-sectional area in square feet, C the drag coefficient, and b an experimental coefficient.[16] Table 21 presents Davis's accepted values for b, C, and A.

Table 21

	b	C	A
Freight cars	0.045	0.0005	80–90
Passenger cars	0.03	0 00034	80–90
Locomotives (steam, electric, and diesel-electric)	0.045	0.0024	
Unstreamlined			
50 tons weight			105
70 tons weight			110
120 tons weight			120
Streamlined		0.0017 *	
Motor and multiple-unit car trains	0 045	0.0024	100–110

* Not originally proposed by Mr. Davis but recommended by the author on the basis of a 30% average reduction in the drag coefficient due to streamlining.

Internal friction in the cylinder, valve motion, pins, and linkages of a steam locomotive causes additional internal resistance losses of an approximate value in pounds equal to *20 times the weight on drivers in tons.* Internal friction is included in the motor and gear efficiency used in determining tractive effort curves for electric and diesel-electric locomotives and need not be otherwise considered.

The Davis formula, based primarily on the studies made by Professor Edward C. Schmidt, University of Illinois, has wide acceptance and is accurate for speeds through the 10–40 mph ranges within which most train resistance studies for tonnage rating and location purposes are made. It is reasonably accurate up to 50 mph speeds. For freight cars operating in the higher speed ranges, curves reflecting the tests made by Professor J. K. Tuthill extending the studies of Professor Schmidt into higher speed ranges should be used (see Fig. 48).

Streamlined passenger-train resistance, especially at high speeds, is best found by use of A. I. Totten's revision of the third term, that for air resistance, in the Davis formula.[17]

Streamlined locomotives:

$$Ra = (0.023\sqrt[3]{L_L} + \Sigma K_L)V^2$$

where Ra = total air resistance of locomotive with length L_L in feet moving V mph, K_L is the sum of appropriate factors from the table following these paragraphs and based on the degree of streamlining.

Unstreamlined vestibuled cars:

$$Ra = \left[0.0031P_c\left(\frac{L_c}{100}\right)^{0.7} + \Sigma K_c\right]V^2$$

where P_c, perimeter of the car, may be taken as 40 ft if not otherwise known and L_c is the car length. ΣK_c again represents the streamline factors.

Streamlined cars:

$$Ra = \left[0.001735P_c\left(\frac{L_c}{100}\right)^{0.88} + \Sigma K_c\right]V^2$$

where P_c is the car perimeter (taken as 35 ft when not otherwise known) of length L_c in feet.

For an entire train including a power unit integral with the train:

$$Ra = \left[0.0020P_c\left(\frac{L_T}{100}\right)^{0.8} + \Sigma K_c\right]V^2$$

where L_T is the overall length of train.

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[16] Tractive Resistance of Electric Locomotives and Cars, by Mr. W. J. Davis, Jr., *General Electric Review*, October 1926, pp. 685–708.

[17] Resistance of Lightweight Passenger Trains, by Mr. A. I. Totten, *Railway Age*, July 17, 1937, pp. 75–79.

[18] John K. Tuthill, High Speed Freight Train Resistance, Its Relation to Average Car Weight, University of Illinois Engineering Experiment Station *Bulletin* 376, Jan 17, 1948.

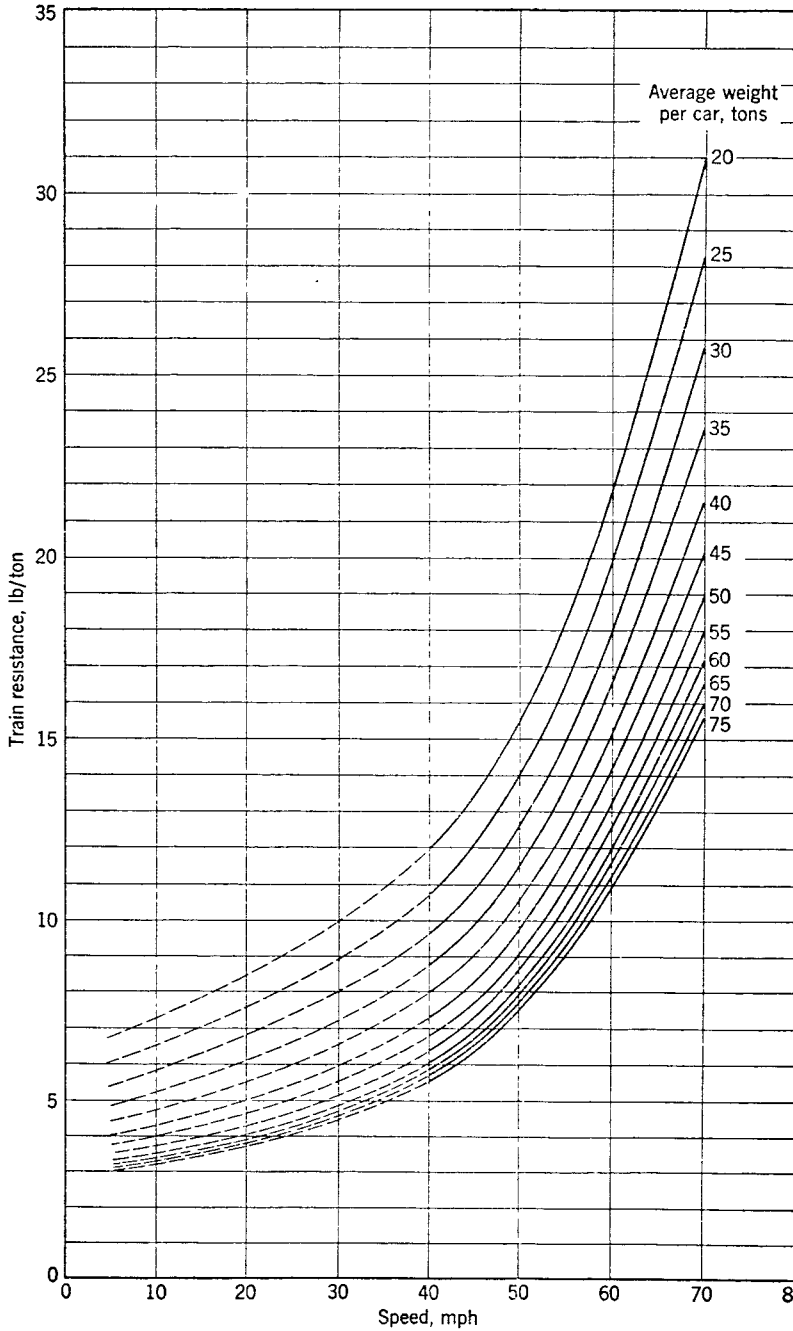

FIG. 48. Schmidt-Tuthill train resistance curves (relation of resistance to speed for various weights per car). Dashed lines show Schmidt's tests, solid lines, Tuthill's tests.[15]

Resistance Factors for Degrees of Streamlining

(As summarized by A. I. Totten from the tests of DeBell and Lipetz[19])

K_1 For power or leading car, nose well streamlined = 0.
For nose bluntly streamlined = 0.000036 × cross-sectional area of nose at full section, including trucks, in square feet.

K_2 For tail shape of rear car well streamlined = 0.
For tail bluntly streamlined = 0.000061 × cross-sectional area of tail at full section, including trucks, in square feet.

K_3 For power-car trucks, both faired = 0.
For unfaired trucks = 0.00026.

K_4 For faired trailing-car trucks = 0.
For unfaired trucks = 0.00013 × number of trailing-car trucks.

K_5 For smooth diaphragms = 0.
For cowled diaphragms = 0.000037 × P_c × number of diaphragms.

K_6 For no bulge of power car = 0.
For bulge of good streamline shape = 0.00032 × cross-sectional area of bulge in square feet.
For bulge of relatively poor streamline shape = 0.00051 × cross-sectional area of bulge in square feet.

K_7 For closed wheel shrouds on streamlined locomotives (all wheels completely enclosed) = 0.
For open shrouds (2 ft by 2 ft 6 in. inspection openings over the driving-wheel journals) = 0.0005 × total number of openings.
For short shrouds (driving wheels and tender trucks completely exposed) = 0.0182.

K_8 For helmet nose on streamlined locomotive = 0.
For straight nose = 0.0021.
For round nose = 0.0026.

K_9 For round-top boiler shape on streamlined locomotive = 0.
For cowled top (domes and fittings enclosed in longitudinal cowl above boiler shroud) = 0.0035.

Extra Resistance of Starting. The resistance of journals and axles is very much greater at the instant of starting and until they have become warmed up. Cars which have been stationary overnight, especially in cold weather, will become "frozen up" and will require a much greater force to start them, but the added resistance is only momentary and consumes but little energy, measured in foot-pounds. However, as much as 15–20 miles of running may be required to warm the lubricants to normal conditions. The frequent practice among locomotive engineers of backing the locomotive for a few feet and then immediately reversing and starting ahead has three effects, all of which help to start the train: (1) the journals are loosened from the somewhat rigid condition they will assume even during a short stop; (2) the springs in many of the couplers are more or less compressed during the backward movement, and their expansion materially assists in starting the cars; (3) if the train is very long, the total slack in the couplers is very considerable, and the locomotive will have moved forward several feet and will have a considerable velocity before the last car starts; the cars are therefore started one by one. Since so much depends on the method of handling the locomotive, there is a corresponding variation in the results of tests which have been made to determine the value of this resistance. Thirty-five tests on the Rock Island system, with trains of 34–45 cars, gave results varying from 10.6 to 18.2 lb per ton. The weighted mean of the values was 14.1 lb per ton. The same tests quoted 30 lb per ton when a train had stood overnight and was "frozen up"; also a resistance of only 6 lb when the stop was merely instantaneous. Other tests have shown an average of 14 lb per ton. An average of 28 lb per ton is often assumed for steam locomotives.

Owing to the above practice in starting trains and to the increased tractive force at starting, this extra resistance is of little importance, i.e., it practically never limits the weight of the train.

Roller bearings decrease journal friction and are particularly helpful in making smooth, easy starts with heavy trains. First cost and danger of losing cars so equipped in interchange have retarded any extensive adoption of roller bearings to freight cars. Actual reductions in journal friction of 80% have been noted at starting with an average reduction in journal friction of 10% between 5 and 35 mph. Average starting resistances of 1.5 lb per ton for cars and 3–4 lb per ton for locomotives have been obtained in tests. At high speeds, the value of roller bearings is debatable.

Grade resistance is 20 lb per short ton (2000 lb) for each per cent of grade. This rule is not mathematically precise but the error is less than 0.5% for a 10% grade, 0.08% for a 4% grade, and is inappreciable for ordinary railroad grades. The **grade of repose** is the

[19] Air Resistance of Passenger Trains, by J. W. DeBell and A. I. Lipetz, *Railway Mechanical Engineer*, December 1935, pp. 496–502.

grade on which the effect of gravity just equals the tractive resistance. If a wagon or car is started down such a grade, it would continue to move indefinitely at a low velocity as long as the conditions are constant. The total resistance up such a grade is precisely twice that on a level.

Curve resistance is considered the equivalent of a 0.04% grade per degree of curve, or 0 8 lb per short ton per degree of curve.

Compensation for Curvature. The principle involved is that the grade should be reduced by such an amount that the saving in grade resistance will compensate for the additional resistance caused by the curvature. The proper rate of compensation evidently

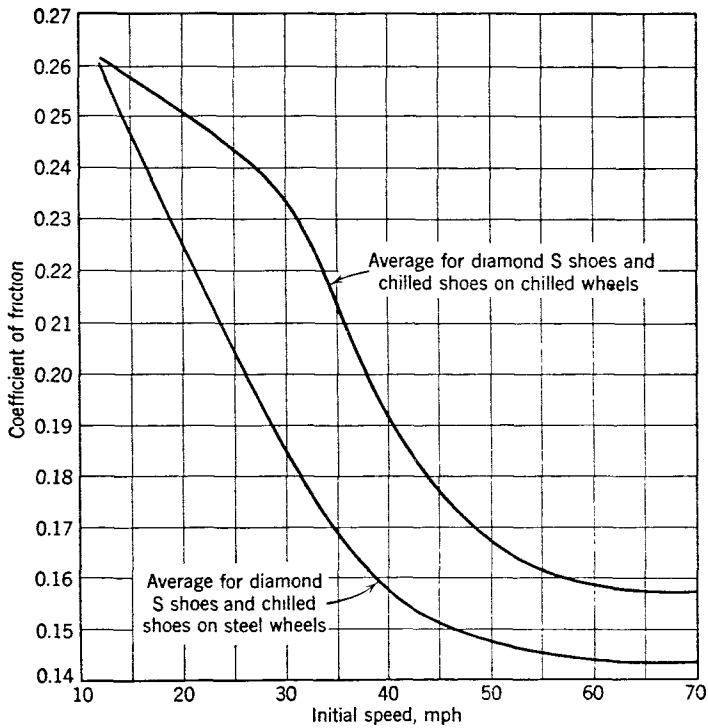

FIG 49 Relation between coefficient of friction and speed for brake shoes on chilled and steel wheels. (Based on The Friction of Railway Brake Shoes, Its Variations with Speed, Shoe Pressure and Wheel Material, by Edward C. Schmidt and Herman J Schrader, *U of Illinois Engineering Experiment Station Bull* 257, June 1933)

depends somewhat on the speed of the train; it also depends largely on the resistance to the rotation of the car trucks about their kingbolts, being very greatly reduced when ball bearings are used under the center and side plates; it is apparently far less per degree of curve on very sharp curvature than on easy curvature. The rate of compensation should be made greater on a curve occurring immediately above a stopping place for trains. Since the added resistance of curvature virtually increases the grade, it is unimportant whether there is any compensation on curves which are on minor grades, provided the total resistance does not become greater than that of the ruling grade. The rules recommended by the AREA are as follows: (1) Compensate 0 03 ft per degree when the length of curve is less than one-half the length of the longest train, when the curve occurs within the first 20 ft of rise of a grade, or when curvature is in no sense limiting. (2) Compensate 0.035 ft per degree when the curve is between one-half and three-fourths as long as the longest train, and when the curve occurs between 20 ft and 40 ft of rise from the bottom of the grade. (3) Compensate 0.04 ft per degree when the curve is habitually operated at low speed, when the length of curve is more than three-fourths that of the longest train, when the elevation is excessive for freight trains, or at all places where curvature is likely

to be limiting. (4) Compensate 0.05 ft per degree wherever the loss of elevation can be spared.

Brake resistance wastes the kinetic energy of a train by transforming it into heat. Unless such energy has been acquired by running downhill it constitutes a double loss, since application of the brakes also requires power. For ordinary stops it may be considered as equivalent to a 7.5% grade or 150 lb per ton, whereas for emergency stops it may reach double that amount. Braking force may be computed as equal to $pRwef$ where p is the ratio between actual brake cylinder pressure and basic train line and auxiliary reservoir pressure, w is the light weight of the car in pounds and R is the ratio between the light weight and the nominal weight, equal to 0.70 to 0.90. The efficiency of the brake rigging system is $e = 95$–98%, and f is the coefficient of brake-shoe friction as shown in the curves of Fig. 49. The actual braking distance, L_b, is determined by using the braking force just derived as F_b in an expression from the principles of acceleration (see p. 6-92) where $L_b = \dfrac{70W(V_1^2 - V_2^2)}{F_b}$. The distance to inaugurate brake action, equal to $1.467Vt$, must also be added, where V is the initial speed in mph and t is the application time in seconds with an average value of about 0.1 second per car for conventional brakes, 0.75–1.0 sec for electropneumatic brakes for the entire train. Since many of the foregoing factors are difficult to determine with any degree of accuracy, a factor of safety of 30% should be used or $L = \left[1.476Vt + \dfrac{70W(V_1^2 - V_2^2)}{pRwef}\right](1.3)$. For most purposes individual railroads rely on braking distance curves obtained from test train runs.

22. MOTIVE POWER

Locomotive Classification. Locomotives may be classified as to source of propulsion energy (steam, electric, gas turbine, or diesel), by service assignment (passenger, freight, yard switching), or by wheel arrangement. The last classification is in widespread use.

An electric or diesel-electric uses numbers to describe the pilot and trailing wheels, but the number of driving wheels is indicated by a letter of the alphabet corresponding in alphabetical position to the number of driving axles. Thus a G-G-1 type electric with two pilot and two trailing wheels and two three-axled-power trucks would be classed as a 2-C-C-2.

For steam locomotives the first number of a series denotes the number of pilot wheels; the second (and third for mallets), the number of driving wheels; and the third, the trailing wheels. A Northern type with four pilot wheels, eight drivers, and four trailing wheels would be described as a 4-8-4. In European practice the same engine would be a 2-4-2, counting the axles instead of the wheels.

Table 22 includes a few representative examples.

Tractive force of a locomotive is its pulling force. It is limited by the capacity of the steam, electric, or diesel generating equipment, by the conversion equipment—cylinders or motors—and by the frictional force or adhesion between drivers and rail. This last is a function of the weight on drivers and of the coefficient of friction between the wheel and the rail. The latter may vary from 0.10 for slippery, frosty rails to 0.33 for clean dry rails or with the use of sand. An average value of 0.25 is normally used in locomotive design although 0.30 or better is sometimes used for rotative motion engines, electric and diesel, where absence of the pulsating force application of steam engines permits a more uniform grip on the rails. The corresponding reciprocal of the coefficient of friction called the factor of adhesion, usually equal to 4, is more often used by motive-power men. A locomotive with a theoretical tractive effort which exceeds the weight on drivers ÷ factor of adhesion is over powered and will slip its drivers on hard pulls. Locomotives must be designed and powered according to the proposed service assignment. There has been a continuing increase in tractive effort in recent years. Modern locomotives have tractive efforts in the 60,000- to 80,000-lb range.

Drawbar Pull. Locomotive tractive effort minus the sum of all resistances (train, curve, and grade) is the net tractive force or drawbar pull.

Diesel-electric locomotives are essentially electric locomotives with self-contained (diesel-driven generators) power plants. Early diesels in switching service used a 4-cycle engine, the weight of which was advantageous in giving more adhesion in switching. General railway use has followed the adoption of a 2-cycle engine with air-intake-compression stroke and a power-exhaust stroke and turbo supercharging of 4-cycle engines. The heart of the diesel is its fuel pump and injector combination, which measures the fuel and delivers it pressurized into the cylinder through an orifice in the injection nozzle.

Ignition is effected when the cylinder pressure reaches 600 lb/sq in. at a temperature of 1000°F. The burning fuel raises the cylinder pressure another 200–300 lb/sq in.

Electric Coupling. Since the diesel is a constant-speed, constant-torque machine with little or no overload capacity, some form of transmission is required to reduce high engine speeds to those suitable for locomotive drive wheels. Electric coupling provides that reduction and flexibility of control. A generator mounted on the engine shaft supplies electric current to traction motors. The usual type of generator is self-excited shunt wound with a variable field current to keep it operating at full engine capacity. The traction motors, normally four per cab and geared to the axles, are d-c series wound.

Tractive Effort. Tractive-effort curves are computed from data on generator traction motors and auxiliaries with an overall limitation imposed by the horsepower rating of the diesel engine. Torque of the motors, and therefore tractive effort, varies with the square of the current. Voltage and speed vary inversely as the current. The tractive-effort curves furnished by the manufacturer are considered to be sufficiently accurate for most purposes. Using the general horsepower formula, $TE_v = \frac{(hp - hp_a) \times 375}{V} \times e$ where hp is the rated diesel engine output, hp_a is that furnished to auxiliaries, e is motor and gear efficiency, usually 80–85%, and V is the speed in mph. For a conventional design of U. S. locomotive this formula may be rewritten as $TE_v = \frac{hp \times 308}{V}$. The general formula for all electric propulsion given on p. 6-83 may also be used, again taking 80–85% as a value for e, the efficiency.

Tractive-effort curves furnished commercially by locomotive manufacturers, Fig. 50*a*, show the theoretical maximum available as well as the continuous ratings for various gear ratios. Characteristic data will also show the maximum length of time the continuous ratings can be exceeded under overload conditions without overheating the motors (usually 15–20 min). Since the diesel engine is not directly coupled to the drivers, its full horsepower is available over all speed ranges. Accelerating capacity is high therefore, throughout the low and medium speed ranges, making the diesel-electric particularly suitable for service involving frequent starts and stops as in switching, yard, and transfer work

Dynamic Braking. On descending grades, the motor leads may be reversed causing the motors to act as generators. The resulting "drag" on the axles can be utilized to control train speed with little or no resort to train brakes and the costs and delays the use of those entails. The generated current is wasted as heat through cooling grids on the cab roof.

Other characteristics include rotative rather than reciprocating torque delivery to the drive wheels with a reduction in dynamic stresses in the track and a noticeable reduction in track maintenance costs. The diesel-electric is smooth starting and relatively easy to handle. Several units can be controlled from one control position. Availability for service is about 85–95% of the time as compared with 50–80% for steam locomotives.

Economies. Diesel-electrics average about twice as much in first cost as equivalent steam locomotives, $300,000–$600,000, depending on the number of units per locomotive. Intensive use must be made to cover these costs and carrying charges. Economies result from greater availability and from a decrease in fuel costs, track maintenance, and delays; from eliminating the extra crews required in double heading and helper service of equivalent steam power; from the elimination of coal, water, and ash facilities (where the division is 100% dieselized); and from a reduction in terminal service time and expense. Authorities are still in argument as to the exact numerical value of these savings. The effects of diesel-electric power on location are discussed on p. 6-97

Mechanical Transmission. Direct-drive diesels have been successful only in small locomotives (less than 500 hp), in contractor and industrial switching service. Heavier loadings involve too much strain and maintenance on main shafts and mechanical transmissions. Hydraulic-torque converter systems have been applied to light-weight rail cars which are meeting with success in handling mail, baggage, and passengers, on branch lines where operating costs of equivalent steam service are too high. There has as yet been no application of this system to road service locomotives.

Gas Turbine Locomotives. Gas turbine locomotives use the burned gases from low-grade fuel oils to operate a turbine. The turbine drives an electric generator furnishing current for traction motors. This design is past the initial experimental stage and is in regular service on one large western railroad Operating characteristics are similar to any electric-drive locomotive. Disadvantages include noise and the necessity of keeping the engine running full speed even when the locomotive is stopped.

Determination of Rated Tractive Force for Steam Locomotives. The tractive force of a locomotive may be determined by actual measurement with a dynamometer car, or at a

Table 22. Representative Locomotive Classification

Steam Locomotives

Whyte Classification	Wheel Arrangement	Name or Type	Notes
4-4-2	◿ ○○ ○ ○ ○	Atlantic	High-speed passenger, old
4-6-0	◿ ○○ ○ ○ ○	10-wheeler	Heavy, passenger, old
4-6-2	◿ ○○ ○ ○ ○ ○	Pacific	Heavy, high-speed passenger
4-6-4	◿ ○○ ○ ○ ○ ○○	Hudson	Heavy, high-speed passenger
2-8-2	◿ ○ ○ ○ ○ ○ ○	Mikado	Freight
2-8-4	◿ ○ ○ ○ ○ ○ ○○	Berkshire	Heavy, freight
4-8-2	◿ ○○ ○ ○ ○ ○ ○	Mountain	Heavy, fast passenger Heavy, fast freight
4-8-4	◿ ○○ ○ ○ ○ ○ ○○	Northern	Heavy, fast freight
4-8-4	◿ ○○ ○ ○ ○ ○ ○○	Niagara	Heavy, high-speed passenger
2-10-2	◿ ○ ○ ○ ○ ○ ○ ○	Santa Fe	Heavy, slow freight
2-6-6-2	◿ ○ ○ ○ ○ ○ ○ ○ ○	Mallet	Heavy, slow freight

Electric Locomotives

Whyte Classification	Wheel Arrangement	Standard or European System	Notes
0-4-0	○○	B	Switching
2-4-2	∘○○∘	1-B-1	
4-4-4	∘∘○○∘∘	2-B-2	
0-4 + 4-0	○○ + ○○	B + B	
2-4 + 4-2	∘○○ + ○○∘	1-B + B-1	
2-6 + 6-2	∘○○○ + ○○○∘	1-C + C-1	Freight
4-6 + 6-4	∘∘○○○ + ○○○∘∘	2-C + C-2	Passenger
2-6-2 + 2-6-2	∘○○○∘ + ∘○○○∘	1-C-1 + 1-C-1	Freight
4-6-2 + 2-6-4	∘∘○○○∘ + ∘○○○∘∘	2-C-1 + 1-C 2	Freight
2-8-2 + 2-8-2 + 2-8-2	∘○○○○∘ + ∘○○○○∘ + ∘○○○○∘	3 (1-D-1)	Virginian Railway

Diesel-Electric Locomotive

0-4-4-0	○○ - ○○	B-B	Drive on 2 trucks 4 axles
2-2-2-2-2-2	○∘○-○∘○	A-1-A-A-1-A	4-axle drive, 2 idler axles
0-6-6-0	○○○ - ○○○	C-C	Drive on 2 trucks 6 axles

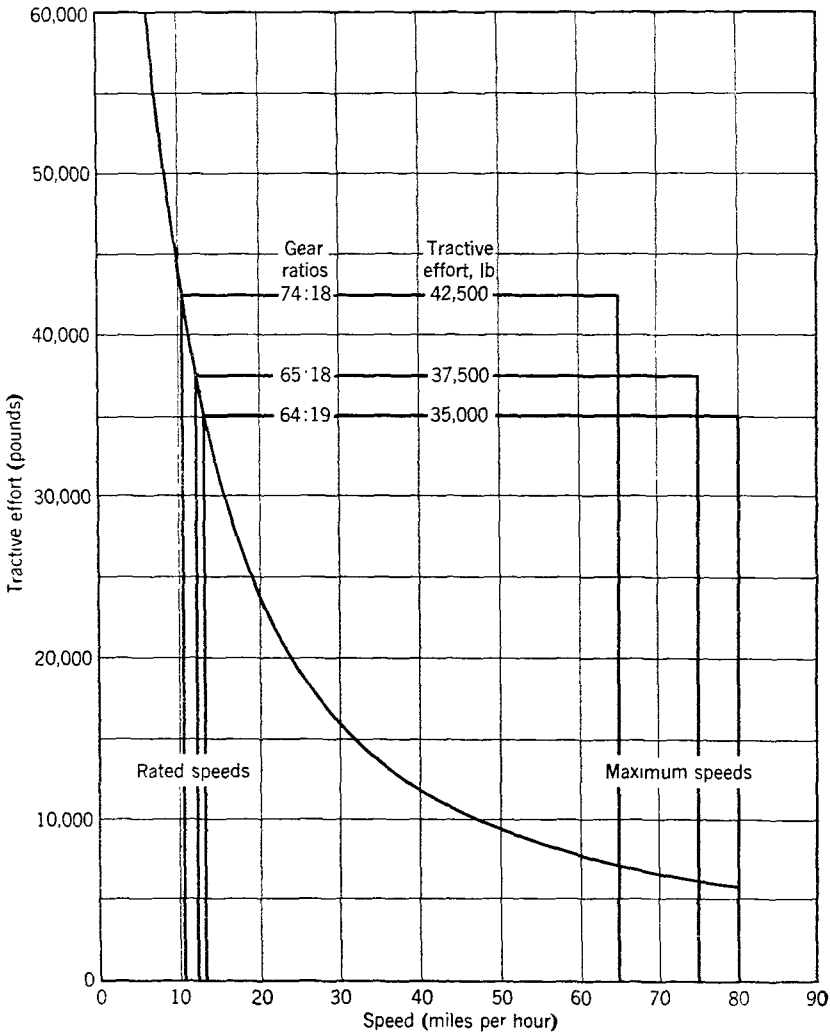

FIG. 50a. Tractive-effort curve for diesel-electric locomotive.

locomotive testing plant, or by computation. With a dynamometer car, speeds, alignment, and grades must be accurately known in order to make the proper allowances. The AREA recommends that speeds be determined to the nearest 0.1 mile per hour. Cylinder power may be determined by means of indicator cards and used to find tractive force. By computation: for low speeds (or maximum tractive force) one may simply take one-fifth to one-fourth the weight on the drivers, assuming reasonably good design. For higher speeds of steam locomotives, recourse must be had to the theoretical formula with empirical coefficients or to empirical methods. Let P = the boiler pressure in pounds per square inch; S, the length of stroke in inches, d, the diameter of piston for simple engines; D, the diameter of drivers; and T.F. the tractive force at the circumference of the drivers. In a simple engine the work done by both cylinders during a complete revolution of the drivers = piston area × effective average cylinder pressure × stroke × 2 × 2. But the work also equals the tractive adhesion developed at the circumference of the drivers × the distance traveled by the drivers during one revolution, which of course equals the circumference of the drivers. Therefore, for simple engines,

$$\left.\begin{array}{r}\text{Theoretical}\\ \text{tractive force}\end{array}\right\} = \frac{\text{Piston area} \times \text{Effective cylinder pressure} \times \text{Stroke} \times 2 \times 2}{\text{Circumference of drivers}}$$

The effective area of the piston is reduced about 1.5% on account of the area of the piston rod. The effective energy at the wheel rim is reduced on account of friction of the piston, piston rod, crosshead, and the various bearings. The effective steam pressure in the cylinder is always considerably less than that in the boiler, even at low speed and full cutoff. These reductions may be allowed for by figuring the steam pressure (effective at the drivers) to be 80–85% of the boiler pressure. Therefore, dividing both numerator and denominator by π(3.1416) we have

$$TF = 0.85Pd^2S/D \qquad \text{(for simple engines)}$$

This is the rated or starting tractive effort and may be used with no appreciable inaccuracy up to speeds of 15 mph. For a multicylindered engine having N cylinders

$$TF = \frac{N}{2} \times \frac{0.85Pd^2S}{D}$$

Thus a 4-cylindered single expansion articulated engine would have a $TE = \frac{1.7Pd^2S}{D}$.

Few compound engines are in use and none have been built in recent years. The surplus steam at starting and low speeds is sometimes used to give additional tractive effort by using it in the cylinders of a small geared booster engine replacing the trailing wheels under the firebox. The increased tractive effort in pounds is

$$T_B = \frac{KPd^2Sr}{D}$$

where r = gear ratio.

K = ratio of mean effective pressure in the cylinder to boiler pressure. It varies from 0.77 for 70% cutoff to 0.80 for 75% cutoff.

Tractive Effort at Various Speeds. At starting and low speeds, the boiler can evaporate water into steam much faster than cylinders can use it so that tractive effort is limited by cylinder size. At higher speeds, the cylinders can use steam faster than the boiler can produce it and boiler design becomes the limiting factor. As greater speed is required, more efficient use of the steam can be made by shortening the cutoff, admitting steam for a portion of the stroke only, and allowing the expanding steam to complete the stroke. As the cutoff is shortened, available tractive force is bound to fall off more rapidly. Locomotive tractive force is thus dependent on the steaming capacity of the boiler as well as cylinder and driving-wheel dimensions. The steaming capacity depends upon the area of evaporative surface (the size of boiler and tubes), the firebox area, and upon the grate area, quality of fuel, and method of firing. A single curve of tractive effort vs. speed will not represent the speed-tractive-effort relation unless a specific cutoff and steam production are assumed. The relation must be represented by the area occupied by a group of curves each of which corresponds to a certain speed-steam relation. Nevertheless, in the interest of simplification, single curves are drawn based on average representative values of evap-

oration and cutoff. These, as given herewith, may not be accurate enough for locomotive design purposes but are sufficiently accurate for use in railroad location studies.

Baldwin Method. Two curves are plotted, one based on cylinder design and the other on boiler capacity. The limiting or lower portions of each are connected by a smooth curve

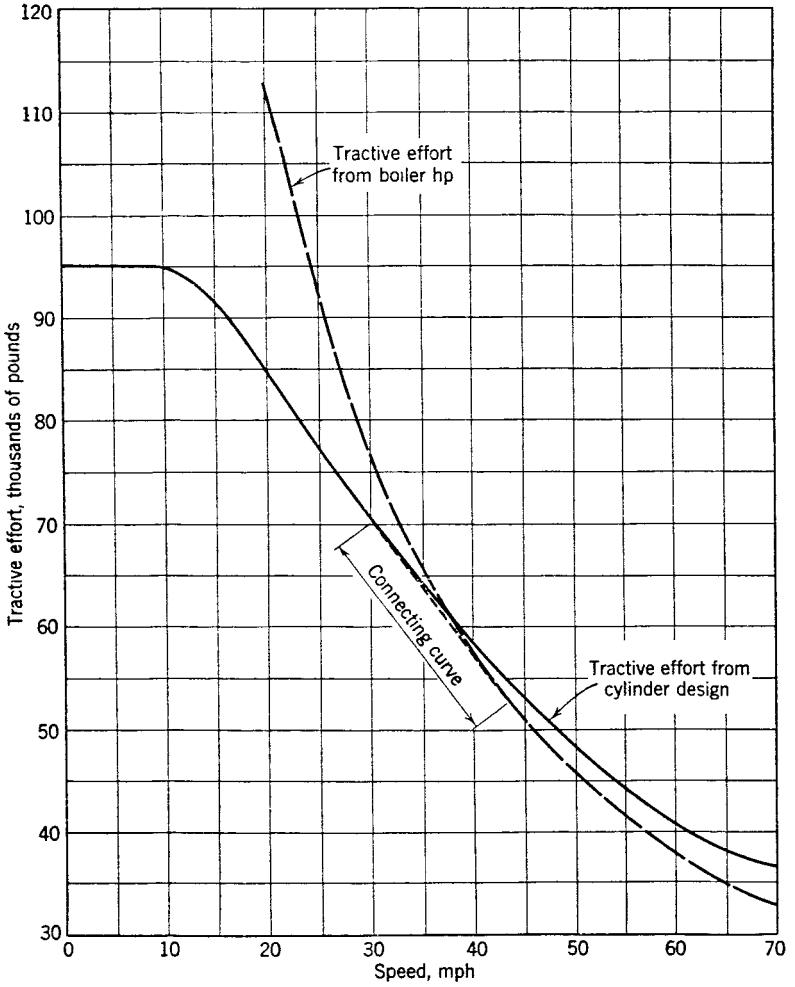

FIG 50b "True" tractive-effort curve (2-10-2 type locomotive, using the Baldwin method)

to form a "true" tractive-effort curve, Fig 50b. The cylinder design curve uses the relation

$$TE_v = \text{tractive effort at any speed} = (mep)\frac{d^2S}{D}$$

where $\frac{d^2S}{D}$ is the engine constant and *mep* is the mean effective pressure at any speed obtained from the curve of Fig. 51. Figure 51 gives the *mep* in the cylinder at any speed in terms of per cent of boiler pressure for various piston speeds which in feet per minute $= \frac{56VS}{D}$, where V is the speed in miles per hour, S is the stroke in inches, and D is

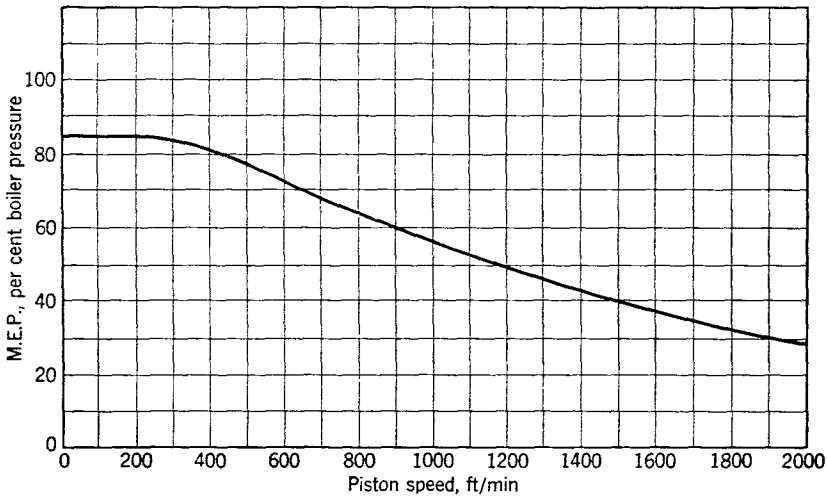

FIG 51 Mean effective pressure curve at various speeds, mph (From *Locomotive Data*, 12th Ed., p 22, Baldwin-Lima-Hamilton Corp., 1944)

driving wheel diameter in inches. Figure 51, based on average cutoff values, is more nearly accurate for locomotive designs produced after 1920.

$$\text{Boiler tractive effort} = \frac{\text{Boiler horsepower} \times 375}{V}$$

where V = speed in miles per hour.

$$\text{Boiler horsepower} = \frac{(A_d e + A_i t)1.08}{F_s}$$

where A_d = sum of direct-heating surface areas (firebox, combustion chamber, siphon, arch tubes) in square feet.

e = assumed evaporation per square foot per hour (55 lb for pre-1920 designs, 80 lb for post-1920 designs).

A_i = indirect-heating surface area (boiler tubes and flues) in square feet.

t = tube factor, the evaporation in pounds per hour from one square foot of indirect heating surface (see Table 23).

F_s = steam factor, pounds of steam assumed to be evaporated per indicated horsepower hour for given degrees of superheat (see Tables 24 and 25).

The 1.08 factor gives 8% additional evaporation when a feedwater heater is used.

Table 23. Tube Factors *

Tube Length, ft	Evaporation, lb/sq ft	Tube Length, ft	Evaporation, lb/sq ft
10.0	13.0	17.5	10.00
11.0	12.55	18.0	9.85
12.0	12.0	18.5	9.70
13.0	11.65	19.0	9.50
14.0	11.25	19.5	9.35
15.0	10.90	20.0	9.20
16.0	10.70	20.5	9.05
16.5	10.50	21.0	8.95
17.0	10.35	21.5	8.80
	10.20	22.0	8.65
		22.5	8.55
		23.0	8.40
		23.5	8.30
		24.0	8.20

* From *Locomotive Data*, 12th Ed., Baldwin Locomotive Works, Philadelphia, 1944, p 23

Table 24. Steam Rates for Maximum Indicated Horsepower Output *

Steam		Pounds of Steam per Horsepower-Hour					
			Superheat, degrees F				
Gage Pressure, lb	Temperature, degrees F	Saturated	150°	200°	250°	300°	350°
175–195	377	28.75 to 28.10	21.65 21.10	20.40 19.80	19.30 18.80		
200–220	388	28.00 to 27.50	21.00 20.65	19.70 19.30	18.70 18.30		
225–245	397	27.40 27.00	20.55 20.25	19.25 18.90	18.25 18.00		
250–270	406	26.90	20.20	18.85	17.95 17.80	17.10	16.30
275–295	414	26.60	20.00	18.70	17.75 17.70	16.80	15.95
300–320	422	26.30	19.70	18.60	17.675 17.575	16.75	15.90
325–345	429	26.00	19.50	18.50	17.55 17.49	16.70	18.58
350–370	436	25.80	19.40	18.40	17.48 17.43	16.60	15.75
375–400	442 448	25.70 25.60	19.30 19.20	18.30 18.20	17.40 17.30	16.50 16.40	15.60 15.50

* From *Locomotive Data*, 12th Ed., Baldwin Locomotive Works, Philadelphia, 1944, p. 25.

Table 25. Average Values of Degrees of Superheat *

Type of Superheater	Tube Combination, dia in in.	Ratio of Small to Large Tubes	Degrees of Superheat
A	5 3/8–2 5 1/2–2 1/4	7 or over 6 or over	150
A	5 3/8–2 5 1/2–2 1/4	5 to 7 4 to 6	200
A	5 3/8–2 5 1/2–2 1/4	Under 5 Under 4	250
E	3 1/2–2 1/4 3 1/2–2 1/4 4 –2 1/4	... ...	250

* From *Locomotive Data*, 12th Ed., Baldwin Locomotive Works, Philadelphia, p. 24.

Horsepower. At low velocities horsepower increases almost directly with the velocity. At speeds above 12–18 mph, it increases very slowly, and at high speeds it decreases somewhat as velocity increases. This limits the rate of acceleration at low speeds as compared to diesel-electric powered units but permits a more rapid recovery from speed reductions within the higher speed ranges.

Table 26. Diesel-Electric Locomotive Data

Courtesy of Electromotive Division, General Motors Corp. (E.M.D.), and General Electric Co. (G.E.)

Line	Item	Locomotives				
1	Model	F-9 lead	F-9 booster	A unit	B unit	GP 9 (single unit)
2	Horsepower	1750	1750	1500	1500	1750
3	Builder	E M.D.	E.M.D.	G.E.	G.E.	E M.D.
4	Classification	B-B	B-B	B-B	B-B	C-C
5	Overall length	50′ 08″	50′ 00″	52′ 00″	50′ 00″	56′ 02″
6	Overall width	9′ 10″	10′ 08″	10′ 06 1/2″	10′ 06 1/2″	10′ 02 1/2″
7	Overall height above rails	15′ 00″	15′ 00″	14′ 10″	14′ 04 1/2″	14′ 11 5/8″
8	Number of motors	4	4	4	4	6
9	Driving wheels	4 pair	4 pair	4 pair	4 pair	6 pair
10	Driving wheel diameter	40″	40″	40″	40″	40″
11	Wheel base between truck centers	30′ 00″	30′ 00″	...	...	31′ 0″
12	Weight on drivers	232,600 lb	224,800 lb	230,000 lb	223,000 lb	235,500 lb
13	Total weight	232,600 lb	224,800 lb	230,000 lb	223,000 lb	235,500 lb
14	Fuel	1200 gal	1200 gal	1200 gal	1200 gal	800 gal
15	Service	Frt and psgr.	Frt. and psgr.	Frt. and psgr.	Frt. and psgr.	General purpose
16	Radius or degree of max. curvature	250′	250′	21°	21°	274′ (locomotive alone, 150′)
17	Tractive effort (max.)	69,780 lb (at 30% adhesion)	67,440 lb	60,000 lb	60,000 lb	70,650 at 30% adhesion
18	Tractive effort, continuous (max. and min.)	Determined by adhesion in most cases		42,500 lb and 42,500 lb	42,500 lb and 35,000 lb	Determined by adhesion in most cases
19	Gear ratios (for max. and min. continuous T.E.)	65/12 and 56/21	65/12 and 56/21	74/18 and 64/19	74/18 and 64/19	65/12 and 56/21
20	Intermediate ratios available	Yes	Yes	Yes	Yes	Yes

Table 27. Steam Locomotive Data *

Line	Item	Locomotives		
1	Railroad	C. & O.	W.M.	A.C.L.
2	Builder	Baldwin	Baldwin	Baldwin
3	Type	4-6-4	4-8-4	4-8-4
4	Service	Passenger	Freight	Passenger
5	Year built	Aug. 1948	Jan. 1947	April 1938
6	Railroad's classification	L-2	J-1	R-1
7	Cylinder diameter and stroke	25″ × 30″	26 1/2″ × 32″	27″ × 30″
8	Valve type	Franklin Poppet	Piston 14″ dia.	Piston 12″ dia.
9	Boiler pressure	255 lb/sq in.	255 lb/sq in.	275 lb/sq in.
10	Firebox	264 sq ft	281 sq ft	272 sq ft
11	Combustion chamber	81 sq ft	139 sq ft	127 sq ft
12	Flues	(3 1/2″) 3245 sq ft	3885 sq ft	...
13	Circulator or syphons	60 sq ft	153 sq ft	169 sq ft
14	Tubes	(12 1/4″ × 18′) 528 sq ft	(2 1/4″ × 21′) 516 sq ft	(2 1/4″ × 21′) 4181 sq ft
15	Total	4178 sq ft	4974 sq ft	4797 sq ft
16	Superheater	E 1785 sq ft	E 2170 sq ft	A 1497 sq ft
17	Feedwater heater	Yes	Yes	Yes
18	Grate area	90.2 sq ft	106.7 sq ft	97.7 sq ft
19	Driver diameter	78 in.	69 in.	80 in.
20	Engine wheelbase	41′ 5 1/2″	46′ 10″	47′ 9″
21	Total engine and tender	93′ 10 1/4″	95′ 0″	97′ 11″
22	Weight on drivers	219,500 lb	290,000 lb	263,127 lb
23	Total engine weight	443,000 lb	506,500 lb	460,270 lb
24	Total tender weight 2/3 full	317,500 lb	340,500 lb	435,500 lb
25	Wheels under tender	12	12	16
26	Tank capacity	21,000 gal	22,000 gal	24,000 gal
27	Fuel capacity	30 tons	30 tons	27 tons
28	Fuel	Coal	Coal	Coal
29	Tractive force	52,100 lb	70,600 lb	63,900 lb

* Courtesy of Baldwin-Lima-Hamilton Corp

23. CHARACTERISTICS OF ELECTRIFICATION

General. The essential characteristic of electric traction which distinguishes it from other forms is the separation of the source of motive power from the rolling stock. This gives it a virtual monopoly for street and interurban systems because it permits the generation of motive power to be concentrated in a single economical plant instead of being distributed over a number of comparatively uneconomical ones, as in the case of steam traction.

Conditions for Electric Traction on Railroads. On electrified railroad trunk lines the generating and distributing systems must have large capacity in order to handle the heavy trains, but are not kept fully employed due to the comparative infrequency of traffic. This results in the investment of a great deal of capital which becomes idle, the fixed charges on which generally exceed the operating economies. Exceptions to this generalization occur under the following conditions:

(*a*) When the cost of coal is very high, the operating economies may be sufficient to counterbalance the fixed charges, as electric traction requires only half the amount of coal needed with steam traction.

(*b*) When cheap water power is available, a similar condition may occur.

(*c*) When it is desired to increase the weight of trains to be hauled on steep grades, the elimination of the power generating system from the locomotive, in electric traction, permits an increased concentration of motors on the locomotive, thereby obtaining the desired increase of hauling power, at a lower cost than by the elimination of grades.

(*d*) When it is desired to increase the capacity of a mountain division and this can be accomplished by increasing the weight of trains as described above, rather than by increasing the number of tracks so as to permit more trains to be hauled

(*e*) When it is necessary to eliminate smoke and gases in tunnels or city streets, the element of economy being necessarily subordinated to safety or convenience.

(*f*) When the traffic density is unusually great, as in certain surburban zones.

(*g*) When by the elimination of smoke and gases, trains through cities can be run underground and the ground surface utilized for lucrative building purposes.

Unfavorable factors include low traffic density, high costs of power, and high interest rates and taxes on initial investment.

Economic Differences between Systems. The stationary equipment of an electric traction system consists of three parts—the generating, transmission, and distribution systems These must have sufficient capacity to carry the peak loads and are therefore likely to be comparatively idle during a large part of the time. The activity of these parts depends upon the number of cars or trains in operation.

The moving equipment of an electric railway system consists of the motors, controllers, and other parts of the rolling stock necessary for electric operation. The activity of these parts does not depend upon the number of cars or locomotives in operation, but only upon the activity of each.

There are several systems of electric traction which differ in the distribution of the investment between stationary and moving equipment. Stationary equipment costs less than moving equipment of the same capacity, because it is concentrated in larger units and its design is not restricted by the limitations of car construction It is, therefore, obvious that for railways operating a large number of cars or trains it pays to have a comparatively large investment in stationary equipment; whereas, railways with few cars or trains must economize in stationary equipment and concentrate their investment as far as practicable on their limited rolling stock.

Technological Differences between Systems. The various systems of electric traction now in use may be classified as follows according to the above principle:

Low-voltage, direct current.
High-voltage, direct current.
Three-phase.
Single-phase.

In every practical electric railway system current is generated for transmission at a high voltage and utilized in the motors at low voltage. In the d-c systems, the transformation from high to low voltage is accomplished at stationary substations, which also change the alternating current from the transmission lines into direct current, which is fed to the trolley or third rail, and thence to the motors. In the a-c systems, the high-voltage current is delivered to the trolley lines and transformed down to a voltage suitable for the motors by means of transformers on the cars or locomotives. Locomotives used with high voltage a-c transmission may have direct-motors, which are supplied with direct current from a

motor generator set in the locomotive. As the d-c systems carry a greater proportion of investment in their stationary equipment, they are especially adapted to urban and important interurban lines, street railways, subways, and elevated lines. The a-c systems, which carry a proportionately greater investment in moving equipment, are especially suitable for trunk lines having infrequent service. The 11,000-volt, 25-cycle a-c current supplying a single-phase motor has received widespread use for line-haul railroads in the United States.

An ignitron-type mercury-arc rectifier changes a-c into d-c current within the locomotive and permits use of 60-cycle current supply from commercial sources. The ignitron locomotive is said to produce 40% more tractive effort for a given weight than an a-c motored unit

The high-voltage, d-c system is intermediate, in the above respect, between the low-voltage, d-c and the a-c systems.

The choice of systems does not depend merely upon the investment, but also upon the operating costs.

24. ELECTRIC LOCOMOTIVES AND CARS

General. Electric locomotives unlike steam locomotives, can be made of practically any desired power capacity without excessive complication and without increase in the number of operators. This arises from the absence of boilers and the possibility of controlling a number of motors or even a number of complete locomotive units as if they were a single unit.

Rating. The rating of electric locomotives is usually expressed in terms of the weight on drivers, the nominal 1-hour tractive effort, the continuous tractive effort, and corresponding speeds.

(*a*) *Weight on Drivers.* The weight on drivers, expressed in pounds, shall be the sum of the weights carried by the drivers and of the drivers themselves.

(*b*) *Nominal Tractive Effort.* The nominal effort, expressed in pounds, shall be that exerted at the rims of the drivers, when the motors are operating at their nominal (1-hour) rating. (See p. 6-83.)

(*c*) *Continuous Tractive Effort.* The continuous tractive effort, expressed in pounds, shall be that exerted at the rims of the drivers when the motors are operating at their full voltage continuous rating.

In the case of locomotives operating on intermittent service, the continuous tractive effort may be given for one-half or three-fourths voltage, but in such cases the voltage shall be clearly specified.

(*d*) *Speed.* The rated speed, expressed in miles per hour, shall be that at which the continuous tractive effort is exerted.

Classification. Locomotives may be classified as follows, with reference to trucks:

1. Rigid wheelbase.
   - (*a*) Without leading and trailing trucks.
   - (*b*) With leading and trailing trucks.
2. Separated bogie truck (the trucks being connected through the upper frames as in a car):
   - (*a*) Symmetrical.
   - (*b*) Unsymmetrical.
3. Articulated (wherein the two trucks are hinged back to back).

All of these types of locomotives are practically steady at speeds under 40 mph, but above this speed marked differences appear, the steadiest riding machines, according to G. Gibbs, having high centers of gravity and long unsymmetrical wheelbases. Considerable difference of opinion prevails in regard to proper mechanical design for high speeds.

Transmissions. Motors are connected to locomotive drivers by the following forms of transmission:

(*a*) Gearless, the motor armatures being mounted direct on the axles.

(*b*) Gearless quill, each motor armature being mounted on a hollow shaft or quill which surrounds the axle, but is free with respect to it. At each end of the quill is a disk having projecting arms which fit into spaces between the spokes of the driving wheels. The arms are connected to the driving wheels by springs which tend to keep the arms centered between spokes but allow sufficient play to relieve the excessive starting stresses in the armature.

(*c*) Direct gears, the motors being mounted either between or over the axles, a motor having a pinion engaging a gear wheel on the locomotive axle.

(*d*) Quilled gears, the gear wheel being on a hollow shaft or quill which engages the drivers through springs, thereby relieving both the armature and gears of excessive starting stresses.

(*e*) Cranks and Scotch yoke, in which two motors are connected together and have crank shafts which operate three sets of drivers

(*f*) Crank and countershaft, in which the motors are located above the trucks and are connected through a countershaft to side rods joining the drivers.

Control. Practically all modern electric locomotives have multiple-unit control, so that two or more locomotives may be coupled together under the control of one operator. Indeed, some locomotives consist of two sections—each of which is virtually a separate locomotive—coupled together permanently.

Selection of Motor. The following information, relative to the service to be performed, is required, in order that an appropriate motor may be selected.

(*a*) Weight of total number of cars in train (in tons of 2000 lb) exclusive of electrical equipment and load.

(*b*) Average weight of load and durations of same, and maximum weight of load and durations of same.

(*c*) Number of motor cars or locomotives in train, and number of trailer cars in train.

(*d*) Diameter of driving wheels.

(*e*) Weight on driving wheels, exclusive of electrical equipment.

(*f*) Number of motors per motor car.

(*g*) Voltage at train with power on the motors—average, maximum, and minimum.

(*h*) Rate of acceleration in miles per hour per second.

(*i*) Rate of braking (retardation in miles per hour per second).

(*j*) Speed limitations, if any (including slowdowns).

(*k*) Distances between stations.

(*l*) Duration of station stops.

(*m*) Schedule speed including station stops in mph.

(*n*) Train resistance in pounds per ton of 2000 lb at stated speeds.

(*o*) Moment of inertia of revolving parts, exclusive of electrical equipment.

(*p*) Profile and alignment of track.

(*q*) Distance coasted as a per cent of the distance between station stops.

(*r*) Time of layover at end of run, if any.

Rating of Motors. The *one-hour rating* of a railway motor shall be the output at the motor shaft measured in horsepower (or kilowatts) which the motor can carry for 1 hr on stand test, starting cold, at its rated voltage and frequency (in case of a-c motor) with the ventilation system as in service without exceeding the temperature limits given in the table of limiting temperature rises.

The *continuous rating* of a ventilated railway motor shall be the output at the motor shaft measured in horsepower (or kilowatts) which the motor can carry for an unlimited period on stand test, at its rated voltage and frequency (in the case of a-c motor), with the ventilation system as in service without exceeding the temperature limits given in the table. Direct-current ventilated railway motors may also be given a continuous rating in amperes at full, three-fourths and half-rated voltage.

The *continuous rating of totally enclosed* d-c railway motors shall be given in amperes at three-fourths, and half rated voltage.

In the absence of any specification as to the kind of rating the *one-hour rating* shall be understood.

The ratings of a *field-control d-c motor* shall relate to its performance with the field connection which gives the maximum motor rating. (This is usually the minimum field.) Each section of the field winding shall be adequate to perform the service for which it is designed without exceeding specified temperature rises.

Voltages. Direct-current motors are usually made for line voltages of 500, 550, or 600 volts and sometimes for 1200 volts or somewhat more. On 2400-volt lines, it is usual to have the motors in pairs connected permanently in series.

Alternating-current single-phase motors are usually made for 400–500 volts, the required voltage being obtained by transforming down the line voltage by means of a transformer or autotransformer on the train.

Weight. Motors vary in weight from 30 lb per horsepower (40 lb per kilowatt) for the largest sizes (200 hp) to 70 lb per horsepower (93 lb per kilowatt) for the smaller sizes (35 hp), the ratings being the nominal.

Controllers. The speed of d-c motors is controlled by connecting resistance in series with the motors, by connecting the motors at first in series and then in parallel, and some-

Table 28. Limiting Temperature Rises

Item		Type of Enclosure	Method of Temperature Determination to Be Employed	Limiting Temperature Rise, degrees C			
				One-Hour Rating		Continuous Rating	
				Class A Insulation	Class B Insulation	Class A Insulation	Class B Insulation
1	Armature and field winding	Ventilated	Resistance Thermometer	100 80	120 95	85 65	105 80
		Totally * enclosed	Resistance Thermometer	110 90	130 105	95 75	115 90
2	Cores and mechanical parts in contact with or adjacent to insulation	Ventilated Totally * enclosed	Thermometer Thermometer	80 90	95 105	65 75	80 90
3	Commutators	Ventilated Totally * enclosed	Thermometer Thermometer	95 105	110 120	80 90	95 105
4	Miscellaneous parts (such as brush holders, brushes, pole tips, etc.) other than those whose location is such that they may injuriously affect the adjacent insulation may attain such temperatures as will not be injurious in any other respect						

* The temperature rises of totally enclosed motors are taken as 10°C higher than the ventilated motors since the cooling on stand test will be inferior to that obtained in service

Note Short-time ratings using modern silicon and glass insulations permit temperatures up to 160°C.

times by varying the strength of the fields. Alternating-current motors are similarly controlled, except that autotransformers are used instead of resistances

The direction of rotation, in both kinds of motors, is changed by reversing the current in *either* the fields *or* armature, this being usually accomplished by rotating an auxiliary handle of the controller.

The type K controller is largely used for light-weight cars, and consists of an operating handle which moves a cylindrical drum with projecting contact pieces which come in contact with stationary fingers. The first three points correspond to accelerating steps, by means of which the resistance in series with the motors is gradually cut out. The fourth step, full series, gives about half speed and is a continuous running point. The following steps restore the resistance and put the motors in parallel. The last step leaves the motors in parallel without any resistance.

Multiple-unit control is used on large cars, especially where the combined motor capacity exceeds 300 hp. The equipment consists of a small master controller which enables a comparatively weak current to operate contactors large enough to make the necessary changes in the circuit, as in the case of type K controllers. The master controller on any car will operate the contactors on all cars, if a jumper cable be run from car to car connecting together all the control circuits.

The weight of control equipment of type K varies from 1200 to 2250 lb, and of multiple-unit equipment from 2800 to 3200 lb.

Tractive effort for an electric locomotive is given by the general expression:

$$TE = \frac{T \times 24 \times N \times e \times G}{D \times g}$$

where TE is tractive effort in pounds, T is torque of a single motor in pounds taken at a 1-ft radius from the motor armature shaft (on the circumference of a circle of 24-in. diameter), N is number of motors, D is driving wheel diameter, G is the number of gear teeth, g is the number of pinion teeth, and e the combined electrical and mechanical efficiency taken as 80–85%. When the manufacturer's curves are based on substation voltage, the tractive effort values should be reduced about 90%. Since, practically speaking, there is no limitation on the power supply to the motors, full horsepower is available over all speed

ranges, permitting rapid acceleration from stops and rendering electric propulsion especially suitable for city and suburban service where fast schedules must be combined with frequent stops.

Table 29. Maximum Overall Efficiency of Motors and Gears at Rated Voltage

Horsepower, 1-hr rating	Kind of Motor	Maximum Efficiency, %
30–100	D-C geared	83–88
100–250	D-C geared	88–89
250–500	D-C gearless	91–93
50–200	A-C series geared	70–80 *
200–500	3-phase induction geared	85–89

* Including step-down transformers.

Power Required at Given Speed. Let

r = train resistance in pounds per ton of total weight of train.
G = per cent grade.
g = degree of curvature.
a = degree of acceleration in mphps.
v = speed in mph.
W = total weight of train in tons.

Then the power required *at the rims of the drivers* is $1.99v(r + 20G + g + 100\,a)$ watts per ton, or $2.67 \times 10^{-3}vW(r + 20G + g + 100a)$ horsepower, total.

The power input, p_i to the car or locomotive is equal to the power at the rims of the drivers divided by the overall efficiency ϵ of the controller, motors and gears, that is,

$$p_i = \frac{1.99\,Wv(r + 20\,G + g + 100\,a)}{1000\,\epsilon} \text{ kilowatts}$$

Approximate Method of Calculating Energy Consumption. The following method is based upon simple kinetic principles, and, if certain characteristics of the run are known, gives the actual energy output at the wheel rims. This fact makes the method useful, not only for rough calculations, but also to check calculations made by the more accurate step-by-step method.

When the method is applied to checking purposes, the column of Table 30 below headed "Actual Energy Output" should be used and the input calculated from the known efficiencies. When applied to rough calculations, the column headed "Approximate Electrical

Table 30. Output at Wheel Rim and Input to Cars in Watt-Hours per Ton-Mile

Energy for	Actual Energy Output at Wheel Rims	Approximate Electric Energy Input
Acceleration	$\frac{V_m^2}{36.2L}$	$\frac{K^2nV^2}{25}$
Frictional train resistance	$\frac{1.99rL_p}{L}$	$\frac{2.9r}{Q}$
Grades	$\frac{39.8GL_p}{L}$	$\frac{57G}{Q}$
Curves	$\frac{1.99gL_p}{L}$	$\frac{2.9g}{Q}$
Total	Sum	Sum

Note: $25 = 36.2\epsilon$, $57.8 = 39.8 \div \epsilon$, and $2.9 = 1.99 \div \epsilon$, where ϵ is the efficiency, taken as 0.7. The formula for energy due to curves assumes each degree of curvature to be equivalent to a train resistance of 1 lb per ton, which is probably high.

Energy Input" should be used. In the latter case the maximum speed and length of run with power on are not known, but it is possible to assume certain values, based upon experience, which will give a rough approximation to the energy required. The total energy in watt-hours per ton-mile will be the sum of the amounts required for acceleration and overcoming frictional train resistance grades and curves. Let

V = average running speed in miles per hour.
V_m = maximum speed in miles per hour.
L = length of run in miles.

L_p = distance traveled, with power on, in miles.
$n = l/L$ = number of stops per mile including one terminus.
r = average train resistance, in pounds per ton (say that corresponding to a speed from 10 to 20% greater than the average speed).
G = average equivalent grade, in per cent.
g = average curvature in degrees.
$K = \frac{V_m}{V}$ = ratio of maximum to average speed; see Table 31 for values of K.
$Q = \frac{L}{L_p}$ = ratio of length of run to distance traveled with power on. See Table 31 for values of Q.

Table 31. Values of K and Q

Stops per Mile, n	K: Locomotive Passenger Trains	K: Single Cars, Multiple Unit Trains, and Freight Trains	Q All Trains
0	1.0	1.0	1.0
0.1	1.2	1.1	1.1
0.2	1.35	1.2	1.25
0.3	1.5	1.25	1.4
0.4	1.6	1.3	1.5
0.5	1.7	1.35	1.7
0.6	1.75	1.4	1.8
0.7	1.8	1.45	1.9
0.8	1.85	1.45	2.0
0.9	1.9	1.5	2.1
1.0	1.95	1.5	2.15
1.2	1.95	1.6	2.2
1.4	1.95	1.6	2.3
1.6	1.95	1.6	2.4
1.8	1.95	1.65	2.5
2.0	1.95	1.7	2.6
2.5	1.95	1.75	2.7
3.0	2.0	1.8	2.8
3.5	2.0	1.85	2.9
4.0	2.0	1.9	2.9
4.5	2.0	1.95	2.95
5.0	2.0	2.0	3.0

More exact methods involve a knowledge of the motor characteristics, gears, brakes, and control.

P.C.C. car is the designation for a relatively new design of high speed (up to 50 mph), streamlined, and modernized car designed by the President's Conference Committee. The car is featured by one-man operation, rapid acceleration and deceleration (the latter aided by magnetic brake shoes which grip the rails), rubber wheel treads to reduce noise, streamlining (mainly for appearance), wide end and central door entrances and exits, and more pleasing and comfortable interior and seating arrangements. To provide smoother operation in conjunction with the higher rates of starting and stopping, 61 to 260 controller steps are used instead of the customary 9 steps.

Multiple unit or **M.U.** cars are used both by suburban and line-haul roads and for subway and elevated service. Superficially the cars resemble single unit street cars, but they have provision for coupling several together in a train with control from the front platform or compartment. Thus the number of cars in a train may readily be varied to meet traffic conditions. Cars vary in length and weight from 40 to 75 ft and from 25,000 to 75,000 lb. Electric control over pneumatic brakes is combined with an empty-load device which automatically varies the braking ratio in proportion to the load. Some designs place a common truck under two adjacent cars to form articulated units. Cars having no motors are operated as trailers coupled to motorized units. Wide doors are pneumatically controlled by the motorman or conductor. Usually the train cannot start until all doors are closed. Ceiling fans or other adequate form of ventilation are important in subway service.

Regenerative Braking. Like the diesel-electric, the motor leads of electric locomotives may be reversed on downgrades permitting the motors to act as generators and exert a drag or braking action on the train. The current thus generated is fed back into the line and may be credited to the railroad's power account.

Additional characteristics of electric operation include freedom from smoke, soot, and cinders, greater availability, ease in starting, and elimination of fuel, water, and ash facilities. Disadvantages are high initial costs and carrying charges, especially for the power production and transmission systems, tie-ups resulting from power failures or storm damage to transmission systems, and inability to operate on tracks not equipped with transmission facilities.

25. POWER STATIONS AND SUBSTATIONS

Power Stations. The power plant of an electric railroad should be located with reference to the load center, the water and coal supply, and the cost of real estate. The load center is usually the least important consideration as the cost of transmission lines and the losses therein do not rank in importance with considerations of plant economy.

Water is required for boilers and condensers. The coal supply should be brought by water where practicable, with auxiliary rail connection.

Substations. A substation is a group of apparatus or machinery which receives current from a transmission system, changes its kind or voltage, and delivers it to a distribution system. Most commonly, substations transform high-voltage alternating current to low-voltage direct current, the former affording the most economical transmission, and the latter the most economical conversion of electrical into mechanical energy. Substations are usually housed in substantial brick and concrete buildings, although where the current has to be changed only in voltage and not in kind, as on single-phase railway systems, the building is often unnecessary, as no rotating machinery is required, and the transformers and switching apparatus can be built for outdoor operation. In some locations, the high tension bus, transformers, and switching equipment may be installed in an outdoor substation adjacent to the building housing the rotating equipment, meters, etc.

Economical Location. Substations are located at intervals of 3–20 miles along the railroad, the spacing being limited by consideration of line voltage, regulation at the cars, and economy, both of first cost and operation. The average permissible voltage drop in trolley and track return is 10%. Theoretically, substations should be located so as to give the least annual expense, taking into account the fixed charges on the investment in substations and feeders, the operating charges, and the cost of energy losses in the system. An investment in copper feeders, especially in the case of bare or weatherproof aerial cables, causes little annual expense beyond the interest and taxes on the investment, whereas a substation involves, in addition, heavy depreciation charges and labor charges for operation. In recent years the use of automatic substations has been spreading rapidly, as it enables the operating force to be reduced, and in many cases eliminated, with the exception of a small force for inspection and maintenance, one group of men being responsible for the care of several substations. Automatic substations have the further advantage that they act instantly in case of trouble, reducing the damage to equipment caused by abnormal conditions within or external to the station.

Automatic stations are usually controlled from a central point by a load dispatcher or supervisor by means of one of several forms of supervisory control which by the use of a very few wires, usually from one to four, enables all switches in a large group of substations to be operated, and all desired indications, such as switch positions, machine temperatures, station output, to be relayed to the supervisor's board

On interurban systems where the traffic density is low automatic stations may be equipped to start automatically as a car approaches the section fed by the substations, and to shut down as soon as the load has dropped.

The permissible distance between substations depends on the voltage of distribution, being greater the higher the voltage. Use of the 22,000-volt a-c system instead of the 11,000-volt system has been considered to reduce the number of substations On an ideal line, doubling the operating voltage quarters the distance between substations. This ratio is not strictly adhered to, however, as the location of substations is also influenced by physical conditions such as availability of a suitable location for the substation, or the location of concentrated loads caused by yards, grades, or sharp curves.

Where a length of track receives its power from only one direction, the distance that can be fed, with a given voltage drop, is one-fourth the distance allowable between substations.

Apparatus. In the usual type of substation for the conversion of high-pressure alternating current into low-pressure direct current, the high-pressure current first passes through oil switches to the high-pressure bus, then through oil switches to the primary windings of the transformers. From the secondary windings of the transformers, the low-pressure alternating current enters the slip rings of the converters, and emerges from the

commutators as direct current of a slightly higher pressure. From the commutators it passes through circuit breakers to the d-c bus bars and thence through circuit breakers to the outgoing feeders. In the design of substations, it is good practice to arrange all the apparatus so as to make the route of the electric current as direct as possible.

All substations are not equipped with synchronous converters, motor generators having been used especially for pressures in excess of 750 volts. Such a motor generator usually consists of a synchronous motor driving a d-c generator.

For voltages below 750 volts, the rotary converter has a virtual monopoly, although in special cases both motor-generator sets and mercury-arc rectifiers are used.

For d-c voltages of the order of 1500 volts, all three clases of equipment are used, the rotary converter and motor-generator set now meeting competition from the mercury-arc rectifier.

For voltages above 1500 volts, motor-generator sets have been used exclusively, but the development of the mercury-arc rectifier is making rapid strides and many installations are now operating at this voltage in Europe.

Portable Substations. Portable substations, that is, substations of small size installed in a box car, are sometimes used where unexpected or occasional loads are liable to occur. They are especially useful to carry holiday loads for which it would not pay to install a permanent substation.

Power Capacity. The power capacity required in a substation depends upon the size, number, acceleration, and speed of cars, on the length of line fed, and upon the number of stops per mile.

Railway substation machines and transformers are usually rated by the kilovolt-ampere output which, having produced a constant temperature, may be increased 50% for 2 hours without producing temperatures or temperature rises exceeding by more than 5°C the standard limiting values established by the American Institute of Electrical Engineers. This is known as the nominal rating.

Buildings. Substations having all apparatus on one level require about 0.2 sq ft per kilowatt of nominal rating. Buildings are usually of brick with concrete foundations for walls and machines, except where the machines are small, in which case the machines are often carried by the floor beams. It is usual to provide a crane which, in important substations, may be equipped for both manual and electrical operation.

Provision must be made for ventilation and good illumination, but roofs must be constructed so that there will be no danger of leaks over the machines and other live apparatus. In locations where the air contains a considerable amount of dust, the air entering the station should be drawn through air filters, circulation of the air being obtained by means of motor-driven fans blowing the air into the station through louvers in the floor near the machine. Lavatory, toilet, and telephone booth should be conveniently near the operators' desks.

Transformer Stations. A transformer station, as used on a-c railway systems, is an aggregation of apparatus for converting high-voltage alternating current from transmission lines into lower-voltage alternating current for use in the trains, the apparatus being usually out of doors and protected only by a fence.

Special designs of oil-cooled transformers, switching apparatus, and lightning arresters must be used for this purpose.

All apparatus is set on concrete foundations, and steel structures are used to support overhead wires and switching apparatus.

Autotransformer System. The New York, New Haven & Hartford Railroad system transmits at 22,000 volts between trolley and feeder, but the locomotives receive current at 11,000 volts between trolley and track rail. This is accomplished by transformer stations about 2 miles apart, where the trolley and feeder are each connected to the track rails through a separate autotransformer. This arrangement not only effects economy in transmission, but also greatly reduces the current in the track rails and earth, with consequent reduction of disturbances to telephone lines.

26. TRANSMISSION AND DISTRIBUTION SYSTEMS

Trolley Systems

The trolley wire is usually of hard-drawn copper but, under unusual conditions, bronze or steel wires are sometimes used. The wire is suspended from insulators some 16–30 ft above the ground, and presents a continuous contact surface to a trolley wheel or bow or to a sliding pantograph shoe attached to the rolling stock or locomotive. The former is limited to currents of 350 amp or less. The pantograph collector has become almost

standard for heavy traction work. The contact shoe is mounted on a heavy collapsible frame which may be raised by air and closed by gravity, or unlatched by air, raised by springs, and lowered by air. One shoe is used for low currents. Two shoes with wearing strip of copper or bronze are used for heavy currents. There are two classes of trolley construction, the span wire and the side bracket; each may have simple or catenary suspension.

Span-Wire and Side-Bracket Construction. In the simple span-wire construction the trolley wire is supported by wires stretched across the tracks between poles or building walls. The side-bracket construction resembles the span-wire except that instead of the supporting wire being stretched between two poles, it is stretched between two supports on the same pole. In both of these types of construction, the trolley wire is supported at intervals of 100 ft or more and sags between supports, making it necessary for the trolley to be in vertical vibration as the cars move.

Catenary Construction. The speed attained upon modern electric roads makes it difficult to obtain satisfactory service with a trolley wire which dips between supports and sways with every impulse. The catenary construction was devised to meet this condition. In general, it consists of a grooved copper trolley wire suspended horizontally from a sagging messenger cable, which is suitably insulated and firmly held in place. The supporting structure employed for interurban single- or double-track roads usually is of the side-bracket type, but for some conditions cross-span construction becomes necessary. The latter method of support differs only in the substitution of a catenary cross span for the bracket arm and doubling the number of poles required for single track.

Applications of Various Types of Construction. The overhead trolley system is used on urban railways, wherever the unsightliness or danger of its exposed construction is not considered objectionable It is used on interurban, suburban, and trunk lines, wherever, by reason of low load or high voltage, the current taken by the trains is not too great to be economically carried on copper wires.

Center-pole construction is approved practice for double-track railways, side-pole construction being generally used for single-track lines. Span-wire construction is used where, for any reason, it is impracticable to have the poles near the tracks or where, as is commonly the case in Europe, the span wires are supported from the walls of buildings. Prejudice against overhead lines is often due to the excessive loading of poles, which is both unsightly and dangerous.

Parts of Overhead Trolley Construction. The trolley wire is secured to an "ear" by soldering, clamps, or other means, and the ear is bolted to a "suspension" which may or may not be provided with an insulating portion. These suspensions are carried by span wires which in turn are fastened to the poles or brackets, or they may be fastened directly to the bracket. If the "suspension" is not insulated, "strain" insulators are inserted in the span wire between the ear and its point of attachment to the poles or brackets. A slightly different form of suspension, called a "pull-off," is used on curves. At turnouts "trolley frogs" must be used to guide the trolley wheel.

Curves. Poles are frequently spaced closer together on curves. Table 32 may be considered typical, the divergence of the wire from the theoretical curve being kept within 3 1/2 in.

Table 32. Spacing of Poles on Curves, Interurban Road

Degree of Curvature of Track	Pole Spacing, ft	Divergence of Trolley Wire from Track Center, in.	Degree of Curvature of Track	Pole Spacing, ft	Divergence of Trolley Wire from Track Center, in.
Tangent	120	0	6	60	2.87
1	120	1.87	7	50	2.34
2	110	3.37	8	50	2.64
3	90	3.06	9	50	2 94
4	80	3.37	10	50	3 24
5	70	3.06			

Simple and Compound Catenary Construction. Catenary construction may be divided into simple and compound. In the former the trolley wire or wires are carried by one or two messenger cables which are supported only at the poles, bents, or span wires, in the latter, the horizontal wire or wires are carried by a messenger cable, which is itself suspended from another messenger cable which is supported at the poles, bents, or span wires. The advantages of the compound catenary are greater flexibility, reduced stresses in the supporting wires, shorter hangers, better lightning protection, and superior curve construction.

The double catenary uses two catenary wires hung from points in the same horizontal plane and supports the trolley wire with double hangers. Inclination of the hangers is kept constant with crossties. This system is infrequently used because of the expense.

To obtain a line which will not require frequent readjustment, the messenger cable must be installed with practically uniform tension throughout its length, making it necessary to have less sag in the shorter spans. For this reason certain definite pole spacings and corresponding hanger lengths have been standardized.

Hangers. Where only one horizontal wire is suspended from the messenger cable, the hangers should hang loosely from the cable and should be screwed fast to the trolley wire. This permits the trolley wire to rise slightly as the trolley passes under it, thereby making the wire equally flexible along its entire length. Where a steel contact wire is clipped to the horizontal conductor, the hangers are usually rigidly attached at both ends, as the duplication of horizontal wires assures uniform flexibility, regardless of the hangers. (See Table 33.)

Table 33. Tangent Construction: Number of Hangers per Span, Pantograph or Bow Trolleys

Length Pole Spacing, ft	Number of Points of Suspension	Length of Hangers, in.										
		6	6 3/4	8 1/2	11	12	13 1/2	14 3/4	16	17 1/2	19 1/4	20 1/2
150	11	1	2	2	2	.	...	2	.	.	2	
125	9	..			1	2	2	..	2		2	
110	8	..		.		.	2	2		2		2
95	7	..					.	.	3	2		2
80	6	.								2	2	2
70	5							.		.	3	2
55	4	.	.	.								4

Wear of Trolley Wheel Bow and Trolley Wire. The wear of the trolley wires is not serious with either wheel or bow collectors. On the lines of the Indianapolis & Cincinnati Traction Company, a copper trolley wire lost less than 1% in weight after it had experienced 39,000 car movements, each car taking an average of about 40 amperes by an aluminum slider

The vertical wear of the steel contact wire on the N. Y., N. H. & H. R.R was 0.028 in. in 30 months, which is practically 4.5% per year of the half diameter of the wire (one-half taken to permit wire to be held in clips) which, even on this vertical diameter basis, indicates a life of over 20 years; but as a matter of fact it will be much more than this, for the reason that as the vertical diameter lessens the breadth of contact increases throughout, thus diminishing the rate of vertical wear. Of further interest is the fact that there is practically no corrosion of the wire as the wire is covered with a film of grease deposited by the pantograph shoe (W. S. Murray).

Change in Length of Trolley Wire. The change in length of copper due to changes in temperature is one of the greatest difficulties in the maintenance of overhead work. A drop of 100°F in temperature will cause a copper bar to contract approximately 1 in. for every 100 ft of length. If it is restrained at the ends this will cause an additional stress of 2500 lb in the case of a No. 0000 trolley.

Third-Rail Systems

Third rails are used for low-voltage railways where the power requirements are in excess of the current-carrying capacity of a trolley wire, and where there is private right of way.

Collector Shoes. Current is collected from the third rail by projecting collector shoes on the cars. These shoes are placed on each side of each truck, and are all alive, when any one is alive.

Rails are usually made of special high-conductivity steel, it being not unusual to have a conductivity 1/6 or 1/7 that of copper.

Types of Construction. Third-rail construction may be classified into the top-contact and under-contact types, each of which is susceptible of important variations in design, especially with reference to the type of protection.

(a) **Interborough Top-Contact Type.** One of the most commonly used type is illustrated in Fig. 52. It is often called the "Interborough type" on account of its use in the subways

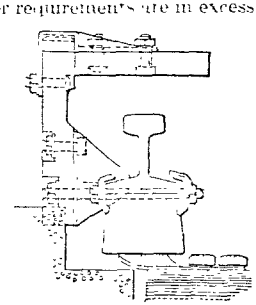

FIG 52 Interborough top-contact type

of New York. The rail is a standard T section and rests on reconstructed granite insulators. A board protection is attached to the rail itself by means of clamps and uprights and is thereby kept in perfect alignment.

(*b*) **Pennsylvania Top-Contact Type.** (Fig. 53.) Another type has the protection supported on separate brackets independent of the third rail itself. It is claimed that this reduces the amount of labor which has to be done on the live rail when repairs are being made, but it cannot be relied upon as well as the New York subways' type to keep the rail and protection in perfect alignment.

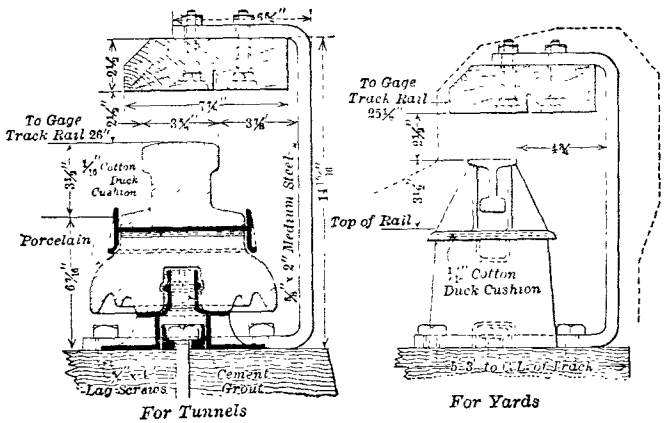

FIG 53 Pennsylvania top-contact type

(*c*) **Under-Contact Types.** (Fig 54.) Although the top-contact types have given first-class service, they are considered to have certain disadvantages for exposed locations as they cannot be wholly protected from snow, ice, and sleet. The lower part is only a few inches from the ties; holding clips generally reduce this clearance, increasing the danger of grounding from accumulation of wet snow, and from flooding The occasional suspension of traffic during sleet and snowstorms and floods, on railroads using the top-contact type of third rail, led to the idea of an under-contact third rail loosely clasped in insulators by hookbolts hung from brackets, with the top and sides of the rail completely sheathed in a flexible insulating material for protecting the rail from accidental contact with man and beast, and from sleet, snow, and spray With this type of rail the protection is of such character that there is no packing of snow between the sheathing and the contact rail, as in some other forms, and in sleet storms no ice forms on the contact surface; some icicles may form at the edge of the petticoats, but, hanging down clear of the edge of the rail, are easily broken off by the passing shoe.

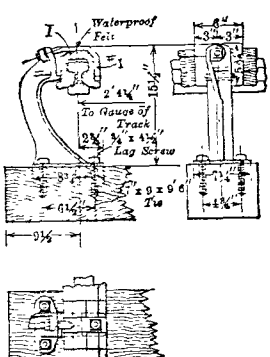

FIG 54 N Y C R R under-contact type

Where the rail is buried in snow, the passage of the contact shoe breaks the snow away, leaving the rail surface clear, instead of ironing the snow down on the rail, as may happen with the top-contact type

Location and Weight of Third Rails. There is no standard gage for contact rails, corresponding to the standard track gage. This unfortunate condition arises from lack of uniformity in the clearance lines of the right of way and in the maximum equipment lines of various railroads. The following standard has been recommended (1911 and 1912) by the AEREA:

The gage line of the third rail to be located not less than 26 in. and not more than 27 in. from the gage line of the track, and the contact surface of the third rail to be not less than 2 3/4 in. or more than 3 1/2 in. above the plane of the top of the track rail.

This standard has since been abandoned, and three standard clearance lines adopted instead. The maximum equipment line for third-rail structures and permanent way, respectively, are given below.

Table 34. Third-Rail Structures Must be Contained within the Line Indicated by the Following Ordinates and Abscissas

Height above Top of Rail, in	Distance out from Gage Line, in.	Height above Top of Rail, in.	Distance out from Gage Line, in
Top of tie	19 3/4	9 7/8	33 3/4
5/8	19 3/4	6 1/8	36 3/4
3 11/16	25	1/4	36 1/4
8	25	−2 1/4	35 1/4
9 7/8	26 7/8	Top of tie	35 1/4

(On curves of less radii than 800 ft and at switches, this line must be moved out 2 1/2 in.)

Table 35. Permanent Way Structures Must not Encroach upon the Line Indicated by the Following Ordinates and Abscissas

Height above Top of Rail, in	Distance out from Gage Line, in	Height above Top of Rail, in	Distance out from Gage Line, in
Top of tie	36 1/4	7 5/16	37 3/4
= 2 1/2	36 1/4	10 7/8	34 1/4
0	37 3/4		

(On curves of less radii than 800 ft and at switches, this line must be moved out 2 1/2 in.)

Inclines. Where it is necessary to break the third rail at crossovers, grade crossings, or electrical section breaks, **end declines** are required to catch the contact shoes and bring them to the normal level of the third rail.

The exact location of the inclines on each side of a track-rail intersection depends upon a number of conditions, an important one of which is the extent to which the car equalizer bar and journal box project outward. If there is a train bus line connecting all the cars, the end inclines may be situated many feet back from the switch point or frog, but if there is no such bus line, the contact rail will probably have to be terminated in a cross incline extending as near as possible to the switch point or frog.

Bonding. In order to provide a continuous path of low resistance for the flow return traction current through rail joints, some form of traction bond must be provided around the joint.

27. LOCOMOTIVE RATINGS

Locomotive tonnage ratings state the maximum load a locomotive of a given type can haul over a specified territory and maintain the required schedule. It depends upon the total tractive resistance to be overcome, of which grade resistance is usually the most important. Hence most tonnage ratings are based on the ruling grade. Other contributing factors are variations in train resistance with speed, curve resistance, time to accelerate, and length of grade. A minimum overall scheduled speed may be determined based on type of train (whether passenger, express, expedited freight, or drag freight) and nature of commodities handled. The length of ruling grade sets the speed over the grade necessary to fit the schedule. Using that speed, the train, curve, and grade resistances are summated and subtracted from the net drawbar pull at that speed to obtain the total gross tons that can be hauled. For comparative purposes this simple rating is usually sufficient. It may also be obtained by working with resistance throughout on a per ton basis.

Adjusted tonnage ratings are necessary to compute the actual performance of a locomotive under practical conditions where account must be taken of the varying weights of individual cars and the corresponding variations in train and grade resistance. The Pennsylvania Railroad method determines an adjusted tonnage factor, K, which is added to the flat tonnage of each car in building a train to its adjusted tonnage.[20] In determining K, the simple tonnage rating for trains of empty (20 tons per car) and trains of loaded cars

[20] Pennsylvania Railroad, *Test Department Bulletin* 26, 1915.

are obtained. By use of the adjustment factor, the adjusted tonnages of each should be equal, that is.

$$W_1 + KN_1 = W_2 + KN_2 \qquad \text{or} \qquad K = \frac{W_2 - W_1}{N_1 - N_2}$$

where W_1 and W_2 are the simple tonnage ratings of the trains with empty cars and the trains of loaded cars respectively, and N_1 and N_2 are the corresponding number of cars in each train. Decimal values of K can usually be ignored and the resulting inaccuracy overcome by using an average of $W_1 + KN_1$ and $W_2 + KN_2$ for the adjusted tonnage rating.

For a given type of locomotive engine and speed on the ruling grade, the grade itself is the only variable. Hence curves can be plotted showing adjusted ratings for that type engine on other grades

Economical Speed. When the velocity of trains is very low (say 5 mph), the fuel used per mile is greater and train wages are greater both on train under consideration and others due to greater interference with traffic, but any saving is small. For high velocities the cost per train mile is likewise comparatively high The most economical speed is about 17 mph for single track and nearly 20 mph for double track according to results of computations by Isaacs and Adams, in *Bul.* 115 of the Amer. Rwy. Eng. and Main Way Assoc These were, of course, based on certain assumptions, but there is no doubt that it is often not most economical to haul maximum train loads or full "ratings" on account of low speeds

With electric and diesel-electric locomotives, a more serious objection to low speed is found in excess heating of the motor. Since tractive effort increases as speed decreases, practical low-speed limitations are set by the continuous, hourly, and short-time ratings

Dynamometer cars are used to obtain by test the performance capabilities of locomotives, obtained with the foregoing computations. The car is placed between the locomotive and train and measures the locomotive drawbar pull (through a hydraulic cylinder and piston), which is recorded as a curve on a moving tape along with other necessary data of speed, throttle or controller position, fuel consumption, etc The area under the curve of drawbar pull represents the work done and, when divided by the time required to cover the distance, measures the horsepower. Electric and diesel-electric, etc., performance can be more readily and accurately determined through instrumentation of the locomotive used along with the builder's curves of motor characteristics.

Temperature has an effect on tonnage ratings by increasing starting and train resistances. Most roads provide some reduction in tonnage, the amount varying with the temperature. Practice on the P.R.R. establishes the tonnage adjustments shown in Table 36.

Table 36 *

Temperature Range, degrees F	Tonnage Rating, %
Above 25°	100
From 25° to 15°	95
From 15° to 5°	90
From 5° to zero	85
From zero to 5° below	80
From 5° below to 10° below	75
From 10° below to 15° below	70
From 15° below to 20° below	65

* Pennsylvania Railroad, Letter of Mr Cover, Chief of Motive Power, Nov. 29, 1950.

The above ratings apply to steam locomotives only. No reductions have been found necessary for electric and diesel-electric locomotives.

28. ACCELERATION AND DECELERATION

A definite force or **tractive effort** is required to accelerate a train to a higher rate of speed. Only that portion of tractive effort not required to overcome other forms of tractive resistance is available for acceleration. From physics, accelerative force, F_a, in pounds, is $F_a = \frac{wv^2}{2gS}$, where w is train weight in pounds, v is speed in feet per second, S is distance in feet to accelerate, and g is the acceleration due to gravity. For practical use and to accelerate from one speed to another, V_1 and V_2 are initial and final velocities in mph, W is weight in tons, and 5% additional force for rotative energy is added: whence $F_a = \frac{70W}{S}(V_1^2 - V_2^2)$. The net accelerative force in pounds per ton is $F_a' = \frac{70}{S}(V_1^2 - V_2^2)$.

The distance to accelerate is $S = \frac{\text{[illegible]}}{F \text{[illegible]}} (V_1 - V_2)$. Similarly the time to accelerate from a speed of V_1 to V_2 mph is $t = \frac{\text{[illegible]}}{F \text{[illegible]}}$. If V_1 is greater than V_2, then these formulas give the time and distance for deceleration (see Art. 5, p. 6-70). The relations between speed, time, and distance expressed as acceleration or deceleration curves would be straight lines if the rate of change of velocity were uniform. Since it is not uniform, points on the acceleration or deceleration curves or in computing a time or distance must be obtained by speed intervals or increments. Size of interval depends on accuracy required. Larger intervals give values of time and distance which are less than actually required. One mile per hour increments [illegible] successfully used with diesel-electrics, but for steam locomotives the [illegible] curve is seldom accurate enough to justify anything less than 3.5 [illegible]. Values of tractive effort, resistances, accelerative force, time, and distance may conveniently be set up in tabular form for each speed increment. Tractive effort and resistance are determined for the *average* value of speed over the speed increment [illegible] and speed-distance curves are plotted for various grades for a given train [illegible] of the acceleration and deceleration curves corresponding to the speeds, grades, and distances involved may be transferred to a profile of the line to show anticipate train or car performance, prepare tonnage ratings, or study the effects of relocations or curve and grade revisions. Before plotting, all stops and speed restrictions due to curvature, city ordinances, railroad crossings, etc., should be spotted on the profile. Units of time plotted against profile distances from a time-distance curve give the overall elapsed time between any two points on the entire territory.

29. ECONOMICS OF LOCATION

Location consists of fixing the horizontal and vertical position of the line.

Economic location is the determination of the proper relation between construction and operating expenses and the discovery, in any individual case, of that location which will afford the most economical combination of cost and operating expenses for the given or estimated traffic. The **principles** are practically applied by comparing proposed plans for new construction or by investigating changes in existing lines including abandonment of existing facilities. For new work the method must include for each proposed route these features: (1) the estimated cost of the line; (2) the annual interest charge on capital at a rate which the circumstances of the project make proper; (3) the estimated annual traffic (not only the gross tonnage, but also the number of trains required to handle it); (4) the effect, if any, of change of location on the traffic obtainable; (5) the estimated annual cost of handling such traffic; (6) the effect, as far as it is possible or proper to predict it, of future changes in traffic or in operating conditions. Having determined the gross revenue, the interest on first cost, and the operating expenses, the line giving the largest net revenue is evidently the most economical line. When revenue is not affected (generally true for small changes) the line having a minimum sum of operating expenses and interest on first cost is the most economical. When studying changes of an existing line, in addition to the features above mentioned, the possible injury to existing facilities must be considered. The proposed plan should, in such a case, so increase the net income that the excess will at least pay a proper return on the entire cost of the change.

The general method of analysis consists in separating the different items of cost of operation and determining the proportion of each item which is affected by each physical element in the proposed change of line. The physical elements are distance, curvature, minor grades, and ruling grades. Table 37 gives the aggregate results for each of the main groups of operating expenses and illustrates the method. Both the grouping and the percentages are now changed (see Table 44, p. 6-104).

The lack of agreement of these figures is partly due to some differences in the basis of computation, partly to the different times at which they were computed, and partly to the fact that such figures can never be constant for all times and conditions, but must be modified to suit local conditions. It is, however, remarkable that the mean values for the large and small changes give 38.1%, 41.4%, 38.5%, and 42% respectively as the percentage of total operating expenses which are affected by changes of length of line.

Earlier location studies reduced all costs to a train-mile basis, assuming thereby that train-mile costs varied little with the size of the train. The AREA recommends use of the 1000 gross ton miles (G.T.M.) unit reflecting train load as the proper basis in comparing effects of distance, curvature, and minor grades. Ruling and pusher grades are compared on a train-mile basis.

Table 37. Distance. Percentage of Operating Expenses Affected by Length of Line According to Four Estimates

Division of Total Cost of Railroad Operation	Wellington, 1887			Webb, 1906			Harriman Line,* 1909			AREA,† 1951		
	Total Percentage	Amount Affected	Per Cent to Total	Total Percentage	Amount ‖ Affected	Per Cent to Total	Total Percentage	Amount Affected	Per Cent to Total	Total Percentage	Amount Affected	Per Cent to Total
Maintenance of way	23.0 §	11 6 ‖ 19.0	50.4 82 7	21.0	15 7 ‖ 19.0	75 0 90.3	24.97	23.45	93 9	18.62	11 18 ¶ 15 39	0.60 0 82
Maintenance of equipment	15.6	5 7 8 2	36 6 52 6	18 0	6 7 7 6	37 2 42 1	20.53	0	0	23 32	14 63 ¶ 17.63	0 63 0 76
Conducting transportation	31.4	7 5 24.2	23 9 77 1	56 6	8.9 25 0	15.8 44.0	50.22	15 02	29.8	49 56	18 42 ¶ 25.53	0 37 0 52
General expenses, miscellaneous and traffic	30 0	0 0	0 0	4 3	0 0	0 0	4.28	0	0	8 50		
Totals	100.0	24 8 51 4		100.0	31 3 51.6	.	100 00	38 47		100 00	44.235 ¶ 58 558	44 235 58 558
									Train wages unaffected Train wages affected	100.00	35 723 ¶ 47.905	35 723 47 905

* Auditor's estimate for one of the Harriman lines.

† Distances of 1 10 miles.

‡ Including several subitems really belonging under maintenance of way or transportation which should decrease materially the percentages for "conducting transportation."

§ Interpretation of first line of figures: the maintenance of way expenses are estimated at 23 0% of total cost of operation; 11 6% of the total are affected by changes of length of line; the 11 6% is 50 4% of 23 0%. Other groups interpreted similarly.

‖ The upper figures show the percentage on the basis of changes so small that they are measured only in feet, which will not appreciably affect several items of expense; the lower figures are for changes measured in miles. The Harriman figures make no such distinction.

¶ Per 1000 G.T.M.

Distance

Short increases or decreases in distance, i.e., those less than a mile, have only a nominal effect on operating costs. As the table indicates, only a small percentage of operating expenses vary with these distances. A greater percentage, 38.3%, are affected when distance variations are between 1 and 10 miles. For distances greater than 10 miles, the per cent affected is 49.6% or 60.4%, depending on whether or not train- and engine-crew wages are affected.

Reduction of distance may also reduce the receipts as well as operating expenses and thus reduce the advantage gained by shorter distance, provided the change of distance is so great that it is measured in miles. The **effect on receipts** is not computable except in the most approximate way; it depends on the following elements. (1) Since the cost of operating additional distance is approximately 40% of the average, whereas passenger receipts and some other kinds are based more or less strictly on actual mileage, there is a real advantage and profit in added distance for such kinds of traffic. (2) For competitive business, where receipts are absolutely independent of distance, any added distance is a pure loss, without any compensation. (3) For through business, where the division of receipts between roads is more or less strictly according to the relative mileage of hauls on the two or more roads, there is profit or loss according to the percentage of length of haul on the home road to the total haul; the larger the percentage, the larger the possible profit, which also depends on whether the rate is competitive or non-competitive. (4) Since a road always has varying proportions of these several kinds of traffic, the net advantage or disadvantage depends on the combined effect. As a final conclusion, there is always some addition to receipts to partially offset the added cost of operating added distance; for a road whose business is very largely non-competitive the increase in receipts due to added distance might be greater than the increase in operating expenses, for a road whose business is chiefly competitive the net loss will be great, in any case there is some reduction on the advantage computed above by decreasing the distance.

Curvature

The basis of the analysis of cost is the determination, first, of the **degree of curve** on which the power requirements are precisely double the power used on a straight level track, and then the determination of the additional cost of the doubled resistance. It is then assumed that train resistance and added cost of curvature are proportional to the number of degrees of central angle and independent of the radius of curvature. The degree of curve on which the resistance is precisely double that on a level tangent has been variously estimated from 10° to 12° 30′. A mile of such continuous curvature will have from 528° to 660° of curvature. Doubling the power requirements on the engine will increase certain items of cost of a train mile by a computable percentage of such cost. A continuous 10° (or 12° 30′) curve will increase items of maintenance of way and equipment by percentages which may be closely estimated. Many of the items of cost of a train mile are unaffected. In this way the total increase in the cost of one mile of such curved track is computed. Dividing the total extra cost by the number of degrees (528 to 660) gives the value of 1°. The chief items affected by curvature are rails, of which the extra wear may be from 200 to 300%; repairs to locomotives, 100 to 200%, and repairs of cars which are increased from 50 to 100%. Some other items of maintenance of way are increased 25 to 50%, and fuel and water would be increased about 50%. The extra cost per degree has been variously computed as shown in Table 38.

Table 38

Author	Date	Degree per Mile	Percentage	Extra Cost per Degree Per Cent
Wellington	1887	600°	35.6	0.0593
Berry	1904	660	35.35 *	0.0536
			29.88 †	0.0498
Webb	1908	528	34.6	0.0655
AREA	1951	528	31.2 ‡	0.0600

* Uncompensated curvature.
† Compensated curvature.
‡ 1000 G.T.M.

The percentage of extra cost per degree chosen should be multiplied by the estimated cost per train mile or 1000 G.T.M. and this value multiplied by the number of trains (1000 G.T.M.), which gives the yearly cost of 1° of curvature. Capitalizing this at the proper rate of interest gives the amount which may justifiably be spent to avoid one degree. For cal-

culations of this kind, estimates must be made on the following items which will vary according to the local conditions. (1) the local cost per train miles or 1000 G.T M.; (2) the percentage by which the various items of cost per train mile are affected by curvature; (3) the number of trains which will use the tracks. For new construction even the last item is an uncertain quantity.

Minor Grades

The basis of computing the cost is to determine, first, the rate of grade which will exactly double the resistance on a level tangent, second, to compute the effect on operating expenses of a mile of such a grade; and third, on the assumption that the cost of lifting a train through so many vertical feet is proportional to the height, the extra cost of 1 ft of elevation equals the added cost of operating a mile of such grade divided by the number of vertical feet in a mile of that grade. It is assumed that the grade which doubles tractive resistance on a level tangent does not limit the length or weight of trains it is designed to haul with one engine, or that any limiting effect is a separate matter, and that the only effect of the grade is to increase the operating expenses over those required for operation over a level track. Minor grades are necessarily divided into three classes on account of a distinction in their effects on the operation of trains. Class A includes those grades which, on account of the rate of grade or the length, do not require any change in the handling of the train, no brakes, and no change in the throttle or cutoff. Class B includes those grades which are steep enough or long enough to require shutting off power (but no brakes) in order to avoid objectionable velocity while descending and to require more power or longer cut-off when ascending. Class C includes the grades which require that power shall be shut off and brakes applied on downgrades and full power used on the upgrades. Since a fast passenger train may safely attain very high speed in a sag and since the effect of a hump is a relatively small reduction of a previously high velocity, a grade which should properly be classified as an A grade for fast passenger trains should be classified as a B or even a C grade for slow freight trains. The average tractive resistance of a train at ordinary velocity is about 10 lb per short ton. For slow freight trains the figure is too high, for fast passenger trains it is too low. Ten pounds per ton is the equivalent of the grade resistance on a 0.5% grade which has a rise of 26.4 ft per mile.

Sags and Humps. This section applies only to the possible elimination of sags and humps, since a long uniform grade connecting two predetermined points is practically unchangeable. The problem therefore becomes the determination of the value of removing a hump or filling a sag whose height is the vertical distance from the vertex of the sag or hump from the uniform grade by which it would be replaced **Value of 1 ft or rise-and-fall**: Class A grades have no effect on the power required of the locomotive, and no effect on operation except a harmless temporary increase or decrease in velocity, they have also no appreciable effect on operating expenses and can therefore be ignored. The effects of Class B and Class C grades have been variously computed as shown in Table 39.

Table 39

		Percentage Affected		Annual Value *	
Author	Date	Class B, %	Class C, %	Class B, %	Class C, %
Wellington	1887	3.03	9.67	84	276
Webb	1908	6.08	7.92	168	219
Berry	1904	3.38	7.13	93	197
AREA †	1951	2.36	8.85	. .	. .

* Annual value in per cent of average cost per train mile for each round trip daily train per foot of rise-and-fall.

† Per 1000 G.T.M.

Table 39 is based on an average train resistance of 10 lb per ton, whereas 6 to 7 lb is more nearly correct for present rolling stock.

Ruling Grades

Ruling grades are those which limit the length or weight of the trains which may be handled by one locomotive. The classification does not include steep grades which will always be operated by two or more engines and are called pusher grades, nor does it include very short steep grades which may always be operated by momentum. The maximum train loads which may be hauled on a given ruling grade and on the proposed lighter grade by an engine of given weight, dimensions, and power may be determined by dividing the tractive force by the total resistance per ton, both taken for the same speed.

The cost per train mile of each additional train for hauling the same tonnage of cars up a somewhat steeper grade has been computed by Webb as about 53% of the average cost of a train mile. J. B. Berry (*Bul.* 49, Amer. Ry. Eng. and Main. Way Assoc.) computes the cost similarly as 43.29%. Isaacs and Adams (*Bul.* 112 of same association) computed that 30.64% of the total operating expenses are "affected by locomotive mileage." They use the figure by multiplying the additional number of train miles required by the higher grade by 30.6% of the average cost per train mile and consider this as the additional cost of the higher grade. The author has computed a value of 39.6

The number of daily trains required for each grade may be found by dividing the given, or estimated, total tonnage by the maximum net train load for each grade; the difference is then the number of daily trains saved.

Table 40. Per Cent of Trains Necessary to Handle a Given Traffic with Single Engine, if Gradients Are Changed from Those in First Column to Any Other Given Gradient

Gradient to Be Reduced, %	Proposed Gradient, Per Cent, with Per Cent of Trains to Handle Same Traffic as on Original Gradient															
	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
2.0	26	30	34	38	42	46	51	55	60	65	69	74	79	84	89	95
1.9	27	31	36	40	44	49	54	58	63	68	73	78	84	89	94	100
1.8	29	33	38	42	47	52	57	62	67	72	78	83	89	94	100	
1.7	31	35	40	45	50	55	60	66	71	77	82	88	94	100		
1.6	33	37	43	48	53	59	64	70	76	82	87	94	100			
1.5	35	40	46	51	57	63	68	75	81	87	93	100				
1.4	37	43	49	55	61	67	73	80	87	93	100					
1.3	40	46	53	59	66	72	79	86	93	100						
1.2	43	50	57	64	71	78	85	92	100							
1.1	47	54	61	69	76	84	92	100								
1.0	51	59	67	75	83	91	100									
0.9	55	64	73	82	91	100										
0.8	61	71	80	90	100											
0.7	68	79	89	100												
0.6	76	88	100													
0.5	87	100														
0.4	100															

This table is taken from *Bul.* 49 of Amer. Ry. Eng. and Main. Way Assoc., by J. B. Berry; calculated for engine and tender weighing 150.3 tons in working order, tractive force 35,268 lb, weight of caboose 17 tons, velocity and frictional resistance 6 lb per ton. As percentages, the figures apply approximately to almost any freight engine

Ruling grade selection is governed in part by the necessity of holding certain key points—initial and final terminals, traffic centers, mountain passes, and river crossings. Between these control points there may be several route possibilities. A ruling grade is selected for each, and the several routes compared. Each leg of the route then contains a potential ruling grade from which one must be selected for the entire route or district. Not only the minimum gradient is sought but also the shortest, since excessive distance on a limiting grade adds time to the schedule. Keeping approximately equal grades on adjacent divisions permits use of similar types of motive power and tonnage ratings. Trains need not be rebuilt at division points. Ruling grades should be free of sidings, stations, water and fueling points, railroad crossings at grade, and any other cause for stops. When these of necessity must be present, the gradient should be reduced to a value on which a tonnage train can be started. Future line changes and improvements should also be kept in mind.

The ability of electric and diesel-electric locomotives to operate for limited periods of time at overload capacities permits the use of heavier grades than indicated by the continuous rating of these units. The grades must be short enough to be negotiated within the time limit for safe overload operation and should be followed by lighter or, better, descending grades to permit cooling the motors. Manufacturer's data specify the safe short period ratings for the various gear ratios and speeds. These characteristics are useful with a rolling profile.

Pusher Grades. A road may be designed for pusher grades by determining a pair of grades such that two engines of a certain type can haul up the steeper grade the same train that will just tax the capacity of one such engine on the lower grade. This pair of grades must be selected according to the character of the country and must be such that all the grades of the line may be reduced, without unreasonable cost, to the limit of the grade for one engine, except such grades as it is intended to operate with pusher engines. Little or nothing is saved by reducing these pusher grades below the limit computed for two engines. In exceptional cases a combination of three grades may be computed, of which the two lower grades are to be operated as above, and the highest grade is such that three engines are required to haul the same train over it. Table 41 is computed for one type of

engine. The precise balance between any through grade and the corresponding grades for one and for two pushers depends on the type of engine, the available ratio of adhesion, and the tractive resistance, but it may be demonstrated that reasonable variations in the type of engine or tractive resistance will only alter the required grades by a few hundredths of 1%. The corresponding pusher grades may therefore be accepted as approximately true for all cases. The accuracy of the table is theoretically impaired by the fact that the rating tonnage on the pusher-engine service is somewhat different from that on the through-engine service. The variation will depend on the ratio of rating-ton resistance to tare-ton resistance. As an illustration, a 1.9% one-pusher grade corresponds to a 0.92% through grade according to the table. Working out the corresponding grade, using the values proposed by Dennis of 2.6 lb as the resistance of a rating ton and 9 lb as the resistance of a tare ton, the corresponding through grade is computed to be 0.99%. Some increase should be expected since the pusher locomotive has a higher resistance per ton than average train resistance. This shows that although no table can be compiled which will suit all conditions, the differences will probably not exceed a few hundredths of 1%. The variations are of less importance with the higher pusher grades. The cost of a pusher engine mile as a percentage of the average cost of a train mile was estimated by Wellington (1887) as 38.3%; by Berry (1904), 34.4%; by Webb (1908), 38.8%.

Table 41. Balanced Pusher Grades for One, Two, and Three Engines

Through Grade, %	Track Resistance, 6 lb			Track Resistance, 8 lb		
	Net Load for One Engine, tons (2000 lb)	Corresponding Pusher Grade for Same Net Load, %		Net Load for One Engine, tons (2000 lb)	Corresponding Pusher Grade for Same Net Load, %	
		One Pusher	Two Pushers		One Pusher	Two Pushers
Level	3868	0.28	0.55	2874	0.37	0.72
0.10	2874	0.47	0.82	2278	0.56	0.98
0.20	2278	0.66	1.08	1880	0.74	1.23
0.30	1880	0.84	1.33	1596	0.92	1.47
0.40	1596	1.02	1.57	1384	1.09	1.70
0.50	1384	1.19	1.80	1218	1.27	1.92
0.60	1218	1.37	2.02	1085	1.44	2.14
0.70	1085	1.54	2.24	977	1.60	2.36
0.80	977	1.70	2.46	887	1.77	2.56
0.90	887	1.87	2.66	810	1.93	2.76
1.00	810	2.03	2.86	745	2.09	2.96
1.10	745	2.19	3.06	688	2.24	3.15
1.20	688	2.34	3.25	638	2.40	3.33
1.30	638	2.50	3.43	594	2.55	3.51
1.40	594	2.65	3.61	555	2.70	3.68
1.50	555	2.80	3.78	521	2.85	3.85
1.60	521	2.95	3.95	489	2.99	4.02
1.70	489	3.09	4.12	461	3.13	4.17
1.80	461	3.23	4.27	435	3.27	4.33
1.90	435	3.37	4.43	411	3.42	4.49
2.00	411	3.52	4.59	390	3.55	4.63
2.10	390	3.65	4.73	370	3.68	4.78
2.20	370	3.78	4.88	352	3.81	4.92
2.30	352	3.91	5.02	335	3.94	5.05
2.40	335	4.04	5.15	319	4.07	5.19
2.50	319	4.17	5.29	304	4.20	5.32

Basis. Through and pusher engines alike, consolidation type, total weight, 107 tons, weight on drivers, 53 tons, adhesion, 9/40, giving a tractive force for each engine of 23 850 lb, normal track resistance 6 (also 8) lb per ton. It is assumed that the engine is so designed and that the velocity and steam pressure are such that the engine can always develop a tractive force of 23 850 lb.

The mileage must be computed as twice the total distance between the sidings at top and bottom of the grade, this distance being always somewhat in excess (and sometimes very much so) of the length of the pusher grade on the profile.

Balanced Grades. The foregoing principles have by implication been applied to grade design for movement in one direction only. Actually trains run in both directions over the same profile. Usually a prevailing tonnage movement exists in one direction only, permitting fewer return trains with more empty cars. But locomotives and crews must be returned also. This suggests the return of lightly loaded trains which do not require as easy gradients as the loaded outbound movement. To be exactly in balance, the return-bound ruling gradient need be no flatter than necessary to permit the same number of cars with less total resistance to be handled as are taken loaded in the opposite direction. The problem is not quite that simple, however The difficulty lies in determining just what per cent of theoretical balance should be used since only in a few limited cases—for example, one-directional movement of iron ore from the pit—do theoretical conditions govern It is difficult to predict what the ratio between prevailing tonnage and adverse traffic is going to be. Usually passenger traffic and tonnages will be the same in both directions. These factors must be considered as urging substantial modifications of a completely theoretical balance ratio.

Momentum Grades and Velocity Heads

Momentum grades are grades which, although somewhat steeper than the ruling grade, can invariably be approached at such a velocity that the kinetic energy of the train will assist the engine sufficiently to carry the train past the summit with a minimum velocity of say 10 mph. If the resistance of trains were independent of velocity, the mechanics of the problem would be much simplified, but the extensive dynamometer tests of Dennis, confirmed by those of Shurtleff, have shown that the resistance per ton of freight trains at velocities between 10 and 35 mph is so nearly uniform that the following method gives a close approximation to accuracy. It is also fortunately true that a very considerable error in velocity height at high velocities is comparatively unimportant. Assuming that a train could run without track resistance, its kinetic energy at a velocity v in feet per second would lift it up a grade to a total vertical height $h = v^2/2g$, in which g is the acceleration of gravity. If V = velocity in miles per hour, $h = 0.03344V^2$; adding 5% for the rotative kinetic energy of the wheels, we have $h = 0.0351V^2$. The values of h (or "velocity heights") are given in Table 42.

An engine properly loaded will haul its train up the ruling grade, and the test of the momentum grade or hump is the possibility of hauling the train up the excess grade with the help of momentum. For example, a train, with the engine loaded for a 0.7% grade, approaches a 1.2% grade 7850 ft long with a velocity of 35 mph. The excess grade is 0.5%, and in a distance of 7850 ft, the excess rise is 39.25 ft. The velocity head corresponding to 35 mph is 43.01 ft; subtracting 39.25, we have left a velocity head of 3.76, which corresponds to a velocity of 10.3 mph, which is a safe speed for going over the summit The secondary question, whether it is safe to assume that no fully loaded freight train will ever need to stop, or even slacken speed, along this grade of 7850 ft, is a matter of practical operation which the engineer is unwarranted in assuming.

The accuracy of this method depends on the assumptions that the resistance is uniform within the range of the velocities, and also that the work done by the engine is uniform. Although the resistance may be considered substantially uniform, the tractive force of the engine decreases as the velocity increases. When, as in the above illustration, the train approaches the grade at a velocity of 35 mph, the engine is not producing as much tractive force as it can produce at a lower velocity. But if the rating of the engine is based on a fairly high velocity on the ruling grade, the excess tractive force at lower velocities may so balance the deficiency at the higher velocities that the method is applicable without material error. Experience has shown this to be true.

Momentum grades should not be introduced in well-designed locations. Rather, momentum should be looked upon as a factor of safety. When momentum grades are used, the actual grade should be no greater than that on which a given locomotive can start half its maximum train load. This permits doubling the hill, i.e., taking one-half the train at a time.

Length of grade must also be considered since velocity head is lost through losses due to sway and to buffer and drawhead concussion. It is assumed to be dissipated at the rate of 30% per mile of grade. As a practical matter, most of the initial velocity head will have been dissipated before the summit is attained.

Sags in Grade Line. Economies are often possible where sags having grades exceeding the ruling grade on one or both sides may be harmlessly introduced. Their harmlessness may be tested by Table 42. It is of course assumed that running through the sag without a stop may be depended on. Deciding first the maximum permissible speed for freight trains in the bottom of the sag, the ability of the train to mount the grade in either direction

Table 42. Velocity Heads in Feet for Speed in Miles per Hour

Formula: $h = 0.035v^2$

Speed	.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.02	0.03
1	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.13
2	0.14	0.15	0.17	0.19	0.22	0.22	0.24	0.26	0.27	0.29
3	0.31	0.34	0.36	0.38	0.40	0.43	0.45	0.48	0.51	0.53
4	0.56	0.59	0.62	0.65	0.68	0.71	0.74	0.77	0.81	0.84
5	0.87	0.91	0.95	0.98	1.02	1.06	1.10	1.14	1.18	1.22
6	1.26	1.30	1.35	1.39	1.43	1.48	1.52	1.57	1.62	1.67
7	1.72	1.76	1.81	1.87	1.92	1.97	2.02	2.08	2.13	2.18
8	2.24	2.30	2.35	2.41	2.47	2.53	2.59	2.65	2.71	2.77
9	2.84	2.90	2.96	3.03	3.09	3.16	3.23	3.29	3.36	2.43
10	3.50	3.57	3.64	3.71	3.79	3.86	3.93	4.01	4.08	4.16
11	4.24	4.31	4.39	4.47	4.55	4.63	4.71	4.79	4.87	4.96
12	5.04	5.12	5.21	5.29	5.38	5.47	5.56	5.65	5.73	5.82
13	5.91	6.01	6.10	6.19	6.28	6.38	6.47	6.57	6.66	6.76
14	6.86	6.96	7.06	7.16	7.26	7.36	7.46	7.56	7.67	7.77
15	7.87	7.98	8.09	8.19	8.30	8.41	8.52	8.63	8.74	8.85
16	8.96	9.07	9.18	9.30	9.41	9.53	9.64	9.76	9.88	10.00
17	10.11	10.23	10.35	10.47	10.60	10.72	10.84	10.97	11.09	11.21
18	11.34	11.47	11.59	11.72	11.85	11.98	12.11	12.24	12.37	12.50
19	12.64	12.77	12.90	13.04	13.17	13.31	13.45	13.58	13.72	13.86
20	14.00	14.14	14.28	14.42	14.57	14.71	14.85	15.00	15.14	15.29
21	15.44	15.58	15.73	15.88	16.03	16.18	16.33	16.48	16.63	16.79
22	16.94	17.10	17.25	17.41	17.56	17.72	17.88	18.03	18.19	18.35
23	18.52	18.68	18.84	19.00	19.17	19.33	19.50	19.66	19.82	19.99
24	20.16	20.33	20.50	20.67	20.84	21.01	21.18	21.35	21.53	21.70
25	21.88	22.05	22.22	22.40	22.58	22.76	22.94	23.12	23.30	23.48
26	23.66	23.84	24.03	24.21	24.40	24.58	24.77	24.95	25.14	25.33
27	25.52	25.70	25.90	26.09	26.28	26.47	26.66	26.86	27.05	27.25
28	27.44	27.64	27.83	28.03	28.23	28.43	28.63	28.83	29.03	29.23
29	29.44	29.64	29.84	30.05	30.25	30.46	30.67	30.88	31.08	31.29
30	31.50	31.71	31.92	32.13	32.35	32.56	32.78	32.98	33.20	33.42
31	33.64	33.85	34.07	34.29	34.51	34.73	34.96	35.18	35.40	35.62
32	35.84	36.06	36.29	36.52	36.74	36.97	37.19	37.42	37.65	37.88
33	38.11	38.34	38.58	38.81	39.05	39.27	39.51	39.75	39.98	40.22
34	40.46	40.70	40.94	41.18	41.42	41.66	41.90	42.14	42.38	42.63
35	42.87	43.12	43.37	43.61	43.86	44.11	44.36	44.61	44.86	45.11
36	45.36	45.61	45.87	46.12	46.38	46.63	46.88	47.14	47.40	47.66
37	47.91	48.18	48.43	48.70	48.96	49.22	49.48	49.74	50.01	50.28
38	50.54	50.81	51.08	51.34	51.61	51.88	52.16	52.42	52.69	52.96
39	53.24	53.51	53.78	54.06	54.34	54.61	54.89	55.17	55.44	55.72
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
40	56.00	58.84	61.74	64.72	67.76	70.88	74.06	77.32	80.64	84.03
50	87.50	91.03	94.64	98.31	102.06	105.77	109.75	113.72	117.74	121.83
60	126.00	130.23	134.54	138.92	143.36	147.87	152.45	157.11	161.84	166.63
70	171.50	176.45	181.44	186.52	191.65	196.88	202.16	207.52	212.94	218.42
80	224.00	229.60	235.33	241.10	246.95	257.88	258.84	264.91	271.04	277.22
90	283.50	289.82	296.23	302.70	309.25	315.86	322.55	329.31	336.15	343.05
100	350.00									

Note: For the table the theoretical heads have been increased approximately 5% to allow for the energy of the rotating wheels. From *Elements of Railroad Engineering*, by W. G. Raymond, H. E. Riggs, and W. C. Sadler, 6th Ed., p. 322.

and have sufficient speed at the top of the grade may be tested. It must also be determined what speed of approach is necessary at the other end of the sag in order to attain that maximum speed at the bottom, and whether it is practicable to approach the sag at that speed.

As an illustration, the ruling grade is 1.2%; economical construction suggests a downgrade of 2.0% for a distance of 4500 ft, followed by an upgrade of 2.0% for 7000 ft. Assume a maximum permissible freight-train velocity of 45 mph, the velocity head being 71.10 ft. The excess grade is 0.8%, which in 7000 ft has a rise of 56 ft. At the top of the grade the remaining velocity head is 15.10 ft, which corresponds to 20.7 mph, which shows that a speed of 45 mph through the sag would be unnecessary. The drop on the other slope of the sag is 90 ft, which would give to the train a velocity of over 50 mph, even without using steam. There is therefore no question in this case of the ability of the train to acquire that speed at the bottom of the sag. It will also be still easier to run the train through the sag in the opposite direction.

Classification of Grades. Table 42 may be utilized to predict the behavior of trains, and therefore to classify the grades into the A, B, and C classes discussed under Minor Grades. The above case should be classified as C grade, since power must be shut off and brakes used to prevent objectionably high speed in the sag, for either direction of movement.

Velocity Profiles. Acceleration and deceleration represent a change in velocity head. In accelerating from a speed of V_1 to V_2 mph, the gain in velocity head is

$$h = 0.0351(V_1^2 - V_2^2)$$

During acceleration a train may be said to be overcoming resistance, the resistance of a grade equivalent to the increase in velocity head. The train will hold that velocity head once gained until there is a change in speed. Figure 55 shows graphically the demand made

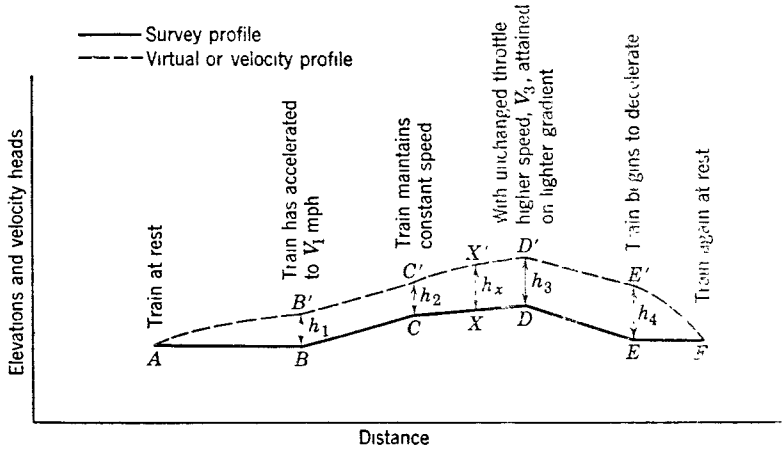

Fig. 55. Virtual profile.

on the locomotive through changing speed and grade conditions by representing that demand as a virtual or velocity profile. The sketch shows the relations between the actual and virtual profiles to be as follows.

(*a*) When a train accelerates, the profiles diverge.
(*b*) When a train decelerates, the profiles converge.
(*c*) When a train maintains a constant speed, the profiles remain parallel at a distance equal to the velocity head for that constant speed.

The *AREA Proceedings* for 1943, Vol. 44, pp. 182–190, describes methods for preparing velocity profiles and their use; these are too lengthy for presentation here.

30. COSTS AND OPERATING STATISTICS

Unit costs are used in preparing estimates for track and structures construction and maintenance. As discussed in construction procedures in Art. 6, unit costs in roadbed construction are determined by the cubic yard of excavation, the gallon or square for compaction, and square feet or square yards for stripping and grubbing. Fences are estimated either as a rod of completed fence or by individual items. Fencing is listed by the yard. Ballast is estimated by the cubic yard, crossties by the piece, bridge timbers by the board foot, and switch ties by the board foot or in sets. Rail, once bought by the long ton, 2240 lb, and scrapped by the short or 2000-lb ton, is now bought by the short ton. Turnouts are listed as complete units or may be detailed by individual items. Crossing frogs and other special fittings are itemized as individual units. Track fastenings are listed by the unit or by the hundredweight. For quick tentative estimating, track is listed at so much a track foot or so much a mile. Various unit costs are shown in Table 43, which follows. To these should be added appropriate percentages for transportation, stores, and other expenses. Current prices are quoted in pertinent trade journals. Most railroads maintain up-to-date price lists both for items of materials and for work units. Formal estimates are usually prepared using account numbers prescribed by the Interstate Commerce Commission in their *Uniform System of Accounts for United States Railroads.* The capital accounts record all investment in roads and equipment or any additions and betterments to the property. Operating accounts for maintenance of way and structures, cars and locomotives, etc., contain the costs of maintaining and operating the properties.

Table 43. Typical Costs per Mile of Track and Other Unit Costs

As of Sept 20, 1954

1A. *Estimated cost to construct 1 mile (5280 track feet) of 132 lb main track using new material throughout, excluding grading*

Item	Unit		Price	Amount	Total
Ties					
Cross, creosote, 7″ × 8″ × 8′ 6″	Each	3250	$ 3 73	$12,122 50	
Store expense 2%				242 45	$12,364 95
Rail					
132 lb new 10560 linear feet	G.T.	207.59	101.95	21,163.80	
Store expense 2%				423.28	21,587 08
Other track material					
132 lb angle bars 36″	Pair	278	6.30	1,751.40	
Bolts, track, 1 1/16″ × 6″	Keg	19.4	22 90	444 26	
Nutlocks, 1 1/8″ spring	M	1 668	89 75	149 70	
132 lb tie plates	Each	6500	1.07	6,955.00	
Spikes, track, 6″ × 5/8″	Keg	56.52	15 40	870.41	
Rail anchors, Fair S.I.	Each	2110	0.38	801.80	
Store expense 6%				658.35	11 630 92
Ballast					
Rock	C.Y.	4000	1 25	5,000 00	
Labor to unload	C.Y.	4000	0.76	3,040 00	8,040 00
Track laying and surfacing					
Lay and surface track	Feet	5280	2 09	11,035 20	11,035 20
				Total	$64,655 15
				Per track foot	$ 12 25

1. *Estimated cost to construct 1 mile (5280 track feet) of 115 lb main track using new material throughout, excluding grading*

Ties					
Cross, creosote, 7″ × 8″ × 8′ 6″	Each	3250	$ 3.73	$12 122 50	
Store expense 2%				242 45	$12,364 95
Rail					
115 lb new 10560 linear feet	G.T.	180.24	101.95	18,375 47	
Store expense 2%				367 51	18,742 98
Other track material					
115 lb angle bars 36″	Pair	278	5 67	1,576 26	
Bolts, track, 1 1/16″ × 5 1/2″	Keg	18 53	22.90	424 34	
Nutlocks, 1 1/8″ spring	M	1.668	89 75	149 70	
115 lb tie plates	Each	6500	0.97	6,305 00	
Spikes, track, 5 3/4″ × 5/8″	Keg	53.72	15 40	827 29	
Rail anchors, Fair S.I	Each	2110	0.38	801 80	
Store expense 6%				605 06	10,689 45
Ballast					
Rock	C.Y.	4000	1.25	5,000 00	
Labor to unload	C.Y.	4000	0 76	3,040 00	8,040 00
Track laying and surfacing					
Lay and surface track	Feet	5280	2.09	11,035 20	11,035 20
				Total	$60,872.58
				Per track foot	$ 11.53

2. *Track, per foot*

Main track	115 lb $11 53	132 lb $12.25	
Secondary and side tracks	90 lb 8.40		

3 *Turnouts, complete*

No 8 non-insulated	90 lb $1,960 00	115 lb $2,680.00	132 lb $2,990.00
No. 10 insulated	90 lb 2,190.00	115 lb 3,640.00	132 lb 4,020.00

4. *Rail, per gross ton*

New	$101 95
Second-hand	64.75
Scrap	33 60

5. *Ballast, per ton*

Crushed stone	$1 25
Slag	1.05
Other	1.00

Table 43. Typical Costs per Mile of Track and Other Unit Costs—*Continued*

6. Ties, average cost	
Sizes 2 and 3	$3.13
Sizes 4 and 5	3.73
7. Joint assemblies, pair (including bolts and locks)	
115 lb	$7.58
132 lb	8.21
8. Track spikes, per cwt	
90 lb	$8.09
115 lb and 132 lb	6.91
9. Rail anchors, each	
Improved, fair	$0.35
Superimproved, fair	0.38
10. Highway crossing, single track, 20-ft highway	
Ballast surface	$175.00
Bituminous surface	200.00
Concrete surface	350.00
11. Section toolhouse	
Branch or main line 12′ × 30′	$3,500.00
12. Automatic block signals	
Per mile	$8,500.00 (varies with location)
Per signal	7,000.00
13. Track laying and surfacing	
Per track foot	$2.09

Note All unit prices of track work are exclusive of grading.

Table 44. Operating Expenses of U. S. Railroads *

Expense Item	Amount, in $1000	Per Cent of Total	Per Cent Change Due to Total Number of Trains
I. Maintenance of way and structures			
Way	830,248	12.046	6.891
Structures	161,449	2.344	0.274
Wire and signal systems	83,561	1.213	0.412
Miscellaneous	179,237	2.600	0.064
Joint facilities	29,034	0.421	...
Total	1,283,529	18.624	7.641
II Maintenance of equipment			
Equipment repairs	1,232.541	17.885	6.555
Retirements	288,638	4.187	...
Machinery	47,315	0.686	0.106
Miscellaneous, deferred, others	38,839	0.564	0.042
Total	1,607,333	23.322	6.703
III. Traffic	194,351	2.820	..
IV. Transportation			
Yard	698 593	10.136	0.083
Trains	1,613,714	23.414	22.817
Miscellaneous	1,011,716	14.681	1.935
Joint facilities	91,771	1.332	..
Total	3,415,794	49.563	24.835
VI. Miscellaneous operations	117,992	1 712	. .
VII. General	272,820	3.959	0.426
Grand total, railway operating expenses	6,891,819	100.000	39.605

* From *Statistics of Railways in the United States*, Table 89, pp. 83–86, 1949, compiled by the Interstate Commerce Commission.

Item V has been discontinued by the ICC.

Table 45. Selected Statistics of Class I Railroads in the United States

From *Statistics of Railroads in the United States*, compiled by the Interstate Commerce Commission, as of December 31, 1951, unless otherwise noted

Item	1941	1948	1949	1950	1951
1. Miles of road owned	219,151	214,512	213,968	213,539	213,401
2. Railroad-owned freight cars	1,703,224	1,759,523	1,753,728	1,721,231	1,752,391
3. Railroad-owned passenger cars	20,709	21,035	19,684	19,109	18,340
4. Rate of return on investment	4.28	4.24	2.86	4.23	3.69
5. Locomotives owned					
Steam	39,353	33,585	30,493	26,575	23,601
Electric	804	760	734	718	705
Diesel-electric	1,007	5,098	6,669	8,643	11,057
6. Crosstie renewals, ×1000					
Treated	41,461	35,024	28,875	29,340	27,938
Untreated	5,763	1,818	1,410	1,154	1,124
7. Rail renewals, in 1000 long tons	1,196	1,366	1,280	1,208	1,131
8. Employees, ×1000	1,140	1,327	1,192	1,220	1,276
9. Operating revenues, ×$1,000,000	5,347	9,672	9,580	9,473	10,391
10. Operating expenses ×$1,000,000	3,664	7,472	6,892	7,059	8,041
11. Operating ratio *	68.53	77.26	80.32	74.52	77.39
12. Train speed, mph	17.0	16.8	16.9	16.2	16.5
13. Net tons per loaded freight car mile	28.5	32.9	31.4	31.7	33.0
14. Car miles per serviceable freight car per day	43.7	47.2	42.9	46.5	47.2
15. Gross ton miles per freight train hour	34,684	42,363	42,346	44,353	46,424
16. Cost per freight train mile *	4.93	9.64	10.26	10.34	11.52
17. Cost per 1000 revenue freight ton miles *	5.89	8.84	9.71	9.05	9.42
18. Cost per passenger train mile *	2.14	4.49	4.66	4.83	5.84
19. Freight locomotive miles, ×1000	645,745	663,244	553,107	576,990	758,680
20. Freight train miles, ×1000	576,727	584,671	498,573	514,971	528,573

* From *Railroad Transportation, A Statistical Record*, published by the Bureau of Railway Economics, Association of American Railroads, March 1953, Washington, D.C.

SECTION 7

SOIL MECHANICS AND SITE EXAMINATION

BY

W. MACK ANGAS AND CARLTON S. PROCTOR

The authors acknowledge with appreciation the value of assistance, suggestions, and constructive criticism offered by Mr. Philip C Rutledge, consulting engineer, of New York, and Professor Gregory P Tschebotarioff of Princeton University.

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SOIL MECHANICS AND SITE EXAMINATION

1. GENERAL

Soil mechanics may be defined as the application of the laws of mechanics to the determination of the strength, stability, and consolidation characteristics of soils. Attempts to apply mathematical analysis to soil and foundation problems before the development of the methods and techniques of modern soil mechanics were devoted largely to theoretical consideration of the anticipated behavior of hypothetical non-cohesive materials. Modern soil mechanics, while utilizing the techniques of mathematical analysis, takes cognizance of the results of extensive laboratory and field investigations of the actual behavior of many kinds of soils under stress. It also takes cognizance of the importance of water as a constituent of soils of many types, rather than treating it as an extraneous, and possibly fortuitous, material. Engineers are indebted to Dr. Karl Terzaghi, the late Daniel E. Moran, and others, many of whom were Dr. Terzaghi's students and associates, for originating many of the concepts of soil mechanics and developing soil mechanics into an engineering specialty the importance of which is unquestioned. For soil mechanics has converted the engineering of foundations and earth structures from an art based almost wholly on empirical rules and the experience of the practitioner to an art based largely on science.

But the methods and techniques of soil mechanics do not alter the fact that the non-homogeneous character of soil greatly complicates the prediction of its behavior under stress. Stresses in soils and the behavior of soils under stress cannot be determined and predicted with the assurance with which the structural engineer deals with stresses and strains in steel, concrete, and timber. Foundation engineering and the engineering of earth structures therefore makes very heavy demands on the experience and sound judgment of those who practice these closely related specialties. Though a knowledge of soil mechanics has become an essential part of the equipment of engineers who deal with foundation and soil problems, the fact remains that they practice an art that is not wholly based on science. Therefore the soundness of many of their decisions must depend largely upon experience and good judgment.

Nomenclature. The following nomenclature is in substantial agreement with reference 8.

A	area.
b	dimension, usually width of a foundation.
c	cohesion per unit area.
C_u	Hazen's uniformity coefficient.
C_p	coefficient of settlement or coefficient of subgrade reaction.
CBR	California Bearing Ratio.
D_d	relative density.
D_e	external diameter of sampler.
D_i	internal diameter of sampler opening.
D_{10}	Hazen's effective size of a sand, the 10% size.
D_{60}	the 60% size of a sand or size such that 60% of the material by weight is smaller.
d	a dimension, usually depth of a foundation, a dial reading, character designated by subscript.
e	void ratio, ratio of volume of voids to volume of solids.
e_c	critical void ratio of sands.
e_f	mechanical efficiency of a pile hammer.
F	force, character indicated by subscript.
F_n	force, or component of force, normal to a plane.
F_s	factor of safety.
G	specific gravity, overall measure of specific gravity of soil particles.
G_b	bulk specific gravity, ratio between weight of gross volume of soil and equal volume of water.
H	thickness, often one-half thickness of a cohesive stratum; height, drop or stroke of a pile hammer.

h	head of water.
h_{cr}	critical height to which a cohesive soil will stand vertically if unsupported.
I_p	plasticity index, difference between liquid and plastic limits.
i	hydraulic gradient.
i_{cr}	critical hydraulic gradient.
j	seepage force per unit volume.
K_a	coefficient of active earth pressure.
K_p	coefficient of passive earth pressure.
k	coefficient of permeability.
L	length.
M	moment, character designated by subscript.
m_v'	modulus of volume change.
n	porosity, ratio of volume of voids to total volume.
p	unit pressure, character indicated by subscript.
Q	quantity, usually quantity of water flowing in a specified unit of time.
q	load per unit area.
q_u	compressive strength of unconfined specimen.
R	recovery ratio.
R_a	allowable bearing capacity of a pile.
R_u	ultimate bearing capacity of a pile
S	degree of saturation; settlement; sensitivity of a clay to remolding.
s	shearing strength per unit area; set of a pile under last blow of the hammer.
T	time factor in settlement estimates.
t	time.
V	volume, character indicated by subscript.
W	weight, nature indicated by subscript.
W_p	weight of a pile.
W_r	weight of drop hammer or ram of steam or diesel hammer.
w	water content, ratio of weight of water contained in a soil to weight of dry solids, usually expressed as a percentage.
w_l	liquid limit, least water content at which cohesive soil acts practically as a liquid.
w_p	plastic limit, least water content at which cohesive soil remains plastic.
w_s	shrinkage limit, least water content completely saturating cohesive soil.
z	a dimension, character indicated by subscript.
α	an angle.
β	an angle.
γ	unit weight.
γ_b	buoyed unit weight of submerged soil.
γ_{md}	dry density, unit weight of a dried sample of a compacted soil.
γ_t	total unit weight of a partially or completely saturated soil.
γ_u	specific gravity of water.
Δ	an increment
δ	an angle of friction between soil and back of retaining wall
θ	an angle.
ρ	a settlement.
σ	a unit pressure or stress.
ϕ	angle of internal friction.

Geology. A knowledge of the geology of the region in which a proposed project is to be undertaken is essential to the handling of foundation and soil problems. The engineer doing foundation work and the engineering of earth structures should therefore have sufficient training in the fundamentals of engineering geology to interpret and use intelligently the information obtainable from geological maps and folios such as those published by the U. S. Geological Survey. He should also be familiar with such sources of geological information as the descriptions and maps listed in the bibliographic bulletins of the Geological Survey. Valuable information on the geology of a region may often be obtained at local natural history museums or from instructors in science and geology at nearby colleges and schools. Excellent summaries of the essentials of geological engineering will be found in refs. 20, 22, and 24. For more complete treatments of the vitally important subjects such texts as refs. 18, 19, and 23 may be consulted. At all events the engineer dealing with soil and foundation problems should appreciate the significance of fundamental characteristics of well-recognized geological formations such as are summarized below.

Glacial deposits are notoriously erratic, having been transported, plowed, and mixed by glacial ice and the water running from melting ice to such an extent that irregular pockets and lenses of materials of different types may follow each other heterogeneously.

Shore deposits and delta deposits were formed where rivers entered seas or lakes and where the resulting sediments may have been eroded by waves and shifted by coastal currents. Such deposits may contain pockets of silt, clay, or even peat.

Floodplain deposits were placed during high-water seasons alongside the lower reaches of rivers. They usually consist of fairly uniform and continuous layers of silt or clay separated by equally uniform layers of sand and even coarser material. However, their normally uniform character may change abruptly where an old river channel has been filled with sediments of various types.

Volcanic formations, coral formations, and combinations of the two frequently uncovered on tropical islands are usually more erratic and unpredictable than glacial deposits.

Bedrock, whether igneous or sedimentary, is usually a reliable material but may contain fissures, folds, and fault zones. These may be filled with compressible soils or may be open as the result of scour. Fissures in the upper surfaces of bedrock are common in glaciated regions. Cavernous conditions are common in soluble limestones.

Marine deposits frequently comprise fine materials, such as clays deposited in bays, seas, and oceans. Thick clay beds are generally of such formation. Coral rocks are a form of marine deposit.

Wind-laid deposits, such as dune sand and loess, are usually remarkably uniform in character but are likely to prove troublesome because they are not densely compacted.

Finally, the engineer dealing with foundations and earth structures should have not only a knowledge of the fundamentals of engineering geology but an appreciation of the limits of his own knowledge. In doubtful cases and frequently where major projects are concerned, a geologist familiar with engineering problems should be called into consultation to advise on the probability of deep-seated and unexpected conditions of which cognizance should be taken.

2. PRELIMINARY EXAMINATION OF THE SITE

Topography and Appearance. To an experienced observer familiar with the geology of the region visual examination of the site and its surroundings often will be strongly indicative, but not conclusive, as to the nature of the subsurface structure. Topographic maps and aerial photographs also may yield valuable information under careful scrutiny by an experienced observer. The natural vegetation found on a site may give information of value to an observer familiar with the characteristics of soils and subsurface structures which locally favor certain types of growth.

Nearby structures, excavations, and fills should be examined and studied. As much as possible should be learned of the history of the foundations of local structures and of any extensive nearby earthwork. Great care should be taken to ascertain definitely whether or not the site being examined may be an old fill. Fills, particularly hydraulic fills, placed on marshes and tidal flats may take on an appearance of permanence in a few years that is most deceptive. Comparisons of old maps with recent maps often provide valuable information.

Preliminary subsurface exploration, comprising at least one dry sample boring and possibly a few soundings or probings, should be made during the initial examination of the site of any major project. The dry sample boring should be made by methods to be described subsequently and should be carried to the maximum depth at which a deposit of soft or compressible material, such as clay or peat, might cause troublesome settlement of the structure under consideration. In any event, the boring or borings should be carried to a depth equal to the least horizontal dimension of the largest proposed structure to be erected on the site.

3. SUBSURFACE EXPLORATION

On major projects and when unusual conditions are encountered on any project, the following brief treatment of the extremely important subject of subsurface exploration should be supplemented by consulting ref. 15.

Supervision of a subsurface exploration should be in the hands of a competent soils engineer under whose personal observation the work should be done. Leaving the supervision of such work to technically untrained personnel, even though they may be experienced members of drilling crews, leads to the misinterpretation of data and may lead to consequent serious error.

The **number and location of borings,** probings, and test pits should be such as to give a reasonably accurate estimate of, and disclose important irregularities in, subsurface conditions. On a large site the first borings should be so located as to furnish data for a general geological survey rather than be arranged in a rigid geometrical pattern. For the initial work on such a site borings spaced as widely as 400 or even 500 ft apart will often be satisfactory provided they are judiciously placed and are not all on a single straight line. Additional borings or investigations may then be made to obtain the data required to bring the general survey to a satisfactory state of completion and to give detailed subsurface information on the sites of proposed structures and excavations. This may involve spacing a number of borings as closely as 50 or even 25 ft and occasionally even more closely, though a spacing of 25 ft will usually be satisfactory for even the erratic subsurface conditions of a glaciated region. This way of developing a plan of subsurface exploration as underground conditions are revealed during the early phases of the investigation is far more satisfactory, especially in a large site, than rigid adherence to a preconceived arrangement of borings. The latter procedure frequently yields inadequate information, and may also waste money on unnecessary work.

In a limited area, such as the site of a building or bridge pier, a subsurface exploration should usually be started by making a few borings near corners of the proposed structure. Unless the subsurface conditions are known to be very uniform, at least three borings should be made. These preliminary borings should then be supplemented by such others as may be required to give sufficient information to permit the effective design of the foundation.

When the construction of a major structure is contemplated on a site where foundation conditions are known to be uncertain, or where they are unknown, preliminary borings on the site should be made prior to its acquisition. Furthermore, a few borings in adjacent areas will often reveal that great economies might be realized by an otherwise unimportant and relatively slight change in the location of the structure. A subsurface investigation of a long narrow area should also include a sufficient number of borings in adjacent areas to permit determination of the dip and strike of important strata.

Where foundations are to be built in cofferdams, which may be difficult to unwater, it is often advisable to locate borings outside the areas to be inclosed, as boreholes inside cofferdams may develop into troublesome leaks. Holes bored inside the area to be inclosed should be plugged with well-tamped impervious soil, but it is preferable to avoid borings inside cofferdam areas.

Depths of subsurface explorations are frequently determined by arbitrary rules, such as that the depth shall be at least equal to the width of the proposed structure. Actually the proper depth of a subsurface exploration is a function not only of the area, dimensions, and weight of the proposed structure but also of its sensitivity to differential settlement, the probable depth of its foundations, and the subsurface conditions revealed by both the preliminary investigation of the site and the initial phases of the subsurface exploration.

Borings should always extend to material of adequate bearing capacity to carry safely the proposed structure, and to sufficient additional depth to clearly demonstrate that such proposed bearing stratum is not underlain by a more compressible or less stable formation. Where the contemplated bearing material comprises a compressible clay, subject to consolidation, the borings should extend to a sufficient depth to insure that the increase in stress, produced by the weight of the structure, will not materially increase the rate of consolidation.

Borings may usually be stopped when rock or other material of a known high bearing value is encountered, but only after it has been thoroughly established that the satisfactory material is not underlain by soft or compressible strata. Where this is not well established, sufficient borings should be extended to the maximum depth to which soft or compressible material could cause unanticipated settlement Where a foundation is to be supported on rock, a boring should be extended at least 10 and preferably 20 ft into it to make sure that bedrock, and not a boulder, has been encountered. Subject to the above general requirements, which take precedence over the specific suggestions enumerated below, it will usually be satisfactory to extend borings on the sites of various types of structures and projects to the depths hereinafter indicated:

1. On the *sites of buildings and structures* borings should extend to a depth such that the increase in the vertical soil stress will not exceed 10% of the vertical stress existing prior to the proposed construction, but the depth should not be less than the width of the structure and should extend to not less than 25 ft below the bottom of the deepest part of the proposed foundation.

2. On the *sites of quay or retaining walls* borings should be carried to a depth beyond which surfaces of failure or sliding will not develop. This depth cannot be determined until the results of the initial phases of the subsurface exploration are available. For purposes of

preliminary estimates and the planning of a subsurface investigation, a depth below the bottom of the foundation, or the tips of supporting piles, equal to the net height of the wall may be used. By the net height is meant the vertical distance from the surface of the earth, or firm harbor bottom, in front of the wall to the top of the wall coping.

3. On the *sites of terraces and fills* borings should extend to depths beyond which surfaces of failure will not develop. For preliminary estimates of the required depths of exploration, it is suggested that 1.25 times the horizontal projection of the slope distances of terraces and one-half the horizontal projection of the slope distances of triangular or trapezoidal fills, be used.

4. On the *sites of deep cuts* borings should be carried to a depth below the deepest part of the proposed cut equal to its bottom width, where this width is less than one-half of the depth of the cut. Where wide cuts are contemplated the stability of the soil under the imposed double terrace loading should be considered. Where the bottom of the cut will be below ground water level the extent of pervious strata and the pore water pressure should be investigated as the stability of the slopes may be affected by seepage.

5. On the *sites of dams and levees* borings should penetrate not only soft and unstable but also permeable strata as consideration of seepage conditions as well as foundation loading must govern design. Seepage and uplift are not only governed by conditions under the dam or levee but also by conditions under the inundated area. For purposes of preliminary estimates depths of exploration of one-half the bottom width are suggested for earth fill dams and levees not exceeding 100 ft in height. For concrete dams not exceeding this height a depth of twice the height from stream bed to crest is suggested. The problem of making subsurface explorations on the sites of large or unusual dams is a subject too complex and highly specialized to be dealt with in a handbook. The required depth of exploration for such structures should be governed largely by the subsurface formations disclosed by preliminary borings and investigations.

6. On the *locations of highways, railroads, and airfields* borings should extend through weathered strata and disclose general stability, drainage conditions, and the maximum effect of frost action. Except where major fills and cuts are necessary, a depth of from 5 to 10 ft is usually sufficient provided unsatisfactory strata are fully explored when encountered.

7. On the *sites of tunnels* preliminary explorations should extend to a depth below the invert equal to the gross width of the tunnel.

Dry sample boring is today the most widely used method of making subsurface explorations. It is, if properly done, a satisfactory method of making a preliminary investigation of subsurface conditions in a wide variety of sites. It will, in many cases, give sufficient information to make more elaborate and expensive methods unnecessary. It can be done fairly effectively with improvised equipment, though on projects of any considerable magnitude both time and money can be saved by the use of specially built boring rigs. The most serious weakness of the method per se is its inability to penetrate rock. This would make the method useless in soils containing boulders, were it not possible to include in factory-built dry sample boring rigs a rotary or percussion drill with which rock may be penetrated.

The essentials of a typical rig for making dry sample borings are shown in Fig. 1. The components of such rigs are sometimes mounted on trucks, sometimes on skids, and more often, as in the outfit illustrated, are arranged for independent transportation to each site where they are set up in necessary juxtaposition. In any event, the actual boring is done with a bit, of which there are many varieties, carried on a hollow drill rod which is raised and lowered and, at the same time, rotated back and forth by means of a tiller clamped to the rod. Water fed down the hollow drill rod issues from orifices in the bit, assists in loosening soil particles, and then carries or "washes" them to the surface through the annular space around the drill rod. The wash water, as it comes out of the casing that usually lines at least the upper part of the bore, is caught in a container or sump and pumped back down the drill rod after the coarser soil particles have settled out. Recirculating the wash water is usually more satisfactory than connecting the drill rod to an independent supply, a procedure that makes it difficult, if not impossible, to determine when porous strata are encountered which take up large quantities of water.

A casing of extra-strong welded black steel pipe, usually 2 1/2 in. nominal diameter and in 5-ft lengths, is used to prevent collapse of the bore. Extra-strong couplings threaded in the standard manner are commonly used for joining the lengths of the casing. The casing is driven in such a way that it follows the progress of the drilling closely enough to prevent caving of the walls of the drill hole and consequent seizure of the drilling tools which may result in a "lost hole." Where samples are to be taken the casing should not be advanced beyond the level reached by the drilling tools even when soft ground makes rapid advance of the casing possible. Occasionally an exception to this rule occurs where sands make it

impossible to advance the drilling beyond the bottom of the casing. When the casing is to be driven, a special driving cap is substituted for the T from which wash water emerges. Driving is usually done with a 300-lb hammer. The weight of the hammer, the drop, and the number of blows required for each foot of penetration of the casing should be recorded.

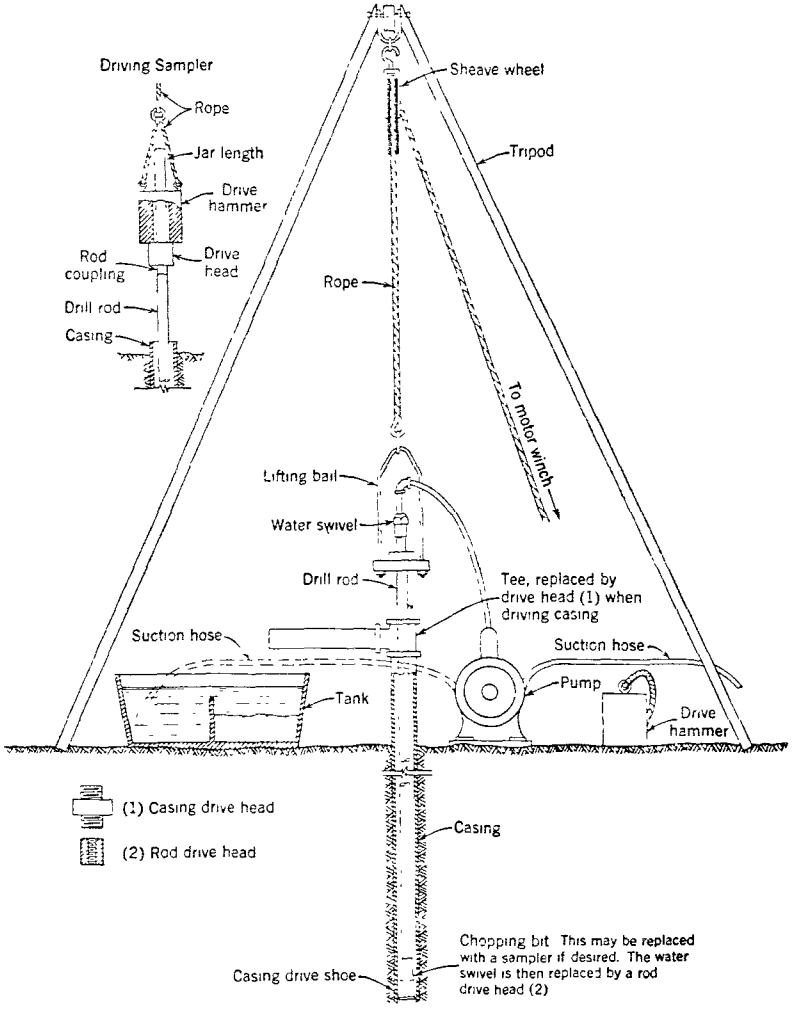


FIG. 1 Dry sample boring rig. The rig as shown is set up for advancing the hole by washing. The inset shows the arrangement for driving a sampling spoon. Courtesy Sprague and Henwood, Inc.

Standard or extra-strong welded 1-, 1 1/2-, or 2-in. black pipe, with couplings recessed to permit the pipe lengths butting at the joints, is usually used for drill rod. Specially made drill rod, such as is used for diamond and core drilling, is, however, preferable on jobs where many borings are to be made. Its extra-heavy weight, great strength, and the rugged square-cut threads of the couplings with which it is fitted insure freedom from many of the troublesome delays that can be caused by drill rod failures.

As boring proceeds with the equipment and by the methods outlined above, an experienced operator can tell by the "feel" of the tiller with which he rotates the drill rod, and by the color of the wash water, when there is a marked change in the character of the mate-

rial encountered by the bit. Whenever there is such a change, and at least every 5 ft whether or not a change occurs, a dry sample should be taken. Prior to taking a sample a cleaning tool providing horizontally directed jets should be substituted for the drilling bit and the casing should be thoroughly cleaned. A strong jet directed downward should not be used for cleaning purposes as it may disturb the material to be sampled at the bottom of

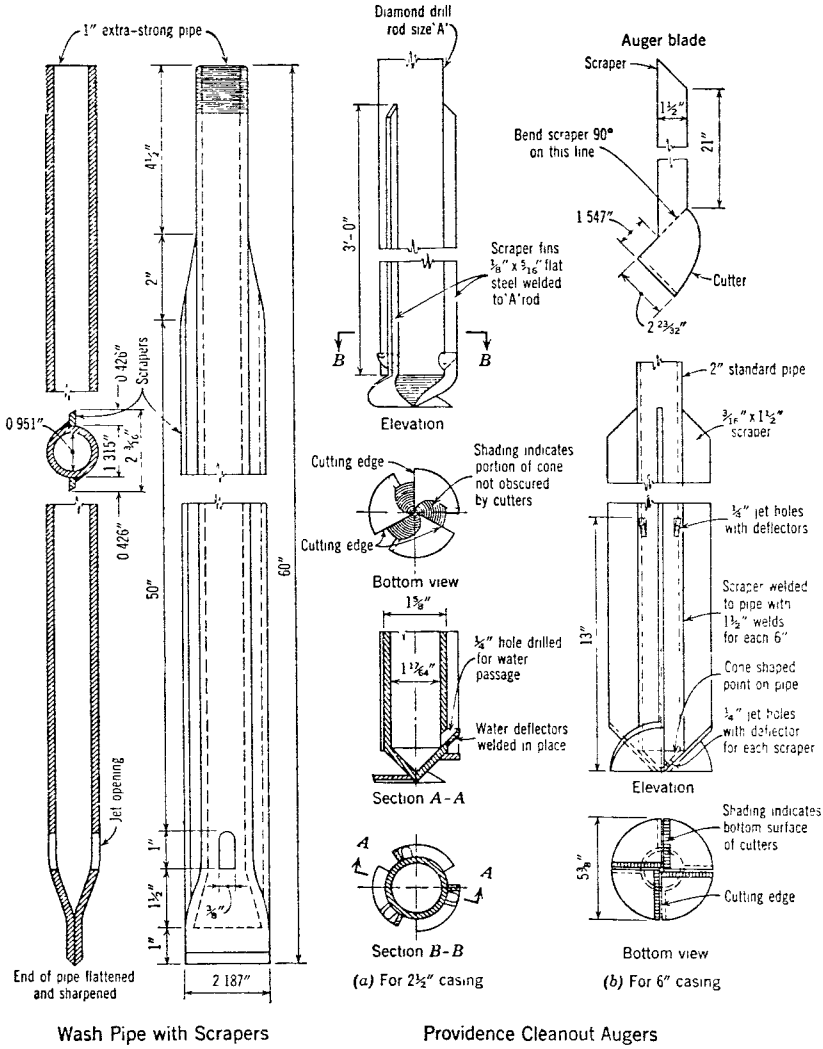
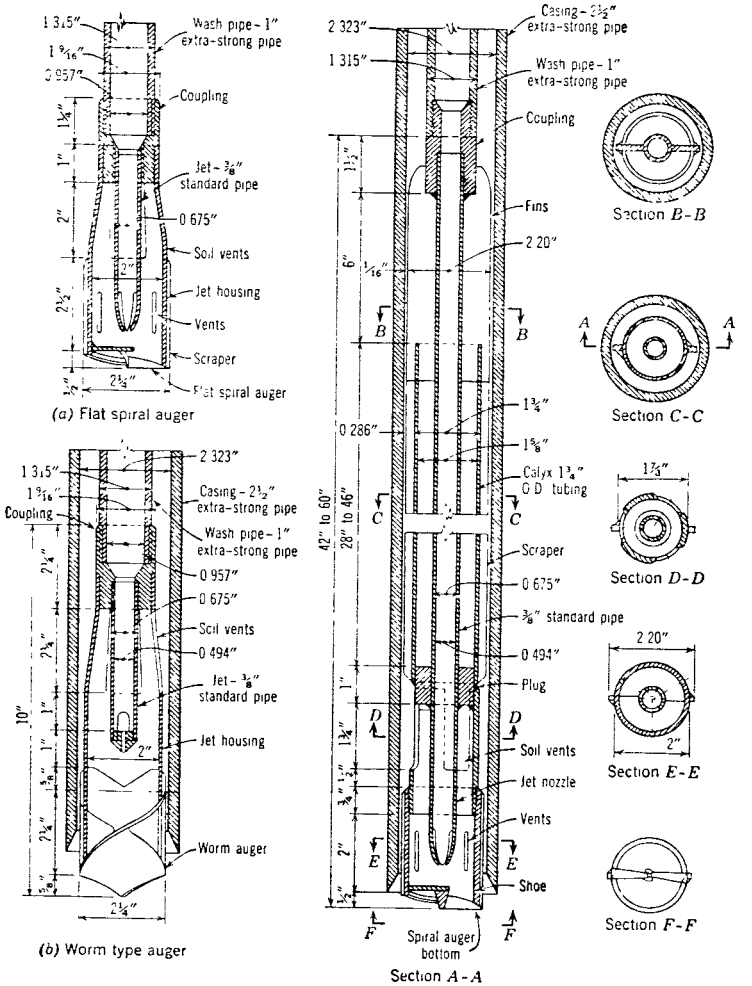


FIG 2 Tools for cleaning boreholes. From *Subsurface Exploration and Sampling*

the bore. Figure 2 shows cleaning tools for casings of several sizes. Those which have an auger, or inclined cutting blade at the bottom, are especially useful through gravel or other coarse material likely to accumulate at the bottom of the bore. Finally a sampling spoon or sampler, Fig. 3, is substituted for the cleaning tool, and the sample is taken by driving the spoon its own length into the undisturbed material below the bottom of the casing and borehole. The spoon or sampler should be driven with a hammer of known weight dropped an established distance. A hammer weighing 140 lb dropped 30 in. is frequently used for

driving 2-in. samplers in 2 1/2-in. casings, and this weight and dimension are recommended for general adoption. The blows per foot required to drive the sampler should be recorded as roughly indicative of the relative density of cohesionless soils or the consistency of cohesive soils. Table 1 may be used to make a rough approximation of these soil characteristics from the behavior of the sampler under the hammer. After being driven the sampler



Shielded Jet Augers

Cleanout Auger with Calyx

of Soils for Civil Engineering Purposes, Engineering Foundation, November 1948.

is withdrawn, removed from the drill rod, and opened. The material it contains is then immediately transferred to a jar with a tightly fitting top and rubber gasket Identifying data should then be entered without delay in waterproof ink on a label firmly attached to the jar—not to the cover. The label should be given a brush coat of a quick-drying lacquer to waterproof it.

Where cohesive soils are encountered it may be desirable to use a thin-walled open-drive sampler consisting of a piece of thin brass tubing or steel Shelby tubing mounted on a head

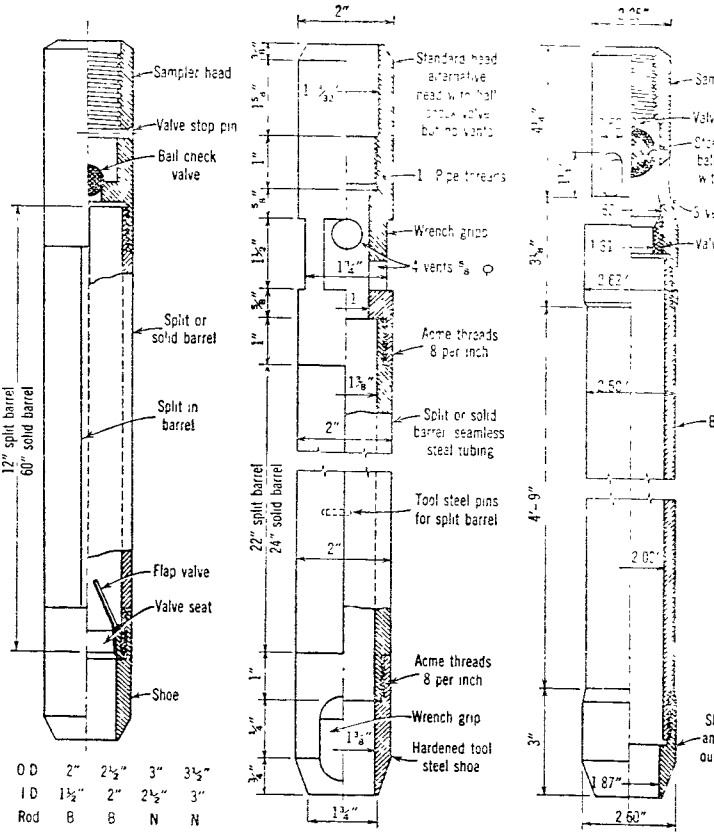


FIG 3 Sampling spoons or samplers. These spoons are driven into the soil to secure so-called dry samples Undisturbed samples cannot be obtained with them From *Subsurface Exploration and Sampling of Soils for Civil Engineering Purposes*, Engineering Foundation, November 1948

Table 1. Presumptive Characteristics of Sands and Clays as Indicated by Standard Penetration Tests *

Sands

Number of Blows N †	Relative Densities
0–4	Very loose
4–10	Loose
10–30	Medium
30–50	Dense
Over 50	Very dense

Clays

Number of Blows N †	Consistency	Unconfined Compressive Strength q_u, tons per ft²
Less than 2	Very soft	Less than 0 25
2–4	Soft	0 25–0 50
4–8	Medium	0 50–1 00
8–15	Stiff	1 00–2 00
15–30	Very stiff	2 00–4 00
Over 30	Hard	Over 4 00

* Data from *Soil Mechanics in Engineering Practice*, Karl Terzaghi and Ralph B. Peck, John Wiley & Sons, 1948.

† N = Number of blows required to drive a 2-in diameter sampler, such as the Raymond sampler of Fig 3, 1 ft with a weight of 140 lb and a height of fall of 30 in The test should be made after cleaning the bore with a water jet or auger, lowering the sampler on the drill rod and driving it about 6 in. into the soil at the bottom of the hole before commencing the test

which screws onto the drill rod, Fig 4, instead of the sampling spoon Samplers of this type should not be driven down with a hammer but forced down with a steady push. This can be accomplished by jacking or by means of a special block-and-tackle device which pulls down a crosshead or fitting attached to the drill rod. Boring rigs of the more elaborate type are sometimes equipped with a hydraulic cylinder or special jacking device for pushing down thin-walled samplers. Such samplers should be pushed straight down.

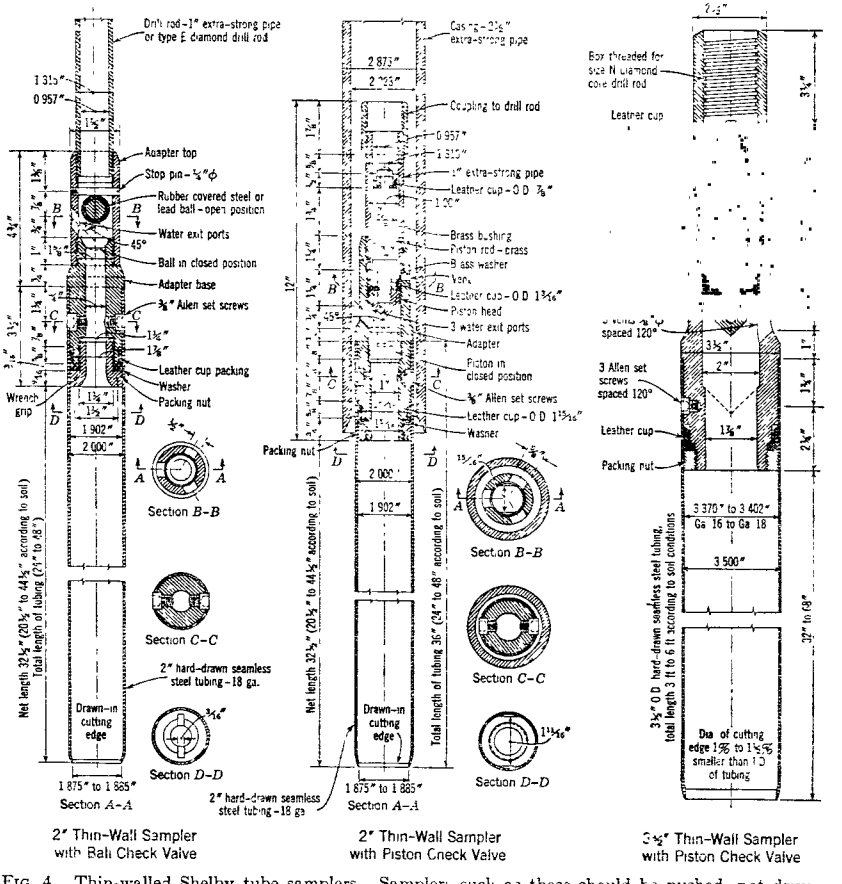


FIG 4 Thin-walled Shelby tube samplers Samplers such as those should be pushed, not driven, into the soil to secure so-called undisturbed samples Undisturbed samples 3 1/2 in or more in diameter are preferable to 2-in samples, which may be of doubtful value if the soil is sensitive to disturbance From *Subsurface Exploration and Sampling of Soils for Civil Engineering Purposes*, Engineering Foundation, November 1948

Under no circumstances should they be pushed and turned so as to "screw" them into the soil. Material is not removed from thin-walled drive samplers in the field. Instead the brass tube or Shelby tube comprising the body of the sampler is detached from its head, and the open ends are cleaned out for a depth of at least 3/4 in. and sealed by pouring melted paraffin or battery-sealing compound into the space from which the soil has been removed. Battery-sealing compound is more effective than paraffin but is messy and difficult to remove. The effectiveness of paraffin seals may be improved by first pouring in paraffin to a depth of 1/4 in , placing a metal disc on this as soon as it hardens, and then completing the seal with another 1/2 in. of paraffin Friction-tape strips should then be placed over the seal and finally bound into place by tape wound around the tube over the ends of the strips. The tape should be given a coat of lacquer The sample is sent to the laboratory in the

sealed tube. Samples taken in this way may be used for a preliminary determination of many of the characteristics of cohesive soils, and are therefore sometimes referred to as "undisturbed samples." However, the term "undisturbed sample" should be reserved for samples of larger diameter which will more nearly approximate a truly undisturbed condition than can be realized with any sampler small enough to go down a 2 1/2-in. casing.

Uncased borings are commonly used in some localities where very stable cohesive soils make it unnecessary to extend casings further down the borehole than is required to recover the wash water and protect the top of the hole from caving and abrasion.

Special drilling fluids or "muds," consisting of suspensions or emulsions of fat clays or bentonites, are sometimes used in lieu of water and are useful in minimizing the caving of boreholes which cannot be effectively driven with water alone as a drilling fluid. Where suitable clays are not available locally, commercially prepared products, such as Volclay, Aquagel, or other highly colloidal gel-forming clays may be procured and used

Rotary drilling is sometimes used in lieu of drilling and washing in making subsurface explorations. The boring in such a case is done by a rotary bit lubricated with a drilling fluid or "mud" consisting of a suspension of fat clay, bentonite, or a similar material in water. The method is now extensively used for drilling wells. When samples are to be obtained, the bit and drill rod are removed and the sample is taken with the same type of equipment and by the methods described above under dry sample boring or later to be described under "undisturbed sampling."

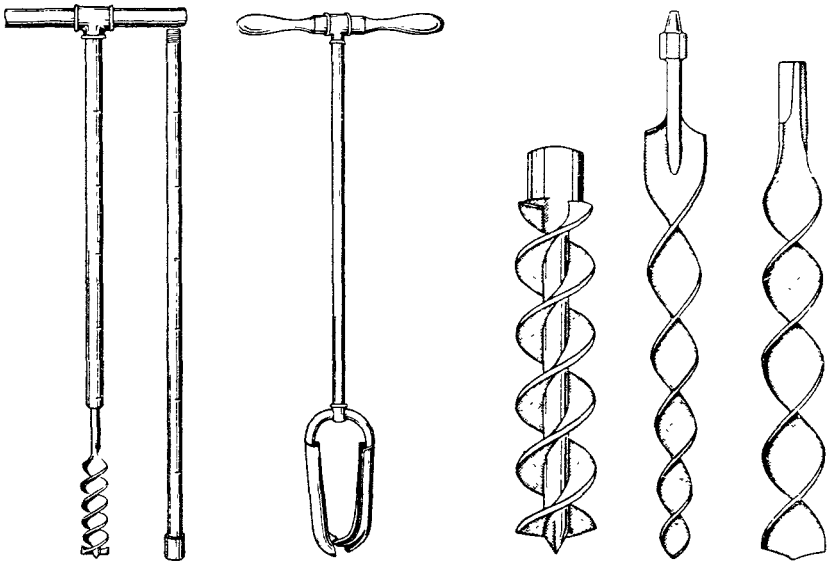
Percussion drilling, the method traditionally used by country well drillers, is sometimes used for advancing a boring between the taking of dry or undisturbed samples. Percussion drilling rigs are usually truck mounted. The actual drilling is done by a heavy bit which is churned up and down in such a way as to disintegrate the material at the bottom of the hole. Water is introduced in limited quantities, if it is not present in sufficient amount as ground water. The loosened material is then removed by means of a bailing bucket substituted temporarily and periodically for the drilling bit. The method can be used to penetrate many types of soil and rock and is therefore useful in differentiating between large boulders and ledge. It is very slow in sticky clays and shales, and impossible in fine, loose, water-bearing sands. Where drilling is done by the percussion method, samples are taken with the types of spoons and samplers used when drilling and washing or rotary drilling advance the bore hole.

Auger borings may be used in making general explorations and are very useful in reconnaissance surveys. Samples of cohesive soils and some types of non-cohesive soils may be obtained by this method if the soils are not too soft. However, the samples brought up on the auger are invariably badly disturbed and consequently of limited value. They may in fact be valueless Fully saturated non-cohesive soils will not be retained on an auger

Hand-operated augers especially made for subsurface explorations are shown in Fig 5. If such augers are not readily obtainable, a fair substitute may be improvised from one of the large augers used by boatbuilders for boring the shaft logs of motor boats. Hand augers are screwed into the earth a few inches at a time and then pulled out, bringing up a sample of the material on the bit. The process is continued until the exploration is extended to the desired depth or, more probably, until the limiting depth attainable by a hand auger has been reached. This limiting depth is usually about 10 or 15 ft, though under favorable conditions it may be as much as 25 or 30 ft.

Power-driven augers, originally developed for boring holes in earth for setting posts and piles, are now used to a considerable extent for making relative shallow exploratory borings. Continuous helical augers shown in Fig. 5 are available for use with equipment of this type. With such augers additional sections of "worm," instead of additional length of drill rod, are added as the depth of the hole increases Material is thus brought to the surface without the interruptions caused by frequently withdrawing the auger from the hole. Materials are, however, very badly disturbed, and it is difficult if not impossible to determine the depth from which a sample came. Such power-driven augers will bore a hole 6 or 8 in. in diameter very quickly. Under favorable conditions they may be able to reach a depth of 100 ft. Therefore, they may be used effectively for drilling holes from the bottoms of which dry or undisturbed samples are obtained by sampling spoons or samplers of types developed for use in holes bored by other means. Short helical augers with interchangeable cutters may also be used on power-driven auger-boring rigs to bore holes in earth from 12 to 16 in in diameter, and disk augers are available with which holes up to 42 in. in diameter may be excavated. In cohesive soils such large-diameter holes of moderate depth are sometimes very useful to permit inspection of the soil in place.

Neither hand-operated nor power-driven augers will penetrate hard or cemented soils, nor will they work effectively in water-bearing sands. This naturally limits their usefulness.



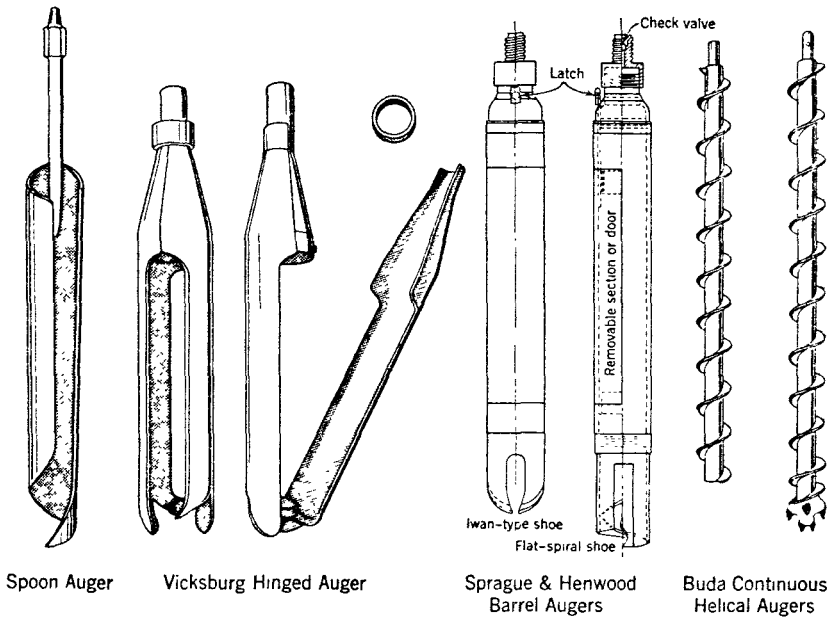


FIG 5 Hand- and power-operated augers From *Subsurface Exploration and Sampling of Soils for Civil Engineering Purposes*, Engineering Foundation, November 1948

Combination drilling rigs, developed by the Materials and Research Department of the California Division of Highways, are equipped for auger boring, percussion drilling, rotary drilling, core boring, the operation of dry samplers, the dewatering of holes, and the making of field permeability tests. Such "all-purpose" drilling rigs are, of course, most useful in making the extensive subsurface explorations required for major projects. Their high cost, however, precludes their use on the average job.

Probing is a method of subsurface exploration that is of minor importance. It is sometimes a useful supplement to borings for determining the approximate elevation of the top of a known underlying firm stratum. Probings may also be useful in large areas to determine the extent and spacing of borings. In its simplest form probing consists of forcing or driving a rod or pipe into the ground. In its more elaborate practical form as used in the United States the probe consists of $^3/_4$- or 1-in. pipe driven by a light drop hammer operated by hand from a portable tripod which is also used as the support for a block and tackle with which the probe is pulled. The number of blows of the hammer, when dropped a fixed distance, say 30 in., required to drive the probe a foot are recorded. The resistance to driving is the combined result of point resistance and skin friction. Some probes or sounding rods are fitted with enlarged points and others are operated in casings to minimize or eliminate the effects of skin friction. For detecting soft spots in shallow hydraulic fills and for determining the bearing capacity of the subgrades of roads and airfields, probes have been developed which include some form of spring dynamometer with which the force required to shove them, by hand, into the soil is measured Methods of using these devices, and more highly specialized types of probe developed recently in Europe, are described in ref. 15.

Wash boring was for many years a widely used method of making subsurface explorations. It is now, however, completely discredited. The equipment with which wash borings were made is similar to that used for making dry sample borings except that no sampling spoons or samplers were used. Instead, samples of wash water were caught as the boring proceeded and an attempt was made to identify and classify the material which settled out of the water The samples thus obtained were so often misrepresentative and misleading that the method is no longer used by progressive practitioners in the fields of soil mechanics and foundation engineering.

Undisturbed Sampling. The method used for obtaining samples at least 3 $^1/_2$ in. in diameter for determination of the strength in shear and the consolidation characteristics of cohesive soils and for determination of certain other characteristics of non-cohesive soils is commonly referred to as "undisturbed sampling." Such samples can be secured from properly drilled boreholes sunk by any of the means previously described, providing the hole and casing are of sufficient diameter to permit free passage of the sampler They can also be secured from test pits The smallest casing diameter through which satisfactory undisturbed samples can be obtained is usually considered to be 4 in. Larger casings are preferable for many purposes, especially for obtaining samples of clay for the determination of its consolidation characteristics. Prior to taking an undisturbed sample, the borehole should be carefully cleaned and any accumulated debris removed from the bottom by means of cleaning tools of the type shown in Fig. 2 The sampler should then be mounted on the drill rod, lowered to the bottom of the borehole, and pushed into the soil either by jacking or by a suitable block-and-tackle downhaul attached to the drill rod. The sampler should not be rotated as it is pushed down nor should it be driven with a hammer.

Samplers of many types are described in ref. 15. The types of sampler commonly used are thin-walled open-drive samplers and piston samplers, the latter type being in general preferable. Figure 4 shows several typical thin-walled open-drive samplers. They consist essentially of a thin brass tube or a steel Shelby tube of 16- to 18-gage material attached by means of countersunk screws to a head which screws onto the drill rod. The head contains a vent and a check valve, or equivalent device, which permits the upward escape of the water or air in the sampler as it is forced into the soil. The check is intended to close when the sampler is withdrawn and thus prevent pressure on top of the soil sample from forcing it out of the tube. The lower end of the tube should be reduced slightly in diameter and sharpened to a cutting edge in such a way that the diameter or "bore" of the opening is 1 to 1 $^1/_2$% less than the internal diameter of the sampler body. Thin brass tubes, because they do not rust, are preferable to Shelby tubes for procuring and storing undisturbed samples.

Some more elaborate types of sampler have a detachable shoe which forms the cutting edge and contains a concealed wire snare for cutting off the sample. Others are fitted with thin spring or hinged fingers or "core catchers" intended to prevent the core from falling out of the sampler tube when it is raised. The thickening of the sampler wall necessary to accommodate such devices is unfortunate and limits their value. Cores of non-cohesive soils are difficult to retain. The introduction of "drilling mud" to the borehole promotes

the retention of sand samples but some fine water-bearing sands require special equipment and unusual methods such as freezing the core before lifting it. For descriptions of these special devices and methods see refs. 15 and 3.

The piston samplers contain a piston or plug which is usually mounted on a long rod which extends the full length of the hollow drill rod. Such samplers are shown in Fig. 6. They are lowered to the bottom of the casing with the piston closing the opening like a plug and preventing the entry of extraneous material and scrapings from the walls of the casing or drill hole as the sampler is lowered. When the sampler comes to rest at the bottom of the drill hole, the piston rod is released from the drill rod and is clamped or held at the top in such a way that it cannot descend as the sampler is forced down into the soil. After the sampler has been forced down, the piston rod is again clamped to the drill rod and the whole affair withdrawn.

Samples are not removed in the field from the bodies of thin-walled open-drive or piston samplers. The thin-walled tubes are removed from the sampler heads and shipped to the laboratory with the samples in them. Immediately after removal from the sampler head the open ends of such tubes are sealed by the method described under dry sample boring. It is advisable to retain the soil removed from the ends of sampler tubes prior to sealing them. These small samples should be placed immediately in glass jars with tightly fitting screw tops and labeled. They will often prove of value for purposes of identification and classification of soils prior to the removal of the undisturbed samples from the Shelby tubes in the laboratory.

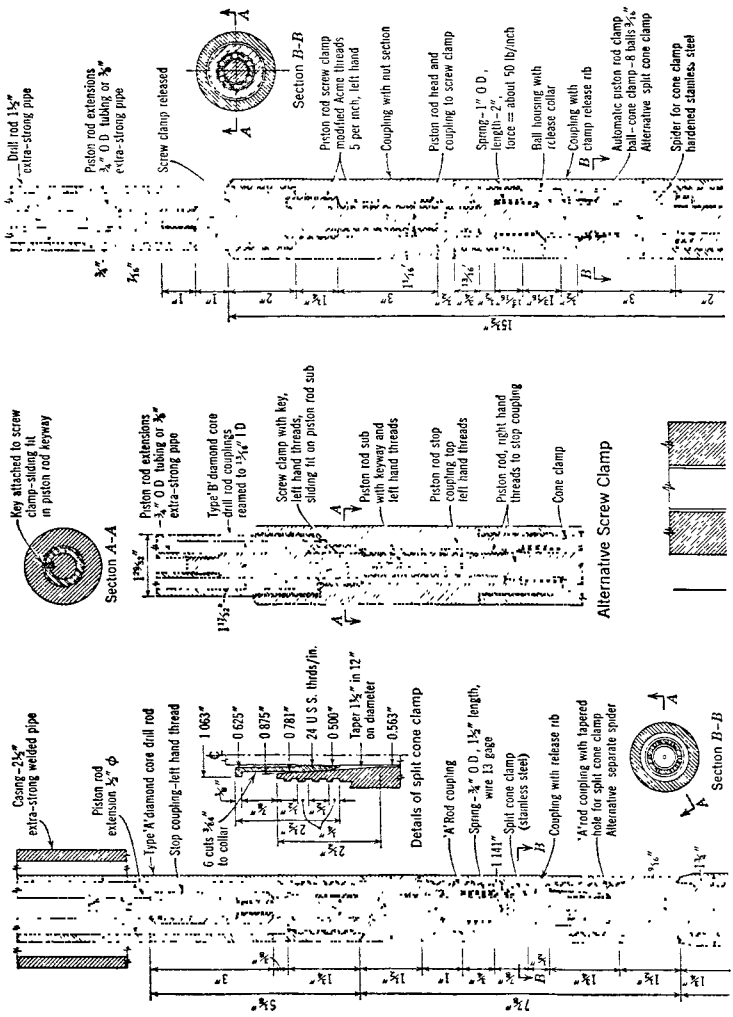
The maximum safe length of sample which may be taken without having the material unduly disturbed by the action of the sampler varies with different soils and sampling devices. In general the ratio between the safe length and the diameter of the sampler decreases as the diameter increases. It is also affected by the thickness of the sampler wall, the character of the cutting edge, and the clearance, or difference between the diameter of the opening formed by the cutting edge and the internal diameter of the sampler body. Generally speaking the safe length in dense to loose cohesionless soils will be from five to ten times the diameter of the sampler. In stiff to very soft cohesive soils it will be from ten to twenty times the sampler diameter. The length of sample that may be obtained without forming a soil cone below the cutting edge and having no more soil enter the sampler far exceeds the safe length. If the safe length is exceeded, soil entering the sampler will be disturbed and will not yield reliable data when subjected to tests in the laboratory.

Because materials in the top and bottom of a sample are inevitably more or less disturbed, and for reasons of economy when continuous sampling is practiced, it is desirable that undisturbed samples should be as long as possible. For this reason the recovery ratio, or the ratio between the length of a sample as recovered and as it was in the ground, should be determined for every sample taken. With piston samplers, which are preferable to open-drive samplers because scrapings from the casing and extraneous soil fragments do not enter the device as it is lowered, the recovery ratio may be considered to be the length from the top of the sample to the cutting edge of the sampler divided by the distance the sampler was driven. With open-drive samplers an attempt must be made to identify and remove disturbed scrapings from the top of the sample before measuring its length. Samples may generally be considered undisturbed when the recovery ratio lies between 1.00 and $1.00 - 2C_i$, where $C_i = \frac{D_s - D_e}{D_e}$, D_s and D_e being respectively the internal diameters of the sampler opening and body (see Fig. 4).

Continuous sampling may be desirable in certain cases, especially where estimates must be made of the prospective settlement of a compressible or cohesive soil. The process, as its name implies, involves taking samples one after another in such a way that a practically unbroken series of samples is obtained, each sample advancing the borehole by its own length. The boring may, in fact, be entirely accomplished by the sampling operations, but it is usually necessary to enlarge the bore and drive the casing after each sample is taken, being careful not to extend the work beyond the bottom of the last sample secured. Drilling and washing, percussion drilling, rotary drilling, or auger boring may be used in this way to enlarge the bore, but it is usually done by drilling and washing.

Test pits are in many cases the most reliable source of information on subsurface conditions and therefore should be used where practicable. Their high cost when carried to any considerable depth and the difficulty of extending them below the water table in permeable soils limit their usefulness.

Test pits of moderate depth which require no sheeting and do not extend below the water table in permeable soils are in common use on sites where bedrock or hardpan is within easy reach. Deeper test pits are frequently employed on the sites of important structures and are commonly used in exploring dam sites. Test pits are furthermore employed in preliminary investigations of many kinds. Shallow test pits are generally dug by hand, but if


Drill rod 1½" extra-strong pipe
Piston rod extensions ¾" O D tubing or ⅞" extra-strong pipe
Screw clamp released
Section B-B
Piston rod screw clamp modified Acme threads 5 per inch, left hand
Coupling with nut section
Piston rod head and coupling to screw clamp
Spring-1" O D, length-2", force = about 50 lb/inch
Ball housing with release collar
Coupling with clamp release rib
Automatic piston rod clamp, ball-cone clamp-8 balls ⅜"
Alternative split cone clamp
Spider for cone clamp hardened stainless steel
Key attached to screw clamp-sliding fit in piston rod keyway
Section A-A
Piston rod extensions ¾" O D tubing or ⅞" extra-strong pipe
Type 'B' diamond core drill rod couplings reamed to 1 3/16" I D
Screw clamp with key, left hand threads, sliding fit on piston rod sub
Piston rod sub with keyway and left hand threads
Piston rod stop coupling top left hand threads
Piston rod, right hand threads to stop coupling
Cone clamp
Alternative Screw Clamp
Casing-2½" extra-strong welded pipe
Piston rod extension ½" ϕ
Type 'A' diamond core drill rod
Stop coupling-left hand thread
6 cuts ¾" to collar
1 063"
0 625"
0 875"
0 781"
24 U S S. thds/in.
0 500"
Taper 1½" in 12" on diameter
0 563"
Details of split cone clamp
'A' Rod coupling
Spring-⅜" O.D., 1½" length, wire 13 gage
1 141"
Split cone clamp (stainless steel)
Coupling with release rib
'A' rod coupling with tapered hole for split cone clamp
Alternative separate spider
Section B-B

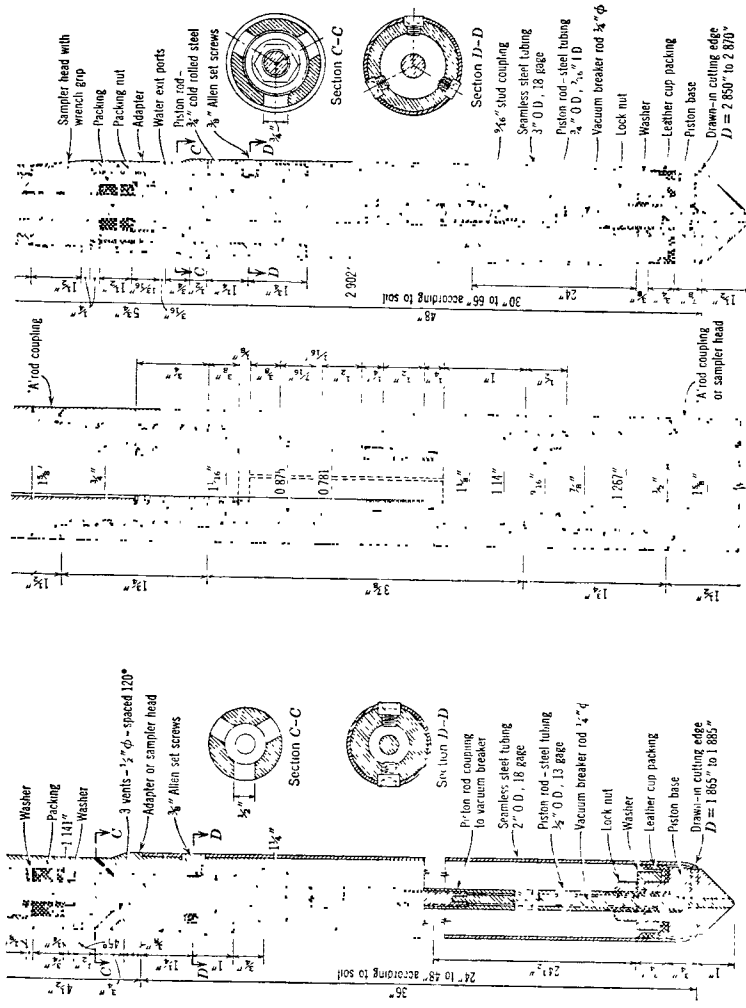


FIG. 6 Piston samplers. This type of thin walled sampler is very useful in soft soils. From *Subsurface Exploration and Sampling of Soil for Civil Engineering Purposes*, Engineering Foundation.

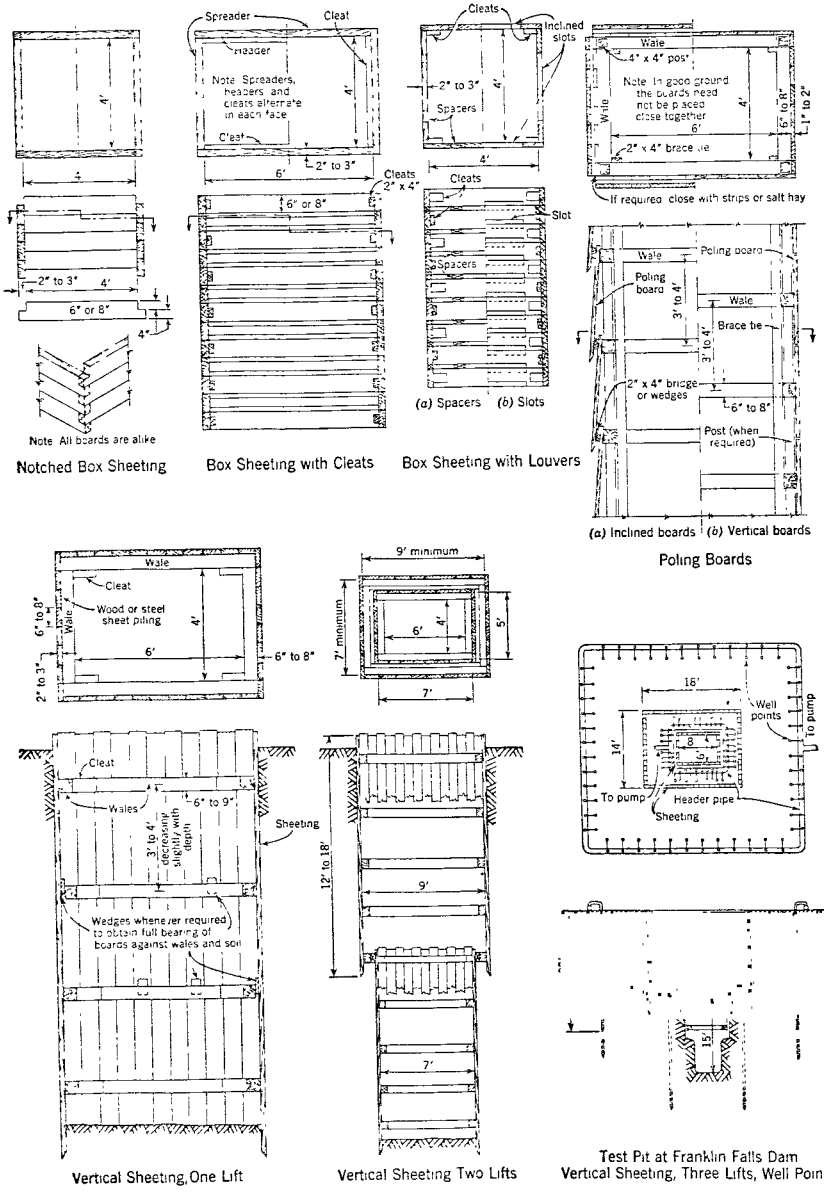


FIG 7 Test pits and test pit sheeting Two tiers of well points were used to lower the water table when the deep test pit was sunk at Franklin Falls Dam (lower right hand corner) From *Subsurface Exploration and Sampling of Soils for Civil Engineering Purposes*, Engineering Foundation.

many are required it will usually be economical to use a clamshell or orange-peel bucket on a small power excavator. Power equipment should not, however, be used when approaching the final depth from which samples, particularly undisturbed samples, are to be taken. Where all the excavating is to be done by hand, dimensions of 4 by 6 ft will frequently prove more convenient and cheaper than smaller pits in which a man does not have room to work effectively. These dimensions are also about the minimum providing adequate working space for taking large undisturbed samples.

Several types of sheeting suitable for use in deeper test pits are shown in Fig. 7. Of these horizontal or box sheeting is the simplest and requires the least excavation for given dimensions of the pit, and has the advantage of permitting examination of the soil strata in

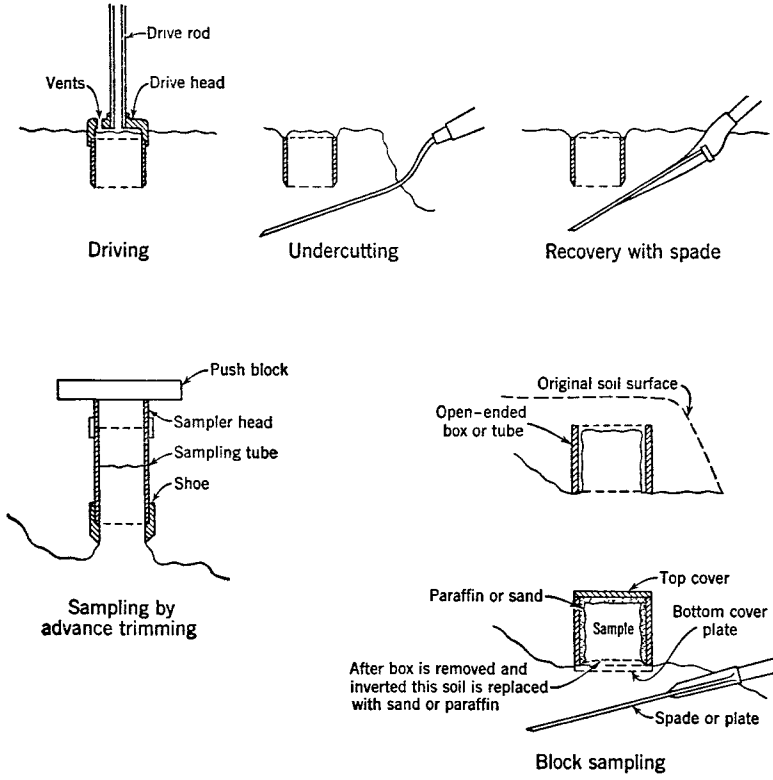


FIG 8 Test pit sampling procedures.

the wall of the pit as work progresses. Small openings or louvers between the boards of horizontal box sheeting are sometimes advantageous. They permit drainage and allow any cavities formed by soil movements or seepage to be packed with hay which usually proves an effective method of controlling a tendency of soil to run into the excavation.

In very soft or loose soils polling boards or vertical sheeting, Fig. 7, will be necessary. Steel liner plates of the type used in tunneling are sometimes used instead of polling boards. The polling-board method is less effective in soft ground than closely driven vertical sheeting but has the advantage, where adequate, of permitting the maintenance of the original dimensions of the pit irrespective of depth. Vertical sheeting may consist of plain boards, tongue-and-groove boards, or Wakefield sheet piling. Where more than one lift of vertical sheeting is required to reach the desired depth, allowance for offsets every 12 or 18 ft should be made. The starting dimensions of such a pit should provide for at least a 3-in. clearance between the wales of the upper lift and the sheeting of the lower one to give room for driving the sheeting. The dimensions of timbers shown in Fig. 7 are based on

experience rather than on theoretical considerations of earth pressures. In excavations of small dimensions open for relatively short periods arching undoubtedly reduces the earth pressures that might otherwise occur. Nevertheless, great care should be exercised to avoid accidents. In firm soils the excavation may go forward concurrently with or even a little ahead of the sheeting, but in such cases the formation of cavities behind the sheeting should be avoided. Wedges should be driven between the boards and wales where spaces show inadequate bearing.

Steel sheet piling, open caissons of steel or concrete, or even pneumatic caissons may occasionally be justified on major projects to permit examination of the soil in place at far greater depths than can be reached by timber sheeting. The use of these types of equipment is described in Sect. 8, Arts. 6, 7, and 8.

The control of ground water in test pits demands great care. In cohesive, mixed, or gravelly soils pumping from a sump and drainage channels may be effective. In sands and silts, however, such methods will be effective only if the depth below the original ground-water table is slight. Even then it may be necessary to protect the drainage channels and sump by partially filling them with a layer of gravel or broken stone. In cohesionless soils it is necessary that during sampling the ground-water level be held below the bottom of required undisturbed samples. For this reason well points, described in Art. 8 under drainage, may be necessary to depress the water table considerably below the bottom of the pit. A suitable arrangement of well points is shown in Fig 7. An advantage of pneumatic caissons is the possibility of so adjusting the air pressure that the water table in the working chamber may be depressed sufficiently to permit taking undisturbed samples of non-cohesive soils in the capillary rather than in the submerged range of saturation.

Sampling should be done near the center of test pits and at least 12 in. from any sheeting that may have disturbed the soil when being driven. Samples may be taken by short open-drive samplers or piston samplers, the former type being the one generally used. The disturbance caused by forcing a sampler into the soil can be largely avoided in a test pit by trimming the sample down to nearly the desired diameter just ahead of the cutting edge of the sampler. This and other sampling procedures commonly used in test pits are shown in Fig. 8. Of the methods illustrated in this figure the most important is block sampling. Strongly cohesive soils, such as stiff clays, are much easier to handle as they are far less susceptible to disturbance by jarring. Blocks of soils of this type may be readily cut from a vertical face or sloping bank by methods illustrated in Fig. 8. These methods require no comment beyond a word of caution as to the advisability of taking samples immediately after the exposure of the face or bank from which they are to be obtained and the necessity of coating them immediately with paraffin to prevent the loss of moisture. The physical characteristics of soils depend to a great extent on their moisture content and it is essential that specimens reach the laboratory with this unchanged.

Determination of the free ground-water level, or levels, and any marked excess hydrostatic pressure in pervious strata are essential steps in subsurface explorations. Ground-water conditions may be regular, with a single closely defined water table which is the contact surface between the free ground water and the capillary zone, Fig. 9; or irregular, with one or more bodies of ground water having water tables perched above impervious strata or held beneath such strata under artesian pressure, as shown in the same figure. Ground-water levels are subject not only to seasonal changes but also to daily, and even hourly, changes caused by pumping from nearby wells, the drainage of excavations and large subsurface structures, and the effects of tidal variations on sites near tide water. The distances over which the effect of the tide on ground-water elevation may be noticeable are, in some cases, surprising. In Florida, for example, the artesian pressure in wells a number of miles inland varies regularly with the rise and fall of the tide. It is therefore of primary importance that records of ground-water observations give specific information not only as to dates and weather conditions but also as to the exact time of each observation.

In a normal boring operation the borehole is kept full, or at least partly full, of water. Reliable determination of the hydrostatic pressure at the bottom of the casing demands that the pressure in the soil and the borehole be given a chance to equalize as evidenced by the establishment of a stable water level in the hole. In pervious gravels and sand such stabilization may occur within a few minutes and the elevation of the water table determined from time to time without delaying the boring seriously. It is difficult to tell when a float lowered down a borehole makes contact with water. Accordingly, a water-surface depth gage consisting of a weighted steel tape, coated with chalk, or a steel rod coated with chalk and suspended from the end of a tape, should be used. The water level in the casing will stabilize at the water table much more rapidly if clean rather than dirty water has to percolate into the soil at the bottom of the casing. For this reason the bore should be thoroughly cleaned and washed with a tool providing horizontally directed jets prior to each determination of the ground-water elevation. The bore should then be pumped out

below the estimated elevation of the water table, and the actual stabilization of the water at this elevation checked by successive measurements of the depth to the water surface.

In sands and silts of low permeability, the time lag or time required for equalization of the water level in the borehole with the hydrostatic pressure at the bottom of the casing may be hours or even days. Such prohibitively long time lags may sometimes be reduced

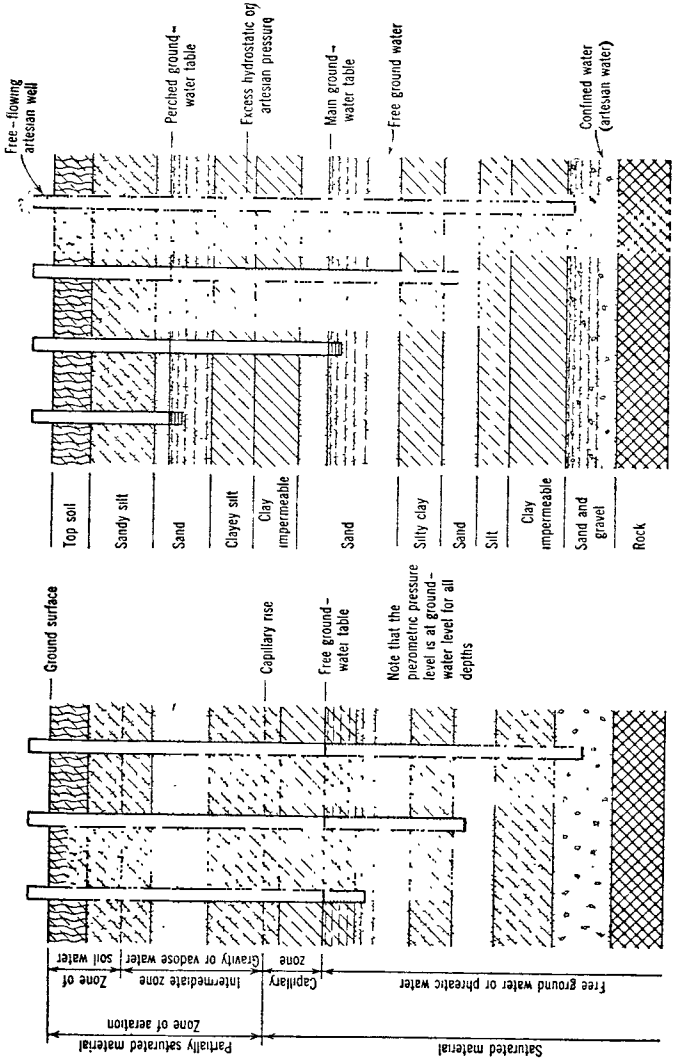
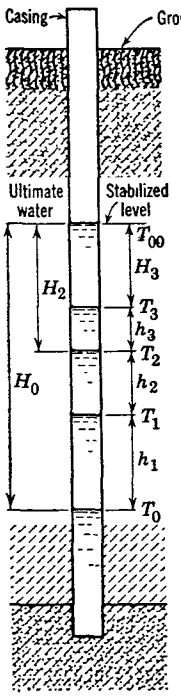


FIG. 9 Regular, irregular, and perched water tables. From *Subsurface Exploration and Sampling of Soils for Civil Engineering Purposes*, Engineering Foundation

to practical limits by the device of progressively bailing water out of a bore in which the water level is going down, or introducing water into a bore in which the water is rising until the opposite effect is produced. In this way the approximate elevation of stability, that is the elevation of the water table, may be found much more quickly than by waiting for inflow or outflow through a soil of low permeability to produce stable conditions. Another method of making a rough approximation of the elevation of the water table is to wash out the bore, pump it out to the estimated water table, and then measure the depth to the water at the beginning and the end of two equal consecutive time intervals. The difference be-

tween the elevation of the water table and the water in the bore at the beginning of the two intervals may be computed by the formula given on Fig. 10.

Determination of ground-water levels and pressures in a simple borehole will at best, however, cause serious interruption of the boring operation unless the material encountered is reasonably pervious. It is therefore advisable to make the best practicable approximation of the water-table elevation or elevations in the first of several borings on a site as the first boring is made. This should show whether ground-water conditions are regular or whether perched or artesian conditions obtain. One or more cased bores may then be left as observation wells in which the water level may be given time to stabilize and thus make possible an accurate determination of the elevation of the water table or water tables encountered.



With the rise or fall determined for two or more equal time intervals

$$T = T_0 - T_1 = T_2 - T_1 = T_3 - T_2, \text{ etc.}$$

The distance to the ultimate stabilized level in the boring can be estimated by

$$H_0 = \frac{h_1^2}{h_1 - h_2} \text{ or}$$

$$H_2 = \frac{h_2^2}{h_1 - h_2} \text{ or}$$

$$H_3 = \frac{h_3^2}{h_2 - h_3}, \text{ etc.}$$

Fill or empty casing to the computed level for final verification or correction

It is assumed that there is no clogging of the intake or change in the outside water levels or pressures during the observations, which condition is fulfilled when

$$\frac{h_1}{h_2} = \frac{h_2}{h_3} = \frac{h_3}{h_4}, \text{ etc.}$$

FIG. 10. Method of approximating water-table elevation in impervious soil. From *Subsurface Exploration and Sampling of Soils for Civil Engineering Purposes*, Engineering Foundation.

Pore pressure determination in an impervious silt or clay, that is the determination of the pressure of the water in the almost microscopic interstices between the soil particles, is more difficult than in sands and permeable silts. The method of allowing, or assisting, the water in a borehole to seek a level such that its static head equals the pore pressure in the stratum under investigation is impracticable because of the prohibitively long time lag. Furthermore the relatively small amount of water that would flow out of, or into, the soil to effect equalization might change the pore pressure materially. Nevertheless the determination of the pore pressure in such soils is frequently necessary in making stability studies of soft soils being consolidated by means of sand drains, and large earth structures such as high embankments, deep fills, and earth dams (see Sand Drains in Sect. 8, Art. 5).

In such cases the method illustrated in Fig. 11 should be used. A borehole is extended down to the stratum in which the pore pressure is to be determined, or a bore drilled during the subsurface exploration may be utilized if not too deep. Backfilling a deep borehole should not be attempted as it is virtually impossible to backfill a bore so tightly that the pore pressure in a stratum above the bottom of the hole may be determined accurately. The bore must be cleaned thoroughly and filled with clean water. A filter point consisting of a short piece of perforated pipe wrapped with several layers of screen cloth, or a short length of permeable ceramic pipe, is then securely connected to a $^3/_8$- or $^1/_2$-in. plastic or metallic tube and lowered to within a foot or so of the bottom of the bore. A non-metallic filter point and tube should be used if there is any chance that electrolytic action might release gases that would vitiate the test. The bore is then backfilled with permeable sand to within 3 or 4 ft of the top of the stratum in which the pore pressure is to be determined. From this elevation the backfilling is carried on with a fat, impervious clay placed by dropping balls of clay, or a mixture of clay and bentonite, down the bore and tamping them in 3- or 4-in. layers with an annular tamper.

If the soil under investigation is a silt with a coefficient of permeability of about 10^{-5} cm per sec or more the tube leading down to the filter point may be used as a piezometer in which the water surface should stabilize at an elevation representative of the pore pressure within 6 hr or less. This elevation may be determined by means of a narrow chalked tape

weighted with slender sinkers that will go down the tube. The measurement should be repeated a number of times at intervals of a few hours to be sure that stabilization has actually occurred.

If, however, the soil is an impervious silt or clay the time lag required for stabilization of the water surface in even a $^3/_8$-in. tube will be excessive. In such cases it will be necessary to attach a mercury U tube manometer, or a bourdon gage reading both pressure and vacuum to the upper end of the tube leading to the filter point and also to provide a vent for filling the tube or permitting the escape of gas. After filling the tube and closing the vent the pore pressure may be read from the gage or manometer as soon as conditions stabilize as indicated by successive readings.

Even this expedient will be ineffective if the soil is an extremely impervious clay, or if the pore pressure is equivalent to that of a water table more than the height of the water barometer below the ground surface. In such cases hydrostatic pressure cells equipped for the remote control of dials at the ground surface must be used.

Realization of the important relationship between pore pressure and the stability of some types of soil has led to the practice of checking the pore pressure in the interior of large earth structures as filling proceeds. For this purpose filter points may be set in the fill as work progresses and connected by tubes to manifolds and gages located in observation stations on the sideslopes of the structure. In these installations it is very important that the tubes from the filter points to the gages run upgrade all the way to avoid air pockets and consequent inaccuracies in pressure measurements. In some installations two tubes are run to each filter point from the observation station and provision is made for circulating water through them periodically to flush them out. Hydrostatic pressure cells operating dials by remote control may also be placed in earth structures as the filling proceeds to determine the pore pressure as the work goes forward.

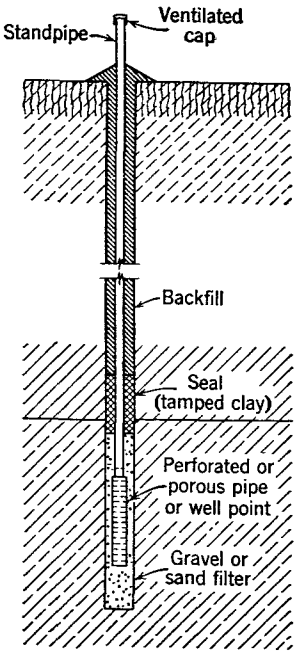


FIG 11. Observation well or piezometer for determining pore pressure From *Subsurface Exploration and Sampling of Soils for Civil Engineering Purposes*, Engineering Foundation.

The approximate yield of water that may be anticipated from a pervious stratum may be determined by bailing or pumping the bore and observing the corresponding stabilized draw-down level. The yield of a single, small-diameter boring cannot be relied upon, however, to furnish data upon which a trustworthy estimate of the pumping requirements for a proposed foundation excavation may be based. Far better results may be obtained by means of a test pit or caisson.

Borings in rock may be made by percussion drilling or core boring with a diamond drill, shot drill, or steel saw-toothed bit. Percussion drilling has been described. It is of extremely limited value as an exploratory method in rock as it yields no good samples. Core boring, on the other hand, if properly done can yield practically continuous samples from which reliable information may be obtained as to the character and soundness of the rock penetrated. Of the three methods of core boring mentioned above, diamond drilling is the most widely used and generally satisfactory for making deep explorations in hard rock. The shot drill and the steel saw-toothed bit are, however, used extensively in softer rocks and are considered satisfactory by many authorities for a number of purposes. For descriptions of the highly specialized equipment and devices used in core boring with these three types of drill and for information as to methods of using the equipment, see ref. 15. Equipment cannot be improvised for making core borings by any of these methods. They demand specially built machinery operated by trained and experienced personnel.

Records of the entire process of making exploratory borings and subsurface explorations must be complete and accurate. It is essential that the record be kept up to date, and that events be recorded promptly as the work progresses. Attempts to write up the records at the end of the day, or even at the end of each shift, will inevitably lead to mistakes.

Samples should be carefully packed for shipment to the laboratory Great care is needed in transporting undisturbed samples of non-cohesive materials if they are to reach

the laboratory in satisfactory condition. It is, in fact, virtually impossible to transport some samples of this type, and it is therefore occasionally necessary to establish a temporary laboratory in the field in which certain types of tests can be made without subjecting samples to the inevitable jarring in handling incident to transporting them to a centrally located laboratory. Jars containing representative samples of soil should be carefully sealed, identified by labels rendered waterproof with a coat of lacquer after data has been entered in waterproof ink, and given adequate protection against breakage during shipment It is important that identifying labels be placed on the jars containing representative samples. If the labels are placed on the jar covers, exchanges and consequent misinterpretations are almost sure to occur. Undisturbed samples should be sent to the laboratory in the tubes in which they were taken. The tubes should be sealed by methods previously described and labeled for identification in the same way as the jars containing representative samples All soil samples, and particularly samples of noncohesive materials, should be guarded against unnecessary handling, jarring, and vibration. For this reason it is much more satisfactory to transport samples to the laboratory on a cushioned support in a service vehicle or truck than to entrust them to the inevitable rough handling they will receive at the hands of a common carrier.

4. SOIL CLASSIFICATION

A preliminary field classification, by soil types, of the material encountered in a subsurface exploration should be made by the soils engineer in charge. The identification of the material encountered, as reported in the records of the exploration, is based on this preliminary field classification. Work subsequently done in the laboratory may, however, lead to the reclassification of materials of doubtful character. The following definitions are given in the interest of standardizing the nomenclature of field classification. The definitions are based largely on those given in refs. 20, 22, and 24.

Boulders are detached and rounded or worn pieces of rock, the least sizes of which are from 6 to 8 in.

Gravel consists of loose, rounded fragments of rock ranging in size from $^1/_4$ in. to boulder size.

Sand is a cohesionless granular soil consisting of particles smaller than gravel. It often occurs as a mixture with gravel. Sand may be further subclassified as. *Coarse sand,* the greater portion of which is retained by a 28-mesh sieve. *Medium sand,* the greater portion of which passes a 28-mesh but is retained by a 65-mesh sieve. *Fine sand* similarly passes a 65-mesh and is retained by a 200-mesh sieve *Compact or dense sands* require a pick for their removal. *Loose sands* can be excavated by shoveling *Quicksand* is any sand rendered unstable by an upward flow of ground water. It is therefore a condition rather than a material. Any sand can be made quick by an upward flow through it of sufficient velocity.

Silty sand is a mixture of sand with fine-grained materials having some plasticity and of the types defined below as silts. These may be of organic or inorganic origin.

Inorganic silt is a fine-grained soil having appreciable though slight plasticity. Silts have little cohesion when dried, and they dust off easily when allowed to dry on the hands. Some fine-grained silts are difficult to distinguish from clays when wet. When such silts are shaken in the palm of the hand, however, their surface becomes glossy due to the expulsion of water from the pores of the soil. Then if the glossy pat of material is suddenly squeezed or bent its surface becomes dull. Clays will not respond in this way to this simple test.

Rock flour is an extremely fine-grained inorganic silt, the particles of which are so fine that the material may have a strong superficial resemblance to clay when wet. It can be distinguished from clay by the shaking test described above.

Organic silt is a fine-grained, more or less plastic soil containing finely divided particles of decomposed organic matter. It ranges from light to dark gray in color and often contains gaseous products of the decomposition of organic matter which give it a characteristic odor.

Clay is an extremely fine-grained inorganic soil, the individual particles of which are so small as to be impalpable when rubbed between the fingers. Many of the particles are in fact of colloidal size. Wet clay sticks to the hands and when allowed to dry on them does not brush off easily. Through a rather large range of water content clay can be rolled into threads that have appreciable tensile strength. Clay has pronounced cohesive qualities. A lump, or pat, of it that is allowed to dry develops considerable strength. Clays may be further subclassified as follows *Fat clays* are finer grained than lean clays. When wet they may have a distinctly soapy feel. They are strongly cohesive and when dry can be broken but not crushed or pulverized by hand. Partially dried fat clays are plastic. *Lean clays* are coarser grained than fat clays but the particles are, nevertheless, too fine to be palpable.

Lean clays are strongly cohesive but less so than fat clays. *Hard or stiff clays* cannot be molded with the fingers nor excavated without a pick. Such clays have been compressed to a low water content and may be good foundation materials. *Soft clays* can be molded easily with the fingers and excavated with a spade. Such clays are relatively compressible, have low shearing strength, and fail under heavy loads. *Varved clay* consists of alternating thin strata of clay and silt deposited respectively in the winter and summer by water from melting ice at the close of a glacial epoch.

Shale is sometimes considered a rock and sometimes a soil. It is material in a state of transition from clay to slate. Some shales decompose rapidly when exposed to the weather.

Marl is a marine deposit consisting of clay, sand, and calcium carbonate. It may occur in the form of a crumbly and very soft rock or a fairly stiff to very stiff grevish green claylike soil.

Marlacious shales, limestones, and clays contain enough marl to give them some of its characteristics. It is exceedingly difficult to define specific distinctions between these

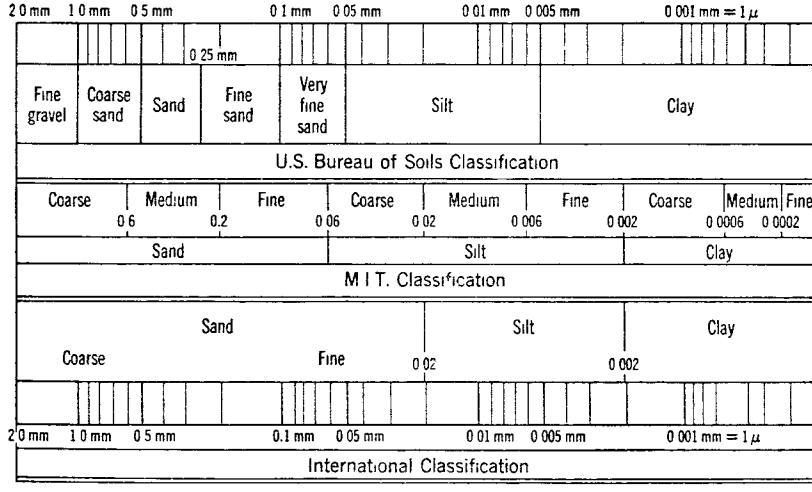


FIG. 12. Classification of soils by grain size.

materials and marl. In different parts of the country the terms are applied to somewhat different materials.

Hardpan is a thoroughly compacted mixture of clay, sand, gravel, and boulders, or a cemented mixture of sand, or sand and gravel, with or without boulders which is difficult to remove by hand-tool excavation. The term is applied to different materials in different localities.

Loess is a fine-grained, granular, slightly cemented, and slightly compacted soil that was deposited by wind. Loess is characterized by a very uniform grain size, by the presence of vertical root holes, and the ability to stand with nearly vertical slopes. The bond, which is due to a calcareous binder, may be destroyed by saturation combined with an increase in load. The result will be rapid settlement. Cuts in loess should be left with very steep, almost vertical, slopes to minimize surface erosion.

Adobe is a term applied in some parts of the country to natural mixtures of fat clays and sand. It is an exceedingly cohesive and sticky soil which is used in the Southwest for making unburned adobe brick.

Peat is an organic soil consisting largely of decomposed vegetable matter. It is plastic and highly compressible. When dried it is light enough to float and will also burn. As the support for a foundation or a material for the construction of an embankment or earth structure, it is probably the most undesirable material with which the engineer has to deal. Deep-seated and unsuspected deposits of peat underlying strata of more desirable material have been the cause of many serious foundation settlements.

Classification by grain size is the basis of several systems which attempt to identify soils by consideration of the size, but not the character, of the particles of which they consist. The methods have the merit of giving definite numerical values for the limiting grain sizes of soil types. On the other hand, the methods have the weakness of ignoring one of the

most important characteristics of a soil. This is its plasticity, which is probably due to the shape rather than the size of the soil grains; plastic soils have a preponderance of thin flat grains, whereas the grains of non-cohesive soils are angular or more or less spherical. Grain-size classifications therefore make no distinction between clays and fine silts such as rock flours, although the materials do not behave similarly when called upon to support a foundation or to remain stable in an embankment or the face of an excavation.

The three most widely used grain-size classification systems are the *U. S. Bureau of Soils Classification*, the *M.I.T. Classification* originated by Glennon Gilboy, and the *International Classification*. The ranges of grain size assigned to various soil types by the three systems are shown in Fig. 12.

The *U. S. Bureau of Soils Textural Classification*, another method based on grain size, is illustrated by Fig. 13. The size ranges used to designate the character of the materials, the

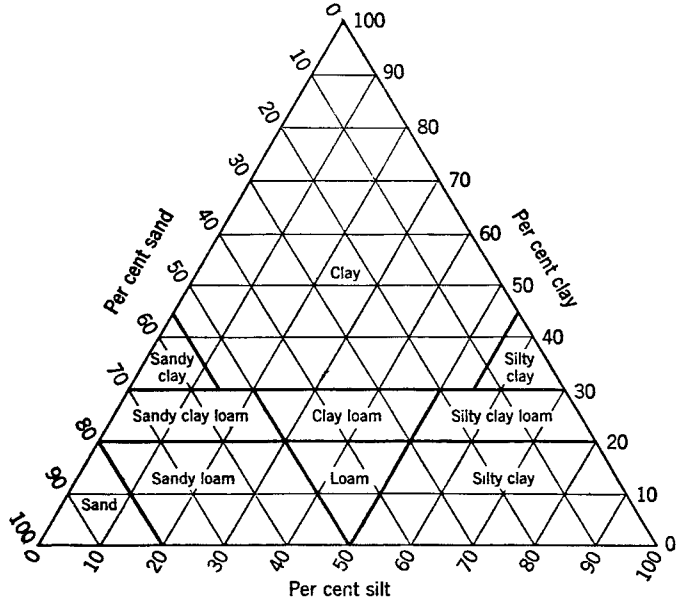


FIG 13. U. S. Bureau of Soils textural classification. From *Fundamentals of Soil Mechanics*, D W. Taylor, John Wiley & Sons, 1948.

percentages of which are plotted on the triangular charts of this system, are those of the U. S. Bureau of Soils Classification, except that sand sizes are considered to extend to 2.0 mm and all coarser cases are eliminated from consideration. A soil with given percentages of material falling into the size categories of sand, silt, and clay is therefore represented by a point on the triangular chart and may be given definitely the "overall" designation assigned the area within which the point falls.

The uniformity coefficient devised by Allen Hazen as a means for quickly and conveniently giving the approximate characteristics of a filter sand is used to a certain extent in soil mechanics and foundation engineering. Hazen called the *effective size* of a sand the grain size such that the weight of all smaller grains in a sample is 10% of its total weight. This grain size is designated by D_{10}. Grain size or diameter, D_{10}, is sometimes called the 10% size and it is recognized practice to designate other percentage sizes by similar notation; for example, the 60% size, designated D_{60}, is such that the weight of the smaller grains in a sample is 60% of its total weight. The ratio of these two effective sizes, that is, the ratio of D_{60} to D_{10} in a sand, was called by Hazen the uniformity coefficient, C_U. The "grain sizes" or "grain diameters" referred to are the dimensions of the openings in square-mesh sieves which will pass or retain the percentages of sand used in determining effective sizes and uniformity coefficients. A uniformity coefficient C_U of 1.0, or slightly more than 1.0, is indicative of a soil in which the grains are all of practically the same size. Ottawa sand is such a soil. A large coefficient means that there is a big spread in grain sizes.

Grain-size curves, such as those shown in Fig. 14, are of value in studying the characteristics of a soil, particularly in making an approximate estimate of its permeability. It is usual to plot such curves on logarithmic paper because the curves for soils having similar uniformity characteristics but differing in coarseness will have the same shape on a logarithmic plot, the curves for finer materials lying to the right of those representing the grain-size distribution of coarser soils. The curves will have little resemblance if the grain sizes are plotted to a natural scale.

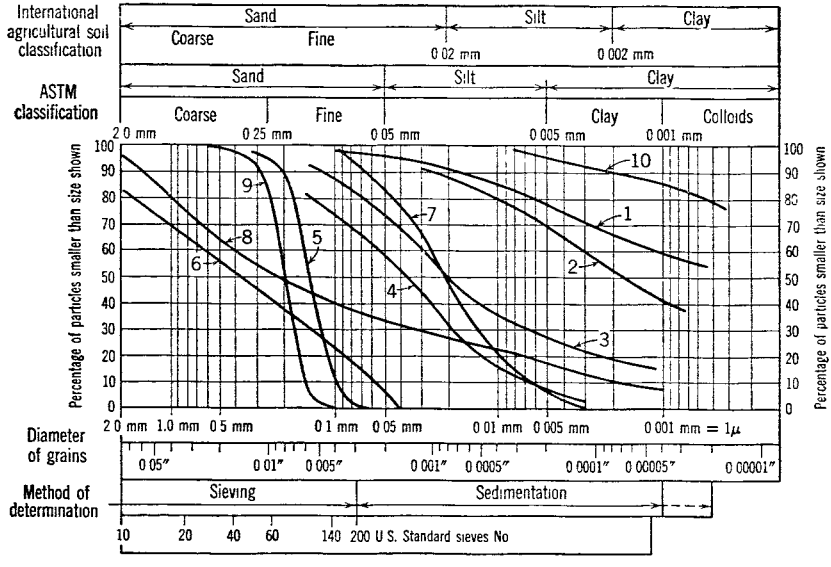


FIG. 14. Grain-size curves By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G P Tschebotarioff, copyright, 1951, McGraw-Hill Book Co.

The *Public Roads Administration Classification System*, the *Soil Classification for Airfield Projects* developed by Arthur Casagrande, and the *Classification of Soils for Airport Construction* appearing in the Civil Aeronautics Administration's pamphlet Airport Paving, published in 1948, are broader systems of soil classification which are widely used for the purposes indicated by their titles. The Public Roads system classifies soils with respect to their desirability for the construction of "dirt" or unpaved roads. The Casagrande and Civil Aeronautics systems classify soils with respect to their desirability as subgrade materials for the support of airport pavements. The essentials of the Casagrande system are given by Fig. 15. The right-hand column of the Casagrande classification chart, it will be noted, gives the Public Roads classification for each type of soil considered. Figure 16 is the CAA classification chart. The four columns at the right of this chart classify each soil group with respect to its quality as the subgrade for both rigid and flexible pavements in localities subject to no frost and severe frost. These classifications run from Fa, the most desirable, to F10, the least desirable subgrade material for the support of flexible pavements; and similarly from R1a to R2e for rigid pavements.

1		2	3	4		5	6
MAJOR DIVISIONS		SOIL GROUPS & TYPICAL NAMES	AFFORDED BY SAMPLES	GENERAL IDENTIFICATION: DRY STRENGTH	GENERAL IDENTIFICATION: OTHER PERTINENT EXAMS	OBSERVATIONS AND TESTS RELATING TO MATERIAL IN PLACE	PRINCIPAL CLASSIFICATION TESTS (ON DISTURBED SAMPLES)
COARSE GRAINED SOILS	Gravel	Well Graded Gravel & Gravel Sand Mixtures, Little or No Fines	GW	None	Gradation, Grain Shape	Dry Unit Weight or Void Ratio,	Mechanical Analysis
	and	Well Graded Gravel Sand Clay Mixtures, Excellent Binder	GC	Medium to High	Gradation, Grain Shape, Binder Exam Wet & Dry	Degree of Compaction,	Mechanical Analysis, Liquid & Plastic Limits on Binder
	Gravelly	Poorly Graded Gravel & Gravel Sand Mixtures, Little or No Fines	GP	None	Gradation, Grain Shape	Cementation, Durability of Grains,	Mechanical Analysis
	Soils	Gravel with Fines, Very Silty Gravel, Clayey Gravel, Poorly Graded Gravel Sand Clay Mixtures	GF	Very Slight to High	Gradation, Grain Shape, Binder Exam Wet & Dry	Stratification & Drainage Characteristics,	Mechanical Analysis, Liquid & Plastic Limits on Binder If Applicable
	Sands	Well Graded Sands & Gravelly Sands, Little or No Fines	SW	None	Gradation, Grain Shape	Ground Water Conditions,	Mechanical Analysis
	and	Well Graded Sand Clay Mixtures, Excellent Binder	SC	Medium to High	Gradation, Grain Shape, Binder Exam Wet & Dry	Traffic Tests,	Mechanical Analysis, Liquid & Plastic Limits on Binder
	Sandy	Poorly Graded Sands, Little or No Fines	SP	None	Gradation, Grain Shape	Large Scale Load Tests or	Mechanical Analysis
	Soils	Sand with Fines, Very Silty Sands, Clayey Sands, Poorly Graded Sand Clay Mixtures	SF	Very Slight to High	Gradation, Grain Shape, Binder Exam Wet & Dry	California Bearing Tests	Mechanical Analysis, Liquid & Plastic Limits on Binder If Applicable
FINE GRAINED SOILS Containing Little or No Coarse Grained Material	Fine Grained Soils	Silty Inorganic Very Fine Sands, Mica Rock Flour, Silty or Clayey Fine Sands with Slight Plasticity	ML	Very Slight to Medium	Examination Wet (Shaking Test & Plasticity)	Dry Unit Weight, Water Content & Void Ratio	Mechanical Analysis, Liquid & Plastic Limits If Applicable
	Having	Clays Inorganic of Low to Medium Plasticity, Sandy Clays, Silty Clays, Lean Clays	CL	Medium to High	Examination in Plastic Range	Consistency—Undisturbed & Remolded	Liquid & Plastic Limits
	Low to Medium					Stratification, Root Holes, Fissures Etc.,	
	Compressibility	Organic Silts & Organic Silt-Clays of Low Plasticity	OL	Slight to Medium	Examination in Plastic Range, Odor	Drainage & Ground Water Conditions,	Liquid & Plastic Limits From Natural Condition & After Oven Drying
	Fine Grained Soils	Micaceous or Diatomaceous Fine Sandy & Silty Soils, Elastic Silts	MH	Very Slight to Medium	Examination Wet (Shaking Test & Plasticity)	Traffic Tests,	Mechanical Analysis, Liquid & Plastic Limits If Applicable
	Having	Clays Inorganic of High Plasticity, Fat Clays	CH	High	Examination in Plastic Range	Large Scale Load Tests,	Liquid & Plastic Limits
	High					California Bearing Tests or	
	Compressibility	Organic Clays of Medium to High Plasticity	CH	High	Examination in Plastic Range, Odor	Compression Tests	Liquid & Plastic Limits From Natural Condition & After Oven Drying
Fibrous Organic Soils with Very High Compressibility		Peat and Other Highly Organic Swamp Soils	Pt	Readily Identified		Consistency, Texture and Natural Water Content	

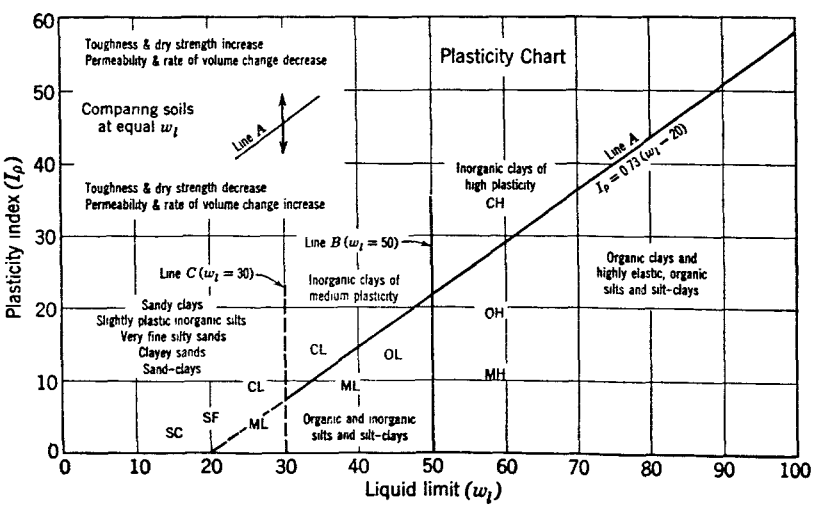
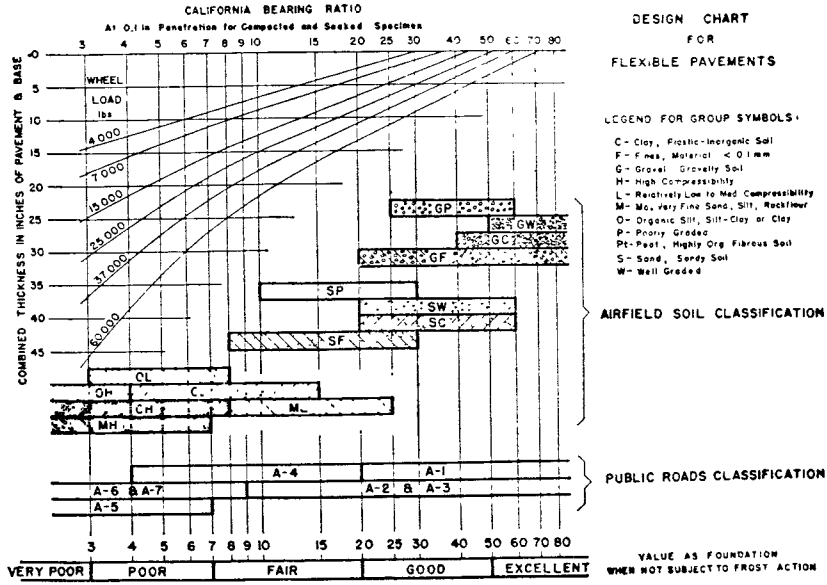


FIG 15. The Casagrande system of soil classification From

SOIL CLASSIFICATION FOR AIRFIELD PROJECTS

7	8		9	10	11	12	13*	14	15
VALUE AS FOUNDATION WHEN NOT SUBJECT TO FROST ACTION	VALUE AS WEARING SURFACE FOR STORE OR EMERGENCY CONSTRUCTION		POTENTIAL FROST ACTION	SHRINKAGE, EXPANSION, ELASTICITY	DRAINAGE CHARACTERISTICS	COMPACTION CHARACTERISTICS AND EQUIPMENT	SOLIDS AT OPT COMPACTION LB PER CU FT & VOID RATIO	CAL BEARING RATIO FOR COMPACTED & SOAKED SPECIMEN	COMPARABLE GROUPS IN PUBLIC ROADS CLASSIFICATION
	WITH DUST PALLIATIVE	4 TH BITUMINOUS SURFACE TREATMENT							
Excellent	Fair to Poor	Excellent	None to Very Slight	Almost None	Excellent	Excellent, Tractor	> 25 e < 0.35	>50	A-3
Excellent	Excellent	Excellent	Medium	Very Slight	Practically Impervious	Excellent, Tamping Roller	>130 e < 0.30	>40	A-1
Good to Excellent	Poor	Poor to Fair	None to Very Slight	Almost None	Excellent	Good, Tractor	> 15 e < 0.45	25-60	A 3
Good to Excellent	Poor to Good	Fair to Good	Slight to Medium	Almost None to Slight	Fair to Practically Impervious	Good, Close Control Essential, Rubber Tire Roller, Tractor	> 20 e < 0.40	>20	A 2
Excellent to Good	Poor	Good	None to Very Slight	Almost None	Excellent	Excellent, Tractor	> 20 e < 0.40	20-60	A-3
Excellent to Good	Excellent	Excellent	Medium	Very Slight	Practically Impervious	Excellent, Tamping Roller	> 25 e < 0.35	20-60	A-1
Fair to Good	Poor	Poor	None to Very Slight	Almost None	Excellent	Good, Tractor	>100 e < 0.70	10-30	A 3
Fair to Good	Poor to Good	Poor to Good	Slight to High	Almost None to Medium	Fair to Practically Impervious	Good, Close Control Essential, Rubber Tire Roller	>105 e < 0.60	8-30	A-2
Fair to Poor	Poor		Medium to Very High	Slight to Medium	Fair to Poor	Good to Poor, Close Control Essential, Rubber Tire Roller	>100 e < 0.70	6-25	A 4 A-6 A 7
Fair to Poor	Poor		Medium to High	Medium	Practically Impervious	Fair to Good, Tamping Roller	>100 e < 0.70	4-15	A 4 A-6 A 7
Poor	Very Poor		Medium to High	Medium to High	Poor	Fair to Poor, Tamping Roller	>90 e < 0.90	3-8	A-4 A 7
Poor	Very Poor		Medium to Very High	High	Fair to Poor	Poor to Very Poor	<100 e > 0.70	< 7	A-5
Poor to Very Poor	Very Poor		Medium	High	Practically Impervious	Fair to Poor, Tamping Roller	>90 e < 0.90	< 6	A-6 A-7
Very Poor	Useless		Medium	High	Practically Impervious	Poor to Very Poor	< 20 e > 0.70	< 4	A-7 A-8
Extremely Poor	Useless		Slight	Very High	Fair to Poor	Compaction Not Practical			A-8

* THESE WEIGHTS APPLY ONLY TO SOILS HAVING SPECIFIC GRAVITIES FROM 2.65 TO 2.75



Fundamentals of Soil Mechanics, D. W. Taylor, John Wiley & Sons, 1948

Soil Group	Mechanical Analysis				Liquid Limit	Plasticity Index	Subgrade Class			
	Retained on No. 10 Sieve, %	Material Finer than No 10 Sieve					Good Drainage		Poor Drainage	
		Coarse Sand Pass No 10 Ret No 60, %	Fine Sand Pass No 60 Ret. No. 270, %	Combined Silt and Clay Pass No. 270, %			No Frost	Severe Frost	No Frost	Severe Frost
E-1	0-45	40+	60−	15−	25−	6−	Fa R1a	Fa R1a	Fa R1a	Fa R1a
E-2	0-45	15+	85−	25−	25−	6−	Fa R1a	Fa R1a	F1 R1a	F2 R1a
E-3	0-45	. .	. . .	25−	25−	6−	F1 R1a	F1 R1a	F2 R1a	F2 R1a
E-4	0-45	. .	. .	35−	35−	10−	F1 R1a	F1 R1a	F2 R1a	F3 R2a
E-5	0-45	. .	.	45−	40−	15−	F1 R1a	F2 R1b	F3 R1b	F4 R2b
E-6	0-55	. .	.	45+	40−	10−	F2 R1a	F3 R2b	F4 R2b	F6 R2b
E-7	0-55	. . .	. . .	45+	50−	10-30	F3 R1b	F4 R2b	F6 R2b	F7 R2c
E-8	0-55	. .	. .	45+	60−	15-40	F4 R1b	F6 R2c	F7 R2c	F8 R2d
E-9	0-55	. .	. .	45+	40+	30−	F5 R2b	F7 R2c	F7 R2c	F9 R2d
E-10	0-55	.	. .	45+	70−	20-50	F5 R2b	F7 R2c	F8 R2c	F9 R2d
E-11	0-55	. .		45+	80−	30+	F6 R2c	F8 R2d	F9 R2d	F10 R2e
E-12	0-55	.		45+	80+	.	F8 R2d	F9 R2e	F10 R2e	F10 R2e
E-13	Muck and peat, field examination						Not suitable for subgrade			

FIG. 16. CAA soil classification system. Classification based on sieve analysis of portion of sample passing No. 10 sieve. When sample contains material coarser than No. 10 sieve in amount equal to or greater than maximum limit shown, a raise in classification may be allowed provided coarse material is reasonably sound and well graded. Good drainage occurs where there will be no accumulations of water that would develop spongy areas in subgrade, where water-table elevation is such that soil will not become waterlogged by percolation from above or capillarity from below and surface water will be removed rapidly. Severe frost condition assumed where depth of frost penetration exceeds anticipated thickness of surfacing, base, and subbase as determined for no frost. See CAA publications on airport paving.

5. SOIL MECHANICS LABORATORY TESTS

The application of soil mechanics to foundation and soil stability problems frequently involves tests by experienced personnel in an adequately equipped soils laboratory. The results of tests which may have been attempted, from necessity, by inexperienced personnel with improvised equipment should be used with great caution. Soil mechanics laboratory tests may be divided broadly into "routine tests" which are of a relatively simple nature and are made more or less as a matter of course on all samples to which they are applicable. These are the determination of the natural water content; the Atterberg tests for the determination of the liquid limit, plastic limit, shrinkage limit, and plasticity index; mechanical analysis; determination of the specific gravity of the solids; and determination of the void ratio. Other tests which may or may not be justified in connection with a specific problem are strength tests; including unconfined compression, direct shear and triaxial shear tests; consolidation tests; permeability tests; and compaction tests. No time, effort, or money should be wasted in attempting tests on soils to which they are inapplicable or in making tests which will not yield information relevant to the problem under consideration.

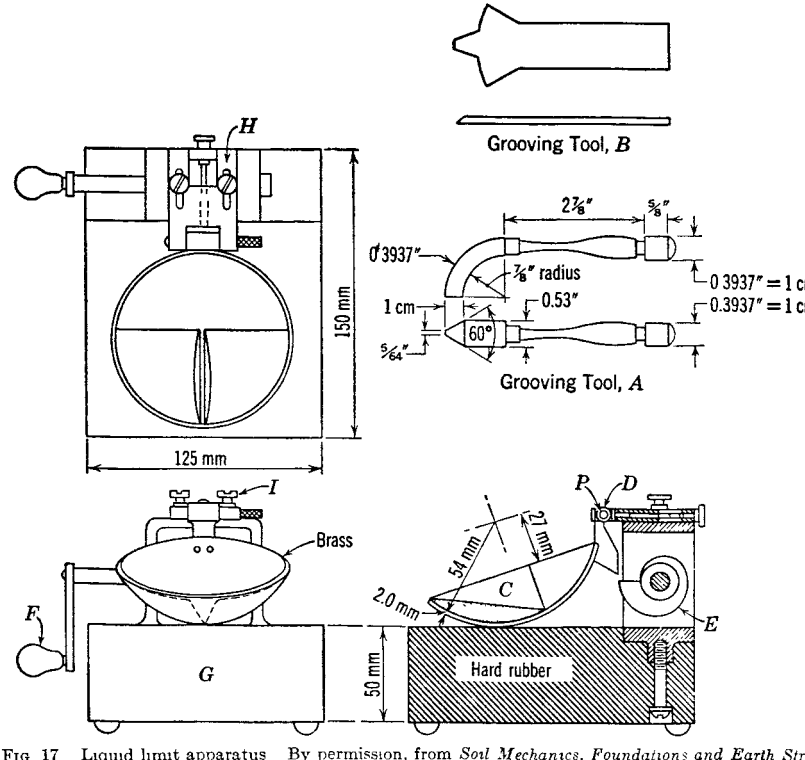


FIG 17 Liquid limit apparatus By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G. P Tschebotarioff, copyright, 1951, McGraw-Hill Book Co.

Brief descriptions of the common routine tests follow. Detailed instructions for making them are given in refs. 1, 2, 6, and 17, which should be consulted by anyone undertaking soils laboratory work.

The **natural water content,** w, is the ratio of the weight of water contained to the dry weight of a soil and is usually expressed as a percentage. It is determined by weighing a sample in a container of known weight, drying the sample in the container, again weighing the sample and container, and computing the water content by the formula

$$w = \frac{W_1 - W_2}{W_2 - W_c} 100 \tag{1}$$

in which w = the water content expressed as a percentage of the dry weight of the soil.
W_1 = combined weight of soil, water, and container.
W_2 = weight of container and soil dried to constant weight in an oven at a temperature not exceeding 110°C; higher temperatures may drive off water chemically combined with constituents of the soil.
W_c = weight of the container.

The **Atterberg tests** determine minimum water contents of cohesive soils causing specified conditions of consistency. The water contents of the *Atterberg limits* are usually expressed as percentages of the weight of the dry soil.

The *liquid limit*, w_l, is the least water content at which a cohesive soil is practically liquid. It is determined by finding the water content at which a groove cut in a pat of the soil will close when the sample is subjected to the shock of 25 blows in the apparatus shown in Fig. 17. The apparatus, it will be noted, standardizes not only the dimensions of the pat and the groove but also the severity of the shocks. It is unnecessary to make repeated trials at different water contents until one is found at which the groove closes after exactly 25 shocks. The number of shocks if plotted to a logarithmic scale against water contents to an arithmetic scale will, for a given soil, give a series of points on a straight line. It is therefore sufficient to make several tests at different water contents, plot the results as in Fig. 18, and accept the ordinate to the resulting straight line at the 25 shock point as the liquid limit.

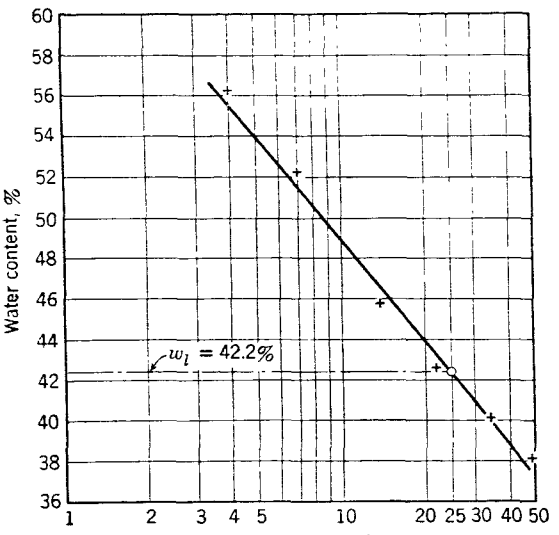


FIG 18 Graphical method of determining liquid limit By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G P Tschebotarioff, copyright, 1951, McGraw-Hill Book Co

The *plastic limit*, w_p, is the minimum water content at which a cohesive soil is plastic. It is determined by finding the minimum water content at which a small sample of the soil can be rolled into threads of about 1/8-in. diameter without crumbling. No special apparatus is required, the soil samples being rolled under the hand on a glass plate.

The *plasticity index*, I_p, is the difference between the *liquid* and *plastic limits* and is computed by the formula:

$$I_p = w_l - w_p \tag{2}$$

Table 2. Unit Weight or Specific Gravity of Distilled Water at Given Temperatures

°C	0	1	2	3	4	5	6	7	8	9
0	0 9999	0 9999	1 0000	1 0000	1.0000	1 0000	1.0000	0 9999	0 9999	0 9998
10	0 99973	0 99963	0 99952	0 99940	0.99927	0 99913	0.99897	0 99880	0 99862	0.99843
20	0 99823	0 99802	0 99780	0.99757	0 99733	0.99707	0 99681	0.99654	0 99626	0 99597
30	0 99568	0.9954	0 9951	0 9947	0 9944	0.9941	0.9937	0.9934	0 9930	0 9926
40	0 9922	0 9919	0 9915	0.9911	0 9907	0.9902	0 9898	0.9894	0.9890	0 9885
50	0.9881	0 9876	0.9872	0.9867	0.9862	0.9857	0.9852	0 9848	0.9842	0.9838
60	0 9832	0 9827	0 9822	0 9817	0.9811	0.9806	0.9800	0.9795	0.9789	0 9784
70	0.9778	0 9772	0 9767	0 9761	0.9755	0.9749	0 9743	0.9737	0.9731	0.9724
80	0 9718	0.9712	0 9706	0.9699	0.9693	0.9686	0 9680	0.9673	0.9667	0 9660
90	0 9653	0.9647	0.9640	0.9633	0.9626	0.9619	0 9612	0.9605	0.9598	0.9591

To get unit weight of water in the English system multiply by 62 424.
Source Fundamentals of Soil Mechanics, D. W. Taylor, John Wiley & Sons, 1948.

The *shrinkage limit* is the minimum water content completely saturating a cohesive soil. It may be determined in two ways. If the specific gravity of the soil is not known, a container of known volume and weight, usually a dish about 1 3/4 in. in diameter and 1/2 in. deep, is filled with the soil in a plastic condition and weighed. The sample is then dried to a constant weight, the drying being accompanied by considerable shrinkage of the plastic soil. The shrunken sample is accurately weighed and its volume determined by mercury displacement. The shrinkage limit is then computed by the formula:

$$w_s = \frac{(W_1 - W_s) - \gamma_w(V_1 - V_2)}{W_s} 100 \tag{3}$$

in which w_s = the shrinkage limit expressed as a percentage of the dry weight of the soil.
W_1 = weight in grams of the specimen in its initial plastic condition.
W_s = weight of specimen in grams after being dried to constant weight at a temperature not exceeding 110°C.
γ_w = unit weight of water in grams per cc which equals its specific gravity at the temperature of the specimen (see Table 2).
V_1 = volume of container in cc
V_2 = final volume of shrunken specimen in cc.

If the specific gravity of the soil has been determined by the pycnometer method, which is described subsequently, the shrinkage limit may be computed by the formula:

$$w_s = \left(\frac{\gamma_w V_2}{W_s} - \frac{1}{G}\right) 100 \tag{4}$$

in which G = the specific gravity of the soil.

Other terms have the same significance as in the previously given formula for w_s.

The opinion of soils engineers vary widely as to the importance of the Atterberg tests. They are virtually ignored by some men. Others make them as a matter of routine on practically all the cohesive soils with which they deal Members of the latter group maintain that the results of the tests give them valuable information which has been called "a feel" for the character of a soil. Undoubtedly the Atterberg limits are of great value in classifying doubtful soils, their utility for this purpose being recognized by the inclusion of plasticity charts, based on the limits, in both the Casagrande and CAA systems of soil classification for airfield projects, Figs. 15 and 16. The Construction and Material Specifications issued by the Department of Highways, State of Ohio, in 1946, included in its requirements for soils used in embankments a provision that a soil "shall have a liquid limit not to exceed 65, and the minimum plasticity index number of soil with liquid limits between 35 and 65 shall be not less than that determined by the formula 0.6 liquid limit minus 9.0."

Mechanical analysis for the determination of grain size distribution is readily accomplished by sieving if the material is a sand or relatively coarse-grained soil. The sieves required are:

1. For preliminary preparation of the specimen in accordance with ASTM Designation D 421-39

No. 4	4760 micron
No. 10	2000 micron
No. 40	420 micron
No. 200	74 micron

2. For the final analysis in accordance with ASTM Designation D 422-39

No. 20	840 micron
No. 40	420 micron
No. 60	250 micron
No. 140	105 micron
No. 200	74 micron

3. For mechanical analyses upon which soil classifications by the CAA system are to be based use the sieve sizes heading the columns of the CAA classification chart, Fig. 16.

It will usually be necessary to wash the material through the 200- and 270-mesh sieves in order that all the fines capable of passing their small openings actually go through.

For the mechanical analysis of a fine-grained soil, such as a clay or silt, or for obtaining data on the grain-size distribution of the finer portions of most soils, analysis by sieving is inadequate if not impossible. In such cases it is usual to resort to methods based on Stokes law that grains of different sizes fall through a liquid at different speeds. The most practical of these methods is one utilizing the specific gravity of a suspension of the soil in dis-

tilled water, determined by a special type of hydrometer as an indirect measure of the amount of material remaining in suspension after given intervals of time and hence of the amount of material of such grain size that will remain in suspension for each of the intervals used. The method, for which detailed instructions are given in ASTM Designation D 422-39, requires much special laboratory equipment and therefore will not be described here.

The results of mechanical analysis may be plotted on a grain-size accumulation curve of the type shown in ASTM D 422-39 from which the percentages listed below are read off and reported:

(*a*) Percentage of particles larger than 2 mm.
(*b*) Percentage of coarse sand, 2.0 to 0.25 mm.
(*c*) Percentage of fine sand, 0.25 to 0.05 mm.
(*d*) Percentage of silt, 0.05 to 0.005 mm.
(*e*) Percentage of clay, smaller than 0.005 mm.
(*f*) Percentage of colloids, smaller than 0.001 mm.

Another form of report largely used today is:

Sieve Analysis

Sieve Size		Percentage Passing
2 in.		
1 1/2 in.		
3/4 in.		
3/8 in.		
No. 4	4760 micron	
No. 10	2000 micron	
No. 40	420 micron	
No. 60	250 micron	
No. 200	74 micron	

Hydrometer Analysis

Size Particle	Percentage Smaller Than
0.05 mm	
0.005 mm	
0.001 mm	

Data from the second form of report may be entered directly on grain-size distribution curves of the type illustrated in Fig. 14.

The results of mechanical analysis are of unquestioned value in determining the suitability of a soil as subgrade material for roads and airport pavements, for determining its fitness for use in earth dams and high embankments, also for estimating its probable stability if required to stand in the exposed face of a deep cut. As data upon which to base an estimate of the ability of a soil to carry loads imposed by a foundation, the results of mechanical analysis are of much more doubtful value. They are not, in fact, used by some engineers in foundation work, although the same men recognize their great importance in the fields mentioned above. They should, however, be made and used in all cases of doubt as to the true character of a soil.

The **specific gravity,** G, of a soil is defined as the ratio between the weight in air of a given net volume of the particles comprising the soil and the weight of an equal volume of distilled water at 20°C. ASTM Designation D 854-45T describes in detail the pycnometer method of determining G. The method is outlined briefly below.

A sample of about 25 grams, dry weight, of the soil is put into a constant-volume flask, called a pycnometer, which is then partially filled with distilled water. Air and gases are driven off by boiling, either by the application of heat or, preferably, by connecting the flask to a vacuum pump capable of reducing the pressure enough to cause boiling. The material in the flask is agitated frequently while being boiled to facilitate the almost complete removal of air and other gases from the soil sample. After boiling, the pycnometer and its contents are allowed to come to room temperature and it is then filled to the reference mark with water at the same temperature. The weight of the pycnometer when filled to the reference mark with water at various temperatures likely to be encountered having been previously determined and recorded, the specific gravity of the soil is determined by the formula:

$$G = \frac{W_s \gamma_w}{W_s + W_a - W_b} \tag{5}$$

in which G = the specific gravity of the soil.
W_s = the weight in grams of the oven dry sample.

W_a = the weight in grams of the pycnometer filled with water at the temperature of the room.
W_b = the weight in grams of the pycnometer containing the soil specimen and filled to the reference mark with water at the temperature of the room.
γ_w = the specific gravity of water at the temperature of the room from Table 2.

The specific gravity of a soil may also be determined by the formula.

$$G = \frac{W_s}{V_1\gamma_w - W_1 + W_s}$$

in which G is the specific gravity and the other terms have the same significance as in formula (3). This method is not as precise as the previously described pycnometer method which is the accepted standard procedure for determining the specific gravity of a soil.

The specific gravity, G, is an overall or average measure of the specific gravity of the particles comprising the soil. The specific gravities of most soils lie between 2.65 and 2.85. Specific gravities below 2.60 are strongly indicative of the presence of organic particles in the soil. Soils containing heavy substances such as iron may have specific gravities exceeding 3.00. The specific gravity of a soil is important because it is required in void ratio determinations which may be necessary in stability studies. It is also of importance in some soil classification problems.

The bulk specific gravity of a soil is the ratio between the weight of the gross volume of a sample and the weight of an equal volume of water. It is determined by the formula:

$$G_b = \frac{W_b}{W_w} \tag{6}$$

in which G_b = the bulk specific gravity of the soil.
W_b = the weight of a known gross volume of the soil, preferably in grams.
W_w = the weight of the same volume of water.

The bulk specific gravity G_b is less frequently used and is less important than the specific gravity G. It is, however, needed in connection with some problems dealing with submerged soils.

The void ratio, e, of a soil is the ratio between the volume of the voids, or intergranular spaces, and the volume of the solid particles. Naturally it changes with varying degrees of compaction or consolidation of a soil. It is usually expressed as a ratio and obtained by use of the formula:

$$e = \frac{V_v}{V_s} = \frac{G\gamma_w V}{W_s} - 1 \tag{7}$$

in which e = the void ratio expressed as a ratio—not a percentage.
G = the specific gravity of the soil.
γ_w = the specific gravity of water at laboratory temperature from Table 2.
V = the volume in cc of an undisturbed sample of the soil, or a sample at the degree of compaction or consolidation under consideration.
V_v = the volume of the voids.
V_s = the volume of the solids.
W_s = the dry weight in grams of the solids in the sample.

The void ratio e is used more frequently in soil mechanics than the porosity n, which is the ratio of the volume of the voids to the total volume of the soil mass, or the degree of saturation S, which is the ratio of the volume of water to the volume of voids. Both n, the porosity, and S, the degree of saturation, are usually expressed as percentages and obtained by the use of the formulas:

$$n = \frac{V_v}{V} \times 100 \tag{8}$$

$$S = \frac{V_w}{V_v} \times 100 \tag{9}$$

in which n = the porosity expressed as a percentage.
V_v = the volume of the voids in the sample.
V = the gross volume of the sample.
S = the degree of saturation expressed as a percentage.
V_w = the volume of water in the sample.

The *void ratio* e and the *porosity* n are different expressions for the same characteristic of a soil. It is usually more convenient to use e, rather than n, because when a given soil

is compacted or consolidated its varying *void ratios* are computed from fractions in which the denominators do not change.

If a soil is initially saturated the *void ratio* may be determined by use of the formula:

$$e = \frac{W_w G}{W_s S} \tag{10}$$

in which W_w = the weight of water in a sample.
W_s = the oven dry weight of the sample
G = the specific gravity of the soil
S = the degree of saturation of the soil expressed as a ratio.

This method of determining e is, however, less precise and reliable than the method previously given, as soils which are apparently saturated may actually contain appreciable quantities of gas.

The *void ratio* is a very important characteristic of soils. It is first of all indicative of the compressibility of both non-cohesive and cohesive soils and, to a certain extent, of the permeability of soils of the former type. It is not in the slightest degree indicative of the permeability of a cohesive soil. Fat clays, which are only slightly permeable, usually have large *void ratios*. The void ratio is of great importance in estimating the stability of saturated non-cohesive soils—see the discussion of critical void ratios below under triaxial shear tests.

The *degree of compaction*, D_d, frequently called the *relative density*, is used to express numerically the looseness or denseness of sandy soils. It is computed by the formula:

$$D_d = \frac{e_{max} - e}{e_{max} - e_{min}} \tag{11}$$

in which D_d = the relative density expressed as a ratio.
e_{max} = the void ratio in the loosest state that can be produced in the laboratory.
e = the void ratio of the soil as found in the field.
e_{min} = the void ratio in the densest state that can be produced in the laboratory.

The relative densities of soils are frequently expressed as percentages Normal ranges of relative densities of sands according to Terzaghi are:

Relative Densities of Sands

Loose sand	D_d varies from 0 to 1/3 (or 0 to 33%)
Medium sand	D_d varies from 1/3 to 2/3 (or 33% to 66%)
Dense sand	D_d varies from 2/3 to 1.0 (or 66% to 100%)

The total unit weight of a partially saturated soil lying above the water table may be computed by the formula:

$$\gamma t = \frac{1 + \frac{w}{100}}{1 + e} G\gamma_w \tag{12}$$

in which γ_t = the unit weight of the partially saturated soil.
w = the water content of the soil expressed as a percentage.
G = the specific gravity of the soil.
γ_w = the unit weight of water.
e = the void ratio.

The buoyed unit weight of submerged soil or a soil below the water table may be computed by the formula.

$$\gamma_b = \frac{G - 1}{1 + e} \gamma_w \tag{13}$$

in which the terms have the same meanings as in eq. 12.

Soil mechanics strength tests frequently employed in the solution of foundation and soil stability problems are *direct shear tests*, *unconfined compression tests*, and *triaxial shear tests*.

Direct shear tests may be used to determine the strength in shear of either cohesive or non-cohesive soils. They are made in three types of apparatus: the single-shear box, the double-shear ring, and the single-shear torsional ring apparatus. Of these the oldest, the best-known, and most widely used is the single-shear box apparatus shown diagrammatically in Fig. 19. Specimens tested in such machines are usually from 2 to 4 in. square and from 1/2 to 1 in. thick. Occasionally specimens as big as 12 in. square and several inches

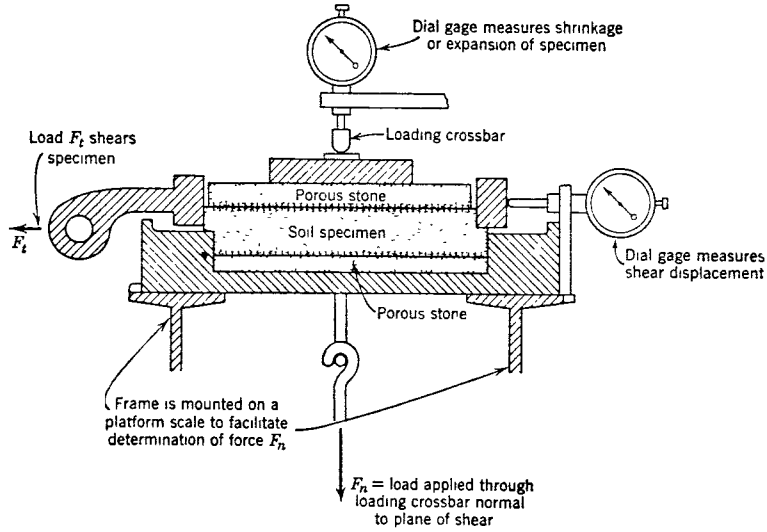


FIG. 19. Single-shear box apparatus

thick may be used if the soil to be tested contains large granular particles and a box shear apparatus of sufficient size is available. The specimen is mounted in a rectangular two-part box the movable, upper part of which slides over the fixed lower part in such manner that the specimen is sheared in the plane of demarkation between the upper and lower parts of the box. In an occasionally used variant of this arrangement the upper part of the shear box is held against horizontal but not vertical movement, while the lower part slides beneath it, but this detail does not affect the principle of operation of the apparatus. The upper box is fitted with a set of interchangeable tops and the lower box with similar interchangeable bottoms. Some of these have porous stone facings through which water can escape from the specimen during a test; others are impervious. Both types are roughened, serrated, toothed, or spiked to inhibit slippage of the specimen. In addition to the arrangements outlined above for permitting or preventing the escape of water from specimens under test, provision is also made for keeping them submerged The top of the upper part of the shear box is free to move vertically except for the support given it by the specimen. Vertical loads of known magnitude may be applied to the top of the shear box and thence to

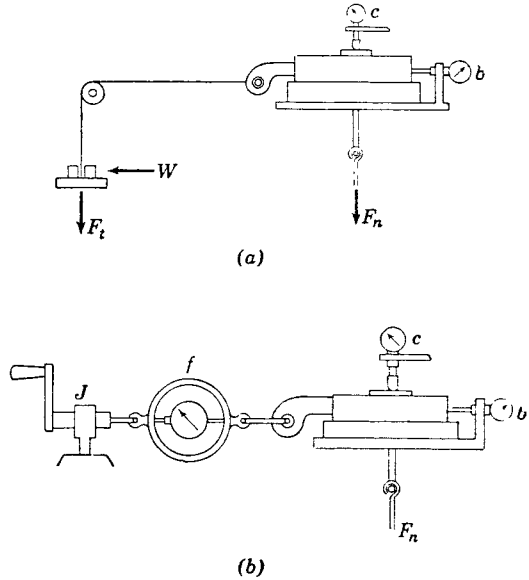


FIG 20 Direct shear test methods (a) Controlled stress test The load F_t is gradually increased by adding weights W Shearing and vertical displacements are measured by the dial gages b and c. (b) Controlled strain test The specimen is sheared at a predetermined rate by the screw jack device J which may be motor driven The shearing force F_t is measured by the proving ring f

the specimen. Variations in the horizontal shearing strength of a specimen produced by varying the vertical stress to which it is subjected may therefore be determined. A dial gage measures the vertical movement of the top of the upper box induced by shrinkage or expansion of the specimen and another similar gage the relative horizontal displacement of the upper and lower parts of the shear box.

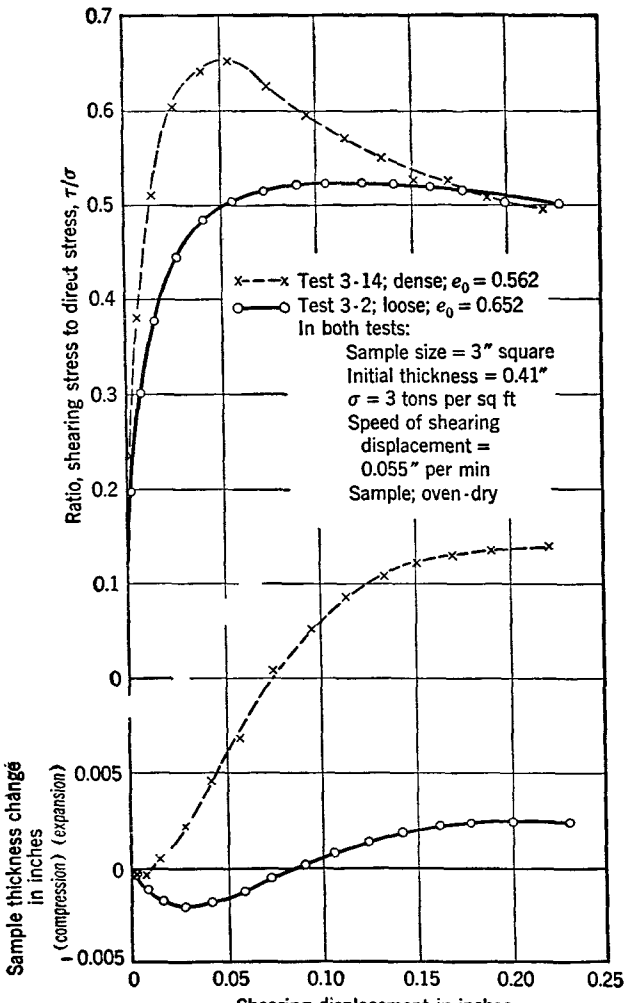


FIG. 21. Shearing strength plotted against shearing displacements. The results of direct shear tests made in a box shear apparatus are usually presented in this way when the controlled strain technique is used. From *Fundamentals of Soil Mechanics*, D. W. Taylor, John Wiley & Sons, 1948.

Two basic methods are used for applying load to specimens in box shear apparatus. In a *controlled stress test* the load is gradually applied by some such device as placing additional weights on a hanger or filling a counterweighted bucket, Fig. 20*a*, the shearing displacement and vertical displacement being read from the dial gages, *b* and *c*, and recorded as functions of the load F_t producing the shearing stress. In a *controlled strain test* the shearing displacement of the upper part of the shear box over the lower part is induced and controlled mechanically in such manner that it increases at a fixed and predetermined rate,

the resistance to displacement offered by the specimen, F_t, being measured by the dial of the proving ring, f, Fig. 20*b*.

Controlled strain tests are easier to make than controlled stress tests and have the advantage of giving data on the shearing strength of specimens after the peak or maximum value has been reached—see typical curves of shearing strengths of soils plotted against shearing displacements, Fig. 21.

The essentials of the direct double-shear ring apparatus are shown diagrammatically in Fig. 22. The specimen, which is cylindrical, is placed in a round shear box consisting of three rings or sections. The central section or ring is movable and can be slid out from between the two fixed ones by application of the force F_t in such manner that the specimen is sheared on the two planes SS. The apparatus may be so designed that the inner tube of a special sampler consists of rings that fit into the shearing device, thus making it unnecessary to transfer the soil from them. However, the undesirable wall thickness of such samplers probably nullifies the admitted advantage of avoiding transfer of the specimen. The ends of the fixed rings are closed with piston-like plugs which may, or may not, provide for drainage of the specimen and through which controlled forces F_n apply stress perpendicular to the planes of shear.

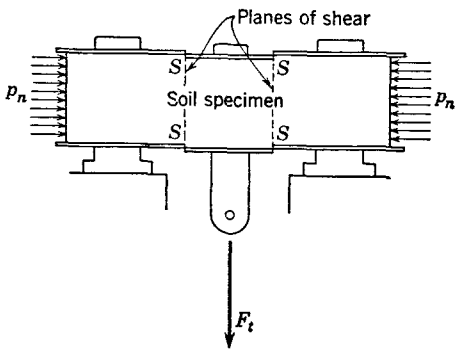


FIG 22. Double-shear ring apparatus The pressure p_n is normal to the plane of shear.

The single-shear torsional ring apparatus provides for the mounting of the specimen, in the shape of a thin "doughnut" of rectangular cross section, in a circular shear box, the upper part of which can be rotated by the application of a controlled moment. Rotation of the upper part of the box shears or "twists off" the specimen on the plane of demarkation between the lower and the upper parts of the shear box. A disadvantage of the apparatus is that the shearing displacements and stresses are not uniform, being greater at the outer than at the inner part of the annular surface of rupture. This complicates the evaluation of test data. Furthermore, it is almost impossible to fit anything but a disturbed or remolded specimen into the hollow ring-shaped shear box. An advantage of the apparatus is that there is no limit to the shearing displacement that can be produced. This makes the torsional ring apparatus very useful in some special research problems.

Three types of shear tests made in direct-shear apparatus are now commonly known by designations first suggested by Arthur Casagrande.

The *quick test* is an undrained test, quickly made without first permitting any consolidation of the specimen or escape of water from it under the load applied perpendicular to the plane of shear. This load will hereafter be called the "normal load" and designated by the symbol F_n. The intensity of the resulting normal stress $\frac{F_n}{A}$ will be designated by the symbol f_n, A being the area of the sheared surface. The difficulty of preventing the escape of water from a specimen in a single-shear box apparatus makes such equipment less desirable for a quick test than a double-shear ring apparatus or a triaxial shear apparatus described in a later paragraph.

The *consolidated quick test* is made after permitting drainage and full consolidation of the specimen to take place under the normal load. When the dial readings indicating vertical movement of the top of the shear box show that consolidation of the specimen under the normal load F_n is complete, which will usually be within less than twenty-four hours, the shearing force F_t is applied to produce rapid failure. This is done without removing or reducing the normal load F_n

The *slow test* is made after permitting complete consolidation of the specimen as in preparation for a consolidated quick test. The actual shearing of the specimen is then done so slowly under drained conditions that any excess pore pressure induced by shearing stresses and deformation will have time to dissipate and additional consolidation can occur before the specimen fails.

With saturated cohesive soils the *slow test* will generally give the highest value of shearing strength, the *quick test* the lowest value, and the *consolidated quick test* an intermediate value.

The results of direct shear tests are usually presented graphically. At least three tests should be made on three specimens from the same sample under varying normal loads F_n. The resulting shearing strengths s, which for each test equal $\frac{F_t}{A}$, are then plotted against their corresponding normal stresses f_n as indicated by the points 1, 2, and 3 in Fig 23 It is essential that the same units—kips per square foot, tons per square foot, or kilograms per square centimeter—be used for plotting s and f_n and that both be plotted to the same scale. The three points should then lie approximately on a straight line, the slope of which is the tangent of the angle of internal friction, ϕ, of the soil. In the case of an undisturbed clay or cohesive soil, the line will usually cross the YY axis, or f_n equals 0 line, at some such point as 4 in Fig. 23. The ordinate 0–4 is then considered the cohesion c in Coulomb's equation:

$$s = c + f_n \tan \phi \tag{14}$$

in which s = the shearing strength of the soil.
c = the cohesion or shearing strength when the normal stress f_n is 0.
f_n = the stress normal to the plane of shear.
ϕ = the angle of internal friction of the soil.

It should not, however, be assumed that the soil would actually have the shearing strength c if the normal stress were 0. The shearing strength of many soils "falls off" as the normal stress approaches 0 in some such way as indicated by the dotted line on Fig. 23.

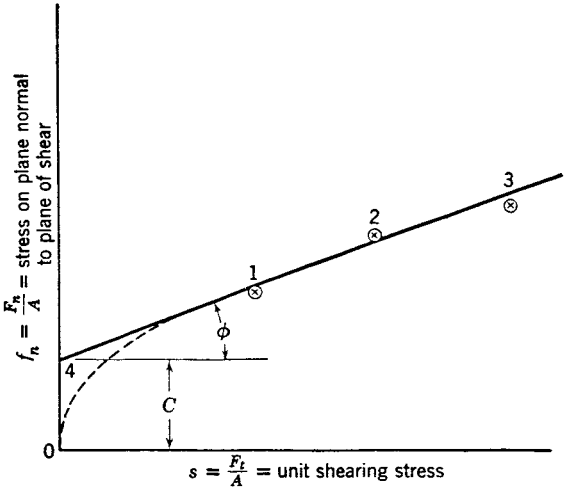


FIG 23 Plot of direct shear test of partially cohesive soil

In the case of clean noncohesive soils such as sands the line plotted through points representing the results of several shear tests made under varying values of f_n should pass through the origin of coordinates. This, however, is not a matter of great importance as direct shear tests of sands are rarely worth while because of the virtual impossibility of taking and mounting specimens in laboratory apparatus without disturbing them so seriously that subsequent tests are of little value.

The triaxial shear test is an indirect, but for many purposes a highly satisfactory, method for determining the shearing strength of a soil. The method is indirect because the triaxial machine measures the compressive strength of a short cylindrical column of soil subjected to a measured vertical load and a uniformly applied lateral supporting pressure. From the intensities of the axial and lateral stresses the shearing strength may be computed or, in accordance with more common practice, determined by means of a graphical method known as a Mohr diagram. The essentials of the triaxial machine are shown diagrammatically by Fig. 24. The cylindrical specimen is usually 1.4 in. in diameter and about 3 in. long, though some recently built machines take specimens 2.8 in. in diameter and as much as 7 in. long. The cylindrical surface of the specimen is covered by a very thin watertight rubber membrane which transmits to the soil specimen the pressure of the liquid in the closed chamber inside which the test is conducted. The specimen rests on a porous stone through which drainage into a burette may, or may not, be permitted by opening or closing the stopcocks g and f. In making a triaxial test the specimen is mounted in the machine, covered with the rubber membrane mentioned above, and subjected to the uniform and controlled pressure of a liquid, usually glycerine, which fills the pressure chamber inside which the test is made. The pressure of this liquid induces an equal uniform stress, designated σ_3, in the specimen. A measured vertical load is then applied axially to the specimen

by means of the piston rod protruding from the top of the test chamber. The stress induced on a transverse cross section of the specimen by this load is designated σ_1. The axial load is increased by either stress-controlled or strain-controlled methods, meanwhile keeping the liquid pressure and σ_3 constant, until the specimen fails, the axial load at failure and corresponding value of σ_1 being recorded. It is usual to repeat the test with several specimens prepared from the same sample of soil, but to vary the liquid pressure and corresponding soil stress σ_3 with each repetition.

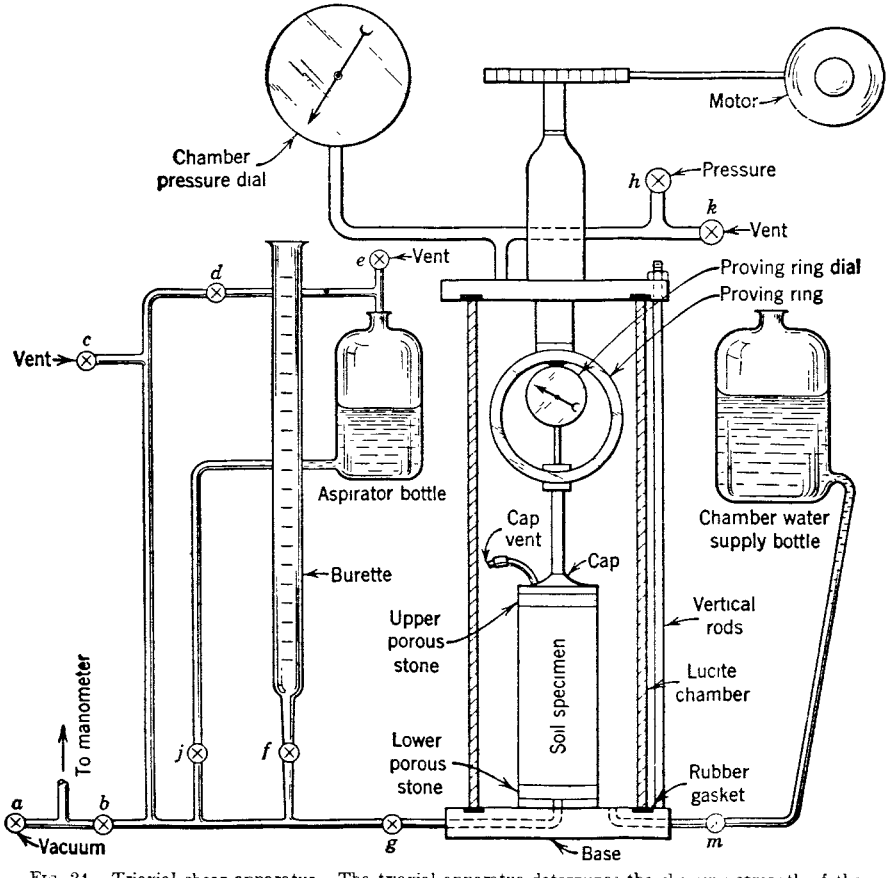


FIG. 24. Triaxial shear apparatus. The triaxial apparatus determines the shearing strength of the specimen indirectly. The apparatus determines the compressive strength of a cylindrical specimen covered with a thin rubber membrane through which pressure is applied by a surrounding liquid to give lateral support. Valves a–m control pressures and flow of liquids involved. From the compressive strength of the specimen when subjected to a known lateral supporting pressure its shearing strength may be determined by means of a Mohr diagram, Fig. 25. For detailed instructions for making test see refs. 2 and 17. From *Soil Testing for Engineers*, T. W. Lambe, John Wiley & Sons, 1951.

A Mohr diagram is preferable to analytical methods for determining the shearing strength of a soil from data yielded by a triaxial test. To draw a Mohr diagram σ_1 and σ_3 from each test are laid off as abscissas on the XX axis of a sheet of rectangular coordinate paper. A "Mohr circle" is then drawn for each test with its center midway between the plotted values of σ_1 and σ_3 and radius $\frac{\sigma_1 - \sigma_3}{2}$ as shown in Fig. 25. Line A-B drawn tangent to the series of circles should be nearly straight—theoretically it should be straight—and will have a slope such that the angle ϕ equals the angle of internal friction of the soil, and the intercept O-A is the cohesion c of eq. 14. A line P-F drawn in any of the Mohr circles represents the probable plane of failure of the soil in shear, α being the angle between this

plane and the horizontal. The line A-B is known as a Mohr envelope. Ordinates of points on this line represent the shearing strengths of the soil when subjected to normal stresses represented by corresponding abscissas. Points on the line therefore represent critical relationships in which failure impends. Points below the line represent safe relationships. The Mohr envelope for a purely cohesive soil should be horizontal and for a completely non-cohesive soil should cut the YY axis at the origin of coordinates.

A complete treatment of the theory underlying the Mohr diagram will be found in ref. 20.

The triaxial machine can be used for making *quick tests, consolidated quick tests*, and *slow tests*. For a quick test, the stopcock controlling drainage is kept closed and the load is applied rapidly to produce quick failure of the specimen. The triaxial machine is far superior to the direct shear apparatus for such tests because of the more satisfactory control of drainage. For a *consolidated quick test* full consolidation and drainage are permitted to take place with σ_1 equal to σ_3. The axial load σ_1 is then rapidly increased to produce quick failure. In a *slow test*, full consolidation is first permitted and the test is then conducted so slowly under drained conditions that any excess pore pressure induced by shearing deformations will have time to dissipate under additional consolidation.

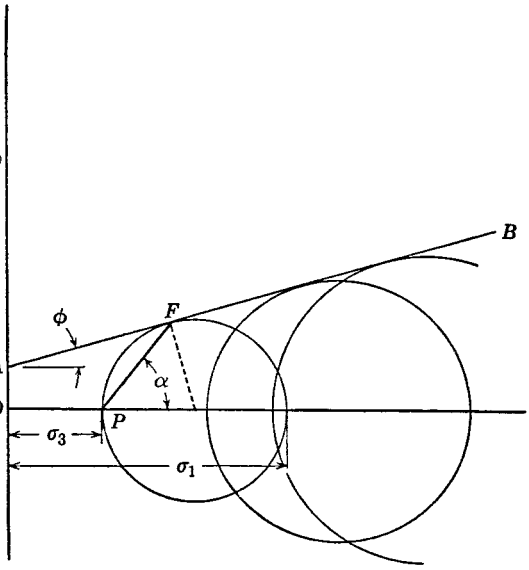


FIG. 25. Mohr circle plot of triaxial shear test.

In all drained tests made with the triaxial machine on saturated specimens of non-cohesive soil, the fluctuations of water level in the burette to which the drainage from the specimen is led give an accurate measure of any change in the volume of the specimen. The machine, therefore, affords the best means yet developed for estimating the important critical void ratio, e_c, of a saturated sand. This is the void ratio at which there is no tendency for a specimen either to increase or decrease in volume when sheared.

The critical void ratio is important because, if a fully saturated loose or inadequately compacted sand is subjected to a sudden shock or heavy shearing stress, there is danger that the grains may rearrange themselves in such a way that they temporarily lose contact with each other. Any load carried by the sand is then temporarily thrown onto the pore water, and the entire mass behaves as a liquid. During such a temporary liquidation a loose saturated sand may flow for a distance that makes the resulting "slide" a major disaster to an earth structure. The critical void ratio, e_c, of a sand can best be determined by means of a series of triaxial shear tests. The tests cannot be made satisfactorily in a box or direct shear machine. Three or more saturated samples of the sand are artificially prepared with varying void ratios e. At least one sample should be as loose as possible and one should be thoroughly compacted. A specimen from each sample is then tested in the triaxial machine, the confining pressure σ_3 being the same for all tests. The tests are made under drained conditions. The increase or decrease in the volume of the specimen is noted by observing the rise or fall of water in the burette. The increases or decreases in the volumes of the specimens when failure occurs are then plotted against their initial void ratios as shown in Fig. 26. Several similar series of tests are then made with different confining pressures σ_3, the results of each series being plotted as shown in Fig. 26. It should be noted from this figure that the critical void ratio of a sand changes if the confining pressure σ_3 changes. It follows that the critical void ratio of a sand in an earth structure is affected by the normal pressure to which the sand is subjected. In spite of the fact that critical void ratios vary in this way, their determination in the laboratory is sometimes justified in order

that sudden and disastrous flows—sometimes erroneously called slides—of saturated non-cohesive materials may be guarded against in the construction of large earth structures.

The unconfined compression test is another indirect method of determining the shearing strength of cohesive soil. The method is inapplicable to sands and non-cohesive materials. The specimen in the form of a short column, about the size of the specimens tested in a triaxial machine, is placed in a small, hand-operated, vertical testing machine in which it is loaded to failure by either stress-controlled or strain-controlled methods. The test may be looked upon as a triaxial test in which the confining pressure, σ_3, is 0. The shearing stress in the specimen is therefore theoretically at all times one-half the direct stress on its cross-sectional area. The ultimate shearing strength of the material is accordingly assumed to be one-half its ultimate strength in compression as determined by an unconfined compression test.

The unconfined compression test, because of its simplicity, is one of the most satisfactory techniques of soil mechanics. Portable machines are available in which these tests can be made, and it is therefore possible to conduct unconfined compression tests of specimens of cohesive soils in the field with a minimum risk of disturbance of the specimen and loss of moisture affecting the results. The unconfined compression test is therefore today the most important method of determining the shearing strength of a cohesive soil.

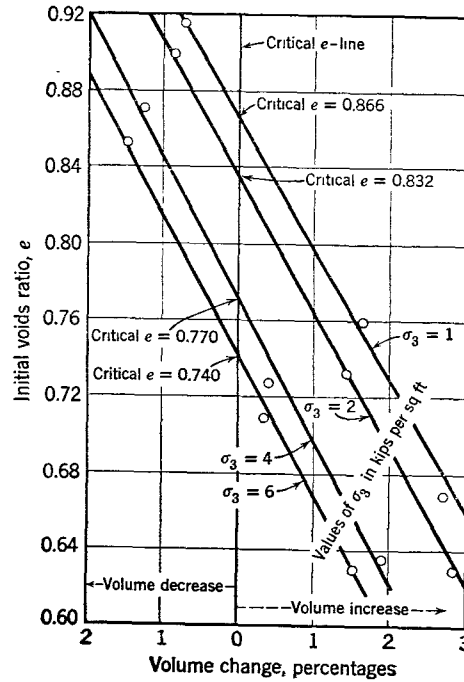


FIG. 26. Determination of critical void ratio of sands. By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G P. Tschebotarioff, copyright, 1951, McGraw-Hill Book Co

The **sensitivity** of a clay is an index of its loss of strength when remolded. The sensitivity may be determined by making an unconfined compressive test of an undisturbed specimen and repeating the test with a remolded specimen, taking care that neither specimen loses a significant quantity of water by evaporation during the tests. The sensitivity is then computed by the formula

$$S = \frac{q_u \text{ (undisturbed)}}{q_u \text{ (remolded)}} \tag{15}$$

in which S = the sensitivity—a dimensionless ratio.
q_u (undisturbed) = the unconfined compressive strength of an undisturbed specimen.
q_u (remolded) = the unconfined compressive strength of a remolded specimen.

Consolidation tests give data needed for estimating the compression and consequent settlement of saturated cohesive soils. The prospective rate of settlement also can be estimated with sufficient precision to warrant guarded consideration of the resulting figures. Satisfactory techniques have not been developed for making and interpreting consolidation tests of unsaturated soils and non-cohesive soils whether saturated or not.

The theory of the consolidation of saturated cohesive soils likens the behavior of the saturated soil to that of a spiral spring supporting a leaky piston in a liquid-filled cylinder, Fig. 27*a*. If load L is suddenly placed on the piston, the immediate effect is to raise the pressure of the fluid in the cylinder and induce its escape through the leak, Fig. 27*b*. During the early phases of the resulting "settlement" of the piston, the fluid pressure is relatively high and the portion of the load carried by the spring correspondingly small. As the leakage continues, however, and the descent or "settlement" of the piston progresses, the spring takes more and more of the load until, when stability is finally achieved, the

entire load is carried by the spring and there is no pressure on the fluid other than that due to its static head, Fig. 27*c*. If now an additional load *L*-2 be placed on the piston there will be a further settlement accompanied by further leakage of fluid, Fig. 27*d*, until a new condition of stability develops, Fig. 27*e*, with the spring again carrying the entire load. During each period of settlement the liquid pressure, rate of leakage, and consequent rate of settlement are at first relatively high. They become lower and finally very low as stability is approached.

Application of this theory to a practical problem of soil engineering involves determination of the maximum consolidation and consequent settlement that will be caused by a given load and the rate at which water will squeeze out of the soil and permit the settlement to occur. Or, to continue momentarily with the spring and leaky piston analogy, it is necessary to determine the load-carrying characteristics of the spring and the rate of leakage through the piston. This is done by testing soil specimens in a type of consolidometer

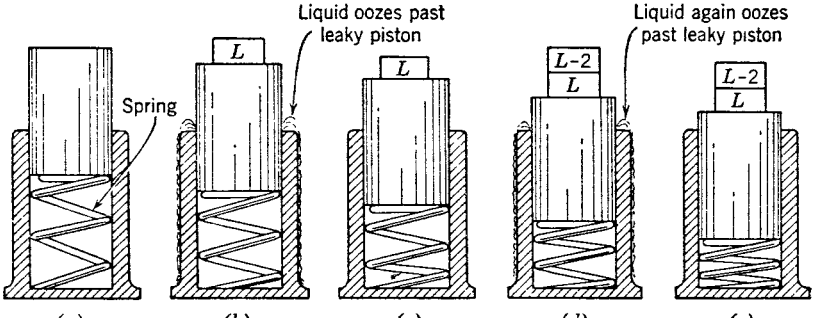


FIG. 27. Terzaghi consolidation theory. (*a*) Piston rests on spring and has no effect on pressure of liquid in cylinder. (*b*) Weight *L* carried largely by increase in liquid pressure until "settlement" of piston is complete. (*c*) Weight *L* and weight of piston carried by spring. Leakage and "settlement" complete. (*d*) Addition of second weight *L*-2 starts a new cycle of leakage and settlement. (*e*) Equilibrium again attained with all weight carried by spring.

developed by Dr. Arthur Casagrande. Figure 28 is a diagrammatic drawing of the apparatus. A short cylindrical specimen, usually 1 in. thick and from 2 1/2 to 4 1/2 in. in diameter is cut from an undisturbed sample of soil, mounted in a close-fitting metal container ring and squeezed between porous stones through which water can escape. The pressure is put on the specimen by means of a controlled load and the resulting consolidation measured by a dial gage reading to 0.0001 in. The load is applied in a series of increments each of which doubles the previous compressive stress in the specimen. The series of compressive stresses used in making consolidation tests has not been standardized but loads causing stresses of 1/4, 1/2, 1, 2, 4, 8, and 16 tons per sq ft or kg per sq cm are widely used. By a fortunate coincidence a stress of 1 ton per sq ft so nearly equals 1 kg per sq cm that the units may be used interchangeably without a conversion factor.

The maximum load applied in a consolidation test should produce a compressive stress in the specimen at least equaling the maximum anticipated compressive stress in the soil due to both natural overburden and superimposed load. As the test progresses, each load increment is added without removing the previously applied load and each increment is kept on long enough to cause practically the maximum consolidation it is capable of producing. Twenty-four hours is generally sufficient for the full consolidation of 1-in. specimens under each load increment. During the 24-hr application of each load increment, the progress or rate of consolidation should be observed and recorded by taking readings of the consolidometer dial gage at some such series of times as: Immediately after the application of the increment and then 15 sec, 30 sec, 1 minute, 2 minutes, 5 minutes, 15 minutes, 30 minutes, 1 hr, 2 hr, 5 hr, and 24 hr after the initial application of the increment. A time-compression curve, of which Fig. 29 is a typical example, should be plotted on semilogarithmic paper as consolidation of the specimen under each load increment progresses. Abnormalities in the shapes of these curves will reveal any tendency of the consolidometer to "stick" or otherwise work badly. Furthermore, one or more of the time-compression curves may be required as a source of data for computing the coefficient of permeability of the soil as will be explained later.

After completion of consolidation under the final load increment the specimen is unloaded in decrements which may be considerably larger than the increments used in the

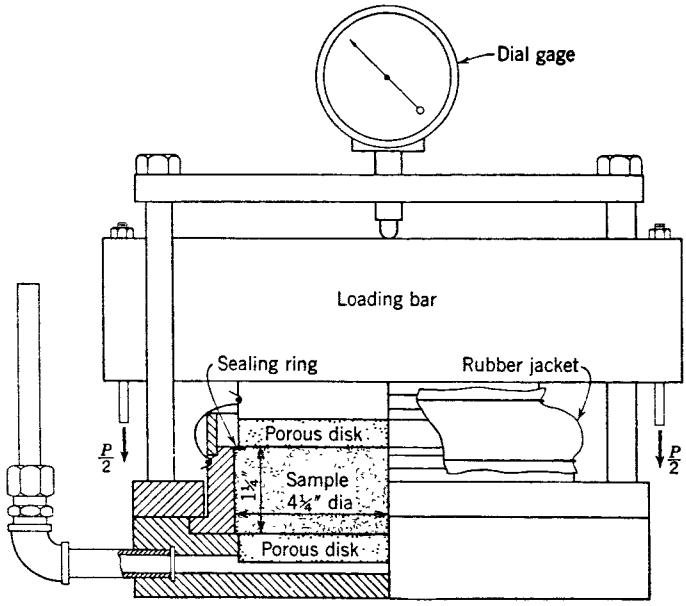


FIG 28 Consolidometer The instrument consolidates a disk-shaped soil specimen (in this case 4 1/4 in in diameter and 1 1/4 in thick) by squeezing the water out of it The specimen is cut from an undisturbed sample and must be saturated cohesive, or partially cohesive, soil It is mounted between porous stone disks through which the water escapes as progressively increasing loads are applied to the upper stone by the loading bar, which is forced down by the controlled forces $P/2$ The progressive decrease in the thickness of the disk is measured by the dial gage Estimates of anticipated settlements of clays and cohesive soils are based on tests made with consolidometers A standpipe and pressure line can be attached to the bottom drainage line to permit use of the instrument as an upward-flow permeameter From *Fundamentals of Soil Mechanics*, D W. Taylor, John Wiley & Sons, 1948.

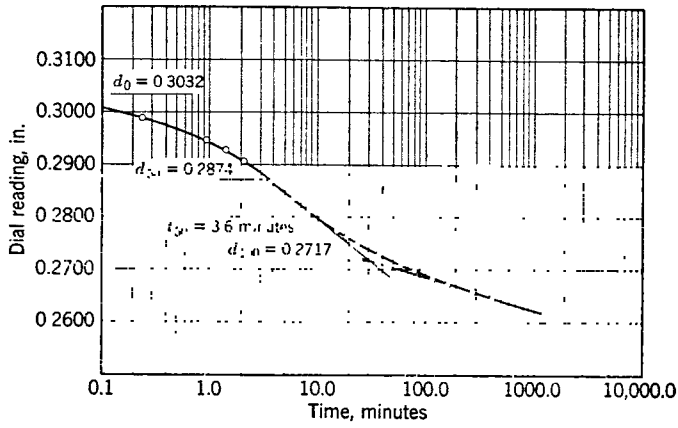


FIG 29. Consolidometer dial reading-time curve The shape of these curves tells whether the consolidometer is working normally The curve for the load increment most nearly approximating the anticipated increase in load under the foundation may be needed for determining the coefficient of permeability k of the soil See Fig 35 for method of determining d_0, t_{50}, and d_{100} and their significance. From *Soil Testing for Engineers*, T. W Lambe, John Wiley & Sons, 1951.

loading phases of the test. For example: If a maximum load of 16 tons per sq ft has been placed on a specimen by the increments previously suggested, the first decrement might reduce it to 2 tons, the second to 1/2 ton, and the third remove it entirely. The specimen should be allowed to expand for at least 24 hr after the removal of each load decrement and should be supplied with all the water it can take up through the porous stones of the consolidometer during each expansion period since expansion can develop fully only as the result of such absorption. It is generally unnecessary to take dial readings at stated intervals and plot time-expansion curves during the periods of expansion following the removal of each load decrement. The dial should be read and the reading recorded, however, at the beginning and end of each expansion period.

Immediately after expansion under no load is complete the specimen is removed, weighed, dried to constant weight, and again weighed. The specific gravity of the soil is determined if this has not already been done. The height in inches of the solid content of the specimen, the height of the water content at the end of the test, the void ratio at the end of the test, the change in void ratio occurring during each period of expansion and compression, and the void ratio at the beginning of the test are computed by the formulas:

$$h_s = \frac{W_s}{AG \times 2.54}$$

$$_{wf} = \frac{W_{wf}}{A \times 1.00 \times 2.54}$$

$$e_f = \frac{h_{wf}}{h_s}$$

$$\Delta e = \frac{\Delta h}{h_s}$$

$$e_i = \frac{h_i - h_s}{h_s}$$

in which h_s = the height of the solid content of the specimen in inches.
W_s = the dry weight in grams of the solids in the specimen.
A = the cross-sectional area of the specimen in cm^2.
G = the specific gravity of the soil.
h_{wf} = the final height of the water content of the specimen at the end of the test in inches.
W_{wf} = the final weight of water in the specimen at the end of the test.
e_f = the final void ratio at the end of the test.
Δe = the change in void ratio produced by a decrement or increment of load. The value of e is computed for each decrement and increment.
Δh = change in thickness of specimen during period of application of a load increment as indicated by dial readings.
h_i = the initial thickness of the specimen—usually assumed to be the height of the container ring.
e_i = the initial void ratio.

Subtracting Δe for the last decrement from e_f gives the value of e before the decrement was removed. Successive applications to this value of e of the appropriate values of Δe for the various decrements and increments of loading, subtracting for decrements and adding for increments, gives the value of e at the beginning of each loading period and finally e_i, its value at the beginning of the test. This should check the value of e_i given directly by the above formula.

As a consolidation test progresses, the initial dial reading and the readings at the ends of the application periods of the load increments should be plotted as ordinates against corresponding soil pressures as abscissas on semilogarithmic paper to produce a dial reading-pressure curve of which Fig. 30 is a typical example. After completion of the test the void ratios should be similarly plotted to produce a void ratio-pressure curve, Fig. 31. The similarity of the curves is apparent. The justification for plotting both is that the dial reading-pressure curve, which is plotted as the test progresses, gives assurance by the normality of its shape that the consolidometer is functioning properly; and the void ratio-pressure curve furnishes data needed for estimating settlements by widely used methods. However, the dial reading-pressure curve, as will be shown later, may form the basis of a reasonably accurate preliminary settlement estimate which can be made without drawing the void ratio-pressure curve.

The most probable value of preconsolidation pressure under which the soil has in the past been compressed and consolidated may be estimated from the void ratio-pressure

curve by Casagrande's widely used empirical method. The method, which is graphical, is illustrated by Fig. 32 and described in the legend of the figure. The construction will also be found drawn to a much smaller scale on Figs. 30 and 31. If the preconsolidation pressure

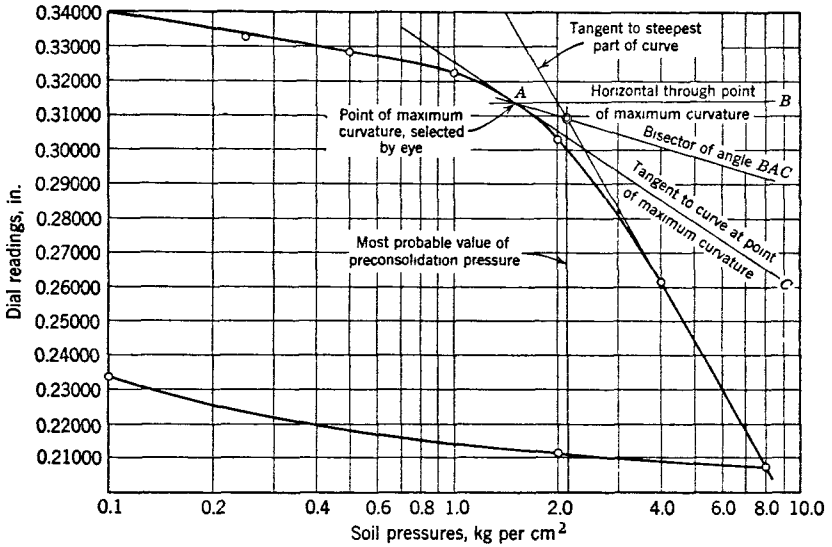


FIG. 30. Dial reading-pressure curve. Casagrande's graphical method of estimating the most probable value of the preconsolidation pressure is shown (See Fig 32 also) Basic data from *Soil Testing for Engineers*, T. W Lambe, John Wiley & Sons, 1951

is materially less than the pressure due to existing overburden, the soil may still be consolidating under its weight. The resulting settlement, it should be noted, will be along the steep part of the consolidation curves (Figs. 30 and 31) and will be relatively large. If, on the other hand, the preconsolidation pressure greatly exceeds the pressure due to existing

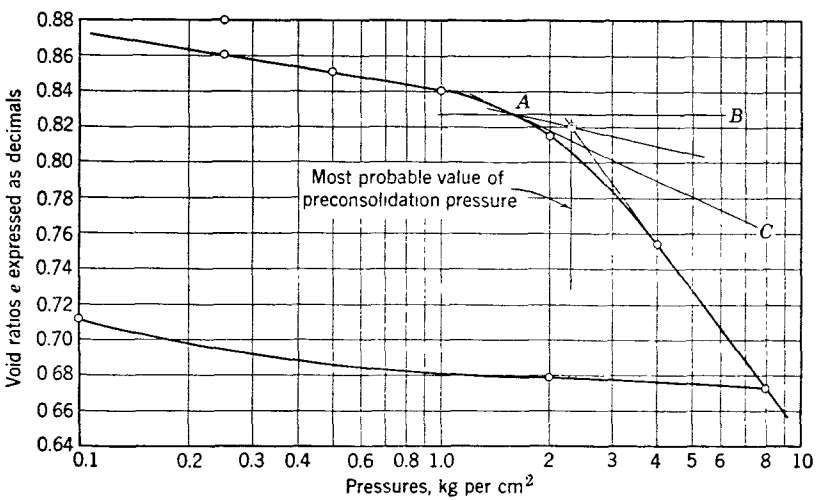


FIG 31 Void ratio-pressure curve. Standard procedure for estimating anticipated settlements of saturated cohesive soils makes use of such curves (See Figs 30 and 32 for explanation of graphical method of determining the most probable value of preconsolidation pressure) Basic data from *Soil Testing for Engineers*, T. W Lambe, John Wiley & Sons, 1951

overburden, and especially if it exceeds the sum of the overburden pressure and the pressure due to anticipated loads, future settlement will be along the flat upper part of the consolidation curve and may be slight and possibly negligible. For this reason the preconsolidation pressure is considered by some authorities to be the most important single item of information yielded by a consolidation test. It is important to bear in mind that the test is extremely sensitive to disturbance of the sample or specimen. Disturbed samples will yield "low" preconsolidation pressures and thus cause undue apprehension as to the probability of excessive settlements. For this reason preconsolidation pressures obtained by testing specimens cut from samples of small diameter, such as may have been taken with a sampler 2 1/2 in. in diameter or smaller, are of doubtful value and should be considered nothing more than rough approximations.

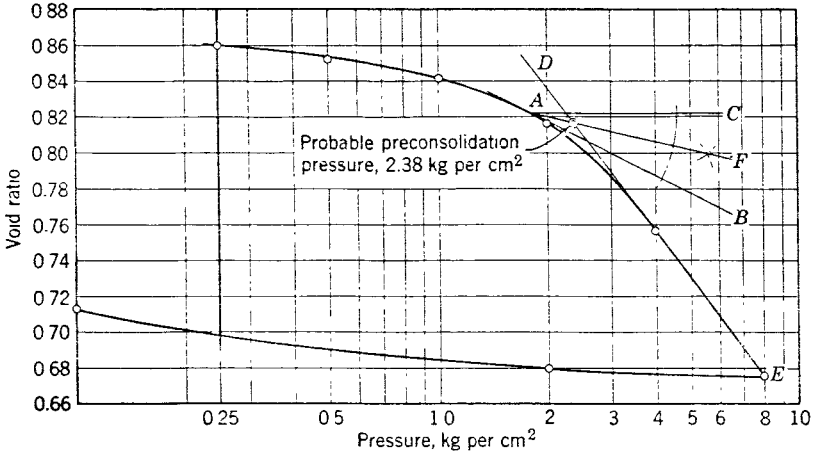


FIG. 32. Casagrande's method of estimating the probable preconsolidation pressure. The point of maximum curvature A of the void ratio-pressure curve is located by eye. At this point a tangent AB to the curve and a horizontal line AC are drawn. The angle CAB is bisected. The intersection of the bisector AF with a tangent DE drawn to the lower straight part of the curve determines the probable maximum preconsolidation pressure.

Preconsolidation pressures exceeding those explicable by a geological history of glaciation or erosion are not necessarily incorrect. Surface drying, and especially repeated wetting and drying in the geological history of a cohesive soil, may induce heavy internal compressive stresses by capillary action and consolidate the material as effectively as a superimposed load. Similarly, chemical action in a soil containing cementatious phosphates and iron oxides may induce internal compressive stresses considerably greater than those to which the soil has been subjected by the weight of overburden.

The total settlement to be anticipated from an estimated increase in the pressure at the mid-depth of a stratum of cohesive soil is computed by the formula

$$S = \frac{(e_1 - e_2)}{(1 + e_1)} 2H \tag{16}$$

in which S = the total anticipated settlement in feet

H = 1/2 the thickness of the stratum of cohesive soil in feet.

e_1 = the void ratio from the void ratio-pressure curve at the estimated pressure due to overburden at the mid-depth of the stratum

e_2 = the void ratio from the curve at the anticipated pressure at the mid-depth of the stratum.

The settlement given by the above formula is the total settlement to be anticipated from the assumed increase in pressure. Methods of estimating pressure increases will be dealt with later. It should be noted that water may squeeze out of a thick stratum of impervious clay so slowly that the rate of settlement during the final stages of consolidation may be extremely slow, so slow in fact that only 90% or less of the total settlement S will occur within the probable useful life of even a so-called permanent structure.

Before attempting to estimate the rate at which settlement will occur, it is necessary to determine the modulus of volume change, m_v', and the coefficient of permeability, k, of the soil.

The modulus of volume change is computed by the formula:

$$m_v' = \frac{(e_1 - e_2)}{(p_2 - p_1)(1 + e_1)} \tag{17}$$

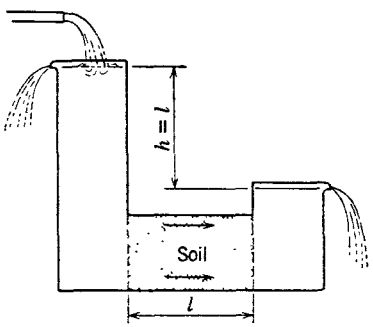
in which m_v' = the modulus of volume change in cm^2 per kg or ft^2 per ton.

e_1 and e_2 have the same significance as in formula (16) for the settlement S.

p_2 = the anticipated pressure at the mid-depth of the cohesive stratum in kg per cm^2 or tons per ft^2.

p_1 = the estimated existing pressure at the mid-depth of the stratum due to oberburden in same units.

Estimation of the time in which any specified percentage of the total anticipated settlement will take place demands determination of the coefficient of permeability k of the soil. This is the imaginary average velocity of the flow of water through the gross area, voids



PERMEABILITY AND DRAINAGE CHARACTERISTICS OF SOILS

Coefficient of Permeability k in cm per sec (log scale)

	10^2	10^1	1.0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
Drainage	Good						Poor		Practically Impervious			
Soil types	Clean gravel		Clean sands, clean sand and gravel mixtures			Very fine sands, organic and inorganic silts, mixtures of sand silt and clay, glacial till, stratified clay deposits, etc.				"Impervious" soils, e g , homogeneous clays below zone of weathering		
					"Impervious" soils modified by effects of vegetation and weathering							
Direct determination of k	Direct testing of soil in its original position—pumping tests. Reliable if properly conducted Considerable experience required											
	Constant-head permeameter. Little experience required											
Indirect determination of k			Falling-head permeameter Reliable Little experience required			Falling-head permeameter. Unreliable. Much experience required			Falling-head permeameter. Fairly reliable Considerable experience necessary			
	Computation from grain-size distribution Applicable only to clean cohesionless sands and gravels									Computation based on results of consolidation tests Reliable Considerable experience required		

After A. Casagrande and R. E. Fadum

FIG. 33 Methods of determining coefficient of permeability The diagram above the table illustrates the significance of the coefficient. The coefficient of permeability of a soil may be considered the imaginary average velocity of flow of water through the gross area of the material, voids plus solids, induced by a hydraulic gradient of unity The flow is considered laminar, and velocities are directly proportional to the hydraulic gradient Table from *Soil Mechanics in Engineering Practice*, by K. Terzaghi and R B Peck, John Wiley & Sons, 1948

plus solids, induced by a hydraulic gradient of unity, the flow being non-turbulent and the loss of head directly proportional to the velocity. Figure 33 illustrates the meaning of the coefficient, shows the normal range of its values for common classes of soil, and indicate. the methods generally considered practical for determining k for each class of materials The figure gives k in cm per sec. As the viscosity of water varies considerably with temperature changes, the results of permeability tests are affected by the temperature of the water with which they are made. Before comparing or using values of k as determined by laboratory tests the values should be multiplied by specific viscosities obtained from Fig. 34 to convert them to values which would have been secured with water at the generally accepted standard temperature of 20°C. In making comparisons of coefficients of permeability cognizance should be taken of the wide variety of units in which k may be given: cm per sec, ft per minute, ft per hr, and ft per year being in more or less common use by different authorities.

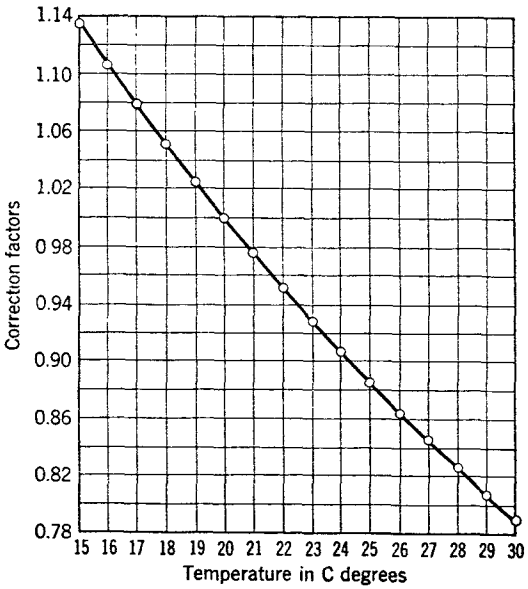


FIG 34 Correction factors to apply to values of k obtained with water at a temperature other than 20°C The viscosity of water varies with changes of temperature Values of k obtained with a permeameter should be corrected by a factor obtained by entering the diagram above with the temperature of the water actually used The corrected permeability will be that which would have been obtained had the water been at the standard temperature of 20°C From *Soil Testing for Engineers*, T W Lambe, John Wiley & Sons, 1951

In dealing with cohesive soils, the estimated settlements of which are based on consolidation tests, decision as to the method to be used for determining the value of k must be based on an advance appraisal of the characteristic of the soil which is to be determined—its permeability. In dealing with clays and silty clays having coefficients of permeability less than 6.0×10^{-6} cm per minute, k should be determined by an indirect method based on data taken from one of the time-compression curves. As a check the coefficient should also be determined directly by using the consolidometer as a falling head permeameter in a manner to be described subsequently. In dealing with more pervious soils, the indirect method fails and the coefficient must be determined by use of the consolidometer as a permeameter.

If the coefficient of permeability is to be determined by the first of these methods, the time-compression curve should be selected which resulted from the pressure increment most nearly approximating the anticipated increase in pressure at the mid-depth of the stratum from p_1 to p_2. The compression recorded on the curve is the total compression of the specimen, due not only to the expulsion of water from voids in the soil, but also to a slight squeezing out of the soil between the loaded filter plate and the container ring, the possible presence of small quantities of gas in the specimen, and the so-called residual or secondary compression of the soil particles It is necessary to estimate the amount of the total compression due to these causes, eliminate it from consideration, and deal with the compression due to expulsion of water only in order to compute the value of k

In many cases this may be done by a graphical method known as the *logarithm of time fitting method* or *log method* This method, which is illustrated by Fig. 35 and explained in the legend of the figure, results in two horizontal lines being drawn on the time-compression curve, the upper one showing what the dial should have read at zero compression and the second one what it should have read when 100% consolidation had been achieved if all instrumental errors and secondary effects had been eliminated.

The log fitting method is, however, unsatisfactory with soils having large secondary compressions and in consequence yielding time-compression curves the lower end of which have no "straight" portion to which a satisfactory tangent can be drawn. In such cases it

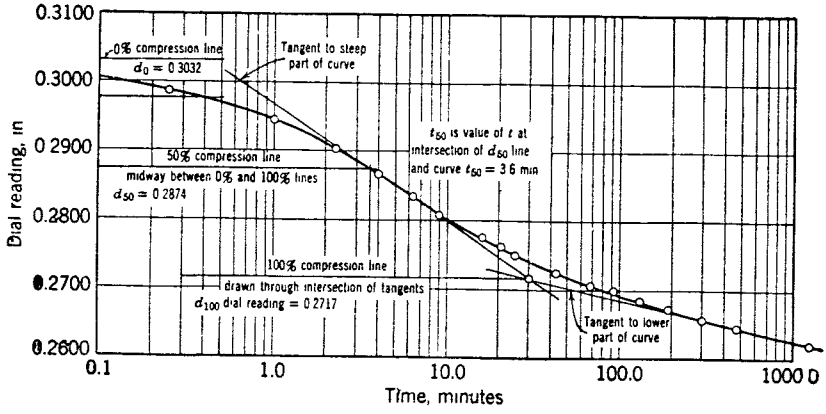


FIG 35 Dial reading-pressure showing logarithm of time fitting method of estimating time required for 50% consolidation. Zero per cent compression line is drawn by laying off above a point in the neighborhood of 0 1 minutes an ordinate equal to that between this point and one at a time four times as great

is advisable to redraw the time-compression curve on ordinary graph paper with abscissas proportional to the square roots of the times in minutes. Figure 36 shows such a "square root of time" plot of a time-compression curve and the graphical construction used to locate the zero compression and 100% compression lines on it.

Having located these lines by either the log fitting or square root fitting method the 50% compression line, midway between the zero and 100% lines, is drawn and the abscissa of its intersection with the time-compression curve noted. This is t_{50}, or the time in minutes

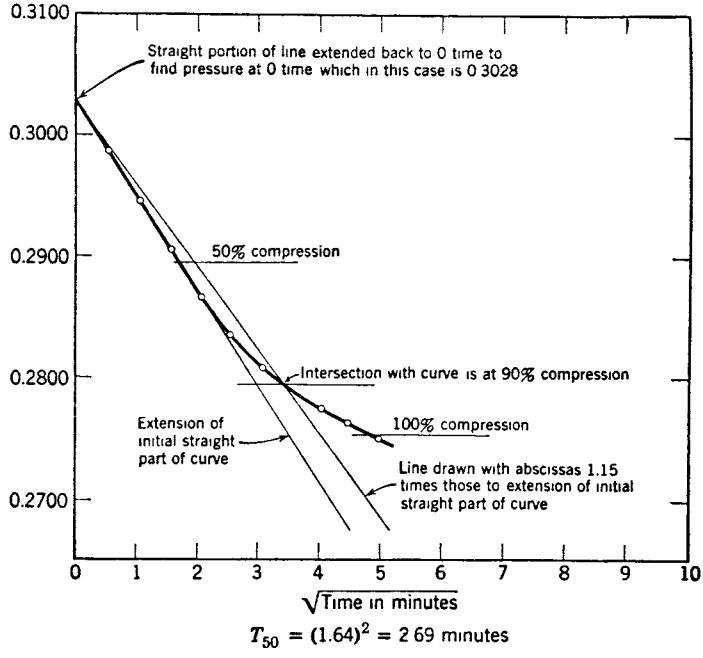


FIG. 36. Square root of time fitting method By the construction indicated the square root of the time at 90% consolidation may be estimated.

required to effect a 50% compression of the specimen. With this factor established the coefficient of permeability is computed by the formula:

$$k = 0.000323 \frac{m_v'}{t_{50}} h_0^2 \tag{18}$$

in which k = the coefficient of permeability in cm per minute.
m_v' = the modulus of volume change in cm² per kg or ft² per ton as previously determined.
t_{50} = the time in minutes required to effect a 50% consolidation of the specimen as determined from the time-compression curve.
h_0 = the thickness in inches of the specimen at the zero point of the compression curve as determined by its initial thickness and subsequent dial readings.

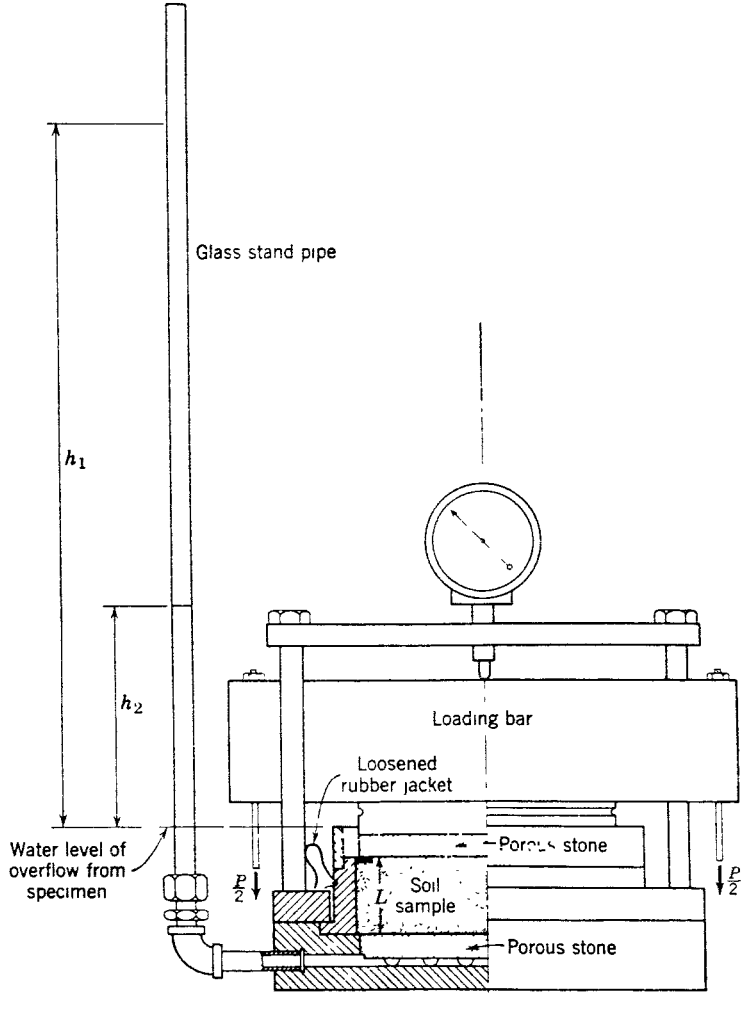


FIG 37. Consolidometer rigged as a falling head permeameter At the end of one of the 24-hour applications of a load increment and without removing the load, this test should be made to determine the coefficient of permeability k of the soil if the coefficient materially exceeds 6.0×10^{-6} or if there is reason to anticipate that it will exceed this figure

In case the coefficient of permeability materially exceeds 6.0×10^{-6} cm per minute or if there is reason to expect that it will exceed this figure, the consolidometer should be rigged as a falling head permeameter, as indicated in Fig. 37, and a direct determination of the permeability made after the end of one of the 24-hr applications of a load increment. The test should be made without removing the load from the consolidometer. The standpipe should be filled with distilled water from which air has been removed by boiling and the time in minutes, t, noted in which the water surface drops from h_1 to h_2. The coefficient of permeability is then computed by the formula:

$$k = 2.3 \frac{La}{At} \log_{10} \frac{h_1}{h_2} \tag{19}$$

in which k = the coefficient of permeability in cm per minute.
L = the thickness in cm of the specimen at the time of the test as determined by its initial thickness and subsequent dial readings.
a = the cross-sectional area of the standpipe in cm^2
A = the cross-sectional area of the specimen in cm^2
t = the time in minutes in which the water surface in the standpipe drops from h_1 to h_2
h_1 = the head of water in the standpipe causing flow through the specimens (see Fig. 37) at the commencement of the time interval measured by the interval t.
h_2 = the head in cm as measured above at the end of the time interval t.

Having computed the modulus of volume change m_v' and the permeability k by the methods outlined above, the time required for the accomplishment of any specific percentage of the anticipated total settlement is computed by the formula:

$$t = \frac{m_v'}{k} 0.929 H^2 T \tag{20}$$

in which t = the time in minutes required to accomplish a specified percentage of the total anticipated settlement S
m_v' = the modulus of volume change in cm^2 per kg or ft^2.
k = the coefficient of permeability of the soil in cm per minute.
H = one-half the depth in feet of the stratum of saturated cohesive soil under consideration if pervious strata provide a means for the escape of expelled water from both the upper and lower faces of the cohesive stratum. If there is drainage from but one face of the cohesive stratum H equals its depth in feet.
T = the time factor, a dimensionless quantity obtained from a graph, Fig. 38, by methods described in the legend of the figure.

A simplified procedure, eliminating the void ratio-pressure curve and permitting preliminary settlement estimates to be made as a consolidation test progresses, gives results that do not differ seriously from those obtained by the previously described and generally accepted method. In using the simplified procedure the preconsolidation pressure is estimated by a direct application of Casagrande's method to the dial reading-pressure curve as indicated on Fig. 30.

The anticipated total settlement S is then computed by simple proportion, the ratio between S and the thickness of the stratum of cohesive soil being the same as that between the consolidation of the specimen under an appropriate increment of pressure and its thickness. This relationship may be reduced to the formula.

$$S = \frac{2H(d_1 - d_2)}{h_s - (d_s - d_1)} \tag{21}$$

in which S = the anticipated total settlement in feet.
H = one-half the thickness of the stratum of cohesive soil in feet.
d_1 = the dial reading in 1/10,000 in. from the curve at the pressure at the mid-depth of the stratum due to overburden.
d_2 = the dial reading corresponding to the anticipated total pressure at the mid-depth of the stratum.
h_s = the thickness of the specimen in inches at the beginning of the test
d_s = the dial reading at the beginning of the test.

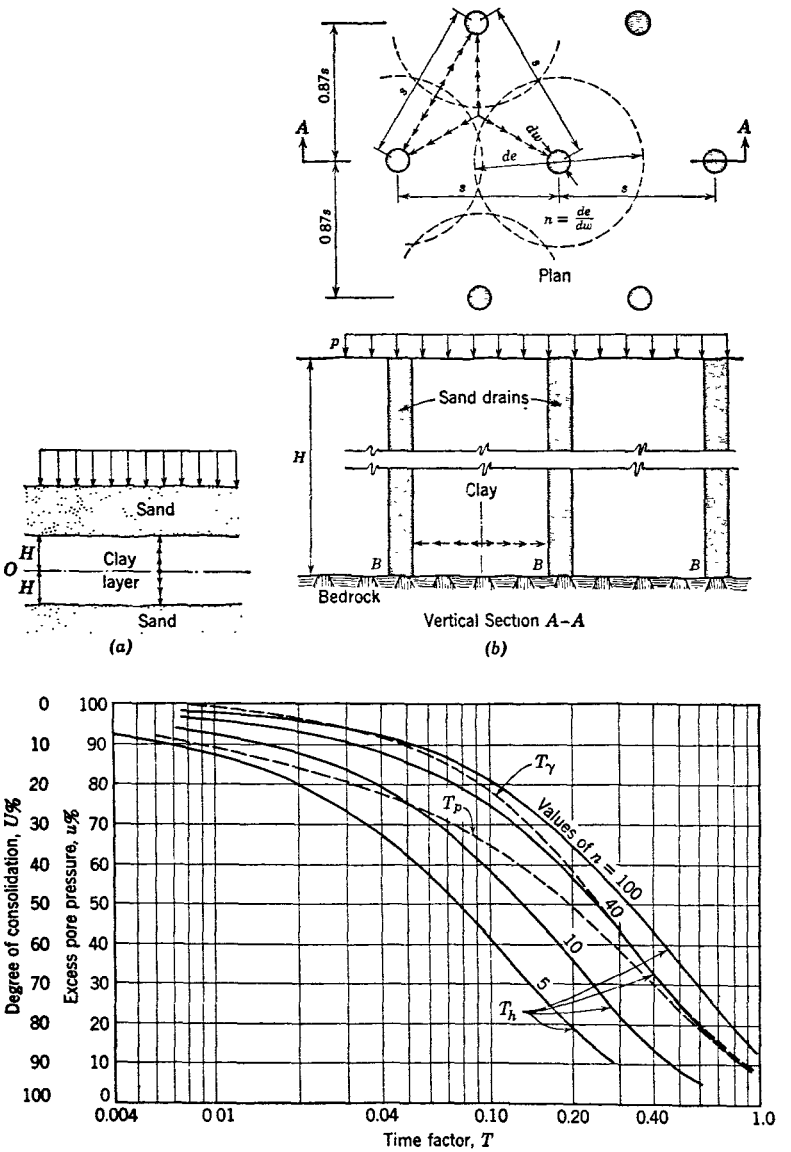


FIG 38 Time factors T for computing settlements The curve T_p is applicable to the usual case of a layer of clay with strata of sand above and below it See diagram *a* above Curve T_γ applies to a bed of clay consolidated under its own weight with no drainage at the bottom and surface drainage at the top. The four curves T_h apply to a stratum of clay in which sand drains have been placed, see diagram *b* By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G. P. Tschebotarioff, copyright, 1951, McGraw-Hill Book Co.

The modulus of volume change is then, like the settlement, computed from data obtained directly from the dial reading-pressure curve, the formula being:

$$m_v' = \frac{(d_1 - d_2)}{[p_2 - p_1][h_1 - (d_1 - d_1)]} \tag{22}$$

in which m_v' = the modulus of volume change in cm² per kg or ft² per ton.
p_2 = the anticipated pressure at the mid-depth of the stratum in kg per cm² or tons per ft².
p_1 = the existing pressure at the mid-depth of the stratum due to overburden in kg per cm² or tons per ft².
d_2, d_1, h_1, and d_1 have the same significance as in the formula for S.

The coefficient of permeability k is determined by one of the two methods previously described and the time t required to effect a given percentage of the total settlement is similarly computed by formula 20.

Constant head permeameters are used for determining the coefficients of permeability of sands and other permeable soils. A simple form of constant head permeameter is shown

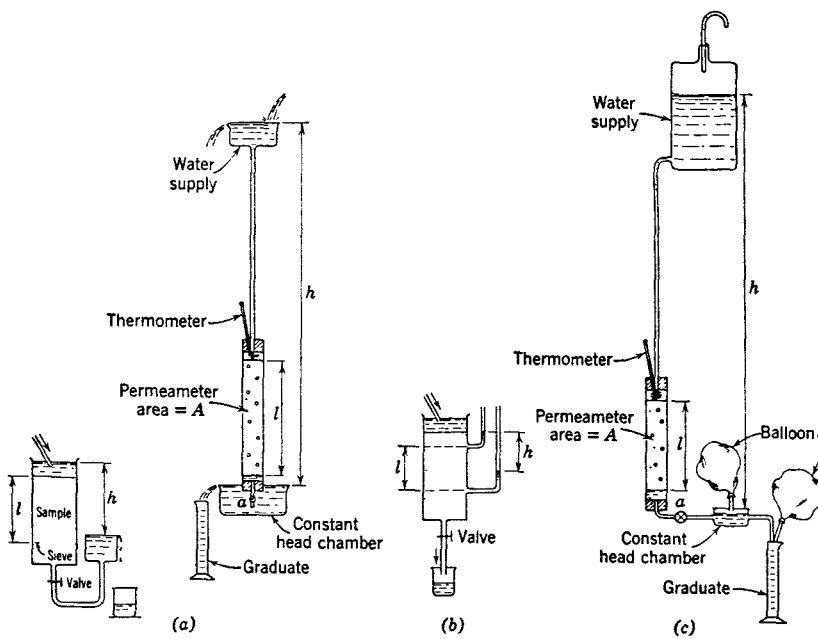


FIG 39. Constant head permeameter. (a) Two simple types of constant head permeameters (b) Constant head permeameter modified to eliminate error caused by silting of the pores at the upper surface of the specimen (c) Constant head permeameter rigged to minimize errors due to evaporation when testing specimen of low permeability From *Soil Mechanics in Engineering Practice*, K Terzaghi and R B Peck, John Wiley & Sons, 1948, and *Soil Testing for Engineers*, T. W. Lambe, John Wiley & Sons, 1951.

diagrammatically in Fig. 39*a*, the apparatus being essentially a small filter in which the specimen is the filter bed. The quantity of water passing through the specimen in a definite time interval is measured and the value of k computed by the formula:

$$k = \frac{Qlc_v}{Ath} \tag{23}$$

in which k = the coefficient of permeability in cm per minute.
Q = the quantity of water passing through the permeameter during the test in cc.
l = the length of the specimen in cm through which the known and constant loss of head occurs.
A = the cross-sectional area of the specimen in cm².

t = the time in minutes in which the quantity of water Q passes through the specimen
h = the head in cm lost in the length l.
c_t = a correction factor obtained from Fig. 34.

Common sources of error in values of k determined with constant head permeameters and recognized methods of minimizing these errors are

(a) Air or gas dissolved in the water may form bubbles in the intergranular spaces of the specimen and materially reduce its permeability. Tests should therefore be made with distilled water which has been deaerated by boiling. Waste of this specially prepared water may be minimized by the arrangement shown in Fig. 39c, but it is essential that the supply bottle be of such large diameter that the head on the permeameter remains practically constant throughout a test.

(b) Fine material may collect on the upper surface of the specimen in a thin layer the permeability of which is relatively low. Measurement of the head loss between two piezometers installed in the body of the permeameter, the distance between them being considered l, as indicated in Fig. 39b, practically eliminates this source of error.

(c) The loss of water by evaporation from the graduate in which Q is measured may cause serious error if tests of relatively impermeable soils, through which the percolation is slight, are attempted in a dry atmosphere. Such loss may be minimized by inclosing the discharge end of the permeameter and using rubber balloons to permit pressure equalization as shown in Fig. 39c, but it is generally preferable to use a falling head permeameter.

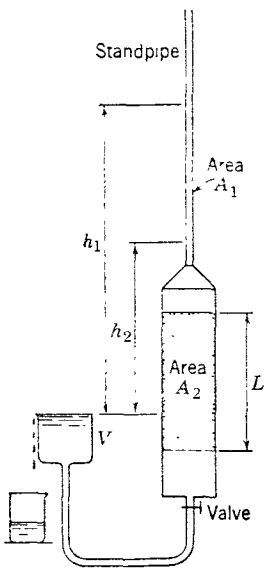


Fig. 40. Falling head permeameter. From *Soil Mechanics in Engineering Practice*, K. Terzaghi and R. B. Peck, John Wiley & Sons, 1948.

Falling head permeameters of the type shown diagrammatically in Fig. 40 may be used to determine the value of k for soils of a wide range of permeability if fitted with a number of interchangeable standpipes, the diameters of which vary from that of the permeameter itself to one-tenth or less of this diameter. Formula 19 is used for computing k. Though a properly proportioned falling head permeameter in the hands of an experienced technician may be used to determine the coefficient of permeability of soils as impervious as clays, it is generally preferable, when dealing with impervious soils such as these, to determine their coefficients of permeability indirectly from data yielded by consolidations tests as described previously.

The permeability of soil is greatly affected by remolding. Also there are usually large differences between the permeability of natural soils in horizontal and vertical directions. Most natural soils contain thin layers of fine material which make their coefficients of horizontal permeability from two to five times, and occasionally ten or more times, their coefficients of vertical permeability. There are, however, notable exceptions in which the vertical permeability of a soil has been found to be many times its horizontal permeability. Good judgment must therefore be used in interpreting laboratory permeability tests and assuming relationships between values of k obtained in the laboratory and the coefficient of permeability of a soil in situ.

The capillary rise of a soil, h_c, is the height to which water may be drawn, or held in it, above the water table by capillary action. It is much greater in fine-grained than in coarse-grained soils, varying between practically zero for gravels and coarse sands to as much as 5 or 6 ft for extremely fine sands. In cohesive soils h_c may be very much larger. In fine silts it may be as much as 20 ft and its value is not limited to the height of the water barometer, or 32 ft. There is, however, no true capillary rise in extremely fine-grained and impervious soils such as fat clays. Their low permeability makes an actual upward movement of water so slow that for practical purposes it does not exist even though the capillary potential due to the extremely small interstices between the soil particles may exceed 100 ft. The capillary rise of water in sand may be demonstrated in the laboratory and the value of h_c approximated by pouring sand, which must be dry, into a vertical glass tube which has a screen across the bottom and placing the tube in water with its lower end just below the free water surface. The height to which water is drawn to fill, or partly fill, the voids is

indicated by the darkening of the sand. Unfortunately the height of the capillary rise obtained by this simple procedure is neither an accurate nor a reliable indicator of the actual height above the water table at which capillary moisture may exist in the soil in situ. This height depends largely upon the past history of the water table. It is usually much greater in soils that have been saturated and in which the water table is falling than in dry soils into which water is rising.

Other methods of determining the capillary rise h_c of non-cohesive soils are described in refs. 1 and 2. These methods without exception require long experience and good judgment on the part of the laboratory technician if results of any practical value are to be obtained. It should be noted that all methods of determining the capillary rise involve the use of disturbed samples and none are applicable to cohesive soils

Compaction tests are made to determine the most desirable or optimum moisture content of soils used in making compacted fills such as embankments or dams. The procedure is:

(*a*) A representative sample weighing at least 6 lb is put through a No. 4 sieve after lumps have been broken up with a rubber pestle. Pebbles and fragments too large to pass the sieve are rejected.

(*b*) A standard Proctor-type mold, Fig. 41, with the collar on, is filled with the sieved material which is placed in three layers each 2 or 3 in. thick. Each layer, after being gently pressed smooth, is tamped with 25 blows of the standard type of hammer shown in Fig. 41, the hammer being moved about to insure uniform distribution of the blows.

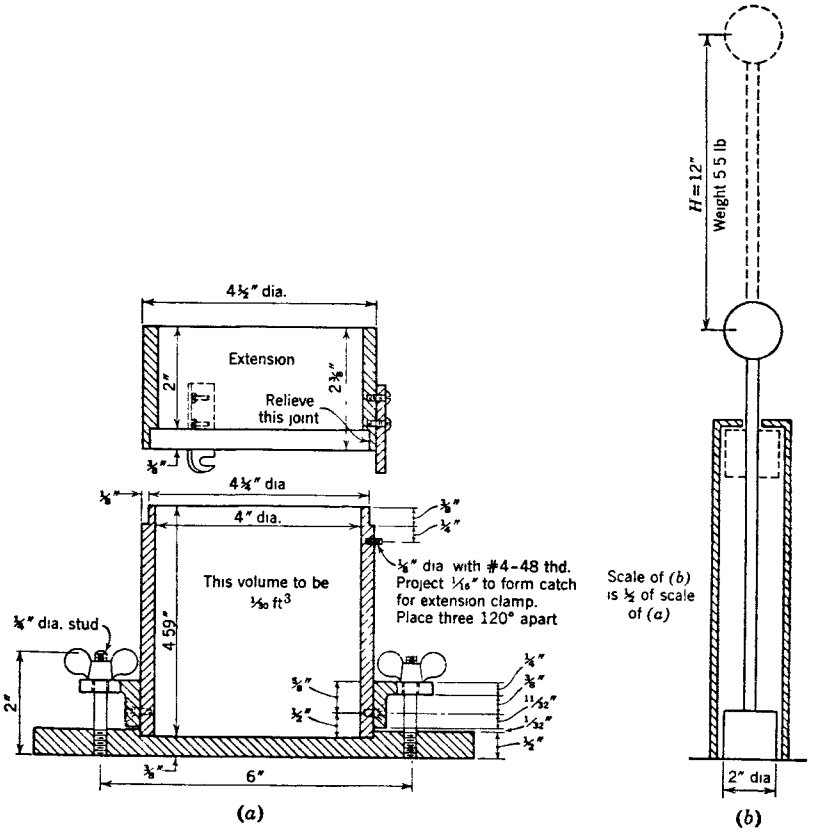


FIG 41 (*a*) A Proctor-type mold for compaction tests From *Soil Mechanics, Foundations and Earth Structures*, G P Tschebotarioff, McGraw-Hill Book Co , 1951 (*b*) Standard Proctor hammer For modified Proctor test of AASHO a hammer of same diameter weighing 10 lb is dropped 18 in.

(*c*) The collar is removed after rotating it to break its bond with the soil which is then trimmed flush with the top of the mold. The mold and soil are weighted to 0.01 lb, and the weight of the compacted soil alone is obtained by subtracting the weight of the mold.

(*d*) The soil is removed from the mold and the water content of a representative 100-gram sample is determined. The dry density of the compacted soil is computed by the formula:

$$\gamma_{md} = \frac{100W}{V(100 + w)} \tag{24}$$

in which γ_{md} = the dry density of the compacted soil in lb per ft³.
W = weight of the moist compacted soil
V = the volume of the mold in ft³ ($^1/_{30}$ ft³ in case of a standard Proctor mold).
w = the water content of the moist compacted soil expressed as a percentage.

(*e*) The water content of the soil is then raised about 3% by adding water with a sprayer and mixing it thoroughly. The test is repeated as often as may be necessary, each time with a larger quantity of water in the specimen. Finally a curve of dry densities in lb per ft³ is plotted against water contents in percentages. Several such curves are shown in Fig. 42. The optimum water content for the particular compaction technique used in making the test is that at which the dry density is maximum

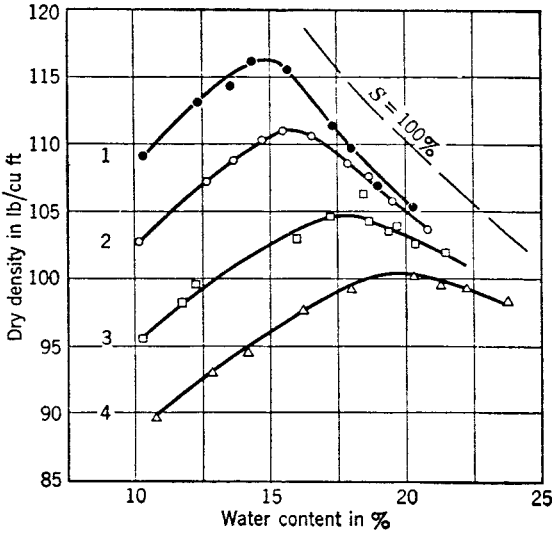


FIG 42 Compaction test results—dynamic compaction curves for a silty clay. The curves show the effect of changes in the energy used in compacting specimens From *Soil Testing for Engineers*, T. W Lambe, John Wiley & Sons, 1951

No	Layers	Blows per Layer	Hammer Weight	Hammer Drop	
(1)	5	55	10 lb	18 in	(mod AASHO)
(2)	5	26	10	18	
(3)	5	12	10	18	(std AASHO)
(4)	3	25	5 1/2	12	

Note: 6 in diameter mold used for all tests.

The optimum water content of a soil as determined by a compaction test will be found to vary with changes in the technique of testing, particularly with changes in the amount of energy used in compacting the material into the mold. Variations in the weight of the hammer, the drop of the hammer, the number of blows struck to tamp each layer of soil, and the number of layers in which the soil is placed in the mold will all affect the result of the test. It is accordingly advisable, when making compaction tests of cohesive soils in important fills and earth structures, to repeat the tests two or three times, using different total energies for compacting each series of specimens, and thus obtain a group or family of curves, 1, 2, 3, and 4, Fig 42, showing the effect on the optimum water content of changes in the energy of compaction. Finally a curve, $S = 100\%$, Fig. 42, showing the theoretical value of γ_{md} which would have been obtained if the specimens had been 100% saturated should be drawn by assuming several values of w and computing the corresponding value of γ_{md} by the formula:

$$\gamma_{md} = 62.4 \frac{G}{1 + \frac{wG}{100}} \tag{25}$$

in which γ_{md} = the dry density of the soil in lb per ft³.
G = the specific gravity of the soil.
w = the water content expressed as a percentage.

If the compaction tests have been properly conducted, none of the curves of the dry densities plotted against water contents can intersect, though they may approach, this theoretical curve.

In making compaction tests reuse of the same sample by the addition of water after each determination of γ_{md} will give results that differ materially from those secured by running simultaneous tests with varying water contents on samples each of which is used but once. The latter procedure is preferable to reusing a single sample if the relatively large amount of soil required is readily obtainable.

The California Bearing Ratio laboratory test determines the relative resistance to penetration between a compacted specimen of soil and a standard specimen composed of crushed rock. A cylinder having an internal diameter of 6 in. and a height of 7 in. is filled with soil compacted by a standard procedure and at an optimum moisture content determined by a preliminary compaction test. The specimen is covered with a perforated plate upon which a light surcharge load rests and while so loaded is submerged for 4 days with a dial gage set to measure to 0.001 in. any swelling of the soil during this period. After the swelling has been measured and recorded the force required to press a flat-ended cylindrical plunger 3 in^2 in area (1.91-in. diameter) into the specimen under strain-controlled conditions at a rate of penetration of approximately 0.05 in. per minute is determined in a testing machine or by means of a loading frame, screw jack, and proving ring. The California Bearing Ratio is the ratio expressed as a percentage between the resistance of the soil to a penetration of 0.1 in. and 1000 lb per sq in. which is assumed to be the resistance of a compacted specimen of crushed rock to a 0.1-in. penetration. Both the swelling of the specimen and the bearing ratio give information of value as to the quality of the soil as a subgrade for flexible highway and airport pavements.

The California Bearing Ratio test is a valuable index test only for those engaged in highway and airport engineering, and the laboratory test requires extensive experience for effective interpretation. There is little or no relationship between the CBR of a soil and its actual practical bearing value as a support for a foundation. See California Bearing Ratio, Art. 6, p. 7-78, and refs. 1, 2, 17, and 24.

6. SOIL MECHANICS FIELD TESTS

Loading tests are advocated, and in some cases required, by building codes. If properly conducted and interpreted in the light of adequate subsurface data they yield information of value in estimating allowable bearing values and predicting settlements. Loading tests should be made whenever available data are inconclusive as to the bearing values of soils.

Recommended apparatus for making loading tests is shown in Fig. 43. The steel bearing plate by which the load is applied to the soil may be either round or square and should have an area of not less than 1 ft^2. Some codes specify plates not less than 4 ft^2 in area. Tests on large areas are preferable to tests on small areas but where high intensities are to be proved, large area tests may be prohibitively expensive. Often, comparative tests on bearing plates of different sizes are desirable. Loads may be applied by a calibrated hydraulic jack as indicated in Fig. 43 or by placing weights on a platform on the top of the column which rests on the bearing plate. As loads are applied in increments the jack arrangement is usually preferable because an increment is more readily applied. It is, however, necessary to keep a man at the site of the test throughout its duration to maintain the jack pressure.

The bearing plate should be set at the elevation of the bottom of the proposed foundation and the backfill around the box or pipe surrounding the column which carries the load down to it should be brought to the intended elevation of the ground around the building. If the soil is non-cohesive and the water table higher than 4 ft below the intended foundation elevation, the water table should be lowered by means of well points and held 4 ft below the bearing plate for the duration of the test. If the water table is higher than 4 ft below the test plate in a non-cohesive soil, much larger settlements will occur under test loads than under finished structures.

The bearing plate should be seated firmly on a smooth, level surface prepared with a minimum disturbance of the soil. If a satisfactory seat for the plate cannot be made by trimming and screeding the soil, a bedding material should be used. This may consist of a layer of medium sand, firmly tamped and screeded and not over 1 in. thick. A mortar of plaster of paris, or plaster of paris and sand, may also be used as a bedding material. The important thing is not the nature of the bedding material but that the plate be firmly seated and level.

Provision should be made for observing settlements to 0.001 in. This is usually done with dial gages mounted as shown in Fig. 43. There should be a gage for each corner of a square plate and at least three gages spaced equidistant around the perimeter of a round

plate. The mean of the gage readings is assumed to be the settlement. It is vital that the gage mountings be well built and firmly supported an adequate distance from the bearing plate. Heavy stakes driven well into the ground at either side of the bearing plate and at least 8 ft from it will usually afford such support. It is also necessary that the gage mountings and the frame or rods that transmit the settlements of the plate to the gages be protected from distortions that might be caused by large differential temperature changes or the alternate wetting and drying of timber.

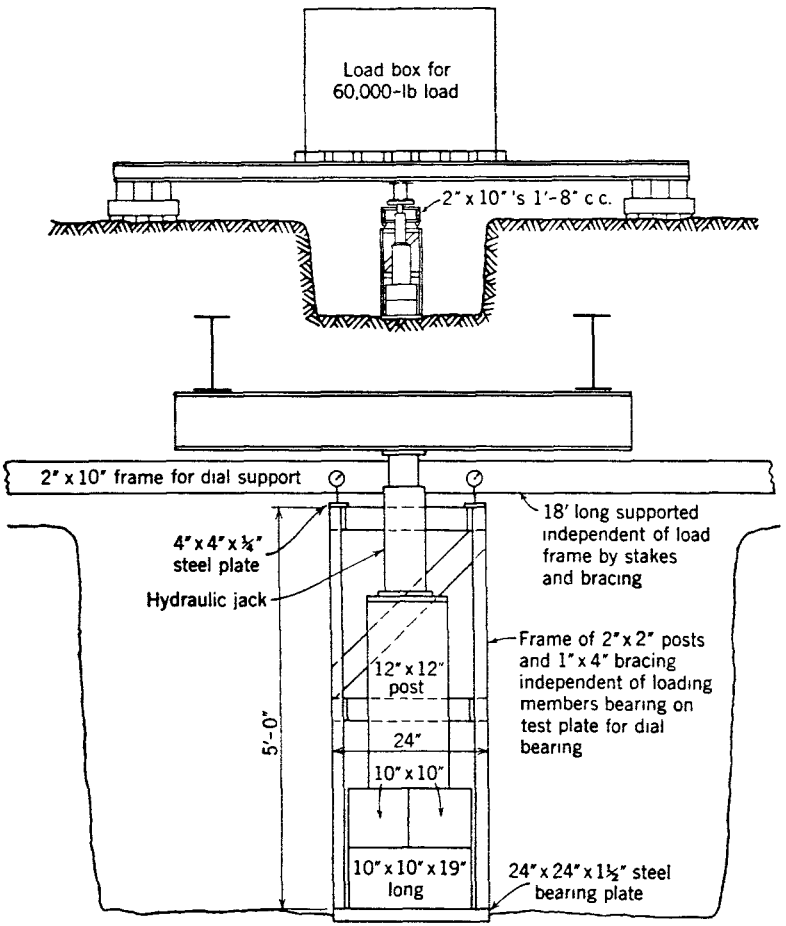


FIG. 43. Load test. The test is applied by a calibrated hydraulic jack with which any desired part of the 60,000-lb load may be transmitted to the bearing plate.

In planning a loading test, or designing a foundation, soil pressures may be reckoned as gross pressures, which are the total pressures under a bearing plate or foundation, in lb or tons per ft^2; or net pressures, which are gross pressures less pressures due to overburden that has been, or is to be, removed. Net pressures will hereinafter be dealt with and referred to merely as pressures unless otherwise specifically stated.

The load should be applied in increments each of which should not produce an increase in the soil pressure under the bearing plate exceeding 25% of the assumed safe bearing value of the soil. Smaller increments may be, and frequently are, used on non-cohesive soils which are not subject to appreciable progressive settlements. On cohesive soils the settlement under each increment may continue for a number of hours or, if the bearing

plate is large, for several days. Large increments must therefore be used on cohesive soils to keep the duration and cost of a test within reasonable limits.

At the beginning of a test a load of 150 lb per ft² should be applied and left on for 5 minutes to seat the bearing plate. This preliminary load should then be released, the dials set to zero, and the first increment of load applied. The dials should be read and settlements recorded ½ minute, 1 minute, 2 minutes, 4 minutes, 10 minutes, 20 minutes, 40 minutes, and 1 hr after the application of each load increment. Thereafter readings should be taken and recorded at intervals of an hour. Settlements should be plotted in the field to an arithmetic scale against times to a logarithmic scale as in the curves of Fig. 44. Each

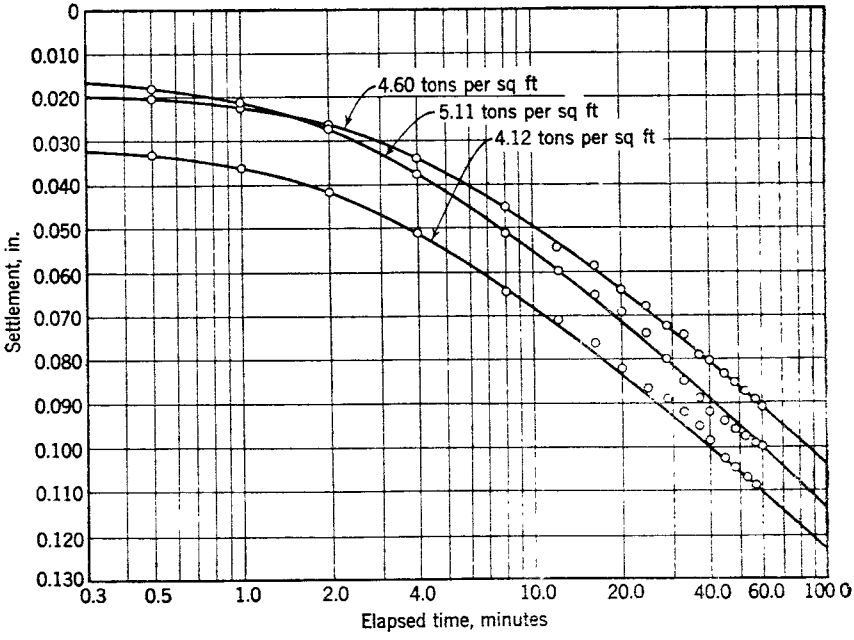


FIG. 44 Time settlement curves. Settlements in thousandths of an inch are plotted to an arithmetic scale The corresponding times in minutes in which the settlements occurred are plotted to a logarithmic scale

intermediate increment of load should be left on until the settlement under it is less than 0.01 in. per hour. The increment causing a bearing pressure equal to the assumed safe bearing value of the soil and the increment producing 150% of this pressure should be left on until the settlements under them do not exceed 0.01 in in 8 hr The heaviest load applied should cause a soil pressure at least 150% of the assumed bearing value of the soil unless a smaller load causes continued settlement and thus shows the assumed bearing value to be too high. Two types of load-settlement curves should be plotted as the test progresses. Final decision as to the maximum load to be applied in the test should in many cases be based on information given by the second of these curves.

Curves of the first type are plots of settlements against total loads, both to arithmetic scales. Four such curves appear in Fig. 45. *A* is typical for a non-cohesive soil. It has an early straight line portion and becomes gradually steeper after a point of curvature is reached, but there is no definite point of failure other than that indicated by excessive settlement. Curve *B* for a purely cohesive soil may not be quite straight, even in its early part, and the steepness of its slope increases rapidly as failure impends There is a well-marked point of failure at which the curve drops vertically. Soils that are mixtures of cohesive and non-cohesive materials yield curves, such as *C*, possessing some of the characteristics of the curves typical of both soil types. A short portion of reverse curvature in the early part of a load-settlement curve, as in curve *D*, is usually indicative of the bearing plate not having been firmly seated at the beginning of the test.

The second, more important, type of load-settlement curve should be started as soon as the early straight-line, or nearly straight-line, part of the first type of curve indicates by

its intersection with the "zero load line" the "zero correction" to be subtracted from all settlement readings to allow for the imperfect seating of the bearing plate and other causes of an apparent slight settlement under no load. Settlements, after the subtraction of this zero correction, should be plotted as abscissas against corresponding load intensities in lb, or tons, per ft^2 as ordinates, both to logarithmic scales, Fig. 46. If a proper zero correction has been made from the arithmetic plot the logarithmic plot almost invariably gives two straight lines the intersection of which may be considered the yield value of a cohesive soil. The bearing value used in the design of the foundation should be kept safely within this yield value. Load tests should be continued sufficiently to determine this yield value which is probably the most useful information given by a load test.

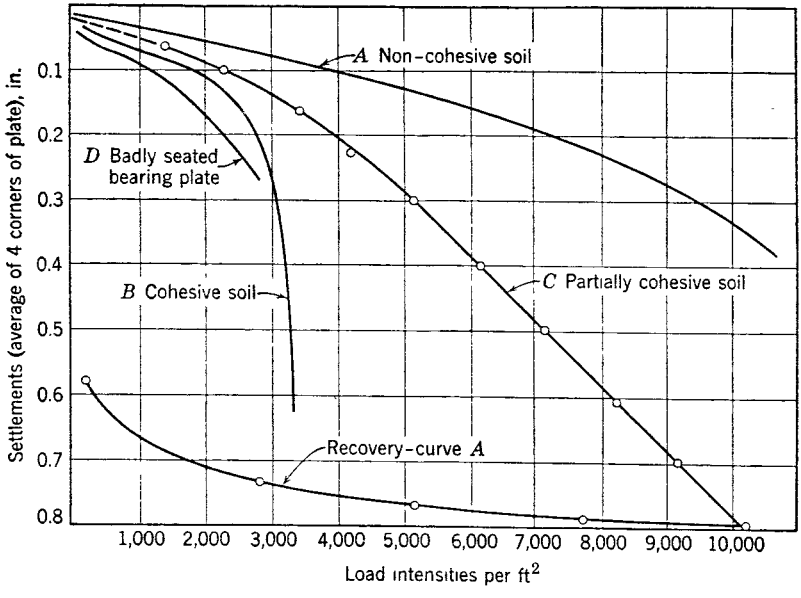


FIG 45 Settlements plotted against load intensities or total loads, both to arithmetic scales

Some building codes provide that the settlement under the assumed allowable bearing pressure shall not exceed a given amount—often 1/2 in.—and that the settlement caused by the increments producing the 50% overload shall not exceed 60% of the settlement obtained under the proposed safe load. The latter provision may, however, lead to the approval of relatively high bearing values and the disapproval of lower values in cases where practically no settlement occurred under a low assumed bearing value and where a slight settlement was produced by the 50% overload.

The comprehensive interpretation of a loading test involves obtaining as much data as possible from it for the design of an economical foundation the total and differential settlements of which will be within acceptable limits. This demands consideration of principles, dealt with more fully in Article 9, which deny the validity of the once widely held belief that settlements of foundations, irrespective of their dimensions and depths and the nature of the soil on which they rest, should equal settlements observed during loading tests conducted on the same or similar soils at unit pressures equalling those under the foundations. Settlements will actually depend not only upon the unit loading but also upon the type of soil—cohesive, non-cohesive, or partially cohesive—its density and state of submergence, and upon the width and depth of the foundation and the ratio of its width to its depth.

The ratio C_p between soil stress and settlement, indicated by the early straight line portion of a load-settlement curve and called the "coefficient of subgrade reaction" by highway and airport engineers, is not therefore a constant but is in general dependent upon the soil type and the ratio of the width of the foundation to its depth. The coefficient may, however, be approximated for different widths and depths of foundation by methods outlined below which are based upon data given by load tests. The coefficient C_p is, it should

be noted, a ratio between stress and settlement. In foot-pound units it may be given in lb, or tons, per ft^2 per in. of settlement. These are ratios, it should be noted, which reduce to loads in units of weight per unit of volume, such as lb per ft^3 or per in.^{3} In metric units C_p is usually given in kg per cm^3. Methods of determining the coefficient vary with the types of soil involved.

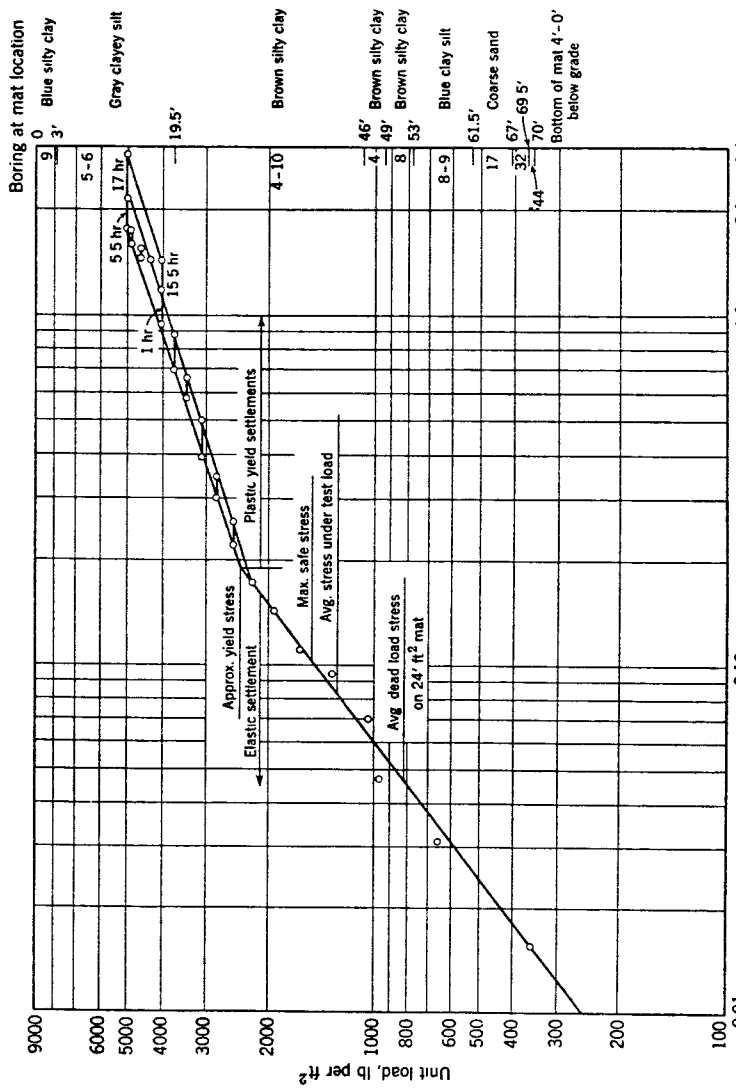


FIG. 46. Soil bearing test results. Figures to left of boring indicate blows required to draw a 2-in. diameter sample spoon 1 ft 0 in. with a 150-lb hammer falling 30 in.

On a partially cohesive soil, which may be considered the general case, two loading tests on bearing plates of different sizes, say 1 by 1 ft and 4 by 4 ft, are necessary and the relationship of settlements to depths and widths of bearing plates and foundations may be expressed by the formula:

$$C_p = \frac{q}{\rho} = C_1\left(1 + \frac{2d}{b}\right) + \frac{C_2}{b} \tag{26}$$

in which C_p = the coefficient of settlement in lb per ft² per in. of settlement.
q = the intensity of the load in lb per ft².
ρ = the settlement in in. produced by the load of intensity q.
C_1 and C_2 are constants for the given soil.
d = the depth below the ground surface of the bearing plate or foundation in ft.
b = the width of the bearing plate of foundation in ft.

Having obtained two values of C_p from two tests, the above equations can be solved simultaneously for the soil constants C_1 and C_2. These constants may then be entered in the equation and values of C_p computed for other values of d and b. It should be emphasized that values of C_p obtained in this way are nothing more than rough approximations giving a general idea of the order of magnitude of settlements that may be anticipated. On an important project it may be worth while to make two or more pairs of large and small area tests. If independent computations are then made of C_1 and C_2 from all possible pairs of areas, the resulting maximum and minimum probable values of C_p should be helpful to the designer.

On a purely cohesive soil, such as clay, the ultimate bearing capacity is independent of the width of the loaded area, but practical allowable bearing values, as limited by acceptable settlements, are inversely proportional to these widths. They are but little effected by depths. The value of C_1 in the equation given above then becomes zero and the equation reduces to.

$$C_p = \frac{q}{\rho} = \frac{C_2}{b} \tag{27}$$

It is therefore possible to determine C_2 and thus obtain a general formula for C_p for a purely cohesive soil by a single loading test.

In dealing with cohesive and partially cohesive soils by this method, however, it is very important to make the previously recommended log-log plot of the load-settlement relationship and determine the yield value of the soil, which should under no circumstances be exceeded. Careful consideration should be given to the possibility of a shear failure—see Art. 9. The importance of the long duration of settlements on cohesive soils should also be kept in mind. In a loading test conducted on a 2 by 2 ft bearing plate most of the settlement will probably occur within a few hours, or, at the most, within a few days, whereas the settlement of a mat foundation 100 by 100 ft would possibly continue for decades.

On a non-cohesive soil the bearing capacity depends more upon the density than grain size or grain-size distribution. Other things being equal, the denser a sand, the greater its bearing capacity. Submergence reduces the bearing capacity of sands markedly. Under a given load a submerged sand—i.e., a sand below the water table—will settle from two to four times as much as the same sand above the water table. The ultimate bearing capacity of a sand is proportional to the width of the loaded area. The practicable allowable bearing value of a sand as limited by acceptable settlements is affected largely by the ratio of the depth to the width of the foundation and very little by its width alone. For purely non-cohesive soils the term C_2 in the general equation given above becomes zero and the equation reduces to.

$$C_p = \frac{q}{\rho} = C_1\left(1 + \frac{2d}{b}\right) \tag{28}$$

Accordingly it is possible to determine C_1 and obtain a formula for C_p for a given non-cohesive soil by means of a single loading test.

In interpreting loading tests consideration should always be given to the fact that the bulb of pressure, or limiting surface of the volume of soil subjected to a significant increase of pressure under a bearing plate of foundation, has a depth about 1.5 times the least dimension (the width) of the plate or foundation, Fig. 47*A*, *B*, and *C*. If the bulb of pressure for a loading test, Fig. 47*A*, lies within a soil of good bearing value which is underlain by a less desirable material into which the bulb of pressure of a proposed foundation will extend, Fig. 47*B*, great care must be used in interpreting the test. If closely spaced footings approximate a mat foundation, Fig. 47*C*, a similar condition may be produced.

For a more complete treatment of the important subject of loading tests see refs. 20 and 22.

Test piles may be used to determine the driving characteristics of a soil and the depth to a firm stratum capable of supporting a foundation. Information obtainable from such piles, particularly if they are test loaded, is helpful in estimating allowable loads for single piles and pile groups irrespective of whether the proposed foundation is to be supported on:

(*a*) Friction piles depending for support upon the frictional resistance and shearing strength of the soil.

(*b*) End bearing piles driven through soft materials to, or into, a stratum capable of supporting them.

(*c*) Piles which get their support by a combination of friction and end bearing.

Test piles are not an adequate substitute for borings. The data yielded by test piles cannot, in fact, be interpreted intelligently without knowledge of the characteristics of not only the soil into which the piles were driven but also the underlying materials. If unsuspected soft material underlies a firm soil in which piles develop a satisfactory resistance to driving, neither their behavior under the hammer nor the test loading of individual piles will reveal a condition which has caused costly foundation failures. Whenever, in such cases, the bulbs of pressure or zones of significant increase in soil stress of individual piles intersect, the safe bearing value of n piles will be less than n times the safe bearing value of a single pile as determined by a pile formula or a test load.

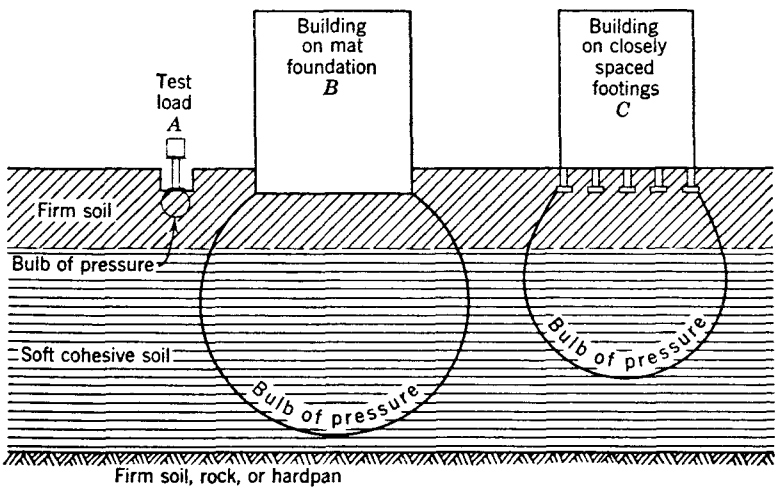


FIG 47 Limitation of load tests A load test A made on firm soil underlain by soft soil will not reveal probability of settlement because bulb of pressure or limiting surface of soil subjected to significant increase of stress does not reach the soft soil. The buildings, however, will settle

Pile formulas for estimating the allowable bearing values of piles are of two fundamental types.

(*a*) Dynamic formulas which equate the energy of the hammer blow to the energy expended in driving the pile into the soil plus the energy wasted in mechanical losses. The resulting equations are solved for the resistance of the soil to the penetration of the pile. This resistance is assumed to be the static bearing capacity of the pile.

(*b*) Static formulas based on assumptions as to the friction between pile and soil and the latter's strength and supporting capacity.

Of the two types of formula the dynamic is the more generally used and important. In the United States the best-known and most widely used dynamic formulas are the Engineering News formulas and modifications thereof. These formulas are:

For drop and single-acting steam hammers

$$R_a = \frac{2W_rH}{s + c} \tag{29}$$

For double-acting steam hammers

$$R_a = \frac{2(W_r + Ap)H}{s + c} \tag{30}$$

in which R_a = the allowable bearing capacity of the pile in lb.
W_r = the weight of the hammer in lb.
H = the fall of a drop hammer or stroke of a steam or diesel hammer in ft.
A = the area of the piston of a double-acting steam hammer in in.^{2}
p = the mean effective pressures on the piston of a steam hammer in lb per in.^{2}

s = the "set" or penetration in inches of the pile due to the last blow (usually the average set for the last 5 blows with drop hammers and the last 20 blows with other types).

c = a term having the following values:

For drop hammers	1.0
For steam hammers	0.1
For steam hammers used on heavy reinforced concrete or steel piles	$0.1 \dfrac{W_p}{W_r}$

in which W_p = the weight of the pile in lb.

The term $0.1 \dfrac{W_p}{W}$ in the last of the three expressions for c is intended to allow for significant energy losses due to inertia effects when a heavy pile is driven with a relatively light hammer. When developed, the formulas given above were supposed to provide a factor of safety of 6. In the light of present knowledge the safety factor probably varies between 2 and 17. In rare cases it may be less than unity.

So-called complete formulas attempt to make allowance for all energy losses. Prominent among these are the Hiley formulas, widely used abroad and strongly advocated by many engineers in the United States. The Hiley formulas are:

For drop, single-acting steam, and diesel hammers:

$$R_u = \frac{12 e_f W_r H}{s + {}^1/_2 (C_1 + C_2 + C_3)} \times \frac{W_r + e^2 W_p}{W_r + W_p} \tag{31}$$

For double-acting and differential-acting steam hammers:

$$R_u = \frac{12 e_f E_n}{s + {}^1/_2 (C_1 + C_2 + C_3)} \times \frac{W_r + e^2 W_p}{W_r + W_p} \tag{32}$$

The terms common to the Engineering News and Hiley formulas have the same significance in both with the exception of W_p, the weight of the pile. The special significance of W_p in the Hiley formula and the meaning of the terms occurring in the Hiley formula only are:

W_p = weight of pile, in lb, including shoe and driving cap for drop and single-acting steam hammer. In the case of double-acting and differential-acting hammers the weight of the anvil should be included. The weight of earth in the spaces between the flanges of H piles and in pipe piles driven open-ended should be included as should the weights of mandrel and followers if used. In the case of driving end bearing piles to refusal on rock only half the actual weight should be used in the formula.

R_u = ultimate bearing capacity of the pile in lb, before the application of a safety factor.

e_f = mechanical efficiency of the hammer. The following values are suggested:
- 100% for drop hammers released by a trigger.
- 75% for drop hammers that overhaul the hammer line and spin the winch drum as they fall.
- 75% for single-acting and differential-acting steam hammers and double-acting hammers in general.
- 85% for double-acting steam hammers having a rated energy of blow based on indicator diagrams.
- 100% for deisel hammers the manufacturers of which have introduced an overall efficiency factor into the rated energy of the blow.

E_n = rated energy of the hammer blows of double-acting and differential-acting steam hammers, in foot-pounds, as published by the manufacturers.

e = coefficient of restitution. Values vary between:
- 0.55 for no cushion—steel on steel—when driving steel pipe piles.
- 0.50 for well-compacted cushion when driving pipe piles and for rams of double-acting hammers striking steel anvils and driving steel or precast concrete piles.
- 0.40 for medium-compacted wood cushion when driving pipe piles; for ram of double-acting steam hammers striking steel anvil and driving timber piles or striking steel helmet containing wood and driving steel piles; and for ram

of single-acting or drop hammers striking directly on head of precast concrete piles not fitted with driving cap.

0.32 for ram of single-acting hammers striking on steel plate cover of wood cap on steel piles.

0.25 for fresh wood cushion when driving pipe piles and for ram of single-acting or drop hammers striking on well-conditioned wood cap of driving cap in driving precast concrete piles or directly on wood pile heads.

0.00 for deteriorated condition of heads of timber piles or of wood cap and excess packing in a driving cap.

C_1 = temporary compression allowance for pile head and cap in in.

C_2 = temporary compression of pile in in.

C_3 = temporary compression allowance for soil.

For estimating the values of C_1, C_2, and C_3 several additional terms are used in conjunction with Tables 3, 4, and 5. The additional terms are:

A_p = average cross-sectional area of the pile at butt and center of resistance to driving, in in.^{2} (shell only in case of steel pipe and Monotube piles, shells and core, or mandrel, for cast-in-place concrete piles). In the case of end bearing piles the center of resistance is at the tip. In the case of reinforced concrete precast piles the area of the reinforcing bars should be transformed into an equivalent concrete area and included.

p_1 = stress in lb per in.^{2} on pile head = $\dfrac{R_u}{\text{Area of pile head}}$.

p_2 = stress on average cross section of pile = $\dfrac{R_u}{A_p}$.

p_3 = stress in lb per in.^{2} on horizontal projection of the tip of an end bearing pile, or a pile of constant cross section, including driving points under steel pipe casings and the total area of the rectangle circumscribed about the cross section of an H pile; or the cross-sectional area at the ground line in the case of a tapered friction pile. p_3, therefore, may equal $\dfrac{R_u}{\text{Area of tip}}$ or $\dfrac{R_u}{\text{Gross area at ground surface}}$.

L = length of pile in ft from head to center of resistance to driving.

With values of p_1, p_2, and p_3 and L Tables 3, 4, and 5 may be entered for estimated values of C_1, C_2, and C_3. These tables are reproduced from ref. 10 with the permission of the

Table 3. Temporary Compression C_1, Hiley Formula

(Temporary compression allowance C_1 for pile head and cap *)

Material to Which Blow Is Applied	Easy Driving, p_1 = 500 lb per in.^{2} on cushion or pile butt if no cushion, in.	Medium Driving, p_1 = 1000 lb per in.^{2} on head or cap, in.	Hard Driving, p_1 = 1500 lb per in.^{2} on head or cap, in.	Very Hard Driving, p_1 = 2000 lb per in.^{2} on head or cap, in.
Head of timber pile	0.05	0.10	0.15	0.20
3–4 in. packing inside cap on head of precast concrete pile	0.05 + 0.07 †	0.10 + 0.15 †	0.15 + 0.22 †	0.20 + 0.30 †
1/2–1-in. mat pad only on head of precast concrete pile	0.025	0.05	0.075	0.10
Steel-covered cap, containing wood packing, for steel piling or pipe	0.04	0.08	0.12	0.16
3/16-in. red electrical-fiber disk between two 3/8-in. steel plates, for use with severe driving on Monotube pile	0.02	0.04	0.06	0.08
Head of steel piling or pipe	0	0	0	0

* Largely from A. Hiley, Pile Driving Calculations with Notes on Driving Force and Ground Resistance, *The Structural Engineer*, Vol. 8, July and August, 1930. For a fuller discussion of the means of obtaining these values see this reference. For purpose of this section values represent average conditions and may be used.

† The first figure represents the compression of the cap and wood dolly or packing above the cap, whereas the second figure represents the compression of the wood packing between the cap and the pile head

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Table 4. Temporary Compression C_2, Hiley Formula

(Temporary compression values of C_2 for piles)

Type of Pile	Easy Driving, p_2 = 500 lb per in.^{2} for wood or concrete piles, 7,500 lb per in.^{2} for steel, net section, in.	Medium Driving, p_2 = 1,000 lb per in.^{2} for wood or concrete piles, 15,000 lb per in.^{2} for steel, net section, in.	Hard Driving, p_2 = 1,500 lb per in.^{2} for wood or concrete piles, 22,500 lb per in.^{2} for steel, net section, in.	Very Hard Driving p_2 = 2,000 lb per in.^{2} for wood or concrete piles, 30,000 lb per in.^{2} for steel, net section, in
Timber pile based on value of E = 1,500,000 *	$0.004 \times L$ †	$0.008 \times L$ †	$0.012 \times L$ †	$0.016 \times L$ †
Precast concrete pile E = 3,000,000 ‡ §	$0.002 \times L$	$0.004 \times L$	$0.006 \times L$	$0.008 \times L$
Steel sheet piling, Simplex tube, pipe pile, Monotube shell, Raymond steel mandrel ‖ (E = 30,000,000)	$0.003 \times L$	$0.006 \times L$	$0.009 \times L$	$0.012 \times L$

* For unusual types of wood piles see Table VI, ref. 10, for values of E.

† L should be considered as length to center of driving resistance, not necessarily full length of pile.

‡ Values of C_2 are in direct proportion to p_2 and inverse proportion to E.

§ May reach 6,000,000 for exceptionally good mix.

‖ When computing p_2 for a Raymond steel mandrel, it is suggested that the weight of the mandrel be divided by 3.4 × length of pile in ft to obtain the average area.

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Table 5. Temporary Compression C_3, Hiley Formula *

(Temporary compression or quake of ground allowance C_3 †)

	Easy Driving, p_3 = 500 lb per in.^{2}, in.	Medium Driving, p_3 = 1000 lb per in.^{2}, in.	Hard Driving, p_3 = 1500 lb per in.^{2}, in.	Very Hard Driving, p_3 = 2000 lb per in.^{2}, in.
For piles of constant cross section ‡, §	0 to 0.10	0.10	0.10	0.10

* All values of p_3 to be taken on projected area of pile tips or driving points for end bearing piles and piles of constant cross section; on gross area of pile at ground surface in case of tapered friction piles; and on bounding area under H piles.

† Largely from A. Hiley, Pile Driving Calculations with Notes on Driving Force and Ground Resistance, *The Structural Engineer*, Vol. 8, July and August, 1930. For a fuller discussion of the means of obtaining these values see this reference. For purpose of this section values represent average conditions and may be used.

‡ It is recognized that these values should probably be increased in the case of piles with battered faces, but insufficient test data are available at present time to cover this condition.

§ If the strata immediately underlying the pile tips are very soft, it is possible that these values might be increased to as much as double those shown.

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author and the publisher. Because C_1, C_2, and C_3 are partially dependent upon values of R_u, the solution of the Hiley formulas for R_u as a function of s involves a cut-and-try process that is laborious and time wasting. This difficulty can be overcome by assuming a number of reasonable values of R_u and solving the equations for s. Curves can then be drawn showing the relationship between the ultimate bearing capacity, R_u; a safe working load after the application of a suitable safety factor; the stress in the pile; and the set, s, Fig. 48. Curves can also be readily drawn showing the same information plotted against blows of the hammer per inch of set, Fig. 49.

A field check of the reasonableness of the estimated values of C_2 and C_3 combined ($C_2 + C_3$) can be made by fastening a piece of bristol board to the pile just prior to the completion of driving, and drawing a pencil line on it along the edge of an independently supported straightedge. If the pencil is in motion when the hammer strikes a simple graph will result, Fig. 50, which gives the value of $C_2 + C_3$ directly.

Most engineers probably consider the Hiley formula fundamentally more accurate than the Engineering News formula and many therefore prefer it and advocate its general adoption. However, a dynamic formula as a means of estimating the bearing value of a pile is only applicable to a pile driven into non-cohesive soil. The applicability of dynamic formulas to soils which are partly cohesive depends upon the extent to which the soils act as non-cohesive or cohesive materials. For this reason, and because the accuracy of com-

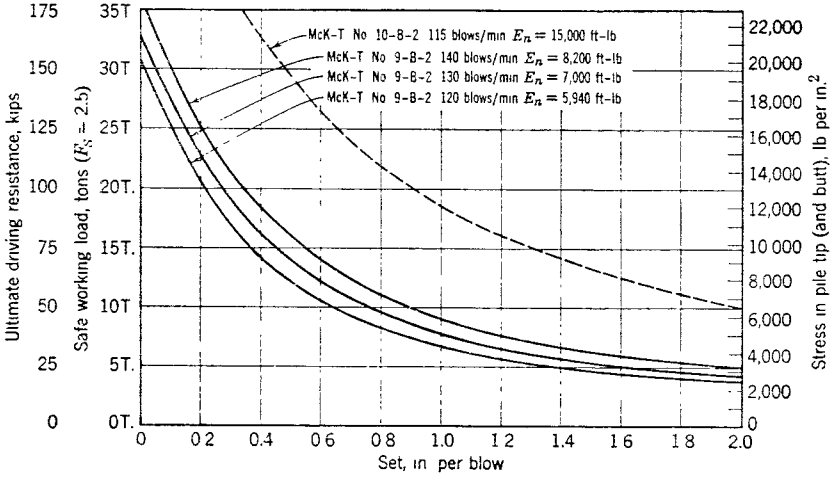


Fig. 48. Hiley formula curves. By permission, from *Pile Foundations*, by R. D. Chellis, copyright, 1951, by McGraw-Hill Book Co.

Conditions of curves: hammer, McKiernan-Terry 9-B-2, driving cap, 650 lb; W_p, 1440 lb; pile, steel pipe 10" O.D. × 1/4" × 30' long, weight 790 lb; soil, pile driven to sand and gravel through soft strata; side friction neglected, and pile assumed as end bearing, effective length, 30'.

Curves of this type have no general applicability. For a given type and weight of pile and a given hammer a special curve must be drawn.

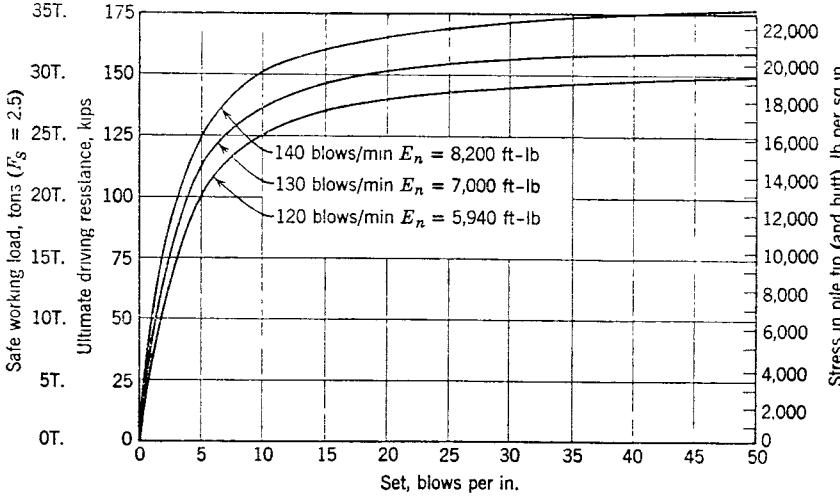


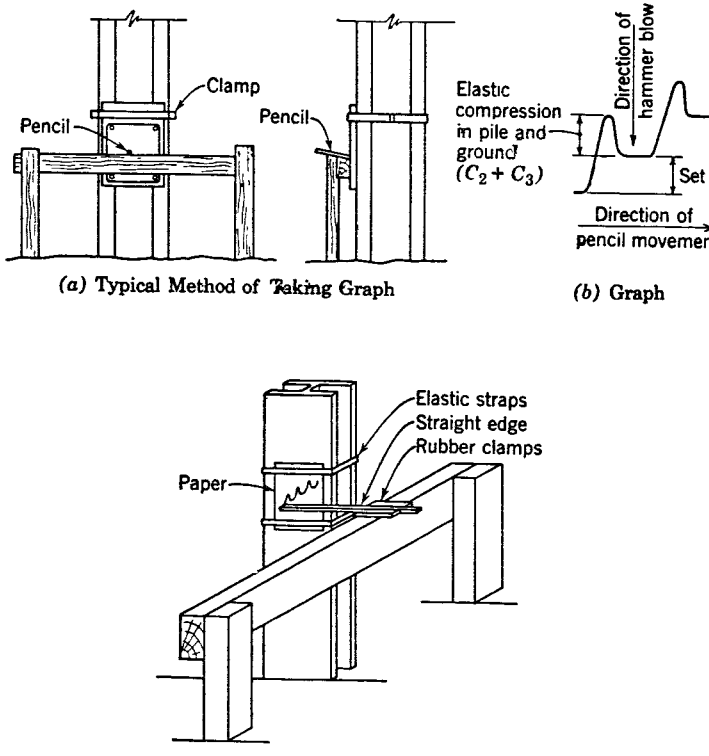
Fig. 49. Hiley formula curves. By permission, from *Pile Foundations*, by R. D. Chellis, copyright, 1951, McGraw-Hill Book Co.

Conditions of curves: hammer, McKiernan-Terry 9-B-2; driving cap, 650 lb; W_p, 1440 lb; pile, steel pipe 10" O.D. × 1/4" × 30' long, weight 790 lb; soil, pile driven to sand gravel through soft strata, side friction neglected, and pile assumed as end bearing, effective length, 30'.

These curves are similar to those in Fig. 48, except that they are entered with blows per inch rather than inches per blow as an argument.

plete formulas, such as the Hiley, depends largely upon the judgment with which empirical coefficients and constants are selected, some engineers maintain that such formulas have no real claim to greater accuracy than simpler ones; see refs. 9, 10, 11, and 12.

Even though the best of dynamic formulas have but a limited value as a means of determining the bearing value of a pile driven into even a purely non-cohesive soil, and are practically valueless when dealing with cohesive soils, they are nevertheless necessary as a means of specifying when to stop driving to obtain reasonably uniform resistances without damaging piles. Complete formulas of the Hiley type are, for such purposes, preferable



(a) Typical Method of Taking Graph

(b) Graph

(c) Alternate Method

FIG 50. Determination of ($C_2 + C_3$) in the field. By permission, from *Pile Foundations*, by R D. Chellis, copyright, 1951, McGraw-Hill Book Co.

to the simpler and older formulas as they yield reasonably accurate estimates of the stresses produced in the pile by driving. When thus used for determining pile stresses the least conservative reasonable assumptions should be made as to hammer efficiencies and constants, rather than the most conservative estimate which should be employed when estimating the bearing values of piles.

Static formulas estimate the bearing values of piles by applying to the embedded surface areas of their sides factors representing the frictional and cohesive resistance offered by the soil; and to their end areas, if there is a partial end bearing effect, factors representing the bearing value of the material on, or in, which the pile tips rest. In practice static formulas have been used extensively in only such localities as New Orleans and Shanghai where thick deposits of soft cohesive soils render dynamic formulas valueless and require the use of friction piles under conditions making consideration of end bearing effects unnecessary. Table 6 gives approximate allowable values of the skin friction on piles. These figures, after being checked against the best available data on successful local practice, may be used for minor projects involving pile foundations on cohesive soils. On large or important projects test piles should invariably be driven and loaded.

Table 6. Skin-Friction Factors for Friction Fills

(Approximate allowable value of skin friction on piles *)

Material	Skin Friction, lb per ft^2		
	Approximate Depth, 20 ft	Approximate Depth, 60 ft	Approximate Depth, 100 ft
Soft silt and dense muck	50–100	50–120	60–150
Silt (wet but confined)	100–200	125–250	150–300
Soft clay	200–300	250–350	300–400
Stiff clay	300–500	350–550	400–600
Clay and sand mixed	300–500	400–600	500–700
Fine sand (wet but confined)	300–400	350–500	400–600
Medium sand and small gravel	500–700	600–800	600–800

* Some allowance is made for the effect of using piles in small groups.

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Test piles should be of the type of pile proposed for use in the foundation, because different types of pile behave differently under the hammer. The cost of the piles themselves is an insignificant part of the total cost of driving and loading test piles. Therefore, the saving that might be realized by substituting timber test piles for concrete or steel

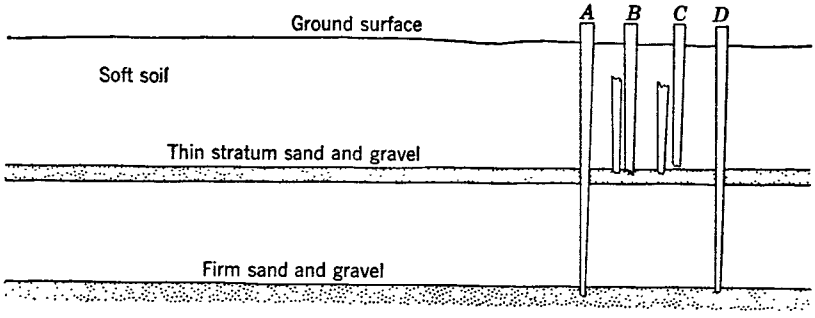


FIG 51. Driving through a thin firm stratum When timber test or foundation piles are to be driven through a thin firm stratum to an underlying firm material, piles frequently break instead of punching through the thin stratum.

piles of the type probably to be used in a foundation is not worth the decreased value of the information obtainable and the possible consequent increase in the cost of the foundation. Some saving in the cost of test piles may, however, be realized by so locating them that some or all may later be incorporated into foundations.

Test piles should be driven with hammers of the type and weight to be used in driving the foundation piles. The weight of the ram should exceed the weight of the pile multiplied by the coefficient of restitution ($W_r > W_{pe}$). Steam hammers should be operated at the pressures recommended by the manufacturers and should be checked to make sure that they are running at rated speed and stroke. Some steam hammers, when the driving is hard, do not work at their full rated stroke nor deliver the energy to the pile at each blow that they are supposed to.

If a drop hammer is used it should weigh not less than 3000 lb, and the height of the drop should not be less than 8 nor more than 15 ft for wood piles and not more than 8 ft for precast concrete piles. Care should be taken that the slack of the hammer line is not taken up prematurely at the end of each drop in such a way that the energy of the blows is reduced.

A few representative piles should be redriven at least 24 hr after they initially develop a satisfactory resistance under the hammer. The usual effect of such a rest period is to greatly increase the resistance. It may, in fact, require a number of hammer blows to start a pile moving after it has become "frozen" into a partly cohesive, or cohesive, soil. On the other hand, soils are occasionally met in which piles develop temporary resistance to driving that disappears after a rest period of a few hours. In such material a pile which has developed practically absolute "refusal" under continuous driving with a steam ham-

mer may again drive easily and exhibit little supporting power after a 24-hr "rest." Such soils have usually been stiff, blue, clay-like muds encountered in the construction of piers and wharves in coastal waters.

If the heads of timber piles broom or split they should be cut back to sound wood and driving continued after an interruption long enough to insure the resumption of normal conditions before the final set per blow is determined.

Where piles are to be driven through a thick bed of soft soil containing one or more firm strata too thin to support the proposed foundation, timber test piles may break instead of punching through one of the firm strata. Unfortunately it is often impossible to determine whether a pile has, in fact, punched through such a stratum or broken, Fig. 51. As breakage of this type may lead to erroneous conclusions and serious mistakes in foundation design, timber test piles, after being driven and possibly loaded, should be pulled and examined if there is anything in their driving records indicative of possible breakage below ground. If a suitable dynamometer is available the pulling resistance should be determined. By assuming the pulling resistance to equal the skin friction on the pile a rough approximation of the point bearing value may be obtained by subtracting the pulling resistance from the ultimate bearing value.

A complete record should be kept of the driving of all test piles, and the data recorded should include.

(1) Date and weather conditions.

(2) Type and weight of hammer and, if the hammer is steam or diesel, its make, model, stroke, rated energy delivered per blow at various working speeds, the pressure of the steam or compressed air used to run it, and the number of blows delivered per minute.

(3) The material and dimensions of each pile, and, for reinforced concrete piles, the date poured.

(4) The location of each pile and the elevation of the ground surface at the location.

(5) The number of blows of the hammer for each foot of penetration and the elevation of the pile butt when any notably hard driving occurred.

(6) The penetration under the last 5 full blows of a drop hammer or the last 20 blows of a steam or diesel hammer.

(7) The final elevations of the butt and tip.

(8) An entry under "remarks" recording any unusual circumstance or event attending the driving of the pile.

(9) The signature of the inspector responsible for the record.

The *original* records of the driving of all test piles, not "smooth copies," should be kept in the permanent files of the engineer responsible for the design of a foundation.

Finally, the data secured by driving each test pile should be plotted as in Fig. 52, with allowable bearing values calculated by the pile formula of the engineer's choice as abscissas against corresponding elevations of the pile tip as ordinates. It is helpful, as in this figure, to show the character of the material, as revealed by borings, on one side of the bearing value diagram to the same scale as the tip elevations.

Test loads are applied by loading a platform attached to the pile, by jacking against a load, or by the "bootstrap method" of jacking against a beam anchored to other piles, Fig. 53*a*, *b*, and *c*.

Of these methods the loading platform, Fig. 53*a*, is usually the simplest and cheapest way of loading a few isolated test piles. The platform must be of adequate strength and steadied laterally by bracing that will not partially support the load. If the soil is cohesive the possibility of a sudden settlement of the pile should be considered in guarding against accidents. The load should be applied in increments the true weight of which must be known. Weighed metal or concrete blocks, iron "pigs," steel rails, structural steel shapes, weighed sandbags, sand in a bin, and water in a tank have all been used.

Where large numbers of piles are to be test loaded the method of jacking against a load with a calibrated hydraulic jack, Fig. 53*b*, may prove economical, especially if the soil is non-cohesive in which the settlement under each increment will develop rapidly, and the load can be made portable by mounting it on a truck or trailer. An objection to the method is the necessity of keeping a man in attendance to maintain the jack pressure and load constant under increasing settlement throughout the period of application of each load increment.

The bootstrap method of jacking against a beam anchored to adjacent piles, Fig. 53*c*, is not in reality a load test, but merely a test of the frictional resistance along the surface of the piles and the shearing strength of the soil between them. No extra load is applied to the area, and no reliable information on compaction effects is yielded by the method. It is not recommended.

Whatever the method of application, test loads should be applied in increments, each of which should be left on until settlement under it ceases or continued settlement indicates

failure. Small increments give more complete information than large ones on the character of settlement curves, Fig. 54, and may enhance the accuracy of settlement predictions, but they prolong tests and increase costs. For general use the requirements of the 1948 Foundation Code of the City of New York are recommended, not only as to the number and size of the increments in which the load is applied, but also as to the decrements in which it is removed and the frequency and precision of settlement and rebound readings.

The New York code provides that: "The test load shall be twice the proposed load value of the pile. The test load shall be applied in seven increments equal to one-half, three-fourths, one, one and one-fourth, one and one-half, one and three-fourths and two times the proposed working load. Readings of settlements and rebounds shall be referred to a constant elevation bench mark and shall be recorded to one one-thousandth of a foot for

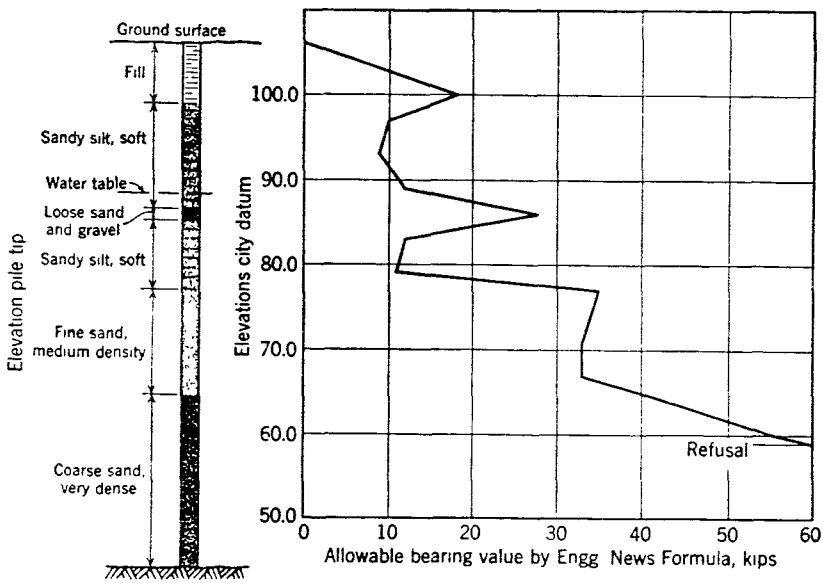


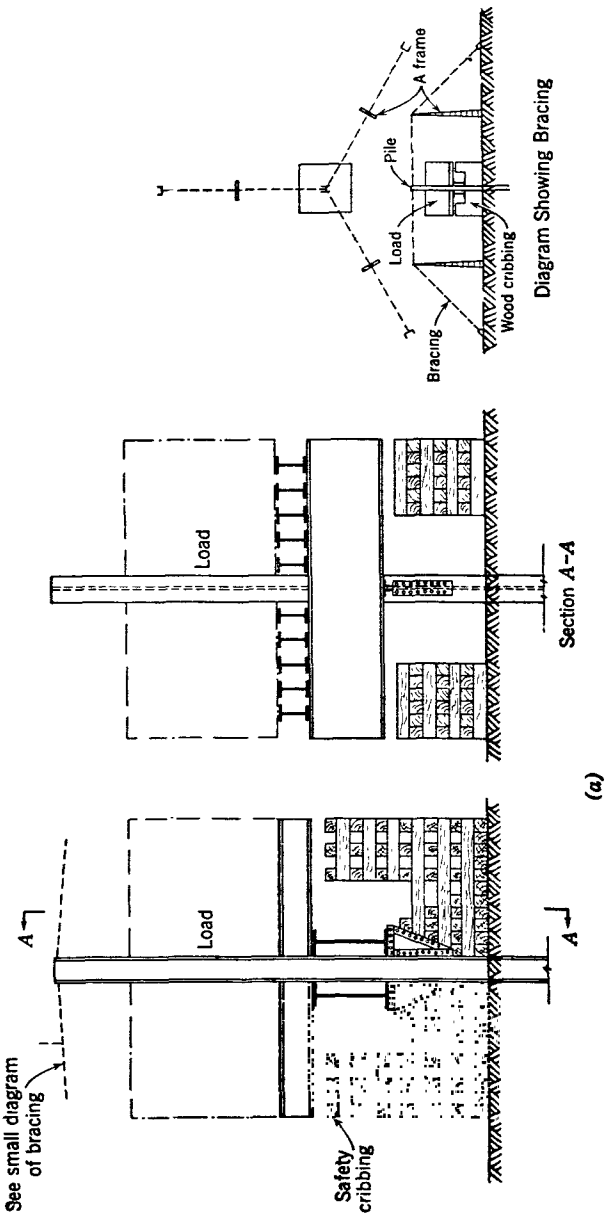
FIG 52 Test pile driving report, graphical presentation Test pile No 17; precast reinforced concrete 16″ × 16″ × 55′; Vulcan No 1 hammer; for pile location, see drawing, formula

$$R_a = \frac{2W_rH}{s + c}$$

each increment or decrement of load. After the proposed working load has been applied and for each increment thereafter, the test load shall remain in place until there is no settlement in a two-hour period. The total test load shall remain in place until settlement does not exceed one-thousandth of a foot in forty-eight hours. The total load shall be removed in decrements not exceeding one-fourth of the total test load with intervals of not less than one hour. The rebound shall be recorded after each decrement is removed, and the final rebound shall be recorded twenty-four hours after the entire test load has been removed. The maximum allowable pile load shall be one-half that which causes a net settlement of not more than one-hundredth of an inch per ton of total test load or shall be one-half that which causes a gross settlement of one inch, whichever is less."

The Boston code provides that the pile, "shall be loaded to at least twice the proposed working load, the load being applied in increments of not over 10,000 pounds. At least four hours shall elapse between the addition of successive increments. Measurements of the settlement, accurate to one thirty-second inch, shall be taken and recorded immediately before and after each increment of load is added. In determining the settlement, proper deduction shall be made for elastic compression of the pile under the test load. The allowable pile load shall not exceed one-half of that causing a total settlement of one-half inch which remains constant for forty-eight hours."

The field reports of load tests should be filed permanently and the data resulting from them plotted as shown in Fig. 54, the loads in tons being plotted as abscissas against corresponding settlements in inches. In Fig. 54 curves A and B are respectively typical


See small diagram of bracing
A
Load
Safety cribbing
A
Load
Section A-A
Pile
A frame
Load
Bracing
Wood cribbing
Diagram Showing Bracing

(a)

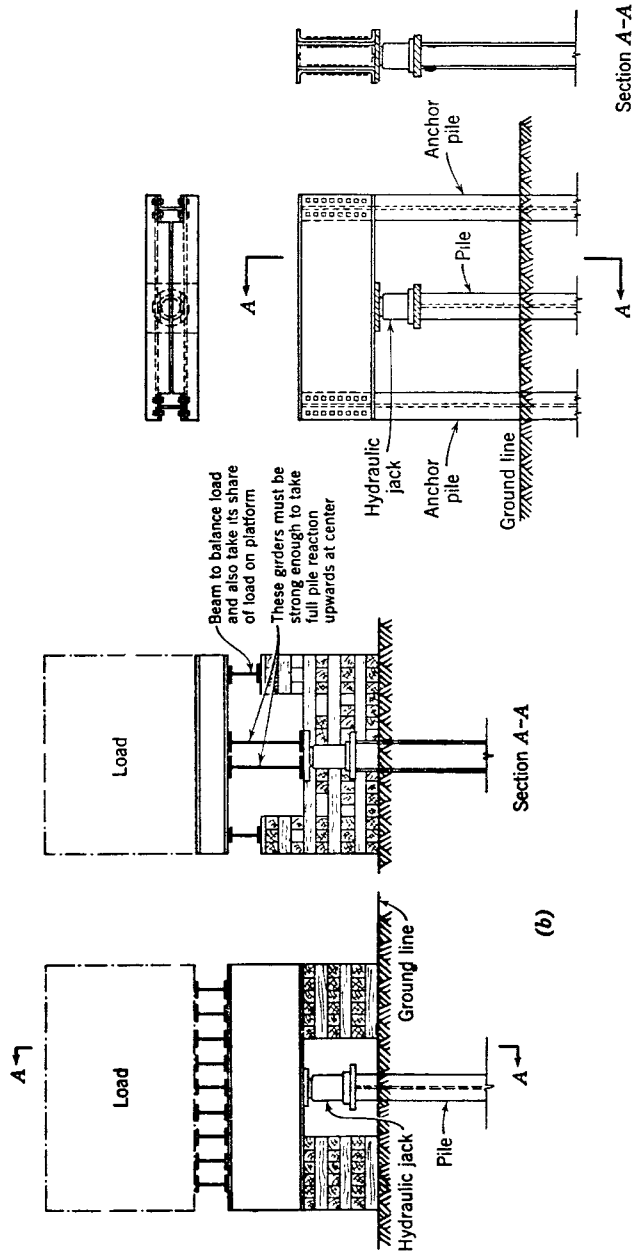


FIG. 53. Methods of loading test piles. (a) Loaded platform supported by test pile (b) Jacking against a load. (c) Bootstrap method Courtesy Carnegie Steel Company.

of load test curves of piles in non-cohesive and cohesive soils. The solid line of curve B indicates the progressive settlement that occurs when a continuously applied load exceeding the ultimate bearing value is placed on a pile in cohesive soil. Curves A' and B' at the top of Fig. 54 indicate the portions of the total, or gross, settlements estimated to be due to the elastic deformations of the piles. In computing these quantities it is usual to assume the axial stress in an end bearing pile to be uniform throughout its length and that, in a friction pile, the stress decreases uniformly from a maximum at the ground line to nothing at the tip.

As the load settlement curve of a pile in non-cohesive soil, Curve A, Fig. 54, shows increasing settlements with increasing loads and no definite point of failure it is usual to limit the load placed on piles in non-cohesive soils to amounts which will not produce excessive settlements. Where the cohesive character of the soil supporting piles gives them

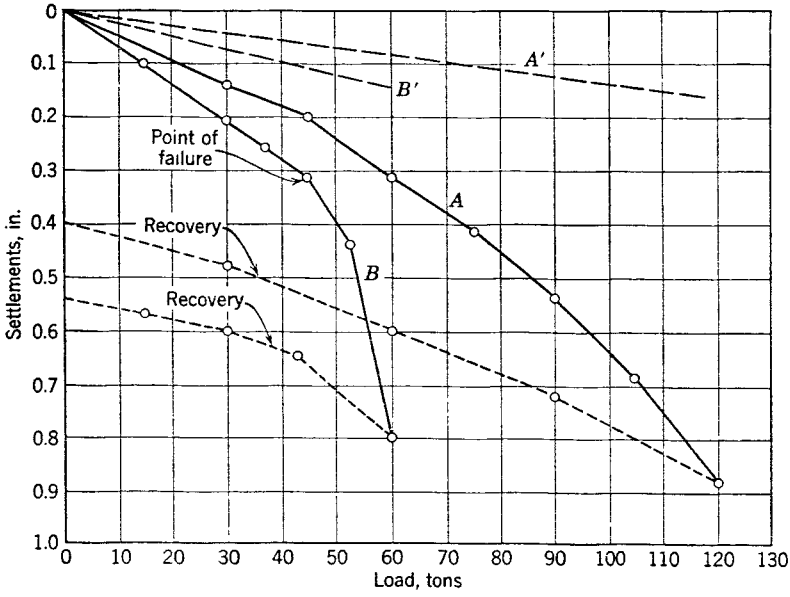


FIG 54 Settlement curves of test piles Curve A is typical of the shape of the settlement curve of a test pile driven into a non-cohesive soil, curve B, into a cohesive soil. A' and B' show the theoretical elastic shortening of the piles When very long piles are driven through fairly firm soil to rock, it is not unusual to have the total settlement less than the theoretical elastic shortening

definite ultimate bearing values, these ultimate values, divided by a factor of safety of 2.5 or 3.0 may yield a safe bearing value less than that indicated by the maximum permissible settlement. For additional discussion of the application of test pile data to foundation design, see Art. 5, Pile Foundations, of Sect. 8.

For further and more detailed discussions of test piles, pile-driving formulas, the test loading of piles and allied subjects, see refs. 9, 10, 11, 12, 13, 16, 20, 22, and 24.

Pumping tests are made to determine the permeability of water-bearing sands and similar relatively pervious saturated soils. Usually they are made to amplify the results of laboratory tests when the coefficient of permeability of a water-bearing stratum must be determined with the greatest practicable precision in order that its yield as a water source, or the inflow from it into a proposed excavation, may be estimated. Pumping tests give much more accurate results in soils relatively free from horizontal variations than in highly stratified materials. A pumping test is, unfortunately, expensive, as it involves drilling a test well, usually 10 in. or 12 in. in diameter, installing a pump of large capacity, and operating it continuously for a considerable time. At least eight observation wells must also be provided. These may be as small as 2 1/2 in. in diameter. Occasionally boreholes drilled during a subsurface exploration may be used as some of the observation wells.

The test well and the observation wells should be drilled through the stratum under investigation to a relatively impervious underlying material as indicated in Fig. 55*a* and *b*. The observation wells should be located on two straight lines intersecting at right angles at

the mouth of the test well, one of the lines being in the general direction of flow of the ground water as estimated by observation of the elevations of the water table in exploratory bores.

The test well should be pumped steadily until the drawn-down elevations of the water in the test well and the observation wells and the yield of the test well have stabilized. It is essential that a condition of stability be achieved before data are taken for computation of the coefficient of permeability or the result will be completely misleading. This may, and often does, involve continuous day-and-night pumping for a number of days. It is also essential that the discharge from the test well pump be piped, or otherwise led away, from the vicinity of the test. If this water is allowed to percolate back into the ground in such a manner as to effect the elevation of the water in one or more of the observation wells the precision and value of the test are destroyed. After stability of the drawn-down water table has been achieved the elevations of the water in the observation wells and the quantity of water delivered from the test well are determined with precision and recorded.

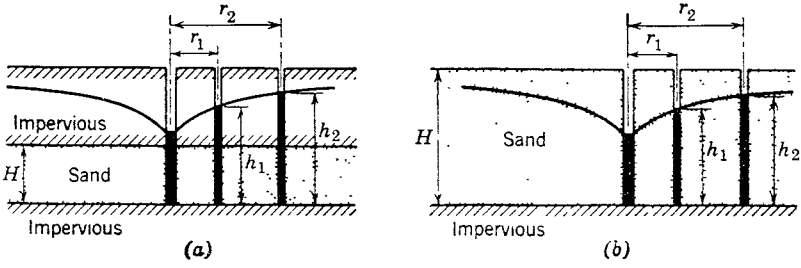


FIG 55 Field determination of permeability Test wells and observation wells (a) Water-bearing stratum underlying an impervious material (b) Water-bearing stratum having a free water table in the stratum. From *Soil Mechanics in Engineering Practice*, K Terzaghi and R B Peck, John Wiley & Sons, 1948

Finally the coefficient of permeability of the soil k is computed by one of the two formulas given below. The first of these applies to the case illustrated in Fig 55*a* in which an impervious stratum overlies or caps the completely saturated stratum of water-bearing soil. The second formula applies to the case shown in Fig. 55*b* in which there is a free water table in the pervious stratum. The formulas are:

$$k = \frac{1.49Q}{H(h_2 - h_1)} \log_{10} \frac{r_2}{r_1} \tag{33}$$

$$k = \frac{2.98Q}{(h_2^2 - h_1^2)} \log_{10} \frac{r_2}{r_1} \tag{34}$$

in which k = the coefficient of permeability of the soil in cm per minute.
Q = the discharge of the test well pump in gallons per minute.
H, h_1, h_2, r_1, and r_2 are the dimensions, in ft. indicated in Fig. 55*a* and *b*

Four values of the coefficient are obtained by using data taken from each of the four pairs of observation wells. Whether the arithmetic mean of the four values should be used in making subsequent computations and estimates, or whether greater weight should be given one or more of the values, is a matter for the judgment of the engineer.

Detailed information on the technique of making pumping tests is given in ref. 25.

In situ Shear Tests. Determination of the in situ shearing strengths of soils has been attempted by field tests based on forcing cone-shaped plungers into the material at the bottoms of drill holes and by measuring the torque required to rotate vane-shaped auger bits which have been forced into the soil at the bottoms of such holes

In the case of sands and non-cohesive soils the shearing strengths of which vary with changes in density and with pressure applied normal to the plane of shear the cone penetration test has little or no value as a means of determining the shearing strength and hence, indirectly, the angle of internal friction ϕ. Nor does there appear to be any reason for anticipating that the penetration of a cone into the material at the bottom of a borehole should generally give information of greater value than the penetration of a sampling device.

The rotating vane has, in a number of cases, given values of the shearing strengths of clays and cohesive soils which agree reasonably with values obtained by other and more

generally recognized methods applied to undisturbed samples. Devices of this type appear well worth further study and development—see ref. 5.

The **California Bearing Ratio field test** is closely allied to the laboratory test of the same name (see Art. 5, p. 7-59). Field tests are made by loading compacted subgrades to determine their bearing capacity. In the California Bearing Ratio field test a plunger of the same area as that used in the laboratory test is forced into a compacted subgrade under strain-controlled conditions approximating as closely as possible those of the laboratory test. As in the laboratory test, the California Bearing Ratio is the ratio, expressed as a percentage, between the pressure required to force the plunger 0.1 in. into the subgrade and 1000 lb per in.^{2}, which is assumed to be the pressure required to force the plunger the same distance into compacted crushed rock.

California Bearing Ratio field tests on an actual compacted subgrade yield information of great value as to the quality of the material, and the CBR tests, both laboratory and field, are the best available measure of soil performance under pavements.

The **Proctor needle,** or plasticity needle, developed by R. R. Proctor, is a device for checking the resistance to penetration and, indirectly, the moisture content of compacted soils. It consists of a small plunger which is placed on the surface of the soil and pressed in a specified distance by means of a calibrated spring and handle. Interchangeable plungers having areas from 1.0 in.^{2} to 0.05 in.^{2} are available. The Proctor needle, like the California Bearing Ratio test, demands extensive experience on the part of the user if consistent results of practical value are to be secured, but in the hands of a skillful and experienced operator it is a useful device for making rapid field checks on the compaction of fills.

7. GEOLOGICAL PROFILES

A geological profile, or profiles, of the site of a project of any importance should be drawn in order that all relevant data on subsurface conditions may be assembled, correlated, and presented graphically in a logical and systematic way to those responsible for major decisions on foundation and earthwork problems. There is no substitute for the preparation of a geological profile as a means of first developing and later presenting the reciprocal relationships between data which may, at first glance, appear to be extremely difficult, if not impossible, to correlate. It is therefore essential that those engaged in the engineering of foundations and earthwork have a thorough knowledge of the construction, interpretation, and utilization of geological or soil profiles.

The development of a geological profile should be started as soon as sufficient data are made available by the early phases of the preliminary subsurface exploration to justify the commencement of work in the drafting room. Information on the classification and recognizable characteristics of soils encountered in the borings and test pits should be entered on profiles of the site, drawn to an enlarged vertical scale, Fig. 56, in order that any systematic stratification may be recognized without delay. There are, of course, cases in which the arrangement of the deposits is altogether erratic and unpredictable. Such conditions are, unfortunately, common in shore deposits, in glaciated regions, and in tropical coastal regions which have been subjected to coral accretions interrupted by intermittent volcanic upheavals. But on most sites, the simultaneous development of profiles on sections at right angles to each other, the occasional use of an isometric sketch drawn to scale with colored pencils, or, more rarely, the construction of a simple three-dimensional model consisting of dowels set in a board and colored to represent soils encountered in the borings and test pits will reveal a probable systematic arrangement of strata that can be confirmed or denied by additional field work or a slight modification of the originally planned program of subsurface exploration.

As the development of the profile proceeds, the topography of the upper boundary of the bedrock or other underlying material which may be considered, for practical purposes, unyielding should, if possible, be ascertained. If there are indications of clay strata of variable thickness, every reasonable effort should be made to ascertain the topography of their upper and lower limits.

The necessity of major changes in the program of subsurface exploration and soil testing will be indicated if the development of the geological profile shows the boundaries of the various deposits to be definitely erratic. In such cases money spent on numerous undisturbed samples and consolidation tests would be largely wasted and an increased number of dry sample borings, and possibly some probings, would be more useful. On such a site a knowledge of the locations of the softest and firmest materials and the general characteristics of these soils is about all the information that can be used effectively in the design of a foundation.

Special profiles on which are plotted important variations in the characteristics of soils are useful in planning certain types of foundation earthwork, and tunneling projects For example, a profile showing variations in the unconfined compressive strengths of an erratic deposit of glacial clays on the site of a tunnel gave information of value to the engineers planning the job. A profile showing variations in permeability may be useful in planning a dam or cofferdam, and one showing variations in penetration resistances as revealed by the logs of dry sample borings or the driving of test piles may be useful to engineers facing an unusual foundation project.

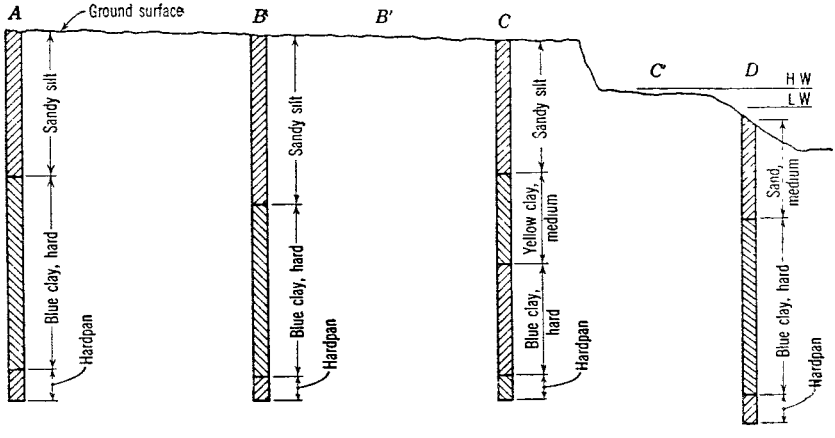


FIG 56 Geological profile—preliminary A rough preliminary geological profile should be drawn as borings are made Borings at A, B, and C indicate the necessity of additional borings at B' and C' to determine the extent of the deposit of medium yellow clay They also indicate the necessity of undisturbed samples of both the hard blue clay and the medium yellow clay

Completed geological profiles for major projects are shown in Fig. 57. A simplified profile sometimes proves valuable as an illustration for a prospectus or non-technical publication intended to give the average intelligent reader some idea of the scope and character of a foundation problem and the methods proposed for its solution.

8. SLOPE STABILITY PROBLEMS

Standard slopes (such as 1.5 to 1 for the sideslopes of excavations, 0 5 to 1 for firm earth, 0.25 to 1 for rock, 4 to 1 for very soft earth, and 1.5 to 1 for earth suitable for use in embankments) were generally used in the design of cuttings, embankments, and earth structures before the practical importance of soil mechanics was realized. That such standard slopes led only occasionally to costly failures is a tribute to the judgment of the men who used them rather than to their intrinsic merit, for standard slopes are in fact safe only under limited conditions In reality there are no safe standard slopes for cohesive soils as their safe angle of slope is a function of many factors, one being the height of the slope. Non-cohesive soils, such as sands, are on the other hand stable at any angle of slope less than the angle of internal friction, irrespective of the height of the slope, unless their stability is affected by water.

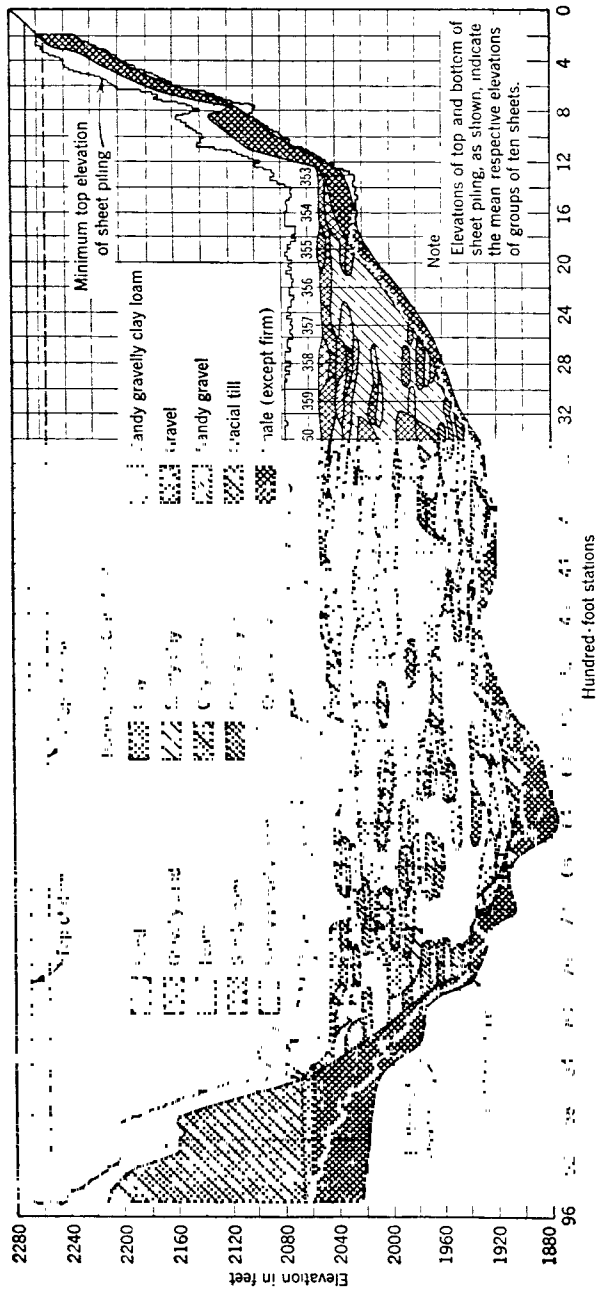
The stability of soils, particularly non-cohesive soils such as sands, is greatly affected by the seepage forces resulting from the percolation of water through them. The force exerted on a unit volume of soil by water seeping through it may be determined by the formula:

$$j = i\gamma_w \tag{35}$$

in which j = the seepage force per unit of volume.
i = the hydraulic gradient.
γ_w = the unit weight of the water.

If foot-pound units are used the above formula gives the value of j in pounds per ft^3 of earth and becomes

$$j = 62.4i$$


Minimum top elevation of sheet piling
Note
Elevations of top and bottom of sheet piling, as shown, indicate the mean respective elevations of groups of ten sheets.
Elevation in feet
Hundred-foot stations

(a)

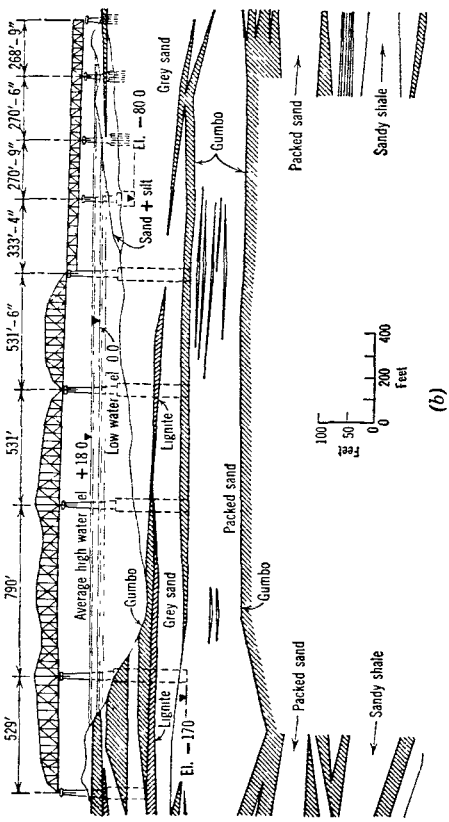


Fig. 57. Geological profiles for major projects. (*a*) Profile along cutoff wall of Fort Peck Dam. From *Fundamentals of Soil Mechanics*, D. W. Taylor, John Wiley & Sons, 1951. (*b*) Geological profile of the Mississippi Bridge, New Orleans, La., from Bridge Foundations, Carlton S. Proctor, *Publications of the International Association for Bridge and Structural Engineering*, Zurich, Vol. V, pp. 261–273, 1938.

Seepage forces act in the direction of flow. If the flow is upward and the resulting hydrostatic uplift is great enough to sustain the submerged weight of the soil, the granular particles lose contact with each other, the material is rendered incapable of supporting a load, and is said to be "quick." Quicksand is therefore any sand in which there is an upward flow the velocity of which is sufficient to produce a hydraulic gradient equal to or greater than the minimum gradient capable of producing a quick condition. This critical hydraulic gradient may be estimated by the formula:

$$i_{cr} = \frac{(G - 1)}{1 + e} \tag{36}$$

in which i_{cr} = the critical hydraulic gradient.
G = the specific gravity of the soil particles.
e = the void ratio expressed as a decimal.

Substituting the specific gravity of quartz sand, 2.65, and the void ratios of submerged sands in their loosest and densest conditions, 0.95 and 0.57 respectively, in the above equation shows that the critical hydraulic gradients of sands lie between 0.85 and 1.05. In practice a value of 1.0 has been found to approximate the critical gradient, but it is usual to design for a gradient of 0.5 or less as sand begins to loosen and thus lose its supporting power before it actually becomes quick.

In the above discussion the direction of flow was assumed to be straight up, a condition readily produced in the laboratory but only occasionally encountered in practice. As seepage forces act in the direction of flow, their vertical components tend to lift soil and their horizontal components to move it laterally. The solution of many soil stability problems therefore demands a reasonably accurate estimate of the direction, velocity, and quantity of the anticipated seepage. Such estimates are usually made by means of flow nets.

A flow net is a graphical device for estimating the paths along which flow will occur and the distributions of the friction losses along such flow paths. Once the flow paths and the distribution of the head losses along them have been determined the quantity of the flow, its velocity and the consequent seepage forces can be estimated if the permeability of the soil is known.

Experimenters have used models of dams, dikes, and cofferdams to study the paths along which water seeps under or through them. Such models are widely used in laboratories for demonstration purposes and are occasionally used in engineering practice as aids to the solution of unusual seepage problems. The models are usually built of sand in glass-fronted flumes, Fig. 58. Dyes introduced into the upstream side of the model produce colored streaks in the soil which are visible through the glass front of the flume and show the direction of the paths of seepage. It is helpful to consider these colored streaks in the soil as "flow lines" and to apply the term "flow paths" to the spaces between adjacent lines. Piezometer tubes set in the walls of the flume show the total head, pressure plus elevation, at many points in the model and give data for drawing equipotential lines, or lines of equal total head, in addition to flow lines on the glass front of the flume. If the model is built of an isotropic soil these equipotential lines invariably cross the flow lines at right angles, the two sets of lines forming a network of "curvilinear rectangles" which, if the spacing of the lines is judicious, become "curvilinear squares." Such a network of flow and equipotential lines is a flow net.

A flow net can be drawn, and in practice flow nets are usually drawn, without the aid of a model by making freehand sketches which are modified and corrected until by a process of trial and error a satisfactory net is produced. Figure 59*a*, *b*, and *c* show three such nets all of which are judged satisfactory because the following criteria are met.

1. The boundary conditions are complied with, inasmuch as certain lines representing fixed physical characteristics of the structures and their sites are incorporated into the nets as flow and equipotential lines.

2. The flow and equipotential lines intersect at right angles to form "curvilinear squares." None of these figures are actually square but the angles at their four corners are right angles and the two lines which divide each square into four smaller ones are of approximately equal length. An exception to the requirement that the figures be square occurs in the first and last area in each flow path. These are usually of irregular form and are known as singular squares. Even they, however, conform to the requirement that all angles are right angles and that lines dividing the areas into four smaller ones are of approximately equal length.

To construct a flow net the boundary conditions are studied and the flow and equipotential lines established by them are identified and drawn. Within these boundaries

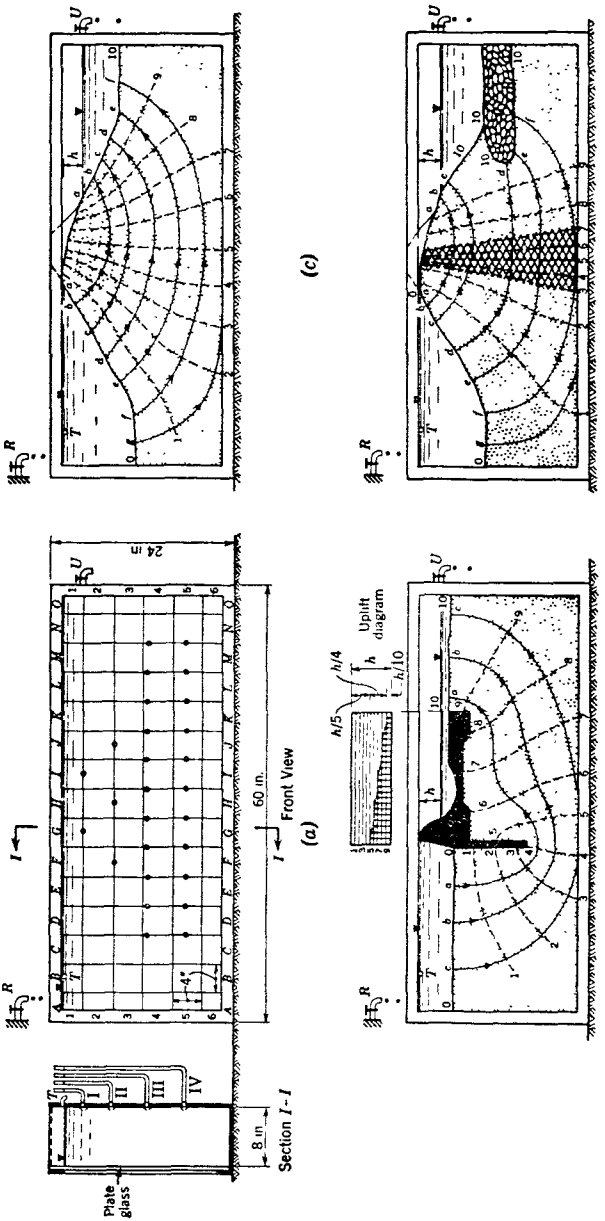


FIG 58. Seepage flume for studying flow through and under dikes, dams, and cofferdams (a) Diagrammatic drawing of seepage flume with rows o piezometer tubes I, II, III, IV in the sheet metal back of the flume (b) Flume used to study flow through pervious soil under masonry dam. (c) and (d) Models of earth-fill dams in seepage flume. The unbroken curved lines a-a, b-b, etc , are flow lines. The dashed lines 1-1, 2-2, etc , are equipotentials. By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G P Tschebotarioff, copyright, 1951, McGraw-Hill Book Co.

are then sketched flow and equipotential lines forming a net which is modified and adjusted until the criteria outlined above are fully met. It should be noted that the first line drawn when the construction of a flow net is started determines the position of every line of the net. D. W. Taylor's Procedure of Explicit Trials, in ref. 20, commended to beginners in flow net construction, is based on this fact. Taylor's procedure is to draw a trial line, preferably a flow line separated from one of the boundaries by a single flow path, and divide

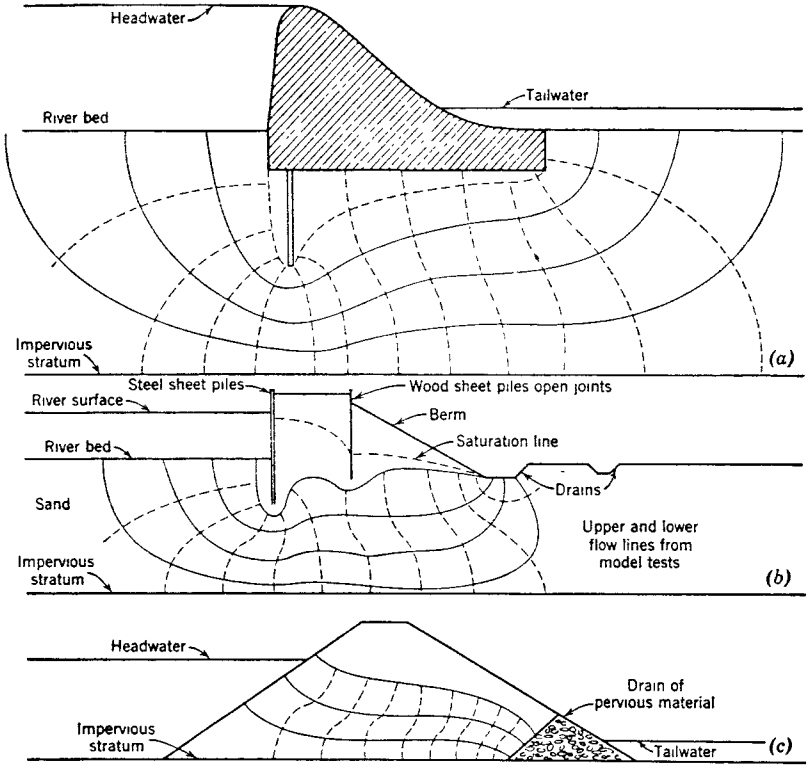


FIG 59 Flow nets (a) Flow net for a concrete dam built on pervious material underlain by an impervious stratum Sheet pile cutoff reduces percolation under the dam (b) Flow net for a large cofferdam The drains and the toe of the slope of the berm should be surfaced with gravel or broken stone to inhibit erosion and sloughing (c) Flow net for an earth fill dam on an impervious clay stratum The drain of pervious material at the toe is necessary to prevent the emergence of flow lines on the slope and consequent sloughing

the path into squares by sketching equipotential lines, Fig. 60*a*. If, as in this case, fractional squares result, the flow path should be widened or narrowed by moving the initially drawn flow line up or down until the number of squares into which the path is divided is an integral number, Fig. 60*b*. The net should then be expanded by adding squares until it is complete or until an inconsistency develops, such as is illustrated in Fig. 60*c* in which a flow line intersects the lower boundary. This shows that the first line was wrong and that its position must be changed and the process of constructing the net repeated until a satisfactory net, Fig. 60*d*, is produced.

Engineers experienced in the art of flow net construction can produce satisfactory nets with surprisingly few trials by sketching the entire net and modifying and correcting it more or less in its entirety instead of constructing it from a trial line. Beginners are apt to draw too many lines. Four or five flow paths are generally sufficient for most practical purposes.

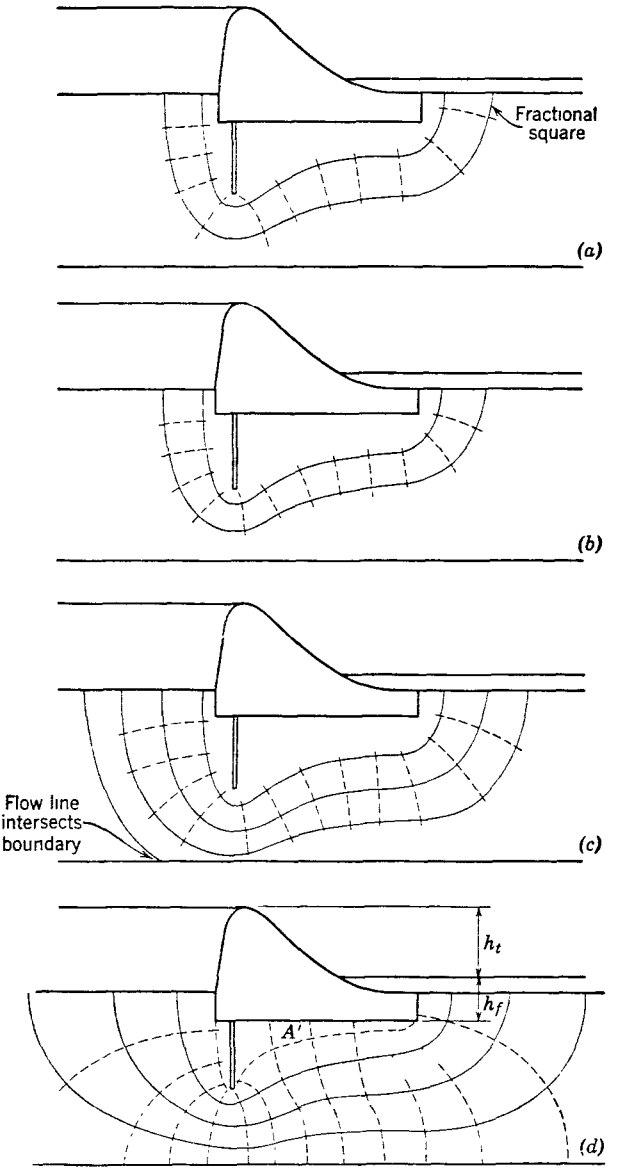


FIG 60 Drawing a flow net—steps taken to produce the net of Fig 59a (a) The dam and boundaries of the net—the river bottom and the impervious stratum—are drawn Two flow lines are drawn and the flow path between them divided by equipotential lines Modification of the path is indicated by the undesirable fractional square (b) A more satisfactory attempt at drawing the first flow path. (c) Expansion of b leads to the intersection of a flow line with the lower boundary This demands a change in the entire net. (d) The final net In drawing flow nets the repetition of mistakes can be avoided by working on tracing paper, each stage of the development of the net being worked out on a sheet laid over the previous trial

The total seepage may be estimated, after a satisfactory flow net has been constructed, by the formula:

$$Q = \frac{n_f}{n_d} k h_t \tag{37}$$

in which Q = the quantity of seepage in ft³ per second per lineal foot of the length of the dam or structure
n_f = the number of flow paths in the net.
n_d = the number of equipotential drops—spaces between equipotential lines—in the net.
k = the coefficient of permeability in ft per sec.
h_t = the head loss in feet between the surfaces of the head- and tail-waters.

The value of Q is therefore the product of three quantities, one the head, one the coefficient of permeability, and one the ratio between the number of flow paths and equipotential drops in the flow net. If the net has been properly drawn this ratio will not be affected by the number of paths and equipotential drops in it.

If the coefficients of permeability of the soil in horizontal and vertical directions differ materially, as they do in most natural soils, the section must be transformed by multiplying all horizontal dimensions by a scale factor before the flow net is drawn. The formula for the scale factor is:

$$\text{Scale factor} = \sqrt{\frac{k_v}{k_h}}$$

in which k_v = the coefficient of permeability in a vertical direction.
k_h = the coefficient of permeability in a horizontal direction.

The formula for the quantity of seepage per lineal foot of dam length then becomes:

$$Q = \frac{n_f}{n_d} h_t \sqrt{k_v k_h} \tag{38}$$

As the true relationship between the horizontal and vertical permeabilities of soil in situ is difficult to determine it is advisable in many cases to draw two flow nets, one with the maximum and the other with the minimum probable difference between the horizontal and vertical permeabilities The net yielding the less favorable solution to any problem under consideration should then be used in the solution of that problem.

If the dam itself is pervious, as are earth fill dams, the flow net extends into the earth structure itself. In such cases the upper flow line must be located by judicious application of such requirements as that it must meet the equipotential line of the upstream face of the dam at right angles and that, as it is an atmospheric pressure line, the only head that can exist along it is elevation head Therefore there must be uniform differences in elevation between the points at which the equipotential lines intersect the upper flow line. Figure 59*c* illustrates this and also shows the virtual necessity of some kind of filter or drain in the downstream toe of an earth fill dam to prevent the emergence of flow paths on the downstream face and consequent sloughing of the soil by seepage unless the slope is very flat

Flow nets are useful in the solution of such stability problems as estimating the uplift pressure on the under side of a dam. Consideration of the significance of the equipotential lines of a flow net shows the intensity of the uplift pressure at any point to be equal to the pressure due to the elevation of the headwater surface less the pressure drop along the flow path to the point in question. Thus in Fig. 60*d* the pressure under the dam at the point A' will be:

$$P = (h_t + h_f - {}^{8}/_{13}\, h_t)62.4$$

in which P = the pressure in lb per ft².
h_t and h_f are the dimensions in feet indicated in Fig. 60*d*.

The fraction $^{8}/_{13}$ is the number of pressure drops (8) in the flow path from its start to the point A divided by the total number of pressure drops in the path (13).

Complex seepage problems, though frequently met in engineering practice, are beyond the scope of this brief treatment. Their solution frequently involves the use of either models or electrical analogy devices. For thorough treatments of the subject see refs. 7, 14, 20, 22, and 24.

Capillarity. Capillary action may augment seepage through earth fill dams, dikes, and cofferdams and thus promote sloughing of their downstream faces. In cold climates it also promotes frost heave and frost boils. On the other hand the presence of capillary water in fine-grained non-cohesive soils, particularly fine sands, may increase their stability materially.

The major portion of the seepage through and under earth fill dams and similar structures follows paths such as are shown on the flow net in Fig. 59*c* and occurs in the saturated soil below the water table The flow line forming the upper boundary of the net represents the sloping water table characteristic of earth structures of this type. However, the soil above the water table is not usually dry, for the zone of so-called capillary saturation extends upward from it to the height of the capillary rise of the soil, h_c. In this zone the soil is nearly, but rarely completely, saturated by water drawn into, or held in, its interstices by capillary action. Above the zone of capillary saturation there lies a zone of contact moisture in which water is held in isolated small interstices and in menisci surrounding contact points of the soil particles. In the zone of capillary saturation, but not in the zone of contact moisture, there is an appreciable seepage flow which may issue from the downstream face of the structure in sufficient quantity to promote sloughing, Fig. 59*c*, unless the drain in the downstream toe is so located as to keep the surface of the zone of capillary saturation below the surface of the soil. Seepage in the zone of capillary saturation may also cause capillary siphoning over an impervious core of inadequate height, Fig. 61. The quantity of seepage through the capillary zones of earth structures is not usually important unless it promotes instability of the soil, though there are cases where raising the height of a core wall may be justified by the resulting reduction in leakage due to capillary siphoning. The quantity of seepage through the zone of capillary saturation of a small laboratory model of a dam or earth structure might, however, be as much as 25% of the total seepage and therefore of great significance if the model were made of fine sand. Models are therefore usually made of coarse sands with a slight capillary rise as their effectiveness is independent of the permeability of the material of which they are built.

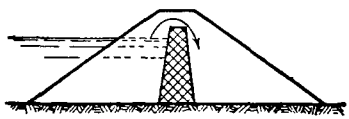


FIG 61. Capillary siphoning The loss or leakage over an impermeable core wall of inadequate height may be considerable even though the crest of the core is above the elevation of the surface of the impounded water From *Soil Mechanics in Engineering Practice*, K Terzaghi and R B. Peck, John Wiley & Sons, 1948.

The stabilizing effect of capillary moisture in fine-grained non-cohesive soils, particularly fine sands, may be marked and important. Tension in the water of the meniscı draws the soil particles together with appreciable force; the resulting friction between the particles gives the soil sufficient strength in shear to stand on steep slopes for heights of 5 or even 10 ft or to present, within the tidal range, the firm surface of a beach at low tide even though the beach sand, when either submerged or completely dry, may be notoriously unstable.

Frost heave and frost boils result from the freezing of water held in the soil by capillary action and the subsequent progressive freezing of additional water drawn up to the growing ice crystals and masses by osmotic action. If a silt, silty sand, or varved clay has a capillary rise such that the upper part of the zone of capillary saturation is above the frost line, heaving and the formation of frost boils are probable Once started, the action continues progressively until thick lenses of ice form, heave up the soil, and reduce it to a mushy boil when the spring thaws occur. Similar action may take place behind a retaining wall, the pressure exerted by the development of the ice lenses so far exceeding that due to the earth pressure for which the wall was designed that it is shoved over, a little at a time, winter after winter, until it ultimately fails. Frost heave does not occur in coarse sands or gravels in which the capillary rise is so small that the zone of capillary saturation lies below the frost line nor in impervious clays which, although they have a large potential capillary rise, are so impermeable that the upward migration of water through them is negligible.

The soils in which frost heave is most likely to be troublesome are uniform soils having more than 10% of grains smaller than 0.02 mm and mixed-grained soils having more than 3% of the total material smaller than 0.02 mm. Varved clays are particularly troublesome.

The prevention of frost heave is best accomplished by the use of subgrade materials of such grain size that they are not subject to heave. Attempts to prevent frost heave by the introduction of a layer of coarse sand or gravel through the large interstices of which capillary migration cannot occur have not in general been successful.

Thermo-osmosis, or the migration of water from warmer to cooler areas, has been observed in fine-grained clays even though the cooler areas were far above freezing temperatures. This phenomenon has been responsible for the swelling of clays and the heaving of earth-supported floor slabs and shallow foundations of light buildings in Africa and Cuba.

Drainage lowers the water table, promotes the stability of soil by eliminating seepage pressure and increasing shearing strength, and reduces the danger of frost heave. Permanent drainage produces these effects for long periods to insure the utility of an area or stability of an earth mass. Temporary drainage is undertaken for limited periods, its usual purpose being to permit subsurface construction work.

Drainage to control permanently the elevation of the water table in an earth structure or in an agricultural area is usually effected by drains of perforated, porous, or open-jointed pipe. The venting of water-bearing strata underlying an earth structure may, in addition, require relief wells consisting of vertical pipes placed by well-drilling methods and slotted or otherwise perforated where they traverse the strata from which water is to escape. In both cases the material adjacent to the pipe should be of such size that it does not enter and, at the perforations or openings, not pass through them in sufficient quantity to clog the drain or pipe. This may necessitate surrounding the pipe with an artificially placed filter, usually of gravel and coarse sand or a mixture of the two. To prevent sealing the openings and, at the same time, to exclude significant quantities of fines, the D_{50} size of the material surrounding the pipe and in actual contact with it should be about equal to the least dimensions of the slots, perforations, or openings. In other words, 60% of the material by weight should have a grain size smaller than the openings. To prevent the soil in contact with the filter from clogging its interstices, the D_{15} size of the filter material should not be more than four or five times the D_{85} size of the finest adjacent soil. Also, to keep seepage forces within permissible limits, the D_{15} size of the filter material should be at least four or five times the D_{15} size of the coarsest surrounding soil. Compliance with these requirements may demand the construction of a graded filter, consisting of concentric "rings"—in reality cylinders—of artificial mixtures of gravels and sands, so graded that the grain-size relationships stipulated above are met between the soil and the outer ring of the filter and then successively between the various filter rings.

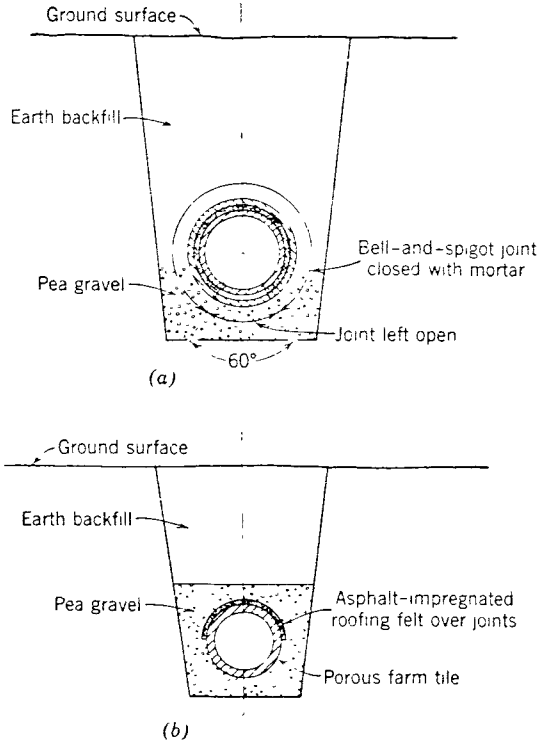


Fig. 62. Subsurface drains. The openings through which water enters a subsurface drain should be at the bottom, or at least in the lower part, of the pipe. Sketch *a* shows a successful method of providing such openings in the joints of bell-and-spigot pipe. Sketch *b* shows a widely used method of laying butt-jointed porous farm tile.

The infiltration of fine materials into drains can be further inhibited by providing perforations or openings in the lower quarter only of the perimeter of the pipe, Fig. 62. This precaution plus a filter of pea gravel has proved satisfactory in some fine Florida sands containing no silt or cohesive matter and in Louisiana sandy silts which contain appreciable quantities of cohesive material.

Predrainage, or drainage in advance of excavation, may be effected by well points, deep wells, a combination of the two, and occasionally by means of sumps. To expedite the drainage of fine-grain soils of low permeability with well points, the so-called vacuum system may be used and soils too fine for even the vacuum system have been successfully drained by using electro-osmosis to accelerate the flow to the well points.

Well points used in predrainage systems are usually either 2 in. or 2 1/2 in. in diameter, 40 in. long, perforated, covered with brass strainer cloth, and open at the lower end to facilitate jetting them into place. A widely used and successful point is fitted with check valves which prevent the jetting water from escaping through the screen while the point is placed and also prevent soil and water from being drawn into the open end of the point

when suction is applied. Well points should be jetted into place in such a way as to wash out a hole of considerable size around the point. This hole is filled with coarse sand to provide a filter around the point. Well points are usually driven from 3 ft to 6 ft apart, depending upon the permeability of the soil, and connected to a 6-in or 8-in. header pipe,

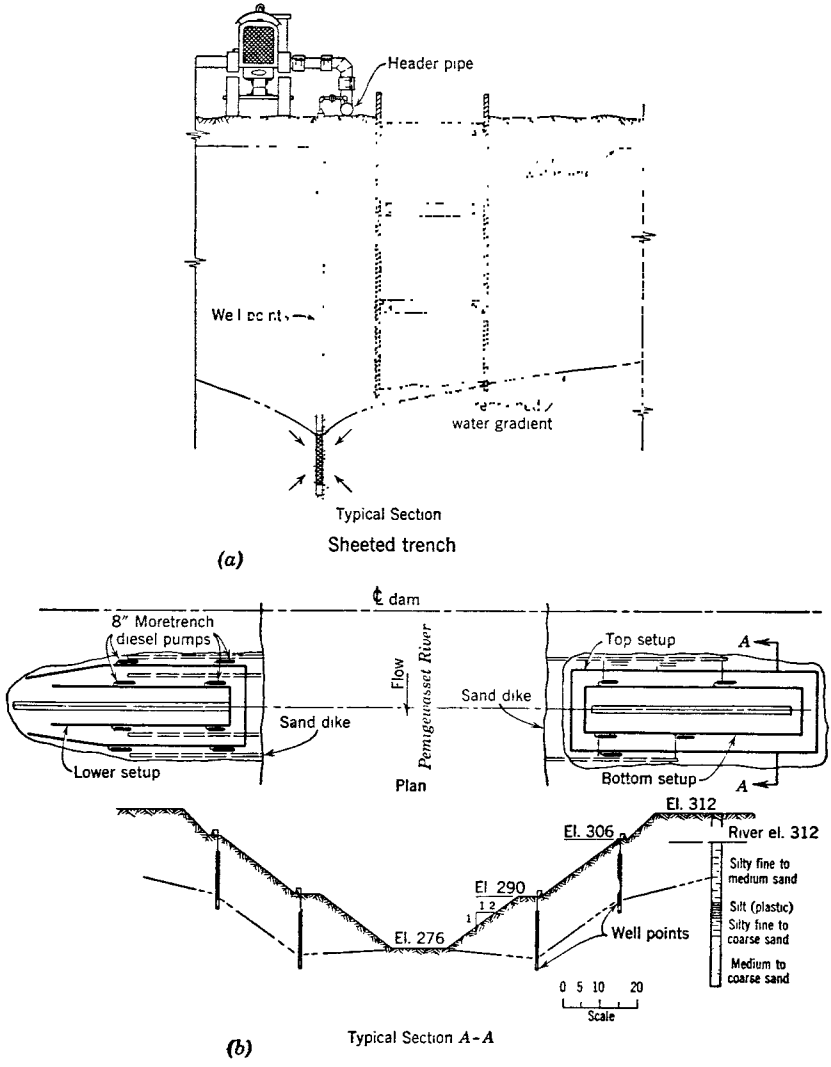


FIG 63. Well point systems (a) Single tier system. (b) Multiple tier systems. Courtesy of the Moretrench Corporation

Fig. 63a, which in turn leads to a centrifugal pump A single row of points will usually suffice to predrain the site of a narrow excavation, as indicated in this figure. Normally a 6-in. pump, having a capacity of from 1000 to 1200 gallons per minute, will handle the flow from 500 ft or 600 ft of header pipe. If, however, there is reason to anticipate that unusually permeable soils will be encountered, a pumping test should be made to determine necessary pipe sizes and pump capacities before planning and installing an extensive well-point system The maximum depth to which the water table can be drawn down by a

single row of well points is 18 ft. Predrainage to this depth can usually be accomplished by pumping continuously for from 2 to 6 days. It may take much longer, perhaps several months, in a soil of low permeability. Drainage to a greater depth than 18 ft may be effected by several tiers of well points installed successively as excavation proceeds as shown in Fig. 63*b*.

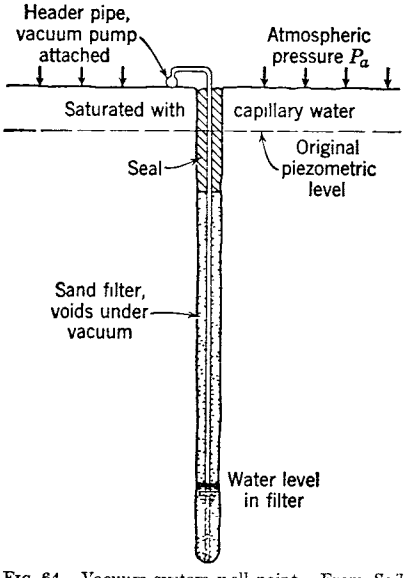


FIG. 64 Vacuum system well point From *Soil Mechanics in Engineering Practice*, by K Terzaghi and R. B. Peck, John Wiley & Sons, 1948.

The **vacuum method** is a variant or refinement of the ordinary well-point method which extends the latter's usefulness to much finer grained soils than it could otherwise drain. If the D_{10} size of a soil is less than 0.05 mm, gravity flow to well points will rarely, if ever, be fast enough to make predrainage practicable unless a vacuum system is used. In setting points for a vacuum system, the upper part of the hole washed out by the jetting water is backfilled with a tightly tamped impervious clay, Fig. 64. The well points are usually set 3 ft apart and a single 6-in. centrifugal pump will normally handle the water from 500 or 600 linear feet of header pipe. One or two vacuum pumps are connected to each header pipe in addition to the centrifugal pump. Positive displacement, gear-driven vacuum pumps are mounted on and incorporated into some centrifugal pumping assemblies built for well-point work. The beneficial effect of the vacuum is not limited to acceleration of the flow to the points. The atmospheric pressure applied to the soil when "a vacuum is drawn on the points" increases its shearing strength greatly and thus enhances its stability.

Electro-osmosis has been used to accelerate the flow of water to well points through extremely fine-grained non-cohesive soils too impermeable for drainage by even the vacuum method. In one successful application of this method by German engineers at Trondheim

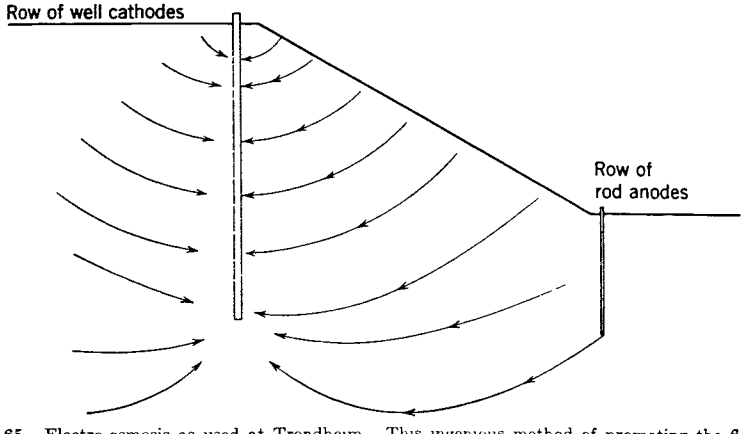


FIG 65. Electro-osmosis as used at Trondheim This ingenious method of promoting the flow of water to drainage wells through soils of low permeability was developed by Leo Casagrande and used successfully to unwater a "difficult" site for the construction of submarine pens for the German navy during World War II

well-point cathodes 21 ft long were spaced 30 ft apart with anodes of the same length located midway between them, the arrangement being shown diagrammatically in Fig. 65. The method is still in the experimental stage and little has been published about the effect

of such variables as the conductivity of the water due to dissolved salts, or the energy required to make the system effective in soils of different types. Indications are that the energy required on large jobs may vary between 0.4 and 1.0 kwhr per yd^3 excavated and may be as much as 10 kwhr per yd^3 on small jobs.

Deep-well drainage is usually resorted to when an excavation must be carried to such depth that predrainage by a single lift of well points is impossible. Figure 66 shows a typical deep-well drainage system. Deep wells are installed by well-drilling methods and are usually 8 in. or more in diameter. They are pumped by motor-driven vertical-spindle centrifugal pumps. The installation of a deep-well system should usually be preceded by a pumping test to determine the necessary sizes and capacities of the wells and pumps. A row of well points, indicated on Fig. 66, at the bottom of the sideslope of the deep excavation is frequently necessary to intercept seepage flow between the deep wells. Such flow

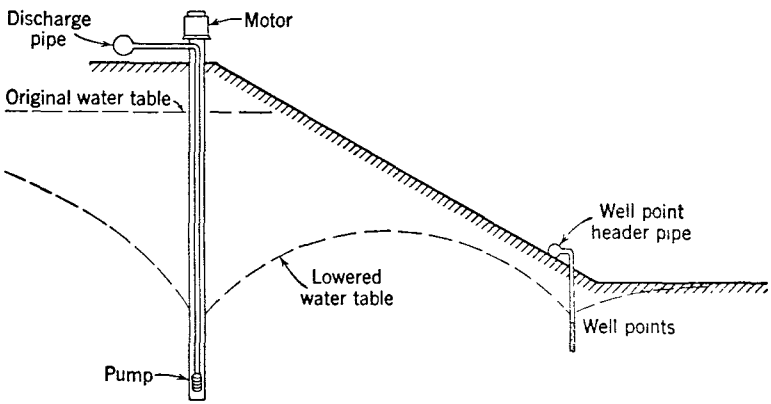


FIG 66. Deep-well drainage system The deep wells dispose of most of the water, the row of well points near the toe of the excavation slope being insurance against the emergence of flow lines and consequent sloughing

if not intercepted will emerge at the toe of the slope and cause annoying sloughing, if not serious slides.

Siemens' method of predrainage is a German predecessor of the well-point and deep-well predrainage systems commonly used in the United States

Sumps as predrainage devices were used occasionally in Europe before the Siemens method was developed in Germany and the well-point method came into use in the United States. Today, however, sumps are an expensive and slow method of predraining as compared to well points and should be used only when necessary equipment for the installation of a well-point system is unobtainable. Sumps for predrainage are in effect surface wells dug by means of small clamshell or orange-peel buckets inside timber or steel sheet pile cofferdams from which water is pumped with any available equipment. Inverted graded filters consisting of layers of progressively coarser materials may be necessary at the bottoms of the sumps to prevent boiling and consequent inflow of sand.

Drainage during excavation is usually accomplished by pumping from sumps in the bottom of the hole. In an excavation of any considerable size drains located near its perimeter lead the water to the sump or sumps. The sideslopes and gradients of the drains should be flat to minimize scour. If the excavation is in a non-cohesive soil such as sand, it may be necessary to line the bottom of the drainage ditches with gravel and to place a thick layer of gravel in the bottoms of the sumps to inhibit scour and "boiling" of the sand due to the upward flow of water through it. The lining material, especially that placed in the bottom of sumps, may take the form of an inverted filter consisting of layers of progressively coarser materials beginning with a coarse sand or fine gravel in contact with the soil and ending with a coarse gravel or crushed rock. Inverted filters inhibit boiling more effectively than simple layers of gravel or crushed rock but are expensive to build initially and must be constantly rebuilt as the excavation is deepened. Sheet piling is usually necessary to support the sides of deep excavations particularly if the soil is non-cohesive. An internally braced cofferdam, resulting from this use of sheet piling, Fig. 67, should be made of sufficient size to permit leaving a berm of earth around its perimeter as indicated in the figure. A circumferential drain should be at the toe of the slope of this berm as

indicated. "Boiling" of non-cohesive soils in the bottoms of cofferdams is much less likely to occur if such berms are provided than if drains are located close to the sheet piling.

Well points, vacuum systems, and bleeder wells and even electro-osmosis may be used as adjuncts to drains and sumps where drainage is being provided as excavation proceeds. It is usual, however, to apply these methods to excavations which are predrained.

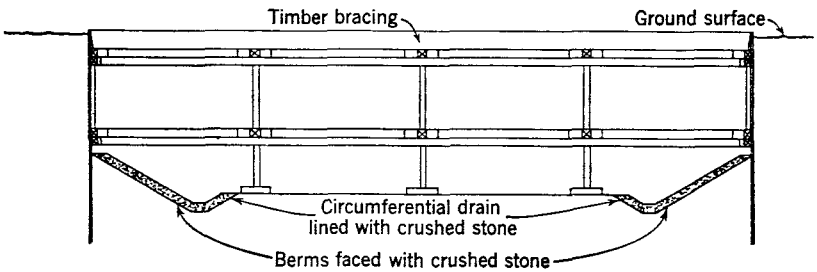


FIG. 67 Internally braced land cofferdam in non-cohesive soil In cofferdams of this type in water bearing non-cohesive soils internal berms are frequently necessary to prevent a concentration of flow lines and consequent upward flow causing boiling of the bottom near the sheet piling This is even more probable in the corners than along the sides of the cofferdam The circumferential drain keeps the working surface dry. The stone lining of the drain and face of the berm inhibits sloughing, boiling, and erosion.

Drainage during excavation is often cheaper than predrainage and, furthermore, has the advantage that the failure of the pumps, due to a power failure or engine breakdown, usually causes nothing more serious than a quiet filling of the excavation or cofferdam. On the other hand, failure of the pumps on a well-point, vacuum, or deep-well drainage system may bring about a rapid development of hydrostatic and seepage pressures sufficient to cause a serious boiling or even a blowing up of the bottom of an excavation.

Cuts in Sand. The stability of the sideslopes of cuts in sand depends on the angle of repose of the soil and the extent to which it is disturbed or stabilized by water or eroded by wind.

The angle of repose, the angle between a horizontal plane, and the maximum slope at which the soil is stable, is a recognized characteristic of non-cohesive soils. The angle of repose of sand is slightly less than its angle of internal friction, but the difference is so small that the angles may be assumed equal. Laboratory determinations of the angles of repose and internal friction of sand are generally unsatisfactory because of the difficulty of taking, transporting, and mounting specimens of sand in laboratory apparatus without disturbing their structure Fortunately the angles of repose and internal friction of sands do not vary widely, and it is accepted practice to assume them to be between 30° for loose and 35° for dense sands.

Both dry and completely submerged sands, if not disturbed by seepage or eroded by flowing water, wave action, or wind, will stand to any height at slopes not exceeding their

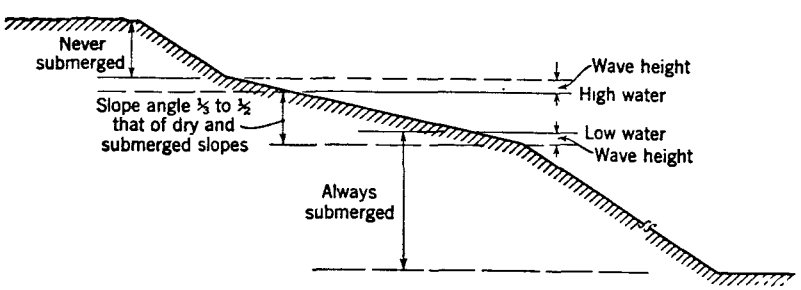


FIG 68 Canal and reservoir banks in non-cohesive soil

angles of internal friction. But, as sands are readily disturbed by seepage and wave action, the slope angles of reservoir and canal banks in sand, within the range of normal fluctuations of water level plus twice anticipated wave heights, should be no steeper than one-half and preferably one-third the angle of internal friction of the soil, Fig. 68.

The disturbing effect of water seeping out of the sand of a reservoir or canal bank during a period of draw down may be estimated by drawing a flow net and calculating the magnitude of the seepage forces. The method is described in ref. 20. Unfortunately, the method involves many assumptions as to characteristics of the soil. Therefore it is generally more satisfactory to assume safe slopes than to estimate them by calculations based on assumptions more difficult to check than assumptions as to the slope angles themselves. Knowledge that similar slopes have proved satisfactory elsewhere under similar conditions is the best guide to the determination of safe slope angles in such cases.

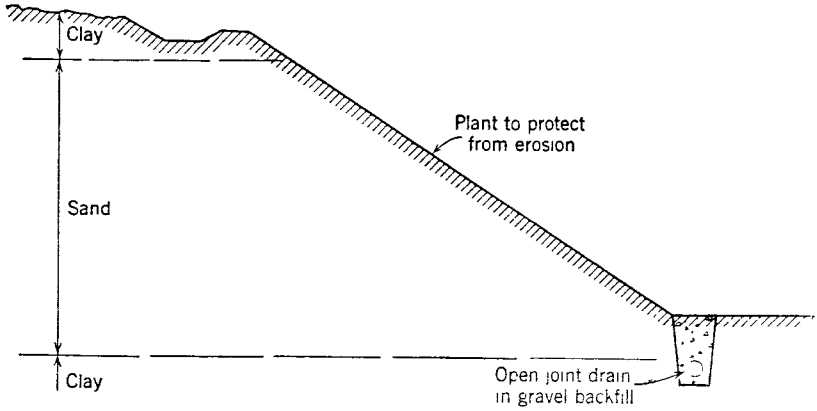


FIG 69 Slope protection in non-cohesive soils Non-cohesive soils must be protected from disturbance by water and wind Planting is always indicated The drains shown will be necessary only when the non-cohesive material is over- or underlain by impervious soil, or the elevation of the water table is such as to cause an outflow from the lower part of the slope

Exposed sand slopes should be protected from erosion by wind and water. The most practical method is suitable planting. Intercepting ditches along the top of sidehill cuttings in sand may be necessary to prevent scour, particularly if the overlying soil is impervious. The emergence of seepage water from sand slopes underlain by impervious soils should also be prevented. This may require a subsurface drain near the toe of the slope. Figure 69 illustrates both expedients.

Seepage water flowing into a sand slope and capillary water, particularly contact moisture in fine damp sand, have marked stabilizing effect This enables some fine-grained damp sands to stand at very steep slopes to heights of 5 ft or more. The stability thus given by contact moisture is, however, temporary. Slides which occur when such sand dries may be dangerous to men laying pipe in trenches not more than 5 ft deep.

For methods of estimating the pressures on braced sheet piling supporting the sides of deep cuts in sand see Art. 10, Earth Pressures.

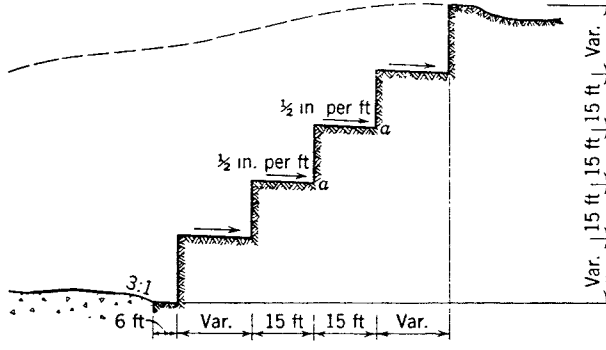


FIG. 70. A cut in loess By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G. P. Tschebotarioff, copyright, 1951, McGraw-Hill Book Co

Dune sands and other wind-deposited sands are poorly compacted and notoriously unstable. The void ratio of such a sand often greatly exceeds the critical void ratio e_c and the soil if saturated and subjected to shock may suddenly "liquefy" and cause serious flows.

Cuts in loess should have practically vertical faces. The particles of this fine-grained, wind-deposited, and loosely compacted soil are slightly cemented, and the material has sufficient shearing strength to stand vertically in unsupported cuts of considerable height. Water destroys the cementing agent in loess, and the soil not only erodes readily but softens and compacts if saturated. Loess must be protected from erosion and combinations of saturation and increase in loading if its stability is to be maintained. Figure 70 illustrates a successful type of deep cut in loess

Cuts in Clay and Cohesive Soils. The stability of the sideslopes of cuts in clay and cohesive soils depends upon the angle of slope, its height, the unit weight of the soil, its cohesion, angle of internal friction, whether or not it is cracked and fissured, the presence or absence of pockets and strata of water-bearing sand, and the presence or absence of seepage.

A cohesive soil of uniform strength and consistency which is neither cracked, fissured, nor subjected to seepage forces but develops a tension crack at the upper part of the surface of rupture will stand vertically to a so-called critical height which may be estimated by the formula:

$$h_{cr} = \frac{1.29}{\gamma} q_u \tag{39}$$

in which h_{cr} = the critical height in feet.
q_u = the unconfined compressive strength of the soil in lb per ft^2.
γ = the unit weight of the soil in lb per ft^3.

This formula is applicable both to purely cohesive soils and to partially cohesive soils which possess appreciable angles of internal friction in addition to cohesion. It is not applicable to submerged cohesive soils which usually soften and lose strength under water if allowed to expand.

Uniform cohesive soils in the absence of seepage will stand on slopes to maximum heights which may be estimated by the formula:

$$H = \frac{c}{\gamma}\left(\frac{\sec^2 i}{\tan i - \tan \phi}\right) \tag{40}$$

in which H = the height of the slope in ft.
c = the cohesion of the soil in lb per ft^2.
i = the slope angle.
γ = the unit weight of the soil in lb per ft^3.
ϕ = the angle of internal friction of the soil.

Where $i < \phi$ the height becomes infinite and the material, like sand, will stand to any height at a slope angle not exceeding its angle of internal friction.

The above formulas are not applicable to stiff fissured clays in which numerous cracks reduce shearing strength to an indeterminate extent, nor to soils containing pockets or strata of water-bearing sand. Such soils defy analytical determination of their stability. The best guides in dealing with them are experience and study of the behavior of similar materials. In such soils the drainage of water-bearing strata by porous drains surrounded by reverse filters may be necessary.

Cylindrical or circular slides, sometimes called "Swedish breaks," in cohesive, or partially cohesive, soils have caused many failures of earth slopes, retaining walls, bulkheads and quay walls, Fig. 71.

Estimating the stability of slopes and structures against cylindrical slides is simple in theory but time-consuming because it involves finding the critical cylindrical surface upon which a slide is most probable by a series of trials rather than by the application of a formula locating its center and giving its radius Essentially the process consists of:

(*a*) Selecting by judgment and experience a possible center of rotation, such as the point *a* on Fig. 71, and drawing the cylindrical sliding surface $b - c$.

(*b*) Estimating the forces, and the moments of the forces, which tend to produce and to inhibit rotation of the earth mass about the axis of the cylinder and consequently to produce or inhibit shear failure on its cylindrical surface. Computing the factor of safety against rotation about the chosen center and entering the factor against the plotted center on the drawing.

(*c*) Selecting other centers of rotation, computing the factors of safety applicable to them, and entering these factors on the drawing alongside the centers to which they apply.

(*d*) Finally by trial and error determining the probable location of the most critical center of rotation and the factor of safety applicable to it.

Many graphical techniques and short cuts are used by those who make circular-slide analyses frequently. Valuable ones are described in refs. 20 and 21. For the practitioner who makes circular slide analyses occasionally, however, the basic method of slices is recommended and described below. Such an analysis should be based on the following data.

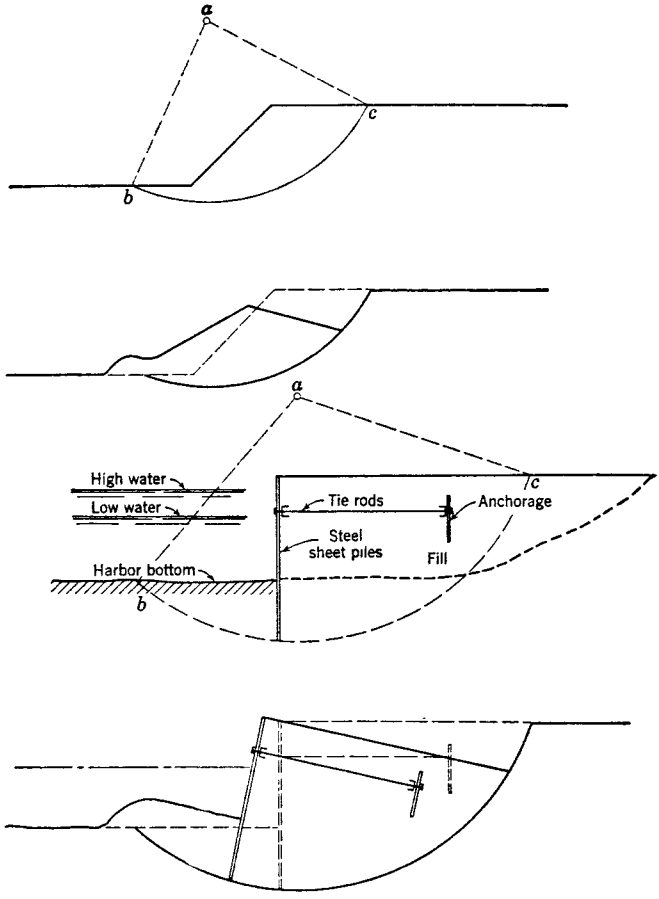


FIG 71. Cylindrical slides, sometimes called Swedish breaks The stability of slopes, embankments, and waterfront structures against this type of failure should be carefully considered when dealing with cohesive or partially cohesive soils

(*a*) An accurate cross section of the slope showing the boundary limits of each classification of soil and for each soil the unit weight, cohesion c, and angle of internal friction ϕ. All structures located on or near the slope should be shown with their weights and loadings.

(*b*) The rate of any appreciable seepage through the section and—if such seepage be present—a flow net from which seepage forces may be estimated.

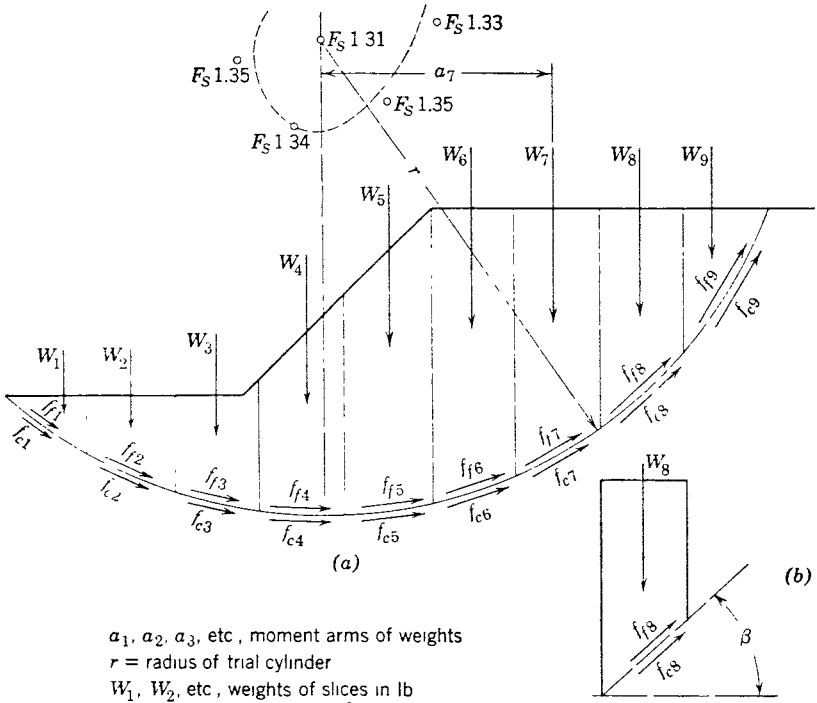
(*c*) The possibility and probability of a slow or rapid draw down of the water in a partly or completely submerged case.

The steps involved in estimating the factor of safety against sliding on a cylindrical surface are then:

(*a*) A center of rotation is selected, and a circular sliding surface drawn.

(*b*) The section is divided into a convenient number of slices, Fig. 72, and the forces acting on each slice are estimated, the following simplifying assumptions being made: (1)

A longitudinal section of the cylindrical earth mass of unit length—usually 1 ft—is studied. (2) No cognizance is taken of the restraining influence of frictional and cohesive forces acting on the end planes of the mass. (3) Similarly, no cognizance is taken of frictional and cohesive forces acting on the vertical planes separating the slices.



a_1, a_2, a_3, etc., moment arms of weights
r = radius of trial cylinder
W_1, W_2, etc., weights of slices in lb
c = cohesion of soil in lb/ft^2
ϕ = angle of internal friction of soil
Assumed length of slice perpendicular to plane of drawing = 1 ft

$f_{f8} = W_8 \cos \beta \tan \phi$
$f_{c8} = l_8 c$
l_8 = length of arc at bottom of slice number 8

Net driving moment M_D = algebraic sum of weights and arms ΣWa
Resisting moment $M_R = r\Sigma(f_f + f_c)$

Factor of safety $F_S = \dfrac{M_R}{M_D}$

FIG. 72 Cylindrical slide analysis by the method of slices. See text for complete description. The method is a cut-and-try procedure. Each of the values of F_S used in sketching the F_S = 1.34 "contour" was obtained by making a complete analysis with the center of rotation at the indicated point.

(*c*) The weight and line of action of the resultant weight of each slice is estimated. Surcharges are considered weights. The buoyed weight, rather than the total weight, of that part of a slice lying below the free water surface alongside the slope in a partly submerged case is considered. In a completely submerged case all weights are buoyed weights.

(*d*) The magnitudes, directions, and locations of the resultants of any appreciable seepage forces are estimated.

(*e*) The algebraic sum of the moments about the center of rotation of the weights and seepage forces acting on the several slices is computed. This is the net driving moment, M_D.

(*f*) The resisting forces due to cohesion and friction acting on the surface of sliding of each slice are estimated. The cohesive force for each slice is computed by the formula.

$$f_c = cl$$

in which f_c = the cohesive force in lb acting tangentially on the cylindrical surface to resist rotation of the mass above the surface.
c = the cohesion in lb per ft^2.
l = the arc length of the bottom of the slice.

The sum of all the cohesive forces Σf_c is the arithmetic sum of the forces f_c for the various slices.

The frictional force acting tangentially on the curved surface at the bottom of each slice is computed by the formula:

$$f_f = W \cos \beta \tan \phi$$

in which f_f = the frictional force in lb acting tangentially on the cylindrical surface to resist rotation.
W = the weight of the slice.
β = the angle with the horizontal made by the cord drawn across the arc at the bottom of the slice.

The sum of the frictional forces Σf_f is the arithmetic sum of the forces f_f for the various slices.

(*g*) The resisting moment is then computed by the formula.

$$M_R = r(\Sigma f_c + \Sigma f_f)$$

in which r = the radius of the sliding circle under investigation and the other terms are as defined above.

(*h*) The factor of safety is computed by the formula.

$$F_S = \frac{M_R}{M_D}$$

Low factors of safety are the rule rather than the exception in such analyses, factors as low as 1.3 being considered reasonably safe when based on adequate knowledge of soil characteristics and seepage forces.

(*i*) The process is repeated with other centers of rotation, the factor of safety applicable to each being entered against its plotted position, Fig. 72, until enough factors of safety have been determined and plotted to permit sketching "iso-F_S lines" or "F_S contours" as in Fig. 72. These may be used to locate the probable position of the center of the most critical circle. Computation of the F_S for this center serves as a check on the entire procedure.

Typical critical circles for several common cases are shown in Fig. 73. Thus where soils having appreciable angles of internal friction ϕ stand on steep slopes critical circles pass through the toe of the slope as in Fig. 73*a*. If the angle of internal friction is slight, critical circles may, and probably do, intersect the base as in Fig. 73*b*. If ϕ equals zero and the slope angle is less than 53° the depth of the critical cylindrical slide is theoretically infinite. Low values of ϕ are undoubtedly provocative of deep-seated slides, but in practice their depth is invariably limited by a firm stratum to which the cylindrical sliding surface is tangent, Fig. 73*c*.

Draw down of the free water surface on a partially submerged slope, or on a completely submerged one, if the draw down causes emergence of the slope, reduces stability. A slow draw down accelerates seepage and increases seepage forces. Making a quantitative estimate of their increase involves drawing a flow net showing the effect on the rate and direction of seepage produced by a given rate of draw down.

A sudden draw down accelerates seepage and changes the effective unit weight of the soil lying between the elevations of the old and new free water surfaces from the buoyed to the saturated weights. Rapid draw downs are therefore provocative of cylindrical slides, and their possibility should be considered in all analyses of slopes that are normally either partly or completely submerged.

The stability of embankments depends upon the stability of their sideslopes and that of the soil upon which they rest.

Embankments of non-cohesive soil must have sideslopes complying with the requirements given for the sideslopes of cuts in sand. To obviate flow slides it is essential that embankment fills be compacted sufficiently to insure a void ratio less than the critical void ratio e_c of the material. Protection from erosion by planting is usually necessary. A large embankment of non-cohesive soil resting on an impervious material in a location where the rainfall is heavy may need subsurface drainage at the toe of one or both slopes to prevent

the emergence of seepage and consequent sloughing as in the analogous case of the cut in sand illustrated by Fig. 69.

Embankments of cohesive soils should be designed with due regard to the limiting relationship between height, slope angle, cohesion, angle of internal friction, and unit weight discussed in Art. 8 under Cuts in Clay and Cohesive Soils. In addition cylindrical-slide analyses should be made to determine the stability of the embankment slopes with respect to slide failures. Proper compaction of the soil is vital, and to this end the moisture content should be kept, if possible, within 2 or 3% of the optimum as determined by compaction tests described in Art. 9 (see also Sect. 9, Art. 8).

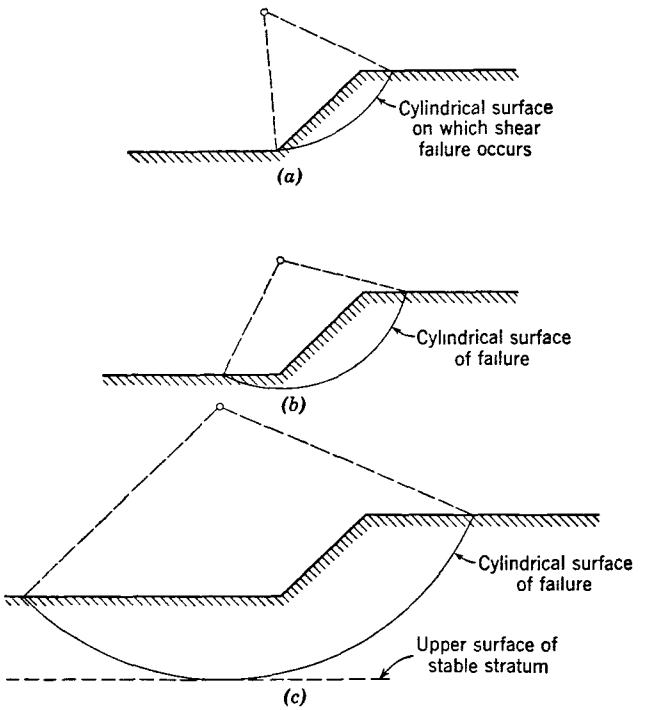


FIG. 73 Cylindrical slides, also called Swedish breaks and rotational slides, have characteristic shapes in different types of soil. The diagrams show the shape characteristics of rotational slides in three types of soil Note that all the soils have cohesive characteristics Slides of this type do not occur in purely non-cohesive soils (*a*) Partially cohesive soils with an appreciable angle of internal friction ϕ produce slides in which the cylindrical surface of failure may be tangent to but does not intersect the base (*b*) In partially cohesive soils with a small value of ϕ the cylindrical surface of failure may intersect the base but will not go far below it. (*c*) In purely cohesive soils having no appreciable angle of internal friction ($\phi = 0$), deep-seated cylindrical slides of this type may occur.

Embankments of either non-cohesive or cohesive soils may be destroyed by failure of the material upon which they rest; in other words, they are, like all structures, subject to foundation failures. The most common type of embankment foundation failure is a shear failure of a soft cohesive subsoil, Fig. 74. Such a failure, if the embankment itself is of non-cohesive material, will probably result from a deep-seated slide in the subsoil as in Fig. 74*a*. If the embankment is built of cohesive soil, the slide may involve both the embankment slope and the subsoil as in Fig. 74*b*. Guarding against such failures involves thorough cylindrical-slide analyses of all embankments resting on cohesive soils. The stability of soft cohesive soils called upon to support embankments or other heavy loads may be improved by expediting their consolidation by means of sand drains or vertical filter wells. The occurrence of cylindrical slides in soft cohesive substrata may also be inhibited by careful control of the spreading of fills and the placing of counterweight berms as the work progresses, Fig. 75.

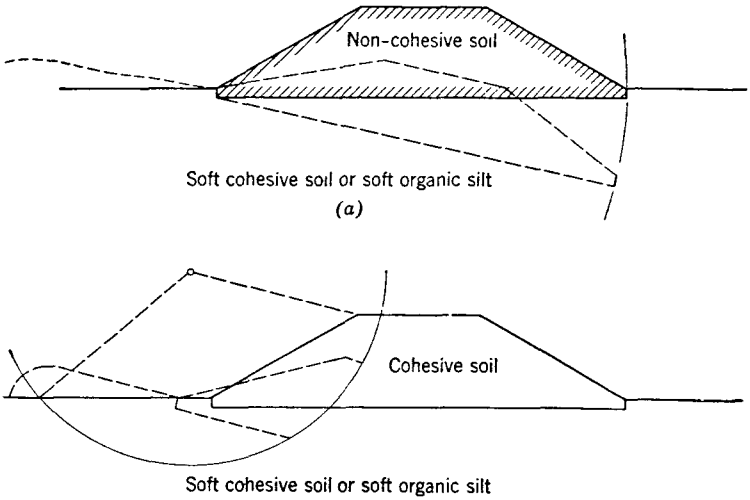


FIG 74. Cylindrical-slide failures of the foundations of earth structures Embankments, dams, and similar earth structures are, like structures built of other materials, subject to foundation failures. The types of soil involved determine the characteristic of the failure as indicated above (a) An embankment of stable non-cohesive soil founded on an unstable soft cohesive soil, organic silt, or muck may rotate as a unit The center of rotation of the cylindrical soil mass will not necessarily be at one corner of the embankment. (b) If the embankment itself is built of cohesive soil the cylindrical surface of shear failure may pass through the embankment and the underlying soft material.

Heavy loads brought onto substrata by embankments may cause not only shear failures but also troublesome settlement. Prediction of the amount and rate of settlement of the foundation of an earth structure, such as an embankment, is undertaken by methods described in Art. 9, Foundation Settlement and Stability Problems.

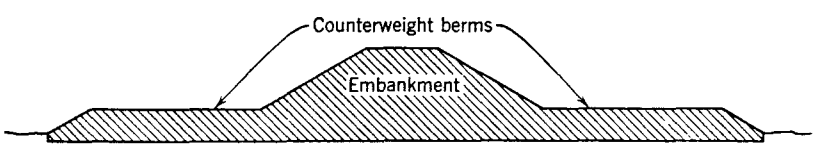


FIG 75 Counterweight berms Cylindrical or rotational slides can frequently be prevented by the use of counterweight berms where an embankment must be built on a soft soil so deep that its removal and replacement by a more stable material is impracticable. A fill of this type should be built in thin layers, each of which should be spread over the entire width to be covered before the next layer is started Similarly it is necessary to avoid large abrupt changes in the depth of the fill longitudinally if deep-seated shear failures, evidenced by the development of mud waves, are to be avoided

9. FOUNDATION SETTLEMENT AND STABILITY PROBLEMS

A satisfactory foundation is reasonable in cost, its total and differential settlements and the rate at which they occur are acceptable, and, if supported on cohesive soil, it possesses adequate stability against shear failure and its concomitant and frequently destructive settling and tipping.

No phases of foundation engineering make greater demands on the experience of the practitioner than do predicting settlements, particularly settlements of cohesive soils, and determining the stability of foundations supported by them. Formulas cannot take the place of good judgment in identifying and estimating the thickness and drainage conditions of each stratum, which should be dealt with independently in preparing a settlement estimate, or in making necessary assumptions as to the physical characteristics of soils which rarely, if ever, possess the homogeneity contemplated by procedures which must be used when the possibility of shear failure is under investigation.

Settlements of non-cohesive soils can best be predicted by methods based on loading tests described in Art. 6. The constant C_1 of the soil and, if it has an ascertainable though slight cohesive characteristic, the constant C_2 should be determined as therein described. The coefficient of settlement C_ρ should be computed by the formulas given in Art. 6 for the width and depth of foundation under consideration. The settlement is estimated by the formula:

$$\rho = \frac{q}{C_\rho} \tag{41}$$

in which ρ = the predicted settlement in in
q = the load on the soil in lb per ft^2.
C_ρ = the coefficient of settlement in lb per ft^2 per in.

Settlements of cohesive soils may similarly be predicted by methods based upon loading tests, the coefficient of settlement being determined by procedures described in Art. 6 and the settlement estimated by the above formula. This formula, when applied to a cohesive, or partially cohesive, soil, yields only an estimate of the total anticipated settlement without giving information as to its probable rate and duration. As settlements of heavy foundations resting on deep beds or strata of saturated soft clays may continue for decades and even centuries, it is generally accepted practice to estimate not only their magnitudes but also their rates and durations. These estimates are based on consolidation tests.

The consolidation test and the basic principles underlying its application to settlement predictions are discussed in Art. 5. That discussion is now amplified to describe the application of consolidation tests to practical problems of settlement prediction. The data needed are

1. An accurate geological profile of the site and, for each stratum of cohesive soil of significant thickness, its:
2. Specific gravity G.
3. Water content w.
4. Void ratio e.
5. Modulus of volume change m_v'.
6. Coefficient of permeability k.
7. Pressure-void ratio curve upon which the minimum, most probable, and maximum preconsolidation pressures have been indicated.
8. The existing pressure p_1 on a horizontal plane at the mid-depth of the stratum.
9. The anticipated pressure p_2 which will occur on a horizontal plane at the mid-depth of the stratum after construction and loading of the proposed foundation.

Methods of obtaining items 1 through 7 enumerated above have been described under Art. 5, Soil Mechanics Laboratory Tests.

The existing pressure p_1 on a horizontal plane a known distance below the surface of the ground is the weight of a vertical prism of soil of unit cross section extending from the ground surface to the plane upon which the pressure is desired. In computing the pressure the total unit weight γ_t of the soil above the water table and the buoyed unit weight γ_b of the soil below it should be used.

The anticipated unit pressure p_2 on a horizontal plane a depth z below the bottom of the foundation and at a point a horizontal distance r from the centroid of its area is estimated by adding the pressure due to foundation loads to the pressure due to remaining overburden and subtracting from their sum the pressure due to overburden removed.

If the greatest dimension of the footing or foundation does not exceed $z/3$, the pressure due to foundation loads may be estimated without significant error by the so-called point load system using the curves of Fig. 76 and the formula:

$$\sigma_z = QI_Q \tag{42}$$

in which σ_z = the pressure in lb per ft^2 on a plane at depth z below the bottom of the foundation and horizontal distance r from the centroid of its area.
Q = the foundation load in kips considered as concentrated at the centroid of its area.
I_Q = a coefficient obtained from the curves of Fig. 76.

If the greatest dimension of the foundation or footing exceeds $z/3$, the point load system may be used by subdividing the footing into areas the greatest dimensions of which do not exceed $z/3$, prorating the entire load Q to these areas and finding the sum of the pressures due to the resulting individual point loads.

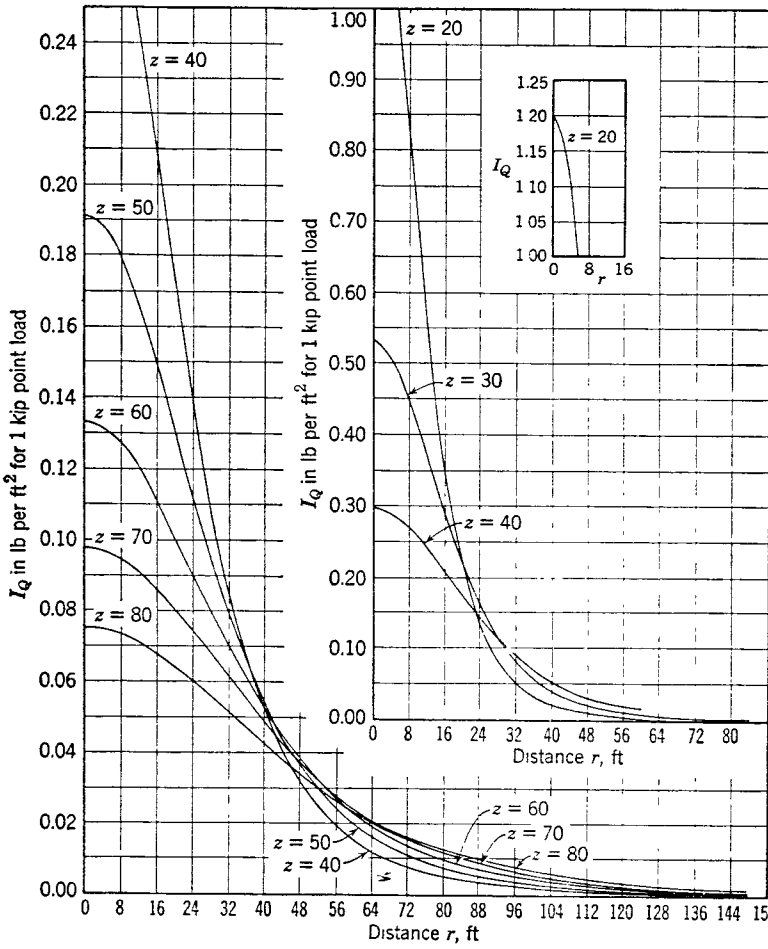


FIG 76 Influence lines for vertical stress for 1-kip point load Boussinesq equation

$$I_Q = \frac{P}{z^2} \cdot \frac{1}{(1 + r^2/z^2)^{5/2}} \cdot \frac{3}{2\pi}$$

Courtesy Philip C Rutledge

Another method of estimating σ_z when the greatest dimension of the footing or foundation exceeds $z/3$ is based on the curves of Fig. 77 and the formula

$$\sigma_z = qI_\sigma \tag{43}$$

in which σ_z = the pressure in lb per ft² on a horizontal plane at a point under a corner of the loaded rectangular area as indicated in Fig. 77.

q = the intensity of the loading on the soil immediately under the foundation in lb per ft².

I_σ = a coefficient obtained from the curves of Fig 77.

When the pressure at a point not below a corner of the rectangular load area is to be determined, the area is divided into four rectangles each of which has a corner directly above the point at which the pressure is desired Thus to determine the pressure at the centroid of a rectangular foundation its area is divided into four equal rectangles each

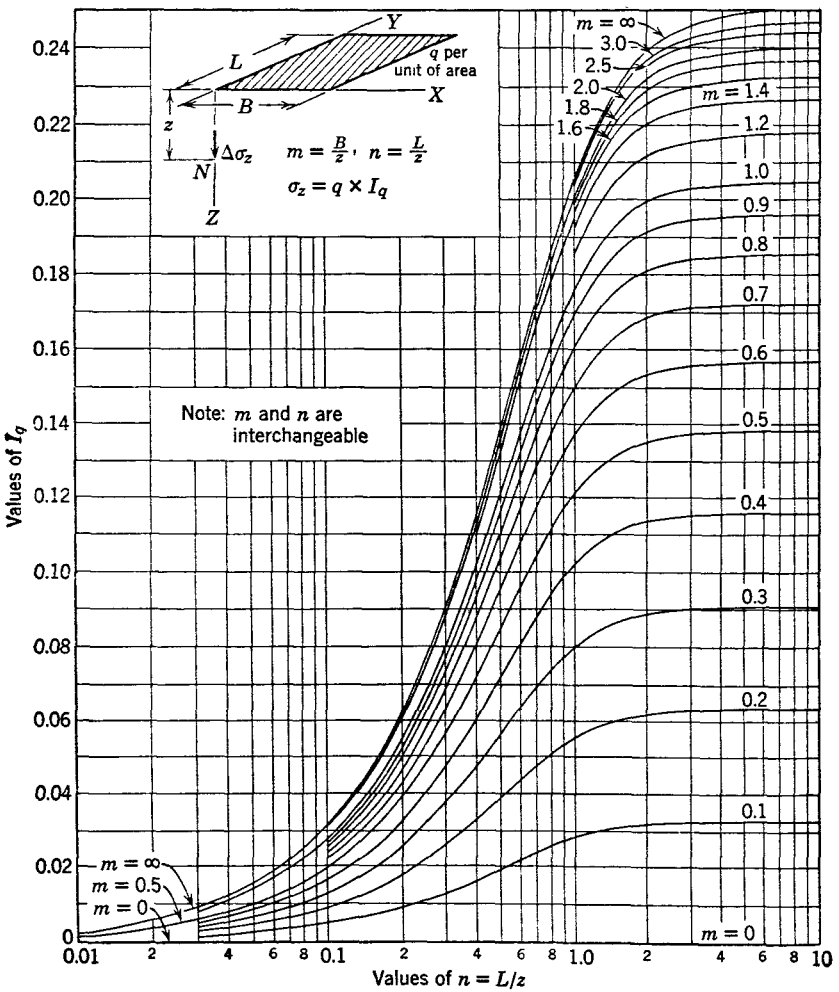


FIG 77. Graph for determining influence value for vertical normal stress σ_z at point N located beneath one corner of a uniformly loaded rectangular area Courtesy R E Fadum.

having a corner over the point at which the pressure is desired. The pressure σ_z is the sum of the pressures due to each of the four rectangles. Similarly if the pressure at a point such as A' in Fig. 78 is to be determined the area is considered to be composed of the rectangles $AFBH$, $AHEG$, $AGDJ$, and $AJCF$ each of which has a corner over the point at which the pressure is to be determined. The pressure is then the algebraic sum of the pressures due to the loads on the individual areas or

$$\sigma_z = \sigma_{AFBH} - \sigma_{AHEG} - \sigma_{AJCF} + \sigma_{AGDJ}$$

If the loaded area is a long narrow rectangle, such as a wall footing the width of which does not exceed $z/3$, the pressure produced on a horizontal plane may be determined by the line load diagram of Fig. 79 and the formulas appearing on the figure. Note that the

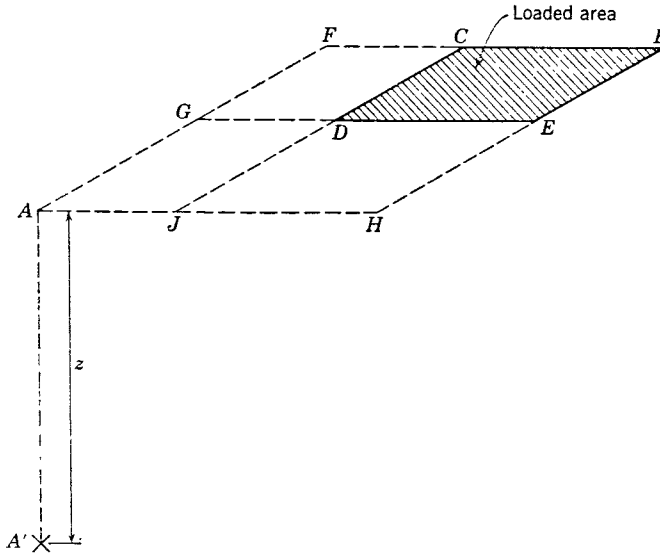


FIG 78 Estimating soil pressure at a point not under a corner of the loaded area by means of Fig 77 and this chart By adding and subtracting stresses produced by imaginary loaded areas, each of which has a corner over the point A', the stress due to a load on the area $DCBE$ may be estimated See text for full explanation.

method gives the pressure at the point having the coordinates x, y, and z, and that this point is abreast of one end of the loaded line. Pressures at points other than those abreast of an end of the loaded line are obtained by methods analogous to those described for using the loaded area diagram of Fig. 77 for finding the pressure at points other than those under corners of a loaded rectangle.

If the pressure on a substratum due to the weight of an embankment of triangular cross section, Fig. 80, is to be estimated the following formulas are useful:

$$\sigma_{zh} = \frac{q_h}{\pi}\alpha \tag{44}$$

$$\sigma_{zt} = \frac{q_h}{\pi}\sin\alpha\cos\alpha \tag{45}$$

in which σ_{zh} = the pressure on a horizontal plane under the heel of the triangular fill at depth z.
σ_{zt} = the pressure on the same plane under the toe of the embankment.
z = the depth of the plane
q_h = the unit load under the heel of the fill.
α = the angle indicated on Fig. 80.

The above formula may be applied to an embankment having a flat top and different sideslopes, Fig. 81, by algebraic addition of the pressures due to assumed extensions of the

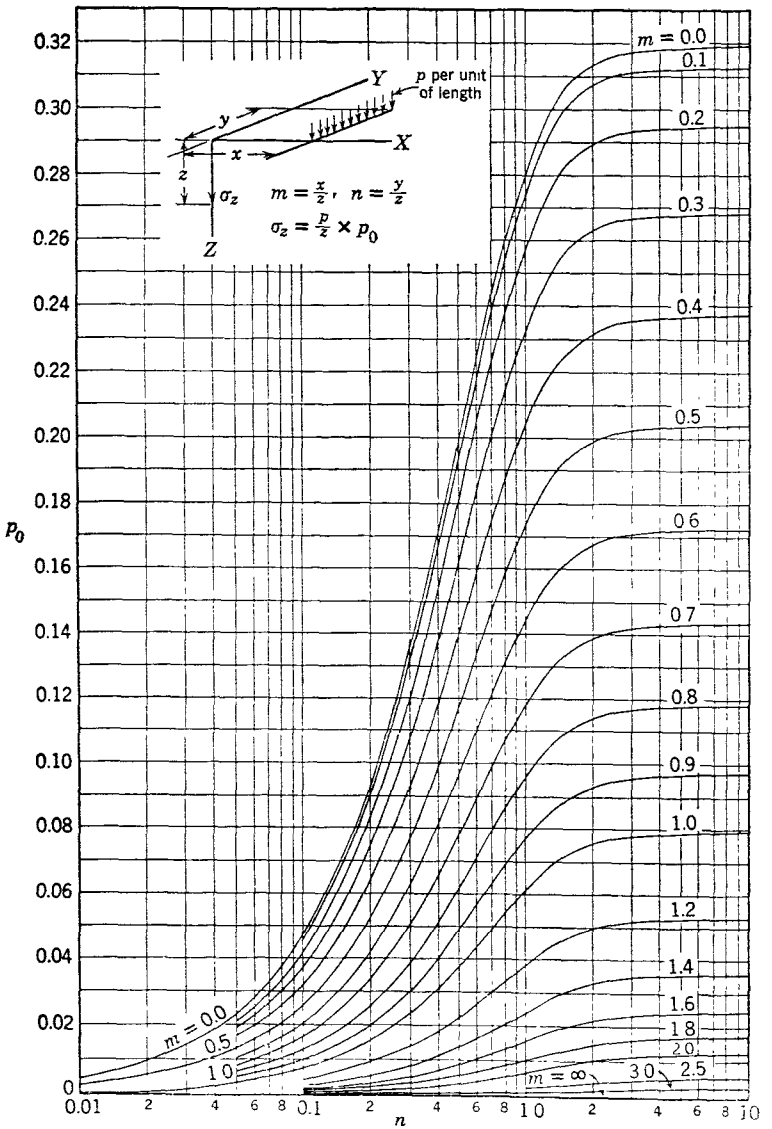


FIG 79 Line of finite length uniformly loaded (Boussinesq case). Chart prepared by Dr R E. Fadum.

slopes. Thus, referring to Fig. 81, the pressure σ_{zA} at A would be:

$$\sigma_{zA} = \frac{q_1}{\pi}\alpha_1 + \frac{q_2}{\pi}\alpha_2 - \frac{q_3}{\pi}\alpha_3 - \frac{q_4}{\pi}\alpha_4 \tag{46}$$

In making settlement estimates it is usually satisfactory to consider the existing and anticipated average pressures p_1 and p_2 acting on each significant stratum to be the pressures at their mid-depths. Slightly greater precision may be achieved by making separate

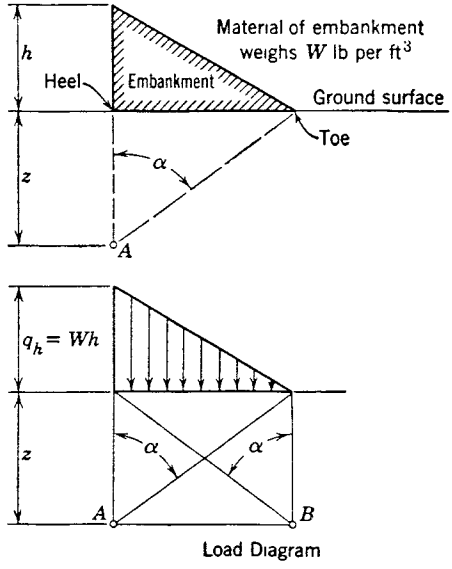


FIG 80 Pressures under a triangular embankment. The angle α in the first formula must be expressed in radians. The practical utility of the formulas is that they can be combined to estimate the pressures under an embankment of normal cross section; see Fig. 81.

$$\text{Pressure at } A = \sigma_{zh} = \frac{q_h}{\pi}\alpha$$

$$\text{Pressure at } B = \sigma_{zt} = \frac{q_h}{\pi}\sin\alpha\cos\alpha$$

computations for the pressures σ_t, σ_m, and σ_b at the top, mid-depth, and bottom respectively of each stratum and averaging them by Simpson's rule

$$\sigma_{avg} = {}^1/_6\,(\sigma_t + 4\sigma_m + \sigma_b)$$

taking the resulting average values as p_1 and p_2.

The estimated existing average pressure p_1 and the anticipated average pressure p_2 on each cohesive stratum of significant thickness should be plotted as in Fig. 82 as abscissas against the mid-depths of the strata as ordinates, and the upper and lower limits of each stratum should be indicated on the plot as in the figure. The probable preconsolidation pressures for each stratum should then be determined by methods described in Art. 5 under Consolidation Tests and plotted alongside p_1 and p_2 as indicated. If, as in Fig. 82*a*, the anticipated pressure p_2 is less than the probable preconsolidation pressure, the settlement will be along the flat upper part of the pressure-void ratio curve and will be slight. If, as in Fig. 82*b*, p_2 exceeds the probable value of the preconsolidation pressure, settlement will be in part along the steeper section of the pressure-void ratio curve and may be considerable. If both p_1 and p_2 exceed the probable value of the preconsolidation pressure as in Fig. 82*c*, the soil is still consolidating under its own weight, consolidation will be accelerated by the weight of the proposed structure, and rapid and extensive settlement along the steep part of the pressure-void ratio curve should be expected.

The amount of the anticipated settlement of each stratum should in any event be estimated by the formulas given in Art. 5 under Consolidation Tests. The times required for the achievement of various percentages of the total settlements should also be com-

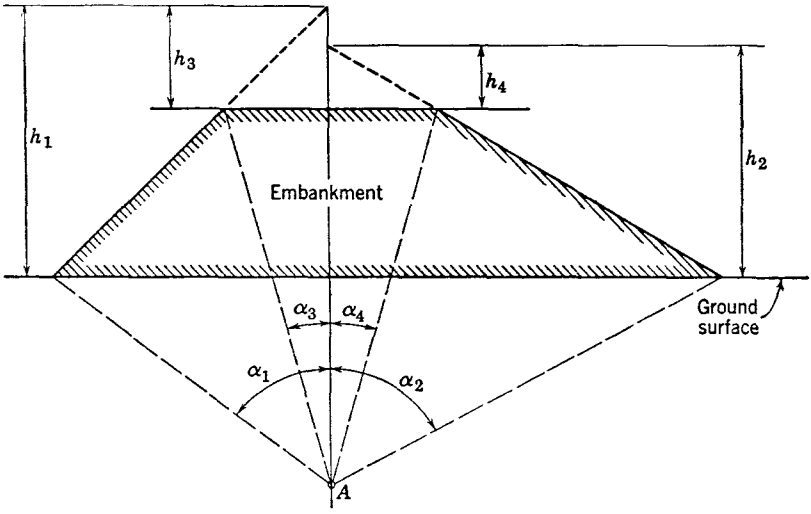


FIG. 81 Pressure under an embankment Note that the sideslopes of the embankment may differ. See Fig 80 and the text for detailed explanation. Pressure at $A = \sigma z_A = \frac{q_1}{\pi}\alpha_1 + \frac{q_2}{\pi}\alpha_2 - \frac{q_3}{\pi}\alpha_3 - \frac{q_4}{\pi}\alpha_4$; all angles expressed in radians. Material of embankment weighs W lb/ft^3; $q_1 = Wh_1$; $q_2 = Wh_2$; $q_3 = Wh_3$; $q_4 = Wh_4$.

puted, and, finally, the anticipated settlements of the various footings and parts of the foundation should be plotted against times in years, Fig. 83, in order that the extent of probable differential settlements may be studied. These differential settlements should be discussed with the designer of the superstructure and the necessity determined of reducing or eliminating them by modifying the design of the foundation.

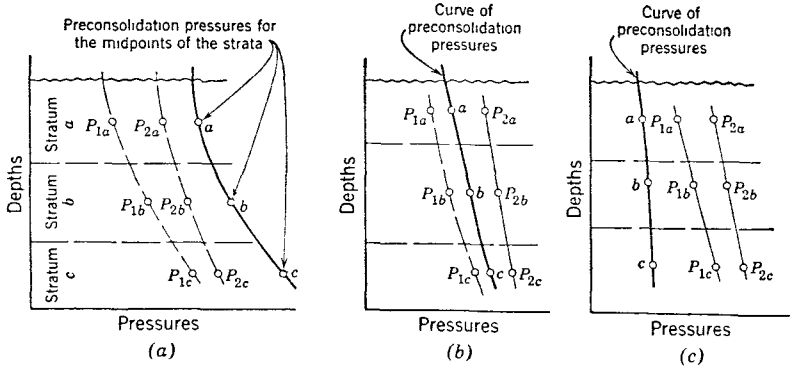


FIG 82 Relationship of estimated existing pressures and anticipated pressures to preconsolidation pressures Studies of this type are of the utmost importance in dealing with cohesive soils The estimated existing soil pressures due to overburden, p_1, and the anticipated pressures p_2 after completion and loading of the proposed structure are computed and plotted for the mid-depth of each stratum The estimated preconsolidation pressure for the material of each stratum is also determined and the values plotted as at a, b, and c. See text for interpretation of the diagrams.

In making estimates of anticipated settlements consideration should always be given to the effect of the progressive lowering of the water table that frequently occurs in urban areas. By increasing the effective unit weight of soil between the foundation and a consolidating stratum from the buoyed weight γ_b to the total weight γ_t such lowering of the water table may greatly accelerate and increase settlements.

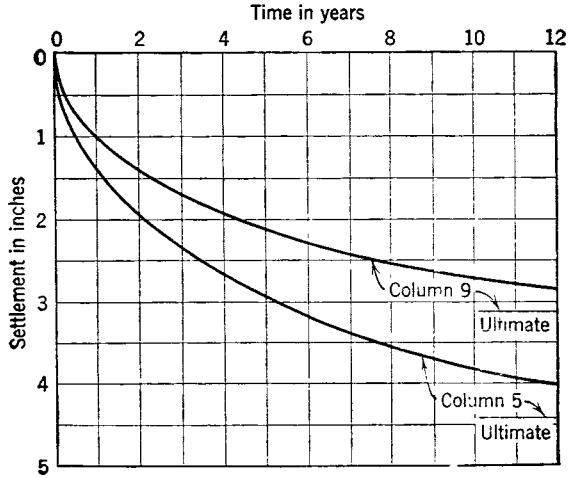


FIG 83 Time settlement curves Curves of estimated settlements of various parts of the foundations of a structure plotted against times permit an intelligent study to be made of the importance of prospective differential settlements. From *Fundamentals of Soil Mechanics*, D W Taylor, John Wiley & Sons, 1948

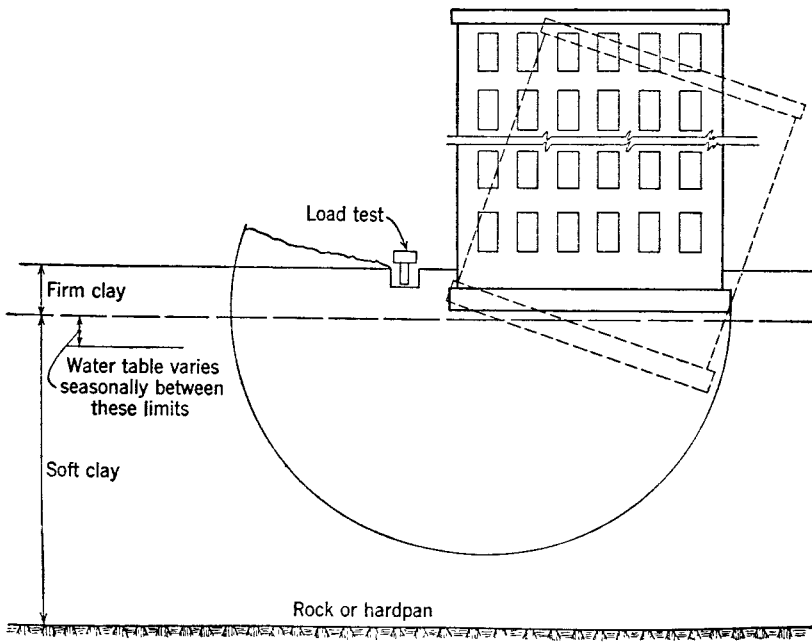


FIG 84. Shear failure of a building foundation The diagram illustrates conditions likely to lead to failures of this type A load test was made at an inadequate depth on the upper part of a stratum of firm clay. Carelessly taken borings revealed the underlying firm stratum but failed to establish the untrustworthy characteristics of the soft clay of low shearing strength which caused the failure. Shear failures of this type occur with dramatic suddenness They are more common under heavily loaded cement and grain silos but are by no means unknown under heavy buildings.

In estimating settlements of foundations resting on alternate strata of non-cohesive and cohesive soils, it is customary to consider only the estimated settlements of the cohesive strata disregarding those of the non-cohesive strata as being so relatively slight as to be of little practical importance.

The **stability against shear failure** of foundations on cohesive soils may be investigated by circular-arc analyses using the cylindrical surfaces of failure indicated in Fig. 84. In such cases, however, shear on the ends of the cylindrical mass of earth which rotates if failure occurs is a real factor in the determination of the stability of the foundation. This factor may be taken into consideration when dealing with rectangular footings by the use of the formula.

$$q = \frac{5.52c}{F_S}\left(1 + 0.38\frac{d}{b} + 0.44\frac{b}{L}\right) \tag{47}$$

in which q = the permissible or "working" bearing value of the soil under the foundation in lb per ft².
c = the cohesion in lb per ft² (usually determined by an unconfined compression test and the assumption that the cohesion equals half the unconfined compressive strength).
F_S = the factor of safety obtained from Table 7.
b = the width of the foundation in ft.
d = the depth of the foundation in ft.
L = the length of the foundation in ft.

The above formula gives a value of the bearing value of a cohesive soil which should insure against shear failure, but does not limit the value to one which will keep settlements within reasonable limits Independent estimates of prospective settlements should therefore always be made for foundations to be carried on cohesive soils.

Table 7. Factors of Safety against Shear Failure to Be Used in Foundation Design

Degree of Sensitivity	Sensitivity Ratio S	Suggested Factors of Safety F_S: Permanent Structures	Suggested Factors of Safety F_S: Temporary Structures
High	≥ 4	3.0	2.5
Medium	2–4	2 7	2.0
Slight	1–2	2.5	1.8
Not sensitive	≤ 1	2 2	1.6

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10. EARTH PRESSURES

The pressure exerted by earth on a retaining wall, sheet pile wall, or other structure which restrains movement of the earth is known as active pressure. The pressure exerted by earth on an abutment or structure the movement of which is restrained by earth is known as passive pressure. Typical surfaces subjected to active and passive pressures are illustrated by Fig. 85. The active pressure is considerably less than the passive pressure produced by a given depth of soil unless the material possesses neither internal friction nor cohesion and accordingly acts as a liquid.

Earth pressures on rigid structures such as masonry and concrete retaining walls and bridge abutments are usually estimated by methods based on either the Coulomb or Rankine theory of earth pressure The former assumes that earth is non-cohesive and that a wedge-shaped prism of soil behind a rigid restraining, or restrained, structure and above a plane of rupture, Fig. 86, is held in equilibrium by forces the directions and magnitudes of which can be determined by the laws of statics if the unit weight of the soil, its angle of internal friction ϕ, and the angle of friction between the soil and the structure are known.

The **Coulomb theory** lends itself better to graphical than analytical solutions. Figure 87 illustrates the widely used *Culmann method* of locating the plane of rupture and determining the earth pressure for either active or passive cases. The angles are plotted as indicated, the slope line BS being drawn with the angle ϕ above the horizontal for the active case and below it for the passive. The trial planes of rupture BC_1, BC_2, BC_3, and BC_4 are drawn arbitrarily and the weights W_1, W_2, W_3, and W_4 of the prisms of soil between the planes

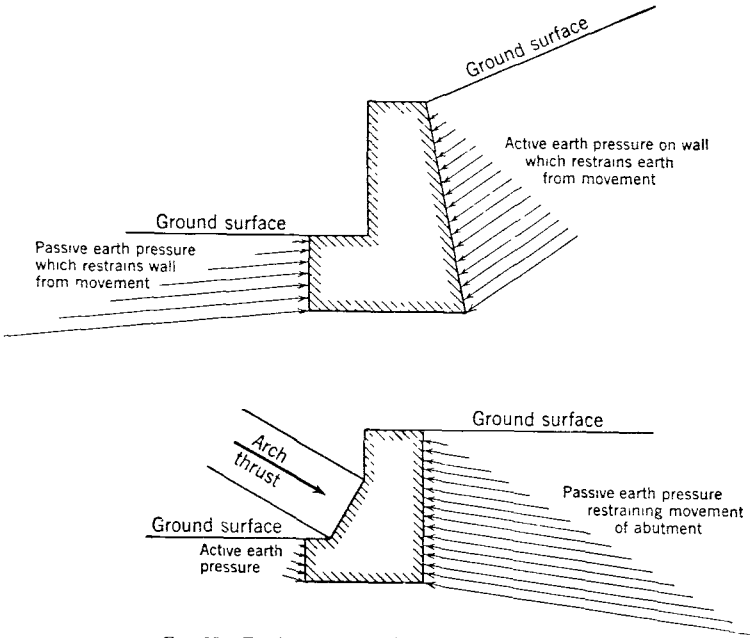

FIG 85 Earth pressures Active and passive cases

and the back of the wall AB are computed. The length of these prisms perpendicular to the plane of the drawing is assumed to be 1 ft To a convenient scale these weights are plotted as the distances Bd_1, Bd_2, Bd_3, and Bd_4 along the slope line from point B From the points d_1, d_2, d_3, and d_4 lines are drawn parallel to the pressure line BL to their intersections e_1, e_2, e_3, and e_4, with the planes of rupture to which they correspond. The lengths of these lines measured to the scale of weights indicate the active pressure consistent with rupture along the arbitrarily chosen planes BC_1, BC_2, etc. A smooth curve, the Culmann line, is

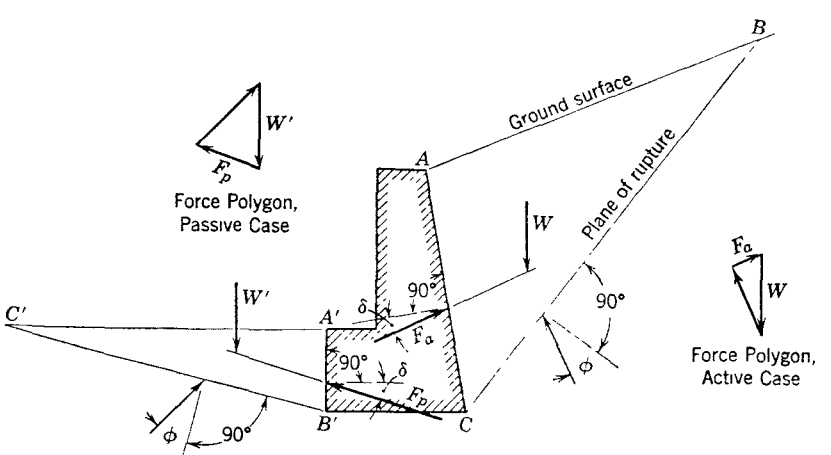

FIG 86 Earth pressure Equilibrium conditions assumed by the Coulomb theory W and W' are the weights of the triangular earth prisms ABC and $A'B'C'$ of unit length ϕ = angle of internal friction of soil δ = angle of friction between soil and wall F_A = equilibrant of active pressure per lin ft of wall F_p = equilibrant of passive pressure per lin ft of wall

then drawn through the points e_1 to e_4 inclusive, and to this curve a tangent parallel to the slope line BS is also drawn. Through the point of tangency e the actual plane of rupture BC is drawn and also the line ed parallel to the pressure line BL. The length of ed measured to the scale of weights is the required pressure. The approximate point of application of the resultant of the pressure may be found by drawing a line, not shown on the figure, from the centroid of the triangle ABC and parallel to BC to its intersection with the back of the wall. Surcharges may be dealt with by adding their weights to those of the trial prisms on which they rest before plotting these as distances along the slope line. For a more complete discussion of Culmann's method see ref. 22.

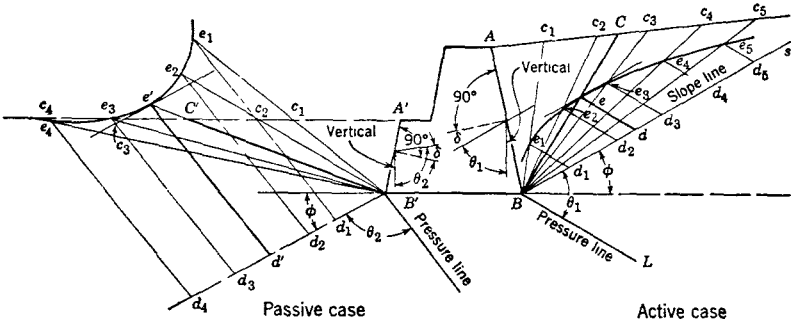

FIG 87. Coulomb earth pressure theory Culmann's construction for finding the planes of rupture—BC in the active case and $B'C'$ in the passive—and the magnitude of the resultants of the active and passive pressures de and $d'e'$ The planes Bc_1, Bc_2, etc , are arbitrarily drawn "trial planes" of rupture The angle θ_1, which determines the slope of the pressure line, is the angle between the vertical and the direction of the resultant pressure on the back of the wall Similarly in the passive case θ_2 is the angle between the vertical and the resultant of the passive pressure

The **Rankine theory** assumes that principles governing the distribution of stress in a homogeneous solid are applicable to earth. It further assumes that earth is homogeneous and cohesionless, that the surface against which it acts is smooth and frictionless, and that the upper surface is a plane of unlimited extent. The first assumption and the principle of conjugate stresses lead to the conclusion that the force acting against the wall must act in a direction parallel to the slope. As the coefficient of friction between earth and wall is assumed to be zero it follows that if the back of the wall is vertical the ground surface must be horizontal if the Rankine theory is to be strictly applicable. Today the theory is usually applied only to cases where the ground surface is horizontal and the back of the wall vertical, other cases being more readily dealt with by some such graphical application as the Coulomb theory or the Culmann method.

A widely used technique for applying the Rankine theory is to compute the pressure which a liquid having the unit weight of the soil would produce, and to this fictitious liquid pressure apply coefficients K_a and K_p which yield respectively the active and passive pressures. The method is illustrated by Fig. 88 upon which necessary formulas appear.

The effect of cohesion in a soil producing earth pressure may be approximated by plotting the pressure distribution which would be caused by a cohesionless soil of the same unit weight and angle of internal friction and then applying graphically the effect of the cohesion by the procedure illustrated in Fig. 89. When using this method the value of the cohesion should *not* be assumed $\frac{q_u}{2}$ or half the unconfined compressive strength, as this term is a function of both the cohesion and the angle of internal friction and equals $2c\left(\tan 45° + \frac{\phi}{2}\right)$. The assumption that $c = \frac{q_u}{2}$ and its subsequent application to the K_a and K_p pressure lines would result in taking cognizance twice of the effect of the angle of internal friction.

Both the Coulomb and Rankine theories yield estimates of earth pressures which should be recognized as approximations, the precision of which does not warrant great refinement in calculations. Of the two methods the Coulomb, which takes cognizance of the effect of friction between the earth and the wall or abutment, is generally considered the more accurate. There is, however, an error in Coulomb's assumption of plane surfaces of rupture, as actual surfaces of rupture are curved. The error is not serious, except in passive cases in which the angle of friction between the earth and wall or abutment structure exceeds 20°. In such cases the Coulomb theory yields unsafely high values of the passive pressure as indicated by Tables 8 and 9. The second of these tables shows the marked

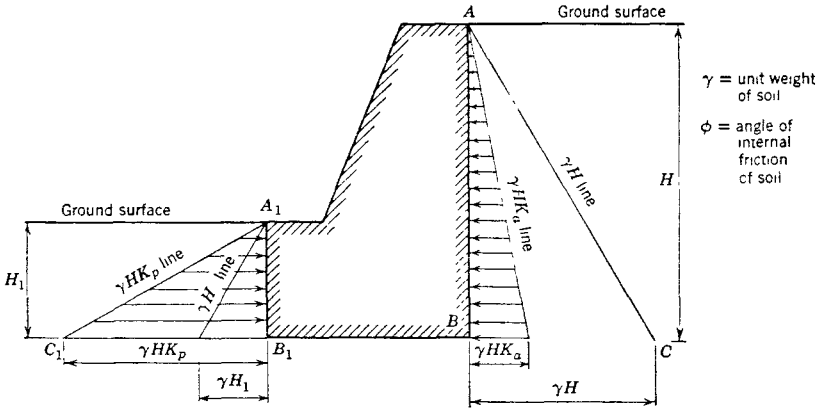

FIG 88. The Rankine theory of earth pressure. *Passive case.* $K_p = \tan^2\left(45° + \frac{\phi}{2}\right)$; resultant of passive pressure, $\frac{\gamma H_1^2 K_p}{2}$, resultant acts at point $\frac{H_1}{3}$ above B_1. *Active case:* $K_a = \tan^2\left(45° - \frac{\phi}{2}\right)$, resultant of active pressure $\frac{\gamma H^2 K_a}{2}$; resultant acts at point $\frac{H}{3}$ above B_1

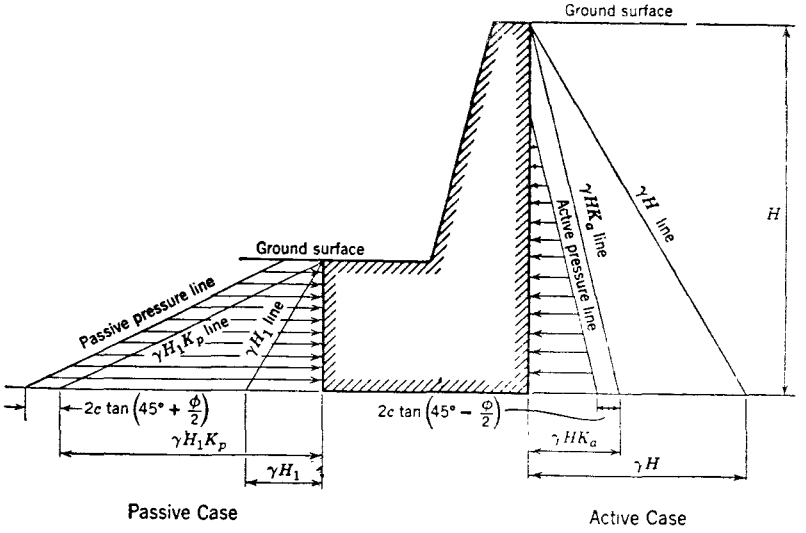

FIG 89 Earth pressures—the effect of cohesion The active and passive pressure lines are shown as determined by the Rankine formula. Other means may, however, be used for establishing these lines prior to determining graphically the effect of cohesion by the method indicated. γ = unit weight of soil, ϕ = angle of internal friction of soil, c = cohesion as determined by triaxial shear test (not by unconfined compression test) if soil has an appreciable angle of internal friction. $K_a = \tan^2\left(45° - \frac{\phi}{2}\right)$, $K_p = \tan^2\left(45° + \frac{\phi}{2}\right)$

Table 8. Earth Pressure Coefficients

Lateral Earth Pressure Coefficients

δ	Values of K_a				Values of K_p			
	φ				φ			
	25°	30°	35°	40°	25°	30°	35°	40°
−40°				0.77 (0.88)				92.3 (17.5)
−30°		0.87 (0.98)	0.50	0.36		10.0 (6.4)	15.3 (9.7)	25.1 (14.6)
−20°	0.61	0.47 (0.48)	0.36	0.28	4.6 (4.1)	6.1 (5.4)	8.3 (7.5)	11.9 (10.4)
−10°	0.47	0.38	0.30	0.24	3.3 (3.3)	4.2 (4.2)	5.3 (5.3)	7.0 (7.0)
±0°	0.41 (0.41)	0.33 (0.33)	0.27 (0.27)	0.22 (0.22)	2.5 (2.5)	3.0 (3.0)	3.7 (3.7)	4.6 (4.6)
+10°	0.37	0.31	0.25	0.21	1.9	2.3	2.7	3.3
+20°	0.36	0.30 (0.30)	0.25	0.20	1.4	1.7	2.0	2.4
+30°		0.30 (0.31)	0.25	0.20		0.87 (0.53)	1.3	1.7
+40°				0.20 (0.22)				0.77 (0.52)

Ground surface assumed horizontal.

δ = angle of friction between soil and retaining wall.

φ = coefficient of internal friction of the soil.

All figures based on assumption that cohesion $c = 0$.

Coefficients in parentheses computed on the assumption that surfaces of failure are curved. Other coefficients based on usual assumption of plane surfaces. Note that differences are significant only in the passive case when δ is large.

Values in boxes are for the widely used value of $\varphi = 30°$ and 35° and $\delta = -30°$, 0°, and +30°.

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effect on earth pressures of slope of the ground surface and the surface against which the earth acts. In dealing with unusual earth pressure problems the extensive tables of earth pressure coefficients based on curved surfaces of rupture of ref. 4 will be found helpful. See also discussion on earth pressures and construction of pressure diagrams in Sect. 27, Volume III, Retaining Walls.

Neither the Coulomb nor Rankine theories of earth pressure are applicable to flexible structures such as steel sheet pile or timber bulkheads or the sheeting and bracing of open cuts in sand or clay. For a discussion of earth pressures on flexible bulkheads see Sect. 21, Harbor Engineering.

Extensive investigations in America and Europe have shown that without doubt the earth pressure on braced flexible sheeting of cuts in both sands and cohesive soils does not by any means follow a "triangular distribution" with minimum pressure at the ground surface and maximum at the bottom of the cut. Actually the pressure distribution diagrams in both materials approximate trapezoids but there are significant differences in the proportions of the trapezoids for non-cohesive and cohesive soils. The pressure distributions recommended for the design of the sheeting and shoring of cuts in sand and cohesive soils are shown respectively in Fig. 90. A factor of safety of at least 2 should be used in designing the sheeting, wales, and struts to withstand the stresses caused by the earth pressures indicated in the diagrams. Points of inflection or "hinges" should be assumed in the vertical members at the indicated locations and knife-edge supports at their lower ends to simplify determination of stresses. The precision of the estimates of earth pressures does not warrant the refinement of treating the verticals as continuous beams partially fixed at their lower ends.

Table 9. Earth Pressure Coefficients

Lateral Earth Pressure Coefficients

	$\omega=$	$-30°$	$-12°$	$\pm 0°$	$+12°$	$+30°$
					1 4 7	1 1 7
$\beta=+20°$	K_a values	0 34	0 43	0 50	0 59	1 17
$+10°$		0 30	0 36	0.41	0 48	0 92
$\pm 0°$		0 26 (0.26)	0 30 (0 30)	0 33 (0 33)	0 38 (0 38)	0 75 (0 85)
$-10°$		0 22 (0 22)	0 25 (0 24)	0 27 (0 26)	0 31 (0 30)	0 61 (0 63)
$-20°$		0 18 (0 18)	0 20 (0 18)	0 21 (0 20)	0 24 (0 22)	0 50 (0 45)
$\beta=+20°$	K_p values	0 50	1 5 (1 0)	2.3 (1.8)	3 1 (2 8)	4 9 (4 6)
$+10°$		0 62	1 8 (1 4)	2 5 (2 3)	3 6 (3 4)	6 2 (5 9)
$\pm 0°$		0 75 (0 40)	2 1 (1 8)	3 3 (3 1)	4 4 (4 2)	8 8 (7 4)
$-10°$		0 92 (0 52)	2 6 (2 4)	3 8 (3 6)	5 9 (5 7)	16 7 (9 3)
$-20°$		1.17 (0 67)	3 4 (3 0)	5 3 (4 8)	9 6 (7 2)	45 7 (11.6)

$c=0$, $\varphi=30°$, $\delta=0°$

All figures based on the assumption that the coefficient of friction between the soil and the retaining wall is 0 and that the cohesion of the soil is similarly 0.

Coefficients in parentheses computed on the assumption that surfaces of failure are curved. Other coefficients based on usual assumption of plane surfaces.

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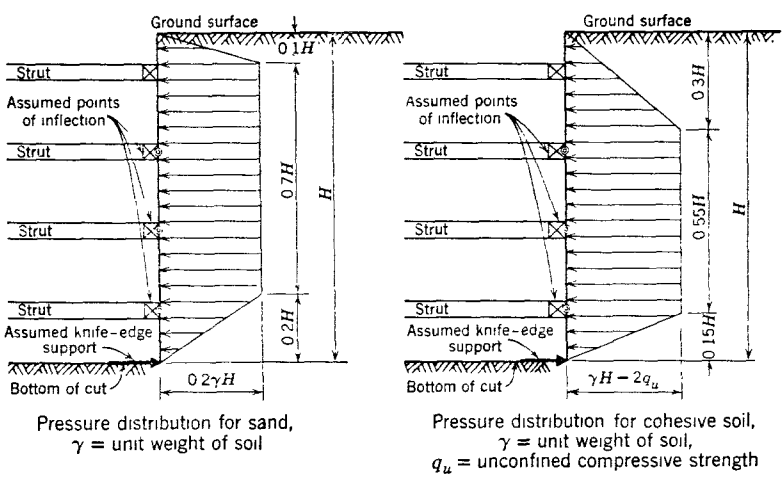

FIG 90. Assumed pressure distributions for design of sheathing and bracing in open cuts Pressure distribution for sand by permission, from *Soil Mechanics, Foundations and Earth Structures*, G P. Tschebotarioff, copyright, 1951, McGraw-Hill Book Co Pressure distribution for cohesive soil from *Soil Mechanics in Engineering Practice*, K Terzaghi and R Peck, John Wiley & Sons, 1948.

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SECTION 8

FOUNDATIONS

BY

W. MACK ANGAS AND CARLTON S. PROCTOR

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FOUNDATIONS

FOUNDATIONS ON LAND

1. GENERAL

Foundation design in cities is in practice governed largely by building-code requirements which stipulate maximum allowable bearing pressures on soils of the types commonly encountered locally. Provisions of a modern enlightened code as to bearing values are given in Table 1. This table combines the Form for the Codification of Allowable Bearing

Table 1. Classes and Bearing Values of Soils Considered as Supports for Foundations *

Class (1)	Material (2)	Allowable Bearing Value, tons per ft^2 (3)
1	Massive crystalline bedrock, such as granite, diorite, gneiss, and traprock in sound condition	100
2	Foliated rocks, such as schist and slate, in sound condition	40
3	Sedimentary rocks, such as hard shales, siltstones, limestones, sandstones, also thoroughly cemented conglomerates in sound condition	15
4	Soft or broken bedrocks of any kind except shale	10
5	Exceptionally compacted or partially cemented gravels, sands, and hardpan	10
6	Gravel, sand-gravel mixtures, compact	6
7	Gravel, loose, coarse sand, compact	4
8	Coarse sand, loose, sand-gravel mixtures, loose; fine sand, compact	3
9	Fine sand, loose	2
10	Stiff clay and soft shales	4
11	Medium-stiff clay	2 1/2
12	Medium-soft clay	1 1/2
13	Fill, silt, organic material, muck, peat, and unusual deposits not provided for herein	0

* This table is based on data, from *ASCE Manual of Engineering Practice* 32, American Standard Building Code Requirements for Excavations and Foundations, approved July 2, 1952, by the American Standards Association.

Values of ref. 2 with allowable bearing values recommended in the same publication by the Sectional Committee on Building Code Requirements for Excavations and Foundations of the American Society of Civil Engineers. Reference 2 contains much information of great value to the designer and constructor of foundations.

Some building and foundation codes permit deviations from stipulated maximum bearing values if properly conducted subsurface explorations, laboratory examinations of specimens, and load tests conducted in a manner satisfactory to the building inspection service convince the responsible authorities of that organization that a proposed foundation design is adequate. Where a code contains such provisions, however, the designer should bear in mind that it also summarizes an immense amount of experience with local soils. The provisions of a well-considered code that has proved satisfactory in a locality, whether or not it is legally enforceable on a site, should be considered by the foundation designer as important but not necessarily conclusive evidence as to the bearing values of materials encountered. But in no case should the appearance of easily recognized reliable materials in shallow test pits be accepted as conclusive evidence as to the foundation requirements of an important structure to be built on a site the underlying strata of which have not been investigated by properly spaced borings of adequate depth. The building code of the City of New York, for example, requires one boring at least 100 ft deep for every 10,000 ft^2 of building area. Furthermore allowable bearing values indicated by even a conservative code with a long record of satisfactory foundations to its credit should be checked by the methods of soil mechanics if the presence of cohesive soils indicate the possibility of troublesome differential settlements or, above all, shear failure.

2. LOADS AND SETTLEMENTS

Dead loads, live loads, wind loads, and, in appropriate localities, earthquake loads should be considered in the design of a foundation. An estimate of the importance of wind and earthquake loads may in many cases lead to a decision that they may be disregarded as inconsequential, but such decision should be reached only after careful consideration of their importance.

Dead loads are usually the most important loads in foundation design because continuity of application makes them the major cause of the settlement of foundations resting on cohesive soils. It is usually, but not invariably, cohesive soils that cause troublesome settlements. In making a first, or preliminary, estimate of the dead loads on the foundation of a building, it is common and accepted practice to use approximate weights per unit area of the roof, floors, exterior walls, and interior partitions. Approximate unit weights of such elements of building construction may be obtained from many handbooks It is usual to make rough but reasonable assumptions as to the distribution of weights to the various parts of the foundation. In preparing and checking the final design, however, the weights of various parts of the structure should be estimated more closely and assumptions as to the distribution of loads should take cognizance of the nature of the building frame and the direction or "run" of such members as joists which may transfer a large part of the weight they carry to opposite sides of a rectangle while delivering little or no load to the remaining sides

Live loads are usually less important than dead loads to the designer of foundations, because their periods of application may be not only intermittent but of short duration. Maximum allowable live loads practically never occur simultaneously on all floors of multistory structures. Building codes therefore usually allow reductions in the live loads used in column and foundation design on some such basis as the following, abstracted from the building code of the City of New York:

(*a*) For multistory storehouses design columns and foundations for 85% of maximum allowable live loads.

(*b*) For other buildings design columns and foundations for 100% of allowable live loads on roof and upper story, 95% of allowable live load on floor immediately below the topmost one, 90% on next lower floor, and so on, decreasing the percentage of allowable live load by 5 for each floor until 50% of the allowable load is reached, which figure is used for all remaining floors.

Judgment must be used in applying such arbitrary rules to unusual buildings and structures. Warehouses used for the long-time storage of heavy materials may have full live loads on all floors for long periods Grain, coal, and cement silos and bins are frequently loaded to capacity for protracted periods.

Impact. The live loads due to overhead traveling cranes should be increased by not less than 25% to allow for impact. Increases of 50% should similarly be made for reciprocating machinery and power units, 20% for light motor-driven machinery, 100% for elevators and their supporting units and hoists, and as much as 300% for such units as rock crushers, ball mills, and cement mills.

Wind loads and earthquake loads usually may be disregarded in designing foundations unless either causes loads on foundations exceeding one-third of the load due to dead and live loads combined. If this ratio is exceeded, the wind or earthquake loads should be included in the design as live loads. In such cases it is usual to allow an arbitrary increase —one-third according to the recommendations of ref. 2—in the allowable bearing values of soils. Whether earthquake and wind loads combined should be considered is a question for the judgment of the designer and depends upon the frequency of high winds and earthquakes in the locality under consideration. Full wind loads should, however, be considered in the design of the foundations of unusually high and narrow buildings, grain elevators, smoke stacks, and other unusually tall structures, the foundations of which may be subjected to uplift or, if they rest on a purely cohesive soil, to shear failure.

Settlements and deformations must be kept within acceptable limits. All foundations settle. The settlement of foundations resting on firm, sound rock will be only the slight elastic deformation of materials stressed within safe limits If the supporting material is gravel, hardpan, firm sand, or a thoroughly preconsolidated firm clay with no softer underlying soil, the total settlement of properly designed foundations should usually be kept within 1 or 2 in. and differential settlements within 1/2 in. Even masonry structures are usually sufficiently flexible to tolerate such settlements. If, however, the supporting material is an unconsolidated, saturated cohesive soil, or if such cohesive soils underlie a firmer material upon which the foundation rests, much larger settlements are usual and differential settlements are likely to be troublesome. In such cases settlements have oc-

curred which are measured in feet and inches rather than in inches and their fractions. The National Theater in Mexico City, for example, has settled more than 6 ft since it was built in 1909. On the same deep deposit of soft clay differential settlements of as much as 4 ft have been observed in long buildings. The prevention of such excessive settlements is

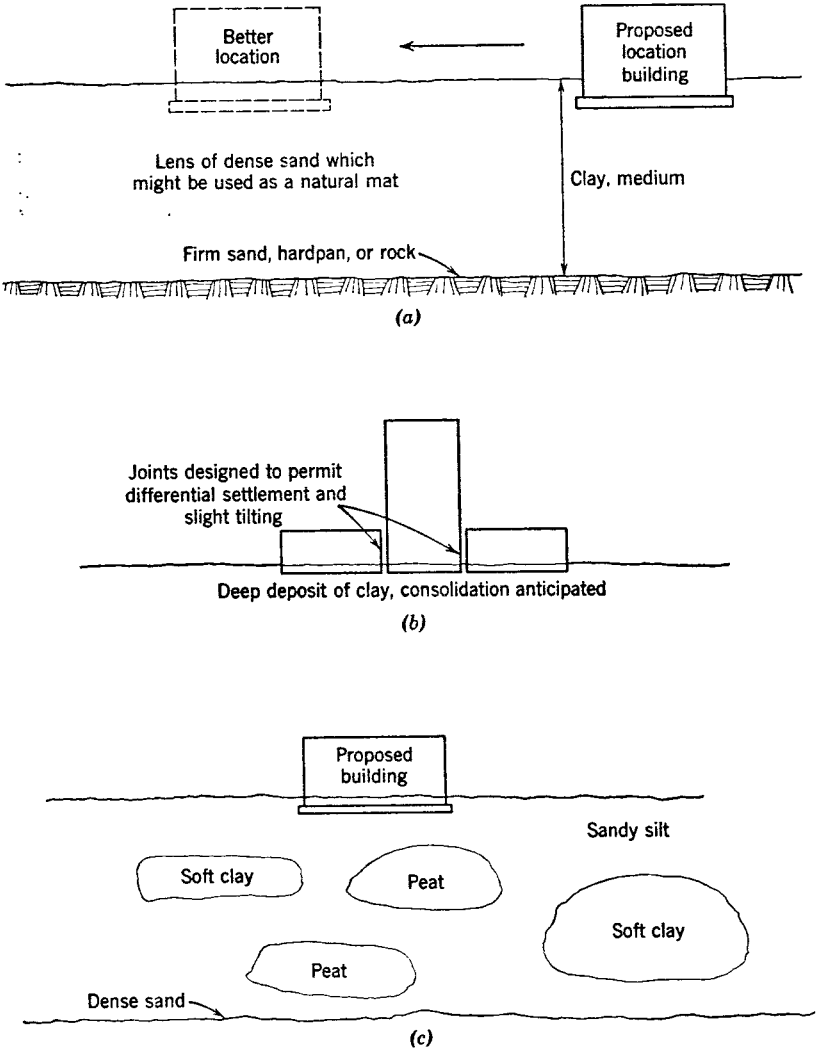


FIG 1 Difficult sites for foundations In *a* the building should if possible be moved to the left to take advantage of the lens of dense sand which might be utilized as a natural mat. In *b* the design of the superstructure should provide for inevitable differential settlements In *c* the cost of a deep foundation reaching the dense sand should be compared with the cost of a better site

naturally a function of the foundation engineer, but it should be kept constantly in mind that it is his duty to keep overall and differential settlements within acceptable limits rather than to attempt the often impracticable task of avoiding settlements. To this end the designer of a foundation must work in close cooperation with the designer of the structure which it is to support and should advise him when and if the subsurface conditions at a site make the prevention of large differential settlements prohibitively expensive.

In some such cases a change in the location of the structure or even the abandonment of the site in favor of a more expensive one may result in overall economy. In other cases it may be advisable to design the superstructure so that large differential settlements may be tolerated. Figures 1*a*, *b*, and *c* illustrate three such cases. In *a* it is possible that moving the structure to the left, as indicated by dotted lines, will permit using the thick lens of sand as a natural mat which will equalize settlements. In *b* the great differences in the weights of the high and low portions of the building make large differential settlements probable and indicate the advisability of providing flexible or articulated joints between sections of the building in order that they may settle independently without damage. These joints should provide not only for vertical differential settlements but for some slight differential tilting of the sections. In *c* deep and irregular pockets of soft clay and peat make the site an extremely undesirable one on which large and unpredictable settlements of buildings on shallow foundations are probable. In such cases the cost of deep foundations going down to firm material should be compared with the cost of a better site.

3. FOOTING FOUNDATIONS

Footing foundations support most light buildings and many heavy and important structures. Their design is worthy of much more serious consideration than it frequently receives. It is not enough to estimate the safe bearing value of the soil and then divide wall and column loads by this value to determine foundation areas. Such procedures frequently lead to large differential settlements if not to more serious types of foundation failure. Foundations designed on the basis of empirical bearing values should be considered adequate and safe only where such values have been thoroughly established by local practice and have proven sound by adequate experience with similar structures in the same localities. Otherwise a complete study of the geology of the site should be made, securing necessary data by borings, test pits, and other means of subsurface exploration. Loading tests are frequently advisable and should usually be made if the contemplated structure is a major one and is to rest on material other than rock. Complex geological formations or unusual foundation conditions frequently require the advice and experience of specialists in foundation design and soil mechanics.

The **depth of a footing foundation** should be sufficient to insure support on soil that will not be heaved by frost or the seasonal variations of the volumes of clays and cohesive soils that occur in some localities. There is no substitute for local knowledge of conditions in determining this necessary depth. Ground rarely freezes to greater depths than 4 or 5 ft in the United States but there are exceptions in the colder parts of the country. Local experience is also of great importance in determining the depth necessary to avoid seasonal heave due to changes in the moisture content of cohesive soils. In most parts of the country no trouble of this kind occurs, but in some localities, notably central Texas, there are soils in which footings must be carried to depths as great as 20 ft in order to avoid difficulties of this kind.

The depth of footing foundations must be sufficient to reach soil of satisfactory bearing value as determined by prospective settlements and stability against shear failure. Determination of necessary footing depths involves consideration of the magnitude of the loads to be carried, data yielded by subsurface exploration, the results of laboratory and field tests, and experience that may be found summarized in local building codes. It also involves consideration of the probable effect of footing dimensions and depths on settlements. Thus under a given unit loading: The settlement of a footing on a partially cohesive soil will be affected by not only the width of the footing but the ratio of its depth to its width as indicated by eq. 26, Sect. 7. On a purely cohesive soil the settlement of a footing will be affected by its width only as indicated by eq. 27, Sect. 7, but both the width and depth of a footing affect its stability against shear failure as indicated by eq. 47, Sect. 7. On a non-cohesive soil settlements are affected by the ratio of the depths to the widths of footings as indicated by eq. 28, Sect. 7. It frequently happens that such considerations as necessary cellar depths dictate greater footing depths than might otherwise be necessary.

Allowable Bearing Values. In practical foundation design it is usual to start by assuming a tentative allowable bearing value of the soil at the depth of the footings indicated as necessary by the subsurface exploration or the cellar depth. If the structure is large and heavy the assumed value should in many cases be checked by loading tests.

Footing Areas. If the soil is a firm non-cohesive material or a thoroughly preconsolidated firm clay, a first approximation of the footing areas may be made by dividing the combined dead and live loads on columns and walls by the assumed allowable bearing value. This simple procedure is justified by the fact that on such soils the intermittent character

of live loads is relatively unimportant insofar as it affects settlement If, on the other hand, the soil is an unconsolidated or partially consolidated clay or a loose non-cohesive material the footing areas should be proportioned by one of a number of methods devised to equalize the effect of intermittent live loads. Of several such methods one devised by the late D E. Moran is recommended and described below

(*a*) The area of the footing receiving the largest ratio of live to dead load is determined by dividing the combined live and dead loads on this footing by the allowable bearing value of the soil.

(*b*) The required area of this footing is then divided by the sum of the dead load and one-half the probable live load to secure a reduced allowable bearing value which is used in determining the areas of the remaining footings.

(*c*) The area of each of the remaining footings is determined by dividing the sum of its dead load plus one-half its probable live load by the reduced allowable bearing value.

Footing Dimensions. Once the areas of the footings are determined their dimensions should be chosen and the footings so designed as to avoid eccentric loading of the loaded soil areas. To accomplish this where the footings are not to extend beyond the face of the exterior wall, it may be necessary to support the wall columns and the row of columns next to the wall on combined footings, three types of which are shown in Fig. 2. Combined

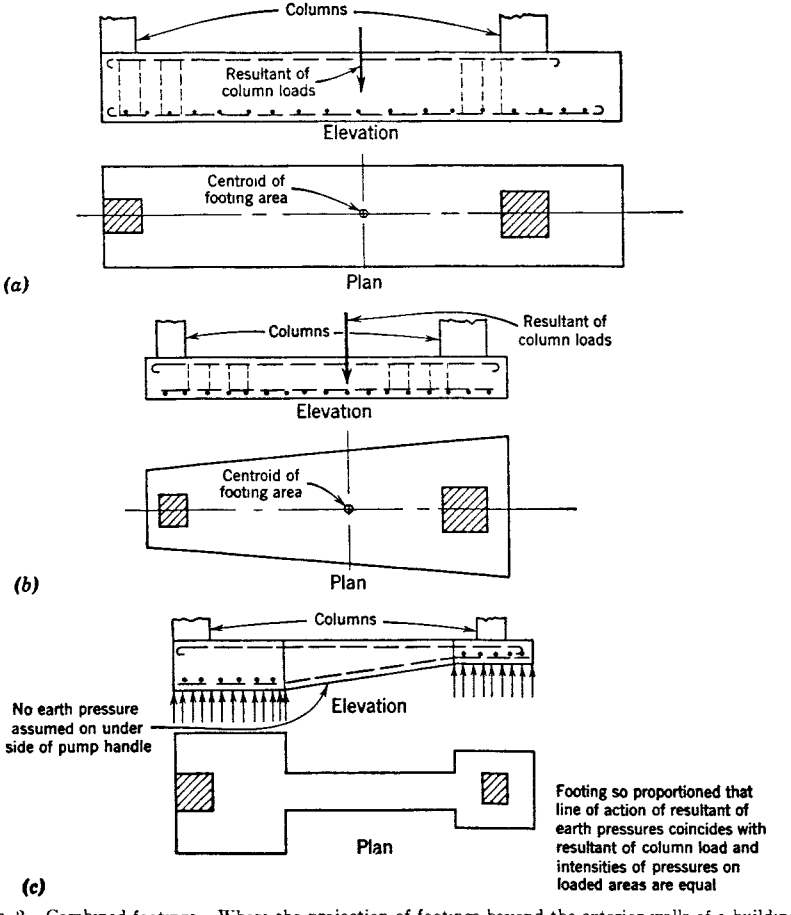


FIG 2 Combined footings Where the projection of footings beyond the exterior walls of a building is objectionable the combined footing avoids the eccentric loading of the soil areas upon which the column foundations rest Three common types of combined footing are shown. (*a*) Combined rectangular footing. (*b*) Combined trapezoidal footing (*c*) Cantilever or pump-handle footing.

rectangular and trapezoidal footings, Fig. 2*a* and *b*, are so proportioned that the centroid of the loaded soil area is located under the resultant of the two column loads. In the cantilever footing, commonly called a pump-handle footing, Fig. 2*c*, the interior footing is proportioned to carry its column load and then, taking moments about the center line of the interior column and assuming a uniform earth pressure under the exterior footing, the exterior footing is so proportioned that the moment is zero. Finally, moments should be taken about the center of the wall column and the reaction under the interior column footing computed. A reduction in the area of the interior footing may then be indicated.

Structural Design of Footings. In all three types of footings the usual methods of reinforced-concrete design are used to determine necessary concrete and steel areas and

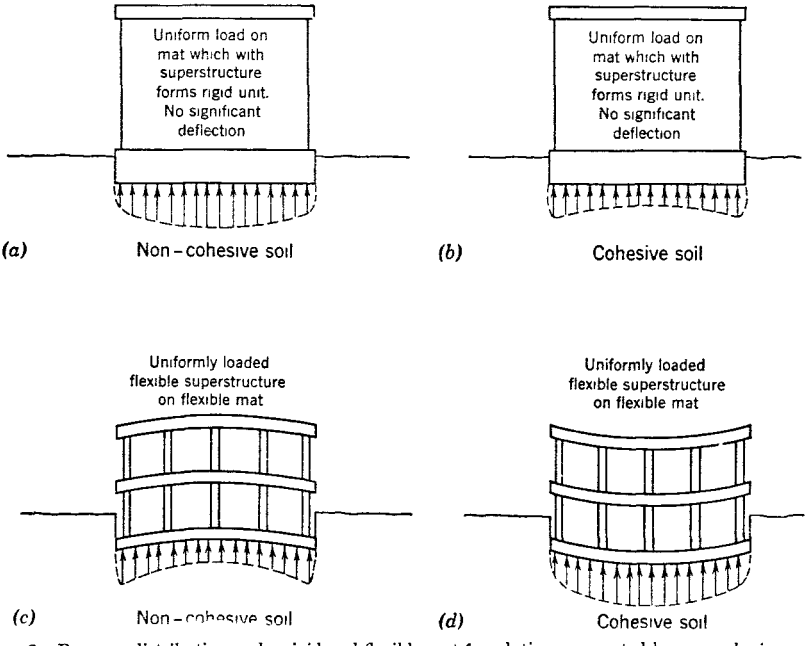


FIG. 3. Pressure distribution under rigid and flexible mat foundations supported by non-cohesive and cohesive soils. A uniformly loaded mat foundation which, with its superimposed structure, is so rigid as to prevent any significant deflection will cause (*a*) maximum soil pressure at its center if the soil is non-cohesive and (*b*) maximum pressure at the edges if the soil is cohesive. Uniformly loaded flexible mats carrying flexible superstructures will, however, have the opposite effect of causing (*c*) maximum pressures under their edges if the soil is non-cohesive and (*d*) maximum pressures under their centers if the soil is cohesive. The divergencies from uniform distribution are, however, dependent on so many variables (soil characteristics, rigidity of structure and so forth) that quantitative evaluation of divergencies is impracticable if not impossible.

stresses. Unless the foundation is waterproofed a minimum thickness of concrete over the reinforcement of at least 4 in. should be maintained even where theoretical considerations indicate lesser thicknesses sufficient. Concrete 2 1/2-in. or at most 3-in. thick will protect adequately the reinforcement of a waterproofed foundation. In designing footings it is usual to assume uniform distribution of soil pressures on their bearing surfaces although the actual pressure distribution is non-uniform. If the soil is non-cohesive, the pressure will be maximum at the center of the footing and minimum at the edges, Fig. 3*a*, whereas if the soil is purely cohesive, the reverse will be true, Fig. 3*b*. The degree of deviation from a uniform distribution is, however, so uncertain that the admittedly erroneous assumption of even distribution is recommended because factors of safety used in normal reinforced-concrete design procedures amply cover errors resulting from the assumption. Tables of value to the designer of reinforced concrete footings and other information of value in foundation design will be found in ref. 19.

Steel-Grillage Footings. Footings consisting of structural steel grillages encased in concrete were widely used before reinforced-concrete design and construction were developed to their present state. Steel grillages may occasionally be preferable to reinforced-concrete

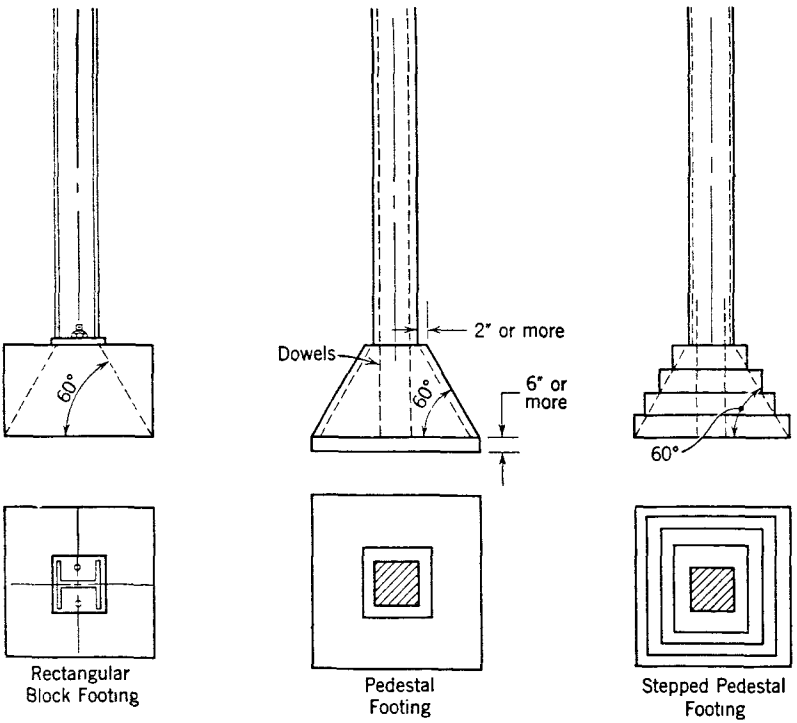


FIG 4 Plain concrete footings Plain concrete footings are rarely used today except under very light buildings Where any considerable bearing area on the soil is necessary it is almost invariably economical to use a reinforced-concrete footing In rare cases where rock or high bearing value is near the surface it may occasionally be advantageous to use plain concrete footings under fairly heavily loaded columns.

footings today. Information on their design may be found in ref. 10 and in many steel handbooks.

Plain concrete-column and wall footings because of their size and weight have been generally supplanted by reinforced-concrete footings. Plain footings are, however, occasionally advantageous and if used their thickness should be not less than 8 in. nor less than twice the maximum offset, Fig. 4.

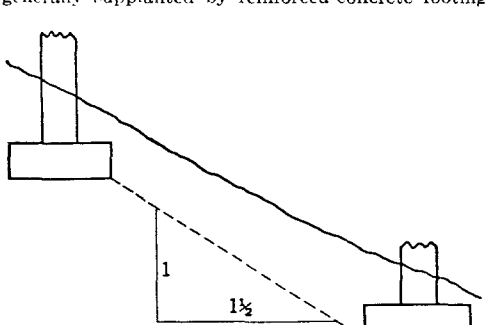


FIG. 5 Footings at different levels. Footings at different levels on a non-cohesive soil should be designed so that the slope of the line joining their adjacent lower edges does not exceed 1½ to 1 On cohesive soil a careful analysis of stability against shear failure should be made, and no simple "rule" applies to such cases.

Footings at different levels, whether they support the same or an adjoining structure, should comply with limitations indicated on Fig. 5, unless adequate provision is made by a retaining wall for the lateral support of the material carrying the higher footing or unless test data on a cohesive soil shows the average stress to be not more than 25% of the shearing strength of the soil.

Settlement and Stability of Footings. Footings resting on cohesive soil should be checked by eq. 47, Sect. 7, for stability against shear failure. In the case of a heavy or

important structure estimates of prospective total and differential settlements should be made before acceptance of the design as satisfactory. If a load test has been made, settlement estimates based on it should be prepared. It should be kept in mind that load tests furnish the best possible data for estimating the settlements of non-cohesive soils. Settlement estimates based on consolidation tests should, furthermore, be made if cohesive soil underlies the foundation.

If estimates indicate the probability of excessive differential settlements, consideration should be given to the possibility of equalizing them by varying the footing dimensions and depths in ways which will be suggested by eqs. 26, 27, and 28 of Sect. 7.

A final adjustment of footing areas may be desirable to insure that prospective slight and acceptable differential settlements of the ends of walls and the corners of buildings may be such as to insure the ends and corners settling a little less than the central portions of walls. Slight differential settlements may produce unsightly cracks in a masonry wall if the ends settle more than the center, Fig. 6*a*, whereas, if the reverse is the case, Fig. 6*b*, the greater area of masonry available to take tensile stresses in the lower and more massive portion of the walls makes the development of cracks much less probable and small cracks, if they occur, will not be obtrusive.

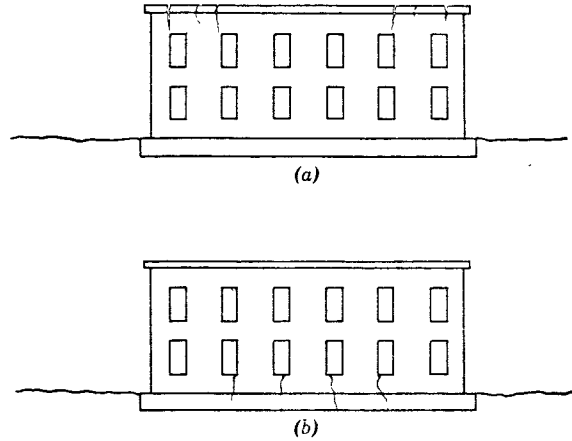


FIG. 6 Some slight differential settlements are more objectionable than others. Where settlements are inevitable and slight differential settlements probable, it is better to have the center of the building settle slightly more than the ends than to have the ends settle more than the center. (*a*) If the ends of a building settle slightly more than the center, unsightly cracks will open from the tops of the windows to the cornice. (*b*) If the center of a building settles slightly more than the ends, the greater cross-sectional area of the material available to take tension minimizes the cracks and those that occur are less unsightly and objectionable.

Footings Subjected to Horizontal Loads. Footing foundations of retaining walls, towers at the ends of transmission lines, and other structures subjected to constantly applied horizontal forces demand special consideration.

In the case of a retaining-wall foundation on non-cohesive soils, or on cohesive soil so thoroughly preconsolidated that appreciable settlements are improbable, stability can be assured by providing for:

(*a*) Sufficient passive pressure to prevent sliding, Fig. 85, Sect. 7.

(*b*) Maximum pressure under the toe of the foundation within the allowable bearing value of the soil.

(*c*) Location of the resultant of all forces acting on the bottom of the foundation within its middle third in order that there may be no uplift on the heel.

(*d*) Adequate drainage of specially selected backfill material placed immediately behind the wall to prevent hydrostatic pressures on the wall or the formation of ice lenses and consequent horizontal thrust in excess of anticipated earth pressures.

Where a retaining wall is to be carried by a partially consolidated compressible cohesive soil subject to progressive settlement requirements, *a*, *b*, and *d* enumerated above should be met and in addition pressures under the foundation should be equalized by a sufficient extension of the toe to insure the resultant of forces acting on the underside of the foundation being at its center rather than merely within the middle third. Furthermore, in such cases settlement estimates should be made and the stability against shear failure of the entire wall and the backfill behind it investigated by cylindrical slide analysis of the type described in Art. 8, Sect. 7.

In designing tower foundations it is desirable, if possible, to provide sufficient weight in the footings on the side away from the direction of any constantly applied horizontal pull to prevent uplift. Such massive foundations are, however, expensive, and economy may dictate the use of footings providing some anchorage in the soil. In such cases friction on the sides of footing should not be relied upon, as maximum uplift will be caused

by a combination of dead and wind loads most probable in a stormy period when soil strength is minimized by saturation. Footings designed to develop resistance to uplift by anchorage should have projecting flanges around their bases, Fig. 7, and the weight of only the soil directly above these flanges should be considered as available to resist uplift. Actually a considerably greater weight of soil will act, but the material will be backfill and therefore somewhat untrustworthy and unpredictable.

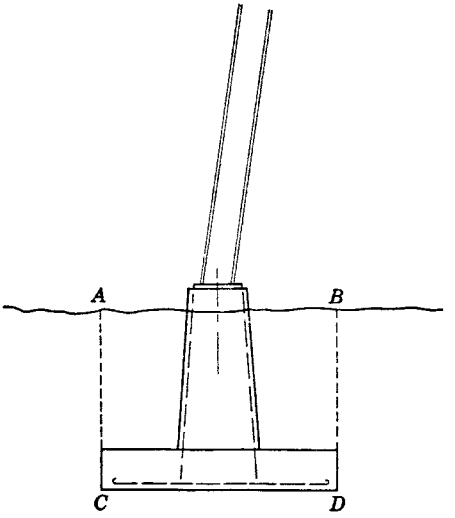


FIG 7 Tower footing subject to uplift In designing an "anchor footing" of this type only the weight of the footing and the soil within the prism defined by *A*, *B*, *C*, and *D* should be considered as available to hold the footing down It is preferable to provide sufficient weight in the concrete of the footing, as thoughtless excavation around an "anchor footing" may destroy its stability against uplift.

Special conditions should be considered in designing footing foundations of unusual structures, particularly if they rest on unconsolidated, saturated cohesive soils, the settlement of which will be accelerated by drainage. The presence of a boiler in a basement or a furnace in a metal-working shop may, for example, raise the temperature of a clay sufficiently to promote rapid migration of water away from heated areas by thermo-osmosis and cause serious differential settlements Similarly, the possibility and probability of the water table being lowered by leakage into sewers and deep basements, drainage into deep excavations, and pumping from wells should also be considered. It should be kept in mind that lowering of the water table promotes settlement by not only accelerating the drainage from saturated cohesive strata but also by increasing the effective loading on such strata through substitution of the total weight of overlying partially saturated soil for the much smaller buoyed weight of the same material.

The **firm seating of footing foundations** on undisturbed soil is important. The surface of the soil at the bottom of the excavation in which a footing is to be poured should be freed of all loose material before the concrete is placed, thus insuring that the footing rests on firm undisturbed material. This is especially important where the soil is a clay or cohesive silt inclined to work up into a soft slurry under the feet of men engaged in setting reinforced steel and doing other work which must be done after the final completion of the excavation In such cases it is highly desirable to increase the depth of the footing a few inches and pour a layer of plain concrete from 4 to 6 in. thick in direct contact with the freshly cleaned earth. The footing is then constructed on the resulting concrete working platform without risk of undue disturbance of the soil. For information on the sheeting of footing excavations and the construction of the land cofferdams in which large and deep footings are occasionally built, see Land Cofferdams, Art. 9.

4. MAT FOUNDATIONS

Mat foundations may be considered a special type of footing foundation in which the footings are so large that they not only touch each other and cover the entire ground area of the structure they support but are furthermore so rigidly connected that differential settlements are minimized. Early attempts to construct mat foundations of masonry or plain concrete usually failed because the material of the mat had insufficient tensile strength to withstand the bending stresses to which it was subjected. The development of reinforced concrete, however, has made mat foundations practicable and they are now a recognized foundation type, the characteristics, advantages, and weaknesses of which should be known to all engineers practicing in the foundation field.

Natural mats consisting of a stratum of firm material overlying soft, compressible soil may be used to reduce differential settlements. Figure 1*a* illustrates such a natural mat in the form of a thick stratum of firm sand at the left side of the diagrammatic drawing. If

this stratum is of sufficient thickness and strength, the proposed building might be moved bodily to the left as indicated by dotted lines and carried on piles driven to the sand Unfortunately it is impossible to determine the strength of such a natural mat with any degree of precision by computation, and dependence must be placed on experience to show the weights and spacing of foundations it can support without serious differential settlements developing. If piles are used to carry the foundation loads down to the firm stratum, care must be used not to drive them too far into the natural mat with the result that they will subsequently punch through it under their applied load In such an analysis the safe shearing strength of the material of the mat should be applied not only to the perimeter area of individual piles and compared with individual pile loads but also to the perimeters of pile groups and compared with the loads they support. A thorough investigation of the stability of such a foundation against rotational shear failure should also be made, and prospective settlements should be studied by estimates based on consolidation tests of the soil below the mat if this soil is cohesive.

Strata of soft limestone which may be used as natural mats are found in many localities. Such strata are in general much more satisfactory than natural mats of firm soil, but care must be exercised to guard against placing heaving foundations over the caverns that frequently occur in many limestones. Where heavy foundations are carried on end-bearing piles resting on limestone, the possibility of dangerous cavernous situations should be thoroughly explored in the subsurface investigation.

Reinforced-concrete mats may be used to obviate serious differential settlements on sites where small localized pockets of compressible soils exist below foundation level but the bearing value of the material underlying the greater part of the area is reasonably satisfactory. Such mats are designed to carry occasional individual column loads without soil support immediately below them and to minimize differential settlements that otherwise might be caused by slight differences in the bearing value of the soil throughout the building area. The preliminary investigation of the practicability of a reinforced-concrete mat should consider the effect of its dimensions on prospective settlement the analysis being made by methods outlined for determining the effect of the widths and depths of footings on probable settlement If the soil is cohesive a thorough study of the stability of the foundation against rotational shear failure should be made as such in an spectacular shear failures have been experienced with large, heavily loaded mats Foundation mats may be designed as reinforced-concrete beam and girder structures or as thick reinforced-concrete slabs, the normal procedures of reinforced-concrete design being used in either case. Flat slabs are usually preferable to beam and girder mats as the latter obstruct the basement floor seriously Beams and girders should not be placed below the slab as the trenches necessary for their construction disturb the soil too seriously and reduce greatly the bearing value of a considerable portion of the available bearing area. Sufficient flexural strength should be provided in a reinforced-concrete mat to prevent serious deformation and consequent differential settlement if the pressure developed under the mat deviates materially from the uniform pressure that will be assumed in the early phases of the design Actually little quantitative information is available as to the extent to which the pressure will deviate from uniformity It is known, however, that the perimeter pressures will exceed the average pressures on a cohesive soil and will be lower than the average on a non-cohesive soil. The structural strength of the mat in any case should be sufficient to tolerate a pressure under the outer bays 25% higher than under the interior bays on a cohesive soil. On a non-cohesive soil the pressure under the exterior bays may be assumed as 25% less than under the interior bays.

The use of thick reinforced-concrete mats to minimize differential settlements of foundations on relatively soft cohesive soils is not recommended as the great weight of a mat of the requisite thickness promotes excessive total settlement. For example, the weight of the reinforced-concrete mat 8 ft thick under the National Theater in Mexico City almost equals the weight of the superstructure and has undoubtedly been at least partially responsible for the building settling more than 6 ft in 40 years.

Floating foundations, consisting of deep reinforced-concrete basements the sidewalls, floors, and interior framing of which are designed as big "barges" stiffened internally by bulkheads that may take the form of Vierendeel trusses are much more satisfactory than thick concrete mats where a heavy building must be carried on a clay which cannot be classified as firm.

An outstanding pioneer foundation of this type which focused attention on its possibilities was constructed for the Albany Telephone Building, see refs 8 and 9. The general features of this foundation are shown on Fig 8. The design provided for a weight of excavated material substantially equaling the combined weight of the foundation and superstructure, with the result that the building caused no increase in the pressure on the stratum that supports it. No serious settlement has occurred and the foundation is an unqualified success.

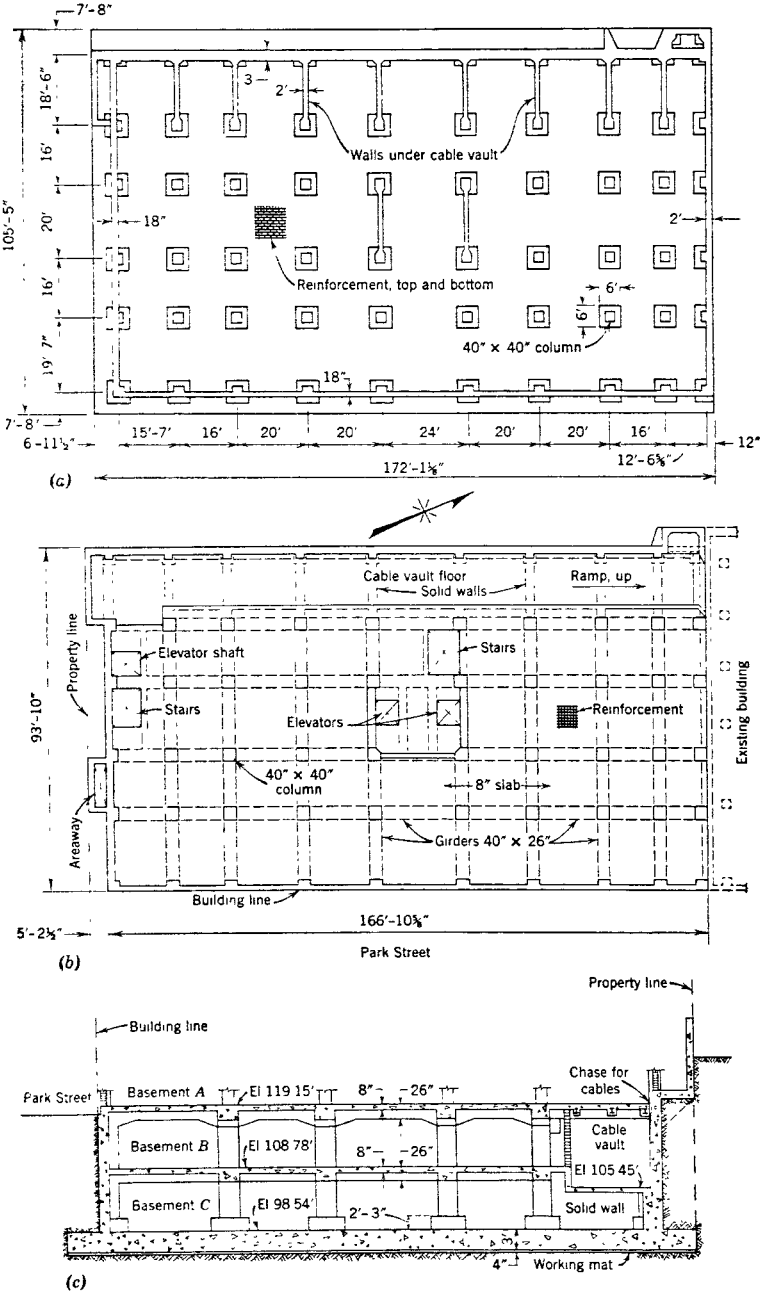


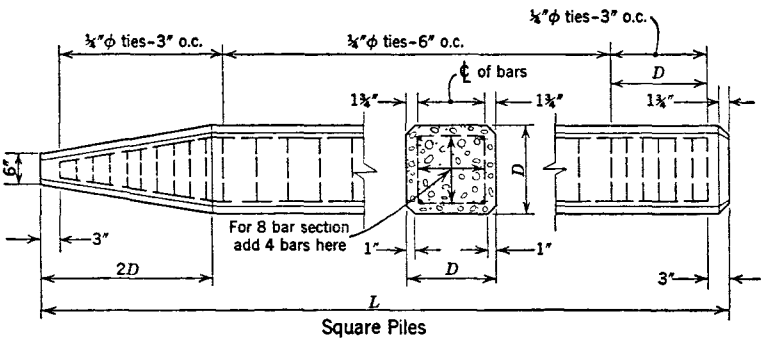
FIG. 8 Foundation of the Albany Telephone Building The basement of the building is a rigid frame concrete structure stiffened with Vierendeel trusses It forms in effect a large reinforced-concrete barge so light in weight that the building does not load the stratum upon which it rests more heavily than it was formerly loaded by overburden (a) Bearing mat and basement C framing Uniform settlement of clay soil and low cost dictated choice of mat (b) Framing of basement B floor system (c) Cross section of rigid frame foundation From *Eng News-Rec*, November 27, 1930.

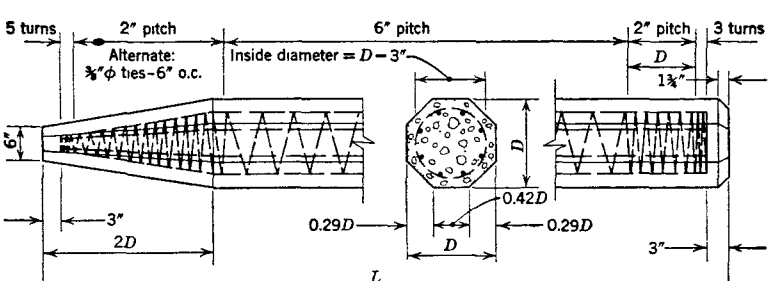
Firm seating of mat foundations on undisturbed soil is as important as the firm seating of footings. The provision of a plain concrete working platform on the bottom of the excavation immediately after the soil surface has been cleaned and freed of all loose material is more often necessary for the satisfactory construction of mat foundations than it is for footings. When an excavation for a large mat is made, the elevation of the soil in the bottom of the excavation should be carefully checked for "heave" or uplift due to the relief of pressure and subsequent plastic flow caused by the weight of the surrounding material. Such uplift, if it occurs, will probably subside when the mat is loaded but cognizance must be taken of the deformation and settlement which will accompany the subsidence.

The large land cofferdams in which mat foundations are usually built are discussed under Art. 9, Land Cofferdams.

5. PILE FOUNDATIONS

Pile foundations support many important buildings and structures, among which are masterpieces of not only modern but medieval and ancient architects and engineers. But the success with which pile foundations support a wide variety of structures on an even





Spiral wire

D	16"	20"	24"
Size	#5	#4	#3

FIG. 9. Precast concrete piles. Precast piles are used extensively in water-front structures and occasionally in foundation work ashore, especially if a considerable length of the pile will be "free standing" between the ground surface and the structure. Precast piles are also used to support many bridges. Courtesy of Portland Cement Association

wider variety of geological formations should not lead to the assumption that piles are a panacea for all foundation problems. Today there is little justification for the once-popular policy of providing pile foundations for practically all heavy structures that did not clearly demand support by piers or caissons carried down to rock or hardpan. This was a policy that without doubt led to the driving of many unnecessary piles, and occasionally to the

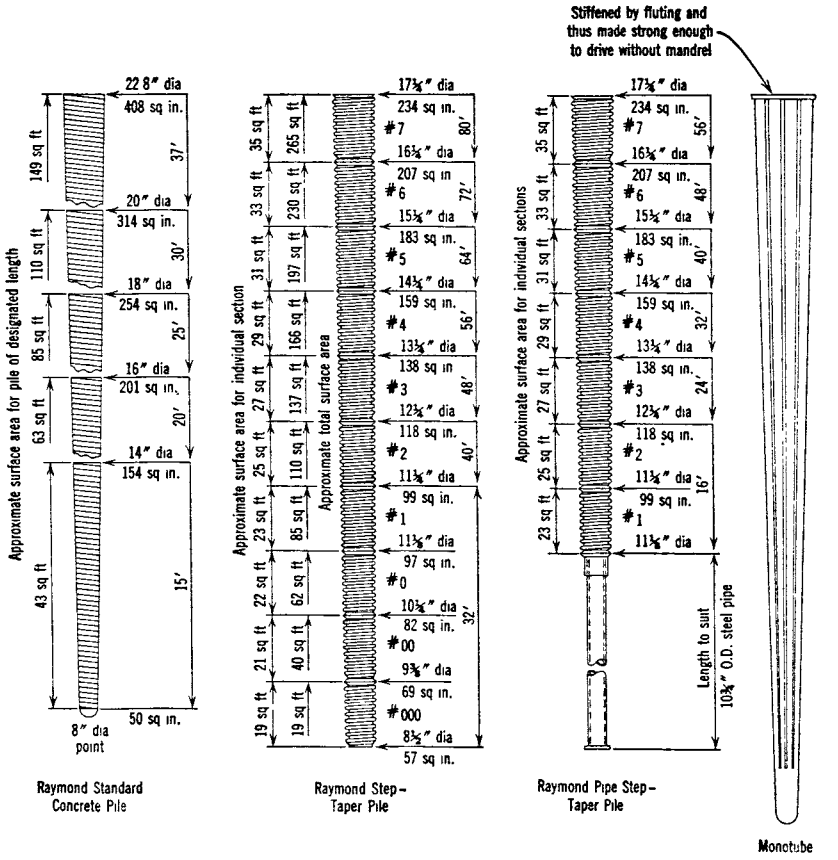


FIG 10 Cast-in-place concrete piles Cast-in-place piles are made by pouring concrete into a shell a close-fitting internal mandrel while being driven Other shells and pipes are strong enough to stand filled All drawings except that of Monotube pile by permission, from *Foundations*

driving of piles that did more harm than good. For piles driven through a stratum of moderately firm soil into an underlying stratum of sensitive clay may, by remolding the latter and transferring the effective foundation load to the lower stratum, reduce the bearing value of the foundation the piles were driven to support. Modern foundation practice makes frequent use of piles but adopts them only after a thorough study of subsurface data indicates their necessity or their structural and economic advantages over alternate procedures. The discussion which follows deals with foundation piles. For information on piles used in the construction of water-front structures, see Section 21.

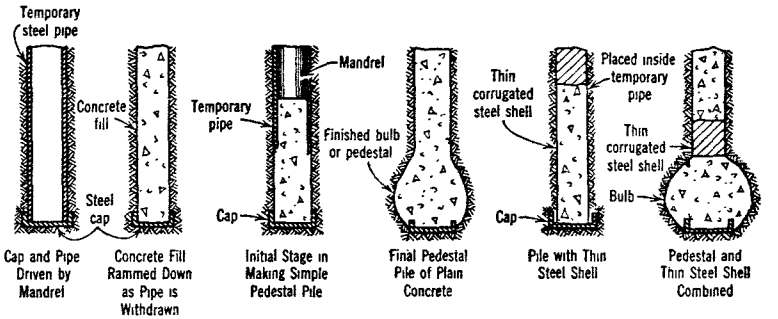
Foundation piles may be classified functionally as.

(*a*) *Friction piles* which by skin friction distribute load throughout their length to the soil into which they are driven.

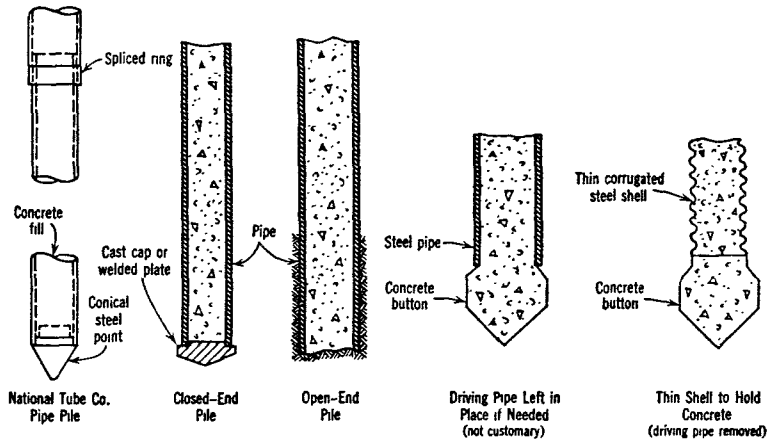
(*b*) *End-bearing piles* which carry a load through soft soil to firmer underlying material on which their tips rest or into which they are driven a short distance.

Both **friction and end-bearing piles may be further classified** according to the material of which they are made and in some cases are subject to subclassification as indicated below. Material classification of piles distinguishes between:

1. *Timber piles* which are further subclassified as:
   (*a*) Untreated piles.



Some cast-in-place concrete piles made by MacArthur Concrete Pile Corp (Similar construction is used by others)



Some steel pipe, concrete-filled piles

Western Foundation Corp. button-bottom piles

or pipe previously driven into the ground Light shells and pipes are temporarily strengthened with driving without such support Some untapered pipe shells are withdrawn from the soil as they are *of Structures*, by C W. Dunham, copyright, 1950, McGraw-Hill Book Company.

(*b*) Treated piles which have been impregnated (usually with creosote) to inhibit damage by decay, boring insects, or marine borers.

2. *Concrete piles* which are further subclassified as.
   (*a*) Precast piles (see Fig. 9 for typical characteristics).
   (*b*) Cast-in-place piles (see Fig. 10 for widely used types).
3. *Composite piles* (see Fig. 11) consisting of a timber lower section surmounted by a concrete section which may be either:
   (*a*) Precast.
   (*b*) Cast-in-place.
4. *Steel piles* which are further subclassified as:
   (*a*) H-beam piles.
   (*b*) Pipe piles.
   (*c*) Drilled-in caissons.
5. *Sand piles*—now more commonly and properly called sand drains.

Timber piles should comply with the specifications of ref. 5 the more important provisions of which are outlined below for convenience.

Untreated timber-foundation piles and untreated timber piles used for the temporary support of falsework may be of any sound timber that will stand driving. Piles more than

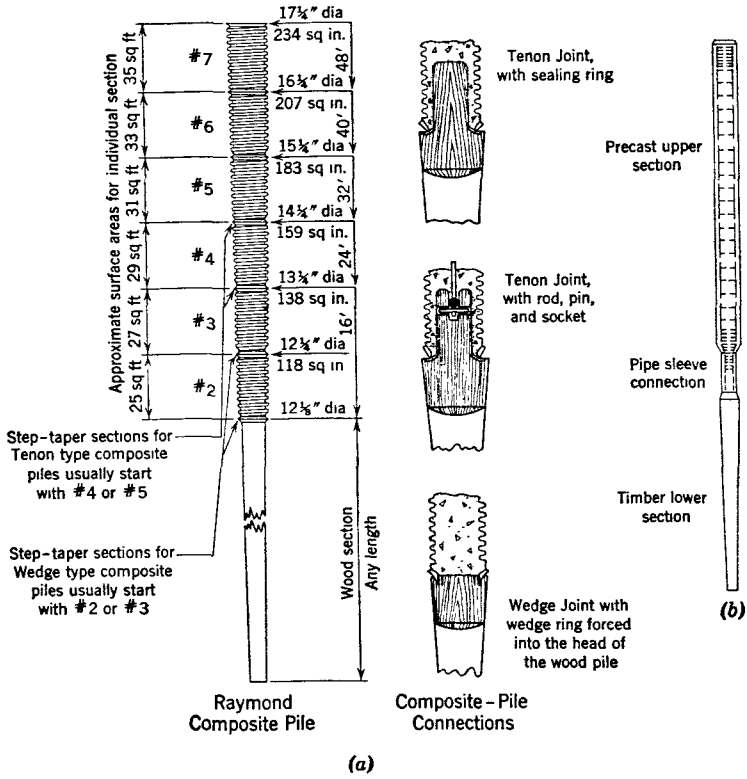


FIG 11. Composite piles Composite piles consist of a concrete upper section and timber a lower section or tip It is essential that the junction between the concrete and timber portions of the pile be below the water table or the timber will rot (*a*) Composite pile with cast-in-place upper section By permission, from *Foundations of Structures*, by C. W. Dunham, copyright, 1950, McGraw-Hill Book Company (*b*) Composite pile with precast upper section If used in water-front structure, the joint between sections must be below mud line.

40 ft long will, however, probably of necessity be southern pine or Douglas fir. Piles should be free from defects which may impair their strength such as decay, splits, twist of grain exceeding one-half the circumference in any 20 ft of length, large knots or holes, knots in clusters or groups, or shakes more than one-third the average diameter of the pile. Piles should taper uniformly from the point of butt measurement to the tip, should be free from short or reversed bends and from crooks greater than one-half the diameter of the pile at the middle of the bend. A line drawn from the center of the butt to the center of the tip should lie within the body of the pile. All knots and limbs should be trimmed or cut smoothly flush with the surface of the pile. Ends should be cut square with the axis of the pile. If peeling is required it should leave the pile smooth, clean, and free from inner bark. Untreated foundation piles need not be peeled unless they are to be stored for a considerable length of time before driving, in which case it is desirable to have the bark removed as it harbors destructive boring insects and their eggs.

The diameter of piles measured 3 ft from the butt should be between 12 and 18 in. The diameter of pile tips should be not less than 8 in for piles less than 40 ft long. Piles 40 ft long and longer should have tips not less than 6 in. in diameter. A tolerance of 0.5 in. less than the given diameter should be allowed in not more than 25% of the pieces of that diameter.

Pile lengths should be as ordered. Piles from 16 ft to 40 ft long inclusive should be ordered in lengths which are multiples of 2 ft; piles longer than 40 ft should be ordered in multiples of 5 ft. A variation of 6 in. should be allowed in the length of any one stick but the average length in a shipment should equal or exceed the billed length.

Treated timber piles should comply with Specification C12-51, Standard for Creosoted-Wood Foundation Piles, of the American Wood Preserver's Association, should be creosoted in accordance with the Association's Specification C3-53, Standard for the Preservative Treatment of Piles by Pressure Processes, and after treatment should be handled in accordance with the Association's Specification M4-52, Standard Instruction for the Care of Pressure-Treated Wood after Treatment.

Treated timber foundation piles should be southern pine, Douglas fir or red (Norway) pine; all of which are acceptable under the above-mentioned specifications. Southern pine is preferable because of its wider sap ring. Creosote does not penetrate heart wood appreciably. Southern pine piles intended for treatment should have a minimum sap ring of 2 1/2 in. Douglas fir and red pine should have a sap ring as wide as possible. Piles should be peeled and if possible air-seasoned for from 4 to 8 months before treating. If this is impracticable the piles should be steamed or boiled in the preservative in accordance with the above-referenced specifications.

The preservative should be creosote complying with Specification P1-53, Standard for Creosote, of the American Wood Preserver's Association and should be applied in accordance with the Association's Specification C1-53, Standard for Preservative Treatment by Pressure Processes—All Timber Products, as amplified and modified by the above-mentioned Specification C3-53.

The heads of treated foundation piles should be given at least two heavy brush coats of hot creosote, the first coat to be applied *immediately* after the pile is cut off to grade, in order that the untreated heart wood may be protected from dry rot. An even better method is to drive a knife-edged ring of heavy sheet steel 4 in. wide into the top of the pile immediately after it is cut off, Fig. 12, fill the resulting shallow basin with hot creosote, and allow it to soak into the end grain of the wood for 24 hr. After the application of the second brush coat or the termination of the soaking-in period the pile head should be given a heavy brush coat of hot coal tar.

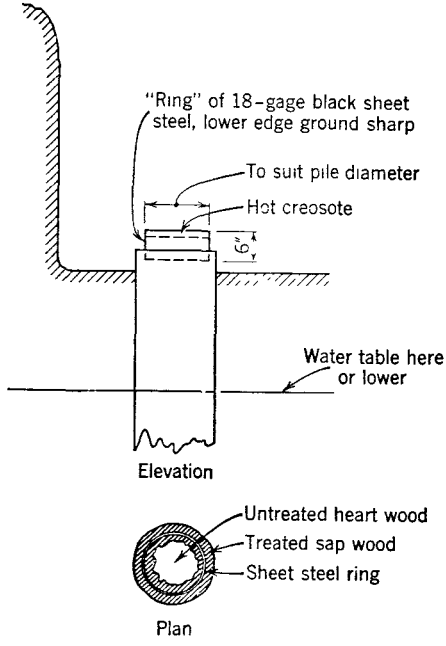


FIG 12. Protecting heads of creosoted foundation piles at cutoff Where creosoted piles are used to support a foundation it is usually because of the difficulty or impracticability of providing cutoff of untreated timber piles below the existing or probable future water table In such cases great care must be used to prevent the infection of the untreated heart wood of the pile with the spores of dry rot A heavy brush coat of hot creosote should be given *immediately* after the pile has been cut off. It is then advisable to follow up the brush coat with the "soaking in" procedure shown above and described in the text

In handling and driving creosoted piles care must be taken not to penetrate or damage the treated sapwood with cant hooks or by rough handling and thus admit spores of dry rot, larvae of boring beetles, or marine borers to the untreated heart wood.

The **splicing of timber piles,** both untreated and treated, is readily accomplished by methods illustrated in Fig. 13. Splices are made after the lower section of pile has been driven practically flush with the ground.

Lagging is sometimes attached to timber piles to increase their frictional resistance and bearing power. Its effectiveness is debatable. Unquestionably it increases the resistance of a friction pile to driving and the bearing capacity of a single pile as indicated by a load test. But whether lagging increases the capacity of piles to carry foundation loads depends upon characteristics of the soil, foundation, and loads. A single lagged pile, or a small group of widely spaced lagged piles, will distribute load to underlying strata over a larger area than if the pile or piles were unlagged and consequently will settle less. But the bulbs of pressure of a large number of closely spaced piles intersect so extensively that

lagging has little effect on the intensity of stress in compressible soil below the pile tips and hence little effect on the settlement caused by long-continued loading of the foundation. Lagging has about the same effect as increasing the diameter of a friction pile. It will increase the bearing capacity of a single friction pile and a small group of widely spaced friction piles driven into and underlain by cohesive soil, particularly if a large percentage of the total load is live load applied intermittently for short periods. Lagging is in general ineffective on piles closely spaced in large groups.

Concrete piles, both precast and cast-in-place, have been the subject of large volumes of technical literature. References 4 and 16 contain much information of practical value on concrete piles.

Precast concrete piles are usually more expensive than cast-in-place piles and are therefore less frequently used in dry-land foundation work. Their carrying capacity will as a rule be limited by the supporting strength of the soil, but consideration should nevertheless be given in their design to the requirements that they be able to carry their service loads without overstress and that they be reinforced sufficiently to permit lifting, handling, and driving without cracking. Insofar as service loads are concerned the design of precast piles which do not protrude above the ground surface may be based on the usual formulas for short concrete columns with either spiral wrapping or lateral hoop ties around the axial reinforcement and with such additional axial reinforcement as may be necessary to give the piles sufficient flexural strength to permit their being lifted and handled without cracking. The spacing of the spiral wrapping or hoop ties should also be decreased at the heads and tips of the piles as indicated in Fig. 9 to inhibit shattering of the concrete during driving. If part of the pile will be free standing above the ground surface it should be designed by a column formula applicable to the condition of fixity that will exist at the top and with an assumed condition of fixity 5 ft below the ground surface if the soil is firm and 10 ft below the surface if the soil is soft. At least a 1 1/2-in cover of concrete should be provided over the outermost reinforcement to protect the steel from corrosion.

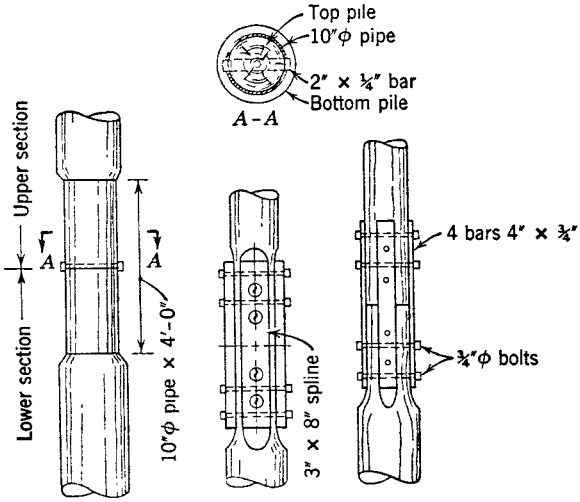


FIG 13 Splicing timber piles By permission from *Pile Foundations*, by R D Chellis, copyright, 1951, McGraw-Hill Book Company

Design of the reinforcement to withstand bending stresses due to lifting and handling is based on standard reinforced-concrete beam formulas and should take cognizance of the number and location of the proposed pickup points for the attachment of slings. A liberal allowance for impact should be made in computing the bending moments, some specifications requiring the addition of as much as 100% to the statically determined moments. The great importance of the number and location of the pickup points is shown by Fig. 14. It is usual to provide inserts in long heavy piles for the attachment of lifting gear. Piles up to 60 ft long are usually lifted with wire-rope slings around them, the corners of the piles being protected by wood lagging.

Precast piles should if possible be made in a casting yard near the job site because of the difficulty and expense of transporting them. The casting platform, usually of wood but occasionally of cement-bound macadam or concrete, must be level, smooth, and on firm ground that will not settle under the considerable weight of the curing piles. Forms, Fig. 15, are generally made of wood, though metal forms are economical on large jobs because they can be reused many times. The forms may be set as close together as convenient and the piles moved to a storage yard as soon as they are strong enough to stand handling. Another method is to space the forms for the first lot of piles cast on centers twice the width

of the piles in order that the first lot cast, with building paper against them, may be used as forms for the second lot. A second and even a third and fourth tier of piles may then be cast on top of the first, making it possible to produce a large number of piles in a small space.

Reinforcement should be assembled into cages. These must be well supported in the forms as they are very likely to get out of position during the pouring of the concrete.

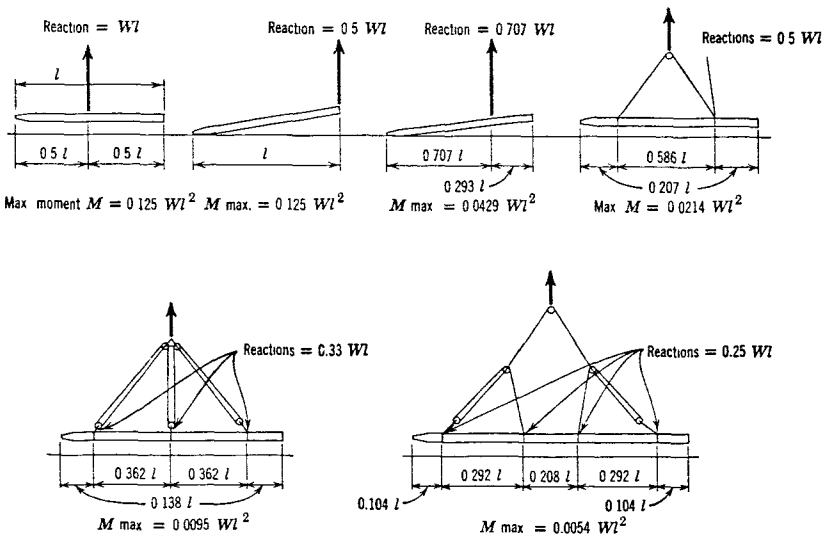


FIG 14 Pickup points and handling gear for precast concrete piles The bending stresses caused by picking up and handling precast concrete piles are greatly influenced by the number of points of attachment of the slings and the locations of the points

The concrete mix should be proportioned to produce material having an ultimate strength of at least 3500 lb per in.^{2} in 28 days. No aggregate larger than 1 $^1/_2$ in. or more than three-fourths the minimum clear space between the reinforcing bars should be used. The concrete should contain not more than 6 gal of water per sack of cement including the water in the aggregate. It should have a 3- to 4-in. slump for hand placing and 1- to 2-in. slump if vibrators are used. Vibration with internal high-frequency vibrators giving 3600–7000 impulses per minute is strongly recommended. To prevent air voids on the undersides

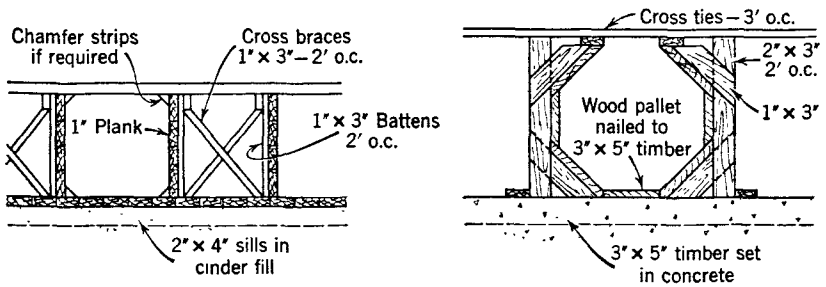


FIG 15. Forms for precast piles. Courtesy of Portland Cement Association

of form surfaces of octagonal piles, this part of the pour should be thoroughly spaded or, better still, vibrated with a blade or a loop of $^1/_4$-in. rod attached to the vibrator head. Top surfaces of piles should be finished with a wood float when the surface sheen has practically disappeared from the wet concrete Test cylinders should be made for at least each set of 25 piles cast and at least one set of cylinders should be made on each day piles are poured. Compression tests should be made on these cylinders in accordance with ref. 1.

High early strength cement may be used in precast piles when necessary to permit early handling and driving.

Precast pile forms should remain in place at least 24 hr after the piles are cast. Piles made with standard portland cement should then be kept soaked for at least 7 days either by ponding or covering with burlap, tarpaulins, or sand kept saturated by frequent sprinkling. This is necessary for only 3 days with piles made with high early strength cement. Piles made with standard and high early strength cements will normally be ready for driving in respectively 28 and 7 days. The usual precautions should be taken against damage by low temperatures when piles are cast or cured in cold weather

Hollow, cylindrical cast-in-place piles have been made by the centrifugal casting process. More recently large hollow centrifugally cast piles have been made in short sections with longitudinal holes cored in the walls through which steel cables are subsequently strung and used to prestress the sections by the method of post-tensioning and assemble them into a pile of any desired length. The prestressing cables are finally grouted into place. Many rectangular prestressed piles have been made in Europe by the pretensioning method. These types of piles are in general patented and are made and driven by the owners of the patent or their licensees

Cast-in-place concrete piles are made by driving a metal form into the ground and filling it with concrete. They may be subclassified as:

Mandrel-driven piles with thin metal shells, of which the Raymond pile is the most widely used in this country. The shell of the standard-type Raymond pile tapers at the rate of 0 4-in. diameter per foot of length and is shipped in 8-ft sections which fit together with a tight overlap in telescopic fashion. The shell is of corrugated sheet steel which is usually of 14 to 20 gage, but metal as heavy as 10 gage and as light as 24 has been used. It is driven with a heavy-steel mandrel inside it. After driving the mandrel is removed, and the shell is inspected by lowering a light into it and is filled with concrete. The standard Raymond pile is available in lengths up to 37 ft 6 in A variant, the step-tapered pile, is available in lengths up to 80 ft. Both standard and step-tapered Raymond piles are shown in Fig. 10.

Cast-in-place cased concrete piles driven without a mandrel have steel casings heavy enough to stand driving. A widely used type, the Monotube, has a tapered steel shell of gages varying from No. 3 to No. 11. The shells are fluted to give them added stiffness. They are driven, inspected internally, filled with concrete, and finally any excess shell at the top is cut off with a torch.

Cast-in-place uncased piles of which the Simplex and MacArthur are commonly used types are placed by driving a heavy untapered pipe casing, filling it with concrete, and then withdrawing the casing. The Simplex pile is driven without a core, a metal shoe closing the end of the casing being left in the ground. The MacArthur pile is driven with a core which fills the casing and is withdrawn before the concrete is placed. If an enlarged foot or "pedestal" is desired it may be formed on the MacArthur pile by replacing the core after the casing is partially filled and has been pulled up 3 ft or so. Driving the core lightly then forces concrete out of the bottom of the casing to form the enlarged foot, Fig. 10. The pile is completed by filling the casing with concrete and withdrawing the casing. Uncased concrete piles are objectionable in soils of such character that driving a pile will deform a nearby pile the concrete of which is still plastic. To offset this difficulty a light casing may be slipped inside the heavy one before the concrete is poured. This light casing is left in place and the pile then becomes a special type of cased pile.

Composite piles consisting of timber lower sections and concrete upper sections, Fig 11, are widely used. The upper sections may be precast, connected by either a pipe sleeve or dowel pin to the previously driven timber section and then used as a follower to drive the timber down until the entire pile is to grade. In other types of composite pile the concrete section is cast in place and is usually of the Raymond type.

Composite timber piles may be made by splicing a treated upper section onto an untreated lower section by one of the methods illustrated in Fig 13. As in other types of composite pile it is essential that the splice be below the water table.

Steel piles are used much more extensively today than in the early decades of the century, the increase being largely attributable to the versatility of H-beam and pipe piles *Drilled-in caissons* are employed for the special purposes for which they were developed.

Steel H-beam piles consist of heavy rolled H sections, the webbs as well as the flanges of which are thick enough to prevent moderate corrosion seriously weakening the pile. H beams drive easily, can be spliced readily by welding, Fig. 16, to make piles of great length, and therefore can be used as end-bearing piles to transmit loads to deep-seated firm strata which it would be impracticable to reach with timber or concrete piles. H-beam piles are occasionally spliced by reveting or bolting, but splicing by welding is standard practice today. A valuable characteristic of H beams is their ability to penetrate deep beds of soft

soil containing thin lenses of firm sand, or sand and gravel, through which it would be difficult if not impossible to drive timber or concrete piles. The small cross-sectional area of H beams furthermore minimizes the volume of earth displaced and thus practically eliminates any tendency of H-beam piles to heave the soil and with it nearby previously driven piles—see subsequent paragraph in this article on pile driving. The foundation of a nearby existing structure is less likely to be disturbed by the driving of H-beam piles

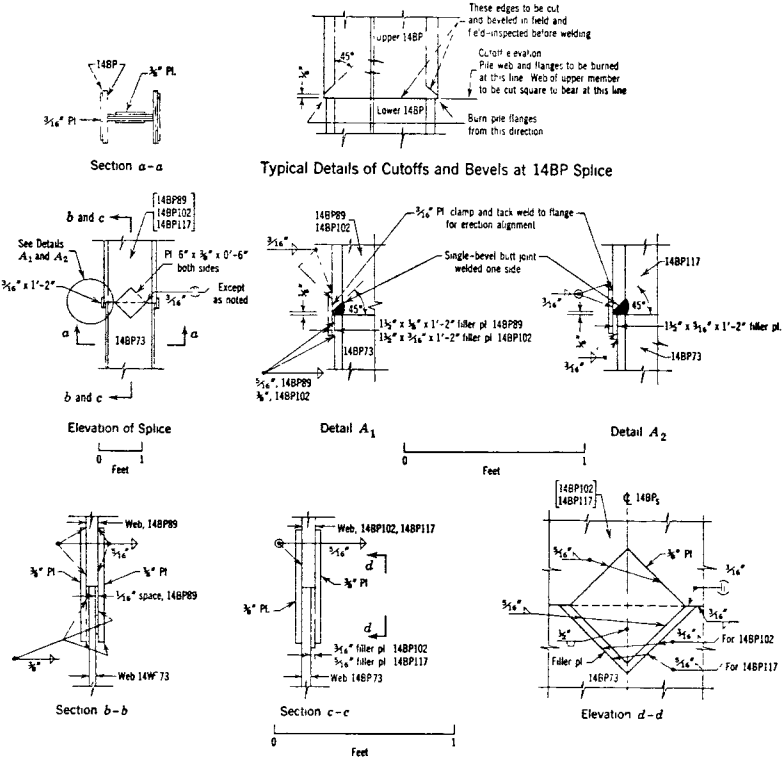


FIG. 16 H-beam pile splices The details are of a splice connecting a free standing upper part of a steel H-beam pile to a lighter lower section Where the two sections are of the same weight the splice is simplified by the omission of fillers

than by the driving of timber, precast concrete, or conventional types of cast-in-place concrete piles. Disadvantages of the H-beam pile are its high cost and vulnerability to corrosion by acid soils and cinder fills.

H beams are more often used as end-bearing than as friction piles. In many cases the bearing value of the material on which they rest is so high that their load-carrying capacity is limited only by their strength as columns. Table 2 gives the column strength of commonly rolled H-beam sections and Fig. 17 shows accepted methods of estimating their effective column lengths as foundation piles. To increase the contact area of H-beam piles that rest on hard rock into which they cannot be driven it is good practice to reinforce the pile tips in some such way as indicated in Fig. 18.

When H beams are used to reach a stratum of firm non-cohesive soil into which they must, however, be driven for a number of feet in order to develop satisfactory bearing value it has on occasion been found advantageous to box the pile by filling the spaces between the flanges, Fig. 19. This may be done by welding a plate between the flanges, by filling the spaces between them with concrete reinforced by rods welded to the steel as shown, or by filling the space between the flanges with timber.

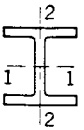


Table 2. Column Strength of H-Beam Piles

Bearing Piles

Elements of CBP and CB sections

Section Index	Nominal Size, in.	Depth of Section, in.	Weight per Foot, lb	Area of Section, in.^{2}	Flange Width, in	Flange Thickness, in	Web Thickness, in	Axis 1-1 I, in.^{4}	Axis 1-1 S, in.^{3}	Axis 1-1 r, in	Axis 2-2 I, in.^{4}	Axis 2-2 S, in.^{3}	Axis 2-2 r, in.
CBP 146	14 × 16	14 18	117	34.42	15 599	0 778	0 779	1233 5	174 0	5 99	492 8	63 2	3 78
		13.99	102	30 00	15 500	0 681	0 680	1060 9	151.7	5.95	423 1	54.6	3.75
CBP 145	14 × 14 1/2	13 86	89	26 19	14 696	0 616	0 616	909 1	131.2	5.89	326.2	44.4	3 53
		13 70	73	21 46	14 518	0 538	0 438	762 2	111.3	5.96	274 5	37 8	3 58
CBP 124	12 × 12	12 12	74	21 76	12 217	607	607	566.5	93.5	5 10	184 7	30 2	2 91
CB 124		12 12	65	19.11	12 000	606	390	533 4	88 0	5 28	174 6	29 1	3 02
CBP 124		11 78	53	15.59	12 046	437	436	394 8	67.0	5 03	127 4	21.2	2 86
CBP 103	10 × 10	10 01	57	16 76	10 224	564	.564	294 7	58.9	4 19	100.6	19 7	2 45
CB 103		10.00	49	14.40	10 000	558	340	272 9	54 6	4.35	93.0	18.6	2 54
CBP 103		9 72	42	12.35	10 078	418	418	210 8	43 4	4 13	71.4	14.2	2 40
CBP 83	8 × 8	8 03	36	10.60	8.158	.446	.446	119 8	29.9	3.36	40 4	9 9	1 95
CB 83		8 06	33	9 70	8 012	.463	300	117 9	29 3	3.49	39.7	9 9	2 02

Column loads in kips for CBP and CB sections

Effective Length, ft	CBP 146 117 Lb	CBP 146 102 Lb	CBP 145 89 Lb	CBP 145 73 Lb	CBP 124 74 Lb	CB 124 65 Lb	CBP 124 53 Lb	CBP 103 57 Lb	CB 103 49 Lb	CBP 103 42 Lb	CBP 83 36 Lb	CB 83 33 Lb
14					326	287	234	239	209	175	135	126
16			393	322	315	281	224	225	197	164	124	116
18	516	450	390	321	300	268	213	211	185	153	113	107
20	506	440	375	309	284	255	202	197	173	143	104	98
22	488	423	360	297	269	242	190	183	162	133	95	90
24	468	407	344	284	253	228	180	171	151	124	86	82
26	450	390	329	272	239	216	169	159	141	115	79	75
28	431	373	314	259	225	204	159	148	131	106	72	69
30	412	357	299	247	212	192	149	137	123	99	66	63
32	394	341	284	236	199	181	140	128	114	92	60	85
34	376	326	271	224	187	171	132	119	107	85		
36	359	311	257	213	176	161	124	111	99	79		
38	343	296	245	203	166	152	116	103	93	74		
40	327	283	233	193	156	143	109	96	87	69		
42	312	270	221	184	147	135	103		81			
44	297	257	210	175	139	127	97					
46	284	245	200	166	131	120	91					
48	271	234	190	158	123	114						
50	258	223	181	151		108						
52	246	213	172	144								
54	235	203	164	137								
56	225	194	156	131								
58	215	185	149	125								
60	205	177										

Courtesy Carnegie Steel Company.

Safe load values above upper zigzag line are for ratios of l/r not over 60, those between zigzag lines are for ratios up to 120, and those below lower zigzag line are for ratios not over 200.

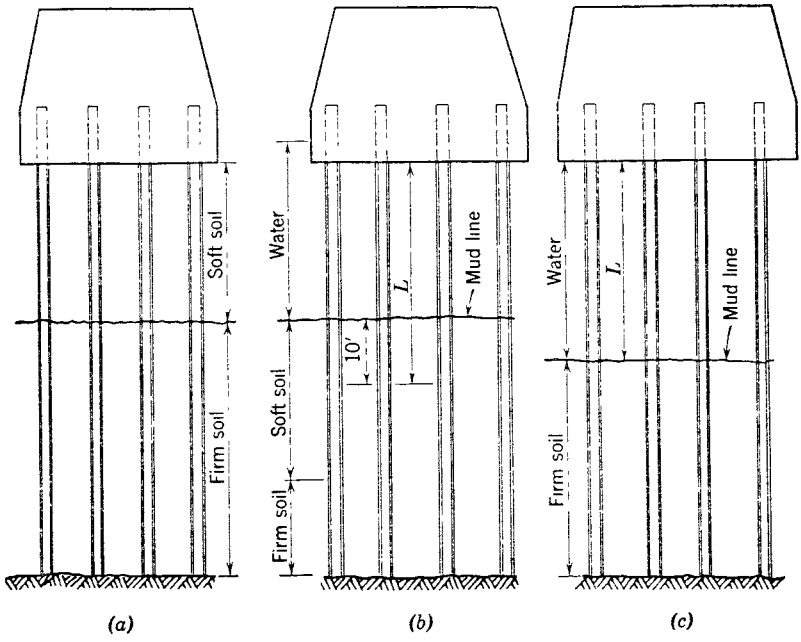


FIG. 17 Effective column lengths of H-beam piles Tests and accumulated experience show that the degree of fixity afforded by even soft soils is considerably greater than the assumptions upon which were based conventional rules for estimating the effective column lengths of H-beam piles The procedure outlined in the sketches above is safe and conservative It should be used unless a cognizant building code provides otherwise. (*a*) Pile fully imbedded in soil—even if soil is *very* soft—consider pile fully supported laterally for entire length (*b*) Pile free standing in water for part of its length and soil below mud line is soft Consider unsupported length distance from upper point of fixity to an assumed point of fixity 10 ft below top of soft soil (*c*) Pile partly free standing in water and soil at mud line firm. Consider pile fixed at the mud line

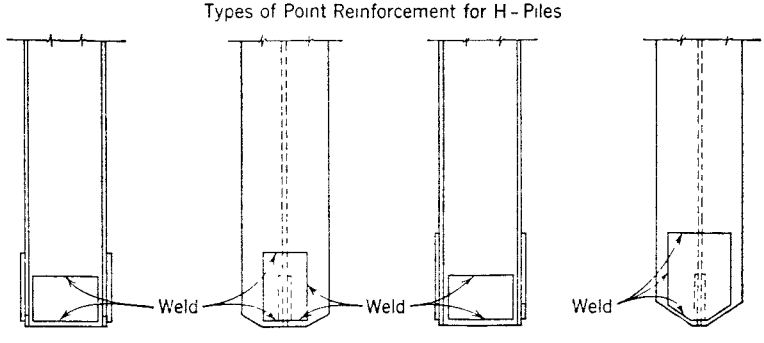


FIG 18 Tip reinforcement for H-beam piles Where H-beam piles are to be driven through sands and gravels to rock it is usual to reinforce the tips of the piles against distortion by welding on stiffening plates as shown Courtesy of Bethlehem Steel Company.

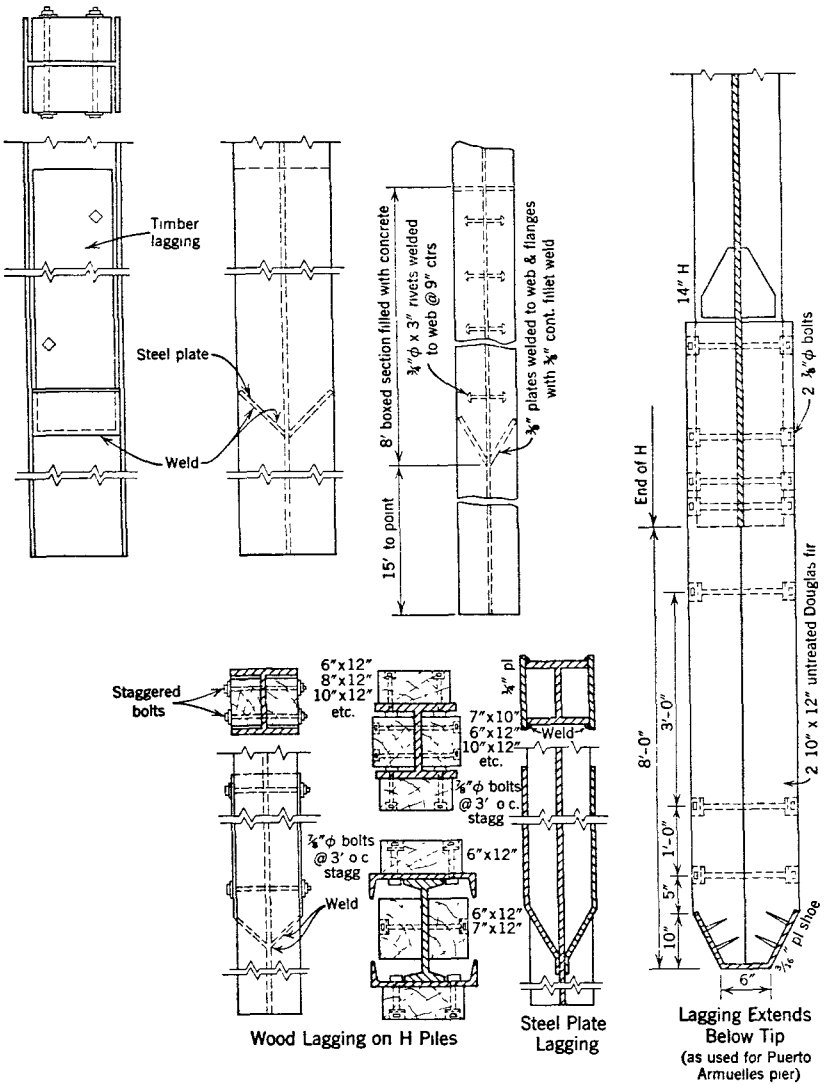
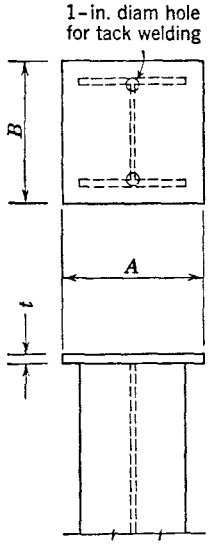


FIG. 19. Boxing H-beam piles. Courtesy of Bethlehem Steel Company.

H beams are occasionally used as long friction piles in cohesive soils. In estimating the probable bearing value of such a pile by a static formula the perimeter of the rectangle circumscribed about the pile section, or "net perimeter," rather than the actual perimeter of the section should be multiplied by the imbedded length of the pile to obtain the area to which the unit shearing strength of the soil is applied. Steel and timber lagging has been attached to H-beam piles to increase their resistance to movement in soil. It is, however, very questionable that money spent on lagging might not have been better spent in making the piles longer.

Bearing plates, unless demanded by building-code requirements, are unnecessary on the heads of H-beam piles embedded in concrete when the area of the pile section multiplied by the safe working unit compressive strength of the concrete plus the imbedded area of

steel multiplied by a safe working unit bond stress equals or exceeds the load on a pile and the concrete is adequately reinforced by spiral wrapping or hoops—see ref. 15. Typical bearing plates are shown in Fig. 20.



Pile Caps

Load, tons	BP 8 × 36				BP 10 × 42 BP 10 × 57				BP 12 × 53 BP 12 × 74				BP 14 × 73 BP 14 × 89 BP 14 × 102 BP 14 × 117			
	A	B	t	Weight, lb	A	B	t	Weight, lb	A	B	t	Weight, lb	A	B	t	Weight, lb
20	8	8	3/4	14												
25	9	9	13/16	19	10	10	7/8	25								
30	10	10	13/16	23	10	10	15/16	27	12	12	15/16	38				
35	11	11	13/16	28	11	11	15/16	32	12	12	1	41				
40	11	12	13/16	30	11	12	1	37	12	12	1 1/8	46				
45	12	12	7/8	36	12	12	1	41	12	12	1 3/16	48				
50	13	13	7/8	42	13	13	1	48	13	13	1 3/16	57	16	14	1 1/4	79
55					13	14	1 1/16	55	13	14	1 3/16	61	16	14	1 5/16	83
60					14	14	1 1/16	59	14	14	1 1/4	69	16	14	1 3/8	87
65					14	15	1 1/16	63	14	15	1 1/4	74	16	14	1 3/8	87
70					15	15	1 1/8	72	15	15	1 1/4	80	16	15	1 3/8	94
75					15	16	1 1/8	77	15	16	1 1/4	85	16	15	1 7/16	98
80									16	16	1 5/16	95	16	16	1 7/16	104
85									16	17	1 5/16	101	16	17	1 7/16	111
90									17	17	1 5/16	107	17	17	1 1/2	123
95									17	18	1 5/16	114	17	18	1 1/2	130
100													18	18	1 1/2	138
105													18	19	1 1/2	145
110													19	19	1 9/16	160
115													19	20	1 9/16	168
120													20	20	1 9/16	177

These caps are based on an allowable bearing value of concrete not exceeding 650 lb per in^2

FIG. 20. Bearing plates on H-beam pile heads Courtesy of Bethlehem Steel Company

Pipe or tubular piles from 10 to 24 in. in diameter with wall thicknesses from 1/4 to 5/8 in. have been widely used, particularly in New York City, as end-bearing piles of great length. The piles are in sections about 20 ft long, the ends of which are machined to insure strong,

straight joints. Couplings are internal cast-steel sleeves, machined to insure snug driving fit, Fig. 21. Occasionally the sleeves and pipes are threaded but this is rarely necessary. Sections are added as driving proceeds with only very short interruptions of actual driving for the addition of sections. Pipe piles are sometimes driven with a cast-steel shoe closing their lower ends After driving they are filled with concrete and thus become a special type of cased concrete pile in which the casing is so heavy that it is counted upon to carry part of the load. More frequently pipe piles are driven open-ended to minimize soil displacement and consequent tendency to produce heave of previously driven piles or disturbance of nearby existing foundations. In this case the soil inside the pipe is usually blown out with compressed air, the end sealed with concrete placed under water with a special bucket, and the water blown out with air and the pipe filled with concrete after the seal has hardened. The New York and Philadelphia building codes permit working stresses of 500 and 6000 lb per in.^{2} respectively on the concrete and steel of such piles and require an assumed reduction of $^1/_{16}$ in. in the thickness of the pipe wall when computing the steel area to allow for possible loss of metal by corrosion. Specifications of the American Railway Engineering Association limit working stresses in the materials of pipe piles to 400 and 6000 lb per in.^{2} respectively in concrete and steel and deduct the outer $^1/_8$ in. of pipe-wall thickness as possibly removed by corrosion. These specifications also require a reduction in the bearing capacity of pipe piles as determined by the above rule when the ratio of the length of a pile to its diameter exceeds 40. For each diameter in length in excess of 40 the reduction is 1%. A reduction of 5% in the allowable load is also demanded by these specifications for each splice below the upper section which is in excess of the number of splices that would be required for uniform 20-ft spacing of the splices.

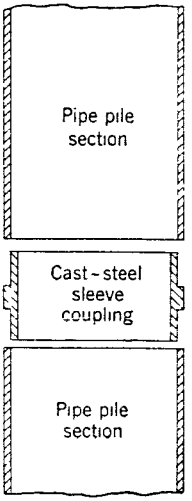


FIG 21. Cast-steel sleeve coupling for pipe piles The coupling is a driving fit in the pipe sections the ends of which are machined, as are all contact surfaces of the couplings.

Sand piles have been used to compact loose non-cohesive soils and poorly consolidated cohesive soils to improve their bearing value. An early method of placing sand piles consisted of driving closely spaced short timber piles, pulling them immediately after they were driven, and filling the resulting holes with sand compacted by tamping. A later method was to drive pipe piles, remove the encased soil, fill the casing with compacted sand, and finally pull out the casing, leaving the core of sand in the ground. The latter methods closely approximate methods used today to place sand drains. The effectiveness of sand piles in cohesive soils is largely due to the fact that they provide escape channels for water and thus accelerate consolidation. To be effective they must be accompanied by a surcharge load to squeeze the water out of the consolidating cohesive soil.

Sand drains are placed by driving thick-walled casings with either loose-fitting shoes or heavy flap valves closing their lower ends, filling the casings with permeable sand, and then pulling the casings in such manner as to leave the core of sand in the ground. The sand is sometimes compacted with internal vibrators as the filling proceeds, and the casings are often capped temporarily before they are pulled in order that compressed air may be used to force the sand core out of the casing as the latter is withdrawn from the ground. A blanket of pervious material is necessary over the tops of the sand drains to provide a means of escape for the water that flows up them. This blanket of pervious material is usually placed before the sand drains are driven, in order that it may be available to carry the pile driver and equipment on the soft soil. The blanket of pervious material, usually sand, also accelerates the early stages of consolidation by its surcharge weight and forms the bottom layer of the fill which is to be placed on the consolidated area. Sand drains have no value unless fill is to be placed over the area where they are driven to consolidate the soft soil by its surcharge weight. The height of the fill is usually increased temporarily over its designed permanent height to hasten consolidation.

Great care must be used in placing fill on an area being consolidated by the sand-drain method in order that mud waves, which are manifestations of deep rotational slides, do not occur. As shown in Fig. 22 the deep rotational slides that cause mud waves will shear off the sand drains and make them ineffective. They will also remold the soil, decrease its cohesive strength, and make it much more difficult to deal with than when in its original condition. To guard against the development of mud waves the fill should be placed in thin layers. Both horizontal and vertical displacement measurements should be made frequently on stakes set in the filled area and around its perimeter to make sure

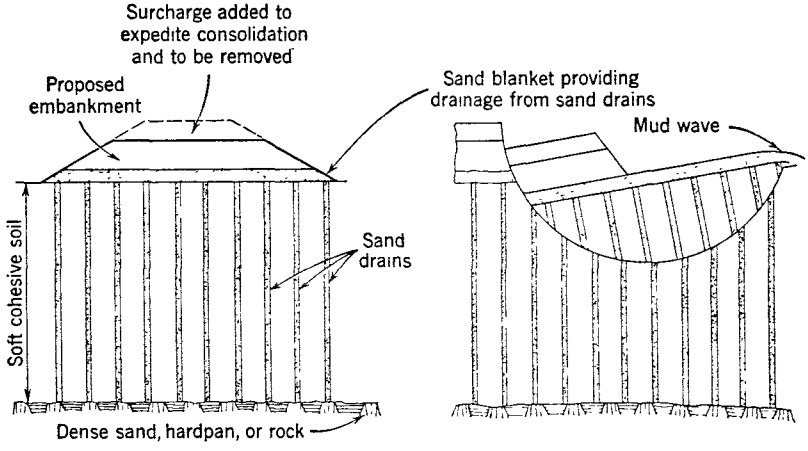


FIG 22 Cylindrical-slide failure and resulting mud wave on a sand-drain project. The top layers of the embankment were placed too soon and carried forward too near its outer end A cylindrical slide, evidenced by the formation of a mud wave beyond the end of the embankment, has occurred. Note that sand drains have been sheared off and made useless

that mud waves are not developing. On important projects pore-pressure measurements, Art. 3, Sect. 7, should be made as the filling progresses to insure that demands made on the shearing strength of the soil as indicated by circular-slide analyses do not exceed the actual shearing strength of the consolidating material as indicated by the formula:

$$s = c + (\sigma - u) \tan \phi \tag{1}$$

in which s = the shearing strength of the soil in lb per ft^2 due to both cohesion and internal friction.
c = the cohesion of the soil in lb per ft^2.
σ = the total pressure on a horizontal plane at the depth of the possible cylindrical surface of shear failure.
u = the pore pressure in lb per ft^2.
ϕ = the angle of internal friction of the soil.

On soft organic silt deposits such as salt marshes mud waves have been started by the placing of even a relatively thin blanket of sand when starting a fill. The use of cinders, rather than sand, for the initial layer of fill on such extremely soft soils is advisable.

The **bearing capacity** of piles is estimated by methods which must take cognizance of both the column strength of the piles and the ability of the earth to support them. In checking the column strength of piles full lateral support may be assumed where piles are imbedded in soil for their entire length, for even very soft soils provide sufficient lateral support to prevent failure of piles by buckling. In all cases of importance test piles of the type and dimensions tentatively selected for the support of the foundations should be driven and loaded by the methods described in Art 6, Sect. 7, under Test Piles If borings have revealed the presence of compressible cohesive material or loose granular soil under the stratum on which end-bearing piles are to rest the strength of such underlying soils and the ability of the stratum to resist failure by punching shear of single piles and pile groups should be estimated.

The bearing value of friction piles should be estimated by first making a rough assumption by the use of Table 6, Sect. 7. If the shearing strength of the soil has been determined another check should be made by applying this unit strength to the imbedded area of the pile and to the area of the smallest envelope that can be circumscribed about a pile group. Test piles should be driven by methods described in Art 6, Sect. 7, under Test Piles, but the fact should be kept constantly in mind that bearing value of n piles in a group is not usually n times the bearing value of a single pile as determined by theoretical considerations or the test loading of a pile. Careful consideration should be given to the sensitivity of a cohesive soil into which the driving of friction piles is contemplated. Above all, the performance records of similar piles in the same or similar soil should be studied as determination of the safe load on a friction pile is, in the last analysis, more a matter of experience and judgment than of applying theories and formulas.

Lateral Loads. The lateral loads that piles will carry without moving appreciably is difficult to estimate. Experience shows that almost any fully imbedded pile will resist a lateral force of 1000 lb without moving appreciably, and many building codes limit permissible lateral loads to this safe and conservative figure. If a foundation is to resist lateral forces of any magnitude, batter piles should be provided. The number of these should be such that the horizontal component of their safe loads equals the horizontal force to be resisted. The sufficiency of the anchorage provided by weight or anchor piles to resist the vertical component of force in the batter piles should also be investigated.

Uplift. Piles are frequently required to provide anchorage against uplift. Many designs have been based on the assumption that a friction pile will safely resist an uplift equal to one-half its safe bearing value or an uplift producing a shearing stress of not over 250 lb per ft^2 on its imbedded surface, whichever is the lesser. Piles which are to resist uplift must have sufficient tensile strength to withstand the upward vertical forces to which they will be subjected. This may require the reinforcement of cast-in-place piles and special consideration of the design of the splices in spliced or composite piles.

Pile-driving formulas are discussed in Art. 6, Sect. 7. Dynamic formulas are valuable "yardsticks" for determining when piles have reached a resistance indicative of a specified bearing value. In no case, however, should the bearing value of a test pile or foundation pile as indicated by a dynamic formula be accepted as adequate data upon which to design a pile foundation. Bearing values yielded by dynamic formulas should be considered merely one of several classes of data, all of which must be studied and correlated if a sound estimate is to be made of the bearing values which should be used in the final design of a foundation. A study of the driving records of test piles makes possible a determination of the bearing value indicated by the chosen pile formula which corresponds to the design bearing value. Piles should be driven to this "bearing value" provided further that they have been driven to the depth necessary to reach the stratum on, or in, which the design contemplated their support.

Pile groups and the manner in which the piles of a group distribute loads to the soil which support them are matters of great importance. Only in the case of end-bearing piles resting on bedrock, a stratum of compacted sand and gravel overlying bedrock, or hardpan of demonstrated bearing value is it safe to assume that a group of n piles will safely carry n times the bearing value of a single pile. In all other cases, and particularly when friction piles are supported by a cohesive or partially cohesive soil, intersection of the bulbs of pressure of the individual piles builds up soil stresses which necessitate reducing the bearing value of individual piles in a group by applying to the estimated safe bearing value of a single pile an efficiency factor the size of which depends upon a number of variables. The Converse-Labarre formula for this efficiency factor is contained in the Uniform Building Code of the Pacific Coast Building Officials Conference and the specifications of the American Association of State Highway Officials. The formula is:

$$E = 1 - \alpha \frac{(n-1)m + (m-1)n}{90mn} \tag{2}$$

in which E = the efficiency factor.
m = number of rows of piles.
n = number of piles in a row.
α = the numerical value, in degrees, of the angle whose tangent is d/s.
s = the spacing, center to center, of the piles in feet or inches.
d = the pile diameter, measured in the same units as the pile spacing.

Values of E computed by this formula for the common spacing of $s = 3d$ may be obtained from Fig. 23.

Another formula giving considerably lower values for the efficiency factor is the Seiler-Keeny formula:

$$E = \left[1 - \left(\frac{11s}{7(s^2-1)}\right)\left(\frac{m+n-2}{m+n-1}\right)\right] + \frac{0.3}{m+n} \tag{3}$$

in which the terms have the same significance as in the Converse-LaBarre formula but in which the term d does not appear. For an extensive treatment of this important subject of the action of groups of friction piles see ref. 11.

The **design of pile foundations** involves the selection of pile types by both functional and material classifications in conjunction with a decision as to the required lengths of the piles. The number of piles required under each footing or pile-supported mat must then be determined together with the most advatageous arrangement of the piles. Finally the probable settlement of the foundation must be estimated, and, if the piles are to be of the friction type driven into a cohesive soil, the stability of the entire foundation against shear failure should be investigated.

In selecting pile types end-bearing piles supported on a reliable material are much to be preferred over friction piles if the requisite lengths of end-bearing piles will not be prohibitively expensive. End-bearing piles are particularly desirable where foundations are required to carry heavy dead loads and where the probable periods of application of heavy live loads will be extended. Where dead loads will be light and the periods of application of live loads will be short, the prospects for small and acceptable settlements of friction piles in a firm cohesive soil are good. Friction piles should under no circumstances be used in soft sensitive clays. They will probably do more harm than good.

The material of which the piles should be made depends upon the size of the loads to be carried, the elevation of the water table, the possibility that the elevation of the water may lower progressively, the character of the soil, the proximity of nearby structures the foundations of which may be disturbed by pile driving, and the availability of special types of piles and the equipment necessary to make, handle, and drive them. The cheapest piles, where the loads to be carried are within their capacities, are usually untreated timber. But it is essential that untreated timber piles be below the water table throughout their entire length or they will rot. In locations where progressive lowering of the water table may result in it being below pile cutoff untreated timber piles should not be used. Properly creosoted piles the heads of which have been carefully treated immediately after being cut off should be permanent even if not completely submerged in ground water, but there is always the chance that damage due to careless handling or overdriving may expose the untreated heart wood and lead to the destruction of the pile. Composite piles, the upper sections of which are of concrete, are permanent if the elevation of the water table remains above the splice.

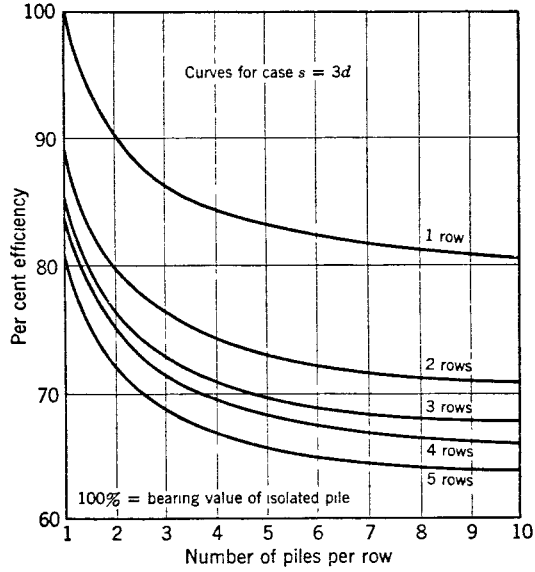


FIG 23 Pile group, Converse-Labarre formula. By permission, from *Pile Foundations*, by R D Chellis, copyright, 1951, McGraw-Hill Book Company

Cast-in-place concrete piles are usually more expensive than timber piles and are immune to damage by rot and boring insects. Uncased cast-in-place piles and cast-in-place piles made with a thin shell which is driven with a mandrel are cheaper than other types of concrete piles and are satisfactory where the driving of nearby piles does not damage piles or unfilled cases that have been recently driven. This objection can be overcome by using cast-in-place piles having a heavy casing that remains in place, but such piles cost more. All cast-in-place piles share the great advantage over precast piles of curing after they are driven, with the result that pile driving and foundation work can usually be started much more quickly with cast-in-place than with precast piles. Furthermore, money is not wasted in casting unnecessary lengths of piling.

Precast concrete piles can be fully inspected before being driven and can be driven through material that will damage the shells of most types of cast-in-place piles. Their relatively high cost, the space and time required to make and cure them, and the heavy equipment needed to handle and drive them are, however, serious disadvantages.

Steel H-beam piles, pipe piles driven open-ended, and drilled-in caissons can be driven as end-bearing piles to great depths and have the great advantage of causing a minimum disturbance of nearby existing foundations and piles already driven. They are all subject to corrosion in acid soils and cinders.

The lengths of end-bearing piles must be sufficient to reach the stratum on which they are to bring up. Lengths of friction piles must be such as to develop necessary bearing value as determined by a study of the geological profile, records of the performance

of piles in the same or similar formations, and the driving and loading records of test piles.

Pile groups should be planned to provide the maximum practicable spacing of piles, especially when the piles are of the friction type The groups should be symmetrical, and the load should if possible be applied at the centroid of the group. A reinforced-concrete footing, designed by the same standard procedures of reinforced-concrete design used for designing footings supported directly on soil, should be constructed to distribute the column loads to the piles When it is impossible to apply the load at the centroid of the pile group, the load on the most heavily loaded pile should be determined by the following method: Plot the proposed layout of the pile group to a convenient scale as in Fig. 24, and indicate the point of application of the resultant load. Draw the XX and YY axes intersecting at the centroid of the group as shown, the coordinates of the point of application of the load being x_0, y_0 as indicated. Compute the moments of inertia of the pile group about the XX and YY axes by the following formulas:

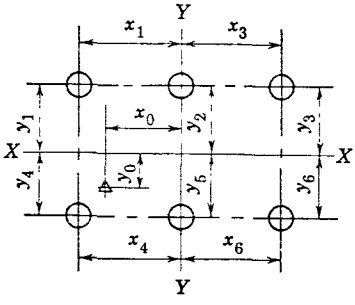


FIG 24 Pile group, load on most heavily loaded pile By permission, from *Pile Foundations*, by R D Chellis, copyright, 1951, McGraw-Hill Book Company

$$I_y = Ax_1^2 + Ax_2^2 + Ax_3^2 + \ldots + Ax_n^2$$

$$I_x = Ay_1^2 + Ay_2^2 + Ay_3^2 + \ldots + Ay_n^2$$

in which I_y and I_x are respectively the moments of inertia about the YY and XX axis, and x_1y_1, x_2y_2, etc , are the coordinates in feet of the centers of the piles The quantities A represent the area of the piles, and as these are all assumed to be equal the quantities A can all be assumed as unity.

Compute the load on any pile in the group by the formula:

$$R_p = \frac{P}{n} \pm \frac{Py_0y_n}{I_x} \pm \frac{Px_0x_n}{I_y} \tag{4}$$

in which R_p = the load on the selected pile in kips.
x_n = the distance of the selected pile from the YY axis in feet
y_n = the distance of the selected pile from the XX axis in feet.
P = the resultant vertical load on the pile group in kips.
n = the number of piles in the group
x_0 = the distance from the point of application of the load to the YY axis in feet.
y_0 = the distance from the point of application of the load to the XX axis in feet.

The most heavily loaded pile in the group will in general be the outermost pile on the same sides of the XX and YY axes as the point of application of the resultant loads, and the most lightly loaded pile the one diagonally opposite it A negative value of R_p for the most lightly loaded pile indicates that it will be subjected to uplift, a situation which should be in general avoided by redesign. The load on the most heavily loaded pile in the group should be within the established safe bearing capacity of the piles. If the loads on individual piles vary greatly the grouping should be rearranged, using a closer spacing on the heavily loaded side or corner until, by trial, an arrangement is obtained which substantially equalizes the pile loads.

A similar method may be used to determine the maximum load on the most heavily loaded pile of a foundation subjected to a horizontal live load such as the wind load on a smoke stack. In this case the XX axis should be chosen in the assumed direction of the wind and the loads on the most heavily and lightly loaded piles, which will be respectively the piles under the leeward and windward sides of the foundation, computed by the following simplification of eq. 4:

$$R_p = \frac{P}{n} \pm \frac{Mx_n}{I_y} \tag{5}$$

in which M is the moment of the resultant of the wind force about the base of the stack in foot kips and other terms are as in eq. 4. In this case equalization of pile loads under a varying wind load is impossible, but the load on the most heavily loaded pile should not exceed its safe bearing value.

Mat foundations are occasionally supported on large groups of friction piles, though in most cases a properly desgined mat renders piles unnecessary. If they are used, however,

the same principles on which the design of pile-supported footings are based apply to the design of the larger group supporting the mat. Where a pile-supported mat or closely spaced pile-supported footings rest on friction piles driven into a cohesive soil, the stability of the entire foundation against shear or area bearing failure and consequent tilting or excessive settlement should be checked. This is done by assuming a boxlike envelope, the sides of which surround the entire foundation and bottom of which lies under the pile tips. The stability of the imaginary rectangular foundation mass is then checked against shear and bearing failure.

Settlement estimates of pile foundations may be made by assuming that the settlement of end-bearing piles under design loads will about equal the settlements of test piles under similar loads. Estimating the settlements of foundations supported by friction piles is not so simple, and there is, in fact, no precise way of making such estimates. About the best that can be done is to assume a uniform distribution of load on a horizontal plane passed through the midlength of the piles and extending beyond the outer limits of the group for a reasonable distance and then make an estimate, based on consolidation tests, of the settlement that would occur if the foundation were a mat of the depth and dimensions of the assumed plane. Such an estimate should, however, be considered nothing more than a rough approximation of what may be expected.

Pile Driving. Foundation piles should if possible be driven by steam hammers. Drop hammers, in comparison with steam hammers, are archaic and inefficient devices which are not only slow and uneconomical but likely to damage piles. The relationship between weights of hammers and piles, limitations as to maximum hammer drop, and precautions necessary to insure the proper working of steam hammers outlined in Art. 6, Sect. 7, under Test Piles should all be maintained in the driving of foundation piles.

The heads of timber piles should be cut square with their axes and trimmed to fit snugly into the driving cap of the drop hammer or the base of the steam hammer. In many localities it is customary to point timber piles with an axe or adze before driving them. A conical or pyramidical shoe of heavy sheet steel is sometimes fitted to the pointed pile when it must be driven through thin strata of sand or gravel. It is undoubtedly advantageous and often necessary to point and shoe timber piles which must be driven through such strata, but in many soils blunt unpointed piles drive as easily as pointed piles and are less difficult to keep plumb during driving.

In driving precast concrete piles a cushion of some kind is necessary between the pile head and the driving cap or follower to prevent shattering the concrete. Cushions are made of soft wood, coiled rope, soft fiber board, old belting, and many other materials. A very satisfactory type consists of three thicknesses of cheap soft wood boards, each 1 in. thick nailed together with the grain of the middle thickness at right angles to the other two. Though some foundation men advocate the reuse of such cushions, it is better practice to use a new cushion on each pile as the use of old cushions that have lost their resiliency may lead to cracked piles. Another very satisfactory type of cushion consists of a single thickness of soft fiber board such as Celotex about 1/2 in. thick. As in the case of soft wood cushions a new one is advisable on each pile.

Jetting may be necessary and is often desirable where timber or precast concrete piles must be driven for some distance into sand. Two jet pipes, one on each side of the pile and neither pipe attached to it are advisable, though one pipe can often be used effectively. The pipes should be 2 or 2 1/2 in. in diameter drawn down at the tip to form a 1-in. nozzle. Each jet should be supplied with at least 250 gal per minute at not less than 150 lb per in.^{2} if the soil contains any cohesive material. In pure sand 60–100 lb per in.^{2} is usually adequate pressure. Quantity of water is more important than pressure, the effectiveness of jetting being more dependent upon lubrication of the sides of the pile than upon disturbance of the soil below the point. If the quantity of water is insufficient to "quicken" the soil around the pile by rising to the surface it is insufficient for effective jetting. By manipulation of the jet pipe, or pipes, much can be done to straighten up a pile that tends to get out of plumb. A jetted pile should be driven without the use of the jet, or jets, for the last 2 or 3 ft to make sure that it is firmly seated.

Steel piles can as a rule be driven without a cushion, the driving cap or hammer base resting directly on the pile head. Jetting is rarely, if ever, resorted to in driving steel piles.

Heave. In driving timber, precast, concrete, or cast-in-place piles it is necessary to keep a close check on the elevation of the heads of piles already driven lest they heave due to the effect of soil displacement when piles are driven near them. Piles which heave should be redriven to their original elevations. In soils where heave is troublesome the use of H-beam or open-ended pipe piles is advisable.

Overdriving. Great care must be exercised to avoid overdriving piles, particularly timber piles. The precautions outlined in Art. 6, Sect. 7, under Test Piles should be followed to guard against and recognize the breaking or brooming of foundation piles. Care-

MORAN, PROCTOR, MUESER & RUTLEDGE
CONSULTING ENGINEERS
420 LEXINGTON AVE. NEW YORK 17, N. Y.

FILE NO._____
DATE_____

PILE DRIVING RECORD

FOUNDATION FOR_____

CONTRACTOR_____ TYPE OF PILE_____
FOOTING NO._____ TIP DIAM._____ BUTT DIAM._____
PILE NO._____ LENGTH OF PILE AS DRIVEN_____
DRIVING RIG NO._____ ELEVATION OF GROUND_____
TYPE OF HAMMER_____ ELEVATION OF CUT-OFF_____
STEAM PRESSURE_____ ELEVATION OF BOTTOM_____
AVERAGE BLOWS PER MIN._____ PAY LENGTH_____

FT. OF PENE-TRA-TION	BLOWS PER FOOT	FT. OF PENE-TRA-TION	BLOWS PER FOOT	FT. OF PENE-TRA-TION	BLOWS PER FOOT	FT. OF PENE-TRA-TION	BLOWS PER FOOT	REMARKS
1		26		51		76		
2		27		52		77		
3		28		53		78		
4		29		54		79		
5		30		55		80		
6		31		56		81		
7		32		57		82		
8		33		58		83		
9		34		59		84		
10		35		60		85		
11		36		61		86		
12		37		62		87		
13		38		63		88		
14		39		64		89		
15		40		65		90		
16		41		66		91		
17		42		67		92		
18		43		68		93		
19		44		69		94		
20		45		70		95		
21		46		71		96		
22		47		72		97		
23		48		73		98		
24		49		74		99		
25		50		75		100		

I hereby certify that this is a true record of the driving of piles in this location and that all piles have been installed in accordance with design and New York City Building Code requirements, except as noted, and that I have personally supervised the installation of these piles.

RECORD CERTIFIED CORRECT
M. P. M. & R.

BY_____
LICENSE NO.

PILE DRIVING INSPECTOR_____
APPR'D. FOR PILE CONTRACTOR_____

FIG 25 Pile-driving report form, test and foundation piles.

less overdriving of piles, particularly timber piles driven with drop hammers, has caused many serious foundation failures.

Inspection. All pile driving should be done under the observation of a competent inspector who should keep a detailed record of the driving of each pile on a form such as is illustrated by Fig. 25. These records—the original field records, not copies—should be filed permanently in the office of the designing engineer. Under no circumstances should the records be destroyed as long as the structure to which they apply exists.

6. PIER FOUNDATIONS ON LAND

Pier foundations are usually employed where a stratum of high-supporting capacity not underlain by unconsolidated or highly compressible soils may be reached economically by any of the pier-installation methods. Piers are usually founded on bedrock, hardpan, compacted sand and gravel, or firm sand. They are generally constructed of concrete, either circular or rectangular in cross section.

Pier foundations are installed by a variety of methods (see refs. 7, 10, and 12). The types commonly employed include concrete piers built in open shafts excavated within braced sheet pile walls, and piers built by the method generally referred to and later described as the Chicago method. Large-diameter pipe piles, drilled-in caissons, open-dredged caissons of concrete or steel, and pneumatic caissons are all devices used in the construction of foundations which are fundamentally of the pier type. The special types of pier foundations constructed with the aid of open and pneumatic caissons are, however, described in Arts. 7 and 8, Open Caissons and Pneumatic Caissons.

Permissible bearing values employed in the design of pier foundations are frequently controlled by building code requirements. Permissible loads as given in Table 1 may be used as a guide in the selection of bearing values, but frequently it is necessary to determine bearing values based on soil-mechanics laboratory tests of soil samples. In all cases where pier foundations are not founded on bedrock, borings should be made to determine accurately the strength and supporting capacity of all soil strata beneath the contemplated bearing surface of the pier foundation.

Open shafts excavated within steel sheet piled walls may be sunk to considerable depths Piling is generally set around a template or frame All piling should be set and interlocked before any driving is done, and usual procedure requires that the sheet piling be driven to its ultimate depth before excavation and internal bracing of the shaft are started. The sheet piles should be driven successively not more than 5 ft at a time, since the driving of single sheets more than 5 ft ahead of adjoining sheets (and in compact medium-to-fine sands more than 2 or 3 ft ahead of adjoining sheets) results in damaged and frequently in stripped interlocks. Open sheet piled shafts may be excavated either by hand

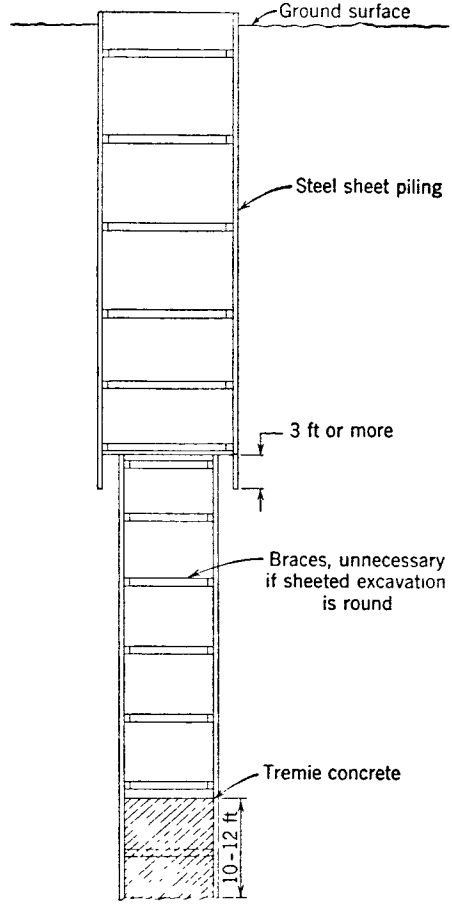


FIG 26 Double-tier sheet pile open well for pier foundation. Open wells of this type can be sunk only in impervious soils or soils above the water table.

or with clamshell or orange-peel buckets, internal bracing of timber or steel being installed as the excavation proceeds. Where the greatest practicable length of sheeting that can be driven does not reach the level of the bottom of the proposed excavation and where the excavation is through impervious soils, an internal offset may be made about 3 ft above the bottom of the sheet piled walls and another set of sheet piling driven as illustrated in Fig. 26. Normally the concrete in open sheet piled shafts is poured against the steel sheeting for the bottom 10–12 ft only, pier bracing in this lower section of shaft being removed as the concrete is placed. After the initial set of the concrete, and before full bond with the steel sheet piling has developed, the sheeting should be tapped by the pile-driving hammer to break its bond with the concrete so that on completion of the pier, the sheeting may be

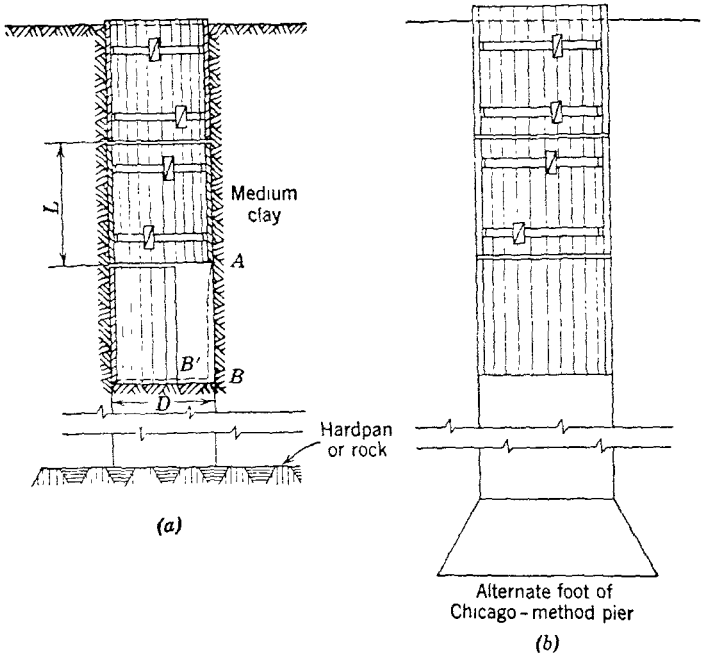


FIG. 27. Chicago method of excavating for pier foundation. By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G. P. Tschebotarioff, copyright, 1951, McGraw-Hill Book Company.

readily removed. Above the base concrete poured against the steel sheet piling, the pier is normally built within conventional forms after which the space between form and sheet piling is backfilled and the steel sheeting removed.

The **Chicago method**, employed extensively in the city of Chicago and in other locations where soil conditions are comparable, provides for the construction of pier foundations through a clay stiff enough to permit excavation, usually done by hand, to be carried 4–5 ft below the bottom of the braced, sheeted portion of the shaft. Figure 27 illustrates the method. After excavating the shaft 4 or 5 ft below the braced and sheeted portion, vertical sheeting of 2 by 6 in. or 3 by 6 in. planks, approximately 4 ft long and beveled at the edges to fit together like barrel staves, are installed to line the circular hole. The newly installed sheeting is then braced by two or three internal hoops of steel bars, angles, or channels, each hoop being made of two semicircular pieces bolted together to form a circle. The shaft is then deepened the length of a section of sheeting, new sheeting and bracing are installed in the newly excavated section and the process continued until firm material at the desired bearing level is reached. In locations where the Chicago method is applicable, the soil penetrated is generally impermeable and only small quantities of water flow into the excavation. This small inflow is handled by pumping. Where the Chicago-method piers bottom on soils of lower supporting value than a hardpan or compact sand and gravel, it is general practice to bell out the bottom of the shaft without sheeting, as shown in Fig. 27*b* to increase the bearing area of the pier. As the shaft is filled with concrete, the sheet-

ing usually, but not always, is removed. This method has proved successful and economical in fairly firm, uniform clays, but should in general be avoided where pockets or strata of water-bearing granular soils are likely to be encountered. However, where borings have demonstrated occasional thin sand lenses interbedded in a massive clay deposit, the Chicago method may be modified by the installation of short sections of steel-plate cylindrical caisson which may be installed and predriven through such water-bearing strata, the shaft then being continued by the normal procedure

7. OPEN CAISSONS

Open caissons comprise three general types which are so called because they are sunk through soil to an adequate supporting medium by open methods, that is without the use of compressed air, and as such are open top and bottom. Open caissons may be.

1 Cylindrical caissons,
2 Single-wall boxtype caissons, or
3. Multiwall caisson piers with dredging wells.

Drilled-in caissons from 2 ft to 4 ft 6 in. in diameter are properly classified, as their name implies, as a type of open cylindrical caisson although occasionally referred to as a type of pier foundation. The shells for drilled-in caissons are of welded steel plate and the caissons derive their name from the fact that they are sunk to rock and excavated, sealed against the inflow of water and soil by a tremie concrete plug, and the bedrock then core-drilled to receive a structural steel H-beam or heavy I-beam section to permit loading the finished caisson as a column to develop higher bearing intensities than would otherwise be obtainable.

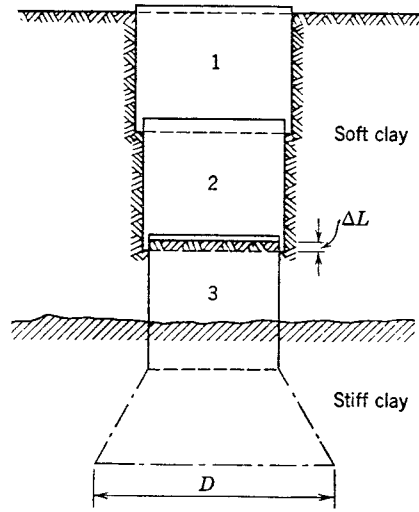


FIG 28 Gow caisson method of excavating for pier foundation ΔL must be sufficient to prevent soil from being squeezed up through the spaces between the caisson sections Numerical value of ΔL varies widely with soils of different consistencies By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G P Tschebotarioff, copyright, 1951, McGraw-Hill Book Company

The **Gow Caisson** is used in localities underlain by shallower clay deposits than are usually dealt with by the Chicago method. Figure 28 illustrates the essentials of Gow's method of sinking a shaft to firm material underlying clay. Excavation of the circular shaft is started without sheeting as in the Chicago method. As excavation proceeds welded or riveted circular liners are slipped down into place, each being from 4 to 8 ft in length and enough smaller than the one previously placed to pass through it. The liner sections therefore fit together by telescoping, and the diameter of the shaft decreases as its depth increases. Any inflow of water is dealt with by pumping as in the Chicago method. The bottoms of the shafts of Gow caissons are frequently belled out in the clay as are the lower ends of Chicago-method piers. The lengths of caisson lining are usually but not always removed as concreting proceeds. Gow caissons are more readily placed than the timber lining employed in shafts sunk by the Chicago method, but the reduction in diameter necessitated by the telescoping of the sections limits the depths to which Gow caissons can be sunk without excessive reductions in diameter.

Single-wall open caissons, Fig. 29, are constructed of timber or concrete and are box-like self-braced structures, partly or entirely open both top and bottom and sunk through soil, or water and soil, to an adequate supporting medium by excavating soil materials below the open floor of the caisson as it is sunk, after which the caisson is filled with masonry or concrete. This type of caisson is used occasionally on land but is employed more extensively for the construction of bridge piers

Multiwalled open caissons of the type described in Art. 14 are used occasionally for the construction of building foundations but are employed much more extensively for the construction of bridge piers.

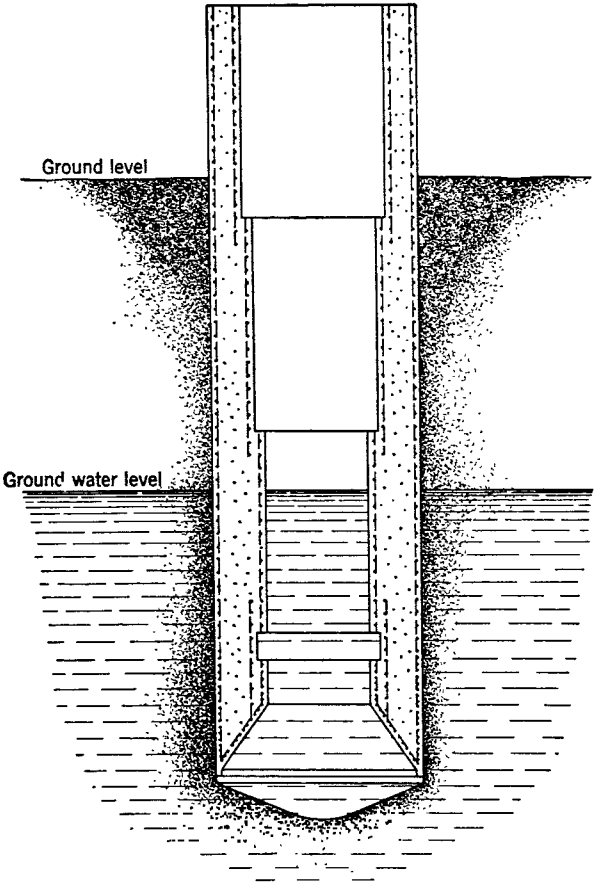


FIG 29 Single-wall open caisson Caissons of this type are built on the surface of the ground and sunk by excavating or dredging through the open well with clamshell or orange-peel buckets In case difficulty is caused by striking an obstruction an air lock can be fitted and sinking continued by the pneumatic process. Open-well caissons are used more widely for bridge foundations than building foundations.

Rotary-Drilling Methods of Sinking Caissons. Other methods of sinking small-diameter cylindrical caissons, have been based on some form of rotary drilling. For example, the concrete pier foundations of the Detroit Post Office were poured in shafts averaging 6 ft in diameter and 118 ft in depth bored by a machine to full depth in firm clay before sectional lagging was installed. In less stable soils caving of the sides of deep-bored shafts has been prevented by keeping the holes filled with a thick, semifluid "drilling mud" as the work proceeded. After the hole reaches full depth, a steel plate liner is installed and the bottom sealed with sand and cement placed outside the casing, or tremie concrete inside it, before the mud is pumped out and the caisson filled with concrete.

Caisson foundations for the Federal Building in New York were sunk to rock through a deposit of sand containing some boulders by an unusual method. Teeth were cut in the lower edge of the steel plate cylinder caisson, bent slightly "in and out" to give them a requisite "set" and faced with Stellite applied by gas welding. The full-length caisson was then lifted into a special rig which rotated it as a unit with a slightly eccentric motion and caused it to bore its way down through sand and boulders like a huge "crown saw." Water fed into the capped top of the rotating caisson escaped through and around the cutting teeth and, flowing up the outside of the cylinder, lubricated its exterior surface and washed

most of the sand out of the interior, so that only a mud slurry remained to be removed after the saw teeth had cut their way into the bedrock far enough to provide a firm seat. Where unusual obstructions caused trouble the boring process was stopped, an air lock fitted, and sinking completed by the pneumatic method. The method proved successful on this job but has not been successful when attempted on sites underlain by deep fills containing heterogeneous obstructions.

8. PNEUMATIC CAISSONS

The **pneumatic caisson,** Fig. 30, utilizes the simple principle that hydrostatic pressures may be counterbalanced by pneumatic pressures. Fresh water weighs 62.4 lb per ft^3 and for each foot of depth below the ground-water level exerts an increment of pressure of 62.4 lb per ft^2 above the pressure of the atmosphere. This is equivalent to $\frac{62.4}{144} = 0.433$ lb per in.^{2} multiplied by the depth in feet below ground-water level. Therefore at any depth d in feet the required pressure is: $0.433d = (0.333 + 0.1)d = \frac{d}{3} + \frac{d}{10}$ lb per in.^{2} Thus at 51 ft below ground-water level the hydrostatic pressure, and hence the required counterbalancing air pressure in a pneumatic caisson, is $51 \times 0.433 = \frac{51}{3} + \frac{51}{10} = 17 + 5.1 = 22.1$ lb per in.^{2}

Whereas the dredging wells of open caissons are open top and bottom, the tops of the working chambers in which excavation is carried on in pneumatic caissons are closed by air-tight covers. These covers are fitted with air locks through which men and materials may enter or leave the working chambers without permitting the escape of compressed air maintained at such pressure as will equal or closely approach the hydrostatic pressure outside the caissons. Workmen, called sand hogs, enter and leave the working chambers and excavated materials are removed from the caissons through the air locks. When a caisson reaches its ultimate supporting medium, usually bedrock, the surface can be thoroughly cleaned and prepared and carefully inspected, thus assuring maximum bearing capacity.

This procedure permits the excavation of water-bearing soils in the dry, since the pneumatic pressure within the caisson may at all times equal and balance the hydrostatic pressure in the soil. As the caisson sinks as the result of excavation over its floor, the hydrostatic head increases and the pressure of the air within the caisson correspondingly increases.

The pneumatic caisson has been used extensively in bridge foundations but its largest contribution has been in the field of foundations for buildings, particularly for heavy structures in locations where water-bearing fine-grained non-cohesive soils overlie bedrock or hardpan at depths up to about 110 ft below ground-water level (see refs. 12 and 18). The development of downtown Manhattan Island with its skyscrapers would have been impossible without the development of the pneumatic caisson. Here water-bearing New York quicksand or "bulls' liver" is encountered from depths only slightly below street grades down to bedrock, or to glacially deposited hardpan overlying bedrock. The water table is relatively close to the ground surface, and no adequate supporting medium is encountered until bedrock or hardpan is reached. Open methods of installing foundation piers to rest on bedrock or hardpan involve the major hazard of permitting the flow of the water-bearing "bulls' liver" to undermine the support of street utilities, subways, and surrounding buildings. The pneumatic caisson eliminated such risks and as a consequence most of downtown New York's high buildings are supported on pneumatic caissons or frequently on pneumatic cofferdams.

When rectangular pneumatic caissons which extend to bedrock are placed consecutively around the perimeter of a building lot, and such caissons are connected by watertight joints between each adjoining pair of caissons, the resulting retaining wall is called a pneumatic cofferdam.

A pneumatic caisson may be most easily visualized by considering a solid concrete or masonry box whose sidewalls are extended downward as "cutting edges" to form one or more rooms, called working chambers, extending over the full bottom area. These rooms are floorless and of sufficient height to permit workmen to excavate the soil materials from the floor of the caisson. From the roof of each such room one or more vertical shaftways (usually steel cylinders of elliptical cross sections) extend up through the concrete-filled box to connect to an air lock. Air is supplied to the caisson working chamber at desired pressure, and conduits for light, telephone, etc., in the working chamber are built into the concrete body of the caisson.

A pneumatic caisson may be built up in its entirety before air is applied and sinking starts, but usually it is built in sections as sinking progresses. The cutting edges of the caisson are landed on its supporting medium (usually bedrock) when excavation and sinking are complete. The bottom is then inspected and accepted, the working chamber and shaftways are filled with concrete, and a solid concrete pier founded on firm material is thus produced.

Frequently the weight of a pnuematic caisson is insufficient to overcome the skin-friction resistance of the soil and cast-iron blocks are placed on the top of successive caisson build-ups to increase the sinking weight.

The most essential element of a pneumatic caisson is the air lock, Fig 30c, placed in each of the vertical shaftways which extend from the roof of the working chamber up through the body of the caisson. The air lock consists of a steel cylinder fitted with a top and bottom door. When workmen or material buckets are being removed from the caisson, the top door is closed and the bottom door is left open until the men or bucket are inside the lock. Then the bottom door is closed and the compressed air in the lock is vented off through a valve until reduced to atmospheric pressure, when the top door is opened, the closed bottom door preventing the loss of air pressure in the working chamber and shaftway. Such venting off must be at a controlled slow rate when men are leaving the caisson. When men or material buckets are entering the caisson, the reverse procedure is followed. After entering the lock, the top door is closed and the lock chamber compressed to the pressure obtaining in the caisson, when the bottom door drops to permit lowering of material and material buckets and entrance of

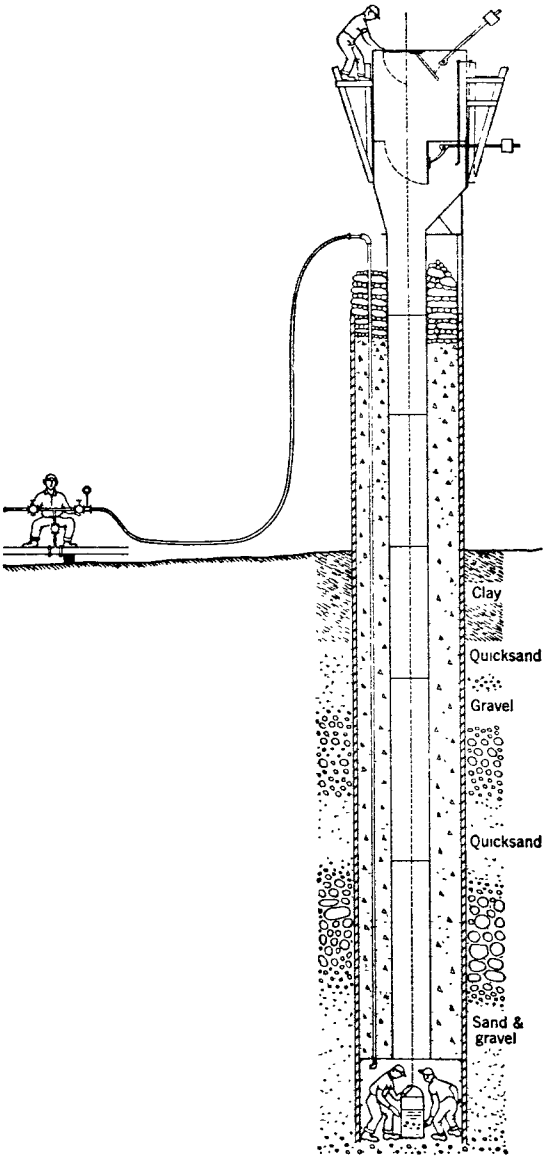


FIG 30 Pneumatic caissons and air lock (a) Diagrammatic drawing of small pneumatic caisson (b) A larger caisson (c) Sketch showing operation of the Moran air lock Courtesy of the Foundation Co, New York City (d) Large pneumatic caisson for bridge pier, by permission, from *Foundations of Bridges and Buildings*, by H S Jacoby and R P Davis, copyright, 1941, McGraw-Hill Book Company

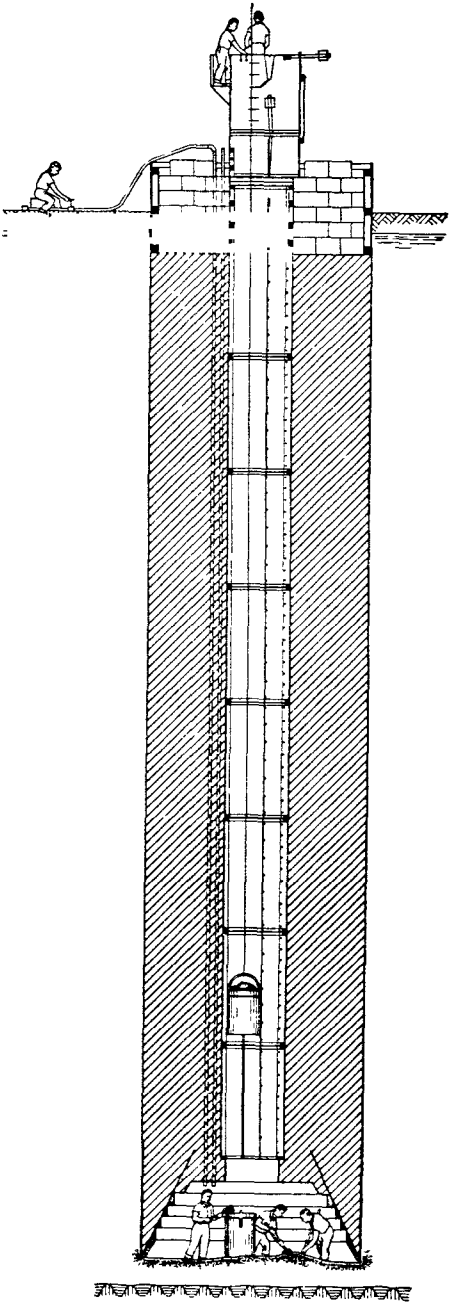
workmen down a ladderway built into the side of the shaftway

In pneumatic caissons for building foundations, there is usually a single shaftway over the center of the working chamber and therefore but one air lock, which is usually placed at the top of the shaftway above the top of the caisson. This single shaftway then serves both men and material handling.

In addition to the principal uses described for pneumatic caissons, they are also employed to provide foundations under dams, as for the Hales Bar Dam across the Tennessee River near Chattanooga Advanced pneumatic tunnel technique also includes the installation of a pneumatic caisson across the line of shield-driven, under-river tunnels. Sunk near each shore line, such caissons provide facilities for the shield assembly and the passage of the shield and liner plates out of the caisson under compressed air in beginning tunnel-driving operating This procedure was highly developed for the Holland Tunnel, built 1921–26 under the Hudson River between New York City and Jersey City.

Safety Requirements. State laws invariably stipulate safety provisions that must be complied with to guard the welfare of men who work in pneumatic caissons. Duplicate air compressors and piping systems to air locks are required and pressure gages showing accurately the air pressure in the air lock and in the working chamber below the lock must be provided. While men are working under compressed air, gage tenders are constantly watching the gages. The laws require a recompression chamber or medical lock and change room on every air job, also one or more licensed physicians who shall examine every man before he enters compressed air and who shall be on hand to take care of men who are affected by the air.

It is dangerous for men who are not in fit physical condition to enter compressed air, particularly under the higher pressures, and particularly dangerous for men to emerge from compressed air without properly passing through the decompression period. Men entering air become charged with compressed air which enters the body tissues, the blood stream, and the entire system so that the condition of the body is much like a bottle of soda water. Men feel exhilarated due to the concentration of oxygen in the air and can easily overexert, although modern labor is not usually so inclined. The principal danger to the workers is when they are coming out of the air. If they emerge too suddenly,

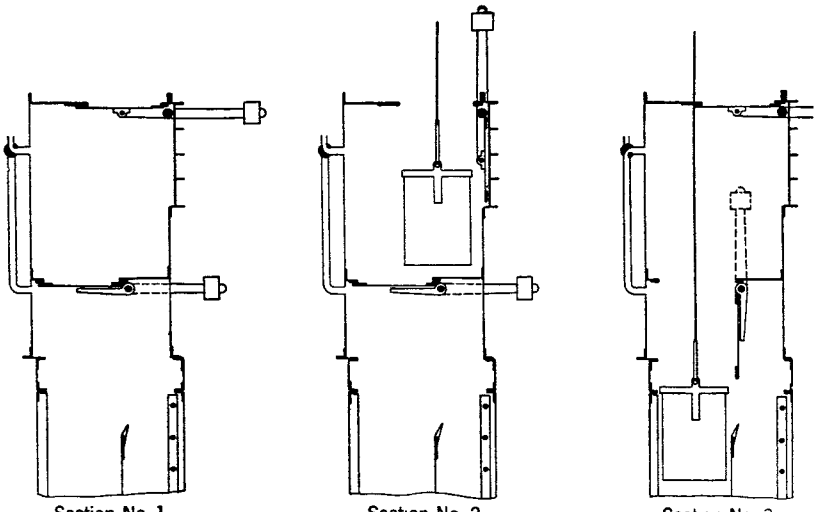


(b)

Fig 30(*b*).

bubbles will form in the body tissues and blood stream, causing cramps, paralysis, or death. This effect may not immediately follow leaving the air; it may come on any time within several hours after leaving compressed air. Again the example of the bottle of soda water applies. If the bottle is opened suddenly, the soda water effervesces violently, whereas if the cap is slowly pried off, very little bubbling occurs and in the case of a slow leak, no bubbling. These same bubbles form in the human body when the worker emerges from air too suddenly.

Each worker in compressed air wears a badge stating that he is employed by a certain contractor at a certain address and that if he is taken sick after he leaves the work he must be immediately returned to the job. He can be relieved only by placing him in the medical lock and putting him under the same air pressure as that in which he had been working.

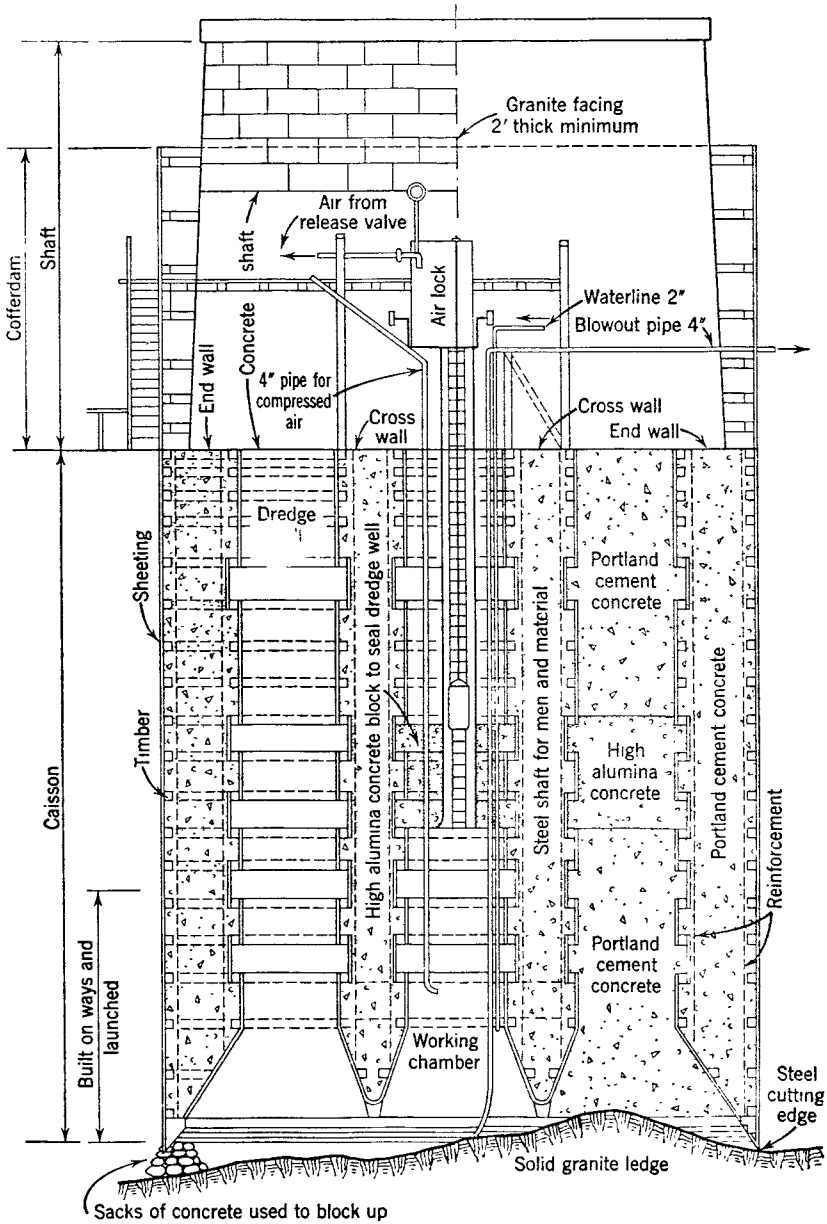


(c)

FIG 30(c).

He is then very slowly decompressed, and usually his troubles can in this way be eliminated. There are, at times, forms of paralysis which occur after men have been working in air which cannot be treated because the paralysis occurs slowly and a long time after the worker has left the air.

Combination open and pneumatic caissons as the term implies are sunk partially by open methods and partially under compressed air. Recent developments in foundation engineering have produced a number of such installations some of which were used in constructing building foundations, though the methods are more usually applied to bridge piers. Economy may be achieved by this combination of open and pneumatic caisson methods: (*a*) where the ground-water level is considerably below sinking level, but where excavation must be carried well below ground water through water-bearing granular soils, such as quicksand; (*b*) where the upper stages of sinking are through cohesive soils, such as stiff clays, but where such soils are underlain by water-bearing quicksands or other water-bearing non-cohesive soils; (*c*) where the water level within a non-cohesive soil may be lowered by artificial means, such as by the use of well points, when the caissons may be sunk by open methods to the lowered ground-water table; or (*d*) where open dredging through deep water-bearing soils is permissible, but where manual preparation and inspection of the finished bottom is desired. See ref. 14.



Kennebec River Bridge

(d)

FIG. 30(*d*).

9. LAND COFFERDAMS

Land cofferdams are usually necessary for the construction of large spread footings and mat foundations. They are frequently necessary for the construction of the basements of large buildings carried on pile, pier, or caisson foundations in congested areas.

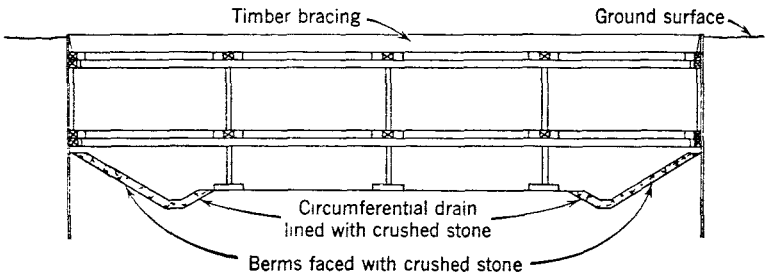


FIG 31 Internally braced land cofferdam in non-cohesive soil In cofferdams of this type in water-bearing non-cohesive soils internal berms are frequently necessary to prevent a concentration of flow lines and consequent upward flow causing boiling of the bottom near the sheet piling This is even more probable in the corners than along the sides of the cofferdam The circumferential drain keeps the working surface dry The stone lining of the drain and face of the berm inhibits sloughing, boiling, and erosion

Land cofferdams are today generally constructed with steel sheet piling internally braced with either timber or steel wales, struts, and framing. Occasionally horizontal wood sheeting fitted between H-beam piles supports the sides of the excavation. The internal bracing should be designed to withstand earth pressures estimated by methods outlined in Art. 10, Sect. 7, with the help of Fig. 90. See Art. 13 for methods of estimating stresses in piling, wales, and braces. Typical arrangement of the sheeting and bracing of land cofferdams is shown in Fig. 31. This figure also indicates the highly desirable and frequently necessary berm around the perimeter of the cofferdam which, if room can be provided for it, will tend to lessen the danger of boils forming in water-bearing sands. In many cases where a deep excavation must be made in a water-bearing granular soil, predrainage by well points, deep wells, or a combination of the two may be necessary to prevent serious boiling of the bottom, involving soil losses from below and outside of the excavation. The necessity of such measures can frequently be predetermined by drawing a flow net representing probable conditions of flow into the cofferdam and estimating the rate of upward flow and the hydraulic gradient inside the cofferdam, Fig. 32 If the indicated gradient exceeds unity, see Seepage Forces in Art. 8, Sect. 7, boiling is probable. Unfortunately two-dimensional flow nets do not adequately represent the conditions existing in the corners of rectangular cofferdams or at the ends of long, narrow cofferdams. It is therefore advisable, if possible, to limit the hydraulic gradient of upward flow in such cofferdams as predicted by two-dimensional flow nets to 0.5 rather than to design for the theoretical critical gradient of unity. See Art. 13, Cofferdams.

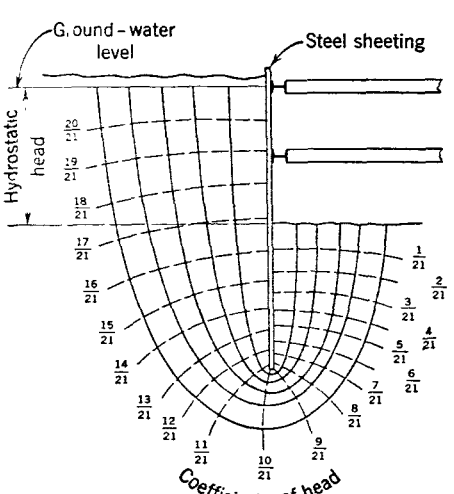


FIG 32 Flow net for cofferdam From *Cofferdams*, L White and E A Prentis, Columbia University Press, 1940

10. FOUNDATIONS ON PERMAFROST

Soils freeze to great depths in arctic climates and remain frozen throughout the year except for an active surface layer a few feet in thickness which alternately thaws in summer and freezes in winter. As in temperate zones, the thickness of the active layer varies with climatic conditions. Because habitable areas in the arctic are generally valleys underlain by permafrost, foundations must be built on it. The fundamental principle governing the design and construction of stable foundations on permafrost is the same as that followed in temperate zones. Foundations should not rest on material that alternately freezes and thaws. But as permafrost foundations rest on soil that *should never thaw*, insulation is necessary to prevent conduction of heat to the material upon which the foundations rest. It is also necessary to avoid adhesion of the active layer to the foundations to prevent their being heaved in winter.

The **depth of thaw** under buildings and pavements supported on permafrost may be estimated in locations where meteorological records are available. A curve of cumulative

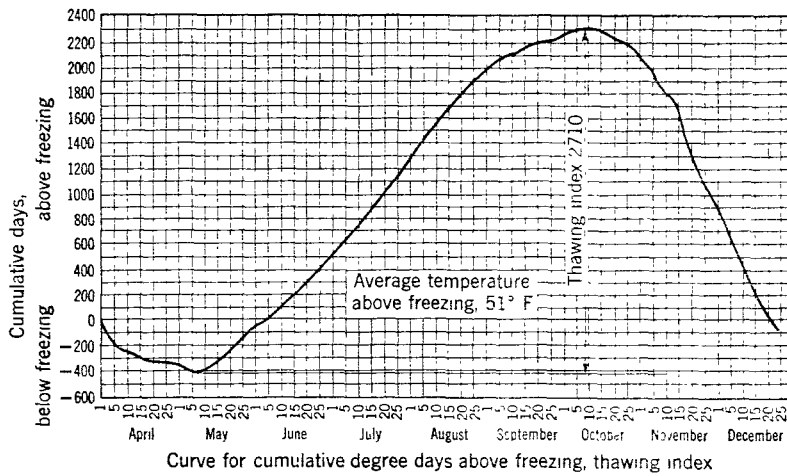


FIG 33 Determination of thawing index for permafrost regions. From *Cold Weather Engineering*, courtesy of Bureau of Yards and Docks, Navy Department, 1948–1949

degree days above freezing, Fig. 33, is drawn, and the thawing index or maximum ordinate to the curve noted. From the curves of Fig. 34 the depth of thaw corresponding to the thawing index is then obtained.

Footing foundations on permafrost have been constructed for small buildings in Siberia as indicated in Fig. 35. The depth of the foundation should be about twice the depth of thaw. The timber grillage on which the concrete foundation rests acts as a thermic insulator and will not rot when frozen. The sloping surfaces, smooth finish, and asphaltic coating of the footings within the active range prevent adhesion of the active layer when it freezes and consequent heaving of the foundation.

Pile foundations on permafrost have been used successfully in Alaska. As it is impossible to provide cutoff below the water table, timber piles should be creosoted in the manner recommended in Art. 5 for foundation piles. The best time of year for pile driving occurs in the fall as soon as the active layer freezes sufficiently to support the pile driver and equipment. The ground must be thawed at the location of each pile to the depth to which it is to be driven. This is done with a steam jet. Steam at 30 lb per in.^{2} delivered through a $^3/_4$- or 1-in. pipe has been found satisfactory for thawing to a depth of 20 ft. For greater depths pressures of 60 and even 90 lb per in.^{2} have been used, the jet pipe being from 1 to 2 in. in diameter. The penetration of the pile into the permafrost should be at least twice the depth of the active layer or depth of thaw. The surface of that part of the pile which will be imbedded in the permafrost should be roughened to promote adhesion. The part of the pile which will be in the active layer should have a smooth surface. Furthermore it should be coated with asphalt, wrapped with felt over which a second coating

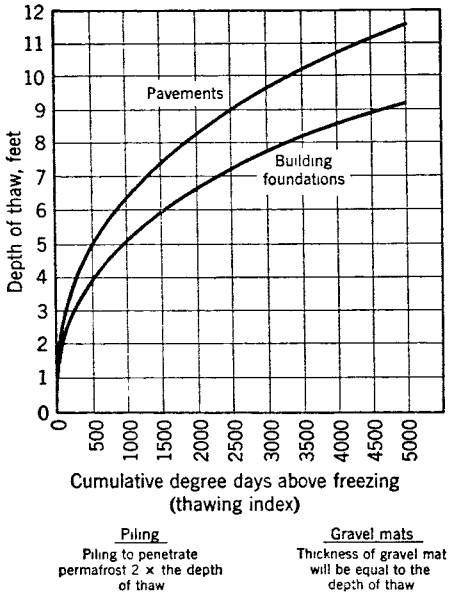


FIG 34 Depth of thaw on permafrost. Curves show the depths of thaw under pavements and building foundations plotted against thawing indices obtained from curves of cumulative degree days above freezing. Courtesy of the Bureau of Yards and Docks, Navy Department.

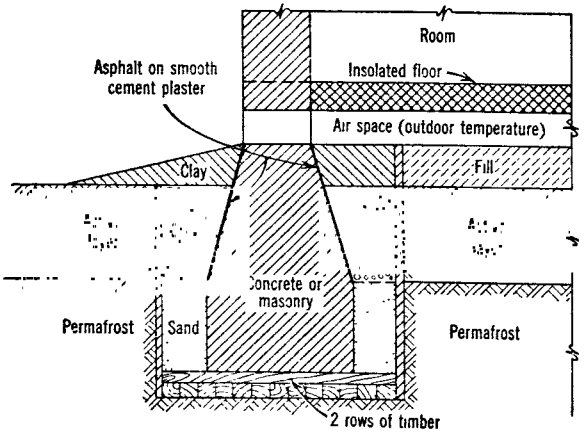
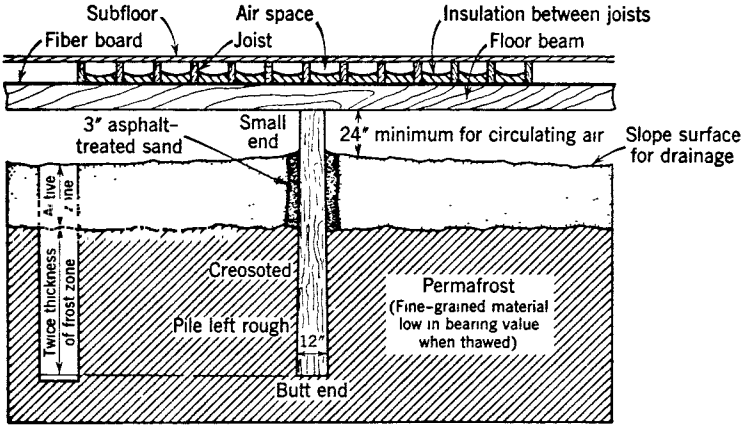
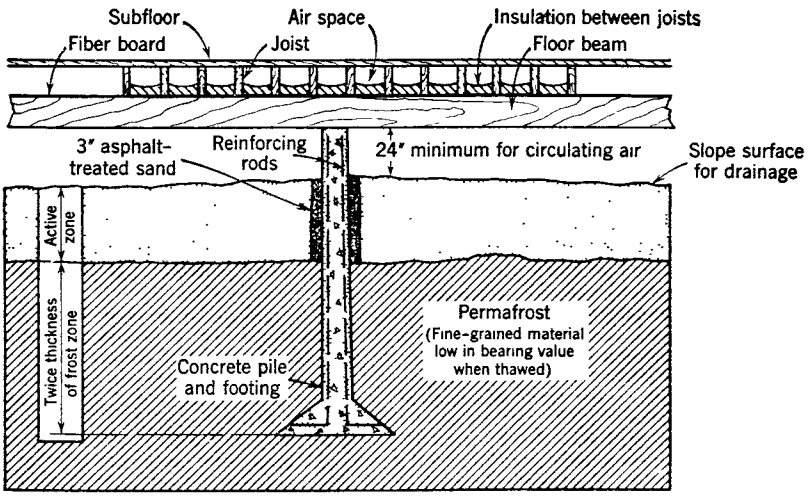


FIG 35 Insulated wall footing for small building in permafrost region of Siberia American experience in Alaska indicates that the air space under the floor of the building should have a clear height of at least 2 ft Original drawing after B D Vassilieff By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G. P. Tschebotarioff, copyright, 1951, McGraw-Hill Book Company.



Method of supporting building on piles with space beneath insulated floor for air circulation to prevent building heat penetrating to permafrost

(a) Wood Piling in Permafrost



(b) Concrete Pile in Permafrost

FIG 36 *(a)* Creosoted timber pile in permafrost *(b)* Cast-in-place concrete pile in permafrost. From *Cold Weather Engineering*, courtesy of Bureau of Yards and Docks, Navy Department, 1948–1949

of asphalt is applied, and finally surrounded by at least 3 in. of asphalt-treated sand, Fig. 36*a*, to prevent adhesion of frozen soil and consequent heaving. Piles should not be spaced closer than 6 ft center to center and should be allowed to freeze into the permafrost for at least a month before being loaded.

Concrete piles of the cast-in-place pedestal type may be used in permafrost. Longitudinal reinforcement, Fig. 36*b*, is necessary as when freezing of the active layer occurs the piles may be in tension. A 3-in. incasement of asphalt-treated sand should surround the pile in the active layer to prevent adhesion and heaving.

Mat foundations on permafrost, Fig. 37, require the complete removal of the active layer throughout the area of the building and the substitution for it of a bed of gravel resting on an insulating blanket of gravel and coarse sand. The concrete slab or mat rests on this bed of gravel.

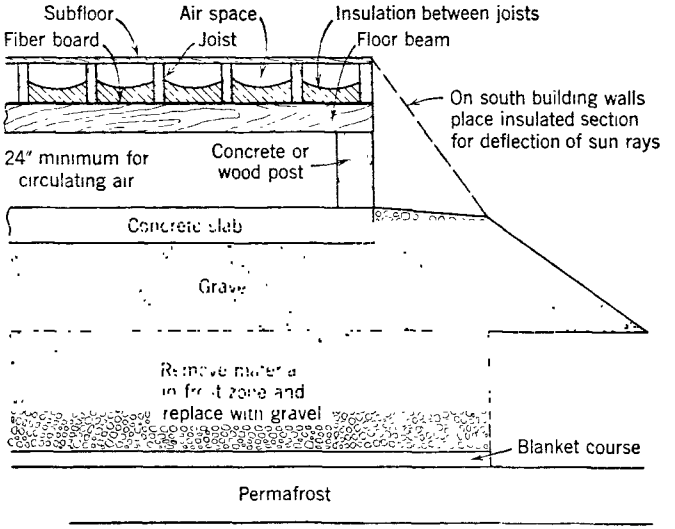


FIG 37 A mat foundation on permafrost From *Cold Weather Engineering*, courtesy of Bureau of Yards and Docks, Navy Department, 1948–1949

Insulation. The design of all buildings on permafrost should provide for adequate insulation of the floor and for an air space at least 24 in. deep and freely vented to the outdoor atmosphere to prevent heat from the building thawing the permafrost.

Fills and embankments on permafrost, especially sidehill embankments supporting roads and air strips, require special treatment and attention. The embankment, acting as an insulator, raises the elevation of the permafrost, Fig. 38*a*, and thus prevents the normal downhill percolation of water through the bottom of the active layer in the fall and early winter. This causes the development of springs along the uphill side of the embankment the water from which forms ice masses that will, at the worst, spread over the pavement, and at best will threaten the stability of the embankment when the spring thaw occurs. Engineers in Siberia construct what are called "freeze belts" some distance uphill from roads, Fig. 38*b*. These collect the ice which might otherwise threaten the road and its embankment. Insulated ditches, Fig. 38*c*, provide means of escape for the water from freeze belts when the ice thaws. Insulating berms alongside highway fills, Fig 39, supplemented by adequate drainage ditches have been used in Alaska to preserve the permafrost under road embankments and dispose of water that might otherwise damage them.

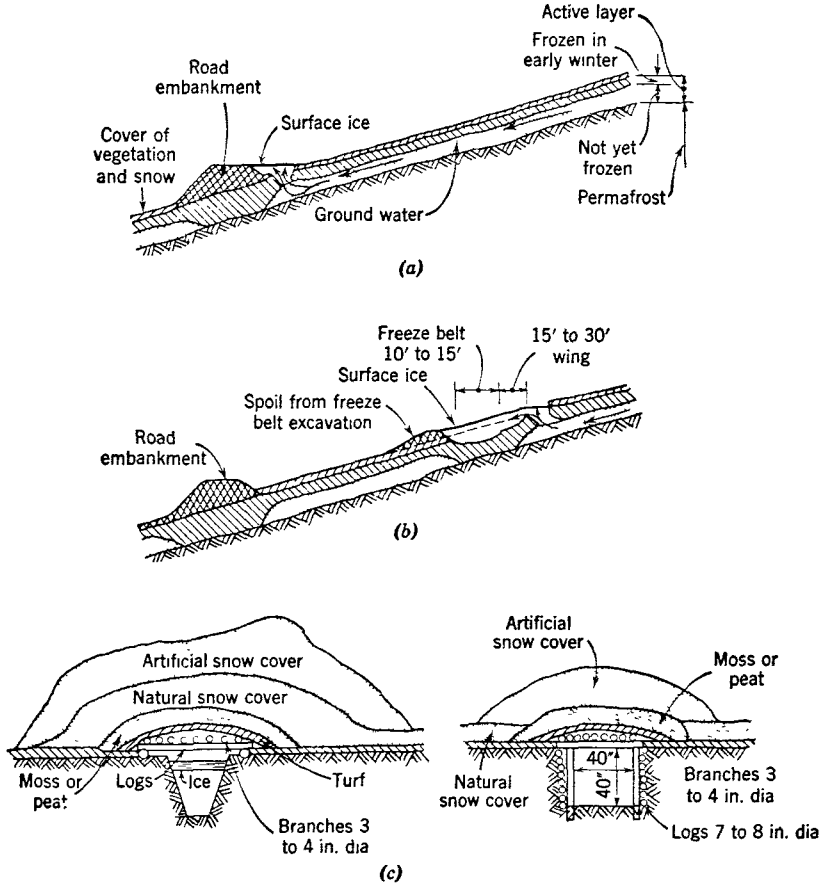


FIG 38 (*a*) Effect of embankment on permafrost The formation of large quantities of surface ice next to an unprotected embankment is detrimental (*b*) A freeze belt as built in the permafrost regions of Siberia These freeze belts protect the roads below them by causing the accumulation of surface ice where it does no harm (*c*) Insulated ditches carry water away from freeze belts during spring thaws By permission, from *Soil Mechanics, Foundations and Earth Structures*, by G. P Tschebotarioff, copyright, 1951, McGraw-Hill Book Company

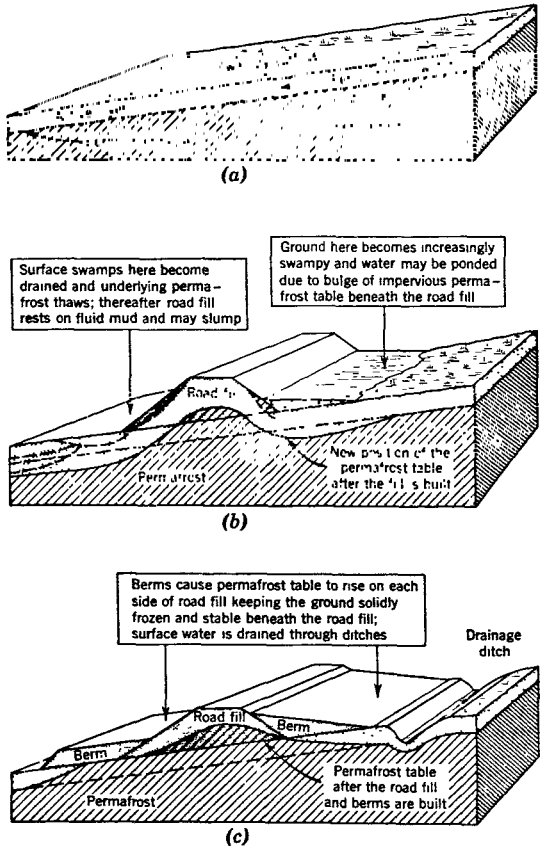


FIG. 39. Insulating berms on permafrost. From *Cold Weather Engineering*, courtesy of Bureau of Yards and Docks, Navy Department, 1948–1949

FOUNDATIONS UNDER WATER

11. GENERAL

The foundations of many structures must be built on submerged sites. Articles 11–15 deal with the design and construction of the foundations of bridges and miscellaneous structures which for one reason or another are built on sites covered by water.

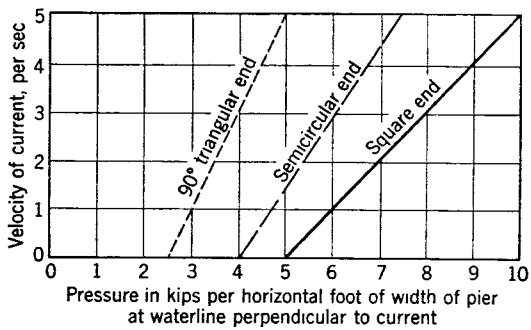
The previously outlined principles of site examination, subsurface exploration, soil-mechanics laboratory and field tests, and procedures for estimating settlements and stability are fully as applicable to the design and construction of foundations under water as to those on land. Accordingly these principles should be applied to both classes of foundations. There are, however, differences in approach to the problems of designing bridge and building foundations. The cost of the foundation of a large bridge is usually a much larger part of the cost of the complete structure than is the cost of the foundation of the average building. In fact where minimum overall cost of a bridge may be sought without regard to many other considerations which usually affect the design of bridges, the cost of a pier, including its foundations, should equal the cost of one span of the superstructure exclusive of the floor system. Furthermore, consideration must be given in bridge design to interference with stream flow and navigation. As this leads at once to a study of the

locations and dimensions of piers and abutments, the question of span length again presents itself. This makes it apparent that close cooperation between the designer of the superstructure of a bridge and the designer of its foundations is if possible even more important than between the designers of the superstructure and foundations of a building. The designer of a building foundation is usually faced with the primary problem of providing a satisfactory support for a specified structure on a given site. He may, and frequently should, make suggestions as to possible modification of the superstructure design which will promote economy in the construction of the foundation, but only rarely will he suggest the radical step of abandoning a site in favor of one affording more favorable foundation conditions. On the other hand, the designer of the foundation should play a major and frequently controlling role in the selection of the site of a large bridge.

12. BRIDGE PIERS AND ABUTMENTS

Loads on Bridge Piers and Abutments. Dead, live, wind, and, where applicable, earthquake loads imposed on bridge piers and abutments should be estimated by the designer of the superstructure. Impact loads may usually be disregarded in designing the foundation. The loads to be carried by the foundation will be the loads imposed by the superstructure plus the dead loads of the piers and abutments and the forces exerted on them by flowing water, wind, ice and earth pressures, and seismic forces. Increases in assumed loading due to earthquake forces should be of the same order of magnitude as those used in the design of the superstructure.

Dead loads of piers and abutments are the total weights of concrete or masonry above water and the buoyed weights (total weights less 62.4 lb per ft^3) of submerged portions.



Empirical values for pressure of ice against end of a pier

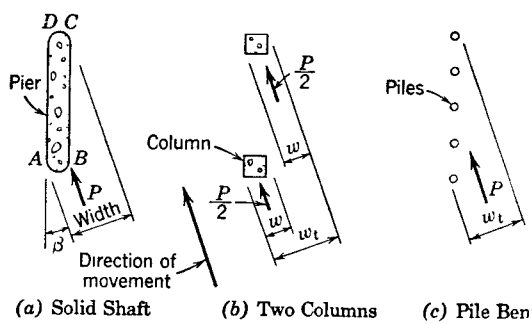


FIG. 40. Ice pressures on bridge piers. Pier widths (*a*, *b*, and *c*) to be used in connection with curves in table. By permission from *Foundations of Structures*, by C. W. Dunham, copyright, 1950, McGraw-Hill Book Company.

Thrust on bridge piers exerted by flowing water may be considerable in fast-flowing streams subject to freshets. It may be estimated by formulas suggested by J. E. Greiner. These are:

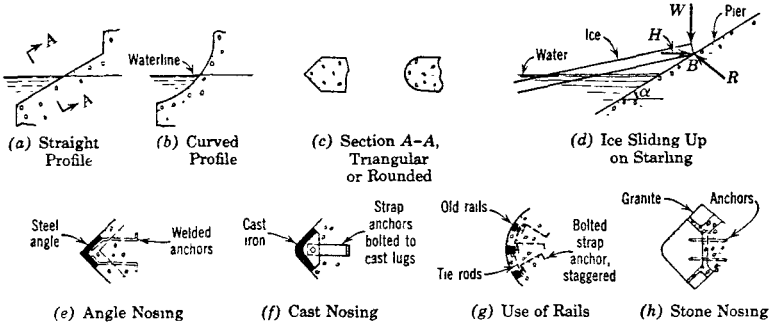
$$P = 1.5V^2 \quad \text{for flat surfaces}$$

$$P = 0.75V^2 \quad \text{for rounded surfaces}$$

in which P = the force exerted on each square foot of the vertical projection of the upstream surface of the pier.
V = the velocity of flow in feet per second

The same authority suggests minimum values of 150 lb per ft² for flat surfaces in streams subject to freshets and 50 lb per ft² for flat surfaces exposed to strong tidal currents.

Wind loads on low piers may usually be disregarded. On high piers and doubtful cases they should be estimated as 30 lb per ft² of exposed vertical surface.



(a) Straight Profile (b) Curved Profile (c) Section A–A, Triangular or Rounded (d) Ice Sliding Up on Starling

(e) Angle Nosing (f) Cast Nosing (g) Use of Rails (h) Stone Nosing

FIG. 41. Bridge pier starlings. By permission from *Foundations of Structures*, by C. W. Dunham, copyright, 1950, McGraw-Hill Book Company.

Ice pressures exerted against bridge piers may be considerable. Pressures in kips per horizontal foot of width of pier at the waterline due to crushing of river ice against the ends of piers may be estimated by Fig. 40. If the pier has a sloping starling which acts as an icebreaker at its upstream end, the pressures indicated by Fig. 40 may be reduced by multiplying them by the sine of the angle α, Fig. 41. Ice pressures due to drifting lake or sea ice may be estimated as 1000 lb per horizontal foot of projected width of pier or pile

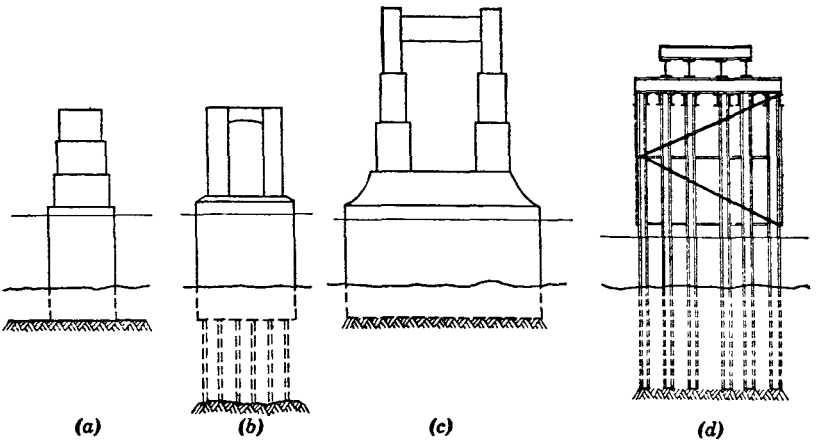


FIG. 42. Types of bridge piers. (a) Simple circular pier on open caisson foundation. (b) Twin piers on pile foundation. (c) Twin piers with starling—caisson foundation. (d) H-beam pile cluster with diagonal welded bracing

bent at the waterline, Fig. 40. Ice jams in rapidly flowing streams may exert a force on a pier which should be estimated as equivalent to a static hydraulic head of 5 ft applied to a length equal to the average lengths of the spans supported by the pier.

Earth pressures on abutments should be estimated by methods discussed in Art. 10, Sect. 7 Earth pressures on bridge piers should be considered if there are marked differences in depth of water at opposite sides of piers located at the edge of the channel or where such differences may be produced by scour.

Bridge Pier Types. Bridge piers and abutments today are built almost exclusively of reinforced concrete. Stone masonry, of which piers and abutments were once usually built, has become prohibitively expensive for the major portions of required sections but is still frequently used as a facing below and a few feet above the waterline to protect concrete from possible damage by water, frost, drift, and ice. Bridge piers are designed as concrete columns, a variety of sections being used, Fig. 42*a*, *b*, and *c*. As indicated in these figures, a pier may consist of a single rectangular or circular column or may take the form of a bent of two or more columns resting on a common base and surmounted by a cap. Piers in swift-running streams are frequently streamlined by the addition of material outside the sections needed to meet structural requirements. Additional material thus added to achieve streamlining or pleasing appearance is called the starling. Occasionally bridge piers are built in the form of bents or clusters of steel, precast concrete, or timber piles surmounted by a suitable cap and possibly strengthened by stiffening diaphragms, Fig. 42*d*.

Interference with stream flow which will be caused by the construction of bridge piers in a quick-flowing stream, particularly if it is subject to freshets, should be estimated. Nagler's formula expressing the relationship between the quantity of water flowing and the increase in depth above the piers caused by the obstruction is.

$$Q = KW_2\sqrt{2g}\left(D_3 - \frac{0.3V_3^2}{2g}\right)\sqrt{H_3 + \frac{\beta V_1^2}{2g}} \tag{6}$$

or

$$H_3 = \frac{Q^2}{2g\left[KW_2\left(\frac{D_3 - 0.3V_3^2}{2g}\right)\right]^2} - \frac{\beta V_1^2}{2g} \tag{6a}$$

in which Q = quantity of water flowing in cfs.
W_2 = mean width of stream at the pier in ft (Fig. 43).
D_3 = mean depth of water in ft/sec below contracted section, Fig. 43.
V_1 = mean velocity of water in ft/sec above contracted section.
V_3 = mean velocity of water in ft/sec below contracted section.
H_3 = increase in depth of water above contracted section, Fig 43.
g = acceleration of gravity in ft/sec^2 = 32 2.
β = a coefficient depending on the percentage of the channel obstructed by the piers, obtained from Fig. 44.
K = a coefficient depending on the shape of the pier and percentage of channel contraction: obtained from Fig. 45.

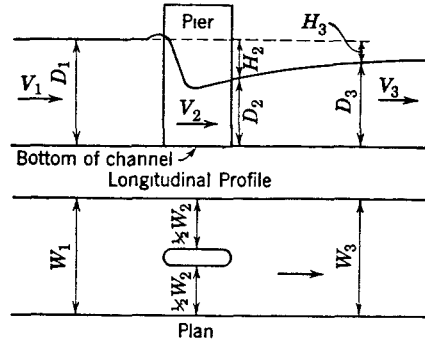


FIG 43 Stream obstruction by bridge piers By permission, from *Foundations of Bridges and Buildings*, by H. S Jacoby and R. P. Davis, copyright, 1941, McGraw-Hill Book Company

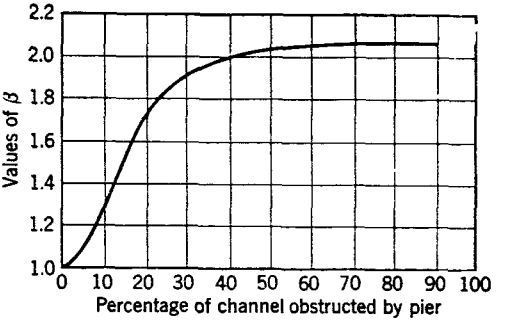


FIG 44. Value of coefficient β in Nagler's formula See eqs 6 and 6a. By permission, from *Foundations of Bridges and Buildings*, by H. S. Jacoby and R. P. Davis, copyright, 1941, McGraw-Hill Book Company.

The values of K given in Fig. 45 are good only if the turbulence caused by the piers is slight. This will be the case where α (see Fig. 45) is less than $\dfrac{1}{0.97 + \dfrac{21V_3^2}{2gD_3}} - 0.13$ which is the case usually met. For the treatment of unusual cases where the turbulence will be moderate to high, see refs. 13 and 23.

100 α	SS	RR	CC	LL	TT	T-T	RS	SR	$P_{55°}$ R	$P_{60°}$ R	$P_{90°}$ R
11 7 L=4W	0 908	0 941	0 954	0 953	0 898	0 907	0 941	0 942	0 945	0 932	0 948
23 3 L=4W	0 864 (0 861)	0 927 (0 923)	0 943	0 943	0 883	0 904	(0 914)	(0 866)			(0 905)
23 3				$P_{90°}$ $P_{90°}$ 0 887	$P_{90°B}P_{90°B}$ 0 906	RL (0 928)	LR 0 932	NC 0 863	NC_1 0 897	NC_2 0 895	NC_3 0 930

FIG 45 Values of K in Nagler's formula See eqs. 6 and 6a The values of K appear below the diagrams of the various types of piers tested in Nagler's model experiments. 100 α represents the percentage contraction caused by the pier, L the pier length, and W the pier width Symbols for the types of pier ends are. S, square; R, round, C, convex, L, lens-shaped; T, cylindrical; and P, pointed Most of the piers had no batter, but those marked $P_{90°B}P_{90°B}$ had a nose batter of 1:12 Those marked NC. NC_1, and NC_2 had pier nose and tail batters of 1:24 Those marked NC_3 had a 1:24 batter all around By permission, from *Foundations of Bridges and Buildings*, by H S Jacoby and R P Davis, copyright 1941, McGraw-Hill Book Company.

A skew of 10° or less between the major axes of bridge piers and the direction of flow of the stream causes little increase in the obstructive effect of the piers. Larger angles of skew begin to be objectionable and angles of 20° or more cause serious increases in the obstructive effect.

Foundations of bridge piers and abutments may be footings on rock, spread footings deep enough to be safe from undermining by scour and supported on soil of suitable characteristics, or they may be pile supported. Methods used for determining required bearing areas, permissible pile loads, prospective magnitudes and rates of settlement, and stability against shear failure are those outlined in Art. 9, Sect. 7, and preceding articles referred to therein. Avoidance of the eccentric application of dead loads to footings and pile groups is important, especially if the foundation is supported or underlain by compressible cohesive soils. The intermittent and brief eccentric application of the resultants of dead, live, and wind loads cannot be avoided but such eccentricity for periods of short duration should not cause troublesome tilting provided it does not produce uplift on one side of a footing or the piles supporting it. Abutment foundations require careful attention in order that unequal bearing pressures and failure to provide adequate means to resist the horizontal

component of the resultant of earth pressures do not cause tilting and movement toward the stream. Where piers are supported by piles, or are composed of bents or clusters of piles, and where piles support an abutment, batter piles should be provided to take the horizontal thrusts caused by earth pressure, heavy ice, or freshets. Differential settlements of pier foundations are of great importance where the superstructure is to consist of spans continuous over a number of supports. Foundations carrying a series of continuous spans should in general be supported on rock or at least hardpan, as differential settlements may cause serious overstress of their superstructures.

Scour. The depth of imbedment necessary to prevent foundations of bridge piers and abutments being undermined by scour is extremely important and, unfortunately, difficult to estimate. The ability of flowing water to pick up and carry solids in suspension varies with about the sixth power of its velocity. Therefore an increase in stream velocity incident to the obstruction of flow caused by bridge piers may cause scour to surprising depths, especially if the stream bed is fine sand or alluvial silt. For example, an increase in depth of 40 ft was reported alongside the sand island cofferdams in which caisson foundations of the Mississippi bridge at Baton Rouge were built. Formulas are no substitute for judgment in estimating probable depths of scour. The best guides are local experience supported by a careful study of any tendency of the stream to shift its channel. Empirical rules should be used with great caution but the policy of the Atchison Topeka and Santa Fe Railway of keeping footings at least 25 ft below the stream bed is worth recording as the safety record of the company's bridges is good; see ref. 20. Pile foundations and fairly deep footings in gravel stream beds have by no means proved immune to failure by scour, both types having failed in major floods. Where bridge piers are built in tidal waters, consideration should be given to the possibility of relatively slight scour exposing untreated foundation piles to attack by marine borers.

Bridge piers are frequently built in small internally braced cofferdams by methods closely analogous to those used in the construction of pier foundations for buildings in open shafts excavated within sheet piled walls. See the paragraph on Open Shafts in Art. 6 and the paragraphs dealing with the design and construction of internally braced cofferdams in Art. 13.

13. COFFERDAMS

Cofferdams are required to exclude water from the working areas in which bridge piers and abutments are constructed unless they are sunk as caissons or built of piling. Cofferdams may be built as earth dikes or constructed of wood or steel sheet piling, the choice of type being dictated by conditions at the site and the availability of materials. For more complete treatments of the important subject of cofferdams see refs 21 and 22.

Earth dike cofferdams are rarely used for the construction of bridge piers. They are occasionally used for the construction of abutments where the water depth does not exceed

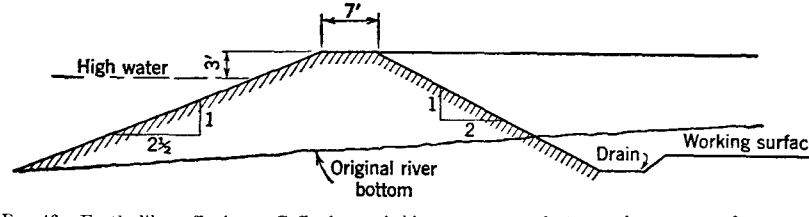


FIG. 46 Earth dike cofferdam Cofferdams of this type are satisfactory only in water of very moderate depth, say not more than about 5 ft, and in locations where there is little current and no high waves to erode the outer slope. Such cofferdams are most frequently used for the construction of abutments

about 5 ft, the bottom is rock or a stable and impervious soil, and neither stream velocity nor waves will cause destructive erosion. Erosion can be reduced if not controlled by facing a dike with earth-filled sandbags or broken stone. Infiltration can be minimized by a core of impervious earth. The tops of earth dikes should be not less than 7 ft wide nor less than 3 ft above extreme high water. Exterior slopes should be no steeper than 2 ½ to 1 and interior slopes 2 to 1. It is essential that enough space be provided inside a cofferdam to allow proper sideslopes for excavation and a circumferential drain to intercept seepage before it reaches the working area, Fig. 46. For estimating the probable inflow and required capacity of pumping plant see subsequent paragraph on pumping cofferdams.

Wood sheet pile cofferdams are usually of the earth-filled double-wall type, Fig. 47*a* and *b*, built with Wakefield sheet piling. Wakefield piling, Fig. 48, is generally made of planks 12 in. wide unless the plank thickness necessary to withstand anticipated bending stresses is less than 1 1/2 in. in which case narrower planks are usual. The planks in the center of the piles should be dressed to insure uniformity of thickness. The outer planks may be rough. The length of tongue should be at least 1 1/2 in. if the planks are 1 1/2 in. or less in thickness. With planks 2 in. or more thick the length of the tongue should equal the plank thickness. Pairs of bolts spaced about 6 ft hold the planks together with heavy spikes spaced 18 in. between the bolts. With 1-in. planks 3/8-in. bolts may be used, with

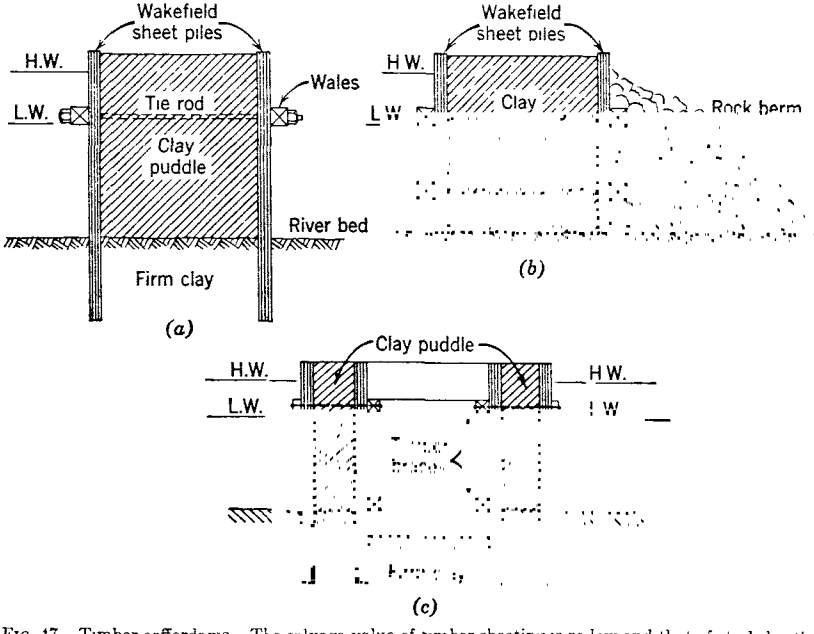


FIG. 47. Timber cofferdams. The salvage value of timber sheeting is so low and that of steel sheeting so high that wood sheet piling can be justified today in only a few exceptional cases. (*a*) Small unbraced cofferdam on clay bottom. (*b*) Unbraced cofferdam on rock bottom. The rock berm would not be necessary in all cases. (*c*) Braced cofferdam. The internal bracing makes possible the use of much narrower clay puddle cells.

planks 1 1/2–2 1/2 in. thick the bolt diameter should be 1/2 in.; and 5/8-in. bolts are usual with planks 3 in. and 4 in. thick. The bottoms of the piles should be cut as shown to assist in keeping the grooves tight against the tongues which should always be the leading edge of the piles as driven. However, where the piles are to be driven to rock the center plank of each pile may be left square to minimize gaps between the pile tips and the rock surface. Internally braced cofferdams are occasionally built of Wakefield piling, Fig. 47*c*, where lumber is cheap and steel sheet piling unobtainable. For methods of estimating stresses in the sheeting and bracing of cofferdams see subsequent paragraphs on the stability of cofferdams.

Steel sheet pile cofferdams are generally more satisfactory than cofferdams built of wood sheeting. Not only can they be made more nearly watertight but steel sheeting drives more easily than wood sheeting, stands driving better, and therefore can be reused many times. Steel sheet pile cofferdams in which bridge piers are built are usually of the internally braced type, Fig. 49 and 51. Cofferdams supported by rock-filled cribs, Fig. 50, are used for the construction of larger structures. Cellular cofferdams, Fig. 52, because of their large size, are rarely if ever suitable for the construction of bridge piers but may be used for inclosing the space in which an unusually large abutment is built. They are frequently employed in the construction of canal locks, dams, and similar large structures. A single large circular cell is occasionally used to inclose a sand island, Fig. 53, in which a bridge pier is sunk as a caisson.

Essential characteristics of steel sheet pile sections commonly used in the United States are given in Table 3. The types fitted for internally braced cofferdams are those having

large-section moduli and great flexural strength such as U. S. Steel Section Numbers MZ 38 to M 115 inclusive, Bethlehem Steel Section Numbers ZP-38 to AP-3, and similar sections of other manufacturers. The types fitted for use in cellular cofferdams are those having high interlock strength as the completed structures will be subjected primarily to hoop tension Low-section moduli are acceptable as the piles in cellular cofferdams will not be required to resist large bending stresses. In theory sections that are perfectly straight between the interlocks are preferable to sections with a slight "bow" as they are

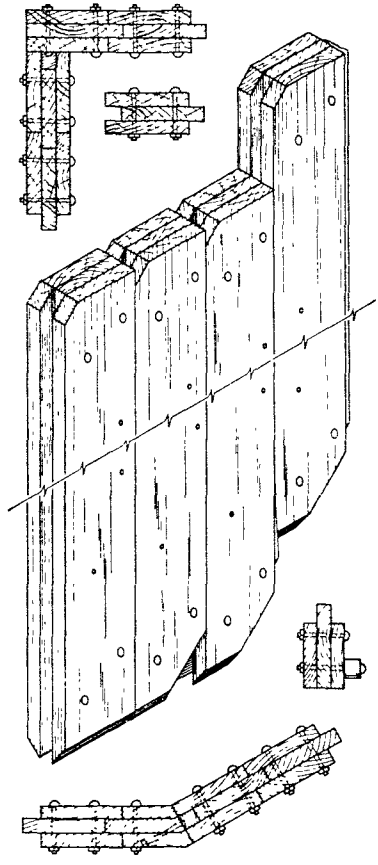


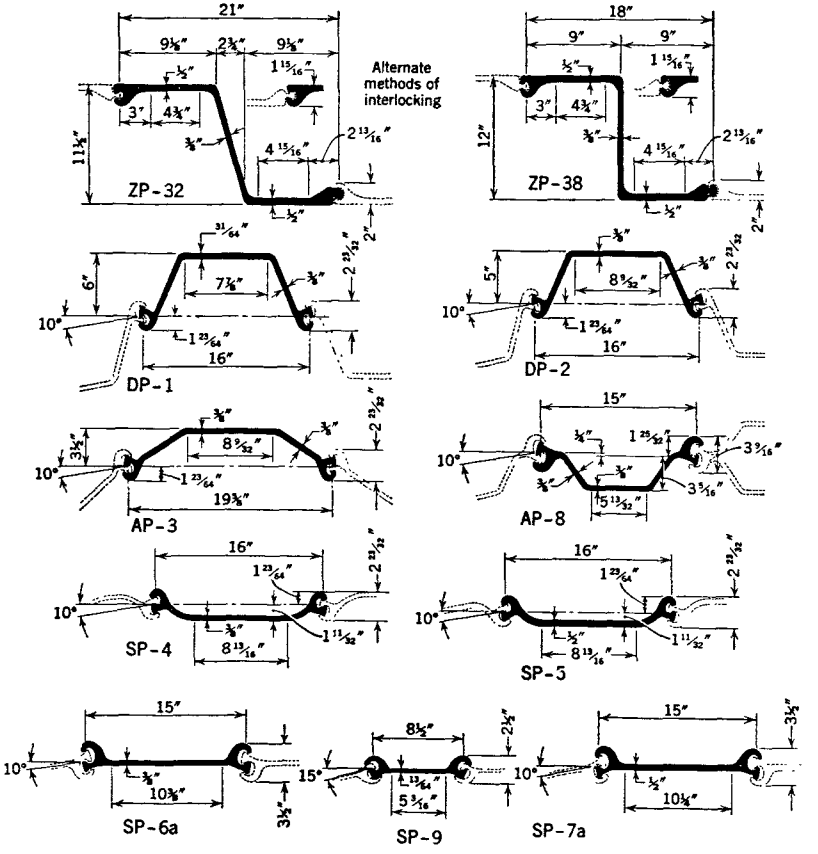
FIG 48 Wakefield sheet piling By permission, from *Foundations of Bridges and Buildings*, by H S Jacoby and R P Davis, copyright, 1941, McGraw-Hill Book Company

stronger in tension. In practice, however, many field men prefer the slightly bowed sections which, although not so strong in hoop tension, are much easier to handle because of the slight flexibility afforded by the curvature between the interlocks which makes it much easier to drive a circular cofferdam without damaging the interlocks or tearing the piling.

The **stability of sheet pile cofferdams** depends upon their ability to resist several types of failure

Earth-filled double-wall cofferdams without internal bracing, sheet pile cofferdams supported by rock-filled cribs, and cellular cofferdams must be stable against overturning and sliding. This can usually be achieved by making the width of double-walled earth-filled cofferdams at least 85% of their height. The overall width or cell diameter of cellular cofferdams should be from 1.0 to 1.15 times their height. In both cases "height" is the vertical distance from firm bottom on which the cofferdam rests to extreme high water. Cells should be filled with a stable granular soil All cofferdams should be analyzed to make sure that the resultants of forces due to water pressure and weight lie within the

Table 3. Steel Sheet Pile Sections



Section No.	Area, in.^{2}	Width, in.	Weight in Pounds: Per Linear Ft of Bar	Weight in Pounds: Per Sq Ft of Wall	Moment of Inertia, in.^{4}	Section Modulus, in.^{3}: Single Section	Section Modulus, in.^{3}: Per Linear Ft of Wall
ZP-32	16.47	21	56.0	32.0	385.7	67.0	38.3
ZP-38	16.77	18	57.0	38.0	421.2	70.2	46.8
DP-1	12.56	16	42.7	32.0	87.0	20.4	15.3
DP-2	10.59	16	36.0	27.0	53.0	14.3	10.7
AP-3	10.59	19 5/8	36.0	22.0	26.0	8.8	5.4
SP-4	8.99	16	30.7	23.0	5.5	3.2	2.4
SP-5	10.98	16	37.3	28.0	6.0	3.3	2.5
SP-6a	10.29	15	35.0	28 0	4.6	3.0	2.4
SP-7a	11.76	15	40.0	32.0	4.6	3.0	2.4
AP-8	11.41	15	38.8	31.0	24.2	8.9	7.1
SP-9	4.38	8 1/2	14.9	21.0	1.0	1.0	1.4

Courtesy Bethlehem Steel Company.

Sections ZP-32 and ZP-38 will interlock with each other, but not with other Bethlehem sections.

Sections DP-1, DP-2, AP-3, SP-4, and SP-5 will interlock with each other, but not with other Bethlehem sections.

Sections SP-6a, SP-7a, and AP-8 will interlock with each other, but not with other Bethlehem sections.

Section SP-9 will not interlock with any other section.

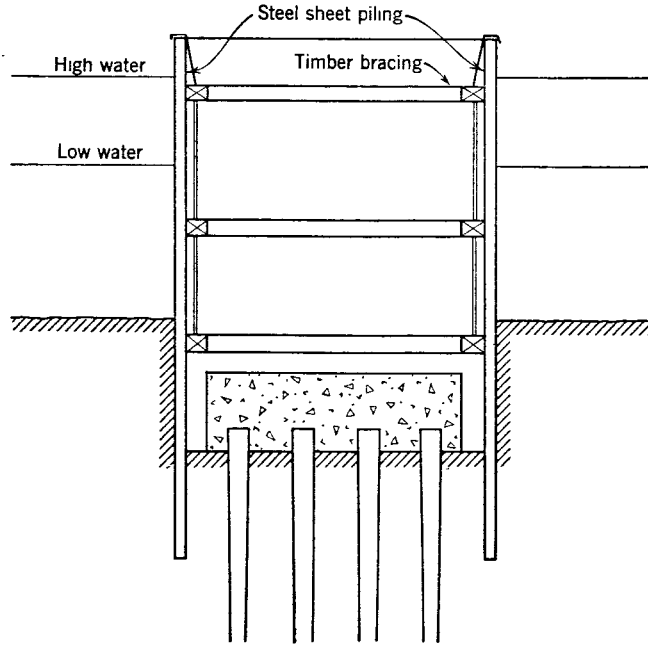


FIG. 49. Small steel sheet pile cofferdam with timber bracing Small cofferdams such as these were once extensively used for the construction of the piers of unimportant bridges Today timber is so expensive and its salvage value so small that steel internal bracing is replacing timber in even small cofferdams.

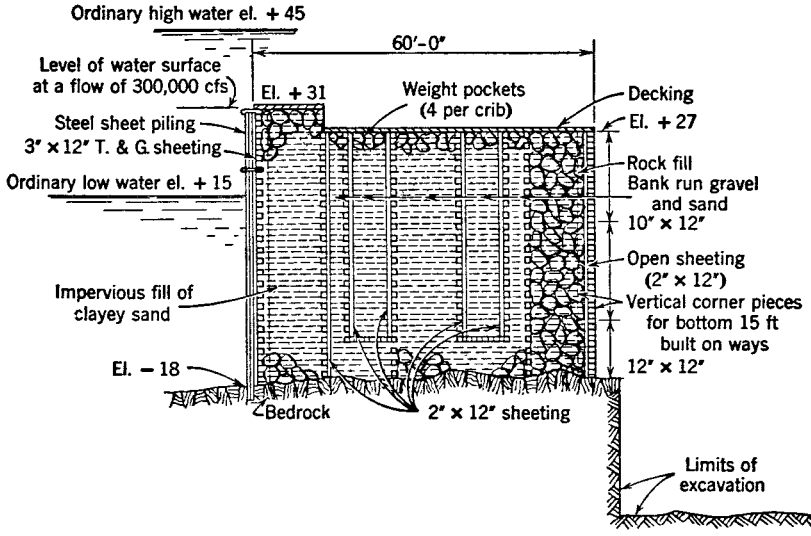


FIG. 50. Steel sheet pile cofferdam supported by rock-filled crib Cofferdams of this type are used in the construction of many important projects where rock bottom makes it impossible to drive sheet piling into the river bed in the conventional way From *Cofferdams*, L. White and E A. Prentis, Columbia University Press, 1941.

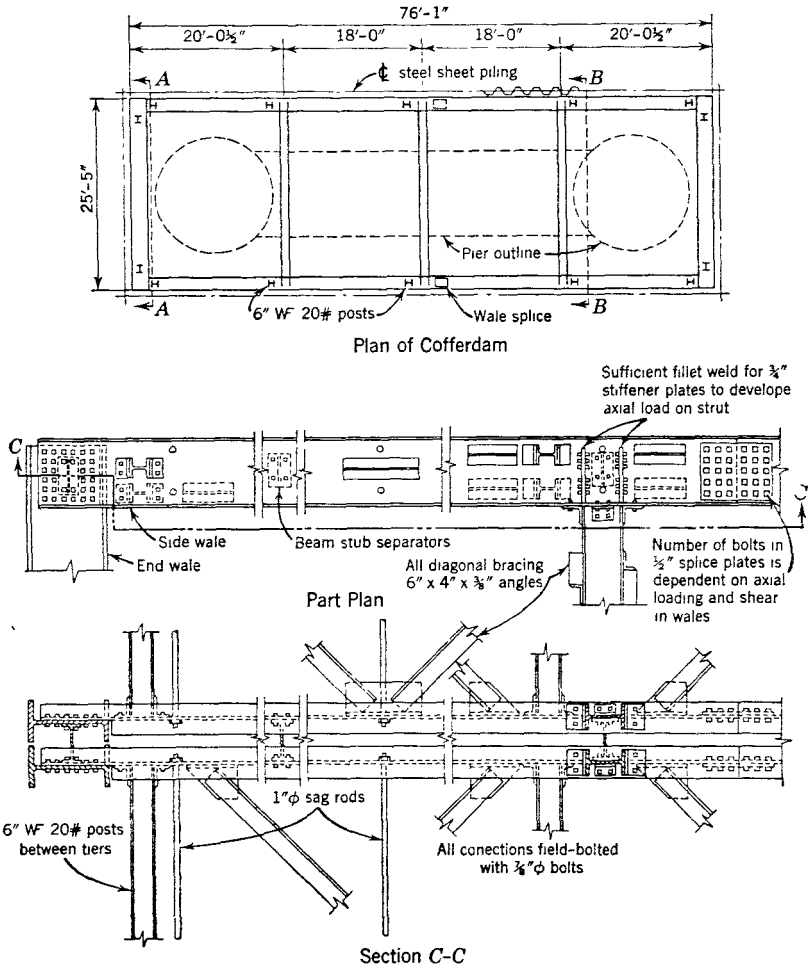
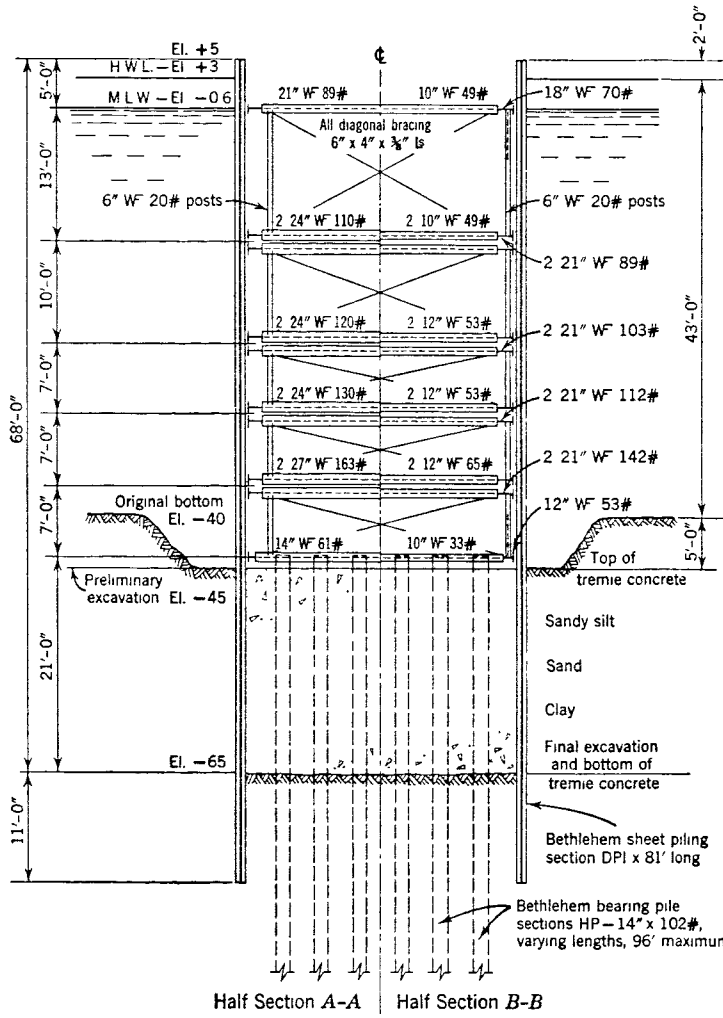


FIG 31 Steel sheet pile cofferdam with

middle thirds of their bases. Stability against sliding is checked by assuming that the coefficient of friction between the filling material and the bottom is not in general more than 0.5.

The sheet piles and wales of internally braced cofferdams must have sufficient flexural strength. It is advisable to compute bending moments as if the piles were simply supported beams spanning from wale to wale and the wales simply supported beams between the struts. The reduction in bending moments due to continuity introduces an additional highly desirable safety factor into the design of a structure that from force of circumstances will be built roughly.

Struts are designed by applicable column formulas. If both longitudinal and transverse timber struts are required, Fig. 31, both should be continuous from end to end. The longitudinal struts and the wales they support should therefore be immediately above or below the transverse struts and their corresponding wales If the two sets of struts and wales are in the same plane the desirable continuity of both sets of struts is impossible.



structural steel bracing Courtesy of Bethlehem Steel Co

If steel struts and wales are used, the details of their connections must obviate excessive stress concentrations and consequent buckling of webbs and flanges

The interlock strength of steel sheet piling in cellular cofferdams must be sufficient to withstand the hoop tension engendered by the active pressure of the earth with which the cells are filled. The tension may be estimated by the formula.

$$T_h = \frac{\gamma H K_a R_c}{12}$$

in which T_h = the force producing hoop tension in one lineal in. of interlock.
γ = the total unit weight in lb/ft^3 of the soil in the cell.
H = the height in ft of the cofferdam fill above the point at which the hoop tension is estimated
K_a = coefficient of active earth pressure of filling material.
R_c = radius of cofferdam cell in ft.

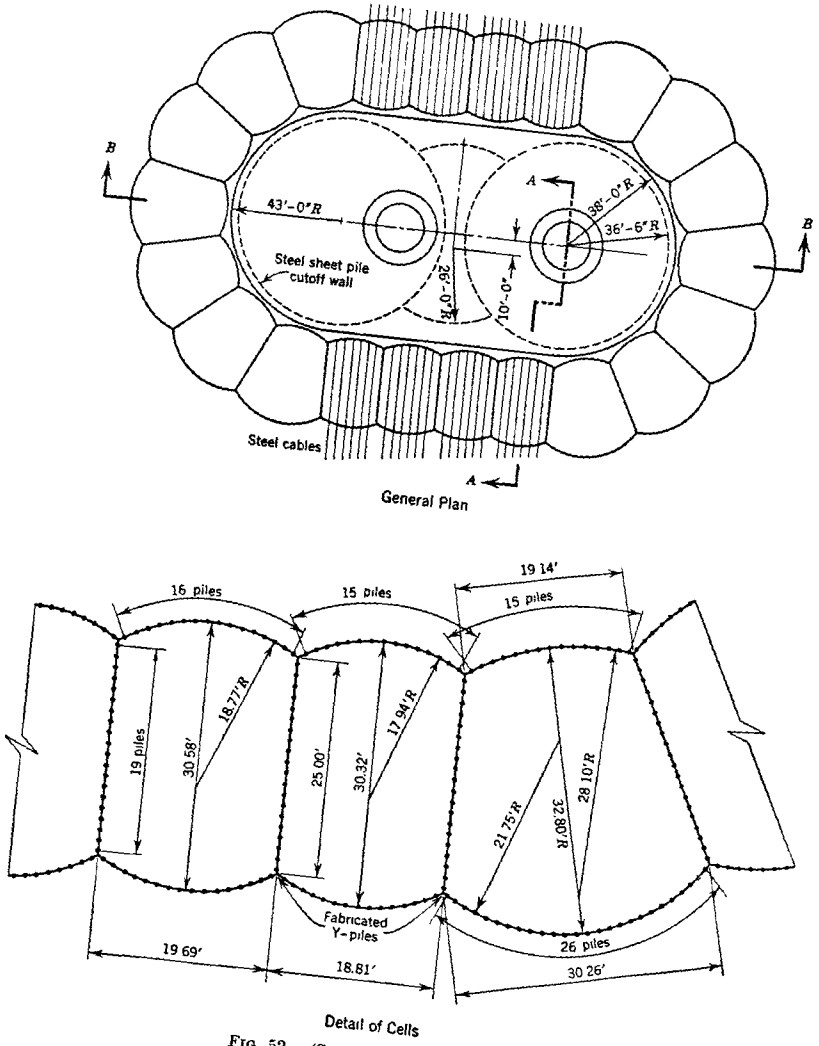
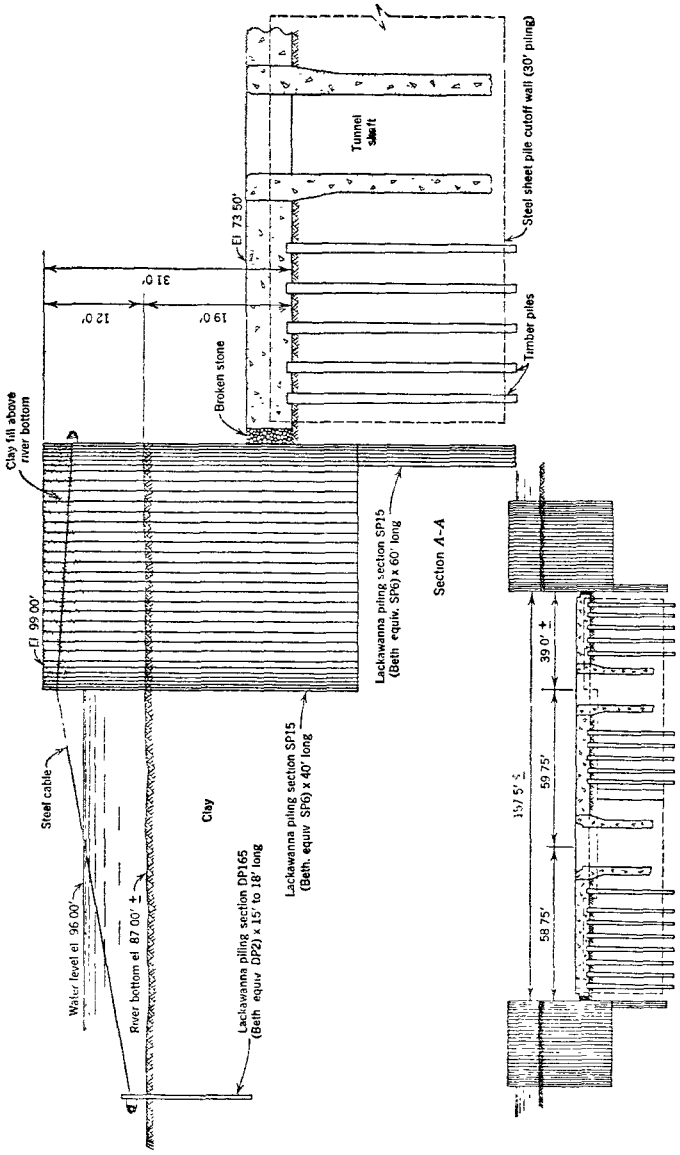


FIG. 52. (See legend on facing page.)



Cellular Cofferdam for Intake System Detroit, Mich.
The George R Cooke Co., Detroit Mich., Contractor

FIG 32 Cellular sheet pile cofferdam. Courtesy of Bethlehem Steel Co.

Stability against shear failure of the bottoms of cofferdams built on cohesive soil should be estimated by circular-slide analyses made in accordance with principles outlined in Art. 8, Sect 7. Consideration should be given to such surfaces of failure as the one indicated in Fig. 54 and cognizance taken of the fact that shear failure of the soil supporting it may involve movement of an entire cofferdam. The cohesive strength c of clays and

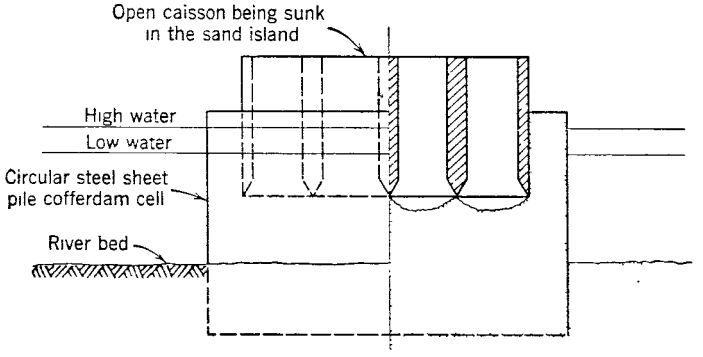


FIG. 53. The sand-island process A sand island is built in the river by driving a large circular steel sheet pile cofferdam and filling it with sand On this sand island an open caisson is sunk by the conventional method of dredging through its wells. After completion of the caisson sinking, the cofferdam is removed and the sand dredged away.

silts underlying the sites of proposed cofferdams should be determined in order that adequate precautions against shear failure may be taken. Stability of sheet pile cells is further discussed in Sect. 21, Volume II.

Stability against quickening and boiling of the bottom of cofferdams built on noncohesive soils such as sand is of great importance. Extensive boiling of the bottom of a cofferdam will not only flood it but may cause its practically complete destruction by removing lateral support given to the sheet piling by the soil. The danger of such trouble occurring can best be estimated by drawing a flow net, Fig. 32 of this section, or Fig. 59*b*, Sect. 7. From it the probable hydraulic gradient of upward flow to the bottom of the proposed finished excavation inside the cofferdam is determined. If this gradient exceeds 1.0

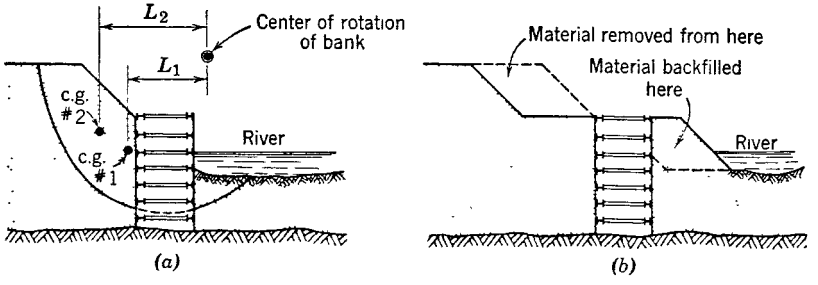


FIG 54 Stability of cellular cofferdam—possibility of shear failure From *Cofferdams*, L White and E. A. Prentis, Columbia University Press, 1950

there is evident danger of the bottom boiling until its entire area becomes quick. As in the case of land cofferdams it is advisable, however, to limit the hydraulic gradient of upward flow as estimated by a two-dimensional flow net to 0 5. It should be noted that excavation inside the cofferdam greatly augments the hydraulic gradient of upward flow. The fact that boiling does not occur when the cofferdam is first unwatered is therefore no assurance that it will not start before excavation is completed. Localized tendency of the bottom to boil when the excavation is practically complete may sometimes be controlled by a blanket of well-graded broken stone or gravel or by building a ring of sandbags around the incipient boil.

If preliminary studies indicate the probability of the bottom of a small cofferdam heaving, boiling, or quickening it is advisable to do necessary excavating through water, with clam-

shell or orange-peel buckets, before the cofferdam is unwatered. Final trimming of the excavation and cleaning out of the corners should be done by a diver who should also level a blanket of broken stone or gravel at least a foot thick laid on the bottom of the finished excavation. An adequately thick seal of tremie concrete, Fig. 51, should then be poured on the stone or gravel blanket and allowed to harden thoroughly before the cofferdam is unwatered.

The rate of inflow into a proposed cofferdam may be estimated by means of the flow net drawn to determine the stability of the bottom against boiling. Having determined the coefficient of permeability k of the soil, eq. 37 of Art. 8, Sect. 7, is used in conjunction with the flow net to estimate the rate of inflow.

The **construction of a cofferdam** is fully as important as its design inasmuch as careless workmanship can nullify the efforts of the most painstaking designer and lead to costly failures. It is essential that all the sheet piling be set around a template or frame and that all of it be interlocked before any driving is attempted. The sheet piles should then be driven successively a few feet at a time as described under open shafts in Art. 6. The internal bracing is assembled in tiers at the top of the cofferdam and lowered into place, the upper tier being suspended from hangers attached to the tops of the sheet piles. As many tiers as possible should be placed before any pumping is done, though the lower tiers must of necessity be installed as the excavation progresses if any considerable part of it is done in the dry. As pumping starts cinders should be sifted into the water outside the sheet piling to seal leaks in the interlocks.

Pumping is usually done with motor-driven centrifugal pumps. Even if current for the pump motors cannot be purchased at the site it is generally preferable to use motor-driven pumps and install diesel-driven generators to supply necessary current. At least three pumps should be used in a cofferdam of any size. Two of these should have ample capacity to handle the prospective inflow, the third being a spare in case a pump breaks down. The pumps should be of the same size and make in order that a single set of spare parts will serve for all three. The pumps should if possible be mounted on a platform suspended by chain falls in order that they may be raised quickly in case the cofferdam floods and also so that they may be lowered to follow the water down and thus obviate troubles due to excessive suction lift. The pump suctions should be equipped with bell-mounted intakes and should be located in a sump surrounded with short lengths of light sheet piling. The bottom of the sump should be tremie concrete to prevent scour. An effective screen should be provided to keep trash out of the sump. The pump discharge lines should terminate below water outside the cofferdam and should be provided with effective check valves to prevent siphoning water back through the pumps in case of power failure. A typical installation for a large cofferdam is shown in Fig. 55.

14. BRIDGE PIER CAISSONS

Several types of caissons are used in the construction of bridge pier foundations and bridge piers. As applied to bridge pier construction the term caisson includes box caissons, frequently called "cribs"; single-wall open caissons, multiwalled caissons, pneumatic caissons; and caissons sunk by a combination of the open and pneumatic processes.

A **box caisson**, Fig. 56, as the name implies, consists essentially of a box, usually of timber or reinforced concrete. Box caissons are employed either where no excavation is required to place them or where all necessary excavation can be done before the caissons are transported to the site and placed. Box caissons are placed by sinking them so that they come to rest on an existing acceptable hard bottom, on a previously excavated or prepared bottom, or on a preinstalled foundation such as a group of piles capped with a timber platform or covered with tremie concrete or a blanket of broken stone. A box caisson, after reaching its prepared bottom or preinstalled foundation, is filled with concrete, boulders, riprap, or masonry. Short span bridges, trestle bents, sea walls, bulkheads, moles, and piers are commonly supported by box-caisson foundations.

Single-wall open caissons of the type occasionally used in the construction of building foundations (see single-wall open caissons in Art. 7) are used extensively as bridge piers and their foundations, Fig. 57.

Multiwalled caissons with open dredging wells, Figs. 53 and 58, may be sunk as bridge foundations to very much greater depths than are attainable by pneumatic caissons. Such caissons have in numerous cases been sunk to depths of 150 ft and in the case of the foundations of the San Francisco-Oakland Bay Bridge to depths of over 240 ft. Multiwalled caissons are usually constructed of reinforced concrete and are built with timber or steel forms. A caisson of this type constitutes an open box, partitioned by vertical, lateral, and transverse walls into cells, called "dredging wells." The lower ends of interior and exterior

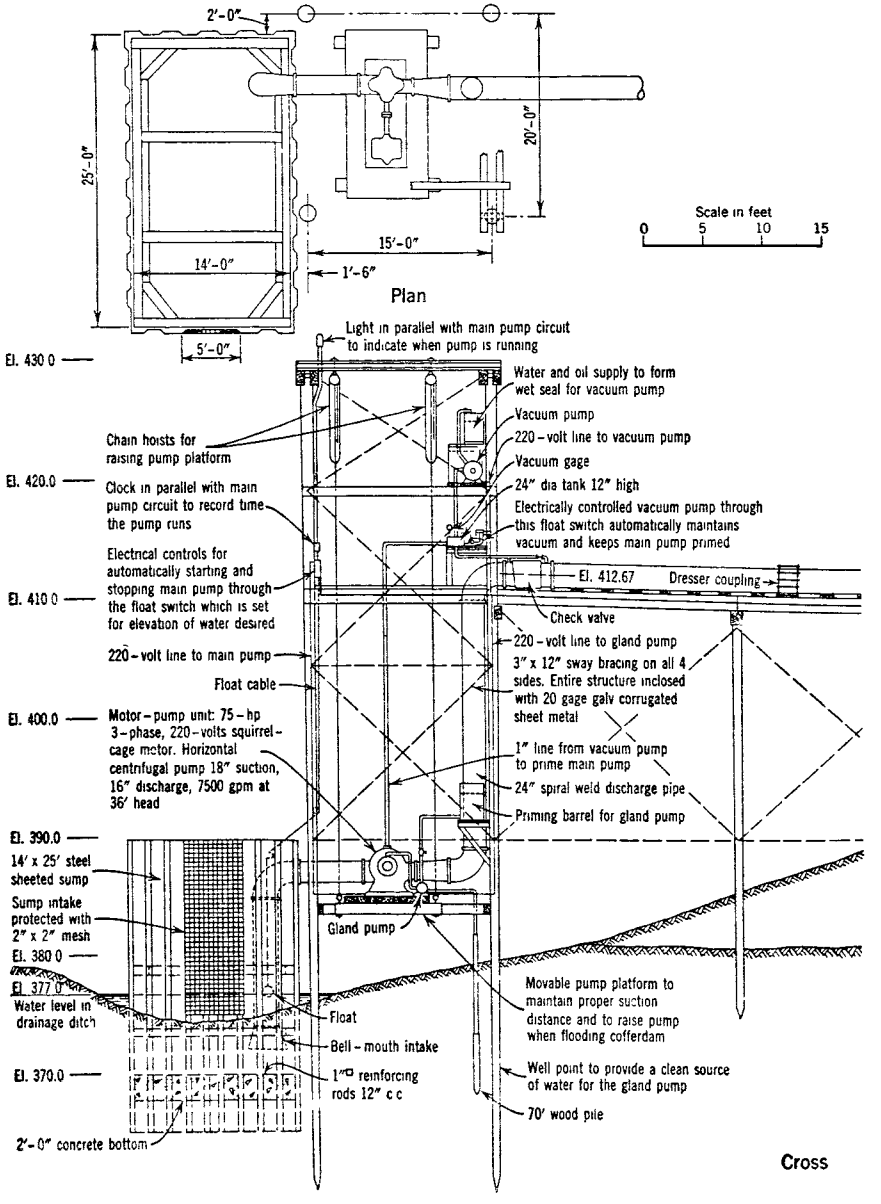
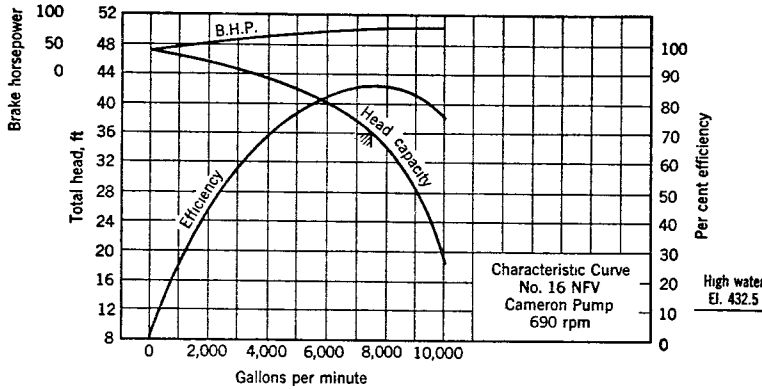
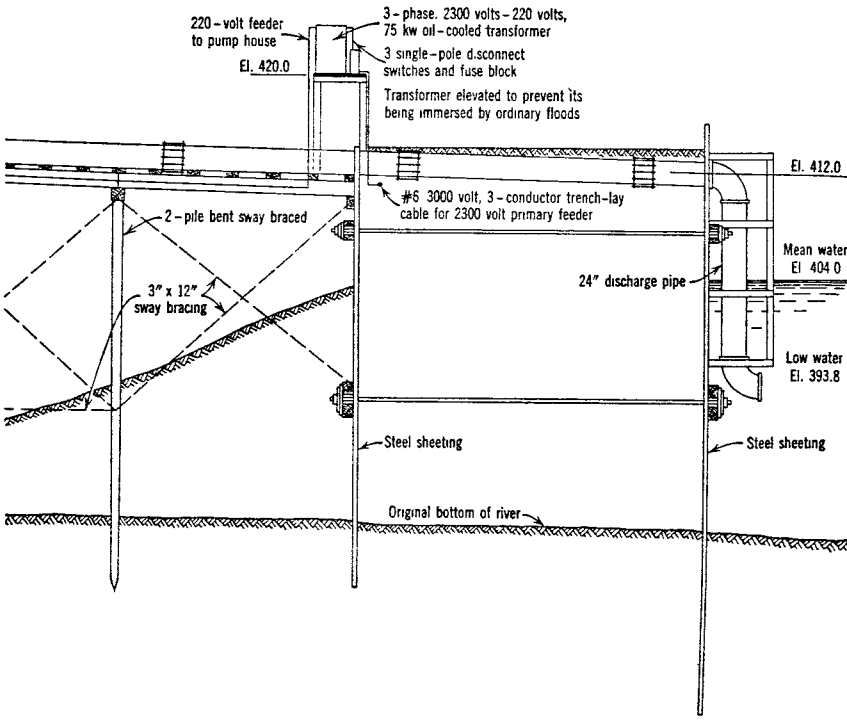


FIG. 55. Cofferdam pumping plant. Automatic pump setup with siphon, Lock and Dam Number.





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26, Alton, Ill. From *Cofferdams*, L. White and E. A. Prentis, Columbia University Press, 1950.

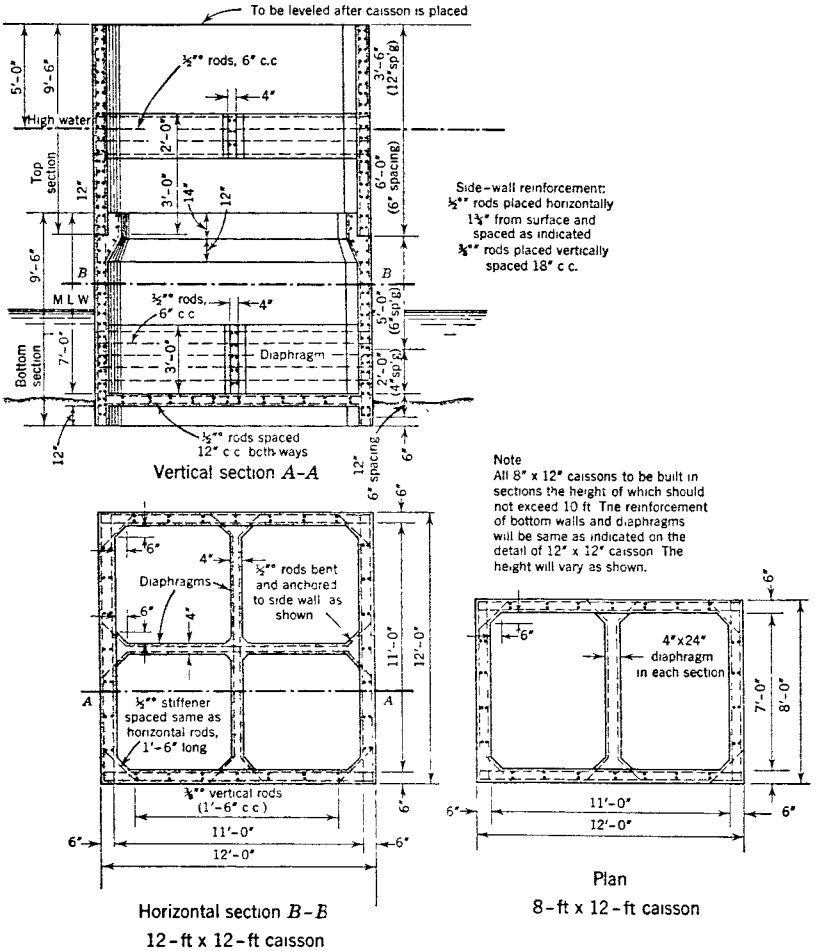


FIG 56 Box caisson for pier Box caisson of reinforced concrete near Glen Cove, Long Island By permission, from *Foundations of Bridges and Buildings*, by H S Jacoby and R P Davis, copyright, 1941, McGraw-Hill Book Company

walls are called the "cutting edges" and are usually formed of structural steel sections but may be of reinforced concrete or hardwood depending upon the type of soil to be penetrated. The interior walls and the interior face of the outer walls may be increased in thickness above their lower sections, to increase the caisson weight so as to assist in overcoming skin-frictional resistance to caisson sinking. Where the interior cutting edges are at a higher level than the exterior edges and where the walls are of increased cross section above the cutting edges, the spaces so outlined are called "dredging chambers" The caisson is excavated by clamshell or orange-peel dredging buckets, and the caisson sinking is normally expedited by caisson weights. When the dredging chambers are cleaned out, they, together with the dredging wells, are filled with concrete.

For bridge foundations, such caissons must be lowered to the river bottom and then sunk to an adequate supporting medium, frequently at considerable depths The principal problems to be met are, first, in the control and lowering of the caisson while it is a floating body—i.e , until it has received its support in firm soil in the river bed; and second, in overcoming the skin friction of the soil against the outer walls as the caisson is forced downward into the excavation made within its dredging wells.

The problem of controlled flotation may be met by building up the lower sections on ways, the cutting edges and lower sections of the walls of the dredging wells being concrete-filled and the upper sections formed but not filled to constitute the superimposed cofferdams. Then when launched from the ways, the caisson floats and is towed to its pier location. By alternately filling the hollow walls with concrete and extending upward the wall forms, the caisson is gradually sunk until the cutting edges are imbedded in firm soil, when dredging is started and the walls extended up and filled as sinking takes place.

Since concrete with its normal reinforcement weighs about 2 1/2 times the weight of water and since the timber forms and bracing with their usual hardware of bolts, nails, and fittings about equal the weight of water, approximately 3/5 of the height of such a caisson must be unfilled to provide the necessary cofferdam buoyancy during flotation. Hence, as the depth of water increases the required cofferdam draft increases so that at considerable water depths this method of seating a caisson becomes more expensive and more hazardous and with sufficient depth, impossible.

Therefore, for great depths of water, other methods are employed. For many years the accepted alternate method was to install false bottoms, generally of timber, under each dredging well of the caisson, to temporarily convert the caisson into a boat When the caisson comes to rest on the bottom, the false bottoms are removed and sinking is started by excavating through the dredging wells.

Examples of bridge foundation installations of open caissons floated by the false bottom method are in the foundations for the Mid-Hudson Bridge at Poughkeepsie, New York, and the Cantilever Bridge section east of Yerba Buena Island for the San Francisco-Oakland Bay Bridge.

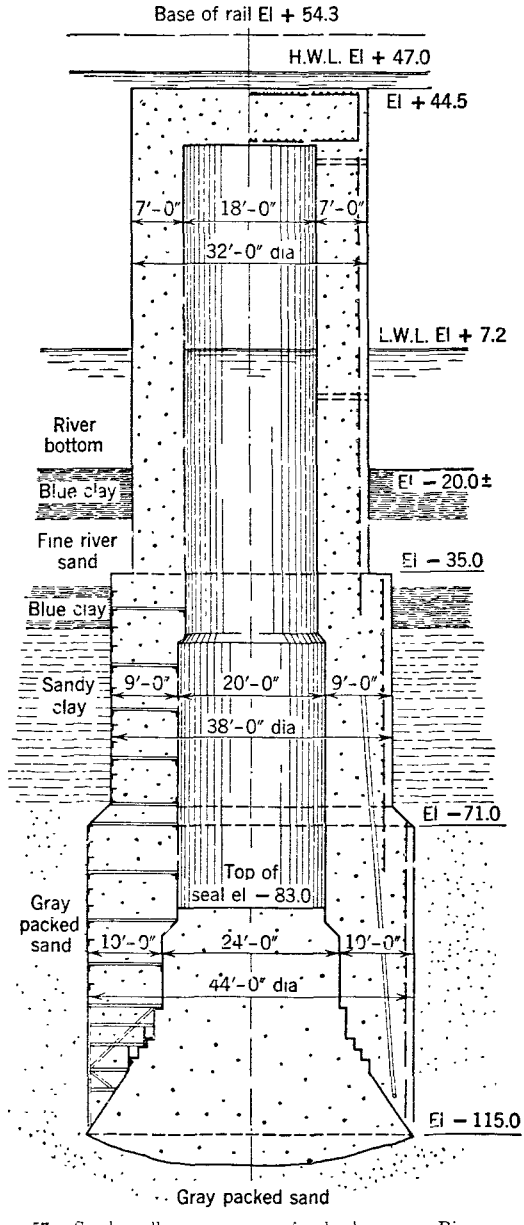
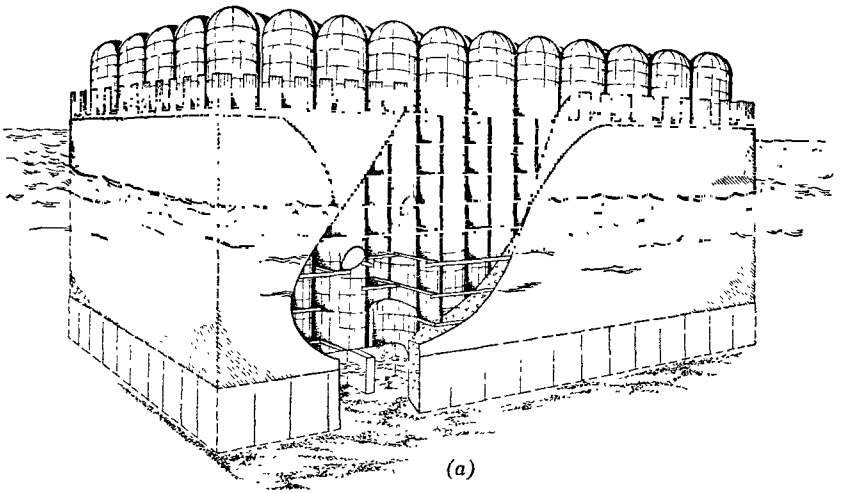


FIG. 57. Single-wall open caisson for bridge pier River pier, Atchafalaya River Bridge, Melville, La Open caissons for large bridge piers may have a number of dredging wells (see Fig. 57).



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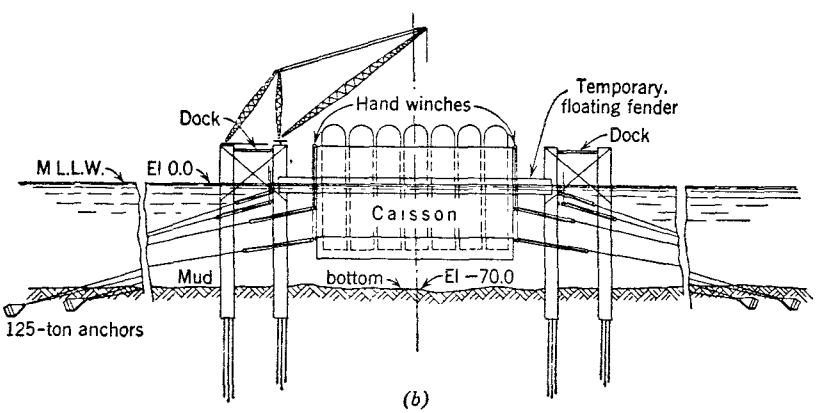
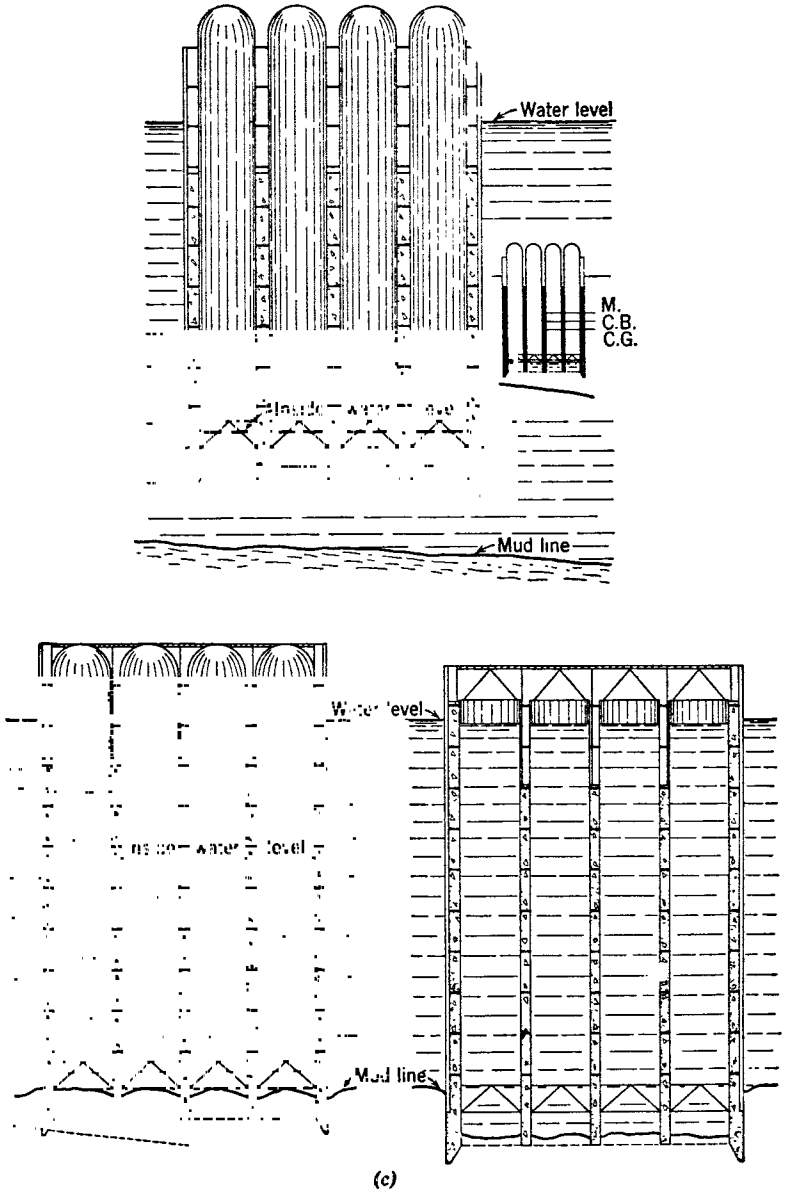


(b)

FIG 58 (a) Cutaway drawing of Moran caisson, San Francisco-Oakland Bay Bridge (b) General picture of caisson us [illegible] Francisco-Oakland Bay Bridge, showing caisson, working docks, and [illegible] em From Constructing Foundations of Trans-Bay Bridge, Carlton S F [illegible] 1935 (c) Floating, landing, and excavating within domed cofferdam fo [illegible] Bridge From Foundation Design for Trans-Bay Bridge, *Civil Engg*, December 1934.

The Moran Caisson. Another development in open caisson design is that employed for the West Bay piers for the San Francisco-Oakland Bay Bridge—see ref. 17 Here it was necessary to meet two unprecedented conditions: caisson sinking depths attained the record of 248 ft below high water as against 180 ft maximum (the previous record depth) at the Huey P. Long Bridge, and caisson flotation depths reached the record of 118 ft as against the previous record of 70 ft at the Hawkesbury Bridge at New South Wales, Australia. No precedented method of installation would meet these conditions.

The Moran domed caisson, Fig. 58, was developed and patented to permit the flotation and sinking of open caissons by a method which eliminated the normal factors of increased hazard and rapidly mounting costs with increased depths of open water. In these caissons the dredging wells are circular and consist of steel cylinders which are carried well above the caisson buildup and capped with hemispherical steel domes. Air pressure, applied to the dredging wells, depresses the water in the cylinders to any desired level, providing effective stability and flotation control during the flotation and sinking stages. During the



(c)

FIG. 58 (*continued*).

flotation stage, as the caisson sinks and is prepared for each buildup, the domes are progressively removed to permit the upward extensions of the steel cylinders, the domes replaced and water within such dredging wells depressed to permit progressive similar operations on all dredging well cylinders. This procedure provides in principle a pneumatic false bottom near the lower end of each dredging well and permits the caisson to be treated as a boat so that the superimposed cofferdam may be maintained as small as is consistent with safe flotation principles, which require that the center of gravity and center of buoyancy be always below the metacenter to provide constant positive righting moment.

After the cutting edges of open-dredged caissons are fully supported in soil there remains the problem of skin friction, common to all types regardless of method of flotation control employed. As the soil beneath the caisson is excavated through the dredging wells, the caisson sinks, but with increased depth of dredged sinking there is a constantly increased frictional resistance of the soil against the outer walls of the caisson Such skin-frictional resistance varies with the type of soil penetrated and has exceeded 1000 lb per ft^2. Skin frictional resistance is reduced by the lubricating and cutting action of "bathing and cutting" jets built into the caisson walls, the bathing jets, in the outer walls only, acting upward to cause an upward flow of water on the outside walls and the cutting jets built into all cutting edges acting downward to expedite dredging and sinking

The **sand-island method,** Fig. 53, is another development of the use of caissons as bridge foundations and piers. In this method a braced steel sheet pile cofferdam or, more usually, a circular self-sustaining steel sheet pile cofferdam is built in the water on the bed soil at the location of each pier. The cofferdams are filled with sand and thus provide "sand islands" upon which caissons of either the open or pneumatic type may be assembled and sunk as land operations, Fig. 53. Examples of the use of the sand-island principle for installing bridge pier caissons are the Ambassador Bridge at Detroit, the Southern Pacific Railway Bridge at Suisun Bay, and the Huey P. Long Bridge across the Mississippi at New Orleans

Pneumatic caissons have been used extensively and with success in the construction of bridge foundations. For example, the anchorages of the Ambassador Bridge at Detroit and the foundations of the Kennebec River Bridge at Bath, Maine, and the Cooper River Bridge at Charleston, South Carolina, were built as pneumatic caissons Pneumatic caissons for bridge foundations are based on the same principles as pneumatic caissons used for sinking building foundations on land and what has been said of pneumatic caissons in Art. 8 applies to bridge foundation caissons. They are, however, usually larger than caissons used on land, Fig. 30*d*, and there are generally more separate shaftways for men and materials. There may, in fact, be several shaftways of each type in a large bridge caisson depending upon its size and the pneumatic pressure requirements. The air locks in such caissons may be at the top, at the roof of the working chamber, or part way up the shaftways.

Combination open and pneumatic caissons (see Art. 8) may frequently be used to great advantage in the construction of bridge foundations.

15. DIVING

Divers are necessary for the prosecution of some phases of underwater foundation and water-front construction work. Their services are occasionally needed in the construction of foundations on land if work must be accomplished in a flooded excavation which it is impossible, hazardous, or inadvisable to unwater. What follows gives general information on the possibilities and limitations of the work which divers can do effectively. It also outlines the precautions, measures, and equipment necessary to provide reasonably safe working conditions for men who engage in a trade that can be decidedly hazardous. As one of the hazards which divers face is the development of "the bends" or caisson disease due to exposure to an artificially high atmospheric pressure and subsequent decompression, the discussion of the physiological effects of compressed air in Art. 8, Pneumatic Caissons, should be read. For a thorough treatment of the complex problems of diving, see ref. 3 An excellent brief treatment is ref. 6.

Depths at Which Divers Can Work. An experienced diver should be able to work effectively at a depth of 90 ft, this being the depth at which second-class divers in the U. S. Navy are qualified to work. Skilled salvage divers are qualified to work at depths as great as 150 ft and exceptionally well-qualified first-class divers and master divers are qualified to work at depths as great as 300 ft when using compressed air and at even greater depths when using an artificial atmosphere of helium and oxygen. The lengths of time divers should be allowed to stay on the bottom vary with the depths at which they are working. Their safe rates of ascent and consequent decompression vary not only with the

depth but with the length of time spent on the bottom. According to the Navy Standard Decompression Table, in ref. 3, a diver who has been working at a depth of 150 ft for 15 minutes requires a total decompression time of 14 minutes which should be provided by halting his normal ascent for 7 minutes when he reaches a depth of 10 ft. If, however, he has worked for 80 minutes at a depth of 150 ft, his total decompression time should be not less than 150 minutes provided by halts in ascent of 18, 23, 32, and 69 minutes respectively at depths of 40, 30, 20, and 10 ft. The practicability of having extensive work done by divers therefore decreases rapidly with increases in the depths at which the work is to be done. The Navy Standard Decompression Table of ref. 3 should be consulted to determine the practicability of having work done by divers at any stipulated depth, but in general it may be said that about 100 ft is the maximum depth at which any extensive activity by divers is practicable. They can do emergency work, such as attaching lifting gear in a salvaging operation, at much greater depths. What has been said above refers to divers working in standard helmets and flexible diving suits. With a shallow water helmet or a face mask skin divers can accomplish minor tasks at moderate depths but skin diving is not nearly as safe as diving in a regulation suit and should not be employed extensively in underwater work.

Adequate air supply is vital to the safety of divers. On small jobs hand-operated compressors are usual but in extensive diving operations the use of power-driven compressors is almost a necessity. In any event, the air after leaving the compressor should be cooled and passed through a suitable oil filter. The air-supply system should include compressed-air storage tanks of such capacity as to provide adequate quantities of air for the safe ascent of divers in case of compressor failure. It should be noted that a man breathing compressed air requires a far greater number of cubic feet of free air per minute than a man breathing at ordinary atmospheric pressure. In fact, a man working in a limited volume of air, such as is available in a diving suit, needs about 1.5 cu ft of air per minute at the pressure at which he is working, irrespective of what that pressure may be. A diver working at a depth of 100 ft, therefore, requires about four times as many cubic feet of free air per minute as a man working at or just below the surface.

A recompression chamber or hospital lock should be provided where extensive diving operations are undertaken, even though the depth at which the work is done is moderate. The exigencies of diving occasionally necessitate bringing a man up quickly and recompression may be necessary to prevent the development of the bends even though the depth may have been slight. A qualified doctor should therefore be kept in attendance and the same precautions observed as for caisson workers. See Art. 8.

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SECTION 9

EARTHWORK AND DREDGING

BY

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EARTHWORK AND DREDGING

EARTHWORK

1. GENERAL

The technique and economics of earthwork have been revolutionized during the past three decades through mechanization Almost anywhere in the United States it is economical and efficient to use power excavators to dig even such small excavations as the cellars of houses or cesspools for single dwellings. There are, however, remote regions in undeveloped parts of the world in which hand excavation is still used on a large scale for such projects as the construction of roads and foundations. Even in such locations hand excavation is generally used in conjunction with motor trucks for hauling. For a complete and adequate treatment of excavation by hand and earth-moving by horse-drawn equipment, refer to earlier editions of this and other handbooks and to books and articles on construction methods written before 1930.

2. HAND EXCAVATION AND HORSE-DRAWN EQUIPMENT

The information given below as to quantities of work that may reasonably be expected of men doing excavation by hand applies to strong, able, and well-fed laborers. Undernourished men who undertake hand excavation in some remote overseas areas cannot do the indicated amounts of work per hour. Their productivity can, however, be materially improved by supplementing their normally inadequate diet by issuing them nourishing food "in kind" in addition to wages in cash. An increase in wage is less effective as the money will be spent largely on things other than food.

Where horse-drawn wagons and scrapers must be used, mules will, in general, be found preferable to horses, as they require less skillful care and attention.

Loosening by picking or plowing is invariably necessary as a preparatory step to hand excavation. A good laborer should be able to loosen with a pick from 3.0 to 5.0 yd^3 of ordinary loam or from 0 5 to 1 0 yd^3 of hardpan or cemented gravel per hour. A plow drawn by two horses or mules will loosen 30 to 40 yd^3 of ordinary loam per hour and will require a driver and plowman for its operation. In stiff clay or gravel four horses will be needed, and in hardpan, six. In both of the latter cases one or two men in addition to the driver and plowman will be needed to ride the plow beam and keep the plow in the ground.

Shoveling. A man can pick and load about 0 5 yd^3 of loam, 0.3 yd^3 of clay, or about 0.1 yd^3 of hardpan per hour. He can shovel about 0.6–0.75 yd^3 of well-loosened earth into a wagon or small truck in an hour. If the wagon or truck is high, the output is reduced. Men cannot shovel effectively into the large trucks normally used with mechanical excavating equipment. The above rates should be attainable by the average vigorous, able-bodied man They have been doubled consistently by many laborers in the days when excavation by hand was standard practice.

Wagons for hauling earth were either bottom or end dump and were drawn by from two to four horses depending upon capacities. Today, where wagons are used for earth moving in remote foreign locations, the best possible use should be made of locally available types. Earth-moving equipment transported from the United States should be motor-driven.

Horse-drawn scrapers of various types and sizes were used extensively for excavating soft and thoroughly loosened materials and hauling them short distances. Drag and fresno scrapers had no wheels but slid over the ground on their bodies or on runners.

Wheeled scrapers consisted of a box-shaped steel scoop mounted on wheels and furnished with levers for lowering, raising, and dumping. They were too heavy for effective use as horse-drawn devices, and the practice of pulling them with tractors undoubtedly suggested the development of the modern large tractor-hauled scraper.

Compacting earth with horse-drawn equipment was surprisingly satisfactory due, no doubt, to the relatively thin layers in which wagons deposited earth and to the beneficial effect of "kneading" by the feet of the animals that hauled the wagons and rollers.

3. POWER EXCAVATORS

Excavating machines commonly used comprise self-propelled cranes fitted with special booms accommodating shovel or hoe attachments; or with ordinary booms from which dragline, clamshell, or orange-peel buckets are operated; Figs. 1*a*, *b*, *c*, and *d*. More highly specialized types of excavators are trenching machines consisting essentially of a number of small buckets mounted on a wheel or endless chain, and loaders which are tractor-drawn scrapers that make a shallow cut and deliver the earth from the scraper blade by means of a power-driven belt conveyor to a truck which runs alongside the loader as it works.

Sizes of Excavating Equipment. The standard sizes of shovel and hoe dippers and dragline, clamshell, or orange-peel buckets operated by commercial cranes used as excavating machines are $^3/_8$, $^3/_4$, 1, 1 $^1/_2$, 2, and 2 $^1/_2$ yd^3, these being the volumes of the dippers' or buckets' struckoff level. The cranes which handle these dippers and buckets vary in maximum capacity from 4 to 60 tons. The size of bucket which can be used with a crane of a given capacity depends upon the weight of the bucket itself, the unit weight of the material to be handled, and the length of boom required. In general, however, $^3/_8$- and $^3/_4$-yd buckets can be handled by cranes of 4- or 5-ton capacity, 1- and 1 $^1/_2$-yd buckets by 10- to 15-ton cranes, and 2- and 2 $^1/_2$-yd buckets by cranes of 20-ton capacity and larger. The rated capacity of a crawler-mounted crane is considered as 75% of the tipover load at a radius of 12 ft from the center of rotation when the machine is standing on firm and level ground. The rated capacity of a crane mounted on rubber-tired wheels is considered to be 85% of the tipping load at a radius of 10 ft from the center of rotation with the unit on firm, level ground. Shovels and draglines of far larger capacity than 2 $^1/_2$ yd^3 have been built and are in use. They are not, however, standard commercial units but machines specially designed and built for the use intended. A shovel for strip mining with a 45-yd^3 bucket has for example been built, and huge draglines of similar orders of magnitude have been produced for levee construction and canal excavation.

Mountings. Commercial models of power cranes suitable for use as excavating machines are available on three basic types of mountings: crawler, truck, and wheel.

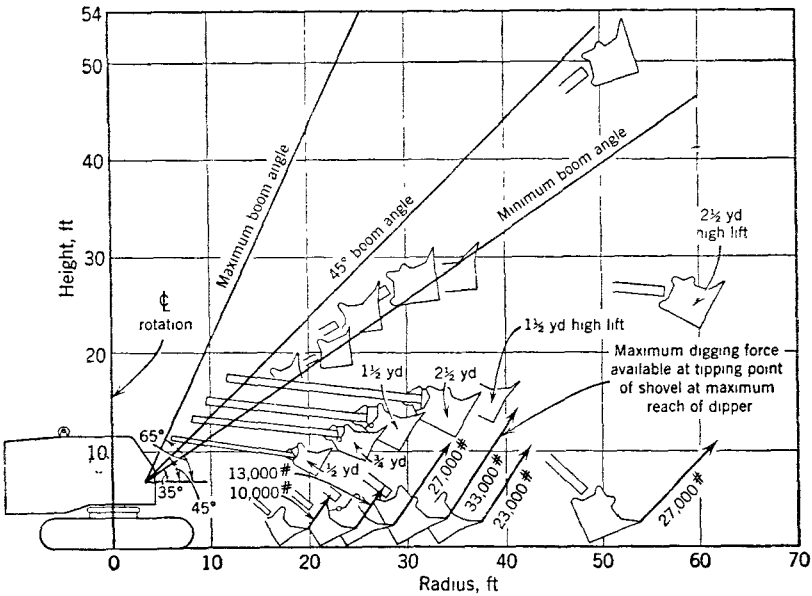
Crawler mountings carry the majority of cranes used as excavators. They are available in various shoe widths and crawler lengths for most models of commercial cranes, in order that machines may be well suited to the work they are to do. On some cranes the overall width of the crawlers can be changed to give greater stability for dragline operation. The bearing pressure under crawler treads usually varies between 720 and 1440 lb per ft^2. The shorter and narrower types of crawlers and shoes are commonly used on shovels which work on firm ground. Long crawlers have the disadvantage of interfering with use of the shovel dipper close to the crane in cleanup work. They also make the machine more difficult to steer. The great merit of the crawler mounting is its versatility on the job site, its ability to carry the heaviest types of crane on soft or rough ground, and its stability. Its more serious limitations are its slow speed when moving (it makes between 1/2 and 2 mph) and the fact that it damages pavements. It is generally necessary to load a crawler-mounted crane onto a low bed trailer if it must be moved on a public road. This is time-consuming and expensive.

A truck mounting consists of a heavy four- or six-wheel motor truck upon which the rotating unit of the crane is carried. The truck has its own motor which is independent of the one which drives the crane mechanism. Truck mounting provides maximum mobility. Truck-mounted cranes can travel at speeds as great as 35 mph on the open road. The stability afforded by truck mounting is inferior to that afforded by a crawler but may be improved by the use of outriggers supported by jacks or blocking. Truck-mounted cranes used as excavators rarely exceed $^3/_4$ yd^3 capacity as width and axle-load limitations imposed by many states would prohibit larger machines traveling on highways and the possible advantage of the great mobility of a large truck-mounted crane is therefore lost.

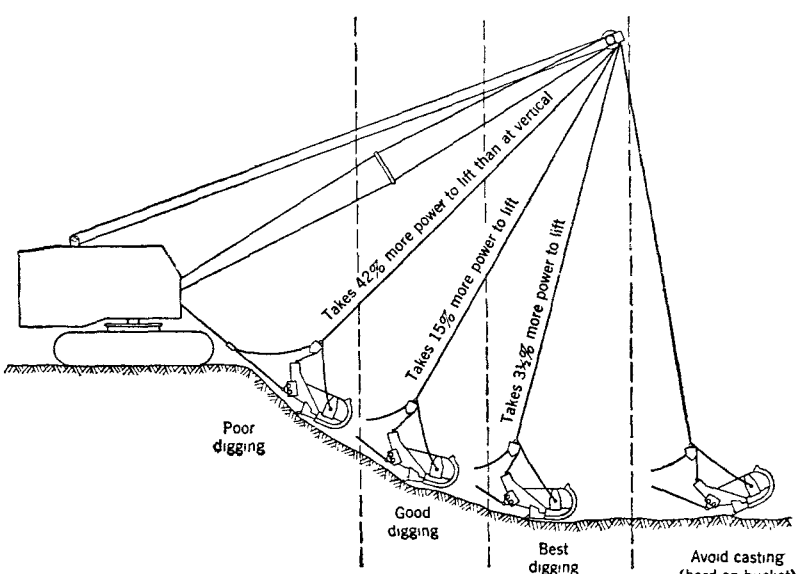
Wheel mountings are somewhat similar to truck mountings but utilize the crane motor to drive the vehicle. A wheel-mounted crane does not steer as well as a truck-mounted crane when on the road, and its speed is usually limited to about 10 mph though some of the smaller models will make 20 mph. Otherwise its advantages and limitations are very similar to those of the truck-mounted crane.

Walking Pads. Some very large excavators built for use on soft ground and employed in building levees and excavating drainage canals are fitted, in lieu of crawlers, with huge pads or feet upon which the machines walk. These are highly specialized pieces of equipment the manufacturers of which should be consulted as to their characteristics.

Railway mountings on standard or narrow-gage railroad trucks were once used largely for excavators, particularly shovels. They are rarely used today as the more versatile and effective crawler mounting has supplanted them.

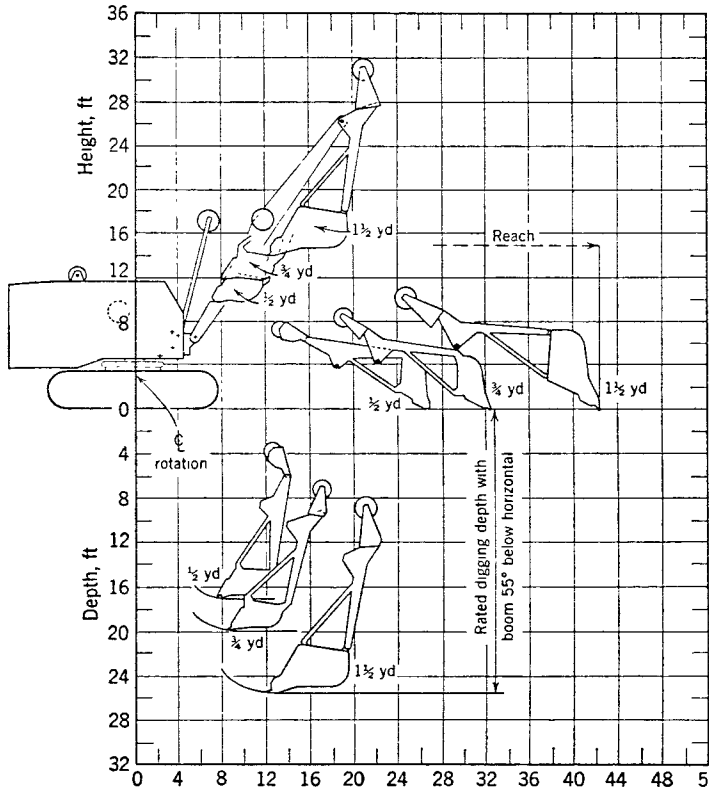


(a)



(c)

FIG. 1. **Power excavators.** (*a*) Shovel digging ranges. (*b*) Hoe (frequently called the back hoe) ment for handling an orange-peel bucket is very similar.



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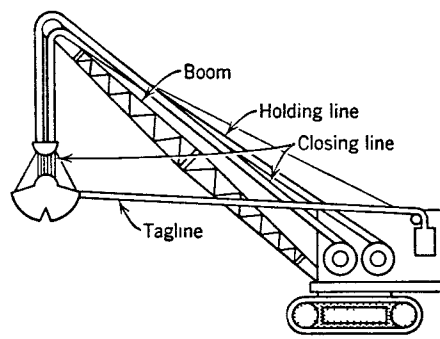

(d)

digging ranges. (*c*) Dragline digging zones. (*d*) Crawler crane with clamshell bucket The arrange-
Courtesy of Power Crane and Shovel Association

Drives. Gasoline engines are used to drive many of the smaller types of crane and a few large ones. Their great advantage is the familiarity of the average mechanic with their operation and maintenance. Diesel engines are, however, rapidly supplanting the gasoline engine on large machines. Their advantages are low fuel cost and, with increasing familiarity of personnel with their care and maintenance, low upkeep and long life. Diesel-electric drive is used on a few very large cranes. These machines are in reality electric cranes which carry their own power plants in their counterweights. Their rather high first cost is justified where the nicety of control afforded by an electric crane is desirable at a location where electric current cannot be purchased. Electric drive with purchased power delivered to the crane through a trailing cable is cheap, economical, and reliable. It has been found highly advantageous where shovels and excavators work for extended periods in the same location such as in quarry operation Steam-engine drive is found on a few old cranes and shovels still in operation. It is less satisfactory than gasoline, diesel, or electric drive, and today is rarely if ever used on new cranes.

Characteristics and capacities of crane-operated excavating equipment are outlined below.

The **shovel** is preferable to other types of excavators where the material is firm or hard and conditions permit the development of a good face or bank against which the shovel can work. A shovel cannot dig to any considerable depth below the surface upon which it stands. Therefore it can be used in deep excavations only when they are large enough to accommodate the shovel in the bottom of the hole and at the same time provide room for a ramp over which trucks can reach the shovel and up which the shovel can travel when its work is finished. Another type of excavator is required for the removal of the ramp.

Shovel Output. The estimated quantities of various types of materials that different sizes of shovels should dig in an hour are given in Table 1. The figures are based on assump-

Table 1. Power-Shovel Capacity in Cubic Yards per Hour

	Shovel Dipper Capacity in Cu Yds								
Class of Material	3/8	1/2	3/4	1	1 1/4	1 1/2	1 3/4	2	2 1/2
Moist loam or light sandy clay	3.8′	4 6′	5.3′	6.0′	6 5′	7.0′	7.4′	7.8′	8 4′
	85	*115*	*165*	*205*	*250*	*285*	*320*	*355*	*405*
Sand and gravel	3.8′	4 6′	5.3′	6 0′	6.5′	7.0′	7.4′	7.8′	8.4′
	80	*110*	*155*	*200*	*230*	*270*	*300*	*330*	*390*
Good common earth	4.5′	5.7′	6.8′	7.8′	8 5′	9.2′	9.7′	10.2′	11.2′
	70	*95*	*135*	*175*	*210*	*240*	*270*	*300*	*350*
Clay, hard, tough	6.0′	7.0′	8.0′	9.0′	9.8′	10 7′	11.5′	12.2′	13.3′
	50	*75*	*110*	*145*	*180*	*210*	*235*	*265*	*310*
Rock, well blasted	*40*	*60*	*95*	*125*	*155*	*180*	*205*	*230*	*275*
Common, with rocks and roots	*30*	*50*	*80*	*105*	*130*	*155*	*180*	*200*	*245*
Clay, wet and sticky	6.0′	7.0′	8.0′	9.0′	9.8′	10.7′	11.5′	12.2′	13.3′
	25	*40*	*70*	*95*	*120*	*145*	*165*	*185*	*230*
Rock, poorly blasted	*15*	*25*	*50*	*75*	*95*	*115*	*140*	*160*	*195*

Conditions (1) Cubic yards bank measurement, per hour. (2) Suitable depth of cut for maximum effect. (3) No delays. (4) 90° swing. (5) All materials loaded into hauling units. (6) Roman figures denote optimum depth of cut; italic figures denote yd^3 per hour.

Courtesy of Power Crane and Shovel Association.

tions that the depth of cut is optimum for each shovel size and type of material and that the shovel swings 90° to deposit each dipperful. The optimum depth of cut for each size of shovel and type of material is given in feet directly above the hourly output in cubic yards. Factors showing the effect of varying the depth of cut and the angle of swing are given in Table 2 An hourly yardage taken from Table 1 when multiplied by a factor obtained from Table 2 should then give the possible hourly output of any standard commercial-size crane equipped as a shovel if the machine works steadily and there are no delays of any kind. Delays of one kind and another are, however, inevitable on an earth-moving job. Table 3 gives job and management factors that should be applied to figures obtained by the use of Tables 1 and 2 in order to estimate the probable actual output of a shovel working for an extended period. The figures are all bank measure and should be increased if truck measure is desired. The increase due to swell will run between 30% for earth to 50% for rock.

Table 2. Effect o Shovel Output of Varying Cut and Swing

(Conversion factors which, when appli to th t tput at 90° swing and optimum depth of cut, will give the outpu other angles of swing or depth of cut)

Depth of Cut, % of optimum	Angle of swing, degrees						
	45°	60°	75°	90°	120°	150°	180°
40	0.93	0 89	0.85	0.80	0.72	0.65	0 59
60	1.10	1.03	0 96	0.91	0 81	0.73	0 66
80	1.22	1.12	1.04	0.98	0 86	0 77	0.69
100	1.26	1.16	1.07	1.00	0.88	0.79	0 71
120	1.20	1.11	1.03	0.97	0.86	0 77	0.70
140	1.12	1 04	0.97	0.91	0.81	0 73	0.66
160	1.03	0.96	0.90	0.85	0.75	0.67	0.62

Courtesy of Power Crane and Shovel Association.

Table 3. Job and Management Factors

Job Factors	Management Factors			
	1. Excellent	2. Good	3. Fair	4. Poor
1. Excellent	0.84	0.81	0.76	0.70
2. Good	0.78	0.75	0.71	0.65
3. Fair	0.72	0 69	0.65	0.60
4. Poor	0.63	0.61	0.57	0 52

Courtesy of Power Crane and Shovel Association.

The size of shovel best suited to a given job should be estimated by consideration of at least the following factors:

(*a*) The output desired and therefore the size of dipper.

(*b*) Radii required to reach digging area and point at which the shovel drops the material. This may be a truck body, a hopper, or a spoil bank.

(*c*) Height range: Can the shovel reach the height of the disposal point and at the same time remove loose material which might otherwise gather on the surface on which the machine stands?

(*d*) Bank height.

(*e*) Clearance required for operation of the machine. This is often the controlling factor in selecting a shovel for work in a small excavation or tunnel.

The **dragline** is preferable to other types of excavator where material is to be cast a maximum distance onto a spoil bank, embankment, or levee. The dragline can dig below the ground on which it stands if the excavation is large enough to permit necessary dragging of the bucket. The bucket works well under water, and the dragline is therefore useful for digging large ditches and drainage canals. It works best with materials that are soft to

Table 4. Dragline Output at Optimum Cut and 90° Swing

Short-Boom Dragline Performance, Cu Yds

Class of material	3/8	1/2	3/4	1	1 1/4	1 1/2	1 3/4	2	2 1/2
Light moist clay or loam	5.0′	5.5′	6.0′	6.6′	7.0′	7.4′	7.7′	8.0′	8.5′
	70	*95*	*130*	*160*	*195*	*220*	*245*	*265*	*305*
Sand or gravel	5.0′	5.5′	6.0′	6.6′	7.0′	7.4′	7.7′	8.0′	8.5′
	65	*90*	*125*	*155*	*185*	*210*	*235*	*255*	*295*
Good common	6.0′	6.7′	7.4′	8.0′	8.5′	9.0′	9.5′	9.9′	10.5′
	55	*75*	*105*	*135*	*165*	*190*	*210*	*230*	*265*
Clay, hard, tough	7.3′	8.0′	8.7′	9.3′	10.0′	10.7′	11.3′	11 8′	12.3′
	35	*55*	*90*	*110*	*135*	*160*	*180*	*195*	*230*
Clay, wet, sticky	7.3′	8.0′	8.7′	9.3′	10.0′	10.7′	11.3′	11.8′	12.3′
	20	*30*	*55*	*75*	*95*	*110*	*130*	*145*	*175*

Conditions: (1) Rate of production in cubic yards per hour (bank measure). (2) When loading into hauling units. No delays. (3) Optimum depth of cut. 90° swing (4) Bucket selected for type of material. (5) Working radius within manufacturer's recommendation for machine stability. (6) Roman figures denote optimum depth of cut; italic figures denote yards per hour.

Courtesy of Power Crane and Shovel Association.

medium hard. It can handle well-blasted rock but not as efficiently as a shovel if conditions permit the use of a shovel. The dragline cannot dig deep excavations in which the bucket must be dropped between cofferdam braces or down dredging wells.

Dragline Output. The estimated quantities of various types of materials that different sizes of draglines should dig in an hour if equipped with short booms are given in Table 4. The figures are based on assumptions that the depth of cut is optimum and that the dragline swings 90° to deposit each bucketful. The depth of cut considered optimum for each size of dragline and type of material is given in feet directly above the hourly output in cubic yards. Factors showing the effect of varying the depth of cut and angle of swing are given in Table 5. An hourly yardage taken from Table 4 when multiplied by a factor obtained from Table 5 should therefore give the possible hourly output of a standard commercial size crane working as a dragline if the machine works steadily and there are no

Table 5. Effect on Dragline Output of Varying Cut and Swing

(Conversion factors which, when applied to the output at 90° swing and optimum depth of cut, will give the output at other angles of swing or depth of cut)

Depth of Cut, % of optimum	Angle of Swing, degrees							
	30°	45°	60°	75°	90°	120°	150°	180°
20	1.06	0.99	0.94	0.90	0.87	0.81	0.75	0.70
40	1.17	1.08	1.02	0.97	0.93	0.85	0.78	0.72
60	1.24	1.13	1.06	1.01	0.97	0.88	0.80	0.74
80	1.29	1.17	1.09	1.04	0.99	0.90	0.82	0.76
100	1.32	1.19	1.11	1.05	1.00	0.91	0.83	0.77
120	1.29	1.17	1.09	1.03	0.985	0.90	0.82	0.76
140	1.25	1.14	1.06	1.00	0.96	0.88	0.81	0.75
160	1.20	1.10	1.02	0.97	0.93	0.85	0.79	0.73
180	1.15	1.05	0.98	0.94	0.90	0.82	0.76	0.71
200	1.10	1.00	0.94	0.90	0.87	0.79	0.73	0.69

Courtesy of Power Crane and Shovel Association.

delays. To the figure thus obtained a job and management factor from Table 3 should be applied to secure a reasonable estimate of the hourly output of a dragline working for an extended period as delays of one kind or another are inevitable on earth-moving projects.

The size of dragline best suited to a given job should be estimated by consideration of at least the following factors:

(*a*) The output desired. This will give a first approximation of the necessary bucket size.

(*b*) Radii required to reach digging area and disposal point. Boom length necessary to achieve required radii will effect output as long booms slow operation of dragline.

(*c*) Bearing pressure under crawlers. As draglines often work on soft ground this may be a controlling factor. If machine is to work near edge of large excavation, possibility of shear failure should be considered.

The **clamshell** is a versatile excavating machine which can do many things well but is less efficient than the shovel and dragline under conditions that permit their effective use. The great merit of the clamshell as an excavator is its ability to dig deep holes extending far below the surface upon which the machine rests. The sides of such excavations may be vertical if properly sheathed. Internal bracing of the sheathing does not seriously interfere with the use of a clamshell if there is space between the braces to drop the bucket. The clamshell is accordingly used for digging sheeted excavations and particularly for excavating in small cofferdams and in the dredging wells of open caissons. It can lift material to a high disposal point and deposit it with accuracy. It works best in soft to medium-hard materials. It is an ideal machine for stock-piling such materials as broken stone and loading them into bins and hoppers.

Clamshell Output. The conditions under which clamshells operate are so varied that tabulated output figures are impracticable. Estimates should be based on bucket capacities and anticipated durations of loading cycles. These will in turn depend upon the depth of the excavation, angle of swing, height of disposal point, and character of material. Cognizance should be taken of the fact that a clamshell bucket will not fill completely when digging firm material.

The size of clamshell excavator and character of bucket best suited to a job should be estimated by consideration of at least the following factors:

(*a*) The size of bucket that will pass readily between the cofferdam braces or down the dredging wells of a caisson.

(*b*) The weight of bucket required. Heavy buckets are needed for firm materials.

(*c*) The radius of action and consequently the boom length required to reach the digging and disposal points.

(*d*) The output desired must of necessity be a more or less secondary consideration. It controls the number of clamshell excavators required rather than their sizes.

Orange-Peel Buckets. The orange-peel bucket is a variant of the clamshell. It opens into three or four sections and closes to form a hemisphere rather than a semicylinder. It is an excellent device for digging circular excavations and shafts. Its general characteristics are similar to those of the clamshell bucket. It is a little less effective than the clamshell as a device for digging clean earth but handles occasional stumps and even fairly large loose rocks more readily.

The **hoe,** frequently known as the back hoe, is primarily a device for excavating below ground level. It will dig harder material than a clamshell, but the depth to which it can dig is limited by the length of the boom and dipper stick. It is an excellent device for digging trenches, small cellars, and other excavations requiring straight sides, plumb faces, square corners, and clean floors.

Hoe outputs, like those of clamshell excavators, are so dependent on many variables that tabulations attempting to show quantities that various sizes of hoe excavators can dig in an hour are impracticable.

The size of hoe excavator best suited to a given job should be estimated by consideration of at least the following factors:

(*a*) Depth of excavation and consequent length of boom and dipper stick.

(*b*) Width of excavation if it is a trench.

(*c*) Clearances necessary for the operation of the machine.

References 8, 14, 15, 16, and 17 give valuable information on the characteristics and capacities of power crane and shovel excavators.

Trenching machines of either the wheel or endless chain type are highly specialized devices. They are more efficient than other types of excavators for digging trenches of moderate depth in material free of large boulders and firm enough to stand without sheeting. They are widely used for digging trenches for the installation of pipes and conduits. They may be used effectively for excavating for wall footings.

Loaders are large tractor-drawn scrapers fitted with belt conveyors that pick up the earth from the scraper blade and deliver it to a large truck which runs alongside the loader until full. The conveyor is driven by an engine—usually a diesel of from 190 to 275 hp—carried on the loader. Loaders will make shallow cuts as wide as 9 ft 6 in. and will make cuts up to 48 in. deep in favorable material if pulled by two tractors or with one tractor pulling and a second pushing. They work best in sand, clay, and other materials that are relatively easy to excavate but will make shallower cuts in firm clay and even soft shale. They are effective only when used in conjunction with large capacity trucks specially built for earth moving and having large flaring bodies affording big targets for the earth delivered by the conveyor belt. Under favorable conditions a loader will fill a 25 cu yd truck or trailer in less than a minute. It is, however, a highly specialized type of equipment the detailed characteristics of which should be obtained from manufacturers.

4. EARTH-HAULING EQUIPMENT

The effectiveness of mechanized earth moving depends not only upon the efficiency of modern power excavators but also upon the ability of recently developed large trucks and trailers to transport earth cheaply from the area of excavation to the area of disposal.

Standard types of commercial truck chassis fitted with dump bodies are used for hauling earth over public roads and may be used for off-the-road hauling on small projects. Such trucks are, however, built primarily for operation on reasonably smooth surfaces and are much less effective for off-the-road hauling than the special types of truck and trailer built for earth moving and able to carry heavy loads over rough unpaved areas in which earthwork is frequently done. Limits imposed by states as to maximum permissible wheel and axle loads and limits as to the overall width of vehicles operated on public roads usually restrict large earth-moving trucks and trailers to off-the-road hauling. Approval of the employment of unusually large or heavy trucks or trailers on a job involving their using or crossing a public road or roads should invariably be secured from the proper governmental authorities before estimates based upon the use of such equipment become part of a plan for the accomplishment of earth work.

Large trucks and trailers for off-the-road hauling of excavated materials are built in sizes from about 10 to 39 tons and truck body capacities from 6.5 to 25 yd^3. Both end and bottom dump bodies are available in a wide variety of capacities, the smaller bodies usually

being of the end dump type, and the larger bottom dump. The end dump truck is the more versatile and generally useful of the two types as it will handle large rock that would jam in the doors of a bottom dump body. The bottom dump is, however, the more efficient machine for handling large quantities of free-flowing material, such as sand, and is specially adapted for use with loaders. Large earth-hauling trucks will make from 25 to 35 mph when carrying heavy loads over fairly rough ground. They provide the cheapest known method of hauling earth for considerable distances, subject always to the limitation that they cannot, in general, be used on public roads.

Railroad cars, once extensively used for hauling earth, have been largely supplanted by the earth-moving truck and trailer. Although earth can be moved cheaply on rails, the cost of laying and frequently moving temporary tracks is so great that rail-borne equipment cannot today compete with trucks and trailers as devices for transporting earth.

Belt conveyors have in the past been used on a few very large jobs for transporting earth and excavated materials. Their high cost of installation has limited their use to projects where the volume of earth to be handled was great, ref. 5. It is doubtful that they could compete today with large trucks, trailers, loaders, and scrapers.

5. EQUIPMENT THAT EXCAVATES AND HAULS

Self-propelled and tractor-drawn scrapers afford economical means of excavating soft to medium-hard earth and hauling it for considerable distances. The exact distances at which separate excavating and hauling units become more economical than scrapers depend upon so many variables that general statements are of little value. The self-propelled and tractor-drawn scraper has, however, an operating range far exceeding its horse-drawn predecessors. The modern scraper equipped with pneumatic tires is frequently used on jobs where the maximum haul exceeds 2 miles. Its speed when traveling off the road on reasonably smooth ground may be as high as 25 to 30 mph. Its wheel loads, axle loads, and overall dimensions prohibit its use on public roads.

Capacities of Power Scrapers. Power scrapers are built in various sizes with bowl capacities from 10 to 20 yd^3 struck measure. The rated struck-measure capacities may be considerably exceeded with material that will stand heaped on the scraper bowl when the machine is moving. Although many power scrapers will load themselves with a favorable material, it is quicker and more economical to use crawler tractors as pushers to expedite loading. This use of pusher tractors for expediting the loading of scrapers is almost universal.

Bulldozers can push earth for short distances economically. Their principal use is for leveling earth dumped at a fill, pushing it into excavations, and for similar jobs involving the movement of large quantities of material very short distances. The tractors on which bulldozer blades are mounted are frequently used as pushers to expedite the loading of scrapers.

The **cableway excavator** consisting of a bucket, usually of the clamshell type, operated from a cable supported by towers at its two ends, was at one time occasionally used on projects involving digging large quantities of material which could be deposited within from 200 to 600 ft from the excavation. Today the shovel, dragline, clamshell, loader, large off-the-road earth-moving truck, and scraper (or a combination of these various types of earth-moving equipment) will usually excavate and transport earth more cheaply than the cableway which has accordingly fallen into disuse. It may occasionally prove economical, however, where a relatively long, narrow and deep excavation must be made under water and the materials deposited close by under conditions permitting the construction of track-mounted towers and cableways spanning both the areas of excavation and deposit. Such an arrangement is illustrated in Fig. 2.

6. SLUICING

Hydraulic sluicing has in the past been used for excavating large quantities of material under conditions favoring the method. Sluicing can rarely today compete with earth moving by modern mechanical excavating and transporting equipment. The sluicing method consists of tearing or breaking down the face of the bank with powerful water jets from 2 1/2-in. to 5-in. diameter nozzles called hydraulic giants. The working pressure at the nozzles may be as high as 150 lb per $in.^2$ and the resulting velocities of the jets as much as 150 ft per sec. The water is usually supplied by centrifugal pumps, the pipe line from the pumphouse to the general location of the giants being from 12 in. to 15 in. in diameter to minimize friction losses. Smaller pipes are used near the giants where frequent handling

is necessary to keep the nozzles near the bank face as the work progresses. The jets are directed against the base of the bank and wash out the material. This causes caving of the bank which furnished loose material which is readily washed away by the jets.

The water and excavated soil which it carries are washed away to the site of the fill along a sluiceway which may be a natural channel or an artificial flume. The supply of water

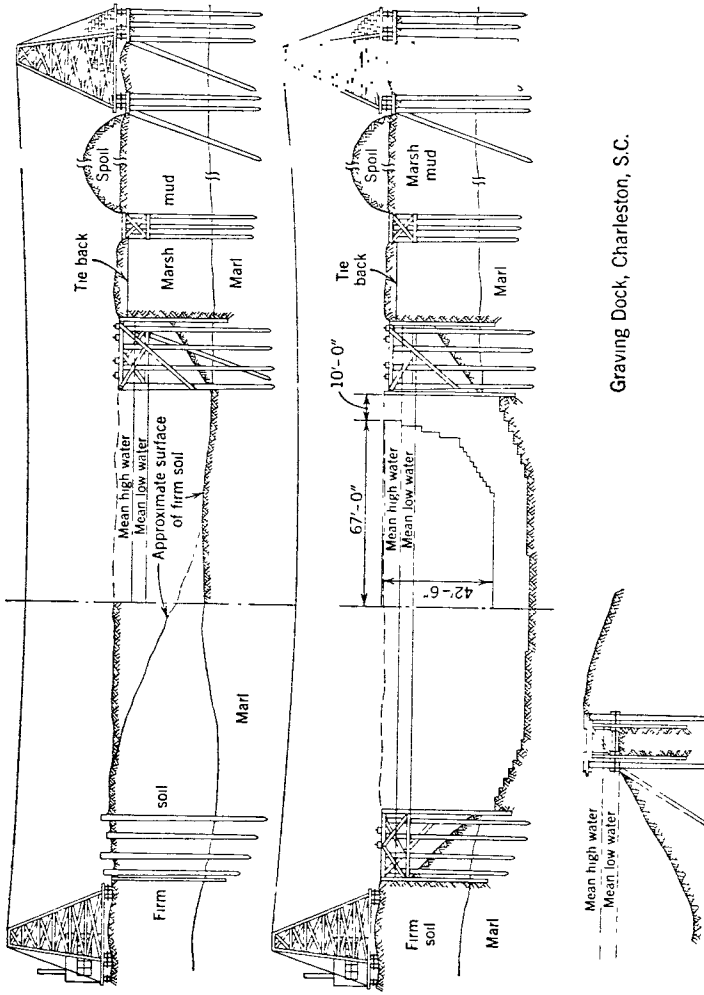

FIG 2 Cableway excavator At one time an excellent and economical method of making large excavations where the disposal area for the spoil was near at hand, the cableway has been relegated to an unimportant position by the development of the crawler shovel, scraper, loader, and large off-the-road earth-moving truck

from the jets is usually insufficient to prevent clogging of the sluiceway and is therefore supplemented by an additional supply furnished by lower-pressure pumps than those which furnish high-pressure water to the giants The sluiceway will usually have to be at a grade of about 4%, but should be kept as flat as possible in order that the giants may be kept at the lowest practicable elevation and the bank height kept high in the interests of getting the maximum amount of material from a single setup of the giants.

Gravel and sand are the easiest materials to excavate and transport by sluicing if there are not too many cobbles and small boulders in the bank. Clay does not sluice as well. Sometimes it requires blasting and often it will not break down under the action of the giants but passes along the sluiceway in large balls which do not make a dense and satisfactory fill.

Sluicing has been used extensively for the construction of earth-fill dams. The method has also been used for large-scale street-grading projects, see refs. 9 and 11.

7. THE EARTH-MOVING PROJECT

Planning an earth-moving project involves organizing a team, the members of which are of such sizes, characteristics, and capacities that each can work at a rate consistent with efficiency, and all working together can accomplish the assigned task at minimum cost within the allotted time. See ref. 16 for a detailed discussion.

Excavating equipment should be selected tentatively that can, first of all, do the work required within the given time. Comparative estimates should then be made to determine the probable relative efficiency of using:

1. Equipment already owned and available, supplemented, if necessary, by additional items that may be (*a*) rented, (*b*) purchased.
2. Rented equipment.
3. Any combination of the arrangments outlined above that promise increased efficiency.

Whether shovels, draglines, or clamshells (or a combination of several types of equipment) should be finally chosen for doing the excavating will depend upon the characteristics of the materials to be handled and the nature of the site. It is, however, essential that the equipment be capable of getting the work done on time and that estimates of the abilities of individual items to cope with difficult conditions and cramped working spaces not be overly optimistic.

Choosing the hauling units needed for transporting the earth from excavation to disposal point and deciding upon the number required demands consideration of a number of factors. The most carefully chosen excavating equipment works efficiently only if served by a sufficient number of haul units of adequate size. It is, however, usually possible to vary the number and size of the hauling units employed on comparatively short notice, and the selection of the hauling equipment is therefore secondary to the primary decision as to the excavating equipment to be used.

Table 6. Cycle Time of Power Shovels

Size of Shovel Dipper, yd	Easy Digging (moist loam or light sandy clay)				Medium Digging (good common earth)				Hard Digging (hard tough clay)			
	45°	90°	135°	180°	45°	90°	135°	180°	45°	90°	135°	180°
3/8	12	16	19	22	15	19	23	26	19	24	29	33
1/2	12	16	19	22	15	19	23	26	19	24	29	33
3/4	13	17	20	23	16	20	24	27	20	25	30	34
1	14	18	21	25	17	21	25	29	21	26	31	36
1 1/4	14	18	21	25	17	21	25	29	21	26	31	36
1 1/2	15	19	23	27	18	23	27	31	22	28	33	38
1 3/4	16	20	24	28	19	24	28	32	23	29	34	39
2	17	21	25	30	20	25	29	34	24	30	35	41
2 1/2	18	22	27	32	21	26	31	36	25	31	37	43

Conditions: (1) Time in seconds. (2) No delays. (3) Digging in optimum depths of cut. (4) Loading trucks on same grade as shovels.

Courtesy of Power Crane and Shovel Association.

Table 7. Cycle Time of Dragline Excavators

Size of Dragline Bucket, yd	Easy Digging (light moist clay or loam)				Sand or Gravel				Good Common Earth				Hard Tough Clay			
	45°	90°	135°	180°	45°	90°	135°	180°	45°	90°	135°	180°	45°	90°	135°	180°
3/8	16	19	22	25	17	20	24	27	20	24	28	31	...	...	..	. .
1/2	16	19	22	25	17	20	24	27	20	24	28	31		. .	. .	. .
3/4	17	20	24	27	18	22	26	29	21	26	30	33	25	30	35	39
1	19	22	26	29	20	24	28	31	23	28	32	35	27	32	37	41
1 1/4	19	23	27	30	20	25	29	32	23	28	33	36	27	32	38	42
1 1/2	21	25	29	32	22	27	31	34	25	30	35	38	29	34	40	44
1 3/4	22	26	30	33	23	28	32	35	26	31	36	39	30	35	41	45
2	23	27	31	35	24	29	33	37	27	32	37	41	31	37	42	47
2 1/2	25	29	34	38	26	31	36	40	29	34	40	44	33	39	45	50

Conditions: (1) Time in seconds. (2) No delays. (3) Digging in optimum depth of cut. (4) Loading trucks on same grade as dragline.

Courtesy of Power Crane and Shovel Association.

Haul units should in general have capacities 4 or 5 times those of the buckets or dippers of the excavators they serve. Units of smaller capacities make impossible demands on the skill and resourcefulness of those responsible for having a truck under the dipper at all times and waste time because they offer too small a target for the excavator operator when dumping the bucket or dipper. The length of time it will take to fill a truck should then be 4 or 5 times the duration of a complete cycle of operation of the excavator. Trucks having capacities that require a fraction of a dipper or bucket to complete their loading are undesirable as they either waste the time of the excavator or travel without a full load.

Determining the number of haul units required to keep the excavator working steadily involves estimating:

(*a*) The time required to fill a unit or the *loading time*.

(*b*) The time required by a unit to make a round trip from the excavator to the disposal area and return. This plus the *loading time* is a haul unit's *cycle time*.

The theoretical number of haul units required is the cycle time divided by the loading time. In practice, however, at least one extra haul unit is usually provided as the haul units are much more likely to be delayed by mishaps than are excavators, and it is wasteful to have an expensive excavator idle for lack of a less costly haul unit. Analysis by the method outlined above will show the great difficulty of operating a large excavator and big haul units economically when the haul is short and the consequent advantage of scrapers, if the material can be effectively excavated by them, on short-haul projects.

Estimates of the loading time for haul units of various capacities serving shovels and draglines can be made with the help of Tables 6 and 7. Estimates of the travel time of haul units are much more difficult to make. They should take cognizance of the practicable speed of the units over surfaces of the type to be traversed, the effect of grades on speeds, unloading and spotting times, and possible delays if the route to be followed traverses a public road or crosses a road or a railroad grade crossing

The effectiveness and efficiency of an earth-moving organization depends largely upon the skill with which the work of the various excavating and hauling units is coordinated. There is no substitute for experience in planning, organizing, and directing an earth-moving project. Mechanical equipment used in modern earth moving is so expensive that it must be used advantageously if economy is to be realized. Experienced and capable supervision is therefore a necessity.

8. FILLS AND EMBANKMENTS

Modern methods produce fills and embankments the settlements of which are at best so slight and at worst so predictable that at a determinable time after the completion of earthwork paving or other construction may be commenced without serious risk of trouble from unanticipated settlement. Naturally the techniques yielding these results differ from once widely used ones which produced fills that might settle for years if built of stable materials, settle for even longer periods if built of less favorable soils, and occasionally fail suddenly by cylindrical slides of cohesive material or liquefaction of a poorly compacted saturated non-cohesive soil. Modern techniques give consideration to the stability of the underlying soil or foundation that supports earthwork, to the stability of the embankment or fill itself, and to the stability of the underlying material and superimposed earthwork if both are cohesive and vulnerable to cylindrical slides. All of these stabilities may be enhanced by care and jeopardized by carelessness on the part of those directing earthwork. For example, it is essential in all filling and embankment building operations that no frozen material be used and that no material be placed on frozen surfaces.

The problem of placing fill on soft cohesive soil is discussed in Art. 8, Sect. 7, under Stability of Embankments, and in Art. 5, Sect 8, under Sand Piles and Sand Drains The measures advocated promote the early virtual completion of appreciable progressive consolidation and consequent settlement. They also prevent shear failure and resulting deep-seated cylindrical slides which may manifest themselves by the formation of mud waves. None of the measures advocated will, however, deal successfully with unreliable compressible materials such as extremely soft organic silts and peat. It is virtually impossible to stabilize such soils, particularly peats of various consistencies, and they must accordingly be removed and more stable materials substituted for them. This has been done successfully by method *a* described below and has been attempted, usually unsuccessfully, by methods *b* and *c*:

(*a*) Excavating or dredging out the soft material and in its place depositing a properly compacted stable soil (usually a sand, or sand and gravel).

(*b*) So placing new fill on the area that the soft underlying material is squeezed out as the filling progresses.

(*c*) Placing at least some of the new fill over the undesirable soft material, and then blowing the soft stuff out laterally by the simultaneous firing of a string of blast charges placed in the soft soil through holes driven through the partially completed fill.

Only the first of these three methods is trustworthy, Figs. 3*a* and *b*. It is usually accomplished with an excavating machine, generally a dragline, working from the top of the advancing fill if the soft stratum does not exceed 10 or 15 ft in thickness. Deeper deposits

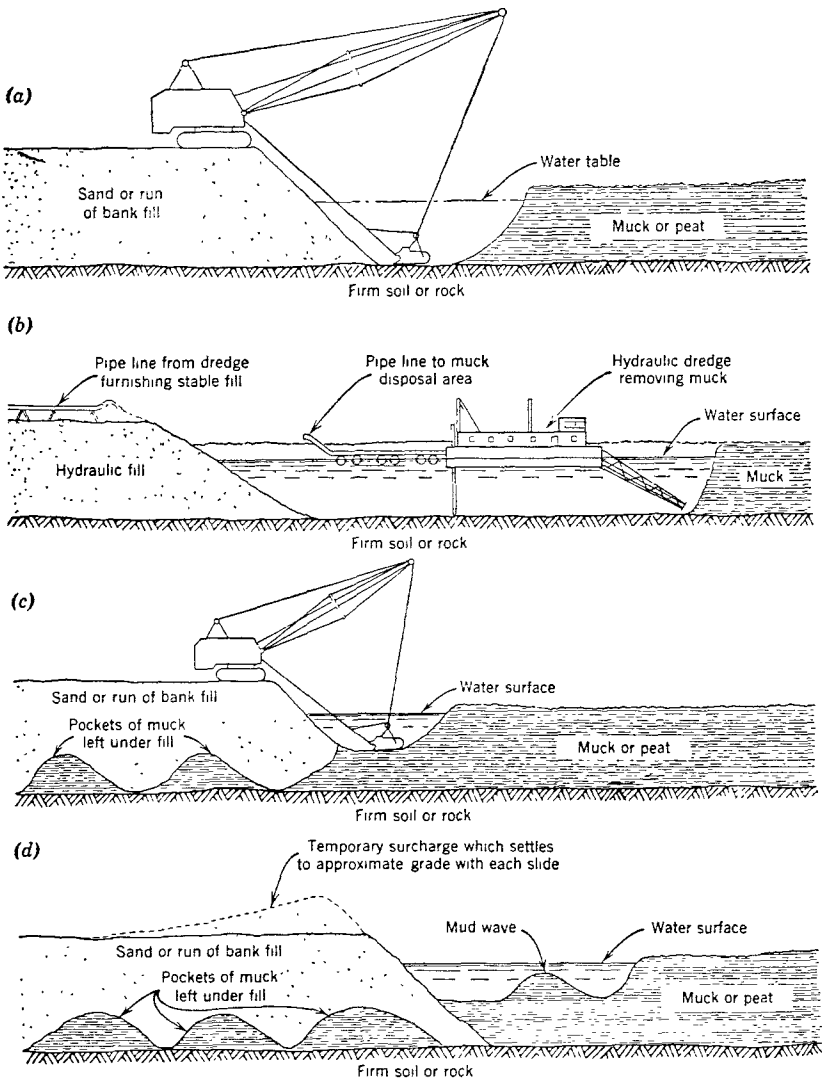

FIG. 3 Embankments in muck or peat The only reliable method of building a stable embankment for a road or railroad in muck or peat is to remove all the soft soil down to a stable stratum upon which the embankment is founded (*a*) Soft soil being removed by a dragline working from the end of the embankment built of stable material deposited by trucks (*b*) Both excavation and fill being made by hydraulic dredges, a method which has proved economical on large projects (*c*) and (*d*) Where excavating equipment capable of removing the full depth of soft soil is not available this method is sometimes attempted. Results are practically never satisfactory as pockets of soft material are invariably trapped under the fill The soft soil in these pockets consolidates under the weight of the fill and causes progressive differential settlements which may be troublesome for years.

require cranes with very long booms to reach beyond the toe of the slope of the fill on which the crane stands. Muck deposits as deep as 60 ft have, however, been dealt with in this way in Florida. Where it is possible to have the muck removed by a dredge, preferably a suction dredge, working ahead of the advancing fill, deep deposits may be removed much more readily than by cranes working from the fill. This practice is not unusual in Florida where the subsequently placed fill is frequently deposited by hydraulic dredging. The removal of soft material by a hydraulic dredge has in general been found more reliable than removal by any form of bucket excavator which may leave pockets of soft stuff which will later consolidate and cause differential settlements.

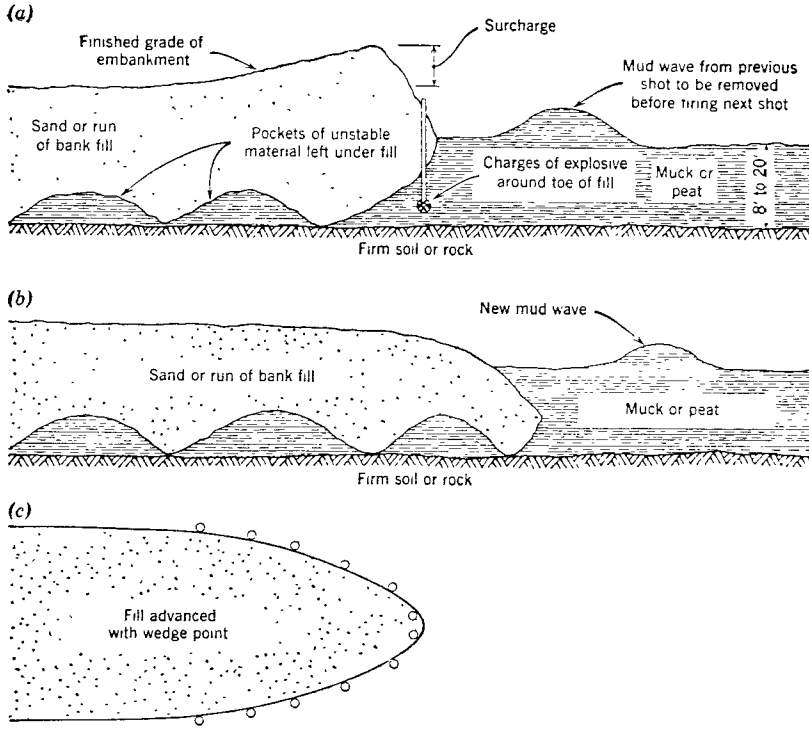

FIG 4 Displacement of soft soil by toe shooting This method can be successful with experienced direction (a) Explosives and earth surcharge before shooting (b) Result of shot (c) Plan showing arrangement of charges Pockets of mud are frequently trapped under the fill as shown but by close attention to detail this can be largely obviated by experienced supervision

Where the site is inaccessible to excavating equipment capable of removing the full depth of objectionable material, the method of displacement illustrated by Figs. 3c and d is sometimes attempted. The excavation ahead of the advancing fill is carried to the maximum feasible depth by means of a dragline. The fill, which should be a non-cohesive material, preferably a mixture of sand and gravel, is dumped without compaction to a height above the finished grade of the embankment and is then consolidated by jetting into it in such manner as to promote outward sliding and displacement of the soft peat or organic silt. This displacement is accompanied by the formation of mud waves in the soft material which are removed as the excavation is advanced ahead of the fill. The front of the advancing fill should be kept wedge shaped in order that lateral as well as forward displacement of the soft soil may promote its displacement by the firm granular material of the fill For further information on the method see ref. 10. Though the method has succeeded in isolated cases it usually leads to differential settlements and costly failures.

The displacement of soft soils ahead of a new embankment as the fill is advanced into a marsh and the displacement of soft soil from beneath a partially completed embankment also has been attempted by blasting. Two methods are used.

The first, known as toe shooting, is illustrated by Fig. 4. In toe shooting, charges placed in the soft material ahead of the wedge-shaped front of an advancing fill blow out soft material by the force of the blast and also promote its liquefaction by shock. This in turn facilitates the subsequent displacement of the soft soil by the heavier material of the embankment. The method should not be attempted where the thickness of the soft soil exceeds 20 ft. The weight and spacing of the charges should be determined by trial. Reference 3 advocates a 10-ft spacing of charges of 40% gelatine weighing 2 lb per foot of thickness of the soft soil at the first trial. Thus if the peat or marsh mud is 15 ft deep the individual charges initially tried should weigh 30 lb.

The second method, known as under-fill shooting, illustrated by Figs. 5*a*, *b*, and *c*, has been used where the soft stratum is considerably thicker. It works best under relatively

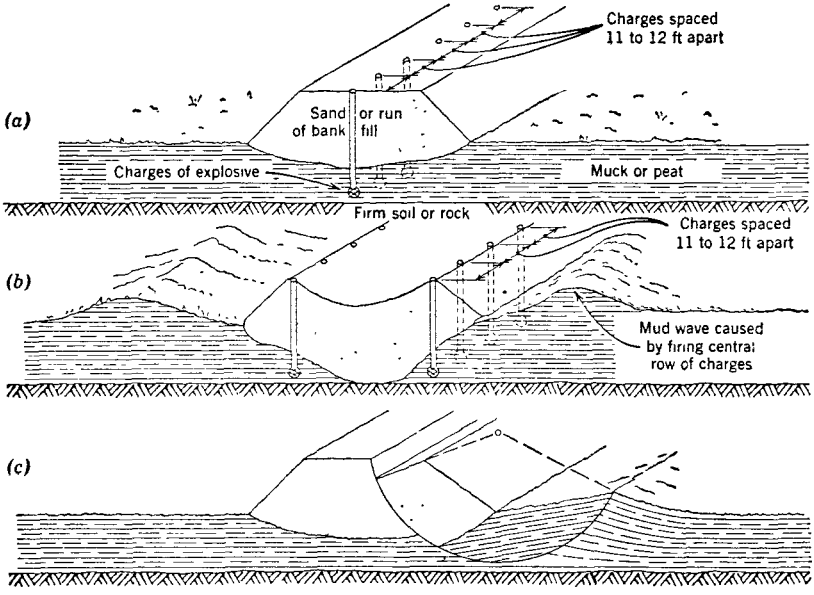

FIG. 5. Displacement of unstable soil by under-fill shooting (*a*) Placing of central row of charges (*b*) Result of firing central row of charges and positions of outer rows (*c*) Type of rotational slide which may develop during placing of fill before first row of charges is fired Estimating the limiting height of fill safe against the type of rotational slide shown in *c* demands determination of the shear strength of the muck Under-fill shooting therefore usually involves repeated lifts of fill and repetitions of the shooting process. Opinions differ as to whether it is better to fire the inner or the outer rows of charges first Like other methods of attempting the displacement of unstable soils under-fill shooting is unreliable

narrow embankments such as highway fills, the top width of which does not exceed 60 ft. Not over about 15 ft of soft stuff can be blown out from beneath the partially completed fill with a single blast, but advocates of the method claim success with deep beds of muck by as many as six repetitions of the blasting operation, each following the addition of fill to the embankment. With wide embankments three rows of charges, Fig. 5*b*, have proved more effective than a single row. Opinions differ as to whether the central or side rows should be fired first. The weights of the charges used at each shooting should be as heavy as possible without blowing holes vertically in the fill. For further information see refs. 2, 3, and 18.

Displacement and blasting methods are at best unreliable methods of dealing with soft material underlying fills. Both are questionable expedients which should be resorted to only when other methods are impossible. Pavements placed on newly built embankments constructed on marsh land by either method should be flexible types that tolerate considerable differential settlement without failing completely. Under no circumstances should concrete pavements be built on such fills until observation over a period of several years permits determination of the character and extent of further impending settlements.

The stability of a fill or embankment itself, apart from the stability of the underlying

material or foundation supporting it, depends upon both the character of the material and the care with which it is deposited and compacted.

The **methods used to compact fills** should be varied to suit the materials dealt with. In the United States soils that are purely non-cohesive are generally compacted either by rolling with pneumatic-tired rollers drawn by crawler tractors, by watering, or by shock loading if the soil is saturated or can be artificially saturated. Compacting by vibration is coming out of the experimental stage and may well become recognized as one of the most effective methods of compacting non-cohesive and slightly cohesive soils. The importance of control of the moisture content of non-cohesive soils that are not submerged during compaction depends upon the grain size and grading of the material. Control of the moisture content is unimportant and unnecessary when fills and embankments are constructed of coarse sands and well-graded mixtures of gravel and sand containing little or no fine material. Control of the moisture content is of great importance when embankments and fills are built of fine sands and silts. Partially and purely cohesive soils are best compacted by rolling with sheep's-foot rollers followed by pneumatic-tired rollers. Control of the moisture content during the compaction of such soils is of first importance.

Site Preparation. The site on which an embankment is to be built or a fill placed should be cleared and grubbed to remove vegetable matter. In areas which have been supporting vegetation the upper 12 in., more or less, will be topsoil containing organic matter. The topsoil should be removed and is usually stored for subsequent use in landscaping operations. On a side hill site steeper than a 4-to-1 slope, the area should be shaped with a blade grader into a series of horizontal benches to improve the bond between the existing soil and the material to be placed on it. The existing soil of the area should then be compacted for a depth of 12 in. before any material is placed on it. If the embankment is to be less than 10 ft high this compaction of the existing soil should be carried to 95% of maximum density—see Compaction Tests in Art. 5, Sect. 7, and California Bearing Ratio Field Test in Art. 6 of the same section. On the sites of higher embankments compaction to 90% of maximum density may be sufficient.

Coarse sand and well-graded mixtures of coarse sand and gravel containing little or no fine material compact readily. They should be dumped and spread by bulldozers into layers not over 8 in. thick, the layers being approximately parallel to the top of the embankment or fill. Each layer should be compacted by rolling with a pneumatic-tired roller consisting of a series of truck or automobile tires mounted on an axle and loaded to not less than 220 lb per lineal inch of tire width. Pneumatic-tired rollers should be hauled by crawler tractors, the vibration of which, no doubt, plays an important part in the compaction of non-cohesive soils. About eight to ten passes of the roller will usually be necessary to compact each layer of fill. The compaction of each layer should be continued until the specified percentage of maximum density is secured. This will, in the case of highway fills and embankments, usually be from 90 to 95% of the maximum, except that within 2 ft of the subgrade elevation it is good practice to require not less than 95% of maximum density. The important thing in building stable embankments and fills is to be sure that the required percentage of maximum density is attained irrespective of the weight of the roller used or the number of passes of the roller found necessary. Control of the moisture content is of little or no importance in compacting materials of this type.

Fine sands and silty soils should be spread in layers and compacted with pneumatic-tired rollers in the same general way as coarser non-cohesive materials, but in addition it is necessary with such soils to maintain a close control of the moisture content. The moisture content of fine-grained sand and silty soils should, in fact, be maintained within 2% of the optimum indicated by compaction tests. This usually demands sprinkling in arid regions or during dry weather in other parts of the country. It may also necessitate the suspension of work when heavy rains raise the moisture content of the soil above the acceptable upper limit. If the moisture content of fine-grained sands and silty soils is not effectively controlled, it is difficult, if not impossible, to compact them effectively enough to prevent the development of flows by sudden liquefaction if the soil becomes saturated. The compaction of each layer should be continued until the specified percentage of maximum density is secured. This will usually be from 90 to 95% of the maximum, except that within 2 ft of the subgrade elevation it is good practice to require not less than 95% of maximum density.

The **compaction of fine-grained sands and silty soils by watering** has been attempted as possibly preferable to compaction by rolling when heavy rainfall makes control of the moisture content impracticable. Two methods of applying water have been tried. In one, the sand is deposited by the haul units in ridges along both sides of the embankment and washed toward the center with jets approximating the characteristics of jets from fire hoses. In the second, water is ponded on the surface where material is being deposited and seeps down through the sand and escapes at the toe of the fill. Neither method has

proved reliable and trustworthy, and neither method is recommended as an acceptable alternative under any conditions to compacting by rolling or vibration.

The Compaction of Hydraulic Fills by Shock. Inundated hydraulic fills of cohesionless soils and some inundated natural deposits of similar materials may be compacted effectively by shock, produced by the firing of explosive charges placed in the fill, see ref. 7. The method is of growing importance but cannot be used unless the fill is inundated or at least completely saturated.

Vibration under certain conditions compacts soils more effectively than rolling. The conditions favoring compaction by vibration are a low percentage of fine and cohesive constituents and vibration at an effective frequency. Experiments in Germany with a compacting device consisting of a base plate or platform 80 ft^2 in area carrying a variable frequency vibrator, the whole affair weighing about 24 tons, indicated that with it layers of non-cohesive soils 7 ft thick could be effectively compacted at a rate of 5000 ft^2 per hr. More recently a variety of vibrating rollers and plate vibrators have been built in the United States. See refs. 1 and 18.

A majority of the vibrators consist of a more or less standard type of self-propelled power roller in which a variable frequency vibrator is fitted to one of the rollers or to a supplementary roller. A non-self-propelled type carries a power-driven vibrator in a roller which is towed by a crawler tractor.

The plate-type vibrators consist of plates, generally shovel- or scoop-shaped, carrying power-driven vibrators. One type carries a number of such units mounted "gang fashion" behind a self-propelled crawler the engine of which also operates the vibrators. Many of the smaller plate units are arranged to propel themselves slowly by a horizontal component of vibratory energy provided for the purpose of moving them.

Practically all types of vibrating equipment have been used with some success in compacting non-cohesive materials. Best results have been given by combinations of heavy vibrators and conventional rolling equipment. The smaller plate vibrators have proved very effective in compacting limited areas, such as backfilling cuts for pipe lines and placing backfill too close to structures to permit the effective use of large compacting equipment. Small units have proved capable of dealing effectively with partially cohesive soils.

Immersion-type vibrators of the kind used in placing concrete are coming into use for the effective compaction of inundated non-cohesive soils.

Cohesive and partially cohesive soils are most effectively compacted by rolling with a sheep's-foot roller at a moisture content within 2 or 2 1/2% of the optimum. The fill should be deposited and then spread by bulldozers or scrapers into layers from 6 to 8 in. thick. Each layer should be compacted by rolling with a sheep's-foot roller to a density of at least 90% and preferably 92% of the optimum except in the upper 2 ft of an embankment in which 95% of the maximum density should be achieved. Rollers should be capable of exerting a footprint pressure of from 60 to 300 lb per $in.^2$ under "feet" 7 $in.^2$ in the case of light rollers and from 200 to 600 lb per $in.^2$ under feet of the same size if classified as heavy rollers. It will usually take from six to ten passes of the roller to produce the required degree of compaction, but as in the case of non-cohesive soils the important thing is not the number of passes of the roller but the degree of compaction obtained.

There is no substitute for intelligent inspection by personnel experienced in soils work when the proper construction of earth structures must be insured. Frequent compaction tests must be made. A Proctor needle may be useful in making rapid approximate density determinations and an experienced man can tell much by the way in which the sheep's-foot roller "runs on its feet" when compaction is nearing completion. Proper control of the moisture content is of the utmost importance. In ordinary fills and embankments built of fine-grained non-cohesive materials or of cohesive materials it is usually satisfactory to maintain the moisture content within 2 or 2 1/2% above or below the optimum. In the case of embankments, dams, and other earth structures above about 75 ft in height, the soil should be kept on the low side of the optimum and pore-pressure measuring devices should be installed and used (see Pore Pressure Determination in Art. 3, Sect. 7, and Stability of Embankments in Art. 8, Sect. 7), to insure against flows or cylindrical slides.

9. BLASTING

Blasting is necessary when rock is to be excavated. It is also economical as a preliminary to the excavation of very firm soils such as shales, marls, and some very firm clays.

In quarrying operations where large quantities of rock are to be brought down from a quarry face 25–40 ft high, it is usual to place the charges in holes 6 in.–7 3/8 in. in diameter drilled with churn drills such as are used by well drillers. Holes are spaced about 20 ft apart and the same distance back from the quarry face. The holes should extend from

4 to 6 ft below the floor of the quarry, and each should be loaded with from 10 to 15 lb of 60% or 75% explosive per foot of hole for not more than about two-thirds of their total depths. The amount of explosive used will depend upon the type best suited to the work. Reference 3 should be consulted as to the types best suited to various kinds of work. For ordinary rock quarrying 60 or 75% dynamite or blasting gelatine is generally satisfactory. It is common practice to fire from 12 to 15 such holes together, but as many as 25 to 30 may occasionally be fired at the same time advantageously. The effectiveness of multiple blasts can often be increased and the objectionable shock they cause can at the same time be diminished by split-second time intervals between the firing of the charges. Millisecond delayed detonators are proving highly effective devices for accomplishing this.

For removing smaller quantities of rock in highway cuttings wagon drills are generally used for drilling the holes for the charges. Wagon drills will readily drill holes as much as 3 in. in diameter and 30 ft deep. Such a hole will take about 4 lb of explosive per foot of length and should not be loaded for more than about two-thirds of its total depth. Jackhammers drill holes up to 1 $^3/_8$ in. in diameter and 6 to 8 ft deep. These are too small to hold the large charges favored in modern blasting practice but are valuable in secondary cleanup work and breaking large blocks of stone that are too heavy to handle.

Explosives of many types are used in modern blasting practice. Characteristics which should govern the selection of an explosive for a particular purpose are: strength, velocity, or speed at which the detonation wave travels through a column of the explosive, water resistance, freezing resistance, inflammability, and, in some cases, the toxicity of the gases released by the explosion. Explosives should be tamped into holes with a wooden tamping pole and never with a metal rod. The effectiveness of even high velocity dynamites is greatly increased by tamping moist soil or other non-combustible material known as "stemming" into the hole on top of the charge. The length of the stemming should be not less than one-third the depth of the hole and half the length of the charge.

Electric blasting detonators are used almost universally for firing charges in construction-work blasting. A variety of detonators are available, some providing a predictable delay betwen the application of the current and the detonation of the charge.

For a complete treatment of the highly specialized and potentially dangerous procedures involved in blasting, see ref. 3.

10. CLASSIFICATION OF EXCAVATION AND EARTHWORK

The classification of excavation and earthwork into rock, loose rock, and earth causes many honest differences of opinion in the interpretation of specifications and contracts. The engineer should write specifications for such work as definitely as possible and use great care and consistency in interpreting them to prospective bidders and contractors. The following suggested definitions and devices are offered as minimizing misunderstandings and misinterpretations:

1. *Solid rock* includes all rock found in ledges or in detached masses exceeding 1 $^1/_2$ yd^3 and which may be best removed by blasting.

2. *Loose rock* includes all other rock and cemented gravel which may be removed without blasting, although blasting may occasionally be resorted to advantageously. It also includes detached stones less than 1 $^1/_2$ yd^3 and more than 1 ft^3 in volume.

3. *Earth* includes all materials of whatever name or character not unquestionably rock as defined above.

4. *Excavation in water* applies to the removal of all materials in water, or all materials thoroughly saturated or impregnated with water except surface water which can be drained away. It includes drainage, bailing, pumping, and all labor, equipment, and expenses connected with such excavation. In changing, deepening, or making water courses all materials above the water level and materials that can be removed without becoming saturated with water shall be classified as dry.

5. *Payment for excavation.* Excavations in all the several classes thereof shall be estimated and paid for by the yd^3 of material removed, measured in its original undisturbed state. Estimates should be based on cross sections, made by the engineer of the undisturbed surface before the work is started, of the uncovered surfaces as the work progresses (when different classes of material are uncovered) and of the completed surfaces.

6. *Payment for embankment.* Embankments shall not be estimated or paid for when made from excavations; but in special cases when made from material furnished by the owner to the contractor embankments shall be estimated and paid for by the yd^3 of material placed in the embankment measured in its finished state. Estimates shall be based on cross sections, made by the engineer, of the surface before the filling is started, and of the finished surfaces.

7. *Haul.* The prices paid for the several classes of excavation should be understood to cover the entire cost of removal and disposal at the location or locations specified, it being understood that no allowance will be made for over-haul.

DREDGING

11. GENERAL

Dredging is the removal of material from under water. It may be undertaken in order to increase the water depth, to procure material for fill, or to permit the extraction of a valuable constituent from the material excavated. The discussion that follows is limited to dredging undertaken for the purpose of deepening water or for procuring material for a fill Dredging undertaken to permit the extraction of a valuable constituent—such as gold or a valuable ore—from the material excavated is in reality a form of mining and is not dealt with herein.

Dredges may be broadly classified as hydraulic and bucket dredges. Each of these basic types is further subclassified into more or less highly specialized types described in some detail in succeeding paragraphs together with brief outlines of the capabilities and limitations of each specialized type. The type of dredge selected for a particular project will depend upon data secured from a preliminary investigation of the project and upon the availability of the type or types of dredge considered most suitable.

12. SURVEYS AND INVESTIGATIONS

Hydrographic surveys and investigations involved in planning and executing a dredging project usually comprise:

(*a*) A preliminary survey yielding approximate figures as to the quantity of material to be dredged and a sufficient subsurface investigation to indicate its probable characteristics. The preliminary investigation should also consider the location of disposal areas where the primary objective of the project is deepening water and the location of areas from which material may be most advantageously dredged where the primary objective is to obtain material for a fill. In both cases possible danger to existing water-front structures should be considered Such structures may be threatened by reduction of passive pressure in front of a bulkhead or quay wall, reduction of the restraining moment where a circular slide may be produced, or the exposure of untreated piles to damage and destruction by marine borers.

(*b*) A hydrographic survey more detailed, precise, and complete than the preliminary one is usual before the commencement of dredging in order that quantities may be computed with requisite precision as a basis for payment to a dredging contractor. Additional borings are also generally necessary in order that accurate estimates may be made of the quantities of various kinds of materials dredged.

(*c*) A final hydrographic survey is necessary after the completion of dredging to insure satisfactory compliance with specifications and, in conjunction with data yielded by the previous survey, to permit reasonably accurate computations and estimates of quantities dredged

(*d*) Methods used in hydrographic surveys are described in Sect. 2, Surveying. On small projects sounding is usually done with a hand lead, the location of each sounding being determined by a range and transit angle, two transit angles, or angles between three fixed objects. In the latter case the angles are measured simultaneously by two sextants operated on the sounding boat. The subsequent plotting is usually done by means of a three-armed protractor. On large projects sounding should preferably be done with a sonic depth finder such as a fathometer. Soundings may be located by the methods mentioned above. On large and important projects, however, it will usually be worth while to facilitate the rapid plotting of the soundings by preparing sextant charts as described in ref. 4 for the expeditious solution of the three-point problem.

(*e*) Quantities dredged may be computed by averaging end sections, by average depths of cuts at corners of square, triangular, or trapezoidal areas, or by taking contour areas with a planimeter if the job is primarily a dredging project undertaken to deepen a channel or harbor. If the primary objective of the project is to make a hydraulic fill, the quantities of material placed usually form the basis of payment. These quantities are determined by the usual methods for measuring earthwork.

(*f*) The surveying work involved in a dredging operation also involves setting up necessary ranges for the guidance of dredge operators

13. BASIC DREDGE TYPES

Dredges may be broadly classified as hydraulic and bucket dredges. Hydraulic dredges consist essentially of a centrifugal pump arranged to draw through a suction line a suspension of solid material in water and to deliver it to a disposal point through a pressure line. This pressure line in some types of dredge may be long enough to reach a fill ashore or an approved submerged disposal area. In other types of dredge it may be so short as merely to reach a bin in the hull of the dredge or a dump scow alongside. At the disposal point the dredged solids are allowed to settle out of the water in which they are suspended.

Bucket dredges are fundamentally excavating machines mounted on suitable floats or hulls. They may be of the dipper, dragline, or clamshell type. A fourth type, which is not extensively used in this country but has proved very satisfactory and popular abroad, is the self-propelled ladder dredge consisting of a number of buckets mounted on an endless chain carried by sprockets on the ends of a hinged frame called a ladder. All the bucket-type dredges have the common characteristics of being unable to deliver material any considerable distance to a disposal area without themselves moving to it. The material they dig is usually moved to the disposal point in dump scows. Some ladder dredges are fitted with bins in which they can transport material to a dumping area. Bucket dredges, except where a long-boom dragline or clamshell is deepening a canal or building a levee under conditions which place the disposal area within boom reach, must be served by dump scows which transport the excavated material to the disposal area.

14. HYDRAULIC DREDGES

The **cutter-head hydraulic dredge** is, in general, the type of dredge most widely used in the United States. Under favorable conditions it will excavate and transport material more cheaply than any other type of dredge or excavating machine.

The cutterhead dredge consists of a hull carrying a centrifugal pump which may be driven by a steam or diesel engine. In the case of large modern dredges turboelectric or diesel-electric drive may be used. Occasionally it may prove advantageous to operate a cutterhead hydraulic dredge with purchased power delivered through a high-tension submerged cable. The suction line from the pump runs down a ladder or frame hinged at the forward end of the hull. The ladder can be swung up and down in a vertical plane and at its lower end carries the intake of the pump suction line and a mechanically driven cutter or agitator. This stirs up the material to be dredged in such a way as to deliver to the pump a suspension of solids in water which can pass through the pump impeller or runner and through the shore line to the disposal points. Cutterhead suction dredges are not usually self-propelled and are moved by towing. They are generally fitted with two spuds at the stern. These are heavy vertical steel or timber columns which may be raised or lowered mechanically through wells provided in the hull and are further supported laterally by massive frame works. The spuds may be lowered into the harbor or river bottom and thus hold the dredge firmly in place if both spuds are down. If only one spud is down the dredge may be swung on it as a pivot.

Dredge pumps are special types of centrifugal pumps having passages through the impellers or runners large enough to pass solids of considerable size. A pump should be capable of maintaining a velocity of not less than 16 fps through the delivery line. Velocities in the delivery lines of as high as 22–26 fps are usual in many large dredges and velocities as high as 32 fps have been used. In general the higher the velocity, the higher the percentage of solids which may be delivered. High velocities, however, cause large friction losses in the line to the disposal point and consequently increase power costs For a discussion of the effect of pipe-line velocities on dredging costs see ref. 4.

The entire operation of a cutterhead hydraulic dredge is usually controlled from an elevated lever room from which the lever man has a practically unobstructed view of the ranges delineating the limits of the area to be dredged and of the activities of the dredge itself and its auxiliaries.

The floating portion of the line through which material is delivered to the disposal area consists of short sections of steel pipe carried on pontoons and coupled by flexible rubber connections. The part of the line on the shore is usually welded steel connected by simple slip joints. Booster pumps are occasionally used on long shore lines in which satisfactory velocities cannot be maintained by the dredge pump alone.

Cutterhead hydraulic dredges are usually rated by the diameter of the shore or delivery line. They vary in size from 6-in. dredges, the pumps of which may be driven by an engine or motor of 150 hp, to 30-in dredges which require as much as 6500 hp to drive the main pump

Disposal areas ashore are usually surrounded by dikes to permit a certain amount of ponding and settling out of solids. The amount of diking necessary depends upon the character of the material dredged. Coarse sands, gravels, and stiff clays, which would come out of the pipe in good-sized balls, do not require extensive dike. Fine sands and silts, however, demand long periods of retention behind well-constructed dikes if a considerable percentage of the material dredged is not to leave the disposal area with the runoff.

Plain suction dredges are very similar to cutterhead dredges but their ladders are not fitted with a cutter. In certain types of sands and silts they work very effectively and are used to considerable extent for maintenance dredging on the Mississippi. They do not, however, possess the general utility of the cutterhead dredge.

The **dustpan dredge** is a special form of hydraulic dredge employed on channel and harbor maintenance work to remove relatively loose deposits of mud, clay, silt, and sand. The dustpan dredge gets its name from the shape of the suction intake which resembles a huge dustpan or the intake nozzle of a vacuum cleaner. There is no cutter, but high pressure jets are provided to agitate the material to be dredged. Dustpan dredges are usually self-propelled and deliver spoil through relatively short floating pipe lines to submerged disposal areas clear of the channel. Occasionally deliveries may be made to the bins of dump scows towed alongside the dredge, but this practice is much more common abroad than in the United States. Dustpan dredges are used extensively by the Corps of Engineers on Mississippi maintenance dredging.

Seagoing hopper dredges are used for maintenance dredging of the channels to harbor entrances They are self-propelled seagoing vessels fitted with powerful dredge pumps which deliver spoil to bins in the hull of the dredge. The material is dredged by suspended suction pipes called drags equipped with scarifiers which loosen the bottom material and permit the pump suction to pick it up Hopper dredges transport spoil in their bins to a disposal area—usually at sea—where it is dumped through mechanically operated doors. Hopper dredges require no tender plant or auxiliary equipment such as dredge pipe and pontoons They are very economical and satisfactory but highly specialized pieces of equipment admirably suited for the purposes for which they are built Practically all, if not all, of the seagoing hopper dredges in the United States are owned by the government and operated by the Corps of Engineers in channel maintenance work.

15. DIPPER, BUCKET, AND LADDER DREDGES

The **dipper dredge** is the most versatile of the several types of bucket dredge used in the United States. It consists essentially of a large, specially built shovel excavating machine mounted on a suitable hull. The hulls of modern dipper dredges are usually steel and are provided with three spuds instead of the two with which suction dredges are commonly equipped. Two of the spuds of a dipper dredge are as close to the bow as possible without interfering with the swing of the boom and dipper stick. There is a single spud at the after end of the hull. The spuds are arranged to be not merely dropped, as in the case of a suction dredge, but to be hauled or pushed down mechanically so that a considerable portion of the weight of the dredge is carried by the spuds. When the spuds are thus forced down the dredge is said to be pinned up on them. This gives the dipper the necessary firm support to permit its effective operation and prevents objectionable rolling of the dredge when the boom is swinging over the dump scow to deposit each dipperful of material.

Dipper dredges vary in size from 1 1/4 to 8 cu yd bucket capacity. On the Atlantic Coast 6 cu yd appears to be a typical size, whereas on the Gulf, Pacific Coast, and Great Lakes most machines are of about 4 cu yd capacity On the Ohio and Mississippi bucket dredges of about 2 1/2 cu yd size appear to be most popular

The dipper dredge will handle almost any kind of material except fine, soft, alluvial silts which will run out of the bucket to such an extent that they cannot be effectively or economically dredged by machines of this type. The depth to which it can dredge is limited by the downward reach of the dipper and by practicable spudding depths.

Bucket-type dredges consist of long-boomed excavating machines mounted on suitable floats and equipped with either dragline, clamshell, or orange-peel buckets. The hulls of well-equipped bucket dredges are usually fitted with three spuds which may be hauled or pushed down mechanically to pin up the dredge in much the same way as the dipper dredge when this becomes desirable.

Dragline buckets have proved useful and effective in canal dredging and levee building. Clamshell and orange-peel buckets are more useful and effective in restricted areas—that is in working close to piers or wharves—and in excavating through the dredging wells of open caissons. The depth to which these dredges can dig is not restricted by their boom length.

The **ladder dredge,** consisting of a number of buckets mounted on an endless chain which operates on sprockets carried by a hinged frame or ladder lowered through a well in the hull of the dredge, is not extensively used in the United States except as a mining device for gold dredging. In this country it is not considered capable of working efficiently under widely varying conditions and therefore cannot be shifted readily from one job to another. If a ladder dredge is built for a job for which it is specially qualified, the volume of work should be such that the entire cost of the dredge can be charged to the project for which it is built.

16. SUBMARINE BLASTING

The dredging of rock and other firm materials often involves blasting. Small isolated amounts of soft rock, such as coral heads, can sometimes be dealt with economically by using heavy charges placed on the surface of the rock, see ref. 13, but, if the amount of explosive required is excessive and if any considerable quantity of rock is to be blasted, the charges should be placed in drill holes, ref. 3. On jobs of any magnitude these are drilled by specially equipped drill barges. These are fitted with spuds with which they can be pinned up and thus held immovable to prevent breakage of the drills. Power-driven drills are mounted along the side of the barge to facilitate the rapid simultaneous drilling of large numbers of holes of various requisite diameters. Drill holes are usually from 2 1/2 to 4 1/2 in. in diameter and, where well drills are used, 5 to 6 in. in diameter. Holes are spaced closer than in blasting on land, 10 ft center to center being a usual spacing. Holes should extend about 10 ft below finished grade of the submarine excavation. A high-percentage dynamite is desirable for submarine work, 60% being commonly used for ordinary rock, 70% or 80% for very hard rock, and 40% or 50% for stratified limestone, shale, or softer rocks. Water-resistant electric blasting caps should be used. In deep water it is good practice to use two caps per hole.

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SECTION 10

TUNNELS

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TUNNELS

1. FUNCTIONAL CHARACTERISTICS

General

Definition. Tunnels are artificial underground passageways, generally constructed for transportation purposes, including such uses as railroad, rapid transit, highway, pedestrian, sewerage, water supply, water power, public utility, and canal

Relative Economy. As compared with alternate types of construction, tunnels may: (1) provide greater convenience and elimination of certain hazards such as snow and icing conditions; (2) reduce total construction costs by reduction of overall length; (3) reduce operating costs and save time by reduction of gradients and distance; and (4) avoid interference with or adverse effect upon other necessary and desirable uses of the surface and air rights above the surface. Any one or combination of these considerations may enter into the determination of the economy of and necessity for constructing a tunnel, as compared to existing structures or possible alternate structures; moreover, in some cases, project requirements or topographical conditions may eliminate any practical alternative to tunnel construction.

Railroad Tunnels

Clearances. Standards used in the design of existing railroad tunnels have varied according to the requirements of individual railroads.

The American Railway Engineering Association recommends the following clearances:

1. The clearances on tangent track shall be not less than those shown.
2. On curved track the clearances shall be increased to allow for the overhang and the tilting of a car 85 ft long, 60 ft between centers of trucks, and 14 ft high.
3. The superelevation of the outer rail shall be in accordance with the recommended practice of the AREA.
4. The distance from top of rail to top of ties shall be taken as 8 in.
5. Legal requirements to govern when in conflict with dimensions shown.

Recommended clearances for third-rail and for pantograph, catenary construction, and adjacent permanent-way structures are available in diagrams published by the Association of American Railroads, Electrical Section, Engineering Division.

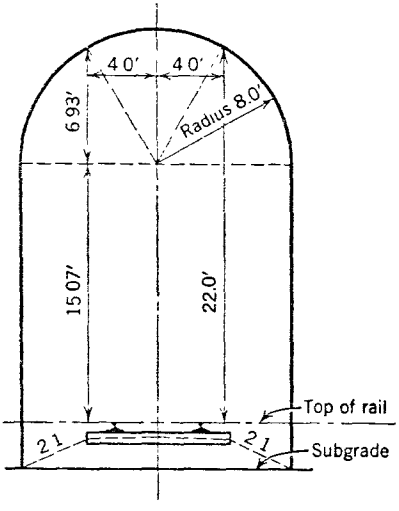
Clearances in shield-driven railroad tunnels or other railroad tunnels constructed by methods requiring a circular cross section would be modified, in the interest of practicability and economy, with respect to the clearance diagrams recommended by the AREA. One of the latest designs for a shield-driven railroad tunnel[1] adopted a section considered adequate for all types and sizes of freight cars in the New York metropolitan area, which provided 18 ft 6 in. from base of rail to crown of arch at center line, based on a 16 ft 6 in. maximum height of load above top of rail, 10 in. for overhead conductor, and 8 in. clearance between load and conductor. A clear minimum height of 16 ft was provided for at 6 ft either side of center line. See Fig 2

Alignment, Grades, and Ventilation. The AREA recommends the following considerations and standards for alignment, grades, and ventilation of railway tunnels.

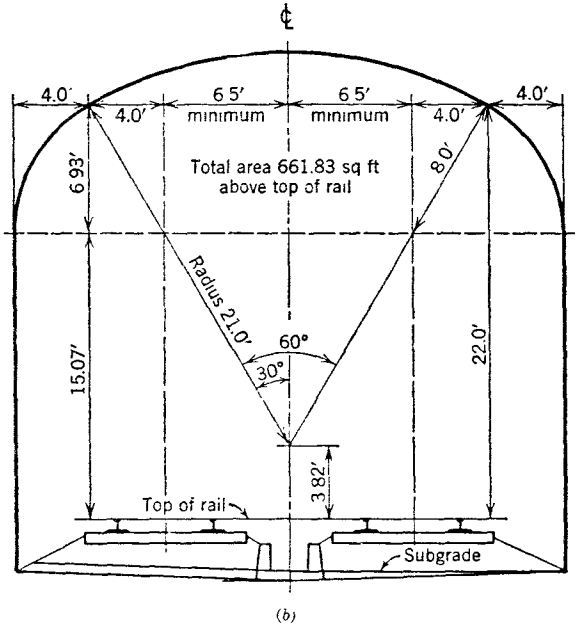
Alignment and Grades. In the location and design of new railroad tunnels consideration should always be given to securing the most advantageous and economical tunnel grade, taking into account as an important, but not necessarily controlling, factor, the increased capital costs of securing a reduced grade, as compared with the capitalized savings, through improved operation and maintenance, which may result therefrom.

Although no hard and fast rule as to the desirable reduction of grade through proposed tunnels and adjacent thereto is possible because of varying conditions of terrain, ventilation, drainage, and operation associated with every tunnel or sequence of tunnels, taking into consideration proper drainage, the rate of grade through proposed tunnels should be

[1] Narrows Tunnel, New York, designed 1921–1924, but not constructed.



(a)



(b)

FIG. 1. (a) Clearance diagram for single-track tunnel on tangent (b) Clearance diagram for double-track tunnel on tangent

kept as low as is economically practicable. As a working basis for comparative study of proposed grades through tunnels exceeding 1000 ft in length, such grade should not exceed 75% of the ruling grade of the district, and this reduced grade should be extended for the full length of a tonnage train, but not less than 3000 ft below the tunnel and one-half that distance above the tunnel.

Comparative studies should be made for greater and less reductions of the proposed grade, in order to ascertain the most suitable grade, within and adjacent to the proposed tunnel. It can be reasonably assumed that the proper percentage of grade reduction for a long tunnel may be greater than for a short tunnel.

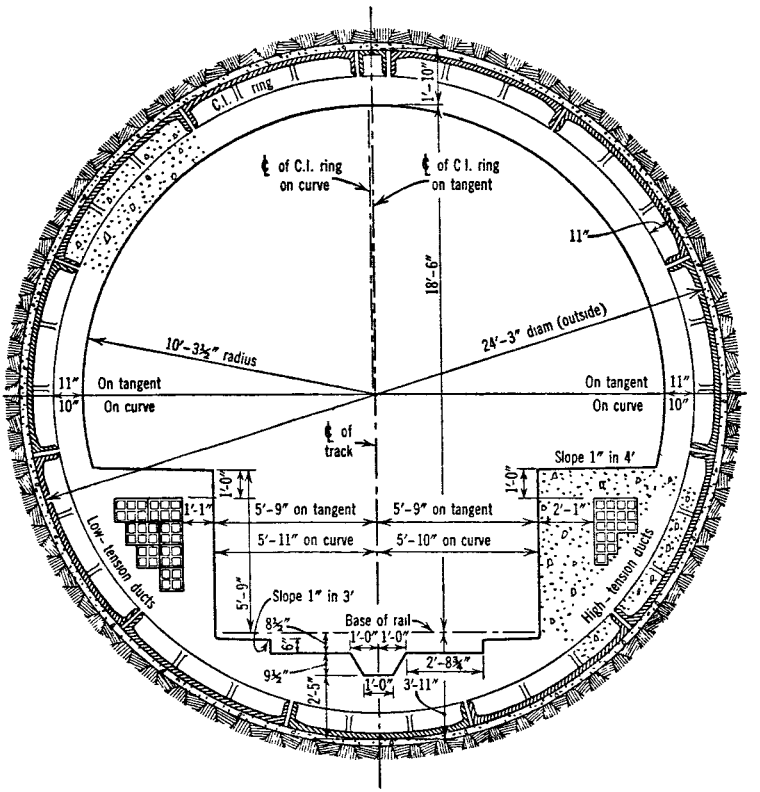


FIG. 2. Clearances designed for shield-driven railroad tunnel.

Summits within railway tunnels should be avoided, if possible, for efficient ventilation.

The alignment through proposed railroad tunnels should be restricted as to curvature as much as is economically practicable, and if curvature is required the regular compensation for such curvature should also be introduced.

Ventilation. Artificial ventilation for railroad tunnels 2500 ft or less in length is considered unnecessary.

For tunnels over 2500 ft in length, a study to determine the proper method of ventilation should be made, giving due consideration to operating and other local conditions.

Shaft ventilation is of limited benefit.

Where electricity is available, it is the most economical type of power for operating a mechanical ventilating plant.

The most practicable, effective, and economical artificial ventilation for tunnels carrying steam-power traffic is to be obtained by blowing a current of air into one end of the tunnel for the purpose of removing, or diluting and removing, the smoke and combustion

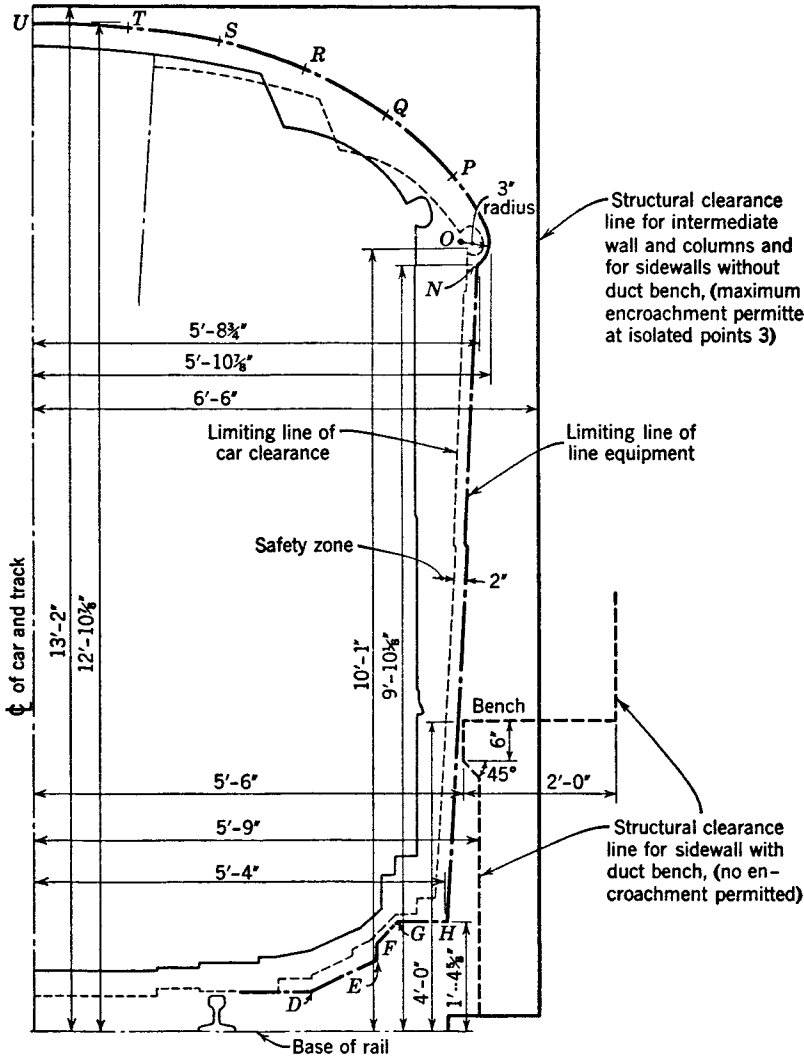


FIG. 3 Diagram showing normal clearance, Independent System, New York Subways.

	From ℄ of Car	From Base of Rail
D	3′ 6″	0′ 5 41/64″
E	4′ 5 1/4″	0′ 11 1/8″
F	4′ 5 1/4″	1′ 1 5/8″
G	4′ 8 1/4″	1′ 4 5/8″
H	5′ 4″	1′ 4 5/8″
N	5′ 8 3/4″	9′ 10 1/8″
O	5′ 7 7/8″	10′ 1″
P	5′ 3″	11′ 1 3/8″
Q	4′ 3″	11′ 11 5/8″
R	3′ 3″	12′ 5 3/4″
S	2′ 3″	12′ 8 7/8″
T	1′ 3″	12′ 10 1/4″
U	0′ 0″	12′ 10 7/8″

gases at the opposite end. As practiced in America, two methods are used to procure ventilation in this way:

(*a*) To blow a current of air in the direction the train is moving and with sufficient velocity to remove the smoke and combustion gases ahead of the engine.

(*b*) To blow a current of air against the direction of the train with velocity and volume sufficient to dilute the smoke and combustion gases to such an extent as not to be uncomfortable to the operating crews and to clear the tunnel entirely within the minimum time limit for following trains.

Rapid-Transit Tunnels and Subways

Clearances. There are no recommended national standards for clearances in rapid-transit tunnels and subways. Clearances are provided to meet the maximum requirements of the rolling stock of the system under consideration, including all of the variations from

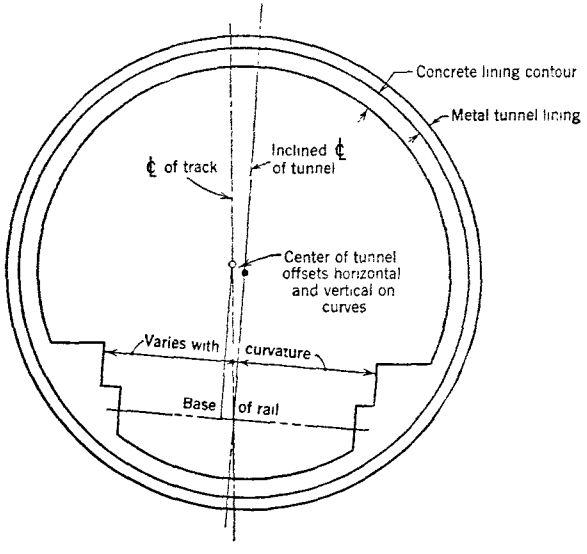


FIG. 4. Method of rotating tunnel section on curve to account for superelevation. Center line of track and center line of tunnel coincide on tangent.

normal midposition of the car which will ordinarily be found, with proper allowances for end and center excesses and excess due to superelevation when the alignment is curved. Allowances for construction deviations of the structure from theoretical lines and grades are also taken into consideration. In addition, it is customary to provide a neutral zone 2 in. wide between the limiting line of car clearance and the limiting line of structure clearance. Figure 3 shows a typical clearance diagram for the Independent System of the New York City Transit Authority. Economy in the design of shield-driven tunnels of circular cross section may be obtained by providing for superelevation as shown in Fig. 4, in which case the center line of the tunnel is adjusted by horizontal and vertical offsets from line and grade of track so that the required diameter of the tunnel is not affected by excess due to superelevation.

Alignment and Grades. Minimum radii of curvature and limiting grades are matters of the operating practice of the rapid-transit system under consideration. The minimum grade should not be less than 0.5%, unless special provision is made for proper drainage. The New York City Transit Authority has adopted a minimum radius of curvature of 350 ft with the requirement that transition curves must be used for all tracks having a radius of curvature of less than 2300 ft, excepting turnouts and crossovers. A maximum grade between stations of 3% has been adopted by this system, and a maximum grade of 1.5% at turnouts and crossovers. Vertical curves of 2-3% are recommended.

Ventilation. It is generally desirable that some form of ventilation be provided in subway systems. In land sections constructed in open cut, the Transit Authority renews the air at and between stations by means of the piston action of the moving trains, the subway being connected by means of flues to gratings, generally placed in the sidewalk at the street level. Air is exhausted through these openings as it is pushed ahead of trains and air is drawn into the subway behind passing trains. The flues and gratings are proportioned for an assumed velocity of the air of 100 fpm and assumed complete renewal of air once in every 15 min, the effective grating area being assumed to be 0.6 of the gross. In order to increase the efficiency of the system a continuous wall is constructed between the tracks of opposing traffic between stations. Ventilation gratings are also placed at the approaches of stations to reduce the draft on the station platforms and stairways caused by incoming trains. In addition to these provisions for ventilation, exhaust fans for emergency use are installed in fan chambers located at the center point of volume between stations. The under-river tunnels are ventilated by means of shafts on each shore. Fans for emergency use are installed at these shafts, in case smoke or fumes due to fire or other cause collect in the tunnel, so arranged with automatic dampers as to be operated either to blow fresh air into the tunnel or draw air out of the tunnel, the fans at the one shaft blowing air while the fans at the other are exhausting air.

Vehicular Tunnels

Historical. Tunnels for vehicular traffic are of comparatively recent date. The first such tunnels of any considerable length were built during the last decade of the nineteenth century, and all such tunnels built prior to about 1920 were designed for horsedrawn traffic without much consideration, if any, given to mechanical ventilation.

With the rapid increase in the number of automobiles there has been created an increased demand for vehicular tunnels for crossing of important waterways and mountain barriers. Since 1920, several major tunnels designed for automotive traffic have been built in the United States and Europe.

Size. Where the volume of traffic is high enough to warrant more than two lanes, it is considered good practice to build a pair of tunnels, each with a roadway of two lanes in one direction, rather than one large tunnel containing more than two lanes, except in cases where ground conditions are unusually favorable. This arrangement usually results in the greatest economy in cost of construction It is the safest type of road for operating high-speed vehicles as there can be no head-on collisions; it also facilitates ventilation. It readily permits the separation of the entrance and exit of the two tubes by a city block or two, which is a traffic advantage in cases other than those where express highways lead directly to the tunnel plazas.

Alignment and Grades. Most vehicular tunnels are located under navigable waterways and an important consideration is to keep the length of the tunnel as short as possible and at the same time use grades which are suitable for the type and volume of traffic. Experience has shown that with a heavy volume of mixed traffic, consisting of passenger cars and trucks, upgrades of 3 1/2% are suitable considering the requirements of the free flow of traffic and the construction costs. Downgrades of 4% are considered satisfactory. In cases where the volume of traffic is relatively low and is expected to continue low for a long period of years, steeper grades are sometimes used to keep the cost of construction to a minimum. Since the need for vehicular tunnels is mostly confined to large cities where property values are high and where the tunnel must be tied into the street and highway systems on the two sides of the waterway, it is ordinarily not possible to follow the generally accepted modern requirements for curvature that prevail on express highways in open country. Tunnels are better policed than open highways, and the maximum allowable speed in the tunnel is usually restricted to 35 to 40 mph. Under these conditions a radius of curvature from 1200 ft to 1500 ft is considered satisfactory and in order to avoid taking of excessively costly real estate, shorter radii are sometimes used.

On curves of short radii the roadway should be widened to provide for overhang of vehicles and superelevation provided corresponding to a speed of about 35 mph.

On shield-driven tunnels it is not practicable to widen the roadway on curves, and the radii must therefore be large enough not to require such widening.

Interior Clearances and Finish. In regions where the local limitations on the width and height of vehicles are 8 ft and 12 ft 6 in. respectively, a roadway width between curbs, for two lanes in the same direction, of 21 ft has proven satisfactory and ceiling heights of 13 ft 6 in to 14 ft are sufficient. In tunnels having a roadway with one lane of traffic in each direction the width between curbs should be not less than 22 ft There should be a berm of not less than 12 in. on each side between the curb and the tunnel wall, and there should be provided on one side of the roadway adjacent to the fast-traffic lane a service

walk for the use of the operating and maintenance personnel of a minimum width of 2 ft 6 in. and preferably 2 ft to 3 ft above the roadway level.

It is standard practice to line the interior of the tunnel walls, as well as the ceiling, with ceramic tile. This not only adds to the appearance and sanitary condition of the tunnel but also increases the degree of illumination per unit of power output.

Lighting. The most recent development in lighting of vehicular tunnels is the use of continuous fluorescent lights. This is economical in power consumption and entirely eliminates the flicker reflection on the polished bodies of automobiles.

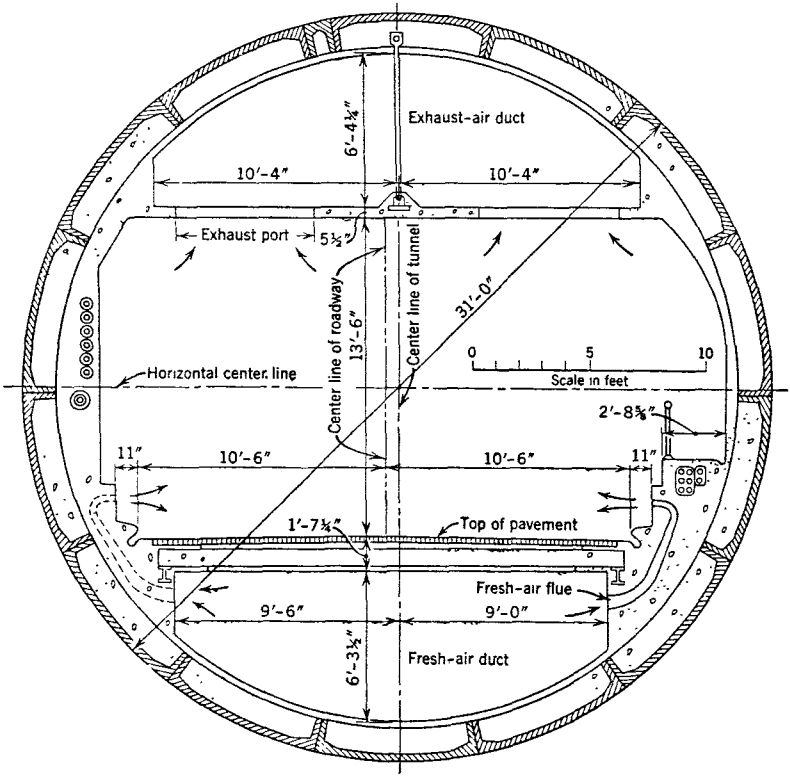


FIG. 5. Typical cast-iron-lined shield-driven vehicular tunnel section in earth, Queens Midtown Tunnel.

At the entrance portal of the tunnel, it is necessary to provide lighting of a greater intensity than the normal tunnel lighting to facilitate, during the daytime, the transition from bright sunlight to the lower-intensity tunnel lighting. This transition lighting has its greatest intensity at the portal with a gradual reduction over a distance of several hundred feet. On long tunnels this transition may extend as much as 1500 ft into the tunnel.

Pavement. To facilitate the repair or renewal of the tunnel pavement, it has been found that the use of a unit type of pavement is desirable. It can be easily repaired or replaced in units one lane at a time during the low-traffic hours at night without interference with the flow of traffic during the daytime. The pavement should have built-in distinctive center-line markers, and traffic should not be permitted to cross the center line.

Emergency Equipment. In a tunnel there is no place to get a stalled vehicle out of the way of moving traffic except by removing the vehicle from the tunnel. For this purpose emergency equipment is provided at each exit of the tunnel to remove a stalled or broken-down vehicle on short notice.

Toll Collection. If one or both ends of the tunnel have adjacent entrance and exit plazas, it is advantageous from the point of view of control, policing, and economy in personnel, to have the toll collection for both directions of traffic at one point. Experience

has shown that it requires three toll collector lanes to handle the traffic for each lane in the tunnel, at peak hour when the roadway is used to capacity.

The toll booths should be located on the driver's side of the vehicle so that the driver may pay his toll with a minimum of inconvenience and delay. The toll-booth islands should have, on the ends facing the oncoming traffic, substantial bumpers to protect the toll-booth structure and the toll collectors from accident by vehicles out of control.

Capacity. The capacity of a two-lane tunnel roadway may be considered as 2400 vehicles per hour, although there will be many peak hours where the volume of traffic will not reach this figure and there will be occasional hours when the traffic on a two-lane road may reach 3000 or more vehicles per hour under favorable conditions.

Ventilation. The first major tunnel designed for automotive traffic was the Holland Tunnel under the Hudson River at New York, which was constructed 1919–1927 jointly by the New York State Bridge and Tunnel Commission and the New Jersey Interstate Bridge and Tunnel Commission. The tunnel project consists of twin tubes with an average length of 8463 ft between portals and was to be used exclusively by vehicles propelled by internal-combustion engines. This presented a problem in ventilation both in character and magnitude which had no precedent on which the method of ventilation could be based, and required the development of original fundamental data through a research program which was under the general direction of the author who was in charge of the design of the tunnel and its ventilation system. Tests were made to determine (1) the amount and composition of exhaust gases from motor vehicles, (2) the physiological effects of exhaust gases from motor vehicles and the dilution necessary to render them harmless, and (3) the method of ventilation and power required for ventilating the tunnel.

Through agreements with the Tunnel Commissions the tests were carried out by the United States Bureau of Mines—in the case of item 2 with the cooperation of Yale University, and for item 3 with the cooperation of the University of Illinois. These tests established the fundamental bases for the following standards and formulas which are used in the design of ventilation for vehicular tunnels and which in addition reflect the operating experiences of the Holland Tunnel and the many subsequently constructed vehicular tunnels which use the same or a similar system of ventilation.

Degree of Ventilation. Based upon the results of tests to establish the amount and composition of exhaust gases from motor vehicles and to determine the physiological effects of exposures of varying periods of time to various concentrations of carbon monoxide in the tunnel atmosphere and with due regard to the experience gained in the operation of vehicular tunnels over a period of years, certain standards have been determined upon as appropriate for the ventilation of vehicular tunnels.

The results of more than 2000 tests on 101 vehicles of various types, makes, speeds, and conditions of loading are summarized in Table 1.

Table 1. Cubic Feet CO per Minute per Motor Vehicle

Speed, mph	Pleasure Cars Average Load and Light			Trucks Under 2-Ton Average Load and Light			Trucks 2 to 5 Tons Average Load and Light			Trucks 5 Tons and over Average Load and Light		
	Up	Down	Level	Up	Down	Level	Up	Down	Level	Up	Down	Level
10	1.40	0.648	1.02	1.44	0.688	1.12	2.82	1.34	2.20	3.03	1.50	2.53
15	1.99	0.962	1.44	1.84	0.926	1.41	2.93	1.58	1.99		0.621	1.90
Accelerating from rest	1.83		1.65	0.924		0.742	3.43		1.70			
Engine idling		0.734			0.685			0.870			0.842	
Engine racing		1.393			1.132			2.155			1.753	

The physiological research established that the effects of carbon monoxide are wholly due to the union of this gas with the hemoglobin. To whatever extent the hemoglobin is so combined, by that amount it is rendered incapable to transport oxygen to the organs and tissues of the body. The combination of carbon monoxide and hemoglobin is a reversible reaction, so that when a person returns to fresh air the carbon monoxide is gradually eliminated. Of all physical signs and tests of carbon monoxide poisoning, headache proved the most definite and reliable.

Concentrations of gas too weak or periods of exposure too short to induce this sign are to be considered harmless. No one had this symptom to an appreciable degree after a period of 1-hr exposure to 4 parts of carbon monoxide in 10,000 parts of air.

In setting the standards considered appropriate for vehicular tunnel ventilation, due regard has been given to the safety and comfort of the operating and maintenance personnel as well as to the traveling public; to the providing in the tunnel of an atmosphere free from smoke and haze which would be considered objectionable by the traveling public or might interfere with visibility or with safe driving.

Based upon all pertinent observations and data, a normal maximum of 2 1/2 parts carbon monoxide in 10,000 parts of air is deemed appropriate and has proven to be satisfactory in operation of important vehicular tunnels.

Based upon this standard of purity of the tunnel atmosphere and with capacity operation of mixed traffic consisting of about 75% passenger cars and about 25% trucks and buses, the quantities of fresh air required per minute per linear foot of tunnel roadway with two lanes of traffic in the same direction may be considered to be as follows:

On level grade	225 cu ft
On 3 1/2% upgrade	300 cu ft
On 3 1/2% downgrade	150 cu ft

These quantities are sufficient to allow for unusual traffic conditions somewhat above normal capacity traffic and for stalled traffic with vehicles crowding close together with engines running.

Transverse Distributive System of Ventilation. In the development of the system of ventilation for the Holland Tunnel, it was established that 3.760,000 cu ft of fresh air per minute would be required at capacity operation. It was concluded that such large quantities of air, with the attendant high velocities, could safely be supplied only by delivery through a special duct separated from the tunnel roadway, with frequent connections to the roadway space throughout the entire length of the tunnel. Similarly, it was concluded that the vitiated air should be withdrawn through a separate exhaust duct with frequent inlets from the tunnel roadway. The size of these openings is determined so as to supply and exhaust the requisite quantities of air appropriate to the grades of the tunnel roadway.

The fresh-air duct is located below the roadway and the exhaust duct overhead. This arrangement is dictated by the fact that the hot exhaust gases, slightly lighter than air, have a tendency to rise. A major consideration also in this determination was the fact that such an arrangement facilitates the quick removal of smoke and heat from the tunnel roadway in case of vehicle fires in the tunnel, thus enabling men with fire-fighting equipment to approach within effective distance of a fire.

The location of the air ducts may vary depending on the shape of the tunnel cross section and the method of construction; but the outlets from the fresh air duct should be near the roadway and the inlets to the exhaust duct near the top of the tunnel.

The system has functioned satisfactorily for more than 25 years' operation of the Holland Tunnel during which time more than 300,000,000 vehicles have used the tunnel. When, in 1949, a truck load of contraband cargo consisting of 4400 gals of carbon disulfide was ignited and burned in the tunnel no lives were lost and the ventilation system continued to function throughout the explosion and fire and the intensive heat which was generated.

The system is in use on nearly all major vehicular tunnel projects built subsequent to the Holland Tunnel.

Power Required for Ventilation. Based upon half-size and some smaller scale-model tests, the following formulas were developed to determine the power required to supply air to and exhaust air from the tunnel through a system of ducts such as described above.

For the supply duct

$$\text{T.P.} = \frac{12y}{D}\left\{\frac{w_1^2}{2g}\left[\frac{aLz^3}{3m} - \frac{1}{2}(1-k)z^2\right] + \frac{bLz}{2gm^3}\right\} + P_0 \qquad (1)$$

T.P. = total pressure inches of water.
y = density of air, D = density of water, in pounds per cubic foot.
w_1 = velocity entering duct in feet per second.
g = 32.2 feet per second, per second.
a = 0.0035, b = 0.01433.
L = total length of duct in feet.
x = distance from entrance to any chosen section.

$$z = \frac{L - x}{L}$$

m = hydraulic mean radius in feet.
k = 0.615, a factor allowing for turbulence.
P_0 = bulkhead pressure in inches of water.

Based on the exhaust-duct tests, a general equation for total suction head was developed as follows:

$$\text{S.P.} = \frac{12y}{D}\left\{\frac{w_1^2}{2g}\left[\frac{aL}{(3+c)m}z^3 + \frac{3}{2+c}z^2\right] + \frac{bL}{2gm^2(1+c)}z\right\} + P_0 \qquad (2)$$

All terms have the same significance as in eq. 1 except the new constant c, which takes into account the turbulence at the exhaust ports. S.P. is the suction head or negative static pressure. For a small model duct with 45° ports, the value $c = 0.25$ gives good agreement between experimental and calculated values. This value is also applicable to the actual tunnel exhaust duct for rates of flow above 200 cu ft per minute per foot of roadway. For lower rates $c = 0.20$ is suggested.

Tests were run for the coefficient of friction by blowing air all the way through with side ports closed. From the results of these tests a general expression for the coefficient of friction was obtained as follows:

$$f = a + \frac{b}{m^2w^2} \qquad (3)$$

where $a = 0.0035$, $b = 0.01433$, m = hydraulic mean radius, and w = velocity of air in feet per second.

These formulas are applicable to ducts with constant area and uniform rate of air supply or exhaust. If on a tunnel duct there occur changes in either the area or the rate of air supply or exhaust the computation has to be made for an appropriate number of subdivisions of the duct and the pressure losses in these subdivisions must be added together.

In the blower-duct system the incidental power losses in addition to those in the main duct usually are the following.

1. Losses through louvers when air enters fan room.
2. Losses in downcast ducts.
3. Losses in bends and elbows.
4. Losses in flues leading from main supply duct to tunnel roadway.

Item 1 with properly designed louvers would ordinarily amount to 0.20 in. of water.

Pressure losses in straight runs of ducts may be determined by the formula:

$$H = \frac{12y}{D} \times \frac{w_1^2}{2g} \times \frac{fL}{m} \qquad (4)$$

where H = loss in pressure in inches of water and the other terms have the same significance as those used in eqs. 1 and 3.

The losses encountered in flow of air in elbows and in ducts with varying cross sections and shapes such as nozzles converging or diverging, as well as the effects of abrupt expansion and contraction, have been determined experimentally by various authorities and organizations and are given in textbooks and fan manufacturers' handbooks.

The minimum static pressure required to discharge the air from the main duct to and across the tunnel roadway depends on the size, shape, and length of the flue, but does not ordinarily exceed 0.25 in. of water.

An examination of eq. 4 for pressure loss in the blowing case showed that the point of minimum static pressure did not occur at the bulkhead end of the duct. The location of this point varies with the dimensions of the duct.

Tests were run on various types of exhaust ports, and the design which gave the least disturbance to air flow in the exhaust duct by the entering air was found to be one making an angle of 45° with the line of flow in the exhaust duct. The pressure required to exhaust air through this type of port for various quantities of air was also obtained from these tests.

In the exhaust-duct system the total pressure losses in addition to those in the main duct consist of the following:

1. Losses at exhaust airports leading to the main duct.
2. Losses in ducts leading from main tunnel duct to fan chamber.
3. Losses in bends and elbows.
4. Losses in fan chamber.
5. Losses or recoveries in discharge stacks.

The loss in the exhaust air port, having an angle of 45° with the direction of air flow in the main duct, is included in the equation for the losses in the main duct and is equal to P_0. P_0 is generally not less than 0.03 in. of water.

The losses in the exhaust-fan chamber may amount to as much as the entire velocity pressure.

The evasé or expanding stacks have the effect of reducing the power loss due to high-velocity pressure as the air is expelled to the outside atmosphere.

After having calculated the losses in pressure in a duct from the fan to the farthest end of the duct, the air horsepower is calculated using the formula

$$\text{hp} = \frac{P \times Q \times 62.4}{33{,}000 \times 12} \tag{5}$$

where hp = horsepower, P = pressure in inches of water, Q = total cubic feet of air flowing per minute.

As the power required for a given duct section varies approximately as the cube of the length of the tunnel section, it is important that the length of a duct section is not too great in relation to its cross-sectional area. An economic evaluation must be made giving consideration to the cost of providing a larger cross-section of the duct or intermediate ventilation stations and the annual power cost.

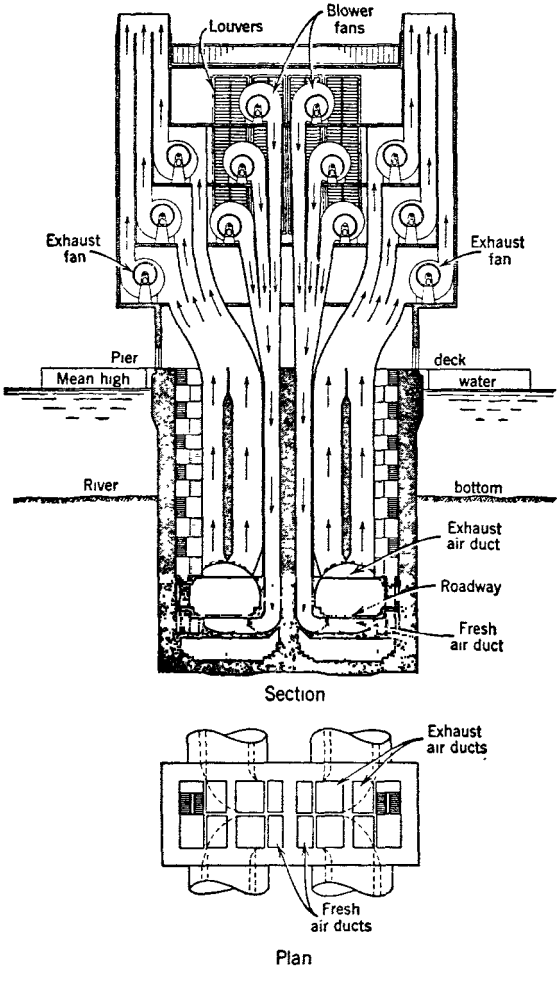


FIG 6 Method of ventilation Building and shaft, Holland Tunnel.

Type and Number of Fans. Horizontal centrifugal fans have been used to provide ventilation for all the major vehicular tunnels in the United States. These fans are of the backward curved blade type suitable for parallel operation and have a non-overloading characteristic.

In the last 15 years axial-flow fans were developed with characteristics similar to those of the backward curved blade horizontal centrifugal fans. This type of fan was first developed and applied to tunnels in Europe and is now being commercially produced in the United States.

Efficiencies of the two types of fans are comparable. Space requirements and limitations in some cases favor the use of axial-flow fans.

Since the traffic varies greatly over the 24-hr period of a day and with the seasons, it is necessary to provide a sufficient number of units with suitable speed variations to provide the proper air requirement for the varying traffic at a minimum expenditure of power. Conservative planning calls for the provision of spare units to allow for shutdowns for repairs of equipment or of the power supply. To guard against interruption in the ventilation and lighting it is necessary to provide for more than one source of power supply.

Ventilation Buildings. Ventilation buildings are usually located near the portals of the tunnel or at intermediate shafts or both as determined by the overall economy of the arrangement in each case.

The general arrangement of the ventilation buildings is to locate the blower fans on floors with large openings to the outside atmosphere with louvers in the openings to protect against rain and snow. The exhaust fans are located in airtight chambers with expanding exhaust stacks to the outside atmosphere.

The floor heights of the buildings must provide for the replacement of the largest equipment units. Space must be provided for the air ducts without the necessity of abrupt bends or elbows and must make adequate provision for electrical and auxiliary equipment.

The ventilation equipment and the lighting of the tunnel are controlled from a central control station which is located either in one of the ventilation buildings or in a separate service building (constructed for that purpose).

Longitudinal System of Ventilation. On shorter tunnels a longitudinal system of ventilation is sometimes used. This method is adaptable to a horseshoe-shaped cross section with flat invert on which the roadway is placed.

The roadway space extends from the roadway to the crown of the arch, providing not only a larger roadway area but also the appearance of roominess and with proper finish and illumination giving the tunnel an attractive appearance.

The air may be supplied by blower fans near the entrance portal, the air being supplied to the roadway at high velocity through one or more nozzles. It then travels longitudinally with the traffic to the exit portal.

One twin-tube vehicular tunnel more than 1 mile long, the Liberty Tunnel at Pittsburgh, Pa., has one ventilation building located on top of the hill through which the tunnel is built at about the midpoint. Here exhaust fans withdraw air entering at the entrance portal. A short distance beyond the shaft, fresh air through nozzles is supplied; this air travels along the roadway with the traffic. Tests have shown that here a very large percentage of the air entering at the entrance portal bypasses the central shaft and continues to the exit portal.

Combinations of Transverse and Longitudinal Systems of Ventilation. Some vehicular tunnels have been built where a combination of the two systems have been applied. The longest of these is the Queensway Tunnel under the Mersey River at Liverpool, England. Here the fresh air is supplied through separate ducts as in the transverse distributive system: the air then travels longitudinally through the roadway section and is exhausted by suction fans located in buildings over four shafts on the main tunnel and one shaft on each of two branches.

Some tunnels have a separate duct through which air is supplied and admitted to the roadway at one point near the middle of the tunnel and finds its way out at each portal.

Ventilation by Natural Draft. In short tunnels it may not be necessary to provide mechanical ventilation, the natural draft being depended upon to renew the air in the tunnel. This is usually satisfactory for tunnels up to several hundred feet long if located on uniform grades and on a straight alignment. In tunnels dipping under an obstruction with changing grades and curved alignment much less natural draft may be expected. It is not possible to establish hard and fast rules for how long a tunnel may be before it requires mechanical ventilation but it is certain that a tunnel with traffic in one direction only may depend on natural draft for a greater length than a tunnel with traffic in both directions on the same roadway.

2. PRELIMINARY INVESTIGATIONS

Surface and Subsurface Surveys. Surface and subsurface surveys should be made, accurately relating to the tunnel plan all topographical features and surface and subsurface structures that would be affected by tunneling operations and that would affect the design of the tunnel. Suitable sounding methods should be employed to establish the relation of the proposed tunnel to the surface and bottom of any body of water under which it is to be built. Supersonic methods of sounding are well adapted to preliminary investigations, especially when an extensive subaqueous area is to be studied.

Geologic Investigation. A geologic section should be prepared along the tunnel line which will show the tunnel profile, depths of cover, ground-water table, and all the kinds and conditions of materials to be penetrated by and surrounding the tunnel bore. Data obtained from the records of geologic surveys, existing tunnels, shafts, mines, excavations, wells, and borings which have been made in the vicinity of the general route of a proposed tunnel, if properly interpreted, may be sufficiently complete and conclusive to form a basis for selection of alignment and preliminary design, but it is usually necessary, in addition, to include exploratory borings as part of the program of geologic investigation, especially after the final determination of the tunnel alignment has been made, for use in

designing the permanent tunnel lining, planning construction methods, and as information for prospective contractors

Exploratory borings should be spaced and located so as to provide data for the preparation of a reasonably accurate and detailed development of the portions of the rock floor and the materials of the overburden that affect the design of the tunnel. Rock gorges, faults, and crush zones, and old stream beds, valleys, and canyons, should be thoroughly explored. It is well to stagger the borings in plan, so as to permit a three-dimensional development of the boundary surfaces of the materials encountered. Where compressed air is to be employed in tunneling operations to keep the excavation dry, it is advisable to locate all borings clear of structure net lines so as to avoid leaving openings through which compressed air might escape from the tunnel. Where rock elevations are a pertinent factor in planning and designing a tunnel, borings should be carried by diamond drilling for at least 10 ft into the rock to establish that the material is bedrock and not a boulder. Diamond drilling should be carried into and below the zone of the face of the proposed tunnel at appropriate intervals.

Samples of the overburden and rock penetrated by the borings should be taken and carefully preserved for inspection, test, and study. Ordinary dry samples of the overburden should be taken so as to show the nature of all of the different strata of overlying and surrounding material especially at shaft sites and other places where excavation is to be made from the surface. Where the tunnel is to be constructed in plastic soils, some samples of the material should be taken by undisturbed-sample methods and the usual laboratory tests made to establish the properties of the soil pertinent to the design of the tunnel.

The elevation of ground water and information as to seasonal fluctuations in groundwater elevations should be obtained from the borings and other available sources. Artesian conditions should be thoroughly investigated.

Methods should be employed in the subsurface exploration to detect any evidence of explosive, toxic, or other gases which create a hazard in tunneling.

The possibility of geophysical methods of exploration, under some conditions, in connection with tunnel planning might well be considered, although it is not suggested that geophysical methods of exploration could be substituted for test borings. The exploration of underground conditions by geophysical methods has long had a recognized field in mining work, and as long ago as 1929 a geophysical method based on electrical resistance for determining the depth of rock was adapted to subsoil studies in foundation work. Since 1936, the geophysical activities of the U. S. Government have been carried on by the U. S. Geological Survey with a broadened scope to include other than mining problems.

The importance of a thorough geologic investigation in connection with planning and designing a tunnel project cannot be overemphasized. Savings made in the cost of this work by curtailing its scope may result in improper design, delays, and unforeseen difficulties in construction which would have been avoided by a comparatively small additional investment in the exploration and study of ground conditions.

The geologic data obtained in connection with a tunnel project should be analyzed by a geologist qualified by training and experience to interpret geological conditions in terms which will be valuable from the standpoint of application to the engineering problems of planning, design, and construction.

If conditions are such that it is impractical or impossible to obtain adequate geological information by the methods discussed in the foregoing articles, where there is a probability that the materials and formations which will be encountered are difficult and variable, and where the tunnel bore is large, it may be advisable to drive an exploratory pilot tunnel in advance of the full tunnel cross section.

3. TUNNELING METHODS IN ROCK

Excavation. Tunnels are driven in rock by repeating in sequence the operations of drilling holes in the rock face, loading the holes with explosive, and blasting, removing, and disposing of the broken rock. The full cross section of the tunnel may be excavated in each sequence, or a smaller drift or drifts may be excavated in advance of the full-size tunnel section, according to the working program adopted

The working program and the type of equipment used in driving a tunnel in rock usually depend on the size and length of the bore and the character of the rock. However, other factors, such as the kind of equipment available, the cost of labor, and the time required for completion may at times have some influence on the selection of the method to be used.

The usual practice is to drill all the holes for the full cross section of the tunnel or of the drift, as the case may be. The number, location, direction, and depth of holes are chosen with due regard to the character of the rock and the size and shape of the bore so as to

maintain suitable progress, break down the rock with the least amount of explosive, break the rock into pieces of a size that can be efficiently handled by the equipment available, keep overbreakage to a minimum, and at the same time avoid, to the greatest extent practicable, the necessity for subsequent costly trimming to net lines. Figure 7 shows patterns of drill holes used on several tunnels.

In American practice in accordance with the pattern adopted, holes for a conical or wedge-shaped cut are generally drilled in the central portion of the face Around the cut holes are drilled the side holes, called relievers, and line or rim holes. All holes are loaded and fired at one time with suitable delay exploders resulting in a rapid succession of blasts. The cut holes which are the first in the series of blasts, break a relieving cut so that fewer side holes and less explosive are required than would otherwise be necessary. Before delay exploders were used, considerable loss of time was occasioned by the necessity of two separate blasting operations, one for the cut holes and the second for the side holes. The plan of cut holes originated on the Hoosac Tunnel work, prior to which time drill holes were placed normal to the face

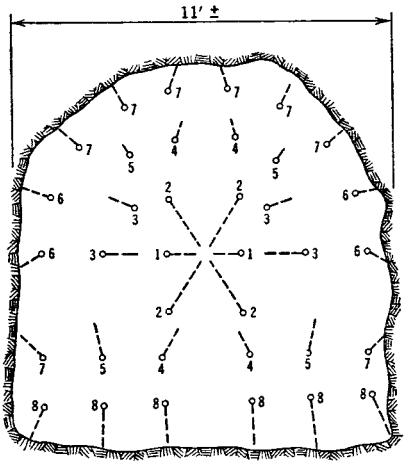
The great improvement in rock tunneling in the last two or three decades comes from the improvement in drills and drill steel, the use of the portable drill carriage or "jumbo" for the drilling operation, the use of delay exploders in blasting, and the development of economical and reliable mechanical muckers. These improvements have made possible new methods and greater flexibility in adjusting methods to the characteristics of a particular job, reflected generally in faster progress and saving in manpower.

Up to about 1925 tunneling methods employing a heading were customarily used. In these methods, a heading, generally with a cross-sectional area of 40–50 sq ft, which is the minimum size for economical driving, is excavated first This heading is subsequently enlarged to the required size by methods which are relatively less expensive than those necessary for the heading excavation The heading may be long or as short as 10 ft, and its placement, whether at the top, center, or bottom of the full-size cross section, depends on conditions of the particular job. The heading, when used, is usually located at the top of the full section, and the drills for this work, when a drill carriage is not employed, are mounted on columns or bars which have to be removed before the blast. The remainder of the section, the bench, can then be excavated by drilling vertical holes. The top heading and bench method is illustrated in Fig. 8. This method has the advantage of convenience in scaling and placing timbering or other support for the roof, when necessary, and the disadvantage of the interference of the bench work in handling materials going to and from the heading. It may still be economical to employ this method in very short tunnels.

For greater speed, the bottom heading has sometimes been used and the enlargement started at frequent intervals along the bottom heading, permitting the full-size work to be completed nearly as soon as the heading. The disadvantage of this method is that the bottom heading has to be completely timbered to protect traffic through the intermediate working faces.

The separate pioneer heading method has been used to advantage for long tunnels which can be attacked from the ends only. This method consists in driving a heading about 8 ft by 8 ft parallel to but 50 ft or more from the main tunnel. At intervals of from 1500 to 2000 ft, cross drifts are driven to the main tunnel, which is there attacked by whatever method is best suited to existing conditions. Where the rock is sound and requires no timbering a favored method is to drive a center heading which can proceed as fast as the pioneer heading, for which progress exceeding 1000 ft per month has been made on some tunnel work. From this center heading radial holes at right angles to the axis of the tunnel are then drilled to the limits of the full-size tunnel and blasted. The center heading between cross drifts is open at both ends and therefore the drilling operations can be carried on from one end while the mucking goes on at the other without interference. The pioneer tunnel was used for the Rogers Pass, Moffat, and New Cascade tunnels, 5, 6, and 7 $^3/_4$ miles long respectively. The use of ring drilling as just described is not confined to tunneling employing a separate pioneer heading but has been used with success on such large rock tunnels as the 32 ft 10 in. diameter Manhattan land tunnels for the Queens Midtown Tunnel project, 1936-1940.

Speed is usually desired in heading work, and much thought and ingenuity have been employed to lessen the loss of time caused by interference of the muck pile after a blast with drilling. A number of drill carriages have been developed for this purpose differing in detail but each equipped with a long arm or boom to carry drills, drill columns, and air and water hose at its outer end. As soon after the blast as practicable, the carriage is moved forward and passes the drilling equipment over the muck pile and drilling is started on the portion of face exposed. The gain is the difference between the time the carriage takes and the longer time required for men to handle the heavy columns and drills over the muck pile.



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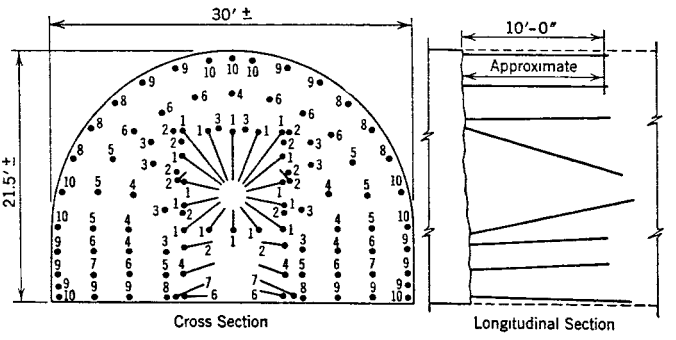

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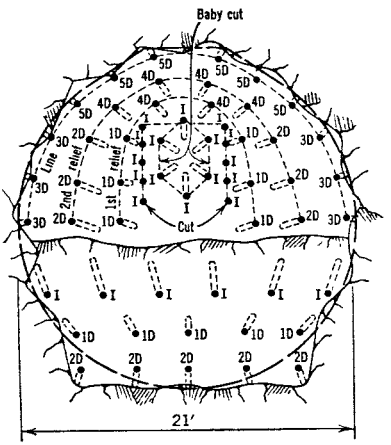

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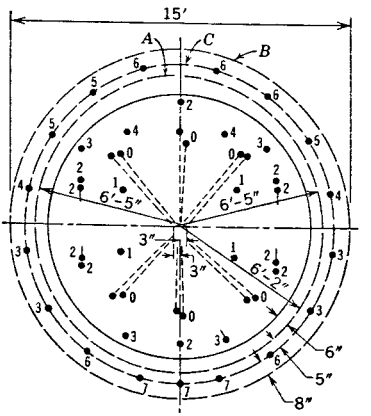

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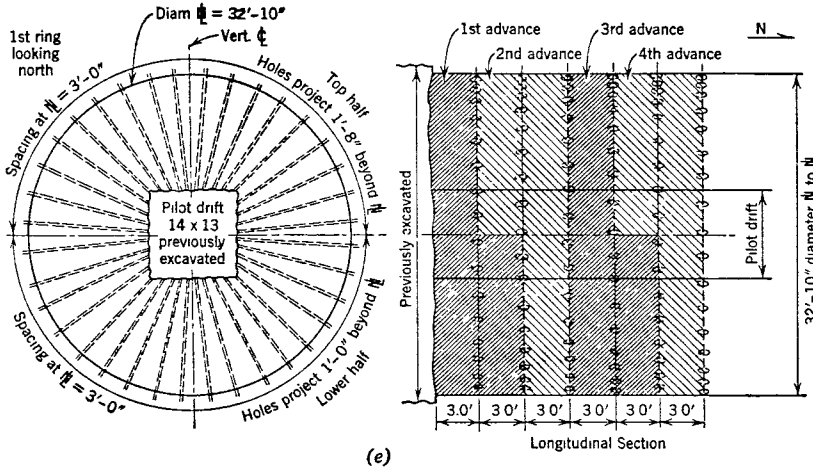

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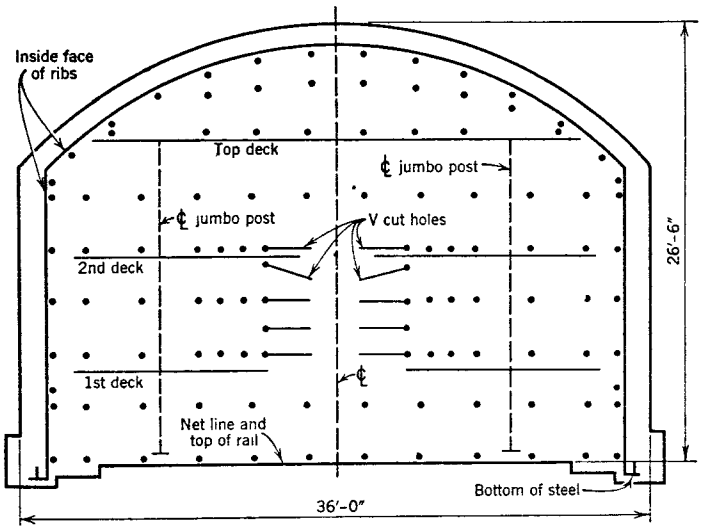


(f)

FIG. 7. (a) Drill pattern, Mono Craters Tunnel. Typical drilling round consisted of 38 holes spaced and fired as indicated, although the number of holes varied between 24 and 40 depending on the character of the ground. Average advance was 5 ft per round but 6–7 ft were pulled in sound rock. (b) Drill pattern, Big Oak Flat Tunnel. Numbers indicate sequence of firing. (c) Drill pattern, City Tunnel 2, Board of Water Supply, New York City. (d) Drill pattern, East Delaware Tunnel. Numbers indicate sequence of firing. A = minimum thickness concrete; B = pay line concrete and excavation; C = average excavation. (e) Drill pattern, Manhattan Land Tunnel, Queens Midtown Tunnel. (f) Drill pattern, Squirrel Hill Tunnel, Pittsburgh. Average drill pattern contains 114 holes, 10 of which are V cut. The firing sequence employs one immediate and 13 one-second delays.

The jumbo is a movable steel frame type of drill carriage on which are mounted all the drills necessary for drilling the heading and which is equipped with platforms for the drillers and drill steel It runs on tracks on the tunnel invert, or it may be mounted on truck or tractor, permitting it, without dismantling, to be moved safely away from the face during the blasting period. To facilitate mobility, the side platforms may be of a folding type. With this equipment it is economical and generally customary, except in very large tunnels, to drill the full section of the tunnel at one time instead of in two operations, heading and bench, and with the delay exploders the whole face may be loaded and fired in one operation.

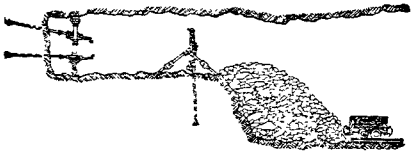


FIG 8 Top heading and bench method

Fast mucking machines are available in all sizes from the small type of mine-car loader, which will operate in headings as small as 4 ft by 7 ft, to power shovels of almost any desired capacity for large tunnels.

The two principal obstacles to continuous fast tunnel driving in rock are the occurrence of faulted, disintegrated, or shattered rock and the inflow of substantial quantities of water.

Roof Supports. The first condition will generally require some form of roof support both to protect the workmen and to prevent undue enlargement of the excavation The supports are usually in the form of an arch—if of timber composed of short segments, if of steel composed of I beams bent to the required curvature. Bents are generally spaced 4 to 6 ft apart, but conditions may require a much closer spacing. If necessary, lagging is used between bents.

When timber is used as the material for the temporary support of a tunnel excavation in rock, the roof support bents are generally made up of timber segments blocked and wedged off the rock at the mitered joints between segments in a series of chords. Lagging is used between sets if required. This segmental arch may be supported on timber posts resting on blocks or sills, or the ends at springing line may be supported on blocking set in hitches cut in the rock. Another method sometimes used for timber is to support the ends of the arch segments at springing line on steel pins or rods inserted in holes drilled in the rock walls The advantages of eliminating the posts, when not required because of side pressures, include savings in materials and excavation and elimination of the interference caused by the posting

Steel ribs for supporting tunnel excavations in rock are widely used in smaller tunnels in the form of the continuous rib type of support in which the complete bent consisting of legs and arch is formed in two pieces with a single splice or butt joint at the crown. For larger tunnels, the segmental-arch type is used with the arch generally formed by two or more segments. These are supported at springing line on steel posts or wall plates. The wall plates may be supported on posts, or, if the rock is of suitable character, on continuous hitches in the rock or on pins set to line and grade in holes drilled in the rock. In the latter case it is generally necessary to post down the wall plates before the roof takes its load, and the method is not adapted to heavy ground requiring immediate support. This is not necessary in the case of the segmental timber arch, in which the ends of the springing line segments crush into the uneven rock wall when the arch is loaded and transfer most of the load to the rock, very little load being carried by the pins provided that blocking is not used between the ends of the springing line segments and the rock.

When there is appreciable lateral pressure, some form of bottom anchorage or strut between posts will be necessary in both timber and steel supports and, if heavy side pressures develop in the rock due to swelling or other causes, or where the material in the bottom due to its character and condition has insufficient strength to sustain the loads carried by the foot blocks or sills, the full circular form of rib support may be required.

For stability, successive bents should be braced longitudinally, with steel tie rods and timber or steel spreaders.

A space will be necessary at the crown, between the rib and the form used for the concrete lining, if it is desired to concrete by means of a delivery pipe at the crown. In smaller tunnels where the thickness of the lining would not provide sufficient space for the pipe, the ribs may be bent to a curve of longer radius at the crown than at springing line, not concentric with the curve of the finished lining, to provide this space, or the rib may end in a short tangent at the crown to obtain the same result.

Lagging may be wooden planks or steel to suit conditions. There are several types of steel lagging, including channels, plates, I beams and liner plates. When lagging is to hold packing between the roof supports and the rock excavation, the pieces of lagging must be placed close enough together to retain the packing; otherwise only as close as necessary

to prevent spalling and hazard to workmen from falling rock. The required strength of the lagging depends on the character of the material supported and the span between bents.

When it is necessary to place temporary supports in a rock tunnel, each bent should be wedged and blocked to bear firmly on the rock or backpacked to keep ground movement to a minimum. If the rock is allowed to move appreciably, very heavy loading may result due to the accompanying increase in height of the rock arch. Packing used to fill the voids outside the timber or steel supports should be thoroughly grouted. This is generally done after the permanent lining has been placed, but, in some cases, packing has been grouted by low-pressure methods prior to placing the permanent lining. In no case should any voids be left between the permanent lining and the rock excavation. If lagging is to be left permanently in place, it should preferably be of steel. Wooden packing left in place outside the permanent lining may compress or decay and so permit the rock to move and increase the load on the permanent lining. The use of steel for the temporary support has several advantages over timber, if steel is available and economical in price. The wood may rot; wooden supports should be kept outside the net line of the permanent lining and, because of this and the larger sizes required, more excavation is necessary than if steel is used, which, in many cases, may be placed partially or entirely within net lines and utilized as part of the permanent lining.

For use in excavations driven through horizontally bedded rock which tends to drop from the roof in large slabs, a support consisting of plates, channels, or other flat steel attached to the roof by long bolts inserted in deep drill holes has been developed and widely used in mines and more recently (cir. 1950) adapted to tunneling. This use of roof bolts is chiefly effective in pinning the overlying strata together to the depth of the bolts and forming a rock beam of the depth of the bolts for the span of the roof. The bolts are generally 1 in. in diameter with one end threaded for about 5 in. and the other slotted for 6 in When the hole for the bolt has been drilled to the proper depth in the roof, a wedge is started in the slotted end of the bolt and the bolt driven in the hole with slotted end up, driving the wedge home and so anchoring the bolt The bolt is then tightened by turning up a nut on the threaded end, using a square roof plate between the nut and the rock roof, or standard roof tie sections or roof channel sections, the latter two types of member generally being engaged by four roof bolts at equal spacing. The bolts are in standard lengths up to 8 ft, and, when longer lengths are considered necessary, two or more bolts may be attached by means of extension couplings. Where conditions have warranted the use of roof bolts in tunneling, this method has effected great savings in materials as compared with the installation of the more conventional types of roof supports

Where the exposed rock is apparently sound after the excavation has been made but tends to deteriorate after exposure to the air, the application of cement grout in the form of Gunite to the arch and sidewall has proved to be effective in preventing the development of conditions which might later require more elaborate support. Bituminous coatings have also been used in some instances for this purpose.

Inflows of water are not only troublesome and sometimes expensive to take care of but are also a detriment and a matter of considerable expense when the concrete lining of the tunnel is being placed. To avoid the difficulties which come from water inflows, the occurrence of water in any drill hole should be the signal for stopping work, drilling additional holes to intercept the water-bearing seam, and sealing the water-bearing seam with cement grout. In case the face of the heading is not strong enough to resist the grouting pressure, it may be necessary to construct a concrete bulkhead against the face. Under some circumstances in order to determine adverse water conditions or other unfavorable conditions, it may be advisable to drill exploratory pilot or feeler holes some distance ahead of the normal depth of drilling. These measures, although the construction is delayed thereby, are essential to the economical construction of the tunnel.

4. TUNNELING IN SOFT MATERIALS

General. Tunnels constructed in soft materials generally require methods which provide for temporary or permanent support of the ground as soon as the excavation is made or shortly after it is made.

Temporary support may be provided in the form of suitably spaced bents of wood or steel with lagging, liner plates, or forepoling placed to retain the material between adjacent bents In these methods, all or part of the temporary supporting structure may be built outside the net lines of the permanent lining and left in place when the permanent lining is constructed. To economize in material, temporary supports may be designed for higher working stresses than those used in permanent design and some account may be taken of arching and other characteristics of the particular soil in question which would result in

smaller loads on the temporary supports than the loads for which the permanent lining must be designed. In order to reduce the possibility of the development of greater loads on the temporary supports than those assumed, it may be necessary to keep the completed permanent lining reasonably close to the face of excavation and to alternate the advance of comparatively short lengths of temporarily supported excavation with the construction of similar lengths of permanent lining.

In the shield method of tunneling, the tunnel shield functions as the temporary ground support, whereas the permanent lining is generally designed of material, such as cast iron, that can meet the maximum loading requirements immediately after it is erected. The shield method reduces to a minimum the quantity of materials used up for the temporary support of the ground. It is also particularly well adapted to subaqueous tunneling, where air pressures above normal are necessary to keep out water, and to tunneling in materials requiring immediate support, where conditions require that settlement of the overlying ground surface be as little as possible.

Tunnels driven through soft materials are generally at comparatively shallow depths below the surface, and, where damage to existing surface and subsurface structures may result due to settlement of the ground, the method of tunneling and types of ground support employed must be such as to reduce ground movement to a minimum. Care must be taken to fill all spaces between the limits of excavation and the back of the lining before the ground has an opportunity to close in. The use of gravel and cement grout ejected into the spaces from inside the tunnel is practiced not only to prevent settlement but also to prevent the development of unbalanced loadings on the lining.

The various methods of tunneling in soft materials, beginning with the elementary operation of mining, are more fully described in the following pages.

Tunneling in Earth by Mining. The elementary operation of mining is illustrated in Fig. 9. Starting with the face of earth which is to be mined, sharpened boards P, called polings, supported by the cross timber C, are driven into the earth. Under the protection of the polings the earth is excavated. The excavation is advanced beyond the end of the first polings by erecting another cross roof timber, C_2, and starting new polings. The mining of soft ground is largely a multiplication of this elementary operation. The sides of the excavation may be treated in the same way; small headings may be enlarged to full-sized tunnels by opening the sides by the same process. In principle this elementary operation covers all conceivable requirements of protecting the sides and roof where pressures are within the strength of the timbering. But there is always exposed a face F which must remain unprotected at least for a short time.

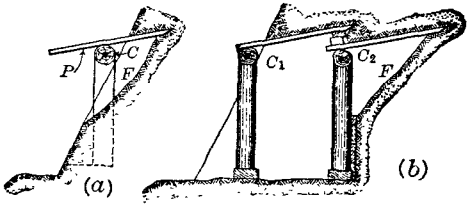


FIG. 9. Mining in earth.

The angle at which an exposed earth face will stand depends: (1) Upon the nature of the earth, clay, in general, standing steeper than sand, coarse sand steeper than fine sand, and a mixture of clay with coarse sand or gravel being better than either alone. (2) Upon the amount of water present. Dampness will increase cohesion, but as complete saturation is approached the angle becomes flatter. If sand, besides being completely saturated, has an unbalanced head of water, that is, if the head or pressure of water in fine sand exposed to the air is greater than the air pressure, then it is quicksand and will take nearly a horizontal slope. Whatever the material, its thorough draining is of the greatest importance for successful mining. (3) The length of time exposed. Time is an important element in tunneling. Some of the most unstable materials will stand even vertical for a little while, so that a small vertical face may be exposed and a breastboard quickly placed. Clayey earth sometimes may move so slowly that the excavation may be made without the protection of the poling, which may later simply be put up against the roof and supported by timbering which will be thoroughly loaded later. (4) The areas of the face exposed. A small area exposed will often stand vertical for a long time when a larger one would cave quickly. (5) The distance below the earth surface. By the ordinary accepted theory of earth pressures the horizontal thrust of earth is a function of the vertical pressure or distance below the surface. But, where small areas are exposed, dry or only damp earth will arch across the area, so that depth does not usually appreciably affect the angle at which the exposed earth face will stand until some time has elapsed. The arching cannot be depended upon in wet, running material. (6) The amount of agitation the earth has received. Earth which may be stable in its natural state may be very treach-

erous after it has been moved. The fear of starting a general movement of the earth makes miners thoroughly pack all voids back of supporting timbering and use the greatest care to prevent the starting of a run.

Timbered Heading. In all methods of soft-ground mining the face is opened with a heading varying in cross section according to the material penetrated, from a one-man burrow to full width and 7 or 8 ft high, and any length (Fig. 10). The heading timbering consists of posts *P*, caps *C*, poling boards *p*, and breastboards *b*. If the ground is stiff, the face of the heading is cut down straight at nearly the forward end of the polings and new posts and a cap are set. If the ground is soft, a small excavation, say 2 ft high, under one or two polings is made and a short board is quickly set vertical under the end of the polings and acts as a combined breastboard and prop for the poling. This is repeated under all polings. As soon as all short vertical breastboards are placed a new cap is set supported by short posts. The remainder of the face is then worked down, usually with horizontal breastboards, to the bottom of the heading.

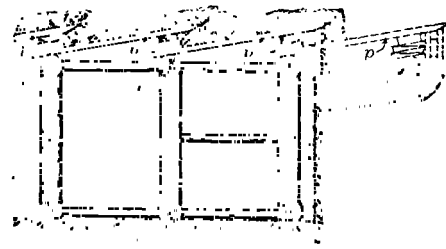


FIG. 10. Timbered headings.

Steel Supports and Liner Plates. Steel supports may be used to advantage, when steel is available and economical in price, instead of the timber posts and caps illustrated in the foregoing. If steel is used, arch ribs bent to required curvature are generally installed to support the roof. The poling, often called forepoling or spiling, which is driven or jacked ahead of the bent or set, may be timber or steel.

Where the ground is of such a nature that a limited area of the roof or arch excavation can be exposed for a short time without raveling or breaking down, steel liner plates may be used to support the ground. Metal liner plates were first used in 1879 in starting the construction of the first tube to be built under the Hudson River between New York and New Jersey. Liner plates as now manufactured are of pressed steel in sizes and weights adapted to convenient hand placing. The plates are often corrugated for added stiffness and may be bolted together by means of side and end flanges to form a continuous steel skin of the desired shape, generally supported from within by steel ribs, against which they are blocked. Liner plates also may be fabricated for installation between the webs of the supporting ribs, or, without flanges, to serve as lagging. In smaller tunnels where ground conditions are favorable, it may be unnecessary to employ rib supports, or, where support is necessary, trench jacks may be used for the purpose instead of ribs. Trench jacks are often used with some form of needle beam which extends back from the face, where it rests on the bench, to a blocked support at the rear. The trench jacks radiate from the needle beam to brace the plates until the permanent lining is placed. This method has the disadvantage, compared with rib supports, of interfering to some extent, with the heading and permanent lining operations.

Liner plates are generally installed in courses, each complete course advancing the heading for the width of plate. Starting at the top of the tunnel, the excavation is carefully advanced sufficiently to install one plate the rear flange of which is bolted to the leading flange of the course of plates previously installed. The plate so placed is temporarily supported by means of blocking. The other plates of the new course are similarly placed, working out and down from the top center plate. The blocking used to support the plates is replaced with trench jacks as the work proceeds. Where ribs are used, the trench jacks are replaced with a rib when the course is completed.

When the ground is stiff and stable enough, excavation may be completed for the full course before placing the plates. Where settlement of the ground is to be kept to a minimum, or where unbalanced loadings may develop on the lining if ground movement is not controlled, all spaces outside the liner plates should be thoroughly filled with gravel, or cement grout, or both. Holes are provided in liner plates for standard connections for this purpose.

Methods Classified. Small bores are driven by simple headings. Large bores are worked in a variety of ways, generally employing one or more smaller headings from which enlargement to the full section is made. In the past, various methods have been classified in general, according to the country in which the method was developed, as American, English, Belgian, German, Austrian, and Italian. There are many variations of these methods, and the use of any of the methods has not been confined to the country for which it is named. The methods, when they were originated, employed wooden supports as

herein described and illustrated, but in the modern variants steel supports and liner plates may be used when it is economical and practical to do so.

American Method (Fig. 11). In the American method the segmental arch timbering and side posts only are used and no interior struts. It is especially adapted to fairly firm material. The face is opened by a top heading, which is poled and timbered in the usual way. In the space between two bents of the heading timber the crown segments of the timber arches are set in position and held in place temporarily by secondary posts or by strips or scantlings spiked to the main posts as shown in Fig. 11. A short length of from 2 to 6 ft, depending on the nature of the ground, section A–A, is then widened out without polings or other roof support, and the segments adjacent to the crown segments are put in position and held by iron dowels and a short prop. If the segmental timbers are not set close together, lagging is inserted above the timbering and cavities between the lagging and the earth are packed. The widening out for the next timber is done in the same way down to the sill. After the two sills are placed, the roof timber is completed and the bench is then removed.

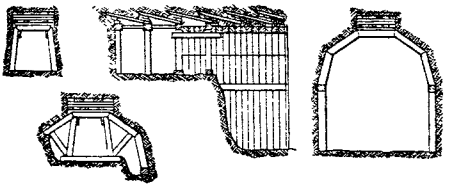


Fig 11 American method

During each operation the sill timbers are underpinned by any one of the several different methods. If the material is very firm a longitudinal cut is made in the bench, leaving a berm on which the sill rests, and the counterforts are excavated at intervals under the sill and posts placed. If the material is too soft for this, pits are sunk at intervals, and, first, short posts and, later, if excavation has proceeded, long posts are placed to support the sills. If the material is firm enough, as for example rock, the sills or wall plates are set in niches at about the springing line and no posts are used. The number of segments in the arch varies from 3 to 7 or even more. The method can be used successfully in firm material which will stand for a short time without support. This type of timbering has been used in American tunnel practice as a semipermanent lining, but is usually replaced within 10 to 15 years by concrete masonry. The chief advantages of this method are the large open area within which the masonry lining can be built continuously from invert to crown and the saving of timber. The method has also been much used in all rock. In rock a top heading is driven and timbered with posts and caps. A short length is widened out down to the springing line of the arch, the sills are placed, and the segmental timber arches erected between the posts and caps, which may then be removed.

English Method. The entire section is removed in short lengths, usually from 12 to 20 ft, in advance of the permanent lining already built, as shown in Fig. 12. The masons and miners alternate in the possession of the face, and the work of excavation and building the masonry is uninterrupted until each is complete for the length. A small drift in wet material is commonly driven at the bottom of the tunnel prism from end to end of the tunnel, both for the purpose of furnishing good drainage facilities and for the establishment of the alignment below ground; also it allows the tunnel to be attacked at several points. The main attack on the face, however, always begins at the top of the section with a heading which in stiff material is often taken out without timbering but in softer material is timbered. Two roof bars, called crown bars, are then placed in this heading with their forward ends resting on posts and their rear ends supported on the completed masonry lining. Transverse polings are then driven over these bars. The heading is then widened out under the transverse polings for the length of the roof bars, vertical breastboards placed under the transverse polings and new side bars are placed; when the ground is stiff enough the polings are not driven over the side bars but placed against the earth roof after excavation. This operation is continued down the sides as far as required by the nature and pressure of the ground, sometimes to the bottom, while at the same time the face is securely breasted and backstrutted to the completed lining. The miners now give way to the masons, who construct the length of lining within the

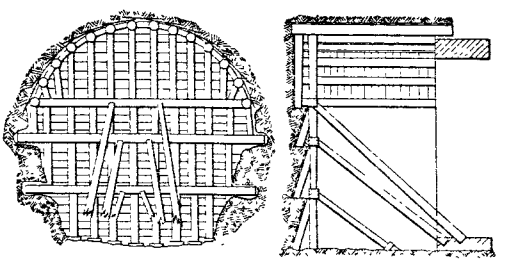


Fig 12 English method.

completed timbering, and the operation is repeated. If the roof bars are not too firmly gripped by the overlying material, the lagging is blocked up on the masonry and the roof bars are barred or pulled forward by jacks and used over again.

Belgian Method. The upper half of the tunnel is excavated much the same as the English method except that the excavation is frequently carried a considerable distance ahead of the masonry, and, as the method is most often used in firm ground, the transverse polings are frequently not driven over the roof bars, but instead are placed as in Fig. 13, against the earth roof after the excavation has been made for a side roof bar.

A cut is then excavated through the center half of the tunnel to the invert, leaving a berm on either side to support the arch of the tunnel lining From this center trench narrow cuts are made at intervals to the side, and the masonry arch is underpinned; the cuts are widened and the underpinning extended until a complete sidewall is built, and finally the invert arch is turned

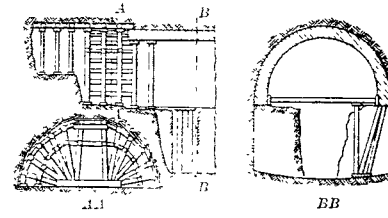


FIG. 13 Belgian method

German Method (Fig. 14). The invert is put in last, but the rest of the lining is built from the bottom up, without removing the center core of earth. In its most characteristic form the method consists in driving two bottom headings one at the foot of each sidewall. In these the sidewalls are built as high as the roof of the heading will allow On top of these headings two others are driven, the sidewalls brought up to their roofs, and so on until the two sidewalls are joined in a center heading at the top. Practically, the number of headings one on top of the other is limited to three, with a top center heading which is

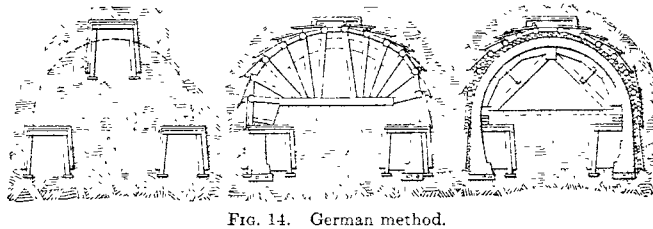


FIG. 14. German method.

widened out to the upper side headings. Usually only two headings on each side have been used, which brings the sides up to the springing of the arch; above this the entire section is taken out by widening out a center top heading down to the side heading by the same plan as in the Belgian method.

Austrian System (Fig. 15). The essential feature is a center cut from top to bottom of the tunnel with timbering as in G–G. The cut is widened out from the top as indicated in section B–B and timbered with segmental arch timbering up to the time the sills are

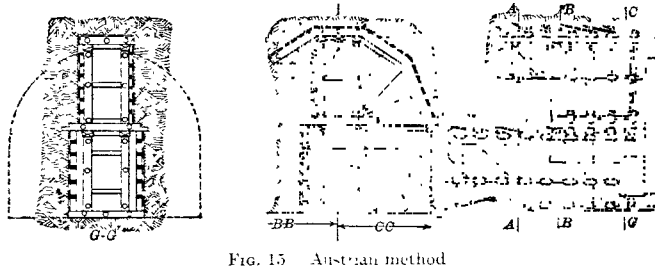


FIG. 15 Austrian method

placed above the horizontal diameter and the timbering is supported by the central frame. The poling boards over the segmental arch timbers are all driven parallel to the axis of the tunnel, and the widening out is done in short lengths, working practically the full face. The variations in the method, depending largely on the character of the material, are practically all in the method of making the central cut. A top heading may be driven, which

then may be cut down in short lengths to the invert of the tunnel, or a bottom heading may be driven, then the top heading, then the core between the two removed in short lengths, the short posts of the top heading being replaced by long ones resting on the cap timbers of the bottom heading.

The **Italian method (Fig. 16)** was especially developed for very soft and treacherous ground and is strictly an emergency method. The success of the system depends on the fact that only small areas are opened and that the material can thus be well controlled. A bottom center drift about one-eighth the height of the tunnel is started and driven a very short distance, 6–10 ft, after which it is enlarged to full width of the tunnel, very heavy and tight timbering being used. The invert and as much as possible of the sidewalls are constructed in the drift, and the open area is backfilled with earth. A center top heading is then driven and enlarged to full width and about one-half the height of the tunnel, leaving a bench the full width of the tunnel and something less than one-third the height. This upper section of the tunnel is very heavily timbered and braced; connection between the upper section and the masonry walls already built is made by trenching through the bench, after which the sidewalls are completed and the arch turned as one operation. The central bench and the backfilled earth are excavated inside the completed tunnel.

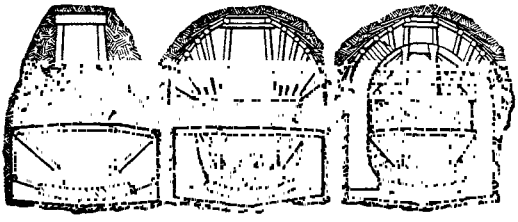


FIG 16 Italian method

The chief advantage of this method is that the workings are so small as to be readily braced and maintained in very treacherous ground. The disadvantage is the excessive cost. Modern shield methods would now be used for all tunnels for which this method was designed, except possibly where only a short length of very treacherous ground is encountered.

5. SHIELD TUNNELING; ROOF SHIELDS

The tunnel shield is a metal casing which is driven in advance of the permanent tunnel lining, serving, in lieu of timbering, to support the ground surrounding the tunnel bore and afford protection for construction of the permanent lining. The principal element of the shield structure is the skin, which is constructed of steel plates, bent to the shape of the tunnel section and slightly larger in dimension. The front end of the skin, where the excavation is made, is reinforced, generally in present practice, with steel castings, to form the cutting edge. In earlier shields, and in smaller shields today, the cutting edge has been formed by reinforcing the leading edge of the skin with stiffening brackets. The intermediate section of the skin is reinforced to add stiffness to the shield; in some of the earlier, smaller shields, stiffening was by means of a metal diaphragm containing an opening or openings for access to the face; in some cases, iron castings have been used for stiffening the intermediate section; more generally in present practice one or more transverse stiffening girders are attached to the inside of the skin around the perimeter of the shield, and, for tunnels of larger section, horizontal and vertical members are used in addition to the girders. Such horizontal and vertical members also serve to divide the cross section of the shield into working pockets which give convenient access at several levels to the face of excavation. The tail of the shield consists of the plain skin and extends back far enough to overlap the completed permanent lining to some extent, while a new course of lining is being built within its protection. It is the practice, especially in subaqueous work, to make the tail long enough to overlap the two leading rings or courses of lining and a few inches of the third when the jacks are retracted for the next advance or shove of the shield. A plate or segment of lining broken in the shoving operation, may then be replaced, still in the protection of the tail. The tail of the shield should be long enough to overlap the completed lining and also provide enough space in the tail to remove and replace shield jacks, when necessary.

The linings of shield-driven tunnels are generally circular in form as this shape is best adapted to the ground conditions requiring shield construction; also, the symmetry of the circular section about a vertical axis is not affected if the shield rotates somewhat as it is advanced, as is often the tendency.

Modern shields are equipped with powerful hydraulic jacks to force them ahead. The jacks are located just inside the skin, generally spaced uniformly around the perimeter of

the shield and housed between the transverse stiffening girders of the intermediate section, bearing, in jack pots, on the web of the forward stiffening girder and the back of the cutting-edge structure. Openings for the jacks, called jack ports, are provided in the web of the rear stiffening girder. The jack plungers, which are equipped with suitable jack shoes to distribute pressure properly on the lining, react against the completed permanent lining to force the shield ahead. Shield jacks are all connected with a battery of valves under the hand of the operator so that he may advance or withdraw any one or any combination of jacks at his will. They are generally operated at a hydraulic pressure of from 5000 to 6000 lb per sq in. A number of spare jacks should be provided, and they should be designed so that they may be easily and quickly removed as they are sure to need repairs and delays are very expensive.

The working pockets of the larger shields are generally equipped with sliding platforms which are operated by means of hydraulic jacks and which can be extended to the face of excavation and used not only for the workmen to stand upon but also to brace the face supports. Where the face is large, hydraulic face jacks, attached to the shield structure, may also be used to brace the face supports. When the shield is advanced, the platform and face jacks are slacked off permitting the jacks to recede as the shield moves forward, at the same time maintaining a constant pressure on the face supports.

The tunneling cycle consists of excavating and supporting the face, advancing the shield, and adding another course or ring to the permanent lining.

The first design for a tunnel shield was patented by Marc Isambard Brunel in 1818. This patent embraced most of the basic features of the tunnel shield as used today. Details of shield design are varied according to the character and condition of the material to be excavated, the size of the tunnel and the methods planned for excavation and disposal.

Where the tunnel is in sand or gravel requiring the face to be supported by breastboards, the upper half of the shield cutting edge is often constructed to extend beyond the lower part to form a projection called the hood, under the protection of which the miners work as the face is excavated. The hood is also an advantage when tunneling with earth in the upper portion of the tunnel bore and rock in the lower, called mixed face, permitting the soft-ground face to be supported by breastboards and bracing in advance, by the length of the hood, of the face of the rock bench, which can then be blasted and excavated without disturbing the supported face.

Where mucking machines may be used to advantage in shield tunneling, the arrangement of shield bracing and pockets is made such as to provide a large bottom pocket on center line, through which the mucking machine has access to the face.

The clearance between the inside of the tail of the shield and the outside of the permanent tunnel lining must be sufficient to allow the direction of the shield to be at some variance with that of the lining without any binding of lining and inside of the tail, especially when negotiating horizontal and vertical curves in the alignment and when correcting construction deviations from theoretical. This space must also be as small as practicable so as to keep excavation to a minimum; also, in subaqueous work in porous ground, the space between tail and lining is a source of compressed air leakage; or, when working under air pressures less than hydrostatic in plastic material, there may be a flow of material into the tunnel through this space.

The **permanent lining for shield-driven tunnels** has usually been cast iron, as a cast-iron lining can be quickly built and has its full strength to resist shield jacking pressures and external loads immediately upon erection. It offers greater assurances of watertightness than other types of lining, and, being thin for its strength, is economical with respect to excavation. Cast-iron lining for tunnels of circular section is made up of successive segmented rings bolted to one another by means of circumferential flanges to form a tube. The circumferential flanges form the stiffening elements of the tube. The rings are made up of segments of convenient length, generally 6 or 7 ft, bolted together by means of end or longitudinal flanges to form the ring. The length of the ring is limited by several considerations including (1) the distance that a miner can reach in advancing the heading from old face to new, (2) foundry practice, (3) weight which can be conveniently and economically handled and erected, and (4) the fact that the length of the tail of the shield is a function of the ring length; the shorter the shield, the more easily it is handled and steered. On the other hand, increasing the ring length increases the tunnel advance per shove and reduces number of bolts and aggregate length of joint between rings in a given length of tunnel. In practice, the ring length has varied from 18 in. in smaller, earlier tunnels, to 32 in. in those more recently constructed.

The contact faces of the flanges of the cast-iron lining are machined to close tolerances for fit and watertightness. Bolts used in erection are, in present practice, generally high-tensile steel and stressed to 30,000 lb per sq in. or more upon erection. Rebates at the inside edges of the segment flanges form calking grooves at joints when the segments are in

contact. Upon erection, the longitudinal or end flanges of the segments making up a ring are staggered with respect to those of adjacent rings for the resulting added ring stiffness. Staggering the joints in this manner is present practice in all segmental linings.

Structural-steel lining made up of flanged segments bolted together similar to cast-iron tunnel lining has also been used in shield-driven tunnels. Special thrust bars of railroad rails or structural shapes must be placed longitudinally between circumferential flanges to take the jack thrusts. In subaqueous work, steel lining is difficult to make watertight. The possibility of deterioration due to corrosion must be weighed in considering steel for the permanent lining of a tunnel.

Taper rings in which the circumferential flanges are not parallel are used in metal-lined tunnels in order to follow designed vertical and horizontal curvature and for correcting tendencies of the lining to depart from theoretical line and grade.

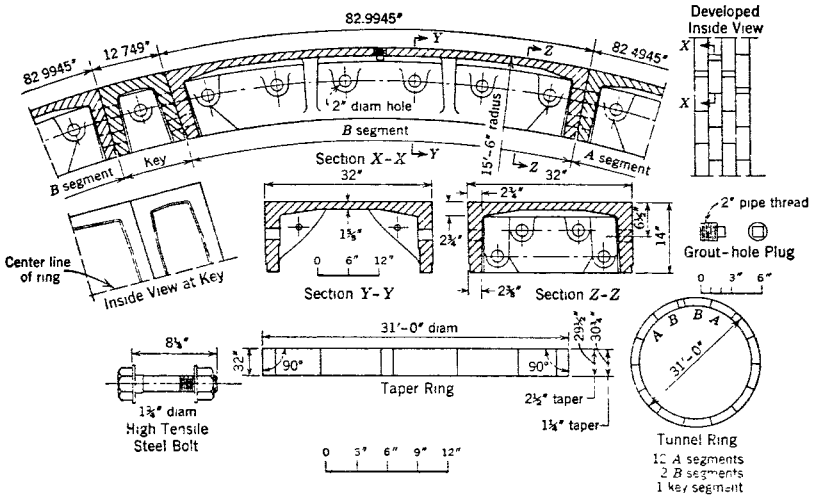


FIG. 17. Cast-iron tunnel-lining details, Queens Midtown Tunnel.

Lining of precast concrete blocks has been used, with some degree of success, in shield tunnels driven through ground above the water table. Concrete block lining is generally considered unsuitable for subaqueous tunnels.

Segmental linings are usually erected by means of a hydraulically operated erector arm which is carried on the axis of the shield or on a working platform behind the shield. The arm can be rotated to any required position and extended or retracted as required.

The bolts of metal lining may be tightened by means of hand wrenches or, where practicable, by means of hydraulic or pneumatic bolt-tightening machines.

Poured concrete lining is seldom considered for shield-driven tunnels because of the many obvious disadvantages involved in its use for this purpose. It has been used, however, employing special cast-iron thrust bars imbedded in the lining opposite the shield jacks and lined up from ring to ring, also employing steel forms designed to take the jack thrusts. Cut-stone masonry and brick have also been used in earlier shield-driven tunnels as have wooden timbers cut to segments, in the case of some small sewer tunnels.

Excavation Methods. There are many variations in the method of excavating the face in shield tunneling, according to the size of the tunnel, the character and condition of the ground, and the relative importance of disturbance to the ground and structures in or upon the ground.

In clay, or other firm material which will stand alone for a short time, the full face is simply excavated, to the length of about one shove, in front of the cutting edge without poling or breasting, and the shield quickly shoved forward. If the earth is not hard enough to injure the cutting edge, only the center of the face is excavated for a depth equal to length of shove and the cutting edge is allowed to break the rest. In softer clay the cutting edge and working floors may be always buried. In still softer materials the face of the shield is sometimes bulkheaded with timbers or steel channels placed across the working pockets, except for one or more relatively small openings which can be readily controlled as to size,

sometimes by means of hydraulically operated sliding doors, from fully closed to fully open; the shield is shoved against the earth face, some part of the material generally being permitted to flow into the tunnel through the openings and the rest being pushed aside as the shield moves forward. If no material is admitted to the tunnel, the shield is said to be shoved "blind" and if some material is admitted it is said to be shoved "partially blind." In driving subaqueous tunnels of large diameter by this method, enough material is admitted to the tunnel to serve as ballast to check the tendency of the lining to rise behind the advancing shield. For example, in driving the south tube of the Lincoln Tunnel, New York, through Hudson River silt, about 20% of the total shield displacement was taken in through two small openings aggregating about 0.5% of the total face area. In other cases, more material than that necessary for ballast may be taken in to minimize any lateral pressures which might be set up against adjacent structures as was true in driving the north tube of the Lincoln Tunnel adjacent to the south tube, which was completed and in operation when the north tube was driven. In constructing portions of the Chicago subways in clay of high water content by the shield method, shoving partially blind under city streets adjacent to heavy, important buildings, the possibilities of heaving or settlement were extremely important and were minimized by adjusting the size of openings in the face of the shield, according to the conditions encountered, so as to admit almost 100% of the displaced material into the tunnel. In this case the openings aggregated from 5 to 20% of the total face area.

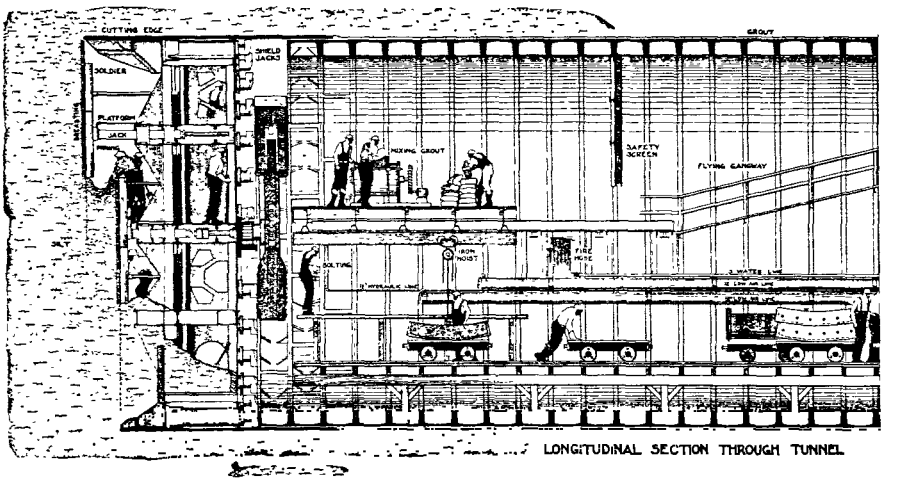
In sand or gravel the face is usually breasted with timber, which may be braced back through the shield against the completed lining, or against the shield by telescoping struts, or by means of the shield platforms and face jacks, as already described. When driving the shield for the Waterloo and City Tunnel, London, through sand and gravel, potholes were raked in front of the cutting edge and filled with soft-tempered clay. This formed a continuous ring of soft clay into which the cutting edge was shoved. The full face was then excavated under cover of the shield.

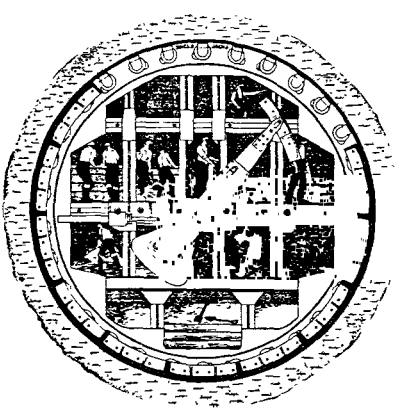
In sand, gravel, or other non-plastic material, it may be necessary to carry the breasting down to the bottom of the face, which is excavated in the plane of the front edge of the hood; when compressed air is used in subaqueous work, it is generally the practice, where settlement of the surface is not important, to breast down the face to about the point where hydrostatic head is balanced by the air pressure in the tunnel, usually in the lower half of the tunnel, and allow the material below to flow in on a natural slope. When the shield is shoved ahead the top breastboards are gradually removed and the material broken down to permit the advance of the hood.

In mixed ground, that is where there is rock in the lower part of the face of excavation and soft ground overlying it, the face is excavated and supported at the plane of the front edge of the hood down to the rock surface and the rock face is excavated by drilling and blasting sufficiently to permit the cutting edge at the bottom of the shield to advance the length of a shove; if the rock extends above the bottom of the hood, it is drilled and shot out around the hood just before making the shove. Where the height of the rock in the face permits, a bottom heading is often driven in advance of the shield, in which is poured a concrete cradle, usually with steel rails imbedded in it, upon which the heavy shield slides true to line and grade as it is advanced. Where the rock is not high enough for a bottom drift, bags filled with concrete may be placed in the rock invert ahead of the cutting edge for the shield to ride upon.

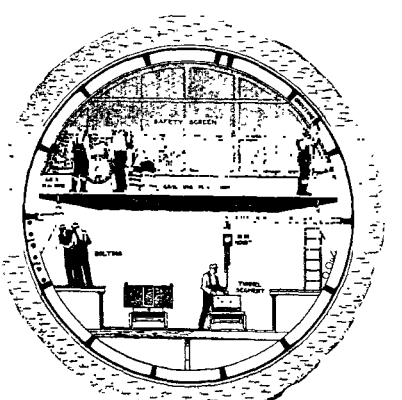
The shield method of tunneling is not ideally adapted to excavating a full rock face, but it is not unusual for some portion of a shield-driven tunnel to be in full rock face, as in passing through a reef in subaqueous work, or in starting a shield from a rock shaft. Under such circumstances, where conditions are such that it is not practicable or safe to excavate the full bore of the tunnel in advance of the shield, as large a bottom drift as practicable and a concrete cradle are employed and the rock is drilled and blasted around the upper part of the shield cutting edge and hood as the shield is moved ahead.

The greatest progress and economy in shield work can be attained in plastic materials which can be displaced by the bulkheaded shield. For example, to compare two large tunnels of equal size (31 ft in outside diameter), the normal average progress in 24 hr per shield of the Queens Midtown Tunnel, built in porous granular materials, rock and mixed face, in the bed of the East River, New York, was: for earth 7.4 ft; for rock 3.2 ft; for mixed face 3.7 ft; whereas normal average shield progress for the Lincoln Tunnel in Hudson River silt was 30 ft per 24 hr, with maximum progress in one 24-hr period of 47.5 ft. When just enough material for ballast is taken into the tunnel in the displacement method, it is not removed until air pressure is reduced to normal; as compared with shield tunneling in materials requiring 100% excavation at the face, there is great economy in the faster progress and much smaller amount of material to be disposed of as well as in the more favorable hours and rates of labor which obtain when disposal can be made under normal





REAR OF SHIELD
ERECTION OF IRON & MUCKING IN PROGRESS

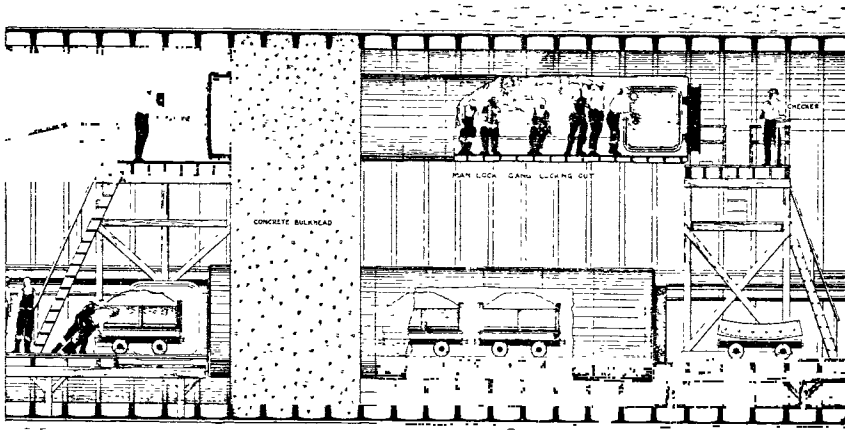


VIEW FROM REAR OF SHIELD
BOLTING & GROUTING IN PROGRESS

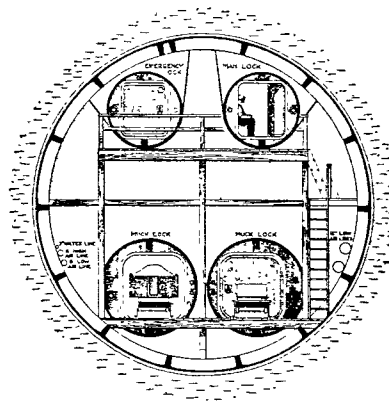
FIG. 18. Shield-tunneling operations

air pressure instead of the reduced hours of labor and higher rates required for work in compressed air.

Space outside the Lining. In shield tunneling, there is a space left between the outside of the permanent lining and the surrounding ground when the overlapping tail of the shield clears the lining as the shield moves forward. This space is equal to the thickness of tail plus clearance and must be filled immediately to prevent the ground from closing in on the lining, causing settlement and possible damage to surfaces and structures. In the stiffer cohesive materials, if the space is not packed promptly, vertical loads may develop on the lining before either active or passive lateral pressures develop, tending to flatten and overstress the lining. In ground of this character temporary tie rods or struts are used to hold the lining in shape until the space is properly filled and consolidated. Cement grout, either neat or mixed in some proportion with sand, as one part of sand to one part of cement, has been used extensively for this purpose. The grout is mixed in a grouting pan and ejected, by means of a hose and air pressure, into the space through the grout holes provided in the lining for the purpose. As grout takes some time to set up and also tends to flow around and out the tail of the shield when ejected too close to the shield, pea



HEADING SHOWING CONSTRUCTION OPERATIONS



EXTERIOR VIEW OF CONCRETE BULKHEAD SHOWING AIR LOCKS

under compressed air.

gravel or other small gravel, such as $^1/_2$ in., has been employed extensively in more modern practice to fill the space outside the lining. The gravel packing is blown through the grout holes in the lining immediately at the shield by means of hose and compressed air, as the shield is shoved ahead and the holes are cleared by the tail, so as to keep the space at all times as completely packed with gravel as possible, right up to the shield. The voids in the gravel are then generally filled with grout, the grouting operation being carried on far enough back in the tunnel to prevent the grout from finding its way into the shield. In some types of ground, and where prevention of settlement is not important, gravel only, to prevent distortion and overloading of the lining, may be employed. When the bulkheaded shield is used to displace soft silts and clays, the ground is of such a nature as to close in uniformly around the lining almost immediately, making the use of grout and gravel of no benefit as well as practically impossible. When only part of the displaced material is taken into the tunnel, a mound is raised above. When it is important to disturb the surface as little as possible by either heaving or settling, a nice balance between the volume of material admitted, volume of annular space, and any change in volume of the disturbed soil must be obtained.

Roof shields are a special form of shield designed and employed to provide temporary support for the roof or arch only of the tunnel excavation. They have been used in subaqueous work, but are better adapted to dry soils or those which can be unwatered or drained while the tunneling work is in progress. They have proved useful under city streets in cases where the cover is small and conditions prohibited excavation from the surface. They are usually semielliptical or semicircular in form and consist of a skin of steel plates riveted to girders which at their lower extremities are tied together with a transverse girder and mounted on sole plates or rollers. Hydraulic jacks supported by the ribs and bearing against the completed permanent lining or the arch centering move the shield forward, as in the case of full shields. The two general methods of using roof shields are: (1) to first drive headings in which the sidewalls are constructed on which the shield is later supported and advanced, (2) to excavate the area of cross section of the shield, build the lining under cover of the tail of the shield, and subsequently underpin with sidewall construction.

Roof shields for masonry-lined tunnels have been used with success principally in France and to some small extent in America, and, in two instances at least, roof shields have been used to construct tunnels of large cross section with cast-iron segments as the arch lining material In the one instance, a very large roof shield was used in constructing 877 ft of 46 ft 4 in. outside diameter semicircular cast-iron-lined tunnel in clay under Dale Street in Liverpool in connection with the Mersey Tunnel project in England, completed 1934. This section of tunnel is the largest on record constructed with a roof shield. In the other instance, a roof shield 36 ft in outside diameter was used in constructing the cast-iron-lined New Union Tunnel in Baltimore, Maryland.

6. USE OF COMPRESSED AIR TO KEEP OUT WATER IN SUBAQUEOUS TUNNELING

Historical. Devices such as the diving bell, which function on the principle of balancing hydrostatic pressure with air pressure, have been known and used since ancient times. It was not until 1839, however, in sinking a cylinder through water-bearing sand in connection with coal mining in France, that compressed air was confined in an excavation to keep out water and the air lock used to permit the passage of men and materials to and from the working chamber. The idea of such use of compressed air and air lock had been patented in 1830 by Sir Thomas Cochrane but never actually applied by him. In 1879, after a relatively extensive development of the idea in sinking shafts and caissons for mines and bridge piers, the use of compressed air to keep out water had its first tunneling application in driving an 18 ft by 16 ft heading in the bed of the Hudson River from a shaft in New Jersey, in the first attempt to build a tunnel between New Jersey and New York City. No tunnel shield was used in this attempt, and after considerable trouble, including a flooding of the tunnel in which 20 men were drowned, the work was suspended in 1882. The work was resumed in 1889, using shields and compressed air, and, after an interruption due to financing difficulties from 1891 to 1892, was successfully completed in 1905. In the same year, 1879, that the Hudson River tunnel was started, compressed air was successfully employed in driving a small tunnel in Antwerp, Belgium, in connection with dock work.

Air Decks, Bulkheads, and Locks. In sinking shafts and caissons, the compressed air is confined by means of an air deck which forms the roof of the working chamber. In tunneling, the compressed air is confined by means of an airtight bulkhead built across the bore of the tunnel When a tunnel is started from a shaft or caisson under compressed air, the work must be carried on through the shaft air deck until a sufficient length of tunnel is driven to permit the installation of the tunnel bulkhead with the proper amount of operating space provided between the face of excavation and the bulkhead and between the bulkhead and the shaft.

The air lock is an airtight enclosure, generally in the form of a cylindrical steel shell with gasketed doors in each end. Air pressure in the lock can be varied by means of air lines and valves. In shafts, caissons, and foundation cylinders, the locks connect to the working chamber by means of shafting leading to openings in the air deck or top of the cylinders. In tunnels, the locks are placed in the tunnel bulkheads which separate areas of higher pressure from those of lower or normal atmospheric pressure. When the pressure in a lock is that of the lower or normal pressure, the door on the high pressure end, which swings outward from the lock when opened, is held tightly closed against the door frame by the difference in air pressures. The door on the low pressure end of the lock swings into the lock. The operation of passing from the lower to the higher air pressure consists of closing the door at the low pressure end of the lock, and, by manipulating a valve, allowing compressed air to enter the lock from either the higher pressure chamber or from a compressed-air

delivery pipe. During the time that the intensity of pressure in the lock is intermediate between the higher and lower pressures, both doors are held closed by the pressure differentials. When the intensity of pressure in the lock reaches that of the higher pressure chamber of the tunnel, the door on that end can be opened. The operation is reversed in order to pass from the higher to the lower pressure, the pressure in the lock being reduced by means of an exhaust valve.

There are many forms and sizes of air locks used in shaft, foundation, and tunnel work, including horizontal, tee, and vertical, according to conditions and requirements. All are operated on the principle described. In small tunnels, a short section of lined tunnel between two bulkheads may serve as a lock, each of the bulkheads, which may be steel diaphragms anchored in the lining, being fitted with a lock door. In larger tunnels, one or more locks are placed in the tunnel bulkhead. The bulkhead may be constructed of brick masonry, structural steel, or concrete, generally the latter in present practice. When space permits, a lock generally 5 to 7 ft in diameter is provided for the exclusive use of the workmen and others who have occasion to go in and out of the workings. This lock is called a man lock, as differentiated from the material or muck lock. Material or muck locks are generally large enough to provide clearance for cars, motors, and other equipment, and are placed in the lower part of the tunnel bulkhead at haulage track level.

Where space permits, in tunnels which may be rapidly flooded by accident, a third lock is placed as high in the bulkhead as is practicable, for the escape of those in the tunnel if such an emergency arises. Other safety measures in tunnels which may be rapidly flooded include the hanging walkway, placed as high in the tunnel as clearances permit and extending from the heading to the emergency lock, and the safety screen or curtain, placed in the upper part of the tunnel near the heading to form an air and watertight trap so that if the tunnel is flooded, air is trapped between the safety curtain and the emergency lock when the water rises to the level of the bottom of the curtain. In the larger tunnels, more than one man lock or more than one muck lock may be installed for added convenience and speed in operations.

Figure 18, Art. 5, illustrates an arrangement of locks used in the Holland Tunnel, New York, and also illustrates the idea of the hanging walkway and safety screen.

Required Air Pressure. The theoretical air pressure required to counteract hydrostatic head at any given depth of water is usually computed on the basis of 0.43 lb per sq in. per ft of depth for fresh water and 0.44 lb per sq in. per ft of depth for sea water. In tunnel practice, the character of the material at the face, the cover, the size of the tunnel, and the method of excavation are also important factors which affect pressure requirements.

In holding back the water which would otherwise flow into the tunnel, the compressed air also reduces the water content of the soil in the face and makes the soil stiffer and easier and safer to handle.

Pressures much less than theoretical may be adequate in some of the less pervious soils, whereas to dry an open, pervious soil completely to a given level would require the full air pressure corresponding to the hydrostatic head at that level. Compressed-air pressure is generally not required for tunneling in stiff, impervious soil.

In tunneling in Hudson River silt with a bulkheaded shield, a gage pressure much less than hydrostatic head is generally adequate, 16 lb per sq in. gage having been used in depths below water of from 90 to 100 ft. Under such low air pressure, the soft plastic silt will squeeze into the tunnel in small quantities through the clearance space between the tail of the shield and the lining and readily flow into the tunnel through the shield bulkhead doors when they are opened as the shield is shoved ahead. Tunneling in this material at these depths with an open face at the shield, or with no shield, would require a much higher pressure, approaching the full hydrostatic pressure.

In tunneling in full face of the more porous materials, such as sand and gravel, as encountered in the bed of certain parts of the East River, New York, hydrostatic head in the larger tunnels is generally balanced at some point in the lower half of the tunnel, averaging a quarter or a third or more the length of the diameter above the bottom. As the hydrostatic head above this point is exceeded by the air pressure in the tunnel, the compressed air will tend to escape from the tunnel through the upper part of the face and the breastboards supporting the face in this area are generally plastered with clay to reduce this loss of air.

Clay Blanket. If the cover over the tunnel is not heavy enough to provide an ample counterweight to the excess of air pressure over hydrostatic pressure at the top of the face, where the difference is greatest, there is danger of a hole being blown through the cover, which might result in a disastrous flooding of the tunnel. This is safeguarded against by depositing a blanket of heavy clay over the tunnel heading to make up the deficiency in weight of the natural cover. The amount of unbalanced air pressure to be accounted for is a function of the full diameter or height of the tunnel in question, as there is generally

the possibility of having to balance hydrostatic pressure at the bottom of the face. Thus for a shield 32 ft in diameter, considering sea water, the unbalanced pressure to be accounted for is about 14 lb per sq in., or about 2000 lb per sq ft. If the natural cover were 10 ft thick and weighed 50 lb per cu ft in water, the blanket would then have to account for 1500 lb per sq ft. If saturated blanket material weighed 60 lb per cu ft in water, a minimum thickness of blanket of 25 ft above existing river bed would be required. The temporary clay blanket deposited on the bed of the west channel of the East River, New York, in connection with the construction of the Queens Midtown Tunnel was 20–40 ft thick, 140 ft wide at the top with 1 on 4 shoulder slopes, and aggregated approximately 290,000 cu yd of clay. When conditions require, an adequate clay blanket is an extremely important safeguard which gives security to the men in the tunnel and a freer hand in heading operations, which will be reflected in better progress.

The **physiological aspects of compressed air work** have resulted in rules and regulations which govern the working hours and the decompression times and generally limit the maximum gage pressure in which tunnel and shaft work may be performed to 50 lb per sq in., except in times of emergency. The working hours and decompression rules for New York State are given below, and represent the standard as generally practiced in compressed-air work throughout the United States at the present time. The limitation of 50 lb per sq in. gage pressure as the maximum in which men are permitted to work would limit the head of water which can be controlled by air pressure to about 115 ft. The much higher heads of water often encountered in deeply overlain tunnels are handled by other means, generally by grouting.

New York State Regulations Governing Pressures, Hours of Work, and Decompression Time for Work under Compressed Air

Two shifts only to be worked during 24 hr with an interval between them in open air.
Maximum working pressures (except in emergency)—50 lb per sq in.

Hours of Work

Gage Pressures, lb per sq in.	First Shift, hr	Minimum Interval, hr	Second Shift, hr
0–18	4	1/2	4 *
18–26	3	1	3
26–33	2	2	2
33–38	1 1/2	3	1 1/2
38–43	1	4	1
43–48	3/4	5	3/4
48–50	1/2	6	1/2

* Union regulations generally require the same hours at these pressures as for 18 to 26 lb per sq in.

Decompression to be by the stage method, dropping to 1/2 maximum gage pressure at the rate of 5 lb per min and completing decompression at a uniform rate so that the total time consumed shall be not less than that computed using the maximum pressure and the rate given:

Decompression Rates

Gage Pressures, lb per sq in.	Rate
0–15	3 lb per minute
15–20	2 lb per minute
20–30	3 lb in 2 minutes
30 or over	1 lb per minute

Required Quantity of Air. The amount of air required to be delivered to a tunnel heading under air pressure is very variable in different tunnels and at different times in the same tunnel. It is affected by the porosity of the soil, the size of the heading, the amount of unbalanced air pressure, and other factors. The compressed air is supplied to the tunnel from a compressor plant on the surface. The air used to counteract water pressure is referred to as low-pressure compressed air as differentiated from the high-pressure supply which is used to operate equipment. Predetermination of the required capacity of a compressor plant to meet exactly the low-pressure compressed-air requirements is practically impossible, especially for large diameter tunnels in porous materials. The tunnel engineer's judgment, in determining the proper capacity to be installed, must of necessity be based on experience and be guided by considerations of a margin of safety to meet emergencies.

Use of Compressed Air with and without the Tunnel Shield. In subaqueous tunneling in water-bearing ground, especially when tunneling under open bodies of water, the tunnel

shield and compressed air generally go together, and their joint use accounts for the success attained in this kind of work on the many such projects which have been completed all over the world since 1888. There are several examples of the successful use of compressed air in driving small or medium-sized tunnels in water-bearing ground without the shield. For example, a portion of the Pennsylvania Railroad tunnels in Long Island City was mined, timbered, and lined in compressed air without use of shields; many thousands of feet of intercepting sewers 7–12 ft in diameter for New York City were built on the margin of the East River using liner plates and compressed air; several miles of Chicago's subway system were built in soft clay employing liner plates, steel ribs and compressed air.

7. SUBAQUEOUS TUNNELS BUILT FROM THE SURFACE

Subaqueous tunnels are sometimes built from the surface, by prefabricating sections of tunnel in dry dock or on launching ways, floating them to the site, and sinking them into a dredged trench with a suitably prepared sand bed, then joining the sections together into a continuous tunnel, and backfilling the trench This method of construction is often economical where conditions are favorable, such as a river bed where the material can be dredged at low cost, where there are no important water-front structures and operations with which the trench would interfere and where there are no important waterfront traffic arteries that would be interfered with by the construction operations.

Generally, in the construction of this type of tunnel, cylindrical steel shells for the tunnel sections are fabricated on shipways Three hundred feet is considered an optimum maximum length for these sections. The ends of the shells are bulkheaded, and the sections are then launched. After launching and as the shells float in a shape-up basin, an interior reinforced-concrete tunnel ring and the interior details of the tunnel are placed. This work is done through openings in the top of the tubes which are later sealed.

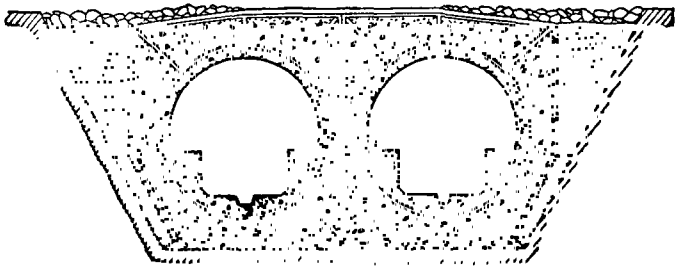
One design employs an outer shell, attached to the inner shell by means of transverse steel stiffening diaphragms, octagonal in shape and spaced about 12 ft apart. The outer shell is made up of steel plates attached to the outer edges of these diaphragms The sections are sunk by placing concrete in the spaces between the inner and outer shells.

Another design employs the single cylindrical shell only, stiffened by angles or channels welded circumferentially to the inside of the shell. This design requires that temporary ballast in some form, water, sand, or concrete blocks, for example, be used in sinking the sections.

The sections are sunk onto screeded sand beds at the bottom of the trench, target masts being temporarily fitted to each end of each tube for alignment and grade observations. The sinking operations are controlled by floating derricks or other means.

The cylindrical shells project as collars for a few feet beyond the dam or bulkhead plates at each end of the sections. Adjacent sections may be locked together by divers by means of pins through flanges and pairs of steamboat ratchets attached to the collars. The joints between adjacent sections are further sealed with tremie concrete and backfill is placed over completed sections. The bulkheads are progressively removed as the laying of additional sections advances. Joints between sections may be finally sealed by special grouting from inside the tunnel.

Refer also to Selected Data Regarding Tunnels, Nos. 3, 6, 21, 30, 31, and 35 under Highway Tunnels and Nos. 1–5 inclusive under Subaqueous Tunnels Built from Surface.



(a)

FIG. 19. (a) Detroit River Railroad Tunnel, Detroit to Windsor. (*Continued on p. 10-34.*)

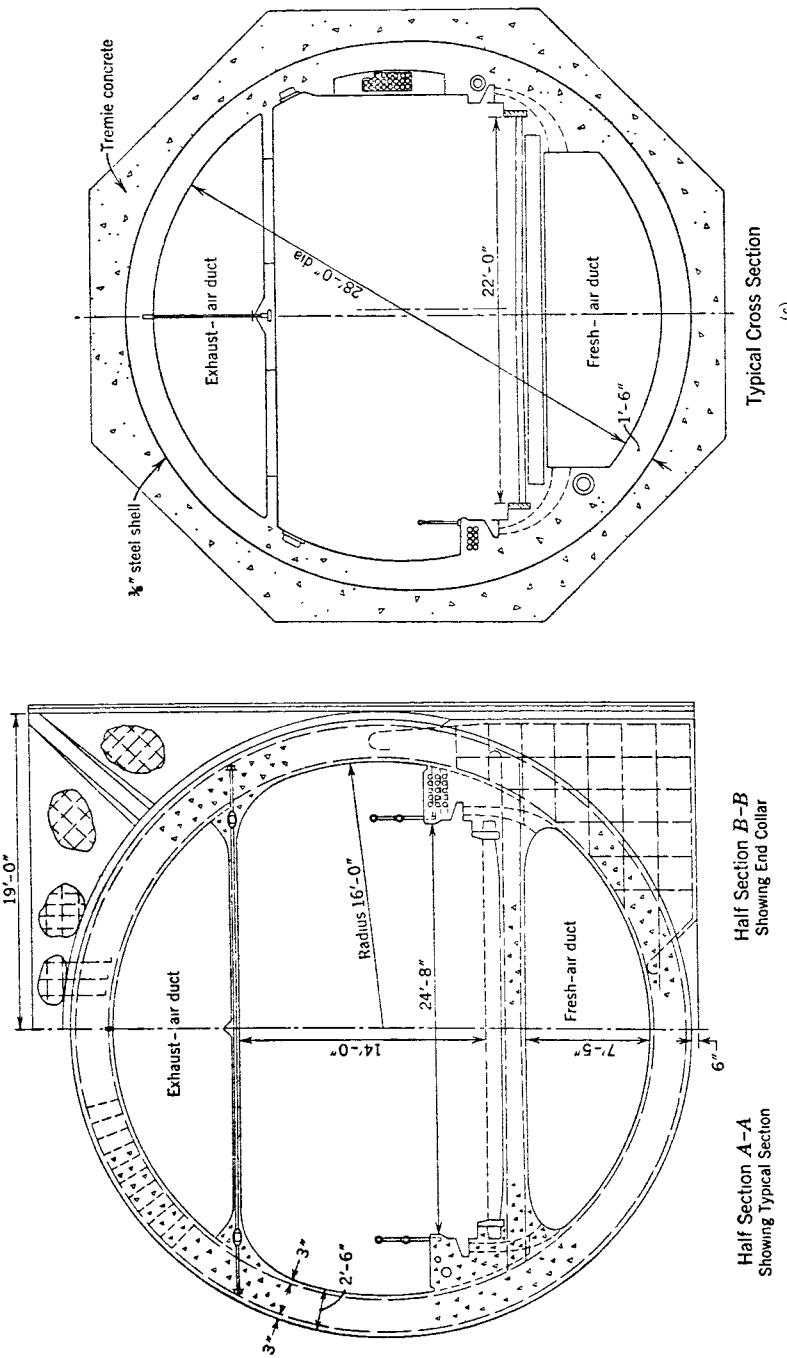


FIG. 19.—*Continued* (*b*) Posey Tube, Oakland-Alameda, California. (*c*) Detroit-Windsor Vehicular Tunnel.

8. CITY SUBWAYS

City subways are built by tunneling or cut-and-cover methods.

The **cut-and-cover method** is usually cheaper than tunneling at depths less than 35 ft and usually is the more practicable method when the cover is shallow, as is often the case with city subways for rapid transit. When traffic will not permit an open cut, the pavement is taken up, if necessary at night, and replaced by a timber deck under cover of which the subway is built. Where wide cutting is objectionable, sidewalls and interior supports are sometimes built in trench and roofed, after which the core is removed and the bottom placed. Where there is ground water, its level frequently may be lowered by well points or draining to sumps and pumping carefully. Where quicksand is met at subgrade, it is especially important to lower the ground-water level, tight sheeting is driven to extra depth and it may be necessary also to excavate for and place the floor quickly in short sections. On the other hand, where the ground is dry and hard enough to stand with but little bracing and where decking is not required, power shovels have been successfully used. Where decking is required, a method was used, for the more recently constructed subways, which permits power shovels to operate under the decking. This method consists in driving vertical steel beams along the sides of the proposed trench, then placing horizontal wood planks or breastboards between the beams as the excavation progresses to support the sides of the cut. Steel decking beams are placed across the width of the trench to support the decking placed thereon and to take the horizontal thrust from the top of the vertical beams. The wide open spaces resulting from this construction make the use of power shovels practicable.

Intercepted sewers may be depressed or undersiphoned, or the sewer system may be modified.

Existing foundations adjacent to the structure are underpinned where required to prevent settlement. Finally, subsurface structures are restored or rebuilt and the street is backfilled and repaved.

Earlier sections usually were of the single-arch type if headroom permitted; where close to the surface the roof was of beams with jack arches between them. Later, beams came to be used also in sidewalls, forming bents, saving in width and facilitating construction. Reinforced concrete requires somewhat less and cheaper steel and permits the use of large forms, but bents may be favored because better adapted to requirements of bracing, piecemeal construction, and support of subsurface structures. The single arch of concrete, plain or reinforced, may be used where conditions of headroom and lateral support are favorable. In soft or wet ground the floor may be formed as an invert or may be reinforced with beams or rods. Waterproofing is commonly several plies of fabric saturated and laid with pitch or asphalt; or it may consist of one ply of asphalt-treated fabric and one or more layers of brick in asphalt mastic. In dry situations, waterproofing may be omitted from floor and sides. Stations are painted, plastered, or tiled in white or light colors.

9. TUNNEL SHAFTS AND CAISSONS

Timbered Shafts. Timbered shafts are sunk vertically by mining much the same as a heading is driven horizontally. A frame of four waling timbers joined at their ends, forming a rectangle, is laid out on the ground. Outside are driven vertical polings or sheeting; inside, as the sheeting is being driven, the earth is excavated to a little above or a little below its lower end, depending upon whether the ground is soft or hard. As soon as the excavation is from 2 to 3 ft below one frame a new frame is placed, and so on to the depth of the length of the sheeting; a new frame is then set with clearance for a new set of sheeting. If the shaft is to be shallow it may be started large enough at the top so that the sheeting may be vertical, and the shaft reduced in size by the thickness of the timbering at the bottom of each set of sheeting which may be from 10 to 20 ft long, as in Fig. 20*a*. If the shaft is to be deep, the sheeting will ordinarily be 5 or 6 ft polings, as in Fig. 20*b*.

The main thing to be avoided in shaft sinking is the starting of a general movement of the earth. The great danger is that cavities will be left or formed back of the timbering or that the timbering though not slack will have to deflect to develop resistance and a small progressive movement of the earth will be started which may extend to the surface. The precautions are, if possible, to allow no cavity to form, securely pack any accidental one, and to wedge all timbering tight against the sheeting so that it will develop resistance before the sheeting begins to move. Perhaps the greatest danger is in water-bearing earth, where leakage is liable to bring into the shaft earth in such small amounts that they are not noticeable but may form cavities back of the sheeting in time.

In rock shafts the principal use of timbering is to prevent the falling of loose pieces from the sides of the shaft, endangering the workmen below. If the rock is fairly good, the timbering is usually kept from 10 to 30 or 40 ft behind the excavation. It is not an uncommon practice to place the permanent lining, usually of concrete, in short lengths 40 or 50 ft above the bottom of the shaft as the work of excavation progresses.

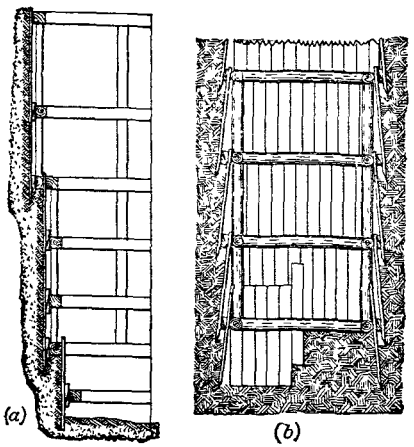


FIG 20 Timbered shafts.

The method of open excavation and timbering or otherwise lining the shaft as sinking progresses is no doubt cheaper, where practicable, than any other method that has been devised, but when quicksand or other soft material is encountered or where rock yielding too much water is encountered, the method may be either impossible or impracticable. For such cases various devices have been used, notably caissons sunk by the aid of compressed air, drop shaft, freezing process, and grouting.

Steel Lining for Shafts. Steel lining for circular shafts in the form of liner plates alone for smaller shafts and liner plates braced by circular horizontal ribs at intervals have proved practicable where the ground may be excavated in increments of depth equal to the width of the liner plates without support. Rectangular shafts may also be sunk by means of steel H beams driven vertically with horizontal lagging, either steel or timber, placed between the H beams. Vertical steel sheet piling may be used for either circular or rectangular shafts, and is adapted to water-bearing ground.

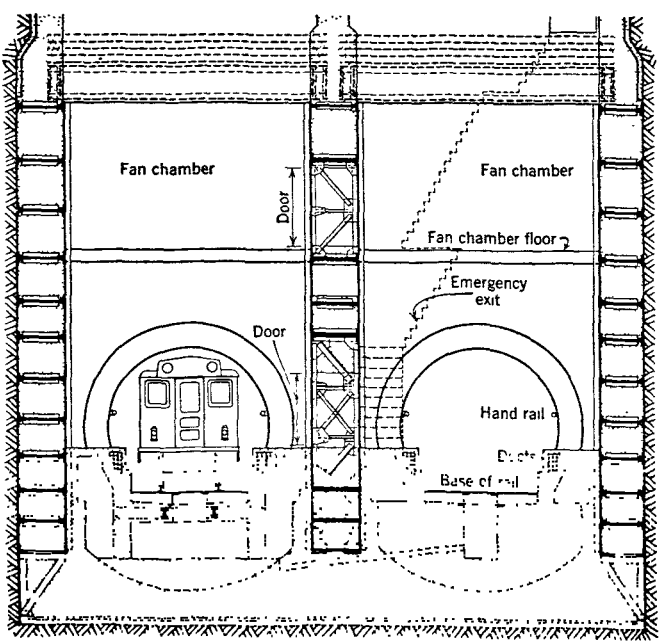


FIG. 21a. Section through completed shaft

Shafts by Caissons. Shafts by caissons are frequently used in tunnel construction because of the requirement for clear space in shaft. The depth of compressed air work is limited by the pressure at which work becomes either too expensive or too dangerous. After a depth of 70 ft below water is reached the cost of labor in compressed air increases very rapidly on account of both greater wages and shorter hours. Very little caisson work has been done under heads of over 100 ft, and the limit is a head of 115 ft.

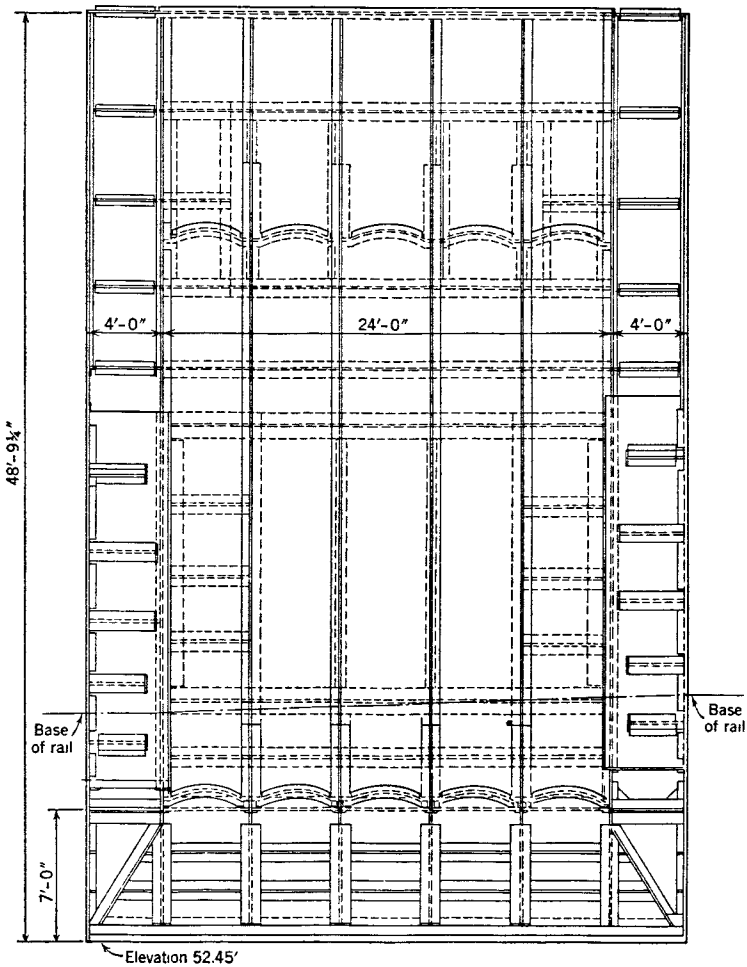


FIG. 21b Longitudinal section of caisson steel construction. (*Continued on p 10-38*)

Drop Shafts. Drop shafts are sunk by building a heavy lining, usually circular, upon the surface, and then excavating the earth inside, allowing the structure to settle slowly into the ground. Drop shafts have been used in a great variety of places, from small wells a few feet deep to mine shafts 20 ft in diameter and over 500 ft deep. The shell or lining may be of timber, masonry, brick, reinforced concrete, cast iron, or steel.

Freezing Process for Shafts. Shafts, mostly in northern Europe (France, Germany, and Belgium), have been sunk by the aid of the freezing process. This method was used for building a large shaft for the Scheldt River Vehicular Tunnel at Antwerp, Belgium.

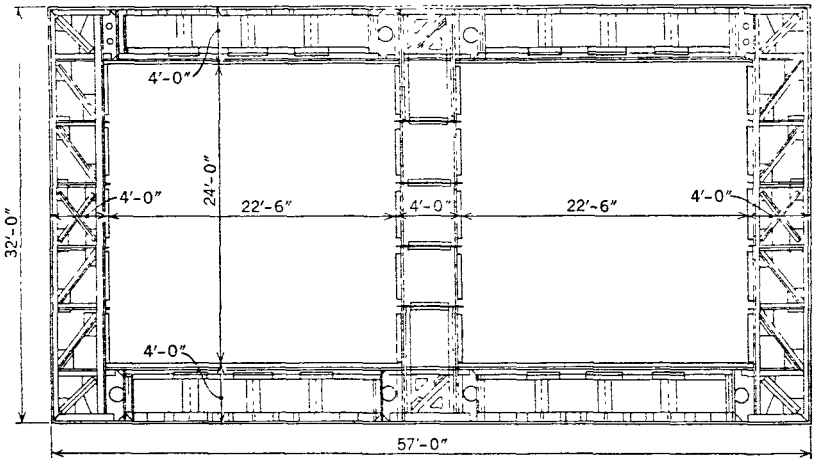


FIG 21c. Horizontal section of caisson steel construction. New York Subway System.

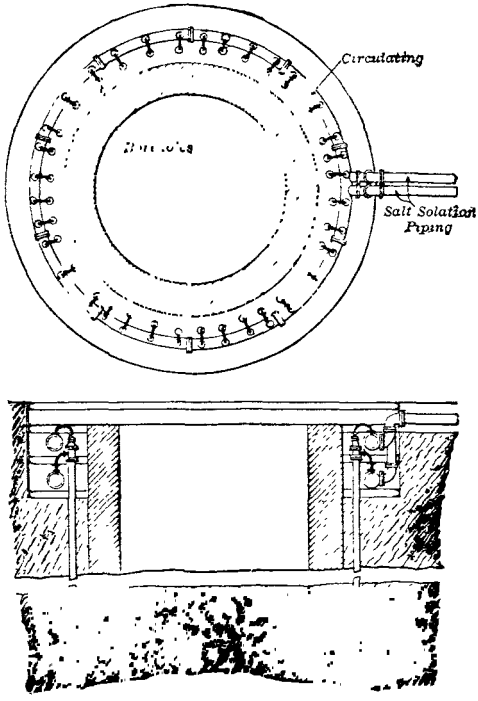


FIG 22. Freezing process for shafts.

The general features of all cases are indicated by Fig. 22. Vertical pipes are sunk, usually forming a circle in plan, completely surrounding the side of the proposed shaft; inside each pipe is placed a smaller pipe, called a circulating tube, opening into the outer pipe at the lower end. These circulating tubes are all joined together at the top by a pipe known as the circulating ring. The freezing pipes are joined together at the top by a similar ring called the collector ring Cold brine at a temperature of from −10° to −30°F is then pumped into the circulating ring, down the circulating tubes, and back up the freezing tubes to the freezing machine.

10. MATERIALS FOR TUNNEL LININGS

General. The principal requirements to be considered in selecting the material for a permanent tunnel lining are strength and permanence In tunnels driven in self-supporting material the lining may serve only as a veneer to provide a smooth friction-reducing surface or a clean, non-dirt-collecting surface and as a protection against spalling or deterioration of the rock due to chemical or physical changes that occur upon exposure. In subaqueous tunnels where the prevention of leakage is an important factor, the lining must be of an impervious type. The availability and cost of the materials under consideration must also be taken into account as well as the availability and cost of the proper quality of labor necessary to construct the lining. The form of the finished tunnel section must also be considered; for example, some materials have the advantage of easier construction of variations in shape than others. There is always the consideration that a cheaper lining requiring greater thickness for strength than a more expensive one may increase the cost of excavation beyond the apparent saving in the cost of the lining itself In shield tunneling, the lining should have the full strength necessary to withstand the shield jack reactions immediately upon being erected, or special forms or thrust bars must be employed to serve this purpose, and the lining must have its full strength to support the surrounding ground as soon as it is cleared by the tail of the shield.

Brick Lining. Brick lining requires a large amount of centering, and the ground may have to stand for a comparatively long time without permanent support. It is not adapted to shield tunneling. Many of the older tunnels, especially those in England, have been brick lined.

Stone masonry is generally more costly than brick work but its greater strength may permit a thinner lining than brick, other things being equal. Stone masonry requires a large amount of centering. It is not adapted to shield tunneling.

Timber lining has been used to a considerable extent in the United States, where timber was easily available and cheap. In tunnels requiring support, timber lining may be effectively employed under some conditions, not only for a temporary support, but as a semipermanent lining to be replaced or supplemented with concrete lining when required. Timber is not adapted to shield tunneling or waterproof construction.

Concrete lining, cast in place, has a wide use in tunnels for all purposes, including water supply, water power, sewerage, railroad, rapid transit, and highway The use of reinforcing permits the use of comparatively thin sections. Modern methods of shooting or pumping the concrete mix into place in steel forms of collapsible type which can be moved ahead through the forms set in place without disturbing them has resulted in almost continuous pouring and rapid progress where conditions are favorable for this type of operation. Under other conditions the concrete may be poured in a sequence of operations, as invert, sidewall, and arch, with economy. Concrete has some degree of watertightness which in some kinds of ground may be improved with grouting. Concrete cast in place is not adapted to modern shield tunneling operations. It is often used as a secondary lining to provide protection and internal structural requirements.

There are several forms of precast concrete blocks which have been adapted to shield tunneling in dry or impervious ground. Concrete block lining cannot be made effectively watertight, and percolating ground water may seep through the joints of this type of lining even when above the general ground-water level. Concrete block linings require centering for the upper portion of the ring until the key is set A patented concrete lining, developed about 1936, of reinforced-concrete segments bolted together, was successfully used in the construction of 2 3/4 miles of running tunnels of the Ilford tube, 13 feet 3/4 in. in exterior diameter, in typical hard, blue, London clay.

Cast-iron tunnel lining is particularly adapted to shield tunneling, but has also been used in major projects such as the Brooklyn-Battery Tunnel, New York, and the Mersey Tunnel, England, where its waterproofing qualities in rock under subaqueous conditions were of special importance. Cast iron has the disadvantage of requiring considerable weight in order to meet strength requirements. It is widely used for the linings of shield-

driven subaqueous tunnels, because it has full strength immediately upon erection; is well adapted to resist jacking thrusts; it may be accurately machined for structural fit and tight joints, and is effectively waterproofed by means of grooves formed at the joints between segments calked from inside the tunnel, and the use of grommets at bolts. Lead wire has been extensively used for calking as have hemp grommets worked up with a paste of red lead and boiled linseed oil for grommeting. An interior lining of concrete is generally used with a primary lining of cast iron for protection, appearance, and easier maintenance. Corrosion of cast iron need not be considered under most conditions met with in tunneling.

Cast-steel lining, similar to cast-iron lining, has been used where additional strength has been required to meet special conditions in tunnels generally lined with cast iron.

Structural steel, besides being used in the form of steel ribs and liner plates, has been used in the form of welded segments of plates and shapes, bolted together through flanges for primary lining. It has the advantage of being lighter than cast-iron lining for the same strength requirements, permitting the use of larger segments rather than smaller segments, with reduction of the footage of joints and number of bolts required per linear foot of tunnel. When used in shield-driven tunnels the length of ring is limited by the permissible distance that the shield can be advanced per shove and steel stiffeners or railroad rails must be added between circumferential flanges to take the shield jack thrusts. Such stiffeners add little or nothing to the strength of the lining for the permanent loading conditions. Structural-steel lining may be field welded for watertightness where seepage conditions in the tunnel permit; attempts at welding under adverse water conditions have not proved successful to date. Structural-steel lining may be designed with machined flanges and calking grooves, similar to cast-iron lining, for waterproofing purposes, but the success of such designs has not been demonstrated in actual practice, and the machining adds considerably to cost. Structural-steel lining may be subject to corrosion and, under such conditions, the possible extent of corrosive action must be accounted for in design.

11. DESIGN OF TUNNEL LINING; MISCELLANEOUS DESIGN NOTES

Loadings. The loadings considered in the design and investigation of the permanent lining of a tunnel fall into the following categories:

a. Weight of lining.

b. Weight of internal structure, if any.

c. Weight and pressures of surrounding ground and water and any superimposed loadings.

d. Loading related to the use of the tunnel, such as weight and impact of traffic, or internal pressures.

e. Loads due to temporary construction conditions, such as compressed-air pressure and reaction of shield jacks; or launching and sinking conditions in the case of prefabricated trench tunnel sections.

A tunnel structure should be designed to withstand the most severe combination of loads that it may reasonably be subjected to.

The loadings of greatest importance in the design of most tunnels are those due to the weight and pressures of the surrounding ground and water. As there is much unknown concerning the earth and rock pressures encountered in tunneling, assumptions as to these loadings must be based largely on engineering judgment, and, due to the uncertainties involved, the assumptions should be made well on the side of safety.

As a guide in estimating earth and rock loadings a careful forecast should be made, not only of the kinds and depths of the materials surrounding and overlying the tunnel bore (see Geologic Investigation under Art. 2), but also of their physical properties and condition and of their probable behavior when the tunnel bore is excavated. Such a forecast is especially important when the final design of the permanent tunnel lining is made before the construction of the tunnel is contracted for, as is generally the present practice in the United States. In many cases, the earth and rock loadings encountered will be caused by gravity alone, but there are other cases where pressures due to physical or chemical changes in the materials exposed in the tunnel excavation are also a factor, as in the case of swelling materials.

Obviously, it would not be either practicable or economical to vary the design of the permanent tunnel lining with every change in loading condition. It is the practice to vary the design for marked changes only. The criterion for each different design adopted should be the maximum loading condition for the particular length or lengths of tunnel involved.

Depending on methods of construction and the kinds and conditions of materials around and over the tunnel, the criterion for maximum loading condition may occur either during construction, as is often the case for linings constructed for shield-driven tunnels, or some time later. The final maximum loads caused by earth or rock pressures in some materials may take years to develop.

In **designing a tunnel lining in earth,** when the cover over the roof of the tunnel is of shallow or moderate depth, the vertical load on the top of the tunnel is generally assumed to be equal to the full weight of the superincumbent materials. If such a tunnel is below the water table and the surrounding material is earth, the vertical load on the top of the tunnel is generally assumed to be equal to the full weight of the earth and water above; e.g., if the water table is below the earth surface, the vertical load would be assumed to be equal to the weight of the saturated earth up to the water table level, plus the weight of the dry or damp earth, as the case may be, above the water table; if the water table is above the ground surface, the vertical load would be assumed to be equal to the weight of the free body of water above the ground surface, plus the weight of the saturated earth below.

When the tunnel is wholly or partly below the water level, it is assumed to be subjected to a lateral pressure the intensity of which at any depth below the water table is equal to the full hydrostatic head for that depth.

The tunnel is also subjected to the active lateral pressure of the surrounding earth, which, if the earth under consideration is below water and the full hydrostatic head has been accounted for, would be based on the weight of the earth in water.

In tunnel design it is generally assumed that the horizontal pressure of the earth varies directly with the depth below the earth surface, and that the horizontal pressure is equal to the vertical pressure multiplied by a constant k which is less than unity. Values of k have been determined by observation, experiment, and theory for many kinds of soils under various conditions. These values of k range from smaller fractions as in the cases of loams, sand, gravel, and stiff cohesive or cemented soils to fractions approaching unity for clays and silts in a plastic state. As the various available data and theories which evaluate k are not all in close agreement, it is customary to take into consideration the several possible values applicable to the soil in question. The Rankine formula, derived for cohesionless soils, expresses values of k in terms of the angle of repose of the soil, and there are many other theories dealing with lateral earth pressures.

The tendency of a tunnel lining to deflect outward under vertical loading may be resisted, not only by active horizontal pressures, but by passive pressures or reactions developed in the soil at the sides of the lining when the tendency of the lining to deflect outward exceeds the active lateral pressures. If the nature and condition of the soil and deflection of the tunnel lining under the other loads to which it is subjected are such that passive pressures would be so developed, these pressures should be taken into consideration in the design of the lining to the extent that they can be relied upon to act permanently.

In soils which have properties favorable to arching, some consideration may be given to the extent to which this action may be relied upon to prevent the full weight of cover from acting as a load on the top of a tunnel. Arch action is nearly always either directly or indirectly a consideration in the design of temporary supports for a tunnel excavation but, because of the uncertainties and possible changing conditions involved, is seldom used as a factor in the final design of a permanent tunnel lining in earth.

When the tunnel is in rock, direct or indirect consideration of arching is generally used to estimate the height of the rock load which may be carried on the roof of the tunnel. In perfectly sound rock, the intrados of the rock arch would coincide with the excavation line and the height of the rock load on the tunnel lining would be zero. From this lower limiting condition, which obtains in sound, self-supporting rock, the height of the rock load on the tunnel roof would be estimated to increase as the strength of overlying rock formations decreases. The terms stratified, schistose, jointed, blocky, seamy, fractured, crushed, etc., approaching the condition of earth, indicate varying degrees of weakness in a formation which would affect the height of the rock load to be accounted for. The height of the rock load is also affected by water conditions, which may, for example, lubricate the joints in the rock and cause a greater load than when the material is dry. In this regard, the final condition of the completed tunnel as to drainage should receive consideration. The height of the rock load is also considered to be a function of the width of yielding material at the level of the top of the tunnel excavation. The minimum yielding width may be considered to be the width of the excavation. If the material at each side of the tunnel bore is considered to yield to some extent, the total yielding width, or span of the rock arch, becomes a function of the height as well as the width of the tunnel excavation. In materials where the loaded wedge of rock at the sides of the tunnel would tend to slide into the

excavation along a plane of rupture, lateral pressures would be developed which should be accounted for in design.

Some of the earlier theories of arching in rock which have merit assume that the rock arch is semielliptic in form and that the planes of rupture are determined by consideration of the angle of repose of the material. Later developments of the theory assume planes of rupture on two-on-one slopes, making the arch-span factor equal to width plus height of tunnel excavation.

In deeply overlain tunnels in hard, brittle rock, active pressures causing strain in the rock may cause thin slabs of the rock exposed in the excavation to become violently dislodged from the sides and roof of the tunnel. This is described as shelling, flaking, or popping rock, whereas evidence of the active pressures encountered in deep tunnels in softer rock may take the form of the crushing and bending in of roof and sidewalls and raising of the invert.

Evaluation of rock loads and pressures is largely a matter of engineering judgment, guided principally by experience and comparative studies, and to some extent by theory based on experimental data and observations of pressures on tunnel supports.

Active Pressures and Reactions Must Be in Static Equilibrium. For all conditions of tunnel loading, active pressures and reactions must be in static equilibrium. Hydrostatic uplift, if a factor, must be safely counteracted by the overburden and weight of tunnel lining.

Tunnel footing loads should not exceed the safe bearing capacity of the supporting material.

Stress Analysis. Observed stresses and deflections in tunnel linings seldom agree with those derived by computation based on assumed loadings and accepted methods of analysis. Among the reasons for this lack of agreement are: (1) The assumed loadings and distribution of reactions may not be in conformity with actual loading conditions. (2) The materials and construction of the lining may not conform with basic assumptions as to elasticity, stiffness, and other properties. (3) The application of the theories used to solve for stresses involves approximations and assumptions that may not be strictly correct. (4) There may be initial stresses, resulting from erection or construction methods, which are difficult to evaluate.

The design and investigation of a tunnel lining must, therefore, be made well on the side of safety, and be made to cover all reasonable loading conditions. The investigation may have its greatest value if it is possible to make an investigation by the same methods and on the basis of similar assumptions, of the lining or linings of existing tunnels which are under analogous conditions and which are safely withstanding their loadings. The stresses computed for the proposed lining can then be compared with those computed for the existing lining or linings as an indication of the relative strengths of the structures, rather than judging the strength of the proposed lining solely on the basis of the magnitude of the computed stresses.

Tunnel linings are generally in the class of statically indeterminate structures. The shape and size of the tunnel and the loading conditions having been determined, the design of the lining may be tentatively adopted by approximations and analogy with existing tunnel structures. Alternate designs may be made for the different types and materials of lining that appear to be suitable.

Analytical methods, for computing thrusts, bending moments, and shear in statically indeterminate structures, are available which permit stresses to be determined quite accurately with respect to the assumptions made as to loading, elastic properties, etc. The most commonly used methods apply the principles of the elastic theory to determine thrusts, bending moments, and shear at the crown of the tunnel arch, from which the thrusts, bending moments, and shear at any other section may be determined by the principles of static equilibrium. Approximate methods of analysis based on algebraic and graphical determinations applicable to linings of variable contour and moment of inertia may be found in *Analysis of Arches, Rigid Frames and Sewer Sections*, Portland Cement Association, March 1943, and other works.

The fundamental formulas for curved beams are:

$$\Delta x = \int \frac{Mh\,ds}{EI} \tag{1}$$

$$\Delta y = \int \frac{Mr\,ds}{EI} \tag{2}$$

$$\Delta \phi = \int \frac{M\,ds}{EI} \tag{3}$$

where Δx = horizontal deflection.
Δy = vertical deflection.
$\Delta\varphi$ = angular deflection.
M = bending moment at any point due to given loading.
h = bending moment at same point due to horizontal load of unity applied at same point.
v = bending moment at same point due to vertical load of unity applied at the point the vertical deflection of which is desired
ds = length of element of the ring.
E = modulus of elasticity of ring material.
I = average moment of inertia with respect to neutral axis for a section of the ring one unit in length.

(The above formulas are based on the approximation that all the fibers of the ring are the same length and equal to a corresponding length of the axis Theoretical deflections computed from the above formulas agree closely with the results of experience and experiment on the deflections of curved beams)

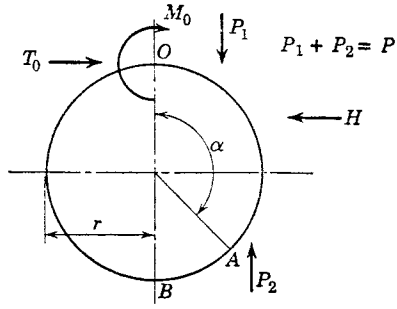


FIG 23 Diagram Neutral axis of tunnel ring symmetrically loaded about vertical axis Ring cut at crown.

Referring to Fig. 23, let circle OAB of radius r represent the neutral axis of a tunnel ring loaded symmetrically about the vertical axis.

A = any point on neutral axis.
α = angle subtended by OA.
r = radius of neutral axis.
H = horizontal component of exterior forces acting on OA.
P = vertical component of exterior forces acting on OA.
M = bending moment at A.
M' = bending moment at A due to P and H.
M_0 = bending moment at crown.
V_0 = shear at crown (= 0 as loading is symmetrical)
T_0 = thrust at crown.
V = radial shear at A.
T = thrust at A.

Consider:

vertically up +
vertically down −
horizontally right +
horizontally left −
clockwise moments + (cause tension in intrados)
counterclockwise moments − (cause tension in extrados)
outward radial shear +
inward radial shear −
thrust-producing compression +
thrust-producing tension −

Considering H, T_0, and P as positive in direction.

$$M = M' + M_0 + T_0 (r - r \cos \alpha) \tag{4}$$

$$T = (H + T_0) \cos \alpha - P \sin \alpha \tag{5}$$

$$V = (H + T_0) \sin \alpha + P \cos \alpha \tag{6}$$

As the ring is symmetrically loaded about its vertical axis, the angular deflection of the crown with respect to the invert is 0 and the horizontal deflection of the crown with respect to the invert is 0.

It may then be shown that:

$$T_0 = \frac{2}{\pi r} \int_0^{\pi} M' \cos \alpha \, d\alpha \tag{7}$$

$$M_0 = -T_0 r - \frac{1}{\pi} \int_0^{\pi} M' \, d\alpha \tag{8}$$

Having determined the value of T_0 and M_0 due to a given loading condition from eqs. 7 and 8, the moment, thrust, and shear at any section can be determined from eqs. 4, 5, and 6.

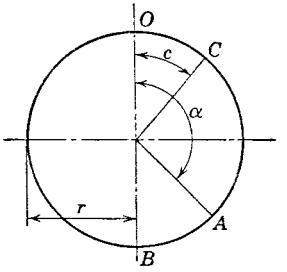


FIG. 24 Diagram. Neutral axis of tunnel ring symmetrically loaded about vertical axis; to determine deflections of C with respect to B.

Referring to Fig 24 in which C is any point subtending angle c for which the horizontal deflection with respect to the invert is desired, it may be shown from eq. 1 that:

$$\Delta x = \frac{r^2}{EI} \left\{ \frac{1}{\pi} [(c - \pi) \cos c - \sin c] \int_0^{\pi} M' \, d\alpha + \frac{1}{\pi} (\sin c \cos c - c + \pi) \int_0^{\pi} M' \cos \alpha \, \partial\alpha + \cos c \int_c^{\pi} M' \, d\alpha - \int_c^{\pi} M' \cos \alpha \, d\alpha \right\} \tag{9}$$

Similar expressions for Δy and $\Delta\phi$ are:

$$\Delta y = \frac{r^2}{EI} \left\{ \frac{1}{\pi} [(c - \pi) \sin c + \cos c + 1] \int_0^{\pi} M' \, d\alpha + \frac{1}{\pi} \sin^2 c \int_0^{\pi} M' \cos \alpha \, d\alpha + \sin c \int_c^{\pi} M' \, d\alpha - \int_c^{\pi} M' \sin \alpha \, d\alpha \right\} \tag{10}$$

$$\Delta\phi = \frac{r}{EI} \left\{ \left(\frac{c}{\pi} - 1 \right) \int_0^{\pi} M' \, d\alpha + \frac{2}{\pi} \sin c \int_0^{\pi} M' \cos \alpha \, d\alpha + \int_c^{\pi} M' \, d\alpha \right\} \tag{11}$$

Ring loading used in typical applications for eqs. 4 to 11 inclusive are shown in Fig. 25.

A method of approximating the effect of passive pressures or reactions due to the horizontal deflection of the tunnel ring under a given combination of loadings may be developed by considering that the soil in question compresses under loading in proportion to the load. This requires the assumption of a modulus of compressibility for the soil in terms of intensity of pressure required to compress the soil one unit, as for example 1000 lb per sq ft to compress the soil 1 in., or 15,000 lb per sq ft to compress the soil 1 in., etc., according to the properties of the soil in question. The analysis is generally made on the basis of several different reasonable assumptions as to soil compressibility. The vertical projections of voussoirs of equal length above and below the horizontal diameter of the ring are assumed to develop average uniform passive pressures over each such voussoir projection dependent on the tendency of the center point of the voussoir to deflect under the active loading combination assumed and on the modulus of compressibility assumed for the soil in question. The lengths of voussoirs may be chosen as large or as small as the desired accuracy of the

approximation indicates is practicable. The complexity of the problem increases very rapidly with the number of voussoirs chosen. The extent of the passive pressure considered to be developed above or below the horizontal axis should be limited by consideration of the zone over which the lining, as it distorts, tends to press outward against the soil to an effective amount. According to one hypothesis substantiated by tests (The Structural Design of Flexible Pipe Culverts by M. G. Spangler, Iowa *Engg. Exp. Sta. Bul.* 151, 1941) passive pressures developed by backfill loading on flexible pipe culverts may be considered

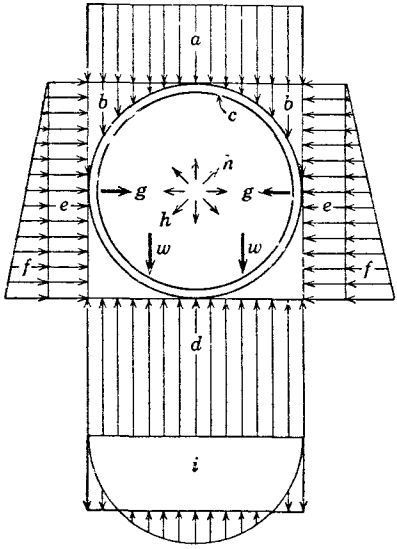


FIG. 25. Typical loading diagram: *a*, downward pressure of soil and water above crown; *b*, downward pressure of soil and water below crown, *c*, weight of tunnel lining, *d*, upward reactions assumed uniformly distributed; *e*, active horizontal loads due to soil and water above crown, *f*, active horizontal loads due to soil and water below crown, *g*, effect of tie rods or struts; *h*, internal pressures, *i*, adjustment to correct for distribution of hydrostatic uplift; *w*, downward internal loads

to have a parabolic distribution effective from 50° above to 50° below the horizontal axis. Analyses of passive pressures have been made on the basis of median passive pressures being developed on two 30° voussoirs above and two 30° voussoirs below the horizontal axis, or on a total arc of 120°, and also on the basis of a uniform median passive pressure acting on the 45° voussoir above and, similarly, as uniform median passive pressure acting on the 45° voussoir below the horizontal axis, or a total arc of 90°. In view of the many uncertainties involved, the latter and simpler basis of investigation would seem sufficiently accurate to approximate the effect of passive pressure in most cases.

Referring to Fig. 26:

Let p_1 = uniform median intensity of passive pressure lb per sq ft on 45° voussoir above horizontal axis

p_2 = uniform median intensity of passive pressure lb per sq ft on 45° voussoir below horizontal axis.

h_1 and h_2 = potential horizontal deflections in inches respectively at 22 1/2° above and 22 1/2° below the horizontal axis due to all active loads considered.

d = modulus of compressibility, of the soil, lb per sq ft per inch of compression.

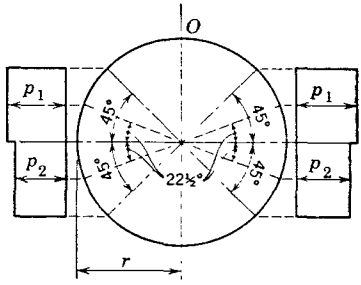


FIG. 26. Diagram (passive pressure). Neutral axis of tunnel ring symmetrically loaded about vertical axis, approximation of passive earth pressures.

Then it may be shown that:

$$p_1 = \frac{dh_1 + d^2(c_2h_2 - c_1h_1)}{(1 - c_1d)^2 - (c_2d)^2} \tag{12}$$

and

$$p_2 = \frac{dh_2 + d^2(c_2h_1 - c_1h_2)}{(1 - c_1d)^2 - (c_2d)^2} \tag{13}$$

where

$$c_1 = -\frac{59.2r^4}{EI} \tag{14}$$

$$c_2 = -\frac{45.9r^4}{EI} \tag{15}$$

The moments and thrusts due to p_1 and p_2 are then included in the analysis.

The extreme fiber stresses in the tunnel ring for each section of the lining to be considered, as at crown, springing line, and invert, are computed by first taking the algebraic sum of moments and thrusts for each such section, and applying the formula

$$S = \frac{P}{A} \pm \frac{Mc}{I} \tag{16}$$

where S = extreme fiber stress, pound per square inch.
P = net thrust, pounds per foot of tunnel.
M = net moment, inch-pounds per foot of tunnel.
A = net area of section, square inches per foot of tunnel
I = moment of inertia of section inches^4.
c = distance of extreme fiber from neutral axis.

This formula is approximately correct for all cases where the radius of the lining is large in comparison to the depth of the lining.

Where the tunnel ring structure is articulated, as in the case of a tunnel lining made up of cast-iron segments bolted together, the joint condition should be investigated and the stresses determined for this special condition.

The data presented in the foregoing are necessarily brief but will serve as basic groundwork for the development of as elaborate treatment as may be desirable.

An investigation of stress in a tunnel lining on the basis of the individual treatment of component loadings, active and passive, as indicated in Figs 25 and 26 makes possible a study of the effects of each of the component loadings on stresses and deflections. A convenient method of studying such effects individually or in any desired combination is especially valuable in the investigation of stresses in the linings of subaqueous shield-driven tunnels, where conditions vary considerably after the lining is erected in the tail of the shield. The lining may have to be temporarily braced with tie rods or struts as it leaves the shield and is exposed to temporary unbalanced loadings before development of final loading conditions and passive pressures; gravel packing is placed in the space left outside the lining by the overlap of the shield as it moves ahead, and may be followed by grouting to fill the voids in the gravel. When no longer necessary, tie rods or struts are removed; air pressure is eventually reduced to normal; temporary clay blanket on the river bed over the tunnel is removed; and finally, interior lining and structure may be placed. The characteristics of the soil may be subject to changes due to the disturbances caused by shield driving, such changes being especially apparent in Hudson River silt or similar material when the soil is largely displaced by the bulkheaded shield, rather than excavated. Under these conditions it has been observed that the vertical diameter of the tunnel lining usually elongates as the lining leaves the tail of the shield, and that there is a tendency for the lining to rise which may be counteracted by admitting some of the silt into the tunnel to act as a counterweight. This distortion of the lining, indicating an approach to fluid pressure conditions, is usually followed by a shortening of the vertical diameter over a period of time, with ultimate return to about its original dimension, indicating a consolidation and stiffening of the surrounding silt, accompanied by decrease in active lateral pressures.

12. SAFETY IN TUNNELING

Tunneling has a comparatively high incidence of injuries and fatalities. The cramped working space, the use of heavy equipment and large amounts of explosive, the sudden development of unexpected hazards, such as water under high pressure or quicksand conditions; the relatively short construction period of each project, accompanied sometimes

with a high turnover rate in employment, together with the necessity for fast progress, are listed in the *U. S Bureau of Mines Bulletin* 439, 1941, as among the contributing factors to the accident rate in tunneling. This bulletin also discusses the principal causes of injuries and fatalities in tunneling, listed in order of importance as follows based on statistics compiled from projects in the United States: (1) falls of rock; (2) haulage accidents; (3) shaft accidents; (4) gas and dust, (5) explosives, (6) falls from stages and jumbos, (7) falling equipment; (8) electrocution. Accidents in which two or more persons have been killed have been caused by explosives, cave-ins, haulage or hoisting failures, methane-gas explosions, sudden flooding, and fires

Accidents in tunneling can be reduced in number by competent, safety-conscious supervision; education; discipline; job safety organization; safety devices and equipment; adequate methods for detection of toxic and explosive gases; control of dust and gas by means of ventilation; adequate lighting; thorough scaling in rock tunnels and prompt timbering where required, strict adherence to explosive and blasting precautions; and fireproof construction.

Wet drilling and other approved methods of dust control are employed in rock tunneling where the character of the rock and conditions of work require precautions against silicosis

Careful selection of men for work in compressed air, enforcement of regulations, proper medical supervision, and the observance of the special rules of hygiene recommended to compressed-air workers have done much to reduce the incidence of compressed-air illness Statistics for compressed-air illness on major tunneling projects of the United States show that, since 1920, severe cases have been very few and that there have been no fatalities due to compressed-air illness, whereas the statistics of earlier projects show a comparatively high rate of severe cases, some of which were fatal or resulted in permanent injury. Because of the high oxygen content of the atmosphere, special precautions must be observed against fire in compressed-air work.

SELECTED DATA REGARDING TUNNELS

13. RAILROAD TUNNELS

(See also data under Art. 17, Shield-Driven Tunnels; Art. 18, Roof Shields; and Art. 19, Subaqueous Tunnels Built from Surface)

(21) Florence-Bologna, Italian State Railways. Across Apennine Mountains. Longest double-track tunnel in the world. Soft schist, shale, and clay formation heavily timbered; there were places where squeezing pressures, explosive gases, and large volumes of water under great heads were encountered. Middle length of 4000 ft widened to 56 ft for four-track passing and switching yard. Driven from portals and two central shafts at enlarged section, 410 ft apart Shafts sunk inclined at 50%, 843 ft deep. Two railway lines, respectively 17 miles and 13 1/2 miles long, were constructed to serve portal operations; central shafts served by aerial tramway 5.6 miles long. Excavation by center bottom drift, crown heading; drift and heading connected by 3 ft by 3 ft shafts; enlargement from drift and heading. Methane gas burned off by electric ignition; maximum water 300 gps at about 200 lb per sq in. Cost $24,735,000. Average progress: 9 ft per day south half; 7.5 ft per day north half.

(22) Moffat Tunnel. 8 ft by 8 ft pioneer tunnel, 75 ft south of main tunnel with 20 crosscuts to it. Heading encountered 1500 gpm flow of water. Much of roof of main tunnel required timbering. Top heading and bench in timbered section; center heading in good rock.

(23) New Cascade Tunnel, Washington. 8 ft by 9 ft pioneer tunnel. Main tunnel enlargement from center heading.

(25*a*) Hagans. Limestone, shale, and sandstone. 4660 ft timbered. Average progress, excavation, 2 shifts per 24 hr, 17.6-ft heading, 11.6-ft bench; lining 328.6 ft per month.

(27) Storm King, New York. Granite and gneiss. Part parallel and adjacent to existing tunnel which was enlarged to 33 ft wide. Proximity of old tunnel and Catskill Aqueduct shaft for siphon crossing Hudson River required great care in blasting An annular cut around the periphery of tunnel contour 6 ft deep. Entire face drilled with 6-ft holes and blasted, using delays.

(30) Western Pacific, 9 tunnels. 5 tunnels in shale and conglomerate. 4 tunnels in volcanic cinders. Three 8-hr shifts. Average 4 ft to 8 ft per day. No shafts or adits. 2 tunnels 6 ft by 6 ft pioneer drift at ceiling grade; 7 tunnels, heading to springing line, bench to subgrade. Timber sets 18 in. to 48 in. centers.

Table 2

Number and Name	Single (S) or Double (D) Track and Date	Length, mi	Clear Height, ft	Clear Width, ft, Straight (S), Curved (C), Straight and Curved (Q)	Material, Excavation Method	Lining	Cost per Linear Foot
(1) Box, Great Britain	D1837-41	1 8	28.0	30.0	Rock E	Brick	$162
(2) Mont Cenis, France-Italy	D1857-72	7.9	24 6	26 2 Q	Rock B	B and M	272
(3) Hoosac, Mass	D1854 76	4.7	19.8 (avg)	25 0 S (avg)	Rock W	Brick	399
(4) St. Gothard, Switzerland	D1872 82	9.3	24 6	26.2 Q	Rock B	B and M	230
(5) Arlberg, Austria	D1880-83	6.5	25.0 [a]	26.3 S	Rock L	Rubble	180
(6) Severn, Great Britain	D1879-86	4.4	24 5	26 0 Q	Mixed E	Brick	208
(7) Stampede, Northern Pacific Ry.	D1886 88	1.9	22.0	16.5	Rock H	B and C	118 exc.
(8) Ronco, Italy	D1882-89	4.8		.	Rock X	B and M	. .
(9) Totley, Great Britain	D1888 92	3.5	26 2	27.0 Q	Mixed R	B and M	. .
(10) Cascade, Great Northern Ry.	S1897-00	2.6	21.5	16.0 S	Rock K	Concrete	. .
(11) Simplon, Switzerland-Italy	D1895 06	12.4	18.0 [a]	16.4 Q	Mixed L	Rubble	240
(12) Pennsylvania Twin, N. Y.	D1905-09	1 8	18.2	16 3 Q	Rock H	B and C	. . .
(13) Pennsylvania 3-Track, N. Y.	1907-09	0.2	18 2	39 7 S	Mixed Y	B and C	.
(14) Bergen, Pennsylvania RR	D1905-09	2.2	18 3	19.0 S	Rock H	B and C	.
(15) Arthurs Pass, N. Z.	S1908-	5.31	16.75	15 S	Rock R	Concrete	. . .
(16) Loetschberg, Switzerland	D1906-13	9.03	22.3	25 0	Rock L		. .
(17) Mt. D'Or, France and Switzerland	1910-15	3.8	20.0	26.2	Rock L	Masonry	. . .
(18) Rogers Pass, B. C.	D1914-16	5.0	21.12	29.0 S	Rock Z	Concrete	. .
(19) Mt Royal, Montreal	D1912-18	3.25	19 75	13.0	Rock L	Concrete	. . .
(20) Tanna, Japan	D1918-	4.85 when complete	22.75	28 0	Variable (volcanic)	Concrete	. . .
(21) Florence-Bologna	D and 4-track 1923-28	11 5		29.0 (56.0 4-track)	Rock and clay Z	Masonry	. .
(22) Moffat, Colo.	S1924-27	6.2	24	16	Rock Z	Concrete	470
(23) New Cascade, Wash.	S1926-28	7 75	24	16	Rock Z	Concrete	. . .
(24) *Illinois Central RR, Edgewood Cutoff*							
(a) Tunnel 1	S1927-28	0.15	23.3 [a]	18.0 C	Rock H	Concrete	324
(b) Tunnel 2	S1927-28 and 1941	1.32	25.3 [b]	18.0 S	Rock H	Concrete	368
(c) Tunnel 3	S1933-35	0.50	22.0 [a]	16.0 arch, 17.5 walls, S	Rock H	Concrete	311
(25) *L&N RR Co.*							
(a) Hagans	S1927-30	1.18	24.0 [a]	16.0	Rock H	Concrete	212.70
(b) Rock House, Creek Branch	S1948	. .	24.2	22.0	Rock	Concrete	462.05
(26) *Missouri Pacific RR*							
(a) Grey Summit	D1928-29	0.30	24.8 [a]	29.0 S	Rock	Reinforced concrete, 1′ 3″ crown, 1′ 9″ walls	295
(b) Labadie	D1928-29	0.10	24.8 [a]	29.0 S	Rock		350

(27)	New York Central System, Storm King, N. Y.	D1928	0 09	23.0 [a]	35.0 C	Rock M	Concrete	411
(28)	Koontz Bend, W. Va , NF&G Ry.	S1930	0 61	22.8 [c]	18.0 S	Rock H	Concrete	210
(29)	Carnifax, NF&G Ry.	S1930	0 18	22.8 [c]	18 0 S	Rock H	Concrete	173
(30)	The Western Pacific RR, 9 tunnels, Calif.	1930–31	1.05 aggregate	22.5 [d]	17 to 18 S and C	5 tunnels, rock Z, 4 tunnels, volcanic cinders Z	12″ × 12″ timber sets	150.29 avg for 9
	(31) *C&O Ry. Co.*							
(a)	Allegheny (westbound), Va. and W Va	S1930–32	0.90	22.0 [e]	18.0 S	Rock K	Concrete	182.60
(b)	Lewis (eastbound), Va.	S1930–33	0.58	22.0 [e]	18 0 S	Rock H	Concrete	172.79
(c)	Big Bend (eastbound), W. Va.	S1930–34	1.17	22.0 [e]	18.0 S	Rock H	Concrete	195.82
(d)	Manns (westbound), W. Va.	S1931–32	0.17	22.0 [e]	18.0 S	Rock H	Concrete	147 40
(e)	Blue Ridge (new), Va.	S1942–43	0.75	22.0 [e]	18.0 Q	Rock M	Concrete	280.39
(f)	Fort Spring, W. Va.	D1945–47	0.53	22 0 [e]	32 0 S	Rock M	Concrete	588.00
(g)	Pine Mountain, Ky and Va.	S1946–49	0.69	22.0 [e]	16 0 S 18.0 S 20.0 S 22.0 C	Rock Z	Reinforced concrete, timber	585.00
(h)	Carver, Ky.	S1948–49	0.29	Steel 24.9 [e] Concrete 22.3 [e]	Steel 21 7 S Concrete 20.0	Rock M	Steel ribs and posts, timber lagging, future, concrete	401.28
(i)	Trace Fork, W. Va.	S1948–50	0.54	Steel 24 9 [e] Concrete 22.3 [e]	Steel 21.7 S Concrete 20.0	Rock M	Steel ribs and posts, timber lagging, future, concrete	379.10
	(32) *B&O RR*							
(a)	Lane Tunnel, W. Va.	S1936	0.45	23.1 [a]	18 0 S	Rock H	Concrete 1′ 11″ min.	.
(b)	Knight Tunnel, W. Va.	S1937	0.10	24.8	20.5 C	Rock H	Concrete 1′ 11″ min.	..
(c)	Lough Tunnel, W. Va	S1948	0.29	24.1	18.0 S	Rock M	Concrete 1′ 10″ min.	...
(d)	Tunnel 1, Clarksburg, W. Va.	D1950–	0 62	28.0	31 0 S	Rock M	Concrete 1′ 10″ min	.
(33)	Shasta Dam, Calif., relocation So. Pacific Co , 12 tunnels	S1939–40	3.60 (12 tunnels)	22.5 [c] and 24 0	17 0 and 18.0, S and C	Rock H and M	Concrete	163–282
(34)	Bozeman, Northern Pacific Ry , Montana	S1944–45	0.58	24.0 [a]	18 0 S	Volcanic tuff M	Concrete	350
(35)	Saltsburg Tunnel, Pa.	D1946	0.50	25.5 [c]	30 0 S	Rock Z	Concrete	714
(36)	Boysen Tunnel, CB&Q RR, Wyo	S1947–50	1 35	25.7	17.0 S	Rock Z	Concrete	
(37)	Altamont, Union Pacific RR, Wyo.	S1949	1 27	24.0	18 5 S	Rock H and M	Reinforced concrete	1000
(38)	New Elkhorn, N&W Ry , W. Va.	D1950	1 33	30 5 [c]	31 0 S	Rock and coal M	Reinforced concrete	800

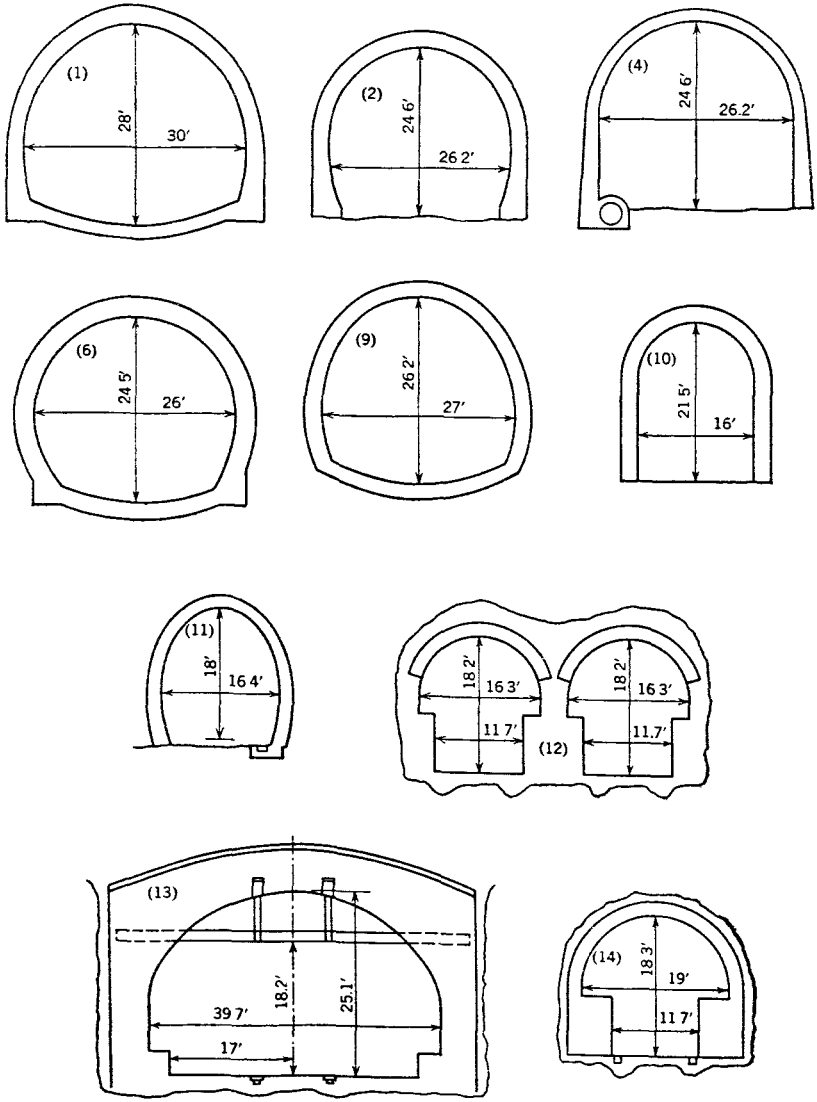
Height [a] Top of rail to crown, [b] Invert to crown, [c] Base of rail to crown, [d] Top of rail to top segment, [e] Top of rail to arch at 4 ft off track center

B and M = brick and stone masonry. B and C = brick and concrete B = Belgian method E = English method H = top heading and one bench K = top heading and two benches L = bottom heading, from which small shafts at intervals are driven to top of tunnel and full width top headings are excavated in both directions, the bench is removed between the top and bottom headings and the lower section of the tunnel is enlarged to full width. M = the full tunnel prism is removed as one operation. R = bottom headings followed successively by their enlargement laterally and upward W = top heading and one bench and English method. X = Belgian and English method. Y = top heading and one bench and cut and cover method Z = see description of methods in the notes

(31*e*) Blue Ridge (new). Greenstone with broad band of granitoid about midway of the excavation. 5 lb 40% gelatin per cu yd. Mucking machines. 60-ft forms, pneumatically placed concrete. Average progress two 10-hr shifts: excavation, 5.78 ft; lining, 18 ft.

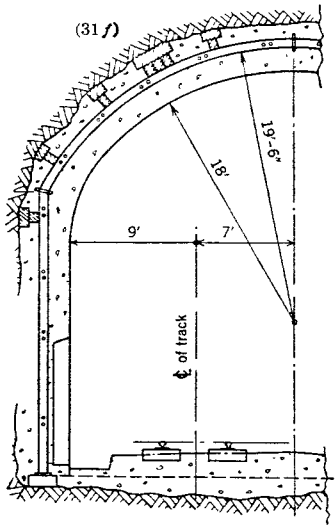
(31*f*) Fort Spring. Limestone Steel posts and ribs spaced 14 in. to 2 ft 6 in. c.c. Driven from both ends. 2 1/2 lb 40% ammonia gelatin per cu yd. Steel posts and ribs 14 in to 3 ft c.c. Muck 1 3/4 cu yd shovel. Average progress three 8-hr shifts: excavation, 10 ft; lining, 30 ft.

(31*g*) Pine Mountain. Large amount of water. Rock bedded 20°, badly broken and faulted, shale, sandstone, soft to hard. Steel arch supports, 2 ft, 3 ft, and 4 ft centers

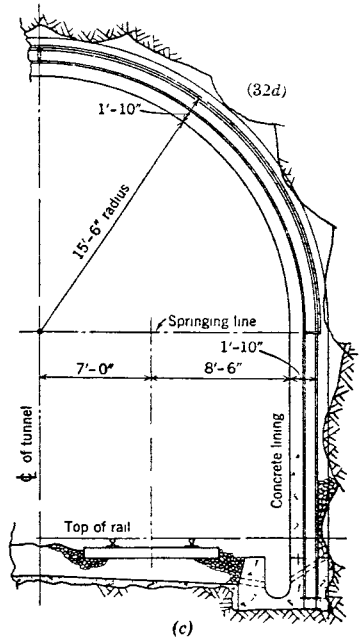


(*a*)

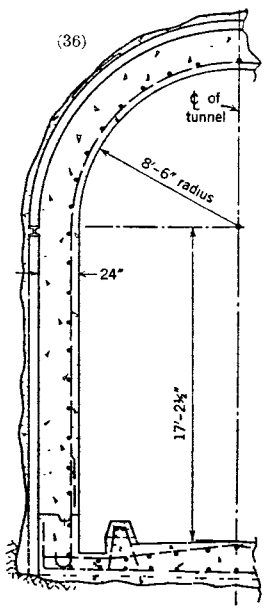
FIG. 27. Sections of railroad tunnels. Numbers in



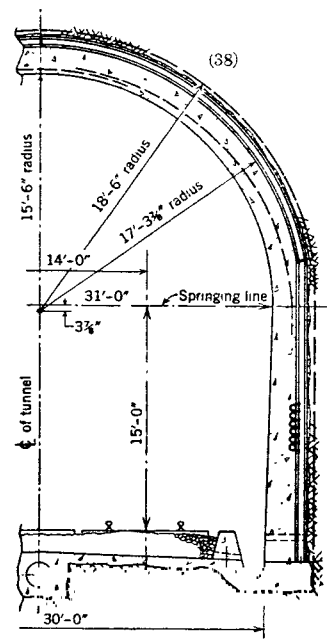
(b)



(c)



(d)



(e)

parentheses refer to tunnels as numbered in Table 2.

when used. Timbered sections 16 ft, 18 ft, 20 ft, and 22 ft wide. All full heading except 800 ft center drift. Two headings from portals. 2 lb 40% dynamite per cu yd shale; 3 lb per cu yd sandstone. Average progress three 8-hr shifts for both headings: excavation full face, 12 ft; excavation heading, 5 ft; lining, 20 ft.

(31*i*) Trace Fork. Sandstone 1274 ft, hard blue shale 1600 ft. Supported full length by steel ribs and posts 18 in. to 6 ft c.c., 4-in. lagging. Two headings from portals. 40% tunnel gelatin, 3.8 lb per cu yd. Average progress two 10-hr shifts both headings: excavation 21 5 ft; lining 21.5 ft.

(33) Shasta Dam Relocation. Generally shale and sandstone. Average advance 10 ft per round, 1 lb dynamite per cu yd. Steel supports used where required. Average progress for 24-hr period, 3 shifts: full face excavation, 8 ft; for drifting in headings, 13 ft. Avg concrete-lining progress· 50 ft per day.

(35) Saltsburg Tunnel. Thin to massive bedded sandstone. Pilot tunnel 14 ft by 16 ft, no support. Main bore: 1 lb 45% dynamite per cu yd. Steel ribs spaced 4 ft c.c. throughout length. Progress, three 8-hr shifts: excavation and ribs, average 10 ft; pilot tunnel, average 13 ft; lining progress, average 17 ft, three 8-hr shifts. No shafts or adits. Constructed by U. S. Corps of Engineers for Pennsylvania Railroad.

(36) Boysen Tunnel, Wyoming. Full heading through major portion of tunnel. Highly folded and faulted sedimentary formation. Wall plate drift method through some heavily faulted areas. Excavation supported with 8-in. WF structural steel supports, variable spacing, 4 ft c.c. through diorite to 8 in. c.c. through faulted and saturated shales. Full face, used 65–85 holes, 10 ft deep, 350 lb 40% gelatin powder per round. Mucking by electric overshot muckers on tractor frames. Invert concrete placed first; 75-ft steel forms for walls and crown; concrete placed by Pumpcrete machine. There were three cut and cover sections, permitting work to proceed at eight headings.

(38) New Elkhorn Tunnel, West Va. Sandstone with thin seams of slate and coal. Excavated full face heading from each portal; jumbo carried 15 water-line drills; detachable bits used. 210–220 holes per shot; depth varied from 3 ft to 18 ft; average pull, 13.5 ft, maximum, 18 ft. 40% and 60% gelatin explosive, avg 3 lb per cu yd; mucking with two $^3/_4$ cu yd shovels; haulage 15 cu yd back dump diesel trucks. 4000 cfm capacity air for drills. Excavation supported throughout with steel ribs and posts normally spaced 30 in. centers; 18 in. spacing for small part of length. Full semicircular arch, reinforced concrete 1 ft 6 in. thick inside 8 in. H beams; walls plain concrete 1 ft 6 in. inside steel at spring line to 2 ft 0 in. at 2 ft below base of rail. 4-in. weep holes 5 ft c.c. each side. Tracks 14 ft 0 in. c.c. Average progress: excavation, each heading, 3 shifts 13.5 ft; lining, two 8-hr shifts, 48 ft; concrete placed in forms by pumping. Ventilation during construction 21,000 cfm through 30-in. pipe each heading. Permanent ventilation, 2 fans at west end, each operated by 400 hp electric motor, supply 1,000,000 cfm forced through tunnel west to east, 12–14 mph.

14. HIGHWAY TUNNELS

(1) The Holland Tunnel for vehicular traffic extends under the Hudson River from the vicinity of Canal Street, in Manhattan, to 14th Street in Jersey City. It was the first tunnel constructed to carry motor vehicles under a body of water in the New York area, and the first for which the transverse ventilation system was designed and installed. In this respect, as well as in many other important features of design, construction, and operation which required research and original treatment, it has served as a pattern for most of the vehicular tunnels built subsequent to it, both in the United States and abroad.

The tunnel consists of twin cast-iron-lined tubes, 29 ft 6 in. and 30 ft 4 in. in exterior diameter, built by the shield method, with steel bent and concrete sections built by cut and cover, and of plazas and approach sections constructed in open cut. The tunnel interior is lined with concrete and the walls are finished with tile. The roadways provide for two lanes of traffic in each tube, the northerly tube carrying westbound traffic and the southerly tube carrying eastbound. 84 motor-driven fans are housed in four ventilation buildings, two on each side of the river, providing for a normal maximum ventilation of 3,760,000 cu ft of fresh air per minute. Each building connects to both tubes by means of shafts which were sunk as steel and concrete caissons. The maximum upgrade in the tunnel is 3.5 and the maximum downgrade is 4.06%. Maximum depths: mean high water to top of tunnel 72 ft; to roadway 93 ft.

There were 13,795 linear feet of shield-driven subaqueous tubes. Of this, 2,412 ft was through mixed river mud, 1,900 ft mica schist rock, 7,650 ft Hudson River silt, and 1,833 ft through sand-clay and cinder fill. In addition, 2,012 ft of cut and cover and 649 ft of open approach were built in New York, and 1.082 ft cut and cover and 970 ft of open approach in New Jersey. The New York powerhouse from which two shields were operated fur-

nished 28,000 cu ft free air per minute at a pressure of 50 lb per sq in. and 3,550 at 125 lb per sq in. The shields were operated by three 25 gpm pumps at 5,000 lb per sq in. pressure. The New Jersey powerhouse from which four shields were operated furnished 40,000 cu ft free air per minute at a pressure of 50 lb and 2,600 at 125 lb. The shields were operated by four 25 gpm pumps at 5,000 lb pressure. All erectors operated at same pressure as jacks. The average progress was 295 ft per month in silt, 142 ft per month in mixed ground, and 57.6 ft per month in rock. This progress included all delays due to starting shields, placing bulkheads and locks, and passing through shafts. In the Hudson River silt the shield could be pushed without taking any material into the tunnel but that procedure distorted grade and alignment of tunnel lining for a considerable distance back of the shield. To control the tendency of the lining to rise, varying quantities of material were taken through the shield ranging from about 25% to 75% of the theoretical displacement per shove. This material was left in the tunnel for ballast until after driving was completed and then mucked out in free air with a monorail handling a clamshell bucket.

The air pressure was seldom above 30 lb per sq in. All lining is cast iron calked with lead, except that for a short distance east and west of the New Jersey shaft the lining is of steel castings. Bolts in the lining are 1 3/4 in. diameter high tensile steel of 110,000 lb per sq in. ultimate strength and 85,000 lb per sq in. yield point. This was a successful innovation as it reduced handling costs and waste due to stripped threads. All shafts were sunk as pneumatic caissons. Each river shaft on the New Jersey side rests on 42 24-in. diameter steel pipe piles filled with reinforced concrete.

The cost of 13,795 ft of shield-driven tunnel was $17,050,000. The four shafts cost $3,100,000. The ventilating machinery cost $572,000. The cost of approaches, street changes, paving, and housing for ventilating machinery and purchase of real estate, which included a number of city blocks, brought the cost of the whole project up to $48,400,000 not including interest during construction.

(3) Posey Tube, under estuary between Oakland and Alameda, Calif. 11 prefabricated reinforced-concrete sections each 203 ft long and 12th section 161 ft long sunk in trench Outside diameter 37 ft, inside 32 ft. 3 sections curved. Outside waterproofed with timber protection. Used water ballast in duct under roadway and 950 tons of sand, 3.5 ft thick, on roadway slab in sinking, sprayed water on sand to obtain desired sinking weight. Typical full transverse method of ventilation, two vent buildings at portals, 4 blower and 4 exhaust fans each building; 40 air changes per hour.

(4) Mersey Tunnel. Under Mersey River between Liverpool and Birkenhead. Generally constructed in fissured and water-bearing sandstone formation. Main tunnel provides 4 lanes, 2 for each direction of travel. Under-river portion 5274 ft long, circular cast-iron lined 46 ft 3 in outside diameter with 44 ft clear diameter inside. Reinforced-concrete roadway deck 18 in. below spring line. Main land approach tunnels practically semicircular, cast-iron lined, 4 lanes. On each side of the river there is a dockside approach, horseshoe section, 26.5 ft inside diameter carrying two lanes of traffic, one in each direction. On both main and dockside approaches the entire space under the roadway is for fresh-air supply; in under-river section, two partition walls form large central compartment for pipe lines or future form of transportation, with outer compartments for fresh air. Lowest point of roadway 150 ft below high water with 60 ft cover over crown. Minimum rock cover at deep point, 3 ft 6 in. Maximum grade, 3.33%. Tunnel driven from two shafts one mile apart; pilot headings 15 ft by 12 ft driven under river and completed before enlargement to full section. For semicircular sections, single heading; for circular section, top and bottom headings. Enlargement to full section and erection of lining begun at several points. In circular section, upper half enlarged and lined first, followed by underpinning top half with lower half. Circular section cast-iron ring 24 in. long made up of 24 segments. Roof shield used for 1,100 ft of main approach in Liverpool where top of tunnel was in clay. (See Art. 18, Roof Shields.) Compressed air to keep out water not used in any tunneling operations. Fresh air from ducts under roadway delivered through slotted opening along face of curbs. Foul air flows along roof space of tunnel and is drawn out by exhaust fans at each vent building. There are 6 fan plants, 3 each side of the river. Centrifugal fans: largest have 28 ft diameter impellers and capacity of 641,000 cfm; smaller 92,000 cfm. Two-speed electric motors with reducing gear with hydraulic coupling between motor and gear for further speed adjustment. All plant in duplicate at each building. Capacity 2,500,000 cfm fresh air for blowers; exhaust approximately same. Project cost $35,389,000.

(6) Detroit-Windsor. Crosses under Detroit River, U. S. to Canada. 2,200 ft prefabricated sunk in trench, 6 sections; shield-driven under compressed air, structural steel lining, 1,243 ft in Windsor and 467 ft in Detroit; steel bent and concrete, 600 ft in Windsor and 627 in Detroit. Full transverse method of ventilation, two ventilation buildings, one each side of river, 6 blower and 6 exhaust fans each building; capacity for tunnel, 1,000,000

Table 3

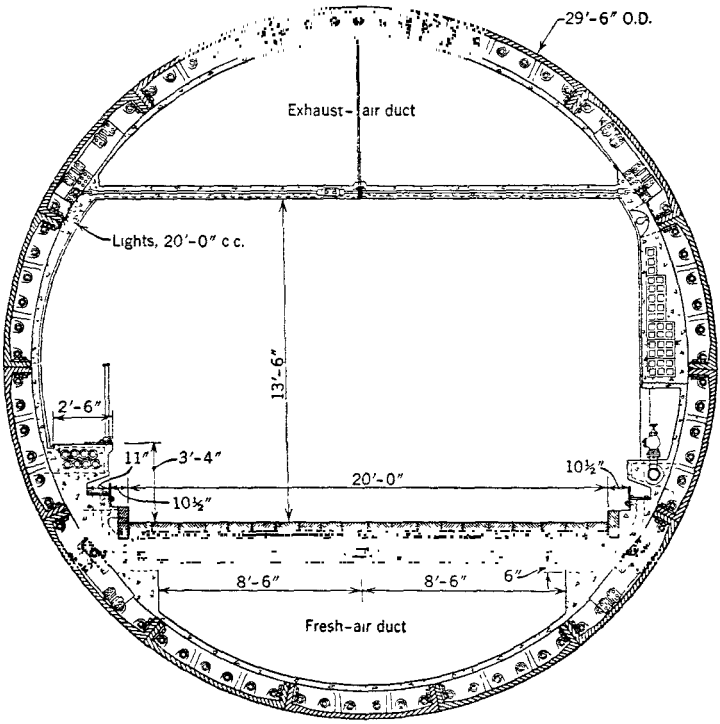
Number and Name	Date Built	Number of Bores	Length Portal to Portal, ft	Width of Roadway, ft	Clear Height, ft	Tunnel Section	Material Excavated	Lining	Cost per Linear Ft or as Noted	Permanent Ventilation
(1) Holland Tunnel, N. Y.	1920–1927	2	8,557 N 8,371 S	20.0	13.5	Circular	Soft ground (silt), mixed face, rock	Cast iron	$48,400,000 total	Mechanical
(2) Liberty, Pittsburgh, Pa.	Opened 1924	2	5,889 including air traps	21.0	20.46 top of arch	Arch and sidewalls	Shale	Reinforced concrete	$5,995,000 total	Mechanical
(3) Posey, Oakland-Alameda, Calif.	1925–1928	1	3,545	22.83	14.83	Circular (trench)	Mud, sand, and clay	Reinforced concrete		Mechanical
(4) Mersey, England	1925–1934	1 4-lane; 2 branches, 2-lane	10,584 main, 3,216 aggregate for branches	36.0 main, 19.0 branches	Arched roof	Circular and semicircular for main; horseshoe branches	Fissured water-bearing red sandstone	Cast iron; inner concrete	$35,389,000 total	Mechanical
(5) Zion National Park, Utah	1927–1936	1	5,613 (including length of 6 galleries)	20.0	16 at ℄	Segmental arch radius = 13.88′, spring line 2.12′ above subgrade	Sandstone	Reinforced concrete 10″ to 12″ min., Gunite	$140	Natural (6 galleries opening on cliff-side)
(6) Detroit-Windsor, U. S.-Canada	Opened 1930	1	5,137	22.0	13.5	Circular (shield and trench)	Clay	Steel and concrete	$23,000,000 total	Mechanical
(7) Antwerp, Belgium	1930–1933	1	5,801	22.17	14.67	Circular (shield)	Heavy clay, sand	Cast iron and concrete		Mechanical
(8) W. 178 Street,	1930–1940	1	2,328	22.0	14.0	Rectangular	Rock	Steel bents and concrete	$1,802	Mechanical
W. 179 Street, Manhattan	1949–1952	1	2,394	22.0	14.0	Arched roof and invert	Rock	Reinforced concrete	$3,647	Mechanical
(9) East Boston, Mass.	Circa 1932	1	5,635	21.5	13.5	Circular (shield)	Clay	Structural steel and reinforced concrete	See description	Mechanical

(10) Wawona Road, Yosemite National Park, Calif.	1932–1933	1	4,233	24.0	19 0 at ℄	Semicircular arch radius = 14′, and 5′ side walls	Solid granite and diorite	Reinforced concrete 15″ to 18″ for 882′; Gunite, 3,351′	$125 including ventilation and light excluding pavement	Mechanical
(11) Breakneck N.Y.	1932	1	580	30	14.33 at curb	Horseshoe height 18 54′, arch radius 29.50′ inside	Gneiss	Concrete 2′ min	$300	Natural
(12) Lincoln Tunnel	1934–1945	2	7,400 N 8,215 S	21.5	13.62	Circular	Soft ground (silt), mixed face, rock	Cast iron; structural steel	See description	Mechanical
(13) Broadway Low Level Tunnel, Oakland, Calif.	1934–1937	2	3,203 and 3,135	22.0	15.67 (at ℄ to ceiling)	Horseshoe 26 67′ wide, 30′ high, inside	Shattered shale	Reinforced concrete, 2′ 1 1/8″ min.	$660 one tube	Mechanical
(14) Yerba Buena Island, San Francisco-Oakland Bay Bridge	1936	1	540	58 0 upper deck. Lower deck: truck lanes 31 0, railroad space 27.0 horizontal	29 0 at ℄ upper deck; 16.0 for truck lane; railroad space, 20.0	Multiradius arch and straight sidewalls	Shattered shale	Reinforced concrete, 3′ min.	$1,580	Natural
(15) Arch Cape, Oreg.	1936–1937	1	1,287	26 0	21.0 at ℄	Semicircular arch, 16′ radius to future lining, ℄ 5 08′ above paving	Sandstone and and shale	Timber (future reinforced concrete)	$157	Natural
(16) Queens Midtown Tunnel, N Y	1936–1940	2	6,414 N 6,272 S	21.6, 23.5 on Manhattan curves	13.5	Circular except cut and cover	Rock, mixed ground, earth, riprap, etc.	Cast iron; structural steel; steel bents and concrete	See description	Mechanical
(17) Big Oak Flat, Yosemite National Park, Calif.	1936–1938	1	2,167	24.0	19.0 at ℄	Semicircular arch, radius = 14′, and 5′ sidewalls	Granite	Reinforced concrete, 18″ min.	$119 (unlined)	Natural

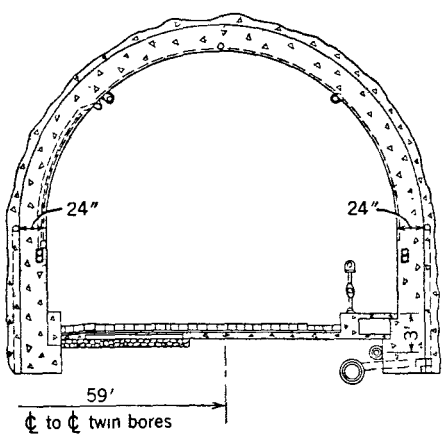
Table 3—*Continued*

Number and Name	Date Built	Number of Bores	Length Portal to Portal, ft	Width of Roadway, ft	Clear Height, ft	Tunnel Section	Material Excavated	Lining	Cost per Linear Ft or as Noted	Permanent Ventilation
(18) Waldo, north approach to Golden Gate Bridge, Calif.	1937	1	1,000	42.0	28.75 at ℄	Semicircular arch, 23.5′ rad, ℄ 5.25′ above paving	Shattered shale	Reinforced concrete, 3′ and 2′ min.	$483	Natural
(19) Elephant Butte, Feather River, Calif.	1937	1	1,187	26.0	16.0					Natural
(20) Funstan Avenue, San Francisco, Calif	1938	1	1,300	44.0	18.0					Natural
(21) Maas Tunnel, Rotterdam	1938–1941	See description	3,512	19.8	13.8					Mechanical
(22) Pennsylvania Turnpike Tunnels, Pennsylvania Turnpike Comm.	1939–1940	1	7 tunnels aggregating 35,296 5	23.0	14.33	Arch and sidewalls, flat invert	Shales, sandstone, and slates, variable	Reinforced concrete, 18″ to net line, 24″ to pay line	$350 avg	Mechanical
(23) Tunnel Under the Croix-Rouse, Lyon, France	1939–1952	1	5,841	39 4 (4 lanes)	14.1 at curb, 19.7 at ℄	Semicircular	Rock (gneiss, schist, granite), soft material at portals	Concrete		Mechanical
(24) Brooklyn Battery Tunnel, N. Y.	1940–1950	2	9,117	21.33	13.5	Circular (except cut and cover sections)	Rock, mixed ground, earth	Cast iron, reinforced concrete, and steel bents and concrete	See description	Mechanical
(25) Battery Park Underpass, N. Y.	1948–1951	4 traffic lanes in one open-cut trench	2,253	22.0 min. 24.0 on curves	14.0 min.	Rectangular	Earth, fill, and rock excavated in trench	Reinforced concrete box	$3,290	Mechanical
(26) Bankhead, Mobile, Ala.	Opened 1941	1	3,108.6	21.0	Arched roof	Circular (trench)		Steel and concrete	Total $4,000,000	Mechanical

(27) West Rock, Conn.	1948–1949	2	1,200	23.0	14.0 at curb, 18.5 at ℄	Arch and sidewalls	Generally trap-rock	Reinforced concrete, 12″ inside steel, 18″ outside	Total $1,934,203	Mechanical
(28) Squirrel Hill, Pittsburgh, Pa.	1948–1953	2	4,225 including ventilation buildings	24.0	14.0	Arch and sidewalls	Shale and limestone	Reinforced concrete	See description	Mechanical
(29) Baytown, Tex.	1949–53	1	3,009	22.0	14.0 min. (arched roof)	Circular (trench)		Steel and concrete		Mechanical
(30) Washburn Tunnel, Pasadena, Tex.	Opened 1950	1	2,900	22 0	Arched roof	Circular (trench), horseshoe (cut and cover)	Silt, sand, clay	Steel and concrete		Mechanical
(31) Elizabeth River, Va	1950–1952	1	3,350	22.0	14.0	Circular (trench)				Mechanical
(32) Broadway, San Francisco, Calif.	1950–1952	2	1,616 including buildings	22.0	22.75 top of arched roof	Semicircular arch and sidewalls	Sand and rock	Reinforced concrete		Mechanical
(33) U N Underpass, B P. Manhattan, N. Y.	Opened 1953	2	1,377	24.0	14.25	Rectangular	Earth and Manhattan schist, excavated from surface	Steel frame and concrete		Mechanical
(34) Memorial Tunnel, West Virginia Turnpike	Opened 1954	1	2,801 including ventilation buildings	24.0	14.0	Semicircular arch, radius = 14 38, 11.42′ sidewalls	Sandstone, shale, coal seams	Reinforced concrete	$1,867 (estimate including mechanical, electrical, and buildings)	Mechanical
(35) Baltimore Harbor Tunnel	1954–1957	Twin tubes carrying 4 traffic lanes	7,650	22.0	14.0	Twin circles (trench), rectangular (steel bent and concrete)	Silt, sand, clay	Reinforced concrete inside circular steel shells	$4,700 per ft of twin-tube tunnel backfilled in trench, including shafts, finish, and paving	Mechanical



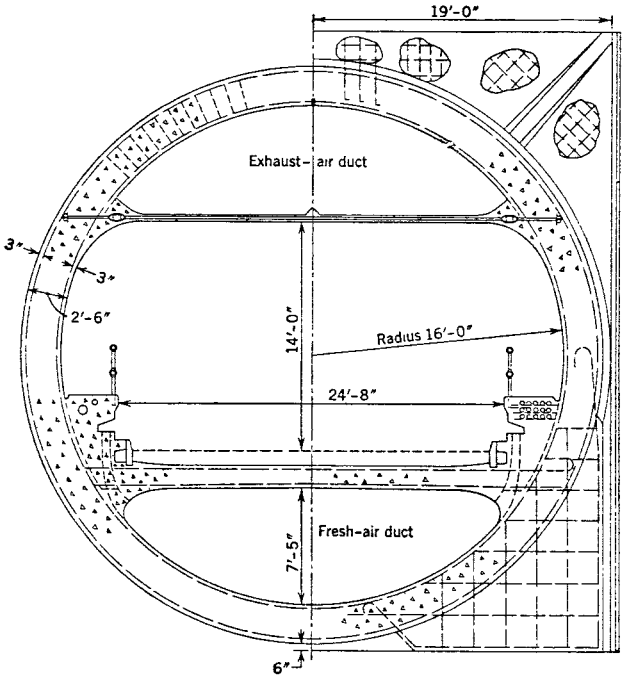
(1)



(2)

FIG. 28. Sections of highway tunnels. Numbers in parentheses refer to tunnels as numbered in Table 3.

(3)



Half Section *A–A*
Showing Typical Section

Half Section *B–B*
Showing End Collar

(4)

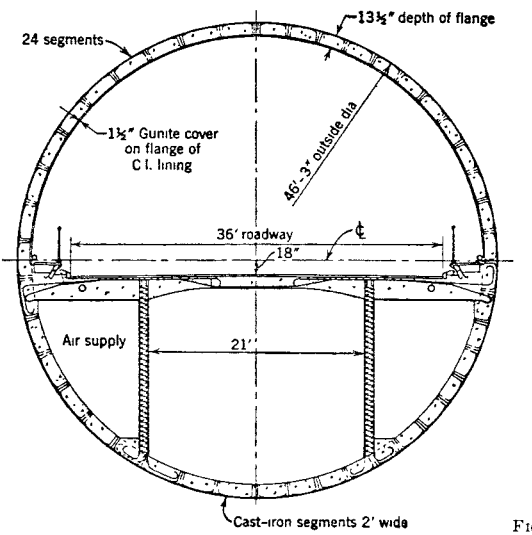
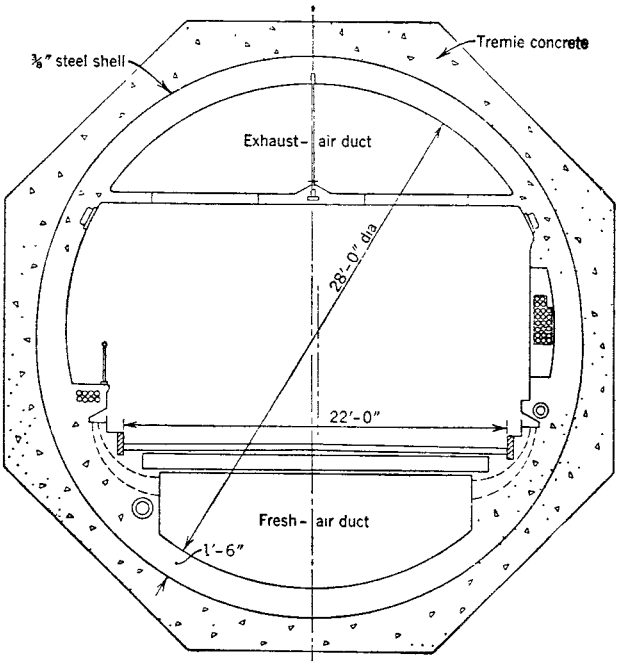
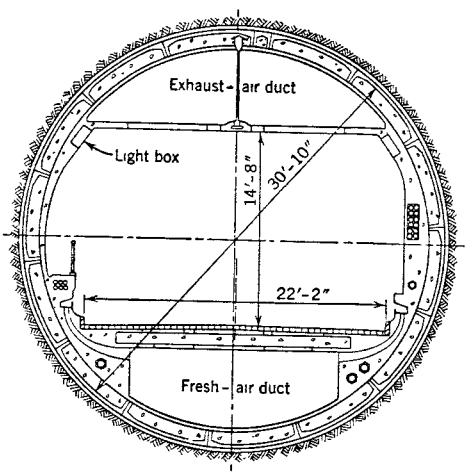


FIG 28—*Continued*

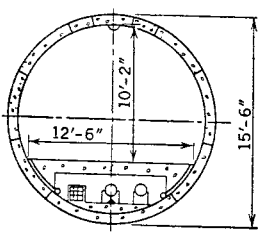


Typical Cross Section

(6)



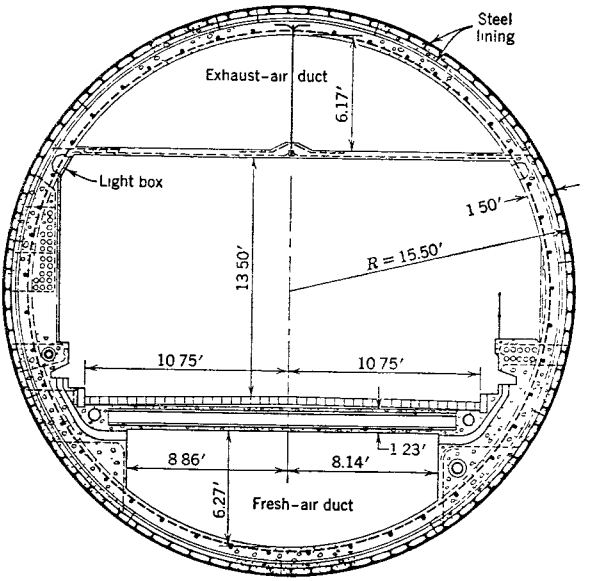
Cast-Iron Tunnel



Pedestrian Tunnel

(7)

FIG. 28—*Continued*



(9)

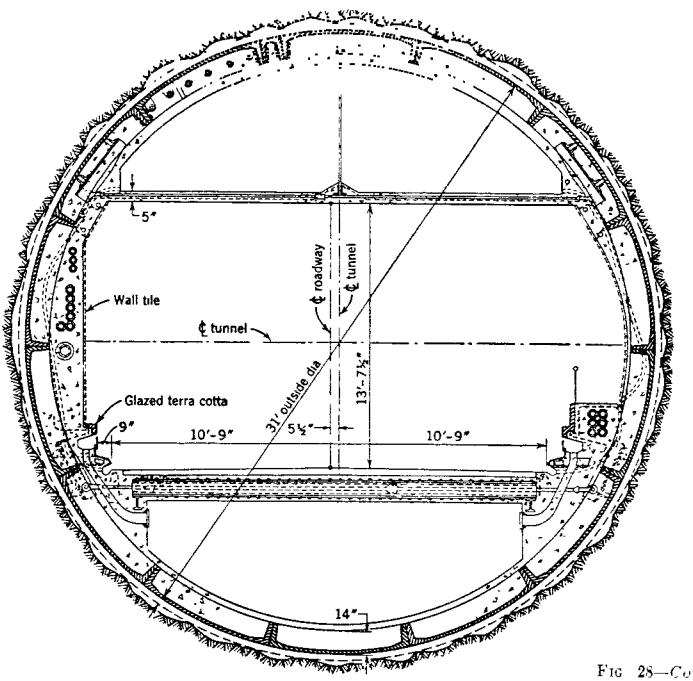
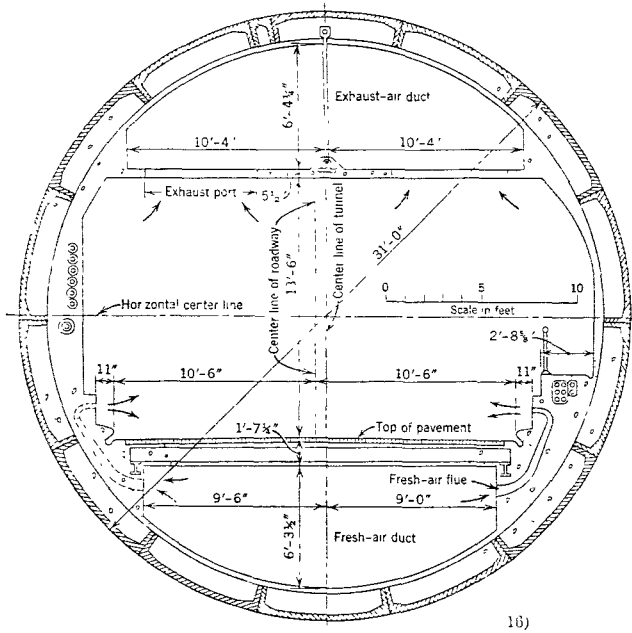


FIG 28—*Continued*

(12)



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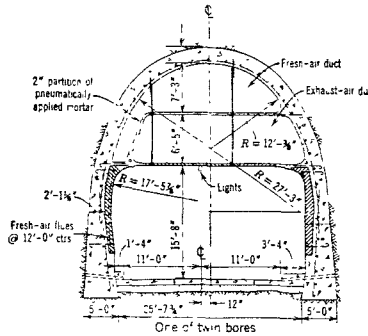

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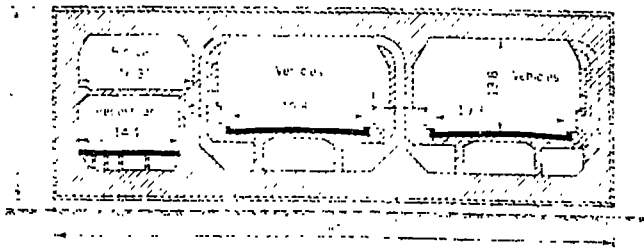
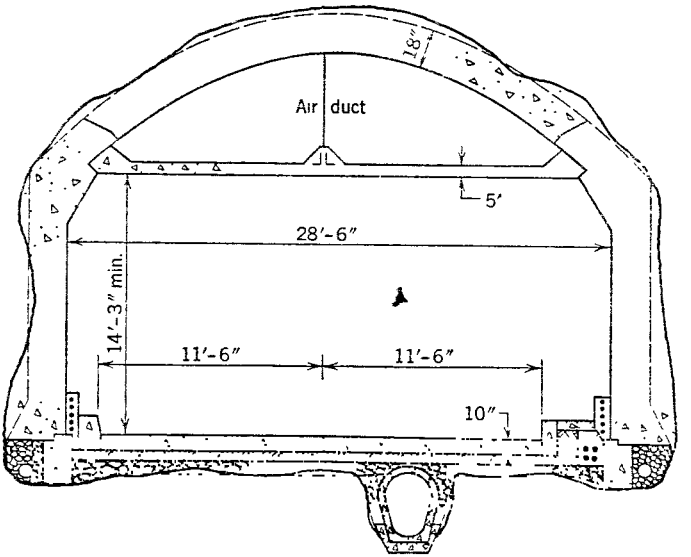
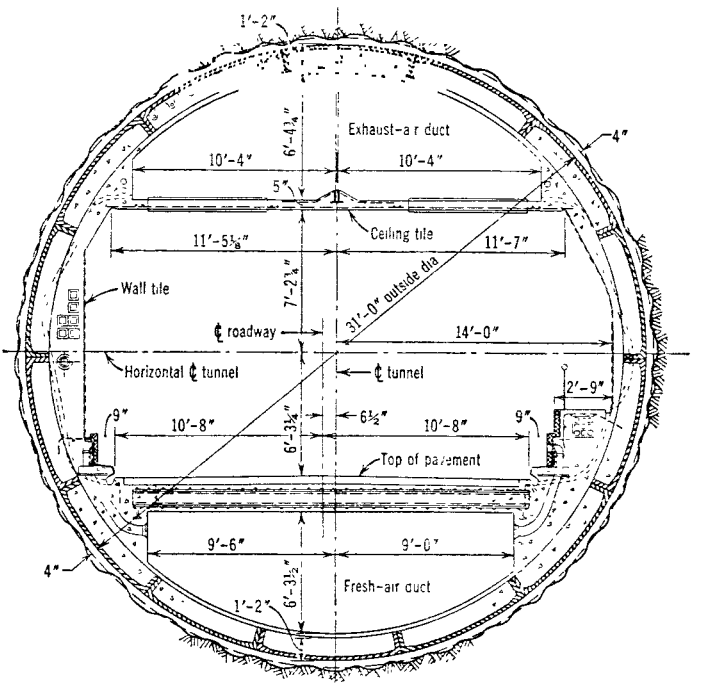


(21)

FIG 28—*Continued*

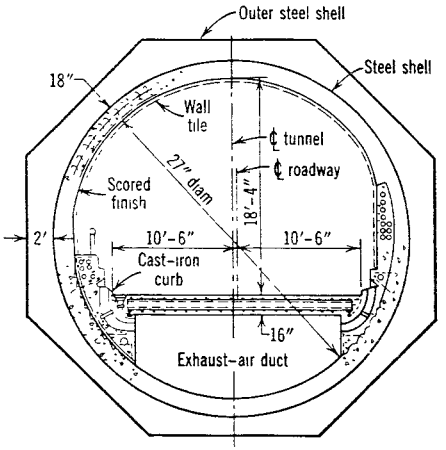


(22)

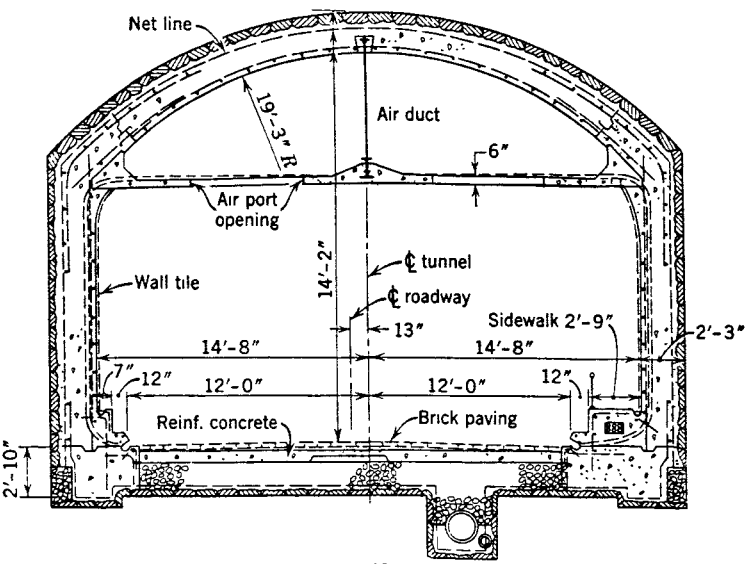


(24)

FIG 28—*Continued*

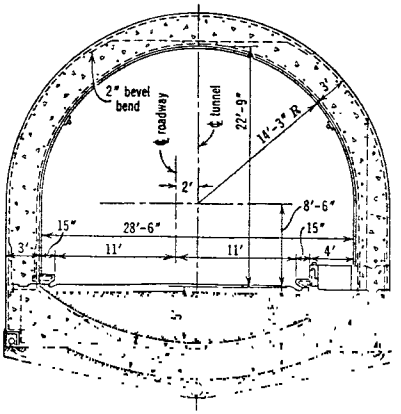


(26)

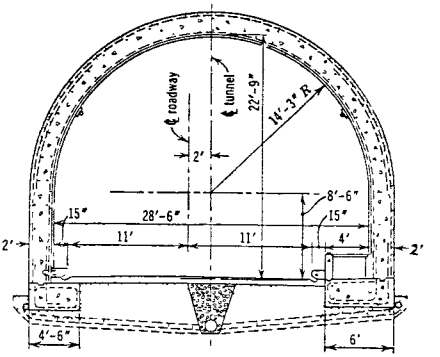


(28)

FIG. 28—*Continued*

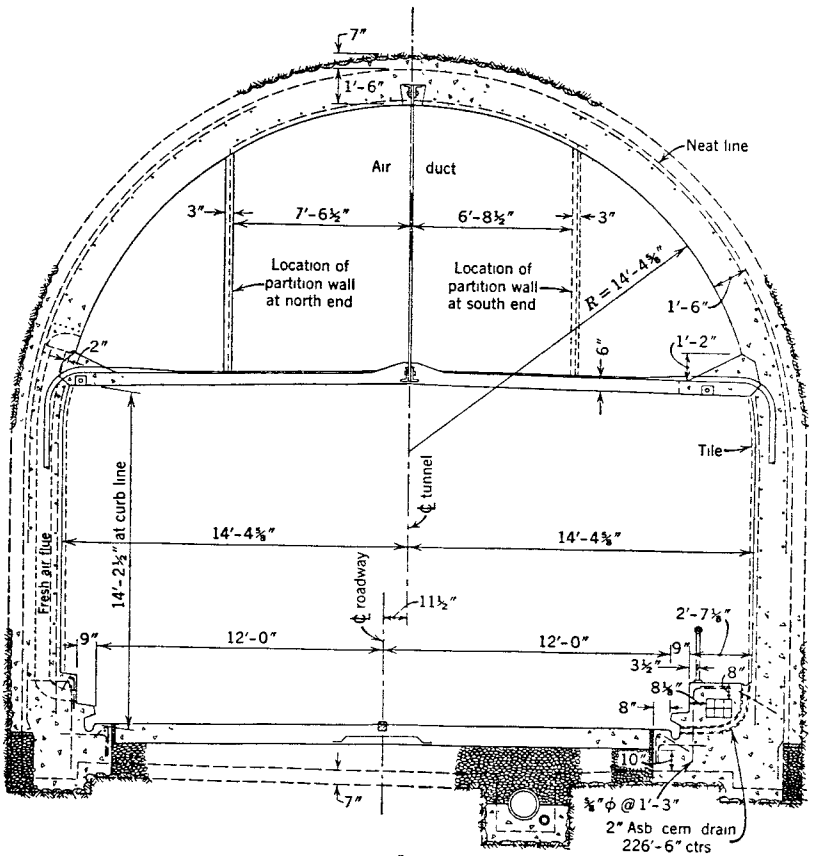


Type A, in soft ground



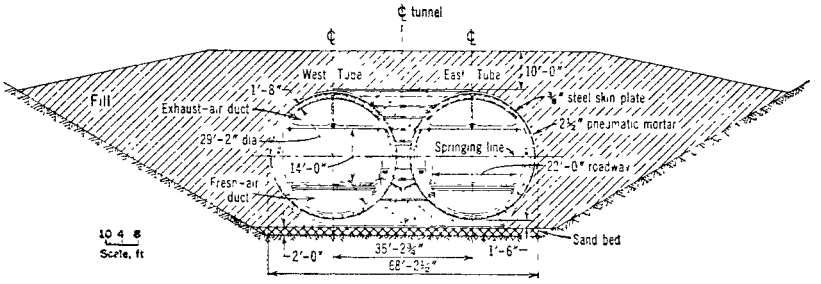
Type B, in rock

(32)



(34)

FIG 28—*Continued*



(35)

FIG. 28—*Concluded*

cfm of fresh air. Trench sections 31 ft diameter 3/8 in. thick steel tube with exterior octagonal stiffening diaphragms 12 ft c.c.; other stiffeners 4 ft c. c. on inside of shell. Tremie concrete poured between shell and forms on exterior stiffeners. 18 in. thick interior reinforced-concrete ring Sections erected on launching ways, after launching, interior concrete placed at fitting-up slip. Special floating screed to level sand bed in trench. Shield section driven in stiff gray clay, maximum air pressure 20 lb per sq in. gage. Maximum progress in 24 hr, 20 ft 5/8 in thick pressed steel lining, 11 segments and key rings 2 ft 6 in. long, 10-in flanges, I-beam stiffeners for jack thrusts Steel lining, designed for 45 ft of liquid clay weighing 135 lb per cu ft, weighed 2.35 tons per lin. ft

(7) Antwerp Tunnel, Belgium. Single tube, under Scheldt River between Antwerp and St. Anne. 913-ft concrete tunnel on St Anne side and 832-ft concrete tunnel Antwerp side 4056-ft cast-iron-lined shield-driven tunnel 2 ventilation buildings 2813 ft apart, one 1348 ft from St. Anne portal and one 1640 ft from Antwerp portal. Maximum depth of roadway below high water, 107.3 ft; bottom of tunnel, 116 ft Maximum grades (ruling), 3.5% Full transverse system of ventilation 24 centrifugal fans total, 2 single-speed motors per fan. Granite block paving. Sodium-vapor lighting. Interior concrete lining in cast-iron section; walls tiled Cast iron similar to Holland Tunnel Two shields driven from opposite sides of river. Air pressure generally 18–20 lb per sq in gage, even when theoretical pressure was 49 lb, due partly to lowering of ground water by pumping and partly to material being solid clay where deeper part of tunnel was located; overlying material fine to coarse sand Ventilating shaft sunk by freezing process to clay bed 85 ft below high water. A cast-iron-lined pedestrian tunnel 1750 ft long between construction shafts in which passenger elevators and escalators were housed was also constructed. Outside diameter, 15 ft 6 in.; walkway, 12 ft 6 in. wide; 10 ft 2 in. roadway to crown. Entirely in massive clay with 10 ft of cover. No compressed air used. Terminal shafts and escalator galleries excavated by freezing method.

(9) East Boston Double-lane single-tube tunnel under Boston Harbor from Boston to East Boston. Shield driven under compressed air Structural steel lining 31 ft outside diameter consisting of 3/8-in. bent plate forming skin and circumferential flanges; 1/2-in. end plates. Flanges 8 in. deep 11 segments and key per ring 30 in. long 88 45-lb rail stiffeners for shield jack thrusts. Inside reinforced-concrete lining 18 in. thick. Granite block paving. Maximum grade 4.2% Bottom of tunnel maximum depth of 85 ft below mean low water 4850 ft driven with one shield from East Boston end Contract cost of shield-driven section $5,697,000. Stiff blue clay with occasional sand pockets and boulders Average shield progress 15.4 ft per day. Conveying system for muck disposal carried muck through automatically controlled twin air locks to truck loading hoppers and bins on surface Cut and cover sections aggregate 785 ft, and open cut approaches 515 ft Two ventilation buildings, one in Boston and one in East Boston; full transverse system of ventilation

(12) Lincoln Tunnel. Provides two lanes of traffic in each direction from approaches in Manhattan located between Ninth and Eleventh Avenues in the vicinity of 38th and 39th Streets to Weehawken, New Jersey, where connections are made to arterial highway routes. The full transverse system of ventilation, similar to the Holland Tunnel, is employed Ventilation plant is located in buildings at three points, two in Manhattan and one in New Jersey. There are 26 blower fans and 30 exhaust fans of the centrifugal type, the normal maximum ventilation being 3,570,000 cfm of fresh air. The greatest part of the length of the project consists of two cast-iron-lined shield-driven tubes, there are also short lengths of cut and cover construction in both Manhattan and New Jersey, and

structural-steel-lined tunnel in rock in New Jersey. Construction contract costs aggregated $46,750,000.

Shield-driven tunnels. Built 1934–38 Twin tubes, generally 75 ft c.c., cast-iron lined, outside diameter 31 ft; everywhere under low water level North tube built after completion of south tube. South tube 6000 ft shaft to shaft, 150 ft in shale in N. J., then eastward 4900 ft through Hudson River silt to ventilation shaft at Manhattan bulkhead. Manhattan side 700 ft Manhattan schist, 250 ft sand and riprap to bulkhead. North tube same as south tube N. J and river crossing, Manhattan 590 ft sand, silt, and riprap from shaft to bulkhead with small amount of rock in invert. Cover 25–40 ft in land section; 40 ft silt cover shoreward ends of river section and 20 ft silt cover midriver. Water depth in river channel 45 ft. Heading and bench method in rock. Shields shoved with faces bulkheaded under the river and silt displaced with about 20% taken into tunnel through two doors in bulkhead in face to serve as ballast to check tendency of iron lining to rise behind advancing shield. South tube built first, taking in 15 cu yd silt per shove. North tube constructed later parallel to south tube about 45 ft clear. Silt taken into north tube controlled to provide ballast and also to minimize lateral pressures on south tube; 17–18 cu yd taken in per shove except close to river shaft, where up to 100% excavated Hydraulic bolt-tightening machines used for tightening bolts in lining under river sections Plant each side of river: 4 5600 cfm low air compressors, 625 hp each; 3 1300 cfm high air compressors, 225 hp each; 2 6000 lb, 25 gpm hydraulic pumps, 100 hp; 1 8 in. by 42 in. hydropneumatic accumulator to operate bolt-tightening machines. Usual air consumption in silt under river, 3000 cfm in the one heading. Usual air pressure in silt, 16 lb per sq in. gage raised to 28 lb per sq in. entering caisson shafts at Manhattan bulkhead. Average progress shield driving, 30 ft per 24 hr (maximum progress south tube in 24 hr, 47.5 ft). Contract cost per linear foot single tube, including cast-iron lining, bolts, interior concrete lining, and shafts, but not paving, tiling, etc., $1044. N. J. shaft 50 ft by 200 ft by 65 ft, deep in rock for two tubes; 2 N. J. construction shafts 20 ft by 20 ft clear by 25 ft deep to top of tunnel lining, in soft ground, cellular cofferdams of steel sheet piling, 2 N. Y. river shafts for ventilation buildings, steel and concrete caissons 42 ft 10 in. by 52 ft 8 in. by 98 ft deep, sunk under compressed air up to 46 lb per sq in. gage. 2 N. Y. construction shafts 32 ft by 36 ft by 65 ft deep, partly in soft ground, most of way in rock.

(16) Queens Midtown Tunnel. Subaqueous vehicular tunnel, crossing East River, New York, between midtown Manhattan and Long Island City, Queens Entrance and exit plazas in Manhattan connect with city streets and are separated to avoid concentration of traffic and conflict of travel to and from the tunnel, combined exit and entrance plaza in Queens, connecting to city streets and express highway. Two lanes westbound in north tube and two lanes eastbound in south tube widen to six lanes in each direction on plazas. 12 booths for all toll collection in Queens. Maximum grades, between portals, 3.5% up, 4% down. Maximum depth of roadway below highwater, 94.3 ft. Structures between portals: two cast-iron-lined shield-driven tubes 31 ft in outside diameter, 3760 ft long; two structural-steel-lined tubes in rock tunnel in Manhattan, 32 ft 10 in. in outside diameter, 1051 ft long; steel bent and concrete structure 387 ft long in Manhattan and 1051 ft long in Queens. Two ventilation shafts and buildings near shore, one each side of river. Tubes have interior lining of concrete, walls and ceiling, tile finish; brick pavement; lighted by means of individual luminaires along walls of tunnels just below tunnel ceiling, each luminaire containing 2 incandescent lamps on independent circuits. Method of ventilation similar to Holland Tunnel 46 ventilation fans installed, generally three speeds each; maximum normal fresh air supply 2,889,000 cfm, 42 air changes per hr. Construction contract costs approximately $39,000,000, under 31 general contracts, of which the contract for river tunnels and shafts 4098 ft between limits was $21,133,000. Aggregate contract construction costs, finished tubes, portal to portal, including rock tunnels and steel bent and concrete structure, $32,520,000, plazas, $3,242,000; buildings, $1,594,000; mechanical electrical equipment including lighting, $1,441,000; paving, $193,000. Construction was started October 1936 and the tunnel put in operation in November 1940.

The river tunnels were driven with four shields, two easterly from Manhattan and two westerly from Queens. The materials encountered were rock consisting of Manhattan schist, Fordham gneiss, limestone, dolomite, and Brooklyn injection gneiss, and overburden consisting of decomposed rock, sand, gravel, silt, clay, boulders, riprap, and fill material. The face of excavation alternated many times through full rock face, mixed face, and full face of overburden, the rock floor being very irregular. Clay blanket was required in both the east and west channels of the river, a riprap protected permanent blanket being constructed in the west channel where the top of the tunnel was several feet above the level of the natural river bed. The most difficult driving was encountered near the Manhattan shore where the top of the tunnel was in extremely porous riprap and fill material resulting in very rapid loss of air from the headings, which at one time required the full air

capacity of the Manhattan plant for one heading. The Manhattan plant consisted of 7 low-pressure-air compressors with total capacity of 40,930 cfm free air and 3 high-pressure-air compressors with a total capacity of 6000 cfm, which could be used to supplement the low-pressure supply. The full capacity of the Queens plant, the same as Manhattan, was at no time necessary. Maximum working pressure in the headings was 37 lb per sq in. gage. Tunnel driving was on a 24-hr, 6-day week basis. Excluding shutdowns and abnormally fast or slow progress, normal average progress for the shield headings per 24 hr was: earth, 7.4 ft; rock, 3.2 ft; mixed face, 3.7 ft. The overall average progress for tunnel driving in the river headings, including time for erecting shields and time for changing operations and for suspensions, including those due to labor disputes, was 18 ft per week per heading.

In excavating the 32 ft 10 in. diameter rock tunnels in Manhattan, 14 ft by 13 ft pilot drifts were driven on the axis of each tunnel, using 6 drifters on jumbo in each heading; 50 7-ft cut and breakdown holes, 8 delays, 125 lb of dynamite. Enlargement by ring drilling; 36 holes per ring, rings 3 ft c c., radial holes spaced 3 ft circumferentially at net line. Drilled upper holes 1 ft 8 in. and lower 1 ft beyond net line. Full ring and half ring fired in one blast, instantaneous and 10 delays, 368 sticks of 60% dynamite. Average excavation progress one heading per 24 hr, including pilot tunnel, enlargement and trimming, 6 ft; pilot tunnel only, 13.1 ft; enlargement only, 10.7 ft. The average overbreakage, net line of steel lining to excavation, was 1.37 ft.

(21) Maas Tunnel, Rotterdam. Begun 1938; work suspended 1940 by bombing; completed 1941 during German occupation. Under-river section 1926 ft long, 9 prefabricated units 81 ft by 28 ft by 201 ft sunk and connected under water. Maximum depth, 73 ft. Section contains two roadway bores and one double deck bore; one deck for cyclists and one for pedestrians. The under-river units were supported on spuds 30 in. clear of bottom, enabling placement of foundation underfill. Sections, fabricated first stage in dry dock and towed to construction dock for completion, weighed 1300–1400 tons more than displaced water and were floated by auxiliary cylinders, and suspended and lowered from floating derricks. Sand was injected under sections by process producing a dense fill. 3-ft jointing spaces separated adjacent sections; units connected by depositing concrete in compressed air by diving bell spanning the full 81 ft width of unit. Temporary control towers were installed on each unit for line and grade. Two ventilation buildings, one at each end of under-river section. Cut and cover ramp tunnels, one 906 ft long and the other 696 ft long. Full transverse ventilation. Maximum grade between portals 3.56%. Estimated cost of entire project, $8,169,600.

(22) Pennsylvania Turnpike Tunnels. The excavation was 32 ft 6 in. wide between pay lines, and 24 ft high. 7 tunnels from 3511 ft to 6070 ft between portals, aggregate 6.7 miles. Followed line of old railroad tunnels started about 1890 and abandoned, saving about one-third excavation Generally driven full face; use of heading and bench to relatively small extent. Steel supports where required spaced 2 ft to 8 ft c.c. Rays tunnel 3398 ft between rock portals, used steel supports throughout, 11 3 1/2-in. drifter drills on jumbo, full face, 81 holes 10 ft deep, 2 2 lb 40% dynamite per cu yd. General for all tunnels, 60–160 holes full face, 6 ft to 13 ft advances, 3–4 lb 40% dynamite per cu yd. Average progress excluding all contracts, previously driven sections, 3 8-hr shifts, 23.2 ft; undriven sections 18.6 ft; average concrete progress, sidewall, arch, and ceiling, about 35 ft per day. Average cost per foot does not include mechanical and electrical equipment for ventilation and lighting, or ventilation buildings. Ventilation: fresh air delivered through ports in tunnel ceiling from duct above ceiling; flows longitudinally in tunnel roadway area out portals.

(23) Croix-Rouse, Lyon, France. Built through hill, maximum depth approximately 280 ft to roadway. Generally excavated by German method, using hand drilling in rock on account of restrictions on power and equipment. Steel rib supports about 5 ft c.c. Five ventilation shafts and buildings, transverse system. Curved ceiling provides duct space above ceiling to structural arch; two fresh-air ducts at sides supply air through flues in walls to outlets at just above curb level each side of 4-lane roadway, 2 traffic lanes, each direction; exhaust duct at center with single line of ports in ceiling on axis of tunnel. Ventilation shafts supply air to, and exhaust air from, adjacent tunnel sections. Axial flow fans, one exhaust, one blower, per building Maximum fresh air capacity 1,750,000 cfm. Fluorescent lighting.

(24) The Brooklyn-Battery Tunnel extends 9117 ft between portals from the Battery at the southerly end of Manhattan Island to Hamilton Avenue in Brooklyn, New York. The plazas are combined exit and entrance, widening to 6 lanes in each direction from 2 lanes in each direction underground. There are 12 booths at the Brooklyn plaza for all toll collections. The maximum upgrade between portals is 3.7% and the maximum downgrade 4%. Maximum depth of roadway is 115 ft below high water. Both plazas

connect to local streets and to express highway systems. The tunnel structure between portals consists of two cast-iron-lined tubes, 31 ft in outside diameter, about 7700 ft long (85% of length between portals), with steel bent and concrete structure about 400 ft long connecting to the portal in Brooklyn and 740 ft of reinforced-concrete structure built in open cut and backfilled and 190 ft of steel bent and concrete structure in Manhattan. The cast-iron-lined tubes have an interior lining of concrete. Between portals the tubes have tile finish on walls and ceiling, brick paving, and are lighted by continuous fluorescent tubes just below the ceiling at the walls on each side of each roadway. The method of ventilation is similar to the Holland Tunnel system with 52 fans installed providing a normal maximum of 4,161,000 cu ft of fresh air per minute, or 42 air changes per hour. Ventilating equipment is located in the Brooklyn ventilation building near the Brooklyn portal, the Governors Island ventilation building, intermediate between portals, and equipment in Manhattan for exhaust is housed underground in Battery Park and for fresh air supply is housed in a building at the Manhattan portal. Construction costs total approximately $67,500,000 under 35 general contracts. Contract construction costs of principal items aggregated: tunnels and shafts, $55,150,000; buildings, $4,500,000; electrical and mechanical equipment including lighting, $4,170,000; plazas, $3,250,000, paving, $335,000. The construction of the project was entirely suspended for 3 years during World War II.

River Tunnels. The river tunnels were cast-iron lined throughout, using "light" lining in rock and "heavy" lining in mixed face and soft ground.

Brooklyn River Tunnels. These tunnels were driven from two construction shafts, one for each tube, in Hamilton Avenue, Brooklyn, passing under city streets and private property in soft ground to the bulkhead line of Buttermilk Channel and continuing in soft ground to the rock reef near Governors Island, and thence in mixed face and rock to the end of the contract near the site of the Governors Island shaft. The total length of single tube in soft ground was 4730 ft; in mixed face, 680 ft, and in rock, 2140 ft, driven with two shields using compressed air in soft ground and mixed face, with the exception of approximately 400 ft in each tube driven in free air from the Brooklyn construction shafts after unwatering the ground by pumping from a well and horizontal radial strainer pipe system. The maximum air pressure required was 37 lb per sq in. gage. The soft materials encountered were clay, sand, gravel, fill, and river mud, and the rock was Manhattan schist. Average progress in shield driving, including Sundays, holidays, ordinary delays, and suspensions, exclusive of 3-year shutdown during World War II, was: in soft ground, 162 ft per month in east tunnel and 192 ft per month in west; in mixed face, 70 ft per month in east tunnel and 75 ft per month in west.

Governors Island Shaft. This shaft was constructed by sinking a steel and concrete caisson from a filled island in the river just off the northeast tip of Governors Island to rock, employing open dredging for most of the depth and using compressed air for the final 12 ft, sealing the caisson in the rock and blasting the shaft through rock below to the tunnels which had already been excavated. The steel caisson was 111 ft 1 3/4 in. by 51 ft 1 3/4 in. and was sunk to rock at approximately 77 ft below high water; the subgrade of the rock shaft was 139 ft below high water.

Manhattan River Tunnels. This part of the Brooklyn-Battery Tunnel project consisted of 31 ft outside diameter cast-iron-lined twin tubes driven mostly in rock consisting of Manhattan schist for 3925 ft. Each tube driven south from shaft on Manhattan shore under mouth of East River to near center of crossing (7850 ft single tube). Shields and compressed air required in about 200 ft length of soft ground and mixed face near river bulkhead, each tube. In rock section bottom drift 24 ft by 24 ft enlarged to full section and shields used to erect cast-iron tunnel lining. Maximum cover 15 ft rock plus 50 ft soft ground; minimum cover 20 ft of rock with 50-ft depth of waterway. Rock generally sound, only a small part of length required support. 9 4-in. drifter drills used in both drift and enlargement. 80 holes 8 ft deep for drift, 5.6 lb 40% dynamite per cu yd and 4.0 lb 60%. 65 holes 8 ft deep for enlargement, 2.7 lb 40% dynamite per cu yd. Mucking machine and scrapers for mucking. Plant 3 5600-cfm low air compressors, 625 hp each; 2 2700-cfm high air compressors, 500 hp each. 2 6000-lb 25 gpm hydraulic pumps, 100 hp each. Progress: bottom drift 14 ft per 24 hr per heading, 3 shifts; enlargement, 20 ft per 24 hr per heading, 3 shifts. Erection of cast-iron lining behind shields (pneumatic bolt-tightening machines) averaged 19 ft per 24 hr per heading, 3 shifts, higher rates of progress up to 980 ft in a month were made under favorable conditions. Where the shields were driven through the soft ground and mixed face sections: progress about 4 ft per 24 hr; pressure, average 17 lb per sq in. gage; maximum 25 lb per sq in.; air consumption 4000 cfm average and 10,000 cfm maximum two headings. 90 ft water level to invert where air was used. Cost per linear foot single tube, including lining, $2300.

(28) Squirrel Hill. Two tubes on tangent 60 ft c.c. Grade 2.5%. North bore tunnel in rock 3785 ft south 3895 ft. Reinforced-concrete lining, walls 27 in. minimum thickness,

arch 24 in. Brick paving, tiled walls, painted ceiling, fluorescent lighting. Air ducts over ceilings; fresh air delivered to exit half of each tube through ports in ceiling, air exhausted through ports in ceiling for entrance half of each tube Reversible dampers permit exhausting air instead of blowing in case of fire. Air traps at exits prevent head wind from stopping outflow of air Two ventilation buildings, one at each portal, 8 centrifugal fans per building, motor operated for 1/3, 2/3, and full speed; 1,162,000 cfm for upgrade tunnel and 740,000 cfm for downgrade. Tunnels excavated from east portals using 3-deck steel frame jumbo each tunnel on 20 ft gage track Hinged interior platforms permitted passage of mucking shovels and trucks South tunnel used 10 drifter drills; north tunnel 16, the extra 6 drills saving time to swing drills to catch rib holes. Drilling, loading, shooting, mucking, and erecting tunnel steel supports synchronized for continuous and alternate operations in both tunnels 24 hr per day. Drill pattern: 85 to 115 holes depending on character of rock 10-hole V cut Straight in holes drilled 8 ft deep, V holes 10 ft deep. 850 ft average of drilled holes per round. All bits side-hole detachable; drills mounted on screw feed for 4-ft steel change The material, shale and limestone, was soft; bits lasted up to 6 rounds 40% gelatin dynamite Typical: 1 instantaneous and 13 one-second delays. 2 short-boom electric shovels and 7 9-cu-yd diesel dump trucks. Wet drilling Wet down muck while mucking. Steel tunnel lagging of 6-in. channel supported on 8 in. 40 lb H-beam ribs generally 4 ft c c; some spacing as little as 2 ft or 1 ft Arch lagged tightly as rock spalled when exposed to air Complete cycle of drilling, loading, blasting, mucking, and raising steel was 8 1/2 hr at start of work reduced to 7 hr 50 min. Average progress 493 ft per month

Drilling and Shooting Data

	North	South
Length	3785 ft	3895 ft
Linear feet of drilled hole per cu yd	3.02	3.14
Pounds of dynamite per cu yd	2 08	2.13

Concreting operations: concrete batching plant for 7 4 1/2-cu-yd truck mixers. Telescopic combination sidewall and arch forms. 3 50 ft 4 in. sections with one traveler Concrete placed with 2 1-cu-yd capacity concrete guns. Reinforcing steel erected from gantry 500 ft ahead of concreting operation. One panel, 50 ft 4 in., placed per 8-hr shift. Worked one shift per day. Average 6 hr 45 min to pour 450 cu yd.

Contract costs: tunnels, $12,372,000 tunnel finish and paving, $743,000, 2 ventilation buildings, $610,000; electrical installation, $855,000; fans and motors, $223,000.

(30) Washburn Tunnel. Provides two-way vehicular traffic in single 2-lane tube under Houston ship channel on route between Houston and Galveston, Texas. The prefabricated section of the tunnel was made up of 4 375-ft lengths of tube, assembled on standard shipways, launched and towed to outfitting basin, where special launching reinforcing was removed and concrete ballast poured. Sections then towed 400 miles to site of tunnel, sunk in dredged trench by addition of tremie concrete between inner and outer steel sheets.

(31) Elizabeth River. Berkley to Portsmouth on route from Norfolk to Portsmouth, Va. 7 prefabricated sections each 300 ft long sunk in trench and backfilled. Inner circular steel shell 31 ft 4 in. in diameter, outer octagonal steel shell 35 ft 4 in diameter. Used temporary bow in launching; sections fabricated at Sparrows Point, Md., and towed 180 miles to shape-up pier at tunnel site where inner shell was lined with 18-in. ring of reinforced concrete, roadway slab, and interior details, placed working through portholes in top of section later sealed with welded plates.

(32) Broadway, San Francisco. Built through hill to eliminate steep surface grades. Rock classified as Franciscan sandstone; part of tunnel length in sand Inside radius of roof arch 14 ft 3 in.; springing line 8 ft 6 in. above roadway level. 4-ft pedestrian walk. 28 ft 6 in. clear width between walls. Reinforced-concrete lining 3 ft thick with heavy invert arch in sand; lining in rock 2 ft thick. Walls and arch tiles. Fluorescent lighting. Longitudinal ventilation; fresh-air fans at each entrance portal blow in direction of traffic. Grade generally 3%. 2 fans can supply 300,000 cfm upgrade; 2 fans 106,000 cfm downgrade; mechanical ventilation augmented by estimated 150,000 cfm each direction carried in by traffic flow.

Contract costs: tunnels, including tile finish $3,334,000, buildings, $672,000; electrical, $153,000; lighting, $118,000; mechanical, $134,000. Bid price per linear foot for tunnel in sand, $1,625; in rock, $870.

(34) West Virginia Turnpike Tunnel. Part of 88-mile West Virginia Turnpike from Princeton to Charleston. Material generally sound sandstone and shale, nearly horizontally bedded, with some coal seams up to 1 foot or more in thickness. The tunnel is on a

3.25% grade descending from northerly portal to southerly, where 2.5% grade is met at southerly portal. One lane of traffic in each direction. Reinforced concrete lining. 1 ft 6 in. minimum thickness, arch and walls Drained structure. Transverse ventilation with both fresh-air and exhaust-air ducts above ceiling; fresh-air flues in wall deliver air above curb on one side of roadway only. 3 exhaust fans in building at north portal and 3 blower fans in building at south portal; fans are axial flow operated by two motors each for two speeds (100 and 7.5 hp respectively); normal maximum ventilation 570,000 cu ft of fresh air per minute. Fluorescent lighting

(35) Baltimore Harbor. The Baltimore Harbor Tunnel for vehicular traffic and its approach highways provide an express route for through north-south traffic which avoids the city streets of Baltimore and the congested highway connections around the head of the harbor and also serves as a direct link between the two Baltimore water-front industrial areas Canton and Fairfield.

The harbor crossing proper, 9215 ft long, including the depressed ramps between portals and grade points and all appurtenant buildings and equipment, is estimated to cost about $50,000,000.

Construction of the harbor crossing, between ventilation shafts in Canton and Fairfield, approximately 6300 ft, consists of the fabrication in shipyards of 21 double-barrelled, welded structural steel sections, each approximately 300 ft long; launching the sections, placing interior concrete and details and exterior protective Gunite, 2 1/2 in. thick, while the sections are afloat at a shape-up basin; towing the sections to the tunnel site; sinking with the aid of derricks onto a carefully prepared foundation course of sand screeded to exact grade at the bottom of a trench dredged along tunnel line, with the further provision that sand or pea gravel shall be forced under pressure to fill any substantial voids between the bottom of the structure and the prepared sand bed; aligning the tunnel sections by means of temporary masts which project above the water surface. Adjacent sections are joined by means of steel pins, wedges, and steamboat ratchets placed and adjusted by divers. After so joining, the section is partially backfilled, to ensure against flotation, and, after placing the next successive tunnel section, an exterior collar of tremie concrete is poured around the junction and backfilling completed.

The cylindrical steel shells are 3/8 in. thick, with exterior transverse 5/16 in. steel plate stiffening diaphragms spaced 12 ft apart and intermediate exterior struts and diaphragms at 4 ft apart. Circumferential 7-in. WF T's are placed circumferentially on the inside of the tubes, 4 ft center to center. The design includes a reinforced-concrete structural ring inside the steel shells, 20 in thick for 4800 ft of tunnel, but on account of greater exterior loading requirements, 24 in. thick for 1500 ft (5 sections) from the Canton shaft to harbor bulkhead line. The twin tubes are 35 ft 2 3/4 in. on centers, the diameter of shell being 32 ft 6 in. for the 20-in concrete lining and 33 ft 2 in. for the 24-in. lining.

Before launching, the tunnel sections are closed at each end with heavy steel plate and girder watertight bulkheads designed to withstand full hydrostatic head in sinking operations; the depth below water level of bottom of tube at lowest point of profile is approximately 100 ft; reinforced "keel" concrete approximately 3 ft thick is also placed in the bottom of the sections for the length between bulkheads; and the Gunite protection is carried high enough to be above water when the section is afloat One-half inch thick steel collar plates, similar to the main steel cylindrical shells, extend 4 ft 3 in. beyond the face of each bulkhead, making the closure between bulkheads of adjacent sections 8 ft 6 in. long. When the exterior seal is made, steel sheet piling or curved steel plate is placed between the vertical edges of the bulkheads as forms to retain the tremie concrete

At the shape-up basin interior concrete is placed, except for tunnel bench and ceiling, while the sections are moored adjacent to a pier, suitably equipped with cranes, runways, etc., for the operations involved. Operations in the tubes are conducted through 5 access manholes provided in the top of each 300-ft tube, and, except where there is such a manhole, through concreting hatches provided in each 12-ft bay between diaphragms. The manholes and hatches are sealed and grouted when work at the shape-up basin is completed, at which time the section is slightly buoyant. The section is towed to the tunnel site where enough exterior concrete is added to give it a negative buoyancy in the order of 100–150 tons. The section is controlled by 2 to 4 floating derricks which lower it to the bottom of the trench, where exterior concreting is completed by tremie, providing a margin of weight over displacement sufficient to insure that the section will remain in place on the bottom.

When several sections are in place and backfilled in the trench and access by ramp has been provided on shore, the work of unwatering the spaces between bulkheads and removal of bulkheads is commenced. After unwatering and removal of the two adjoining bulkheads the joint between collar plates of adjacent sections is spanned by a steel plate bent in the form of a channel with 3-in. legs and 1 ft 3 in. web, welded to create an annular space over the joint. This space is grouted to complete the seal. Concrete lining and interior

details are then completed across the junction, followed by concreting bench and tunnel ceiling, omitted at the shape-up basin.

The walls and ceiling of the Baltimore Harbor Tunnel are finished with ceramic tile; the paving is brick; lighting is by 2 lines of fluorescent lamps in each tube located just below the tunnel ceiling at each side of the roadway. Ventilation is by the transverse distributive system. There are 8 fresh-air fans and 8 exhaust-air fans in each of 2 ventilation buildings, one in Canton and one in Fairfield.

The maximum grade in the tunnel is 3.5%.

15. AQUEDUCT, FLOOD-CONTROL, AND POWER TUNNELS

Descriptive Data

(1b) Catskill System, First Stage, N. Y. 13.9 miles of grade tunnels, 35 miles of pressure tunnels, and 6.3 miles in siphons. The pressure tunnels and siphons were circular.

Rondout, 4.47 miles, 14.5 ft diameter. Rock tunnel driven from 8 shafts by top heading and bench method. Cost to city per linear foot based on contract prices: $180.

Wallkill, 4.43 miles, 14.5 ft diameter. Rock tunnel driven from 6 shafts in Hudson River shale, by top heading and bench method.

Moodna, 14.2 ft diameter. Tunnel is shale and granite. 7 shafts.

Hudson, 5.68 miles including Moodna; 14.0 diameter. Tunnel is sound granite, 1100 ft below surface of river. Hydraulic gradient 400 ft above river. Shaft at either end. Water-bearing fissure sealed by building concrete bulkhead 8 ft thick heavily reinforced with drains and grout under pressures up to 1000 lb per sq in. forced through pipes set in bulkhead and leading to fissures.

Yonkers, 2.46 miles, 16.6 ft diameter. Tunnel in gneiss, driven from 4 shafts.

City Tunnel 1, 18.11 miles, 11 ft diameter. Tunnel under New York City in gneiss, schist, and limestone from 200 to 750 ft below surface. Driven from 25 shafts by heading and bench method. Electrically operated machinery generally used.

(1c) Catskill, Shandaken Tunnel. Short section in earth at outlet end. Except for northerly 3.5 miles it is a grade tunnel, horseshoe section, 11.5 ft high, 10.25 ft wide. Northerly 3.5 miles tunnel depressed below hydraulic grade because of rock condition. Driven from 8 shafts from 1.3 to 2.7 miles apart, aggregate depth 3413 ft.

(1d) City Tunnel 2. Hillview reservoir, Yonkers, Westchester County through Bronx, Queens, and Brooklyn. Pressure tunnel, driven through gneiss and limestone, generally 500 ft below surface. Grouting at 300 to 500 lb per sq in. Heading and bench method. 19 shafts, 9 being sunk under compressed air to bedrock as circular concrete caissons, 19 ft interior diameter; bedrock 68 ft to 140 ft deep.

Progress per heading per week:

Excavation: average, 60 ft; maximum, 145 ft.

Lining: average, 68 ft; maximum, 248 ft.

Cost per Linear Foot

	Unsupported		Supported	
	Cost	Length, ft	Cost	Length, ft
21 ft dia	$402.86	1,771	$515.50 to $692.88	1,868
17 ft dia	278.97	65,856	297.97 to 407.85	2,716
15 ft dia	232.98	6,211	321.77 to 326.89	119

(1e) Delaware System. Delaware Aqueduct placed in limited service 1944, comprising 85 miles of circular-lined pressure tunnel driven in rock from Rondout reservoir to Hillview reservoir, connecting to City Tunnel 2, making aggregate of 105 miles of continuous tunnel, longest in the world for any purpose. 31 shafts at 26 locations; shafts, 1.8 to 5.2 miles apart, 313 ft to 1551 ft deep (total aggregate depth 3 2/3 miles), 14 ft to 19 1/2 ft in diameter, lined with concrete as sunk. Tunnel, 300 ft to 1000 ft below general level of ground surface, 2500 ft below crest of Shawangunk range; 600 ft below sea level under Hudson River; 660 ft below sea level under Kensico reservoir. At several locations, particularly at the Hudson River and Kensico reservoir undercrossings, deep earth-filled preglacial gorges encountered in the exploratory boring, or geological faults where the rock is crushed and decayed, required that the tunnel be located deep enough so that the cover of sound rock and overburden would resist bursting pressure; under the Hudson River this pressure was in excess of 500 lb per sq in. In general, construction progress on the 13 1/2 ft

and 15 ft diameter tunnels, averaged, in the different kinds of rock encountered and under the different contracts, from 135 ft to 270 ft per week; concrete lining from 290 ft to 1000 ft per week. Work was on a 24-hr basis. The rock surrounding the tunnels was grouted as required, with neat cement, or cement and sand grout usually under pressures of from 100 to several hundred lb per sq in., using pressures as high as 1000 lb per sq in. in zones of water-bearing, disintegrated rock.

(1f) Connecting Tunnels. Two pressure tunnels from Prepacton reservoir to the Neversink and Rondout reservoirs.

The East Delaware Tunnel is 25 miles long, constructed from the two ends and two intermediate shafts. Driven through shales and sandstones, horizontally stratified. Full-face excavation: jumbo on regular tunnel track carrying 6 drills. Average excavation progress about 35 ft in 24 hr. Owing to the stratification, the rock tends to break square in the quarters instead of circular. The roof is generally supported by means of steel roof bolts and channel sections, averaging 5 rows of bolts on 4-ft centers or approximately 50 lb of steel per linear foot of tunnel. Plain concrete lining 6 in. minimum thickness, 11 in. average, 19 in. pay. The arch forms are telescopic type and concrete placed continuously with a pneumatic placer.

(7l) Hoover Dam Diversion Tunnels, Colorado River, in Nevada and Arizona. Four tunnels, two each side of dam. Inlets about 1600 ft upstream and outlets about 2400 ft downstream. 8 ft by 10 ft adits driven on Nevada and Arizona sides about midway of length of tunnels, 12 ft by 12 ft pioneer headings at top of 56 ft diameter sections turned both ways from adits. Enlargement from portals. Main bench 30 ft deep and wings each side of heading taken out in next operation full width; invert section about 14 ft deep final. No timbering necessary. Rock: schist, quartz diorite and granite free from faults and seams and generally dry. Truck-mounted drill carriage used to drill main bench, modified for invert. Bench drilled with 120 holes including wing holes. 15 delays. Average advance 16 ft. Invert round 28 holes, 1 1/2 hours. Average advance, 20 ft. Overall powder consumption, 2.38 lb per cu yd. Mucking by electric shovels with 3 1/2 cu yd buckets; truck removal. Average cycle: drilling and shooting, 3 hr; clearing up, 1 hr; mucking, 7 to 9 hr. 8 headings used 4 carriages and 8 shovels.

(9a) Montebello-Druid. 85% in rock, 1464 ft in mud and water-bearing sand requiring compressed air. Top heading and bench method in rock. Mucking by machines and air shovels. 6 shafts, 14 ft by 18 ft and 14 ft by 28 ft, 70 ft to 100 ft deep. Average progress in rock 12.4 ft per day, 2 12-hr shifts.

(9b) Gunpowder Falls-Montebello. Coarse granite to black gneiss. Three shafts. Full heading. Mostly unsupported. Jumbo drill carriage, 5 pneumatic drills. Mucking by electric-powered loaders. Excavation progress: average 20 ft per day, 3 8-hr shifts, 6 days per week.

(9c) Patapsco-Montebello. Granite, pegmatite, gneiss, and quartzite. Full section excavated. 2.4% required support, steel ribs and wood lagging. Progress 3 8-hr shifts, 6 days per week, average 30 ft per day. Concrete, steel forms, invert, and arch. Forms telescoped to pass through each other. On 10 ft diameter tunnel, used continuous advancing slope method. Costs: Tunnel 1, 10 ft diameter, $200 per ft, 7 ft diameter with 4 1/2 in thick pipe, $260 per ft, with 8 in. thick pipe, $297 per ft. Tunnel 2, unlined 11 ft diameter, $155 per ft, finished 10 ft diameter tunnel, $240 per ft.

(11) Mono Craters. Variable: rhyolitic tuff, sandstone, quartzite, diorite, granite, basalt. Wide variety of conditions: hard, self-supporting rock; squeezing material. Built with owner's forces. Heavy glacial material with large flows of water encountered. Flow of carbon dioxide gas from water averaged 1000 cfm requiring extensive ventilation equipment. 3 shafts, 6 headings. Parts of tunnel were driven through old necks of Mono Craters. Progress: excavation average 15 ft per day, 3 shifts; maximum 90 ft, 2 shifts. To stop water and gas, grout at 1000 to 1100 lb per sq in. forced through holes drilled to 40 ft in advance of face; heading advanced 20 ft, then holes 40 ft from new face, etc. Tunnel supported for 67.4% of length, spiles used for 23.2%; 18.5% of face advanced behind breastboards. Maximum flow of water from all headings, 20,000 gpm.

(12) Colorado River Aqueduct Project. Grade tunnels generally lined with plain concrete, pressure tunnels reinforced as required. The 14 ft and 9 ft 9 in. tunnels have continuous steel cylinders backed with concrete and lined with reinforced Gunite 2 in. thick. All tunnels were excavated through igneous rocks, generally classified as granitic, with exception 3 tunnels portions of which were in sedimentary sandstone and conglomerates; also 2949 ft dry partially consolidated sand and gravel, 2100 ft moist sand and clay; some cemented gravel. Excavation practically all full section using drill carriage or jumbo. Support during excavation, average 59%. Average 2.7 lb of explosive per cu yd. Excavation progress main aqueduct tunnels, 4 ft to 5 ft per 8-hr shift. 17 adits and shafts for work aggregated 16,931 ft.

Table 4

Number and Name	Date Built	Length, mi	Tunnel Section	Size	Material Excavated	Material and Thickness of Lining, in.	Cost per Linear Foot or as Noted
(1) *NYC Board of Water Supply*							
(*a*) New Croton	1890	22.6 7.2	Horseshoe Circular	13.5′ wide 10.5 dia	Rock	Brick	$112
(*b*) Catskill, First Stage	1907–1917	55.2	See description	See description	Rock	Concrete	See description
(*c*) Catskill, Shandaken Tunnel	1917–1927	18.1	See description	See description	Rock	Concrete	
(*d*) City Tunnel 2	1928–1934	20	Circular	15′—17′—21′	Rock	Concrete	See description
Delaware System							
(*e*) Delaware Aqueduct	1936–1944	45 27 13.6 85.6	Circular	13 1/2′ 15′ 19 1/2′	Rock	Concrete	
(*f*) Connecting tunnels: Neversink	Under construction 1953	5	Circular	10′	Rock	Concrete	
East Delaware	Under construction 1953	25	Circular	11′ 4″	Rock	Concrete	
(2) Buffalo, New York	1910	1.14 0.81	Horseshoe Horseshoe	12′ by 11′ 3″ 9′ by 8′	Rock	Concrete	
(3) Los Angeles, Calif., Aqueduct	Completed 1913	42.7 (164 tunnels)	Arch and vertical sidewalls	8.25′ high, 7.5′ wide		Concrete	
(4) Apulian, Italy	1915	60 (99 tunnels, longest 12 mi)	Horseshoe	10.3′ high, 9.3′ wide		Concrete	
(5) *Hetch Hetchy, City and County of San Francisco*							
(*a*) Mountain Tunnel	1914–1925	19	Horseshoe	10′ 3″	Rock	38% unlined remainder concrete, min. 6″	$11,500,000 total
(*b*) Moccasin (power)	1922–1925	1.01	Horseshoe	13′ 0″	Rock	Concrete, min. 9″	$1,340,000 including inlet tower and surge shaft
(*c*) Foothill	1926–1928	15.8	Circular	10′ 3″	Rock	Concrete	$8,000,000 total
(*d*) Coast Range	1927–1934	25, continuous tunnel; 0.6, pipeline; 3.5, tunnel	Circular	10′ 6″	Rock	Concrete, 6″ to 3′ thick	$24,000,000 bond issue

	(6) *So. California Edison Co. Tunnels*								
(a)	Huntington-Shaver	1925-1928	4.85	Horseshoe, 7′ radius arch	14′ wide, 13′ 2″ high	Rock	Timber and concrete, 12″ thick		$106.76
(b)	Mono-Bear Tunnels	1926-1927	1.44 0.75	Rectangular	9′ high, 8′ wide	Rock	Unlined except concrete in short lengths		$79.37 99.95
(c)	Big Creek 4	1949-1950	2 sections	Horseshoe, 12′ radius arch	24′ dia, 24′ radius walls, 100′ radius floor	Rock	Unlined except concrete invert and short lengths concrete or Gunite		$275.00
	(7) *Bureau of Reclamation (selected list of canal tunnels built 1928-1942, from total of 50)*						Concrete		
							A *	*B* †	
(a)	Owyhee 1	1930-1932	3.55	Horseshoe	16.58′	Tuff, soft cinder, dense black rock	8	16	$120.60
(b)	Owyhee 5	1930-1933	4.15	Horseshoe	9.25′	Tuff, basalt, sand, water	6	10	69.74
(c)	Kendrick 6	1936-1937	1.03	Horseshoe	13.75′	Sandstone and shale, some water	7	12	97.37
(d)	Shoshone Canyon	1936-1938	2.64	Horseshoe	12.0′	Limestone, sandstone, granite, numerous caves with sulphur gas	6	10½	77.66
(e)	Sanpete, Ephraim	1935-1939	1.35	Horseshoe	6.5′	Limestone and shale	3	6	24.41
(f)	Roza 1	1936-1938	1.56	Horseshoe	17.0′	Basalt	8	16	94.91
(g)	Roza 3	1936-1938	1.81	Horseshoe	17.0′	Lava, rock, basalt clay seams, some water	8	16	102.21
(h)	Provo River, Alpine, Draper	1939-1941	2.85	Horseshoe	6.5′	Shea, andesite, clay, gravel, slick gouge, 190 gpm water, mud flow	3	7	83.00
(i)	Klamath, Tule Lake	1940-1941	1.25	Horseshoe	5.75′	Basalt, volcanic tuff agglomerate, diatomite	3	7	$35.95
(j)	Tucumcari 2	1940-1941	1.50	Horseshoe	11.5′	Sand, clays, sandy shale	5	10	
(k)	Tucumcari 3	1941-1942	1.85	Horseshoe	11.5′	Soft sandy shale, sandstone, clay			
(l)	Hoover Dam Diversion Tunnels (4 tunnels)	1931-1933	15,904 aggregate	Circular	56′ excavated, 50′ inside	Rock	36		About $1700

Table 4.—*Continued*

Number and Name	Date Built	Length, mi	Tunnel Section	Size	Material Excavated	Material and Thickness of Lining, in	Cost per Linear Foot or as Noted
(8) Hawks Nest (power), W. Va.	1930–1933	3.08	Circular	30′ 32′	Rock	Concrete, short sections of steel	$244 60
			Circular Horseshoe	42′ 46′ horizontal × 43′ vertical		Unlined except paving	
(9) *Baltimore, Md.*							
(*a*) Montebello-Druid Conduit	1931–1933	2.51	Circular	7′ inside dia	Rock, mud, and sand	84″ precast concrete pipe concreted into tunnel	$88.74
(*b*) Gunpowder Falls-Montebello Tunnel	1937–1940	6.55	Circular	12′ inside dia	Rock	(1) Reinforced concrete precast pipe (2) Steel plate with Gunite (3) All Gunite 6″ min. concrete	$5,389,312 total
(*c*) Patapsco-Montebello Tunnel	1946–1951	16.84	Circular	12′ inside dia and 7′ inside dia, unlined 11′ dia	Rock	Monolithic reinforced concrete	See description
(10) *Fort Peck Dam and Reservoir, Mont.*							
No. 1 (power)	1934–1937	1.11	Circular	24′ 8″ clear dia	Shale, thin layers of clay	Reinforced concrete 45″ downstream, 36″ upstream, additional 1″ steel plate in 1	$525
No. 2		1 14					$525
No. 3		1.26					
No. 4		1.37					
(11) Mono Craters, Calif., Los Angeles Dept. of Water and Power	1934–1940	11.30	Modified circular	9′ 7 1/2″ dia	Variable	Concrete	$231.82 incl. shafts
(12) Colorado River Aqueduct and distr. system, Metropolitan Water District of So. California	1935–1941	83.7 (27 tunnels) 8.36 (2 tunnels) 0.44 (1 tunnel) 15.2 (8 tunnels) 0.54 (8 tunnels) 0.71 (1 tunnel)	Horseshoe Horseshoe Circular Circular Circular Circular	16′ 15′ 3″ 14′ 10′ 9′ 9″ 6′	Rock	See description	$143 90 115.50 189.50 78.60 78 60 43.50
(13) *Chicago, Ill., Water Supply Tunnels*							
(*a*) Chicago Ave. Lake and Land Tunnel	1935	11.5	Horseshoe	15′ 4″ wide, 16′ high 12′ 0″ wide, 13′ high 9′ 6″ wide, 10′ high	Rock	Concrete	See description
(*b*) Desplaines St.	1940	1.6	Horseshoe	12′ 0″ wide, 13′ high	Rock	Concrete	See description
(*c*) Stewart Ave. Tunnel	1943	4.28	Horseshoe	9′ 6″ wide, 10′ high	Rock	Concrete	See description
(*d*) Tunnels for So. District Filtration Plant	1945	2.80	Horseshoe	15′ 4″ wide, 16′ high 12′ 0″ wide, 13′, 12′, 9′ high	Rock	Concrete	See description

(14) Tionesta Dam Outlet, Pa.	1938	0.35	Circular	19′ dia clear	Shale	Concrete 21″ thick	$335 00
(15) Youghiogheny Dam Outlet, Pa.	1940	0.34	Circular	18′ dia clear	Shale	Concrete 27″ thick	$280.00
(16) Kanopolis Dam Tunnel, Kansas	1940–1941	0.46	Circular	14′ dia clear	Rock	Reinforced concrete 1.5′ thick	$167.00
(17) *Nantahala Hydro Development, N. C.*							
(*a*) Diversion Tunnel	1940–1942	5.58	Circular	16′ dia inside	Sandstone with much feldspar	12″ min. concrete	
(*b*) Power Conduit	1940–1942		Horseshoe	12′ × 12′	Sandstone with much feldspar	Unlined	
			Horseshoe	13′ wide, 14′ high		Unlined	
			Horseshoe	14′ wide, 13′ high		Unlined	
			Horseshoe	Variable width, 12′ high		Concrete	
			Circular			Steel and concrete	
(18) *Tennessee Valley Authority*							
(*a*) Ocee	1941–1942	2.46	Horseshoe	12′ 6″	Rock	Concrete	$2,384,200 total
			Circular	12′ and 12′ 6″		Concrete and steel liner	
(*b*) Appalachia	1941–1943	8.25 (including 0.57 mi of 22′ dia)	Circular	18′ 0″ inside 20′ and 22′	Rock	Concrete 12″ min. Unlined	$9,216,640 total
(19) *Pacific Gas & Electric Co.*							
(*a*) Cresta, Feather River Project, Calif.	1948–1949	4.06	Horseshoe	27′ dia	Granite	Unlined	
			Horseshoe	26′ dia		Unlined	
			Horseshoe	22′ dia		Concrete	
			Circular	19′		Concrete	
			Circular	18′ 6″ dia		Steel	
(*b*) Dutch Flat, Bear River, Calif.	1941	1.46 2.66	Horseshoe and circular	Gen. 8′ to 9′ 2″	Rock	Unlined Lined concrete	
(*c*) Electra, Mokelumne River, Calif.	1946–1947	6.38 1.77	Horseshoe	12′ 10″ × 15′ 6″	Rock	Unlined and concrete and Gunite lined	
(*d*) Lower Bear River, Calif.	Started 1950	2.59	Horseshoe	8′ 6″ × 11′ 0″	Granite	Unlined	
(*e*) Pit River 5, 2 tunnels, 1 and 2	1942	0.97 4.38	Circular Horseshoe	19′ 0″ dia	Rock	Concrete	
(*f*) Rock Creek, Feather River Project	1948–1949	6.27	Horseshoe	25′ dia	Granite	Unlined concrete	
			Horseshoe	22′ dia		Unlined concrete	
			Horseshoe	19′ dia		Unlined concrete	
			Circular			Steel lining	
(*g*) West Point Tunnel, Mokelumne River Development, Calif.		0.31 2.40	Horseshoe Circular	10′ 10″ × 13′ 6″ 12′	Granite	Concrete Unlined	

Table 4—*Continued*

Number and Name	Date Built	Length, mi	Tunnel Section	Size	Material Excavated	Material and Thickness of Lining, in.	Cost per Linear Foot or as Noted
(*h*) North Fork	1926	3 66		12′ × 12′	Granite	Unlined	
(20) El Mirador Irrigation, Mexico	1942–1945	7	Circular	16′ dia clear	Rock	Concrete 12″ min.	Contract cost $12,000,000
(21) *Lerma Aqueduct, Mexico*							
(*a*) Valley of Toluca	1942 1951	13 75	Circular	10′	Rock	Concrete	$25,000,000 total
(*b*) Las Cruces Mt. Range		9					
(*c*) Valley of Mexico		14					
(22) *Nottingham Intake, City o Cleveland, Ohio*							
Shore tunnel	1947 1949	1.55	Circular	10′ dia inside	Shale	Concrete plain and reinforced where supports were necessary	$232 80 per ft including shaft
Lake intake		3.5	Circular	10′ inside dia pipe	Trench in lake bed	Reinforced concrete 12″ thick	$157 50 per ft including intake and breakwater
(23) Tennyson Supply Conduit, Cincinnati, Ohio	1949 1951	2.58	Circular	8′ final	Rock	Concrete min 8″, min avg 12″ in 4′	$198.82
(24) Garrison Dam Outlet Work (power and flood control), N Dak.	1949 1951	8 tunnels, each 1200′ long, 1.82 miles total	Circular	29′ power, 26′, 22′ flood control clear dia	Clay and lignite	Concrete 2′ 6″, 2′ 9″, 3′ 6″	22′, $1,050 26′, 1,302 29′, 1,713
(25) Ft. Randall Dam Tunnels, Missouri River Mile 921.9, 8 power tunnels, 1 regulating tunnel, 3 flood-control tunnels	1949–1951	12 tunnels, each 873′ long, 1.98 mi total, 22′ dia, 4554′, 28′ dia, 5922′	Circular	9 22′ and 28′ dia, 3-22′ dia clear	Chalk	Reinforced concrete 2 1/2′ thick	$988 74 contract (gov. cost add 7%)
(26) Bombay, India	1949–1952	3.9 (3 tunnels)	Horseshoe	10′	Trap Rock		
(27) Owens Gorge, Calif.	1950–1953	10.6 (5 tunnels)	Horseshoe with circular interior	10′ 1 1/4″ inside	See description	Concrete	
(28) Eklutna Tunnel, Alaska (power), USBR	1951–1954	4.5	Circular	9′	Rock and volcanic tuff	Reinforced concrete	$17,350,000 contract (including appurtenant facilities)

* *A* = net line
† *B* = pay line

(13) City of Chicago Water Supply Tunnels. Niagara limestone, generally self-supporting. Costs: 1939–1941, 10,083 ft of 16-ft section, $125.00 per linear foot. 1943–1945, 3,000 ft of 16-ft section, $180.00 per linear foot; 1,150 ft of 13-ft section, $170.00 per linear foot. 1937–1940, 8,390 ft of 13-ft section, $167.00 per linear foot. 1938–1942, 22,610 ft of 10-ft section, $124.00 per linear foot.

(20) El Mirador. Limestone conglomerate 1 3/4 miles and balance shale. Driven from two portals and adit at midpoint. Full face method. Continuous support in shale; sets of 12 in. by 12 in. timbers, generally 4 ft c.c. Plastic flow in shale caused movement to center and heaving of floor as much as 4 ft. Maximum cover, 525 ft. 3 8-hr shifts per day: average excavation progress, conglomerate, 28.4 ft; shale, 17.6 ft. Concrete lining, 3 shifts per 24 hr: invert, 225 ft; arch, 142 ft.

(22) Nottingham Intake. Hard shale. Built in connection with new 150 mgd filtration and pumping plant. Lining 1 ft thick in self-supporting ground; reinforced where supports were necessary. Excavation full section. Holes 6 ft deep, 2 lb 40% gelatin per cu yd. Excavation and form setting, 3 8-hr shifts, 24 hr per day; 8 hr per day placing concrete. Progress 13 ft to 15 ft per 24 hr. Permanent shaft existing at terminus 86 ft diameter, 102 ft deep. Temporary shaft midway 22 ft diameter liner plates 44 ft to surface of shale; solid shale to grade at 102 ft from surface. 22 ft diameter permanent shore shaft at Lake Erie, liner plates and 18 in. concrete inside plates; total depth 87 ft. Ventilated during construction by means of electrically driven exhaust fans and 24 in. diameter corrugated steel pipe.

(23) Tennyson Supply Conduit. Limestone, shale, and sandstone. Shale swelled when exposed to dampness. Alternating layers shale and sandstone. Excavation drilled circular, became rectangular. 10 ft wide, 10.5 ft high.

(24) Garrison Dam Outlet Work. Highly compressible clayey material, horizontal bed and interbedded with lignite seams. Excavation full face. 8-ft draw. Hand-held air drills driving coal augers. 20% dynamite, 9 delays, 0.45 lb per cu yd. Excavation supported with 4 segment steel ribs and lagging.

(26) Bombay, India. Tunnels for water supply, Tansa Lake reservoir to Vaitarna reservoir. 20,500 ft required 3 yr. Hand loading and hoisting. Machine drilling.

(27) Owens Gorge Tunnel, Department of Water and Power, City of Los Angeles. 300 miles northeast of Los Angeles. 5 bores driven from 6 headings. Jumbos carrying 5 drills each. Rock: pink rhyolite, some consolidated volcanic ash, andesite, decayed rock, glacial moraine. Soft ground. Horseshoe excavation 11.5 ft diameter in good ground, 12.25 ft diameter with steel supports During a 31-day period in volcanic ash formation record driving progress was made, average daily 73.77 ft; 3.055 lb dynamite per cu yd (17% and 45%). Difficulties later required abandonment of section of tunnel which was bypassed with new bore.

(28) Eklutna Tunnel. Rock is argillite and graywacke, fairly hard and massive to badly broken. Some volcanic tuffs and flows. Roof bolts used where suitable; steel ribs used for supports in poorer ground. Minimum thickness of lining, 9 1/2 in.

16. SEWER AND MISCELLANEOUS TUNNELS

(2) East River Gas, New York. Gneiss, mica schist, veins of decomposed schist, and black mud. Average progress, 69 ft per week in good rock in each heading; full face excavated. Compressed air 15 to 48 lb per sq in. used while in soft material. Good rock not lined; soft material shield-driven, lined with cast iron.

(3) Chicago Telephone and Express Subway. Full-size excavation with spades and draw knives. Compressed air used, 9 lb per sq in. Floor 30 ft below street.

(4) Gunnison, Colorado. Irrigation Canal. Mostly black shale and metamorphic granite; other materials penetrated, clay, water-bearing gravel and sand, sandstone, limestone, coal, and marble. 1 working shaft. Top heading and bench method. Average length of tunnel completed per month, 566 ft.

(5) Laramie-Poudre, Colorado. Irrigation tunnel. Granite syenite and quartzite. Pneumatic hammer drills. Excavated full face. Holes fired and fuses cut to proper length to discharge them in prearranged order. Average monthly advance, 473.7 ft for 19 months.

(8) City of St. Paul sewer tunnels. Glacial drift; timber sets for poling, breastboards at face. One 9-hr shift per 24 hr; progress, 4.25 ft (one tunnel set). (The St. Paul City Sewer System comprises 55 miles of tunnel in sandstone, 1.35 miles in glacial drift, and 1.02 miles in limestone, sizes 2 ft 6 in. by 6 ft 0 in. to 12 ft diameter or equivalent.)

(9) Minneapolis-St. Paul Sanitary District, intercepting sewers. St. Peter sandstone. One operating shaft 75 to 210 ft deep per section, 1 to 1 1/2 miles long. Considerable water encountered, up to 4000 gpm from one section, aggregate 15,000,000 gpd. The St. Peter

Table 5

Number and Name	Date Built	Length, mi	Tunnel Section and Size	Material Excavated	Lining	Cost per Linear Foot or as Noted
(1) Netherton, Great Britain, cana	1856–1858	1 7	Horseshoe, 24.3 ft high × 27 0 ft wide	Mixed	Brick	$63.00
(2) East River Gas, N. Y.	1891–1894		8 5 ft high × 10 5 ft wide	Mixed	Part unlined and part cast-iron lined, 10.83 ft dia	
(3) Chicago Tel. and Express	1901	65.0	7.5 ft high × 6 0 wide	Stiff clay	Concrete	
(4) Gunnison, Colo.	1905–1909	5.8	11.4 ft high × 10.5 wide	Mixed	Concrete	
(5) Laramie-Poudre, Colo	1909	2.3	7.5 ft high × 9 5 ft wide	Rock		
(6) Roosevelt, Colorado, mine drainage	1907	4	75 to 80 sq ft	Rock	None	$20.00 to $27.00
(7) Rove, France, canal		4.0	40.0 ft high × 72 ft wide	Dolomite and limestone		
(8) City of St Paul, sewer tunnels	1929–1931	1 35	Semielliptical, 10 ft wide	Glacial drift	Concrete	$82.00
(9) Intercepting Sewer Tunnels, Minneapolis-St. Paul Sanitary Dist.	1934–1936	5.1	Circular, 11′ 9 1/2″	Sandstone	Reinforced concrete, 1′ thick	$67.50 to $82.50
(10) Manhattan Intercepting Sewers, N.Y.C.	1935–1937	1.04 in rock, 3.42 in soft ground	Horseshoe, 5′ 9″ × 3′ 8″ to 8′ 2″ × 10′ 0″	Rock and soft ground	Reinforced concrete, 8″ to 16″	Total tunnel cost $3,500,000
(11) *Main Intercepting Approach Tunnels*						
Manhattan	1935–1938	0.76	Circular, 8′ 0″	Rock	Concrete 18″ thick	$163.00
Bronx	1935–1937	0.95	Circular, 10′ 6″	Rock	Concrete 18″ thick	
(12) Detroit Intercepting	circa 1937	1 08	Circular, 22′ dia shield	Soft clay	Concrete blocks and concrete interlining	
(13) Leadville, Colo.	1943	2.1	9′ × 10.5′	See description		

sandstone pulverizes readily when wet and was generally pumped out with the infiltrating water. On one contract the outside excavation line was cut in the sandstone by high-pressure water jet, in several others by flattened pipe with longitudinal slots using jet action of compressed air and sand to cut the sandstone. 2 headings from each shaft, 3 shifts per 24 hr, average excavation progress, 25 ft per heading per 24 hr. Concrete average 40 ft per 24 hr.

(10) Manhattan Intercepting Sewers, N. Y C Rock Manhattan schist. Soft ground: clay and sand, fill and riprap. Rock excavation full face, timbering where required. Soft ground, subaqueous, full face, liner plates, poling plates, breastboards, hand mucking, 1 cu yd cars; maximum high water to invert 30 ft, cover 8 ft to 30 ft; maximum air pressure 15 lb per sq in gage, average 8 lb per sq in. Plant each shaft Low air supply. 3 compressors at 750 cfm each, 1 at 1200 cfm. High air supply: 1 compressor at 550 cfm. Bulkheads and locks: 3 ft by 4 ft openings with lock doors in steel diaphragms in tunnel lining In soft ground, concreting carried close to heading. 4 6-hr shifts per day; average progress per heading per 24 hr. solid rock, 19 6 ft; clay, 16 5 ft; fine sand, 10 4 ft; coarse sand, 6.5 ft; mixed face, 5.0 ft when rock was below springing line, 8 0 ft when rock was above springing line.

(11) Main Intercepting Approach Tunnels, Manhattan and Bronx, N. Y. C. Limestone, gneiss, and schist. The Manhattan tunnel contains an inverted siphon to bypass large chloritic decomposed zone; siphon dropped 220 ft below planned grade to 550 ft below high water of East River. The tunnels deliver sewage from interceptors to Wards Island treatment plant.

(12) Detroit Intercepting Sewer Tunnel. Shield-driven tunnel in soft clay, compressed air. Precast interlocking concrete block lining with interlining of concrete. Blocks 18 in. thick, ring 30 in. long, 10 segments, one segment split for keying and locked with shear pin. Shield 22 ft diameter, no hood, 14 ft 11 in long: steel bulkhead with 6 2 ft by 2 ft openings controlled by hydraulic gates. Traveling jumbo with 6 in. extensible I beams to support blocks in upper half of ring. 100% of displaced ground admitted through openings in bulkhead. Progress shield driving, 50 ft per 20-hr day (4 5-hr shifts). Maximum progress, 67.5 ft.

(13) Tunnel constructed to drain deep workings of mines in Leadville, Colorado, district. Iron Hill lateral, 2400 ft long; Downtown lateral, 3400 ft. Excavated full section, 700 ft in unconsolidated glacial drift, 400 ft mixed face, most of tunnel in shale, sandstone, and Leadville limestone. A run of water, mud and sand, decomposed porphyry, and rock fragments necessitated driving a bypass heading The zone was bulkheaded and grouted, using hot water and calcium chloride to accelerate setting, and oats, the latter successfully.

17. SHIELD-DRIVEN TUNNELS

Notes Regarding Shield-Driven Tunnels

Numbers refer to numbers in Tables 6 and 7, Tunnel Shields and Cast-Iron Lining in Tunnels.

(1) Thames, England. 1825–1842. London clay. Maximum progress 2 ft, average 0.5 ft per day.

(2) Tower, England. 1869. London clay Maximum progress 9 ft per day.

(4) City and South London, England. 1886. Mostly clay, also several lengths of water-bearing sand and gravel. The average progress of each shield was 333 ft per month during one-third of work

(5) Mersey, England. 1888. Clay, coarse ballast, and running sand. Maximum progress was 57 ft per week.

(6) St. Clair River, Michigan. 1889. Almost entirely in soft damp clay. Maximum progress per month was 382 ft, average 231 ft per month.

(7) Hudson River, New York. 1879–1905. Mostly in silt; at New York side some sand and short length in rock. From May, 1890, to August, 1891, the average progress per month was 126 ft in the first tunnel; in the second tunnel the average progress per day was 15.3 ft.

(8) Glasgow Harbor, Scotland. 1890. One-third in clay with boulders, the rest in sand and gravel The average progress in sand was 2 ft per day.

(9) Glasgow District Subway. 1891–1895. Clay, sand, gravel, and quarry waste. The average progress was 100 ft per month.

(11) Blackwall, London. 1892. London clay, sand, and one-third in open gravel. The maximum progress was 12 1/2 ft per day. The average progress under the river was 98 ft per month.

Table 6. Tunnel Shields

Number and Name of Tunnel	Number of Shields	External Diameter of Shields, ft	External Diameter of Shields, in.	Thickness of Skin, in.	Length, ft	Length, in.	Number of Shield Jacks	Inside Diameter of Jacks, in.	Maximum Thrust, tons *a*	Weight of Shield, tons *a*	Distance Driven, mi
(1) Thames, Great Britain	1	*b*		...	9	0	10	.		134	
(2) Tower, Great Britain	...			1/2	4	9	6	*c*	.		. .
(4) City and South London	18	11 11	1 4 1/2	1/2 1/2	5 6	11 6	6 6	6 1/2 6 1/2	179	...	6.20
(5) Mersey, Great Britain	1	10	3	3/4	11	7	10	7	770	. .	0 25
(6) St. Clair River, Mich.	2	21	6	1	15	3	24	8	1795	92	1 14
(7) Hudson River, N. Y.	2	19	11	1 1/4	10	6	16	8	1540	92	1.60
(8) Glasgow Harbor	1 2	17 17	3 3	3/4 1	7 8	0 6	13	7	250	...	0.41
(9) Glasgow District	16	12	2 1/2	1/2	6	6	6	6 1/2	100	7	3.70
(11) Blackwall, Great Britain	1	27	8	2 1/2	19	6	28 6	8 10	5785	224	0 59
(12) Clichy Siphon, France	1	8	4 1/2	3/4	6	8	5	6 1/2	80	73	0.29
(13) East River Gas	2	11	0 3/4	7/8	7	2 1/4	12	5	595	12	*d*
(15) Waterloo and City, Great Britain	2 2 1	13 13 24	2 9 10	1/2 1/2 1	7 9 10	0 6 0	7 7 22	7 7 7	135 135 845	 112	2 84
(16) Concorde Siphon	1	6	9	3/4	6	8	4	6 1/2	65	...	0.15
(17) Central London	...	12 22	8 10	1/2 1	7 6	0 10	6 22	7 7	185 680	.	13.00
(19) L'Oise Siphon, France	1	8	7 1/2	9/16	16	2	10	6	430	. .	0.18
(20) Baker Street and Waterloo	1	13	0	1	9	8	14	6	475	33	6.28
(21) Greenwich, Great Britain	1	13	0	1	14	1	13	7	840	84	0.23
(22) Lea, England	1	12	5	1	11	6	11	6 1/2	. .	.	0.21
(23) Battery, East River	6	16	11 1/4	1 1/8	9	6	14	8	1750	55	1.67
(24) East River, PRR	8	23	6 1/2	2 1/4	18	0	27	9	7730	240	2.87
(25) North River, PRR	4	23	6 1/4	2 1/8	17	3 3/8	24	8 1/2	3300	193	2.31
(26) Brackenagh, Ireland	1	6	2	1/2	6	9	4	6 1/2	.	.	0.12
(27) Rotherhithe, England	2	30	8	2 1/4	18	0	40	9	5600	...	0.69
(28) Kingsway, England	1	16	2	1	8	9	16	8	..	...	0.09
(29) River Dee, Scotland	1	7	4 5/8	1/2	6	9 3/4	7	6 1/2		.	0.06
(30) Charing Cross, England	...	12	8	...	8	9	10				
(31) Great Northern Strand, England	...	12	8	1/2	6	11	8	. .	.	. .	
(32) 14 St. East River	8	18	5 1/2	2	15	3 3/4 *e*	17	8	125 *f*	116 *g*	1.88
(33) Whitehall St., East River	4	18	6	2 1/4	16	4 *h*	17	8	125 *f*	104 *g*	1.60

(34) Lawrence Street, Brooklyn	2	18	6	2 1/4	16	4 *h*	17	8	125 *f*	104 *g*	1 00
(35) Old Ship, East River	6	18	0	2 1/4	16	4 *h*	17	8	125 *f*	98 *g*	2 31
(36) 60 St., East River	2	18	6 7/8	2 7/16	16	0 3/4 *e*	20	8	125 *f*	142 *g*	0.68
(37) Holland Vehicular	5	30	2	2 3/4	18	10	30	10	6000	400	2.46
	1	31	0	2 3/4	18	10	30	10	6000	.	0 15
(38*a*) Cleveland Water Intake		12	2 1/8		15	1	12	9	1020	. .	3.05
(39) 53 Street, East River, Rapid Transit, N. Y.	2			No data	No data		18	10	.	.	1.07
(40) Fulton-Cranberry Street, East River Rapid Transit Tubes, N. Y.	8	18	10	2 1/2	15 2	0 10 hood	18	10	4140	103 struct., 163 tot.	
(41) Harlem River, N Y.	2				No data		18	10			0 56
(42) Rutgers-Jay Street, East River Rapid Transit Tubes, N. Y.	8	18	10	2 1/2	15 2	0 10 hood	18	10	4140	103 struct., 163 tot	2.07
(43) Detroit-Windsor Vehicular	2	32	3 5/8		15 no hood	3 1/2	30	...	7500	275	0.32
(44) Prospect Park, N Y C. Rapid Transit	2	20	5		18	0				160	1.3
(45) Detroit, Mich., Dearborn Ford Plant Water Supply	4	21	6	2 1/4	14 hood to tail	0	20	10	3120 avg	...	2.27
(46) East Boston, Vehicular	1	31	7				30	...	7070	.	...
(47) Lincoln Tunnel, New York City to Weehawken, N. J. Vehicular	4	31	8	2 1/2 + 1/2 plate in tail	15 2	6 10 hood	28	10	6440	214 struct., 304 tot N Y ; 316 tot N. J	2 21
(48) 6th Ave Subway, N.Y.C.	1	17	10 1/2	2 1/2 mid 2 tail	18 including hood	0	14	10	2800	66 106 tot.	0.36
(49) Queens Midtown Tunnel, N.Y.C.	4	31	8	2 1/2 + 1/2 in tail	15 2	10 10	28	10	6440		1 43
(50) Chicago Subway, Chicago, Ill.	4	25	3		19	6	24	10	4800	225	4.1
(51*a*) Brooklyn-Battery Tunnel, N.Y.C., Manhattan River Tunnels, Vehicular	2	31	8	2 1/2 + 1/2 in tail	15 2	6 10 hood	28	10	6440	225 struct., 315 tot.	1.48
(51*b*) Brooklyn-Battery Tunnel, N.Y C., Brooklyn River Tunnels	2	31	8	2 1/2 + 1/2 in tail	15 2	10 10 hood	28	10	6440		1.43

a Tons of 2000 lb. *b*. Height, 22 ft 3 in ; width, 37 ft 6 in *c* Hand-operated screw jacks. *d*. Driven a very short distance *e* Includes 2-ft hood. *f* Per jack. *g*. Includes hydraulic equipment which weighs about 40 tons on 60th Street shield and 28 tons on other shields. *h*. Includes 2 ft 2 in hood.

Table 7. Cast-Iron Lining in Tunnels

Number and Name of Tunnel	External Diameter of Ring ft	External Diameter of Ring in.	Length of Ring, in.	Thickness of Web, in.	Depth of Flange, in	Number of Segments Exclusive of Key	Weight, lb per linear ft.	Bolts per Ring C	Bolts per Ring L	Diameter of Bolts, in	High Water to Base, Maximum, ft.
(2) Tower, London	7	1 3/4	18	7/8	3	3	971	31	.	3/4	. .
(3) Antwerp		[a]	19 3/4	. . .	4-6	4	. . .	14	4		26
(4) City and South London	10	10 3/4	19	1	4 3/8	6	1,880	47	. .	1	75
	11	3	20	7/8	4 1/2	6	1,880	47	.	7/8	
	22	6	18	1 1/8	7 3/4	12	.	83	. .	1 1/8	. . .
	32	0	18	1 1/2	12	16	. . .	113	. . .	1 1/4	. . .
(5) Mersey, Great Britain	10	0	18	1 11/16	6	10	3,061	28	22		54
(6) St. Clair, Mich.	21	0	18 1/4	2	7	13	9,333	157	56	7/8	78
(7) Hudson, N. Y.	19	6	20	1 1/2	9	11	6,055	66	36	1 1/4	100
	18	9 1/4	20	2 1/2	8	. . .	. . .	66	36	. .	
(8) Glasgow Harbor	17	0	18	1	6	13	4,559	66	28	. .	62
(9) Glasgow District	12	0	18	3/4-1	6	9	2,241	46	20	1	70
(10) Kingston, N. Y.	9	0	18	7/8	4	6	1,580	37	14	1	. . .
(11) Blackwall, Great Britain	. . .	. . .	. . .	1 1/2	10	14	9,408	. .	. .	. .	. .
	27	0	30	2	12		13,265	70	75	1 1/2	80
(12) Chehy Siphon, France	8	2 1/2	19 3/4	1	4	5	1,453	36	24	1	58
(13) East River Gas	10	10	16	1 1/4	4	9	2,400	46	20	1	122
(14) Mound of Edinburgh	17	6	18	1 3/4	7	14	7,100	. .	.	1 3/8	. . .
(15) Waterloo & City	13	0	20	7/8	5 1/8	7	2,326	57	24	1	62
	13	7 1/4	20	7/8	5 1/8	7	2,431	57	24	1	
	24	6	18	1 1/4	9	13	. . .	92	.	. .	. . .
(16) Concorde Siphon	6	6 1/2	19 3/4	7/8	3 5/8	4	895	21	15	7/8	38
(17) Central London	12	6	20	7/8	4 7/8	. . .	. . .	. . .	. .	7/8	. . .
	22	6	18	1 1/8	8	12		81	39	1 1/8	.
(18) Spree, Germany [b]	13	1 1/2	25 5/8	3/8	4	9	1,270	72	36	.	35
(20) Baker St. and Waterloo	12	9 3/4	18	1	4 7/8	6	2,643	53	14	7/8	70
(21) Greenwich, England	12	9	20	1 1/4	6	8	3,080	47	.	1 3/8	70
(22) Lea, England	12	3 3/4	21	7/8	4 7/8	6	.	29	21	7/8	30
(23) Battery, East River	16	8 1/2	22	1 1/8	7 1/2	8	4,000 4,540 5,130	49	27	1	94
(24) East River, PRR, Land	23	0	30	1	8	11	5,166	67	60	1 1/4	. . .
	23	0	30	1 1/4	9	11	6,776	67	60	1 1/4	. . .
(24) East River, PRR, River	23	0	30	1 1/2	11	11	9,102	67	60	1 1/2	68
	23	0	30	1 1/2	11	11	9,102	67	60	1 1/2	93
(25) North River, PRR	23	0	30	2	11	11	12,127	67	60	1 3/4	98
	23	0	30	1 1/2	11	11	9,273	67	60	1 1/2	. . .

(26) Hilsea Creek, Portsmouth, England	12	6	20	. .	. .	6	2,240	. . .	. . .	. . .	58
(27) Rotherhithe, Great Britain	30	0	30	2	14	16	16,600	. .	. .	. . .	.
				1 3/4	14	16	14,700	85	79	1 1/2	97
(28) River Dee, Scotland	8	6	18	1	5	5	1,960	42	18	1	50
(32) 14 St., East River, N. Y.	18	0 [c]	26	1 3/8	9	9	6,323	55	50	1 1/4 [d]	97
	17	2 [e]	26	1	7	9	3,801	55	40	1	115
(33) Whitehall St., East River, N. Y.	18	0 [c]	26	1 3/8	9	9	6,323	55	50	1 1/4	87
	18	0 [f]	26	1	7	9	4,048	55	40	1	80
	17	2 [e]	26	1	7	9	3,180	55	40	1	90
(35) Old Slip, East River, N. Y.	17	6 [c]	26	1 3/8	9	9	6,166	55	50	1 1/4	89
	17	6 [f]	26	1	7	9	3,949	55	40	1	89
(36) 60 St., East River, N. Y.	18	0 [c]	26	1 3/8	9	9	6,323	55	50	1 1/4	116
	18	0 [f]	26	1	7	9	4,048	55	40	1	85
(37) Holland Vehicular	29	6	30	1 5/8 to 2 3/4	14	14	17,300	85	75	1 3/4	101
(38*b*) Mersey Tunnel (Vehicular), England	46	3 [e]	24	1 1/2	13 1/2	24		96	120	1 1/2	197
(39) 53 St., East River, Nott Ave., N. Y. C.	18	3	30	1 3/8 to 2	9	9	6,275	55	50	1 3/8	95
(40) Fulton-Cranberry St, East River Rapid Transit Tubes, N. Y. C.	18	3	30	1 3/8 to 2	9	9	6,275	55	50	1 3/8	92
(41) Harlem River, 161 St, Bronx	18	3	30	1 3/8 to 2	9	9	6,275	55	50	1 3/8	56
(42) Rutgers-Jay, East River Rapid Transit Tubes	18	3	30	1 3/8 to 2	9	9	6,275	55	50	1 3/8	93
(47) Lincoln Tunnel, N Y C. to Weehawken, Vehicular	31	0	30	1 5/8 to 2 3/4	14	14	17,640	85	60	1 3/4	106
(48) 6th Ave. Subway, N. Y. C.	17	5	32	1 1/4	7	8	5,600	49	36		
(49) Queens Midtown Tunnel, mid-Manhattan to L. I. City, Queens	31	0 [g]	32	1 5/8 to 2 3/4	14	14	17,000	85	60	1 3/4	103.5
	31	0 [h]	32	1 1/8 to 1 5/8	11	14	9,350	85	60	1 3/8	103
(51*a* and *b*) Brooklyn-Battery Tunnel, N. Y. C.	31	0 [g]	32	1 5/8 to 2 3/4	14	14	17,000	85	60	1 3/4	107
	31	0 [h]	32	1 1/8 to 1 5/8	11	14	9,350	85	75	1 3/8	125

[a] Height, 5 ft 7 in.; width, 4 ft 11 in.
[c] Earth or earth and rock.
[e] Rock without shield.
[g] Heavy in earth and mixed face.
C = circumferential; L = longitudinal.

[b] Steel lining.
[d] Bolts upset 1/8 in. for thread; also upset 1/8 in. near the head end.
[f] Rock with shield.
[h] Light in rock.

(12) Siphon de Clichy, France. 1892. Open water-bearing sand. The maximum progress was 10 ft per day and the average 6.5 ft per day.

(13) East River Gas, New York. See item 2 of Art. 16, Sewer and Miscellaneous Tunnels.

(15) Waterloo and City, England. 1893. Clay and ballast. 10 ft per day was the average progress while the work was in full swing.

(20) Baker St. and Waterloo, England. 1898. About 350 ft was in gravel under the Thames River; the rest was in London clay. The average progress per month was 117 ft in gravel and 137 ft in clay.

(21) Greenwich England. 1898. Mostly in clay with the crown in close gray sand; about one-third length is in close gray sand and a small part in ballast. The maximum progress was 269 ft per month and the average 148 ft.

(23) Battery, East River, New York. 1901. 500 ft almost entirely in rock, 1500 ft in fine sand with some clay; the rest was coarse sand. Rates of progress follow: In normal air above water line in sand and gravel, average 140 ft per month; under river in rock, fine sand and clay, and sand and gravel, 91 ft per month.

(24) East River, Pennsylvania RR. 1903–1909. Land portion in rock, soft top in places. Under river 18% in fine sand stretched with clay, stiff clay, sand and boulders, 35% sand and boulders, sand and clay, and fine sand; 22% all in rock; and 25% in rock with sand, gravel, and boulders in the crown. Rates of progress were: Best day's work 17.5 ft. Average for month in earth 176 ft, in rock and earth 56 ft, and in rock 61 ft.

(25) North River, Pennsylvania RR. 1903–1909. Mostly in silt, also short lengths in sand and in rock. The maximum month's progress was 545.2 ft. The average progress in rock was per shield: 2.1 ft per 24-hr day where no drift had been excavated in advance of the shield; progress in mixed face, 2.6 ft per day; sand and gravel, 5.1 ft per day; silt (based on 8777 ft of tunnel in silt), 17.5 ft per day.

(30) Charing Cross and Hampstead. 1904. London clay. Maximum progress 180 ft per week.

(32) New York Subway. 1916–1919. B. R. T. system from North 7th St., Brooklyn, to 14th St., Manhattan. Shaft at each side of river. 2 land shields, 2 river shields from each shaft (8 shields in all). Tunnel part in rock and part in sand and part in clay. Average progress per month river section, in earth 147 ft, in earth and rock 44 ft, land section, in earth 168 ft.

(33) New York Subway. 1914–1919. B. R. T. system from Montague St., Brooklyn, to Whitehall St., Manhattan. Shaft on each side of river. From Brooklyn shaft two shields driven each way. River shields met rock tunnel driven without shields under river from Manhattan shaft. Material on Brooklyn side sand with some clay. Average progress per month river section, in earth 192 ft, in rock and earth 80 ft, land section in earth 209 ft. Maximum progress per month, river section 345 ft; land section 454 ft.

(34) New York Subway. 1914–1919. B. R. T. 2 shields driven under streets of Brooklyn at a depth of from 43 ft to 75 ft. All above tide water. Extension of the Whitehall-Montague street tunnel. Material sand. Compressed air not used.

(35) New York Subway. 1914–1919. I. R. T. Shafts on each side of river. Two shields in each direction from Brooklyn shaft; 2 shields toward river from Manhattan shaft. Material sand on Brooklyn side. Tunnels part in rock, part in sand on Manhattan side. Average progress for month, river section, in earth 184 ft, in earth and rock 74 ft. Land section in earth 167 ft.

(36) New York Subway. 1916–1919. B. R. T. from North Jane St., Queens, to East 60th St., Manhattan. One shaft on each side of river and shaft in rock on Blackwells Island. From Queens Borough shaft, 2 shields were driven under east channel of East River, pushed through rock tunnels, previously excavated under Blackwells Island and continued through earth under west channel to rock on Manhattan side. Average progress for month, river section in earth 188 ft, in rock 119 ft.

(38a) Cleveland Water Intake. Completed about 1917. Shield-driven intake tunnel under Lake Erie, 3 miles long. Stiff clay with sand seams; gas bearing. After disastrous explosion, regular coal-mine precautions observed. Compressed air heading from land was 14,000 ft long. Shield was a simple riveted-steel double-ring structure without interior partitions or diaphragm. No hood. Cast-steel cutting edge. Lining concrete blocks, 11 1/2 in. thick, rings 18 in. long, 6 blocks to the ring, all alike, wood segments used for tapers. Inside diameter 10 ft. Half of excavation with machine; other by hand clay knives. Upper half excavated to 4 1/2 or 6 ft, then lower half. Maximum progress 886 ft in one month. Depth 100–110 ft below lake level. Cover in lake 50 to 70 ft thick, 20 lb per sq in. air pressure, gage.

(39) 53rd St., East River, Nott Ave., March 1927–July 1930. Length 5,585 ft (single track 11,170 ft): masonry-lined rock tunnel, 5,021 ft; cast-iron-lined shield-driven, 5,654 ft;

steel bent and concrete, 151 ft Contract cost, $10,255,150; $918,01 per ft of single track including shafts. Three shafts: Manhattan, Welfare Island, and Queens. Shields driven westerly from open cut in Queens. Permanent blanket in west channel Progress, feet per day, including Sundays, holidays, delays: Shield and compressed air soft ground 8.6; mixed face 4 9, rock 4 1 Rock tunnel, normal air, 5.0.

(40) Fulton-Cranberry St. East River Rapid Transit Tubes, built 1928–1934. Cast-iron-lined twin tubes 18 ft 3 in. outside diameter. 9,100 ft (total single tube 18,200 ft). Maximum grades 4.1%, minimum curvature, radius 335 ft. Material. Manhattan, fine sand, some silt or clay; Brooklyn, medium sand, boulders Cover: land sections 30 ft to 87 ft; under river generally 20 ft. Full face excavation breastboards to near bottom. Everywhere below water except short distance in Brooklyn above water. Hand mucking; storage battery locomotives. Disposal to scows. Plant each side of river. 5 5,600 cfm low air compressors, 625 hp each; 3 1,300 cfm high air compressors, 225 hp each; 3 6,000 lb 25 gpm hydraulic pumps, 100 hp each; 1 hydropneumatic accumulator. Full capacity of air supply often required: average 60%. Worked 24 hr per day, 6 days per week. Maximum air pressure 47 lb per sq in gage when sinking sumps; maximum tunneling 37 lb per sq in.; minimum 5 lb and some free air. Average progress per 24 hr per heading:

	Manhattan	Brooklyn
Land headings	14 4 ft	15.0 ft
River headings	13.5 ft	12.5 ft

Two steel and concrete caissons to accommodate both tubes; one sunk each side of river to depth of 80 ft. Average contract cost per linear ft of single tube $905, including cost of interior lining, ducts, etc., and of shafts but not of special station construction. Total single track including 3 stations 19,048 ft, contract cost $20,934,730, $1,099,00 per ft of single track.

(41) Harlem River, 161st St., Bronx June 1928 to May 1932 7795 track feet concrete-lined tunnels; 5060 track feet open cut including two shafts: 2935 track feet shield-driven cast-iron-lined, 3 tubes Length of contract, 5410 ft; distance between shafts, 1535 ft. Cost $8,063,352 total; $510.66 per linear foot single track. Progress shield-driven tunnel, feet per day, including Sundays, holidays, and delays: north tunnel 8 9; center tunnel 7 2; south tunnel 8.2.

(42) Rutgers-Jay St., East River Rapid Transit Tubes. Built 1930–1934. Cast-iron-lined twin tubes 18 ft 3 in. outside diameter, 5,474 ft (total single tube 10,948 ft . Maximum grade 3.3%, minimum curvature radius 2300 ft. Material: Manhattan, fine sand with some clay or silt, Brooklyn, medium sand, some boulders, tunnels passed through ridge of mica schist near Manhattan pierhead line. Maximum air pressure 42 lb per sq in. when sinking sumps, maximum tunneling 38 lb per sq in.; ground water lowered by pumping in Brooklyn so these land headings were in dry. Plant same as Fulton-Cranberry. Average progress per 24 hr per heading:

	Manhattan	Brooklyn
Land headings	19.5 ft	18.0 ft
River headings	16.0 ft	12.0 ft

One single shaft to accommodate both tubes put down on each side of river, 70 ft deep. Average contract cost per linear foot of single tube $898, including cost of interior concrete lining and of shafts, but not station construction. Total contract cost including one station $11,108,749; $1,014 88 per foot single track including station Manhattan shaft was 60 ft by 32 ft by 70 ft deep steel and concrete caisson sunk and sealed under compressed air; Brooklyn shaft, steel and concrete structure 59 ft 2 in. by 30 ft 6 in. by 70 ft deep built in free air in sheeted excavation.

(44) Prospect Park, N. Y. C. Rapid Transit. Express, 1551 ft, and local, 1900 ft, both northbound and southbound constructed in sandy soil above water table under Prospect Park, Brooklyn, in 1930, using two shields, each of which was reused by jacking out of shaft at end of first run and transporting and lowering in shaft at beginning of second run. Reinforced concrete block lining made up of 10 standard blocks 6 ft 1 in long, 18 in. thick, making up ring 30 in long 16 ft 9 in. inside diameter; 19 ft 9 in. outside diameter. Blocks weighed 3200 lb each.

(45) Dearborn Ford Plant Water Supply. Built about 1931. 12,000 ft long; 60 ft deep; 10 curves. Excavated in clay with 4 shields. Concrete block lining erected in shields. 18 in. thick. 10 blocks per ring, 2 of which were half-length key blocks. Inner lining 18 in. thick, inside diameter 15 ft. Shield bulkheaded; 4 12-in. by 12-in openings admitted displacement of shield each 30 in. shove, 32 cu yd. Street raised 1/4 to 1 in . no gravel was ejected outside the lining. There was no distortion of lining.

(48) 6th Avenue Subway, 1936–1938. A single shield was used to drive the two tunnels, east tunnel, 861 linear feet; west tunnel, 1046 lin ft Tunnels started through bulkhead walls of existing subway, ending in open-cut excavation. Material: full rock face, mixed face, and full earth: very fine sand, silt, and clay above rock. Compressed air, maximum gage 18 lb per sq in., average 8 lb per sq in , minimum 4 lb per sq in.; low air compressor plant, 3 compressors, total 7400 cfm. Pea gravel used to fill void outside lining; grouting through lining about 30 ft behind shield. Cost of tunnel driving: excavation, cast-iron lining, and gravel and grout $620 per linear foot exclusive of overhead and profit. Shield progress per 24-hr working day. east tunnel 8 35 ft; west tunnel 6.54 ft; average 7.25.

(50) The Chicago Subway (see also Art. 20, City Subway Systems). South of the Chicago River several miles of tubes were built by the shield method. The shield method was adopted here, in preferance to the liner-plate method, because the clay was soft and the adjacent high buildings required that the safer, though more expensive, of the two methods of tunneling be used. Compressed air was used at gage pressures of 8–15 lb per sq in. Each shield was provided with a hood which projected beyond the lower half of the shield cutting edge. Horizontal and vertical struts stiffened the center section of the shields and divided the face into 6 working pockets, the shields generally being advanced with 5 to 20% of the face area open, the clay squeezing into the shield in ribbons. Primary lining, erected in the tail of the shield, consisted of structural steel rings· 2 ft 9 in. long, weighing about 5 1/2 tons per ring or 2 tons per linear foot of tunnel. The lining was 24 ft 9 in. in exterior diameter and was made up of six segments, about 12 ft long, and a key segment; circumferential flanges were 10 1/2 in. deep, and end flanges were 12 3/4 in. 3/4 in diameter bolts were used to connect segments and adjacent rings. 4-in. T stiffeners placed longitudinally at approximately 12-in. centers around the circumference of each ring provided for the reaction of the shield jacks. A reinforced-concrete lining 2 ft 2 in. thick was placed inside the steel lining, the concreting operation usually following 500 ft behind the shield. The interior diameter of the arch above benches of the finished shield-driven tubes was 20 ft 5 in.

Note: For description of Nos. 37, 38*b*, 43, 46, 47, 49, and 51*a* and *b*, see Art. 14, Highway Tunnels.

18. ROOF SHIELDS

(6) Started 1901. Material penetrated, blue clay and boulder clay, with some sandy and silty permeable clay. Air pressure, 18 to 20 lb average, 25 lb maximum. Average progress 4.8 ft per day.

(7) The roof shield moved on steel rollers on subway walls already placed. It was advanced 32 in. at a time. Concrete lining; the jacks reacted against 5 in. diameter iron pipes filled with concrete which were imbedded in the finished arch.

(8) New Union Tunnel, Baltimore, Md., July 1933 to September 1934. Double-track railroad tunnel. 2085 ft driven with roof shield, joining cut and cover sections. Clear inside dimensions 33 ft wide, 24 1/2 ft high, 40 ft out to out of wall drifts. 30-in. concrete invert. Built adjacent to old brick-lined 2-track tunnel built 1873 in which service was maintained. (51 ft c.c., 14 ft clear between walls.) Roof-shield section, cast-iron-lined semicircular arch 17 ft 8 in. radius outside, spring line 6 ft 7 in. above base of rail, cast iron to 2 ft 9 in. below spring line. No interior concrete lining above spring line; concrete bench and invert below spring line. Two bottom sidewall drifts 11 ft diameter shield-driven, lined with steel plates: 11 segments per ring, 16 in wide, 2-in. flanges, 3/16 in. thick. The sidewall shields were made up of cast-iron segments bound inside 1-in. steel plate, 11 ft 4 in. outside diameter, 8 ft 3 1/2 in. overall length including 18 in. hood; tail 2 ft 9 in., propelled by 8 6 in. diameter jacks. Sidewalls to 2 ft 9 in. below spring line concreted in drifts; roof shield advanced on sidewalls on inverted rails set in concrete. Roof shield was structural steel with cast-steel cutting edge. Skin made up of three 7/8 in. plates Tail-lapped completed lining 43 in. when in position to be advanced another 30-in width of arch ring. Shield rode on 3 3/4 in. steel sole plates 7 ft 3 in. long. Two platform working levels. 9 6 in. diameter platform rams. Weight of shield. structural 151 tons; hydraulic equipment 72 tons. Cast-iron lining weighed 8880 lb per linear foot. 1 3/4 in. diameter bolts; staggered joints Flange depth 1 ft 2 in., 1 1/2 in. web, 3-in. grout holes, 10 plates and key per course; 5 types of segments; lead calking. Material excavated: glacial drift (sand, gravel, clay, and mud) overlying soft gneiss on hard rock, irregular water level from above crown to below, drained by sidewall drifts.

Sidewall drifts usually 500 ft ahead of roof shield. Concrete placed through boreholes from surface at 175-ft to 200-ft spacing. Sidewall shields started October 23, 1933; holed through July 23, 1934; 500 ft in gneiss, remainder glacial drift. Roof shields all soft ground, average progress when job was well under way 15 ft per day.

Table 8

Number, Name, and Date of Tunne	Number of Shields	Width of Shield ft	Width of Shield in.	Height of Shield * ft	Height of Shield * in.	Skin, in. thick	Length † ft	Length † in.	Number of Jacks	Inside Diameter of Jacks, in.	Maximum Thrust, short tons	Distance Driven, miles
(1) de Clichy, 1895	1	23	9	9	8	9/16	17	3	6	9 1/2	616	0 78
(2) Boston, Tremont, 1897	1	29	4	8	7 1/4 overall	1	14	0	10	6	1250	0.11
(3a) Orleans Railway, 1898	1	34	9	12	6	1	19	8	10 10	10 4 1/2	1120 168	0.54
(3b) Extension, 1898	1	32	0	11	8	3/4	23	0	8	9 1/4	1021	0.22
Collecter de Brevre, 1898:												
(4a) Chagnaud type	1	16	2	6	6 3/4	3/4	18	0	6	6 3/4	228	0.11
(4b) Chagnaud type	1	16	2	7	6 3/4	3/4	18	0	6	6 3/4	228	0.18
(4c) Dioudonnat type	1	16	3	10	6 overall	9/16	13	4	5	6 1/4	224	0.07
Paris Metropolitan Ry.:												
(5a) Champigneul type, 1899	4	28	3	8	9 1/2	7/10	23	2	8	9 1/4	923	0.95
(5b) Dioudonnat type, 1899	3	28	3	8	7	1	22	2	8	8 1/2	896	0.33
(5c) Weber type, 1899	2	28	3	8	7	1	16	1	4 8	5 3/4 8 1/2	179 896	0.90
(5d) Lamarre type, 1899	2	28	3	8	7	1	19	9	9	7 1/2	1149	0.19
(6) Boston Harbor, 1901	2	28	10	14	5	1 1/4	13	0	16	3 1/4	75	0 81
(7) Flatbush Avenue Rapid Transit, Brooklyn, N. Y., Ca. 1916–1917	1	36	0	14	0	1 1/2	12	0	14	8 1/2	1540	
(8) New Union, Baltimore, Md., 1933	1	36	1/4	20	9	2 5/8	13 hood 2′ 6″	9 3/4	20	10	4000	0.40
(9) Mersey Tunnel, opened 1934	1	46 plus clearance	3				12	6	24			0.17

* Height = from spring line to crown unless noted

† Length = distance over all.

19. SUBAQUEOUS TUNNELS BUILT FROM SURFACE

(1) Detroit River Tunnel, Michigan Central Railroad. Built 1906–1910. 2-track tunnel, 1.55 miles portal to portal. The entire tunnel was built in blue clay. The under-river section, 2965 ft long, was built by sinking steel tubes of 3/8 in. thick plates, 23 ft 4 in. in diameter in pairs 26 ft 4 in. c c. in a dredged trench each tube being for a single track. The sections were 262 ft 6 in. long with two exceptions, one section containing sumps was 238 ft 6 in. long, and a closure section was 64 ft 6 in. long. The tubes were reinforced by means of 4 by 3 by 3/8 in. circumferential stiffener angles riveted inside the plate at 12-ft centers, and were fitted, at 12 ft intervals, with 1/4 in. thick steel diaphragms. Wood sheathing was fastened to the upright edges of the diaphragms to retain the tremie-placed concrete when the sections were placed in the trench. Thickness of concrete outside steel was 3 ft at sides and 4 ft 6 in. top and bottom. Depth of trench 60–80 ft below mean water surface: average depth in river bed 36 ft; 18 ft minimum, 48 ft maximum. For several hundred feet, top of tunnel was 3–7 ft above river bottom. Tubes were bulkheaded, floated into position and filled with water to sink. Sinking controlled by auxiliary air cylinders, two anchors of 22 tons each, and derrick scow. Adjacent sections were joined by means of pilot pins in sockets and bolts placed by divers. Tremie concrete was placed around outside of section, trench backfilled, and tube unwatered. Interior reinforced concrete lining minimum 20 in. thick placed working from Detroit end. Radius of arch was 10 ft; 18 ft top of rail to intrados of arch. 11 ft 6 in. clear between benches on both sides of track; benches 5 ft 3 in. above top of rail.

The easterly and westerly approach tunnels, respectively 3549 ft and 2159 ft long, were of twin construction. Portions were built with use of center wall and side shields, and compressed air was used on part of the work. Normally 18 ft top of rail to intrados of concrete arch of 8 ft 3 in. radius; other clearances same as river section.

(2) Chicago River Tunnel. Single length of two parallel steel cylinders with a longitudinal dividing wall, lined with reinforced concrete and sunk on prepared foundations in a trench dredged in the river bottom. Tubes of 3/8 in. steel plate, stiffened by plate and angle gussets above and below the central wall every 7 1/2 ft. Concrete lining 20 in. to 40 in. thick. Central wall 3 ft thick. Length of tubes 278 ft, width 41 ft, depth 24 ft. Weight when sunk 8000 tons. Sand backfill and cover. Connection to shore tunnels made by cofferdam. Built 1910–1912.

(3) Paris Subway under Seine, Pont Mirabeau crossing. The river portion is 644 ft long and made up of 5 sections 116.8 ft to 144.4 ft long. Each section was a pneumatic caisson consisting of a structural-steel frame covered inside and out with steel plates and filled with concrete, with a double-track tube of similar construction above the working chamber. Overall width 29.9 ft, overall height 30.1 ft. These sections were sunk in position 15 3/4 in. apart, this space filled with tremie-placed concrete through which an opening of required tunnel section was later excavated. A tunnel having a total caisson length of 1313 ft was built under the Seine by a very similar method in 1906–1907.

(4) Harlem River crossing, New York Subway, Lexington Avenue Line. 4-track tunnel built very much the same as the Detroit River tunnel noted above. The sections as assembled had an overall width of 76 ft and height of 24 1/2 ft. Four of the sections were 220 ft long and the remaining one was 200 ft long. Contract price $1500 per linear foot for 4-track structure. Built 1911–1912.

(5) Harlem River Tunnels, New York Subway, Lenox Avenue Branch. Twin tube sections 610 ft long of cast-iron lining, inside diameter 15 ft. Across-the-river trench was excavated, bottom of which was a little below the middle of the finished tunnel. Two lines of heavy sheet piling were driven outside the line of the finished tunnel and cut off at the springing line of the arched roof. A floating timber box was then built in which the arched roof of cast-iron segments was erected and covered with concrete with proper internal bracing and bulkhead ends. The floating box was sunk from underneath and the roof left floating like a diving bell. This with steadying tackle was sunk and left resting on the sheet piling and bearing piles previously driven in the trench. After divers connected this to the section already in place, the lower half of the tunnel was built in compressed air. Built 1903–1904.

Note: For descriptions of other subaqueous tunnels constructed from surface see Art. 14, Highway Tunnels, Nos. 3, 6, 21, 30, 31 and 35.

20. CITY SUBWAY SYSTEMS

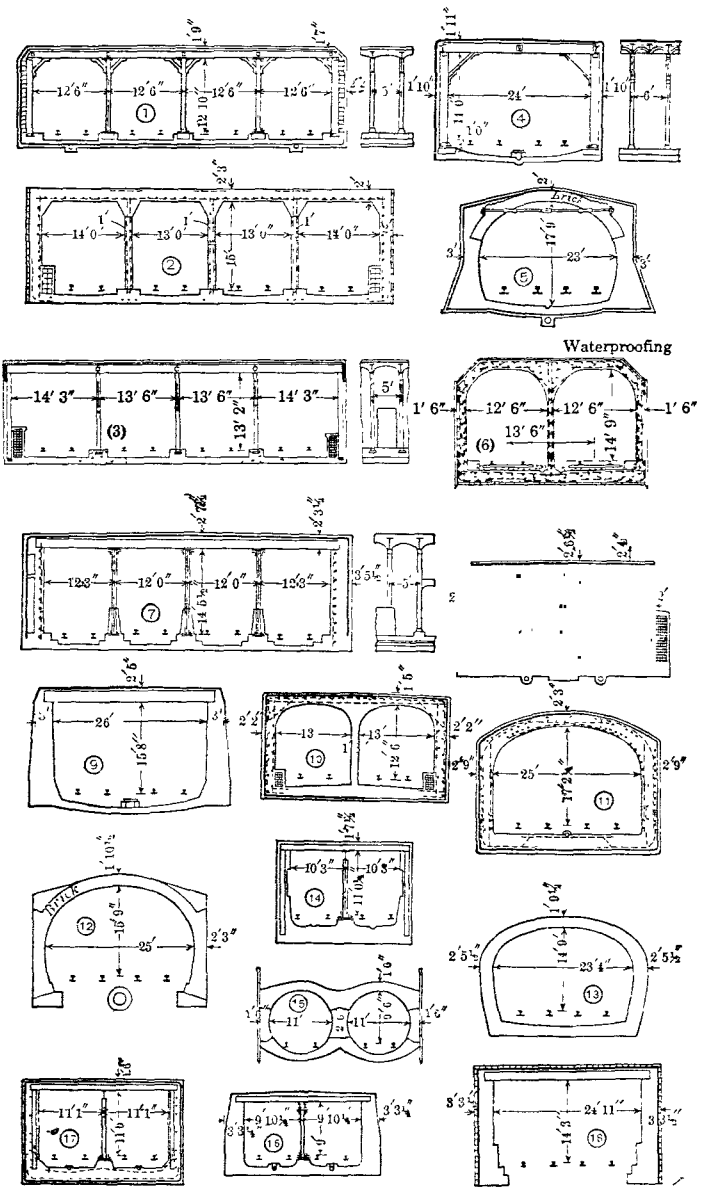


FIG 29. Subway sections.

United States

1. *The New York Subway System*, the largest, has 132 miles of underground 2- or 4-track structure under municipal operation. Portions first built (1900–1903) are nearly all of bent type, Fig. 29, (1), and some built later (1902–1912) are of reinforced-concrete type (2) or similar. Type (3) provides a partition between the middle tracks to promote ventilation by train movement. It applies to the most recently constructed subways. The roof and where necessary the sides and floor are waterproofed. The inside tracks are for express and the outside for local trains. At express stations the platforms are between local and express tracks, serving both, and at local stations at the sides. In some parts the four tracks are built in two tiers. Excavation was partly in rock. Ground water and very fine quicksand were frequently met. Methods varied widely. The system includes several miles of arch tunnels in rock, cast-iron-lined tubes, and steel viaduct extensions. See Art. 17, Shield-Driven Tunnels, and Art. 19, Subaqueous Tunnels Built from Surface, for description of subaqueous tunnels in the system.

2 *The Boston Subway System*, the first in America, with 32 miles underground trackage, radiates to Cambridge, Dorchester, East Boston, and Brookline. It is mostly 2-track subway and principally of bent type (4) and arch type (5) in earlier (1895–1908) portions and of reinforced-concrete type (6) in a later portion. Where traffic was dense a short portion somewhat like (5) was tunneled with roof shield. The Cambridge portion (1909–1912) is of arched reinforced type (11), but where cover is shallow the roof is made flat. Excavation was nearly all in sand, clay, and gravel. The system is for both surface cars and trains. It includes two shield-driven subaqueous tunnels and steel viaduct extensions or connections.

3. *The Philadelphia Subway System.* Private capital built a subway section, east and west in Market Street, 1903–1907, 2 1/4 miles long, which is in equal parts a 4-track (7) and a 2-track, (8) structure.

During 1924–1928 the city constructed the 4-track Broad Street and Grange Avenue subway 7 miles long and in 1932 completed a 2-track spur 2.3 miles long. The city also completed a southern 2-track extension to the Broad Street Subway for a distance of 1.4 miles.

The city is now (1955) constructing an extension of the old Market Street subway comprising a 4-track subway (with tunnel under Schuylkill River) to accommodate both subway trains and streetcars, 0 7 miles long; a 2-track subway for subway trains, .4 mile long; and a 2-track subway for streetcars, 1.2 miles long.

All of the city-built subway sections are of steel bent construction supporting concrete jack arch walls and roof. Roof sections have been weatherproofed with either asphalt mastic or 3-ply fabric and asphalt. The sidewalls are not waterproofed. Subway is drained by longitudinal drains under subway floor with lateral drains extended outside of sidewalls to control the height of ground water.

4. *Long Island Railroad Tunnel on Atlantic Avenue, Brooklyn*, (9), built 1901–1907, 2 miles long, is for electric trains of the Long Island RR. The sides are retaining walls and with the roof are waterproofed with tarred felt. Excavation was mostly in sand and gravel.

5. *Hudson and Manhattan Railroad Tunnel on Sixth Avenue*, New York (10), built 1906–1911, 1 mile long, is for Hudson and Manhattan RR trains entering the city by tunnel under the Hudson. Roof, sides, and, where in earth, floor are reinforced. Waterproofing was mostly of asphalted burlap but in a short portion a waterproofing compound was mixed with the concrete. Excavation was partly in rock. Some quicksand was met.

6. *The Chicago Subway*, constructed 1938–1941 and 1947–1949, aggregates approximately 8 miles of 2-track line comprised of pairs of single-track tubes and station section. Most of the subway was built by tunneling methods in plastic blue clay under city streets. The maximum cover was 35 ft and the minimum 15 ft, ground-water level was a minimum of 40 ft above the invert. The water content of the clay varied from 10% to 20%. Compressed air was employed in tunneling, pressures ranged from 8 to 15 lb per sq in.

Where the clay had relatively high bearing values, the liner-plate method of tunneling was used. These sections were horseshoe in shape, constructed by mining in a series of side drifts and benches or levels. Reinforced-concrete lining.

For description of shield-driven tunnels see item 49, Art. 17, Shield-Driven Tunnels.

Foreign Subways

7. *The London Metropolitan and District Subway*, (12), an irregular oval line termed the Inner Circle, was the first city subway for rapid transit. It was built 1860–1884 and later extended from its original length of 13 miles, part of which is open cut. The arch and walls are of brickwork. The extrados is waterproofed with asphalt. Where cover is shallow, cast- or wrought-iron roof girders were used. With soft or wet foundation, an invert was added. Excavation was mostly in gravel, sand, and clay. Later extensive underground lines are in deep-level tubes, shield-driven in London clay. Parts of system consist of two tubes 11 ft 8 1/4 in. diameter 5 ft apart in clear about 40 ft below surface. At stations diameter is 21 ft 2 1/2 in. and on sharp curves 12 ft.

8. *The Paris Underground Railway*, (13), a network of belt and intersecting lines, begun 1898 and opened 1900, had in 1928 a length of about 60 miles, mostly subway. A link south of the Seine, begun 1912 and halted by the First World War, was resumed 1923, the plan being to provide about 80 miles of double-track line within the city proper, in addition to a large mileage of extensions on viaduct or surface. Short portions first built were tunneled with roof shield or by the Belgian method. Watertightness was secured by grouting. Excavation was mostly in sandy soil and in some parts in soft rock.

9. *The Berlin Underground Railway*, (14), an original east-west trunk, begun 1896 and opened 1902, with later extensions or connections to suburbs, has about 25 miles of 2-track line, of which about half is subway. A new north-south trunk, 7.7 miles long, built by the municipality, was opened for operation by the existing company early in 1923. The section shown is that of the Schoneberg branch, opened 1910, of the bent type. The longitudinal girders are discontinuous, each supporting 5 roof beams, and rest on two columns over which they cantilever. A feature was the use of small I beams, driven as piles at each side every few feet, to the inner flanges of which were clipped breastboards, retaining the earth and serving as back forms for the concrete of the sidewalls, in such manner as to permit the final withdrawal of the beams. The beam piles were braced apart only at the top, leaving a free working cut. An earlier type has thicker walls without beams. A late type has no columns. Waterproofing is asphalted paper. Excavation was mostly sandy and much of it water-bearing.

10. *The Glasgow District Subway*, (15), an irregular oval line, built 1891–1897, is 6 1/2 miles long, comprising subway, tunnel, and open cut. For the subway, a heavily sheeted trench was excavated deep enough to build the roof, which was next waterproofed with layers of asphalt and covered with the restored pavement. The roof, supported by the sheeting, was then undermined with drifts in which were built the walls and floor, completing the structure. A short section in soft ground was built under air pressure. Excavation was in sand, clay, mud, and rock.

11. *The Budapest Subway*, (16), for surface cars, a line 2 miles long, built 1894–1896, was the first underground electric railway. The I beams of the roof, which is close under the pavement, are supported at the middle on a continuous girder of two I beams resting on columns spaced about 13 ft apart. Waterproofing consists of sheets laid in pitch. The system is being expanded, 1953, with an east-west line with 8 stations Two tracks are carried in separate tunnels 5 meters in diameter.

12. *The Hamburg Subway*, (17), begun 1907, forms 5 miles of a ring with external branches opened in part in 1912 and since extended. Excavation was in varied soil, in parts water-bearing. In dry ground, openings are left in the floor under the tracks and only the roof and sides were waterproofed. Beam piles were used at the sides, as in Berlin. Where depth permitted, a single arch section with invert was used.

13. *The Buenos Aires Subway*, (18), for surface cars, was begun 1911 and opened 1913 for a length of 2 miles, with extensions in progress or planned. It is of the earlier German type without steel in sidewalls. Waterproofing is tar paper and asphalt. Excavation was in hard clay. Electric shovels were used.

14. *The Madrid Subway*, opened in part in 1919 and extended in 1926 to a length of 9.3 miles, is roughly cross-formed in plan. The double-track arch and the rectangular section are both used. For the latter, the sidewalls were first built in separate trenches, the pavement was then removed, the roof built, and the pavement replaced, after which the core was excavated, and the floor and middle support placed.

15. *The Sydney, Australia, Subway*, a 2-track loop 3 6 miles long, from the Central Terminal station through the business district, with extensions, was opened in part late in 1926. Some viaduct and some open cut are included. Steel framing is used where head room is restricted.

16 *The Tokyo Subway System* was opened for a length of 1 1/2 miles late in 1927, and is being expanded. The structure is 2-track and of the bent type.

17. *Moscow, Russia.* In 1939 26.7 miles of subway were complete with 28 stations, about two-thirds of mileage being in tunnel. Subway construction was started in 1933 and 7 miles put in operation 1935. Maximum grade is 3.33%. Stations are finished with polished marble and granite, tile mosaics. Air pressures up to 35 lb per sq in gage were employed in some of the tunnel work. Shield-driven sections of tunnel lined with concrete blocks, 20 ft 4 in outside diameter. Shields weighed 132 tons

18. *The Osaka Subway, Japan.* Construction started 1930; 1.6 miles in operation 1933, and 2 6 miles with 5 stations in operation 1935. The subway is 2-track and is being expanded.

19. *Yonge Street Subway.* Toronto, Canada. 2-track subway to replace trolley lines. Reinforced-concrete box section. System 4 1/2 miles long complete 1953. Partly in open cut bridged by intersecting streets Between stations subway box is 32 ft 6 in wide by 17 ft 8 in. deep; at stations 52 ft 4 in. wide by 18 ft 3 1/8 in deep. Top of rail to roof is 13 ft Station platforms 500 ft long for 8 54-ft cars. 12 stations. Material excavated was clay overlying shale, one-third in the shale.

SECTION 11

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MATHEMATICAL TABLES

NUMBERS

1. LOGARITHMS

Logarithms (abbreviation *log*).—The log of a number is the exponent of the power to which it is necessary to raise a fixed number to produce the given number. The fixed number is called the *base*. Thus if the base is 10, the log of 1000 is 3, for $10^3 = 1000$. There are two systems of logs in general use, the *common*, in which the base is 10, and the Napierian, or *hyperbolic*, in which the base is 2.718281828. . . . The Napierian base is commonly denoted by e, as in the equation $e^y = x$, in which y is the Napierian log of x. The abbreviation $\log_e$ is commonly used to denote the Napierian log.

In any system of logs, the log of 1 is 0; the log of the base, taken in that system, is 1. In any system the base of which is greater than 1, the logs of all numbers greater than 1 are positive and the logs of all numbers less than 1 are negative.

The modulus of any system is equal to the reciprocal of the Napierian log of the base of that system. The modulus of the Napierian system is 1, that of the common system is 0.4342945. The log of a number in any system equals the modulus of that system × Napierian log of the number. The *hyperbolic* or *Napierian* log of any number equals the common log × 2.3025851.

Every log consists of two parts, an integral part called the *characteristic*, or index, and the decimal part, or *mantissa*. The mantissa only is given in the usual tables of common logs, with the decimal point omitted The characteristic is one less than the number of figures to the left of the decimal point in the number whose log is to be found. The characteristic of numbers from 1 to 9.99+ is 0, from 10 to 99.99+ is 1, from 100 to 999+ is 2, from 0.1 to 0.99+ is −1, from 0.01 to 0.099+ is −2, etc. Thus,

log of 2000 is 3.30103;	log of 0.2	is −1.30103,	or	9.30103 − 10
" " 200 " 2.30103;	" " 0.02	" −2.30103,	"	8.30103 − 10
" " 20 " 1.30103;	" " 0.002	" −3.30103,	"	7.30103 − 10
" " 2 " 0.30103;	" " 0.0002	" −4.30103,	"	6.30103 − 10

The minus sign is frequently written above the characteristic thus: log 0.002 = $\bar{3}$.30103. The characteristic only is negative, the decimal part, or mantissa, being always positive.

When a log consists of a negative index and a positive mantissa, it is usual to write the negative sign over the index, or to add 10 to the index, and indicate the subtraction of 10 from the resulting logarithm. Thus log 0.2 = $\bar{1}$.30103, may be written 9.30103 − 10. The difference between a logarithm and 10 is its *arithmetical complement* or **cologarithm.**

In tables of logarithmic sines, etc., the −10 is generally omitted, as being understood.

RULES FOR USE OF THE TABLE OF COMMON LOGARITHMS.—To Find the Log of a Decimal Fraction or of a Whole Number and a Decimal.—First find the log of the quantity as if there were no decimal point, then prefix the index according to rule; the index is one less than the number of figures to the left of the decimal point.

EXAMPLE	log of 3 14159	log of 3.141	= 0 497068.	Diff. = 138.
From proportional parts		5	= 690	
" " "		09	= 1242	
		log 3.14159	0.4971494	

If the number is a decimal less than unity, the index is negative and is one more than the number of zeros to the right of the decimal point. Log of 0 0682 = $\bar{2}$ 833784 = 8 833784 − 10.

To Find the Number Corresponding to a Given Log.—Find in the table the log nearest to the decimal part of the given log and take the first four digits of the required number from the column N and the top of the column containing the log which is the next less than the given log. To find the 5th and 6th digits subtract the log in the table from the given log, and multiply the difference by 100, and divide by the figure in the Diff. column opposite the log; annex the quotient to

the four digits already found, and place the decimal point according to the rule; the number of figures to the left of the decimal point is one greater than the index. The number corresponding to a log is called the anti-logarithm.

Find the anti-log of..... .... 0 497150
Next lowest log in table corresponds to 3141.. .. . 0 497068 Diff. = 82.
Tabular diff. = 138; 82 ÷ 138 = 0.59+

The index being 0, the number is therefore 3 14159+

Multiplication by Means of Logarithms.—Add together the logs of the two numbers to be multiplied. The sum is the log of the product.

a. Where both factors are greater than unity.

EXAMPLE. 31 × 1274 = 39,494.

Solution. log 31 + log 1274 = 1 491362 + 3 105169 = 4 596531 = log 39,494.

b. Where one or more factors are less than unity, the logs with a negative characteristic can be handled most conveniently by adding and subtracting 10.

EXAMPLE. .000028961 × .084507 = 0000024474.

Solution. log 000028961 + log .084507 = $\bar{5}$.461814 + $\bar{2}$.926893.
= (5 461814 − 10) + (8.926893 − 10) = 14 388707 − 20 = $\bar{6}$ 388707
= log .0000024474.

Division by Means of Logarithms.—Subtract the log of the divisor from the log of the dividend. The remainder is the log of the quotient.

a. When the divisor is smaller than the dividend.

EXAMPLE. 2987 ÷ 63 = 47 41284.

Solution. log 2987 − log 63 = 3 475235 − 1 799341 = 1.675894 = log 47.41284.

b. When the divisor is larger than the dividend, add and subtract as many tens as may be necessary to the log of the dividend and proceed as before.

EXAMPLE. .000672 ÷ 263 = .00000255513.

Solution. log .000672 − log 263 = $\bar{4}$ 827369 − 2 419956
= 6.827369 − 2 419956 − 10 = $\bar{6}$.407413 = log .00000255513.

c The log of a fraction is obtained by subtracting the log of the denominator from the log of the numerator. Thus,

$$\log \frac{a}{b} = \log a - \log b.$$

To Raise a Number to Any Given Power.—Multiply the log of the number by the exponent of the number, and find the number whose log is the product.

a. Where the exponent consists of one figure

EXAMPLE. 16.2^3 = 4251.528.

Solution. 3 × log 16 2 = 3 × 1 209515 = 3 628545 = log 4251 528.

b. Where the exponent consists of two or more figures, it is best to multiply the characteristic and mantissa separately.

EXAMPLE. $005624^{.37}$ = .147067

Solution. .37 × log .005624 = .37 × $\bar{3}$ 750045 = 37 × (− 3) + .37 × 750045
= − 1 11 + .277517 = $\bar{1}$ 167517 = log .147067.

c. Where the number is a fraction, first find the log of the fraction and then multiply it by the exponent.

EXAMPLE. $\left(\frac{276}{.032}\right)^{.72}$ = 681.9396.

Solution. .72 (log 276 − log .032) = .72{ 2 440909 − (8 505150 − 10) }
= .72 × 3.935759 = 2.833746 = log 681.9396.

To Extract Any Root of a Number.—Divide the log of the number by the index of the root, and find the number whose log is the quotient.

To extract the root of a decimal: *a* When the root index is positive and evenly divisible into the negative characteristic of the log of the number, the division may be performed with the negative characteristic written in its usual place.

EXAMPLE. $\sqrt[4]{.0006954}$ = .16239

Solution. log .0006954 ÷ 4 = $\bar{4}$ 842235 ÷ 4 = $\bar{1}$.210559 = log .16239.

b. When the root index is positive and not even divisible into the negative characteristic, add to the log of the number, and indicate the subtraction from it, the smallest integral multiple of the root which will eliminate the negative characteristic. Divide the result by the root index and ascertain the number whose log corresponds to the quotient.

EXAMPLE. $\sqrt[3.1]{.00002785}$ = .03393

Solution. log .00002785 = $\bar{5}$ 444825; $\bar{5}$ 444825 ÷ 3.1 = {(2 × 3 1) + $\bar{5}$.444825 − 2 × 3 1} ÷ 3.1
= {1.644825 − 6.2} ÷ 3.1 = $\bar{2}$ 530589 = log .03393.

c. When the root index is negative, determine the excess of the negative characteristic over the positive mantissa. Divide the result by the root index and ascertain the number whose log corresponds to the quotient

EXAMPLE. $\sqrt[-4]{000003976}$ = 22.394.

Solution. log .000003976 = $\bar{6}$.599446 = .599446 − 6 = − 5 400554;
− 5 400554 ÷ (− 4) = 1 350138 = log 22.394.

Solution of Exponential Equations.—In an exponential equation, the unknown quantity is the exponent; thus $a^x = b$ This may be transformed to $\log a^x = \log b$, or $x \log a = \log b$, whence $x = \log b \div \log a$

a When the base is greater than unity, put the equation in the form $x = \log b \div \log a$. Then $\log x = \log (\log b) - \log (\log a)$.

EXAMPLE. $32.6^x = 14\,632$.

Solution. $\log x = \log (\log 14\,632) - \log (\log 32\,6) = \log 1\,165303 - \log 1.513218$
$= 066439 - .179901 = \bar{1}\,886538$; $x = .77008$.

b When both the known quantities are decimal, put the equation in the form $x = \log b \div \log a$. Subtract the positive mantissa from the negative characteristic in both divisor and dividend, obtaining negative remainders. Change the signs of divisor and dividend and proceed as in Case *a*.

EXAMPLE. $.0729^x = .2693$.

Solution. $x = \log .2693 \div \log\ 0729 = \bar{1}\,430236 \div \bar{2}\,862728$
$= (-\ 569764) \div (-\ 1\,137272) = 569764 \div 1\,137272$;
$\log x = \log\ 569764 - \log 1.137272 = \bar{1}\,755695 - .055864 = \bar{1}.699831$
$x = .50099$.

c. When only one of the known quantities is a decimal, put the equation in the form $x = \dfrac{\log b}{\log a}$. Subtract the positive mantissa from the negative characteristic of the numerator or denominator as the case may be, and rewrite the fraction with the remainder so obtained as the new numerator or denominator. Make both numerator and denominator positive, but write a minus sign in front of the fraction, to signify that the result will be a negative quantity. Solve the fraction by logarithms and write a minus sign in front of the result.

EXAMPLE. $.726^x = 802.7$.

Solution. $x = \dfrac{\log 802.7}{\log\ .726} = \dfrac{2.904553}{\bar{1}.860937} = \dfrac{2\,904553}{-\ .139063} = -\dfrac{2\,904553}{.139063}$.

$\log x = -\ (\log 2.904553 - \log .139063) = -\ (\ 463079 - \bar{1}\,143211) = -\ (1.319868)$.
$x = -\ 20.8867$.

d. When the exponent is negative and one of the known quantities is less than unity, put the equation in the form $(-\ x) = \dfrac{\log b}{\log a}$. Subtract the positive mantissa from the negative characteristic, as in Case *c*, and multiply both sides of the resulting equation by (-1). Find the value of x as in Case *c*.

EXAMPLE. $10\,78^{-x} = .09431$.

Solution. $x = \dfrac{\log\ 09431}{\log 10\,78} = \dfrac{\bar{2}\,974558}{1.032619}$; $x = \dfrac{1\,025442}{1.032619}$.

$\log x = .010911 - .013940 = \bar{1}\,996971$. $x = .99305$.

Table 1. Logarithms of Numbers from 1 to 100

N	Log	N	Log	N	Log	N	Log	N	Log
1	0 000000	21	1 322219	41	1 612784	61	1 785330	81	1 908485
2	0.301030	22	1.342423	42	1 623249	62	1 792392	82	1.913814
3	0 477121	23	1 361728	43	1 633468	63	1 799341	83	1.919078
4	0 602060	24	1 380211	44	1 643453	64	1 806180	84	1 924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1 662758	66	1 819544	86	1 934498
7	0 845098	27	1.431364	47	1 672098	67	1.826075	87	1 939519
8	0 903090	28	1.447158	48	1 681241	68	1 832509	88	1.944483
9	0.954243	29	1.462398	49	1 690196	69	1 838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1 707570	71	1 851258	91	1.959041
12	1 079181	32	1 505150	52	1 716003	72	1 857332	92	1 963788
13	1.113943	33	1 518514	53	1 724276	73	1 863323	93	1 968483
14	1.146128	34	1.531479	54	1 732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.375061	95	1.977724
16	1.204120	36	1.556303	56	1 748188	76	1.880814	96	1 982271
17	1 230449	37	1 568202	57	1.755875	77	1 886491	97	1 986772
18	1.255273	38	1 579784	58	1.763428	78	1 892095	98	1 991226
19	1 278754	39	1.591065	59	1 770852	79	1.897627	99	1 995635
20	1 301030	40	1 602060	60	1 778151	80	1 903090	100	2 000000

Table 2. Common Logarithms of Numbers

N	0	1	2	3	4	5	6	7	8	9	Diff.
100	**000000**	**000434**	**000868**	**001301**	**001734**	**002166**	**002598**	**003029**	**003461**	**003891**	432
1	004321	004751	005181	005609	006038	006466	006894	007321	007748	008174	428
2	008600	009026	009451	009876	010300	010724	011147	011570	011993	012415	424
3	012837	013259	013680	014100	014521	014940	015360	015779	016197	016616	420
4	017033	017451	017868	018284	018700	019116	019532	019947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	024486	024896	412
6	025306	025715	026125	026533	026942	027350	027757	028164	028571	028978	408
7	029384	029789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	034227	034628	035029	035430	035830	036230	036629	037028	400
9	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998	397
110	**041393**	**041787**	**042182**	**042576**	**042969**	**043362**	**043755**	**044148**	**044540**	**044932**	393
1	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830	390
2	049218	049606	049993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524	383
4	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	064083	376
6	064458	064832	065206	065580	065953	066326	066699	067071	067443	067815	373
7	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514	370
8	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182	366
9	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819	363
120	**079181**	**079543**	**079904**	**080266**	**080626**	**080987**	**081347**	**081707**	**082067**	**082426**	360
1	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004	357
2	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552	355
3	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
4	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562	349
5	096910	097257	097604	097951	098298	098644	098990	099335	099681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	103462	343
7	103804	104146	104487	104828	105169	105510	105851	106191	106531	106871	341
8	107210	107549	107888	108227	108565	108903	109241	109579	109916	110253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	113609	335

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130 2	173.6	217 0	260 4	303.8	347 2	390 6
432	43.2	86.4	129 6	172.8	216 0	259.2	302.4	345.6	388 8
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
428	42.8	85.6	128.4	171 2	214.0	256.8	299.6	342 4	385.2
426	42.6	85.2	127.8	170.4	213 0	255 6	298.2	340.8	383.4
424	42.4	84.8	127.2	169 6	212 0	254 4	296.8	339.2	381.6
422	42.2	84.4	126 6	168 8	211 0	253 2	295.4	337 6	379.8
420	42.0	84.0	126.0	168 0	210.0	252.0	294.0	336.0	378 0
418	41.8	83.6	125.4	167.2	209.0	250 8	292.6	334.4	376 2
416	41.6	83 2	124 8	166 4	208 0	249 6	291 2	332.8	374.4
414	41.4	82.8	124 2	165 6	207 0	248 4	289 8	331.2	372.6
412	41.2	82.4	123 6	164 8	206 0	247.2	288.4	329.6	370 8
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
408	40.8	81.6	122.4	163.2	204.0	244 8	285 6	326.4	367.2
406	40.6	81.2	121.8	162.4	203 0	243.6	284 2	324 8	365.4
404	40.4	80.8	121 2	161 6	202 0	242 4	282.8	323 2	363.6
402	40.2	80.4	120.6	160.8	201 0	241.2	281.4	321.6	361 8
400	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
398	39.8	79.6	119 4	159.2	199 0	238 8	278.6	318.4	358 2
396	39.6	79.2	118 8	158.4	198 0	237.6	277 2	316 8	356.4
394	39.4	78.8	118 2	157 6	197 0	236 4	275 8	315.2	354.6
392	39.2	78.4	117 6	156.8	196 0	235 2	274.4	313 6	352 8
390	39.0	78.0	117.0	156 0	195.0	234.0	273.0	312.0	351.0
388	38.8	77.6	116.4	155 2	194.0	232 8	271.6	310 4	349.2
386	38.6	77.2	115 8	154 4	193 0	231 6	270.2	308 8	347.4
384	38 4	76.8	115 2	153 6	192 0	230 4	268 8	307 2	345 6
382	38 2	76.4	114 6	152 8	191 0	229 2	267.4	305 6	343.8
380	38.0	76.0	114 0	152 0	190 0	228 0	266.0	304.0	342.0
378	37.8	75.6	113 4	151 2	189 0	226.8	264.6	302 4	340 2
376	37.6	75.2	112 8	150 4	188 0	225 6	263 2	300 8	338 4
374	37.4	74 8	112 2	149.6	187 0	224 4	261 8	299 2	336 6
372	37.2	74.4	111.6	148 8	186.0	223.2	260.4	297.6	334.8
370	37.0	74.0	111.0	148.0	185.0	222 0	259.0	296.0	333.0
368	36 8	73.6	110.4	147.2	184 0	220 8	257.6	294 4	331.2
366	36 6	73 2	109.8	146 4	183 0	219 6	256 2	292.8	329 4
364	36.4	72 8	109 2	145 6	182.0	218 4	254 8	291.2	327.6
362	36 2	72 4	108 6	144 8	181 0	217.2	253 4	289 6	325.8
360	36 0	72 0	108 0	144 0	180 0	216 0	252 0	288.0	324.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
130	**113943**	**114277**	**114611**	**114944**	**115278**	**115611**	**115943**	**116276**	**116608**	**116940**	333
1	117271	117603	117934	118265	118595	118926	119256	119586	119915	120245	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	123525	328
3	123852	124178	124504	124830	125156	125481	125806	126131	126456	126781	325
4	127105	127429	127753	128076	128399	128722	129045	129368	129690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219	321
6	133539	133858	134177	134496	134814	135133	135451	135769	136086	136403	318
7	136721	137037	137354	137671	137987	138303	138618	138934	139249	139564	316
8	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
9	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818	311
140	**146128**	**146438**	**146748**	**147058**	**147367**	**147676**	**147985**	**148294**	**148603**	**148911**	309
1	149219	149527	149835	150142	150449	150756	151063	151370	151676	151982	307
2	152288	152594	152900	153205	153510	153815	154120	154424	154728	155032	305
3	155336	155640	155943	156246	156549	156852	157154	157457	157759	158061	303
4	158362	158664	158965	159266	159567	159868	160168	160469	160769	161068	301
5	161368	161667	161967	162266	162564	162863	163161	163460	163758	164055	299
6	164353	164650	164947	165244	165541	165838	166134	166430	166726	167022	297
7	167317	167613	167908	168203	168497	168792	169086	169380	169674	169968	295
8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	173186	173478	173769	174060	174351	174641	174932	175222	175512	175802	291
150	**176091**	**176381**	**176670**	**176959**	**177248**	**177536**	**177825**	**178113**	**178401**	**178689**	289
1	178977	179264	179552	179839	180126	180413	180699	180986	181272	181558	287
2	181844	182129	182415	182700	182985	183270	183555	183839	184123	184407	285
3	184691	184975	185259	185542	185825	186108	186391	186674	186956	187239	283
4	187521	187803	188084	188366	188647	188928	189209	189490	189771	190051	281
5	190332	190612	190892	191171	191451	191730	192010	192289	192567	192846	279
6	193125	193403	193681	193959	194237	194514	194792	195069	195346	195623	278
7	195900	196176	196453	196729	197005	197281	197556	197832	198107	198382	276
8	198657	198932	199206	199481	199755	200029	200303	200577	200850	201124	274
9	201397	201670	201943	202216	202488	202761	203033	203305	203577	203848	272

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	254.4	286.2
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
292	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257.4
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
160	**204120**	**204391**	**204663**	**204934**	**205204**	**205475**	**205746**	**206016**	**206286**	**206556**	271
1	206826	207096	207365	207634	207904	208173	208441	208710	208979	209247	269
2	209515	209783	210051	210319	210586	210853	211121	211388	211654	211921	267
3	212188	212454	212720	212986	213252	213518	213783	214049	214314	214579	266
4	214844	215109	215373	215638	215902	216166	216430	216694	216957	217221	264
5	217484	217747	218010	218273	218536	218798	219060	219323	219585	219846	262
6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
7	222716	222976	223236	223496	223755	224015	224274	224533	224792	225051	259
8	225309	225568	225826	226084	226342	226600	226858	227115	227372	227630	258
9	227887	228144	228400	228657	228913	229170	229426	229682	229938	230193	256
170	**230449**	**230704**	**230960**	**231215**	**231470**	**231724**	**231979**	**232234**	**232488**	**232742**	255
1	232996	233250	233504	233757	234011	234264	234517	234770	235023	235276	253
2	235528	235781	236033	236285	236537	236789	237041	237292	237544	237795	252
3	238046	238297	238548	238799	239049	239299	239550	239800	240050	240300	250
4	240549	240799	241048	241297	241546	241795	242044	242293	242541	242790	249
5	243038	243286	243534	243782	244030	244277	244525	244772	245019	245266	248
6	245513	245759	246006	246252	246499	246745	246991	247237	247482	247728	246
7	247973	248219	248464	248709	248954	249198	249443	249687	249932	250176	245
8	250420	250664	250908	251151	251395	251638	251881	252125	252368	252610	243
9	252853	253096	253338	253580	253822	254064	254306	254548	254790	255031	242
180	**255273**	**255514**	**255755**	**255996**	**256237**	**256477**	**256718**	**256958**	**257198**	**257439**	241
1	257679	257918	258158	258398	258637	258877	259116	259355	259594	259833	239
2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
3	262451	262688	262925	263162	263399	263636	263873	264109	264346	264582	237
4	264818	265054	265290	265525	265761	265996	266232	266467	266702	266937	235
5	267172	267406	267641	267875	268110	268344	268578	268812	269046	269279	234
6	269513	269746	269980	270213	270446	270679	270912	271144	271377	271609	233
7	271842	272074	272306	272538	272770	273001	273233	273464	273696	273927	232
8	274158	274389	274620	274850	275081	275311	275542	275772	276002	276232	230
9	276462	276692	276921	277151	277380	277609	277838	278067	278296	278525	229
190	**278754**	**278982**	**279211**	**279439**	**279667**	**279895**	**280123**	**280351**	**280578**	**280806**	228
1	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075	227
2	283301	283527	283753	283979	284205	284431	284656	284882	285107	285332	226
3	285557	285782	286007	286232	286456	286681	286905	287130	287354	287578	225
4	287802	288026	288249	288473	288696	288920	289143	289366	289589	289812	223
5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
6	292256	292478	292699	292920	293141	293363	293584	293804	294025	294246	221
7	294466	294687	294907	295127	295347	295567	295787	296007	296226	296446	220
8	296665	296884	297104	297323	297542	297761	297979	298198	298416	298635	219
9	298853	299071	299289	299507	299725	299943	300161	300378	300595	300813	218

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
270	27.0	54.0	81.0	108.0	135.0	162.0	189.0	216.0	243.0
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
262	26.2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	230.4
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
250	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
230	23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
200	**301030**	**301247**	**301464**	**301681**	**301898**	**302114**	**302331**	**302547**	**302764**	**302980**	**217**
1	303196	303412	303628	303844	304059	304275	304491	304706	304921	305136	216
2	305351	305566	305781	305996	306211	306425	306639	306854	307068	307282	215
3	307496	307710	307924	308137	308351	308564	308778	308991	309204	309417	213
4	309630	309843	310056	310268	310481	310693	310906	311118	311330	311542	212
5	311754	311966	312177	312389	312600	312812	313023	313234	313445	313656	211
6	313867	314078	314289	314499	314710	314920	315130	315340	315551	315760	210
7	315970	316180	316390	316599	316809	317018	317227	317436	317646	317854	209
8	318063	318272	318481	318689	318898	319106	319314	319522	319730	319938	208
9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	**322219**	**322426**	**322633**	**322839**	**323046**	**323252**	**323458**	**323665**	**323871**	**324077**	**206**
1	324282	324488	324694	324899	325105	325310	325516	325721	325926	326131	205
2	326336	326541	326745	326950	327155	327359	327563	327767	327972	328176	204
3	328380	328583	328787	328991	329194	329398	329601	329805	330008	330211	203
4	330414	330617	330819	331022	331225	331427	331630	331832	332034	332236	202
5	332438	332640	332842	333044	333246	333447	333649	333850	334051	334253	
6	334454	334655	334856	335057	335257	335458	335658	335859	336059	336260	201
7	336460	336660	336860	337060	337260	337459	337659	337858	338058	338257	200
8	338456	338656	338855	339054	339253	339451	339650	339849	340047	340246	199
9	340444	340642	340841	341039	341237	341435	341632	341830	342028	342225	198
220	**342423**	**342620**	**342817**	**343014**	**343212**	**343409**	**343606**	**343802**	**343999**	**344196**	197
1	344392	344589	344785	344981	345178	345374	345570	345766	345962	346157	196
2	346353	346549	346744	346939	347135	347330	347525	347720	347915	348110	195
3	348305	348500	348694	348889	349083	349278	349472	349666	349860	350054	194
4	350248	350442	350636	350829	351023	351216	351410	351603	351796	351989	193
5	352183	352375	352568	352761	352954	353147	353339	353532	353724	353916	
6	354108	354301	354493	354685	354876	355068	355260	355452	355643	355834	192
7	356026	356217	356408	356599	356790	356981	357172	357363	357554	357744	191
8	357935	358125	358316	358506	358696	358886	359076	359266	359456	359646	190
9	359835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	**361728**	**361917**	**362105**	**362294**	**362482**	**362671**	**362859**	**363048**	**363236**	**363424**	188
1	363612	363800	363988	364176	364363	364551	364739	364926	365113	365301	
2	365488	365675	365862	366049	366236	366423	366610	366796	366983	367169	187
3	367356	367542	367729	367915	368101	368287	368473	368659	368845	369030	186
4	369216	369401	369587	369772	369958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	371991	372175	372360	372544	372728	184
6	372912	373096	373280	373464	373647	373831	374015	374198	374382	374565	
7	374748	374932	375115	375298	375481	375664	375846	376029	376212	376394	183
8	376577	376759	376942	377124	377306	377488	377670	377852	378034	378216	182
9	378398	378580	378761	378943	379124	379306	379487	379668	379849	380030	181

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205.2
226	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	203.4
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.4
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
240	**380211**	**380392**	**380573**	**380754**	**380934**	**381115**	**381296**	**381476**	**381656**	**381837**	181
1	382017	382197	382377	382557	382737	382917	383097	383277	383456	383636	180
2	383815	383995	384174	384353	384533	384712	384891	385070	385249	385428	179
3	385606	385785	385964	386142	386321	386499	386677	386856	387034	387212	178
4	387390	387568	387746	387924	388101	388279	388456	388634	388811	388989	
5	389166	389343	389520	389698	389875	390051	390228	390405	390582	390759	177
6	390935	391112	391288	391464	391641	391817	391993	392169	392345	392521	176
7	392697	392873	393048	393224	393400	393575	393751	393926	394101	394277	
8	394452	394627	394802	394977	395152	395326	395501	395676	395850	396025	175
9	396199	396374	396548	396722	396896	397071	397245	397419	397592	397766	174
250	**397940**	**398114**	**398287**	**398461**	**398634**	**398808**	**398981**	**399154**	**399328**	**399501**	173
1	399674	399847	400020	400192	400365	400538	400711	400883	401056	401228	
2	401401	401573	401745	401917	402089	402261	402433	402605	402777	402949	172
3	403121	403292	403464	403635	403807	403978	404149	404320	404492	404663	171
4	404834	405005	405176	405346	405517	405688	405858	406029	406199	406370	
5	406540	406710	406881	407051	407221	407391	407561	407731	407901	408070	170
6	408240	408410	408579	408749	408918	409087	409257	409426	409595	409764	169
7	409933	410102	410271	410440	410609	410777	410946	411114	411283	411451	
8	411620	411788	411956	412124	412293	412461	412629	412796	412964	413132	168
9	413300	413467	413635	413803	413970	414137	414305	414472	414639	414806	167
260	**414973**	**415140**	**415307**	**415474**	**415641**	**415808**	**415974**	**416141**	**416308**	**416474**	
1	416641	416807	416973	417139	417306	417472	417638	417804	417970	418135	166
2	418301	418467	418633	418798	418964	419129	419295	419460	419625	419791	165
3	419956	420121	420286	420451	420616	420781	420945	421110	421275	421439	
4	421604	421768	421933	422097	422261	422426	422590	422754	422918	423082	164
5	423246	423410	423574	423737	423901	424065	424228	424392	424555	424718	
6	424882	425045	425208	425371	425534	425697	425860	426023	426186	426349	163
7	426511	426674	426836	426999	427161	427324	427486	427648	427811	427973	162
8	428135	428297	428459	428621	428783	428944	429106	429268	429429	429591	
9	429752	429914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	**431364**	**431525**	**431685**	**431846**	**432007**	**432167**	**432328**	**432488**	**432649**	**432809**	
1	432969	433130	433290	433450	433610	433770	433930	434090	434249	434409	160
2	434569	434729	434888	435048	435207	435367	435526	435685	435844	436004	159
3	436163	436322	436481	436640	436799	436957	437116	437275	437433	437592	
4	437751	437909	438067	438226	438384	438542	438701	438859	439017	439175	158
5	439333	439491	439648	439806	439964	440122	440279	440437	440594	440752	
6	440909	441066	441224	441381	441538	441695	441852	442009	442166	442323	157
7	442480	442637	442793	442950	443106	443263	443419	443576	443732	443889	
8	444045	444201	444357	444513	444669	444825	444981	445137	445293	445449	156
9	445604	445760	445915	446071	446226	446382	446537	446692	446848	447003	155
280	**447158**	**447313**	**447468**	**447623**	**447778**	**447933**	**448088**	**448242**	**448397**	**448552**	
1	448706	448861	449015	449170	449324	449478	449633	449787	449941	450095	154
2	450249	450403	450557	450711	450865	451018	451172	451326	451479	451633	
3	451786	451940	452093	452247	452400	452553	452706	452859	453012	453165	153
4	453318	453471	453624	453777	453930	454082	454235	454387	454540	454692	
5	454845	454997	455150	455302	455454	455606	455758	455910	456062	456214	152
6	456366	456518	456670	456821	456973	457125	457276	457428	457579	457731	
7	457882	458033	458184	458336	458487	458638	458789	458940	459091	459242	151
8	459392	459543	459694	459845	459995	460146	460296	460447	460597	460748	
9	460898	461048	461198	461348	461499	461649	461799	461948	462098	462248	150

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
162	16.2	32.4	48.6	64.8	81.0	97.2	113.4	129.6	145.8
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
290	**462398**	**462548**	**462697**	**462847**	**462997**	**463146**	**463296**	**463445**	**463594**	**463744**	
1	463893	464042	464191	464340	464490	464639	464788	464936	465085	465234	149
2	465383	465532	465680	465829	465977	466126	466274	466423	466571	466719	
3	466868	467016	467164	467312	467460	467608	467756	467904	468052	468200	148
4	468347	468495	468643	468790	468938	469085	469233	469380	469527	469675	
5	469822	469969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	471585	471732	471878	472025	472171	472318	472464	472610	146
7	472756	472903	473049	473195	473341	473487	473633	473779	473925	474071	
8	474216	474362	474508	474653	474799	474944	475090	475235	475381	475526	
9	475671	475816	475962	476107	476252	476397	476542	476687	476832	476976	145
300	**477121**	**477266**	**477411**	**477555**	**477700**	**477844**	**477989**	**478133**	**478278**	**478422**	
1	478566	478711	478855	478999	479143	479287	479431	479575	479719	479863	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	
3	481443	481586	481729	481872	482016	482159	482302	482445	482588	482731	143
4	482874	483016	483159	483302	483445	483587	483730	483872	484015	484157	
5	484300	484442	484585	484727	484869	485011	485153	485295	485437	485579	142
6	485721	485863	486005	486147	486289	486430	486572	486714	486855	486997	
7	487138	487280	487421	487563	487704	487845	487986	488127	488269	488410	141
8	488551	488692	488833	488974	489114	489255	489396	489537	489677	489818	
9	489958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	**491362**	**491502**	**491642**	**491782**	**491922**	**492062**	**492201**	**492341**	**492481**	**492621**	
1	492760	492900	493040	493179	493319	493458	493597	493737	493876	494015	139
2	494155	494294	494433	494572	494711	494850	494989	495128	495267	495406	
3	495544	495683	495822	495960	496099	496238	496376	496515	496653	496791	
4	496930	497068	497206	497344	497483	497621	497759	497897	498035	498173	138
5	498311	498448	498586	498724	498862	498999	499137	499275	499412	499550	
6	499687	499824	499962	500099	500236	500374	500511	500648	500785	500922	137
7	501059	501196	501333	501470	501607	501744	501880	502017	502154	502291	
8	502427	502564	502700	502837	502973	503109	503246	503382	503518	503655	136
9	503791	503927	504063	504199	504335	504471	504607	504743	504878	505014	
320	**505150**	**505286**	**505421**	**505557**	**505693**	**505828**	**505964**	**506099**	**506234**	**506370**	
1	506505	506640	506776	506911	507046	507181	507316	507451	507586	507721	135
2	507856	507991	508126	508260	508395	508530	508664	508799	508934	509068	
3	509203	509337	509471	509606	509740	509874	510009	510143	510277	510411	134
4	510545	510679	510813	510947	511081	511215	511349	511482	511616	511750	
5	511883	512017	512151	512284	512418	512551	512684	512818	512951	513084	133
6	513218	513351	513484	513617	513750	513883	514016	514149	514282	514415	
7	514548	514681	514813	514946	515079	515211	515344	515476	515609	515741	
8	515874	516006	516139	516271	516403	516535	516668	516800	516932	517064	132
9	517196	517328	517460	517592	517724	517855	517987	518119	518251	518382	
330	**518514**	**518646**	**518777**	**518909**	**519040**	**519171**	**519303**	**519434**	**519566**	**519697**	131
1	519828	519959	520090	520221	520353	520484	520615	520745	520876	521007	
2	521138	521269	521400	521530	521661	521792	521922	522053	522183	522314	
3	522444	522575	522705	522835	522966	523096	523226	523356	523486	523616	130
4	523746	523876	524006	524136	524266	524396	524526	524656	524785	524915	
5	525045	525174	525304	525434	525563	525693	525822	525951	526081	526210	129
6	526339	526469	526598	526727	526856	526985	527114	527243	527372	527501	
7	527630	527759	527888	528016	528145	528274	528402	528531	528660	528788	
8	528917	529045	529174	529302	529430	529559	529687	529815	529943	530072	128
9	530200	530328	530456	530584	530712	530840	530968	531096	531223	531351	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
154	15.4	30.8	46 2	61 6	77 0	92 4	107 8	123 2	138.6
152	15 2	30.4	45 6	60 8	76.0	91 2	106 4	121 6	136 8
150	15.0	30 0	45 0	60 0	75.0	90 0	105 0	120 0	135 0
148	14 8	29.6	44.4	59.2	74 0	88 8	103 6	118.4	133.2
146	14.6	29.2	43 8	58 4	73.0	87.6	102.2	116 8	131 4
144	14.4	28.8	43.2	57 6	72.0	86 4	100.8	115 2	129.6
142	14.2	28 4	42 6	56 8	71 0	85 2	99.4	113 6	127 8
140	14.0	28.0	42 0	56 0	70.0	84.0	98 0	112 0	126 0
138	13.8	27.6	41.4	55.2	69 0	82.8	96 6	110.4	124.2
136	13.6	27.2	40 8	54.4	68 0	81.6	95 2	108 8	122.4
134	13.4	26.8	40 2	53.6	67.0	80 4	93.8	107 2	120 6
132	13.2	26.4	39.6	52 8	66 0	79 2	92.4	105 6	118 8
130	13.0	26 0	39.0	52 0	65 0	78 0	91.0	104 0	117.0
128	12.8	25 6	38 4	51 2	64 0	76 8	89 6	102 4	115 2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
340	**531479**	**531607**	**531734**	**531862**	**531990**	**532117**	**532245**	**532372**	**532500**	**532627**	
1	532754	532882	533009	533136	533264	533391	533518	533645	533772	533899	127
2	534026	534153	534280	534407	534534	534661	534787	534914	535041	535167	
3	535294	535421	535547	535674	535800	535927	536053	536180	536306	536432	126
4	536558	536685	536811	536937	537063	537189	537315	537441	537567	537693	
5	537819	537945	538071	538197	538322	538448	538574	538699	538825	538951	
6	539076	539202	539327	539452	539578	539703	539829	539954	540079	540204	125
7	540329	540455	540580	540705	540830	540955	541080	541205	541330	541454	
8	541579	541704	541829	541953	542078	542203	542327	542452	542576	542701	
9	542825	542950	543074	543199	543323	543447	543571	543696	543820	543944	124
350	**544068**	**544192**	**544316**	**544440**	**544564**	**544688**	**544812**	**544936**	**545060**	**545183**	
1	545307	545431	545555	545678	545802	545925	546049	546172	546296	546419	
2	546543	546666	546789	546913	547036	547159	547282	547405	547529	547652	123
3	547775	547898	548021	548144	548267	548389	548512	548635	548758	548881	
4	549003	549126	549249	549371	549494	549616	549739	549861	549984	550106	
5	550228	550351	550473	550595	550717	550840	550962	551084	551206	551328	122
6	551450	551572	551694	551816	551938	552060	552181	552303	552425	552547	
7	552668	552790	552911	553033	553155	553276	553398	553519	553640	553762	121
8	553883	554004	554126	554247	554368	554489	554610	554731	554852	554973	
9	555094	555215	555336	555457	555578	555699	555820	555940	556061	556182	
360	**556303**	**556423**	**556544**	**556664**	**556785**	**556905**	**557026**	**557146**	**557267**	**557387**	120
1	557507	557627	557748	557868	557988	558108	558228	558349	558469	558589	
2	558709	558829	558948	559068	559188	559308	559428	559548	559667	559787	
3	559907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
4	561101	561221	561340	561459	561578	561698	561817	561936	562055	562174	
5	562293	562412	562531	562650	562769	562887	563006	563125	563244	563362	
6	563481	563600	563718	563837	563955	564074	564192	564311	564429	564548	
7	564666	564784	564903	565021	565139	565257	565376	565494	565612	565730	118
8	565848	565966	566084	566202	566320	566437	566555	566673	566791	566909	
9	567026	567144	567262	567379	567497	567614	567732	567849	567967	568084	
370	**568202**	**568319**	**568436**	**568554**	**568671**	**568788**	**568905**	**569023**	**569140**	**569257**	117
1	569374	569491	569608	569725	569842	569959	570076	570193	570309	570426	
2	570543	570660	570776	570893	571010	571126	571243	571359	571476	571592	
3	571709	571825	571942	572058	572174	572291	572407	572523	572639	572755	116
4	572872	572988	573104	573220	573336	573452	573568	573684	573800	573915	
5	574031	574147	574263	574379	574494	574610	574726	574841	574957	575072	
6	575188	575303	575419	575534	575650	575765	575880	575996	576111	576226	115
7	576341	576457	576572	576687	576802	576917	577032	577147	577262	577377	
8	577492	577607	577722	577836	577951	578066	578181	578295	578410	578525	
9	578639	578754	578868	578983	579097	579212	579326	579441	579555	579669	114
380	**579784**	**579898**	**580012**	**580126**	**580241**	**580355**	**580469**	**580583**	**580697**	**580811**	
1	580925	581039	581153	581267	581381	581495	581608	581722	581836	581950	
2	582063	582177	582291	582404	582518	582631	582745	582858	582972	583085	
3	583199	583312	583426	583539	583652	583765	583879	583992	584105	584218	
4	584331	584444	584557	584670	584783	584896	585009	585122	585235	585348	113
5	585461	585574	585686	585799	585912	586024	586137	586250	586362	586475	
6	586587	586700	586812	586925	587037	587149	587262	587374	587486	587599	
7	587711	587823	587935	588047	588160	588272	588384	588496	588608	588720	112
8	588832	588944	589056	589167	589279	589391	589503	589615	589726	589838	
9	589950	590061	590173	590284	590396	590507	590619	590730	590842	590953	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
122	12.2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	109.8
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0
118	11.8	23.6	35.4	47.2	59.0	70.8	82.6	94.4	106.2
116	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.4
114	11.4	22.8	34.2	45.6	57.0	68.4	79.8	91.2	102.6

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
390	**591065**	**591176**	**591287**	**591399**	**591510**	**591621**	**591732**	**591843**	**591955**	**592066**	
1	592177	592288	592399	592510	592621	592732	592843	592954	593064	593175	111
2	593286	593397	593508	593618	593729	593840	593950	594061	594171	594282	
3	594393	594503	594614	594724	594834	594945	595055	595165	595276	595386	
4	595496	595606	595717	595827	595937	596047	596157	596267	596377	596487	
5	596597	596707	596817	596927	597037	597146	597256	597366	597476	597586	110
6	597695	597805	597914	598024	598134	598243	598353	598462	598572	598681	
7	598791	598900	599009	599119	599228	599337	599446	599556	599665	599774	
8	599883	599992	600101	600210	600319	600428	600537	600646	600755	600864	109
9	600973	601082	601191	601299	601408	601517	601625	601734	601843	601951	
400	**602060**	**602169**	**602277**	**602386**	**602494**	**602603**	**602711**	**602819**	**602928**	**603036**	
1	603144	603253	603361	603469	603577	603686	603794	603902	604010	604118	108
2	604226	604334	604442	604550	604658	604766	604874	604982	605089	605197	
3	605305	605413	605521	605628	605736	605844	605951	606059	606166	606274	
4	606381	606489	606596	606704	606811	606919	607026	607133	607241	607348	
5	607455	607562	607669	607777	607884	607991	608098	608205	608312	608419	107
6	608526	608633	608740	608847	608954	609061	609167	609274	609381	609488	
7	609594	609701	609808	609914	610021	610128	610234	610341	610447	610554	
8	610660	610767	610873	610979	611086	611192	611298	611405	611511	611617	
9	611723	611829	611936	612042	612148	612254	612360	612466	612572	612678	106
410	**612784**	**612890**	**612996**	**613102**	**613207**	**613313**	**613419**	**613525**	**613630**	**613736**	
1	613842	613947	614053	614159	614264	614370	614475	614581	614686	614792	
2	614897	615003	615108	615213	615319	615424	615529	615634	615740	615845	
3	615950	616055	616160	616265	616370	616476	616581	616686	616790	616895	105
4	617000	617105	617210	617315	617420	617525	617629	617734	617839	617943	
5	618048	618153	618257	618362	618466	618571	618676	618780	618884	618989	
6	619093	619198	619302	619406	619511	619615	619719	619824	619928	620032	
7	620136	620240	620344	620448	620552	620656	620760	620864	620968	621072	104
8	621176	621280	621384	621488	621592	621695	621799	621903	622007	622110	
9	622214	622318	622421	622525	622628	622732	622835	622939	623042	623146	
420	**623249**	**623353**	**623456**	**623559**	**623663**	**623766**	**623869**	**623973**	**624076**	**624179**	
1	624282	624385	624488	624591	624695	624798	624901	625004	625107	625210	103
2	625312	625415	625518	625621	625724	625827	625929	626032	626135	626238	
3	626340	626443	626546	626648	626751	626853	626956	627058	627161	627263	
4	627366	627468	627571	627673	627775	627878	627980	628082	628185	628287	
5	628389	628491	628593	628695	628797	628900	629002	629104	629206	629308	102
6	629410	629512	629613	629715	629817	629919	630021	630123	630224	630326	
7	630428	630530	630631	630733	630835	630936	631038	631139	631241	631342	
8	631444	631545	631647	631748	631849	631951	632052	632153	632255	632356	101
9	632457	632559	632660	632761	632862	632963	633064	633165	633266	633367	
430	**633468**	**633569**	**633670**	**633771**	**633872**	**633973**	**634074**	**634175**	**634276**	**634376**	
1	634477	634578	634679	634779	634880	634981	635081	635182	635283	635383	
2	635484	635584	635685	635785	635886	635986	636087	636187	636287	636388	
3	636488	636588	636688	636789	636889	636989	637089	637189	637290	637390	
4	637490	637590	637690	637790	637890	637990	638090	638190	638290	638389	100
5	638489	638589	638689	638789	638888	638988	639088	639188	639287	639387	
6	639486	639586	639686	639785	639885	639984	640084	640183	640283	640382	
7	640481	640581	640680	640779	640879	640978	641077	641177	641276	641375	
8	641474	641573	641672	641771	641871	641970	642069	642168	642267	642366	
9	642465	642563	642662	642761	642860	642959	643058	643156	643255	643354	99

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
110	11.0	22.0	33.0	44.0	55.0	66.0	77.0	88.0	99.0
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.4
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6
102	10.2	20.4	30.6	40.8	51.0	61.2	71.4	81.6	91.8
100	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
440	**643453**	**643551**	**643650**	**643749**	**643847**	**643946**	**644044**	**644143**	**644242**	**644340**	
1	644439	644537	644636	644734	644832	644931	645029	645127	645226	645324	
2	645422	645521	645619	645717	645815	645913	646011	646110	646208	646306	
3	646404	646502	646600	646698	646796	646894	646992	647089	647187	647285	98
4	647383	647481	647579	647676	647774	647872	647969	648067	648165	648262	
5	648360	648458	648555	648653	648750	648848	648945	649043	649140	649237	
6	649335	649432	649530	649627	649724	649821	649919	650016	650113	650210	
7	650308	650405	650502	650599	650696	650793	650890	650987	651084	651181	
8	651278	651375	651472	651569	651666	651762	651859	651956	652053	652150	97
9	652246	652343	652440	652536	652633	652730	652826	652923	653019	653116	
450	**653213**	**653309**	**653405**	**653502**	**653598**	**653695**	**653791**	**653888**	**653984**	**654080**	
1	654177	654273	654369	654465	654562	654658	654754	654850	654946	655042	
2	655138	655235	655331	655427	655523	655619	655715	655810	655906	656002	96
3	656098	656194	656290	656386	656482	656577	656673	656769	656864	656960	
4	657056	657152	657247	657343	657438	657534	657629	657725	657820	657916	
5	658011	658107	658202	658298	658393	658488	658584	658679	658774	658870	
6	658965	659060	659155	659250	659346	659441	659536	659631	659726	659821	
7	659916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
8	660865	660960	661055	661150	661245	661339	661434	661529	661623	661718	
9	661813	661907	662002	662096	662191	662286	662380	662475	662569	662663	
460	**662758**	**662852**	**662947**	**663041**	**663135**	**663230**	**663324**	**663418**	**663512**	**663607**	
1	663701	663795	663889	663983	664078	664172	664266	664360	664454	664548	
2	664642	664736	664830	664924	665018	665112	665206	665299	665393	665487	94
3	665581	665675	665769	665862	665956	666050	666143	666237	666331	666424	
4	666518	666612	666705	666799	666892	666986	667079	667173	667266	667360	
5	667453	667546	667640	667733	667826	667920	668013	668106	668199	668293	
6	668386	668479	668572	668665	668759	668852	668945	669038	669131	669224	
7	669317	669410	669503	669596	669689	669782	669875	669967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	670988	671080	
9	671173	671265	671358	671451	671543	671636	671728	671821	671913	672005	
470	**672098**	**672190**	**672283**	**672375**	**672467**	**672560**	**672652**	**672744**	**672836**	**672929**	
1	673021	673113	673205	673297	673390	673482	673574	673666	673758	673850	
2	673942	674034	674126	674218	674310	674402	674494	674586	674677	674769	92
3	674861	674953	675045	675137	675228	675320	675412	675503	675595	675687	
4	675778	675870	675962	676053	676145	676236	676328	676419	676511	676602	
5	676694	676785	676876	676968	677059	677151	677242	677333	677424	677516	
6	677607	677698	677789	677881	677972	678063	678154	678245	678336	678427	
7	678518	678609	678700	678791	678882	678973	679064	679155	679246	679337	91
8	679428	679519	679610	679700	679791	679882	679973	680063	680154	680245	
9	680336	680426	680517	680607	680698	680789	680879	680970	681060	681151	
480	**681241**	**681332**	**681422**	**681513**	**681603**	**681693**	**681784**	**681874**	**681964**	**682055**	
1	682145	682235	682326	682416	682506	682596	682686	682777	682867	682957	
2	683047	683137	683227	683317	683407	683497	683587	683677	683767	683857	90
3	683947	684037	684127	684217	684307	684396	684486	684576	684666	684756	
4	684845	684935	685025	685114	685204	685294	685383	685473	685563	685652	
5	685742	685831	685921	686010	686100	686189	686279	686368	686458	686547	
6	686636	686726	686815	686904	686994	687083	687172	687261	687351	687440	
7	687529	687618	687707	687796	687886	687975	688064	688153	688242	688331	
8	688420	688509	688598	688687	688776	688865	688953	689042	689131	689220	89
9	689309	689398	689486	689575	689664	689753	689841	689930	690019	690107	
490	**690196**	**690285**	**690373**	**690462**	**690550**	**690639**	**690728**	**690816**	**690905**	**690993**	
1	691081	691170	691258	691347	691435	691524	691612	691700	691789	691877	
2	691965	692053	692142	692230	692318	692406	692494	692583	692671	692759	
3	692847	692935	693023	693111	693199	693287	693375	693463	693551	693639	88
4	693727	693815	693903	693991	694078	694166	694254	694342	694430	694517	
5	694605	694693	694781	694868	694956	695044	695131	695219	695307	695394	
6	695482	695569	695657	695744	695832	695919	696007	696094	696182	696269	
7	696356	696444	696531	696618	696706	696793	696880	696968	697055	697142	
8	697229	697317	697404	697491	697578	697665	697752	697839	697926	698014	87
9	698100	698188	698275	698362	698449	698535	698622	698709	698796	698883	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
500	**698970**	**699057**	**699144**	**699231**	**699317**	**699404**	**699491**	**699578**	**699664**	**699751**	
1	699838	699924	700011	700098	700184	700271	700358	700444	700531	700617	
2	700704	700790	700877	700963	701050	701136	701222	701309	701395	701482	
3	701568	701654	701741	701827	701913	701999	702086	702172	702258	702344	
4	702431	702517	702603	702689	702775	702861	702947	703033	703119	703205	
5	703291	703377	703463	703549	703635	703721	703807	703893	703979	704065	86
6	704151	704236	704322	704408	704494	704579	704665	704751	704837	704922	
7	705008	705094	705179	705265	705350	705436	705522	705607	705693	705778	
8	705864	705949	706035	706120	706206	706291	706376	706462	706547	706632	
9	706718	706803	706888	706974	707059	707144	707229	707315	707400	707485	
510	**707570**	**707655**	**707740**	**707826**	**707911**	**707996**	**708081**	**708166**	**708251**	**708336**	
1	708421	708506	708591	708676	708761	708846	708931	709015	709100	709185	85
2	709270	709355	709440	709524	709609	709694	709779	709863	709948	710033	
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	710879	
4	710963	711048	711132	711217	711301	711385	711470	711554	711639	711723	
5	711807	711892	711976	712060	712144	712229	712313	712397	712481	712566	
6	712650	712734	712818	712902	712986	713070	713154	713238	713323	713407	
7	713491	713575	713659	713742	713826	713910	713994	714078	714162	714246	84
8	714330	714414	714497	714581	714665	714749	714833	714916	715000	715084	
9	715167	715251	715335	715418	715502	715586	715669	715753	715836	715920	
520	**716003**	**716087**	**716170**	**716254**	**716337**	**716421**	**716504**	**716588**	**716671**	**716754**	
1	716838	716921	717004	717088	717171	717254	717338	717421	717504	717587	
2	717671	717754	717837	717920	718003	718086	718169	718253	718336	718419	83
3	718502	718585	718668	718751	718834	718917	719000	719083	719165	719248	
4	719331	719414	719497	719580	719663	719745	719828	719911	719994	720077	
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	720903	
6	720986	721068	721151	721233	721316	721398	721481	721563	721646	721728	
7	721811	721893	721975	722058	722140	722222	722305	722387	722469	722552	
8	722634	722716	722798	722881	722963	723045	723127	723209	723291	723374	
9	723456	723538	723620	723702	723784	723866	723948	724030	724112	724194	82
530	**724276**	**724358**	**724440**	**724522**	**724604**	**724685**	**724767**	**724849**	**724931**	**725013**	
1	725095	725176	725258	725340	725422	725503	725585	725667	725748	725830	
2	725912	725993	726075	726156	726238	726320	726401	726483	726564	726646	
3	726727	726809	726890	726972	727053	727134	727216	727297	727379	727460	
4	727541	727623	727704	727785	727866	727948	728029	728110	728191	728273	
5	728354	728435	728516	728597	728678	728759	728841	728922	729003	729084	
6	729165	729246	729327	729408	729489	729570	729651	729732	729813	729893	81
7	729974	730055	730136	730217	730298	730378	730459	730540	730621	730702	
8	730782	730863	730944	731024	731105	731186	731266	731347	731428	731508	
9	731589	731669	731750	731830	731911	731991	732072	732152	732233	732313	
540	**732394**	**732474**	**732555**	**732635**	**732715**	**732796**	**732876**	**732956**	**733037**	**733117**	
1	733197	733278	733358	733438	733518	733598	733679	733759	733839	733919	
2	733999	734079	734160	734240	734320	734400	734480	734560	734640	734720	80
3	734800	734880	734960	735040	735120	735200	735279	735359	735439	735519	
4	735599	735679	735759	735838	735918	735998	736078	736157	736237	736317	
5	736397	736476	736556	736635	736715	736795	736874	736954	737034	737113	
6	737193	737272	737352	737431	737511	737590	737670	737749	737829	737908	
7	737987	738067	738146	738225	738305	738384	738463	738543	738622	738701	
8	738781	738860	738939	739018	739097	739177	739256	739335	739414	739493	
9	739572	739651	739731	739810	739889	739968	740047	740126	740205	740284	79
550	**740363**	**740442**	**740521**	**740600**	**740678**	**740757**	**740836**	**740915**	**740994**	**741073**	
1	741152	741230	741309	741388	741467	741546	741624	741703	741782	741860	
2	741939	742018	742096	742175	742254	742332	742411	742489	742568	742647	
3	742725	742804	742882	742961	743039	743118	743196	743275	743353	743431	
4	743510	743588	743667	743745	743823	743902	743980	744058	744136	744215	
5	744293	744371	744449	744528	744606	744684	744762	744840	744919	744997	
6	745075	745153	745231	745309	745387	745465	745543	745621	745699	745777	78
7	745855	745933	746011	746089	746167	746245	746323	746401	746479	746556	
8	746634	746712	746790	746868	746945	747023	747101	747179	747256	747334	
9	747412	747489	747567	747645	747722	747800	747878	747955	748033	748110	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.6
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
560	**748188**	**748266**	**748343**	**748421**	**748498**	**748576**	**748653**	**748731**	**748808**	**748885**	
1	748963	749040	749118	749195	749272	749350	749427	749504	749582	749659	
2	749736	749814	749891	749968	750045	750123	750200	750277	750354	750431	
3	750508	750586	750663	750740	750817	750894	750971	751048	751125	751202	
4	751279	751356	751433	751510	751587	751664	751741	751818	751895	751972	77
5	752048	752125	752202	752279	752356	752433	752509	752586	752663	752740	
6	752816	752893	752970	753047	753123	753200	753277	753353	753430	753506	
7	753583	753660	753736	753813	753889	753966	754042	754119	754195	754272	
8	754348	754425	754501	754578	754654	754730	754807	754883	754960	755036	
9	755112	755189	755265	755341	755417	755494	755570	755646	755722	755799	
570	**755875**	**755951**	**756027**	**756103**	**756180**	**756256**	**756332**	**756408**	**756484**	**756560**	
1	756636	756712	756788	756864	756940	757016	757092	757168	757244	757320	76
2	757396	757472	757548	757624	757700	757775	757851	757927	758003	758079	
3	758155	758230	758306	758382	758458	758533	758609	758685	758761	758836	
4	758912	758988	759063	759139	759214	759290	759366	759441	759517	759592	
5	759668	759743	759819	759894	759970	760045	760121	760196	760272	760347	
6	760422	760498	760573	760649	760724	760799	760875	760950	761025	761101	
7	761176	761251	761326	761402	761477	761552	761627	761702	761778	761853	
8	761928	762003	762078	762153	762228	762303	762378	762453	762529	762604	75
9	762679	762754	762829	762904	762978	763053	763128	763203	763278	763353	
580	**763428**	**763503**	**763578**	**763653**	**763727**	**763802**	**763877**	**763952**	**764027**	**764101**	
1	764176	764251	764326	764400	764475	764550	764624	764699	764774	764848	
2	764923	764998	765072	765147	765221	765296	765370	765445	765520	765594	
3	765669	765743	765818	765892	765966	766041	766115	766190	766264	766338	
4	766413	766487	766562	766636	766710	766785	766859	766933	767007	767082	
5	767156	767230	767304	767379	767453	767527	767601	767675	767749	767823	
6	767898	767972	768046	768120	768194	768268	768342	768416	768490	768564	74
7	768638	768712	768786	768860	768934	769008	769082	769156	769230	769303	
8	769377	769451	769525	769599	769673	769746	769820	769894	769968	770042	
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	770778	
590	**770852**	**770926**	**770999**	**771073**	**771146**	**771220**	**771293**	**771367**	**771440**	**771514**	
1	771587	771661	771734	771808	771881	771955	772028	772102	772175	772248	
2	772322	772395	772468	772542	772615	772688	772762	772835	772908	772981	
3	773055	773128	773201	773274	773348	773421	773494	773567	773640	773713	
4	773786	773860	773933	774006	774079	774152	774225	774298	774371	774444	73
5	774517	774590	774663	774736	774809	774882	774955	775028	775100	775173	
6	775246	775319	775392	775465	775538	775610	775683	775756	775829	775902	
7	775974	776047	776120	776193	776265	776338	776411	776483	776556	776629	
8	776701	776774	776846	776919	776992	777064	777137	777209	777282	777354	
9	777427	777499	777572	777644	777717	777789	777862	777934	778006	778079	
600	**778151**	**778224**	**778296**	**778368**	**778441**	**778513**	**778585**	**778658**	**778730**	**778802**	
1	778874	778947	779019	779091	779163	779236	779308	779380	779452	779524	
2	779596	779669	779741	779813	779885	779957	780029	780101	780173	780245	
3	780317	780389	780461	780533	780605	780677	780749	780821	780893	780965	72
4	781037	781109	781181	781253	781324	781396	781468	781540	781612	781684	
5	781755	781827	781899	781971	782042	782114	782186	782258	782329	782401	
6	782473	782544	782616	782688	782759	782831	782902	782974	783046	783117	
7	783189	783260	783332	783403	783475	783546	783618	783689	783761	783832	
8	783904	783975	784046	784118	784189	784261	784332	784403	784475	784546	
9	784617	784689	784760	784831	784902	784974	785045	785116	785187	785259	
610	**785330**	**785401**	**785472**	**785543**	**785615**	**785686**	**785757**	**785828**	**785899**	**785970**	
1	786041	786112	786183	786254	786325	786396	786467	786538	786609	786680	71
2	786751	786822	786893	786964	787035	787106	787177	787248	787319	787390	
3	787460	787531	787602	787673	787744	787815	787885	787956	788027	788098	
4	788168	788239	788310	788381	788451	788522	788593	788663	788734	788804	
5	788875	788946	789016	789087	789157	789228	789299	789369	789440	789510	
6	789581	789651	789722	789792	789863	789933	790004	790074	790144	790215	
7	790285	790356	790426	790496	790567	790637	790707	790778	790848	790918	
8	790988	791059	791129	791199	791269	791340	791410	791480	791550	791620	
9	791691	791761	791831	791901	791971	792041	792111	792181	792252	792322	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
620	**792392**	**792462**	**792532**	**792602**	**792672**	**792742**	**792812**	**792882**	**792952**	**793022**	70
1	793092	793162	793231	793301	793371	793441	793511	793581	793651	793721	
2	793790	793860	793930	794000	794070	794139	794209	794279	794349	794418	
3	794488	794558	794627	794697	794767	794836	794906	794976	795045	795115	
4	795185	795254	795324	795393	795463	795532	795602	795672	795741	795811	
5	795880	795949	796019	796088	796158	796227	796297	796366	796436	796505	
6	796574	796644	796713	796782	796852	796921	796990	797060	797129	797198	
7	797268	797337	797406	797475	797545	797614	797683	797752	797821	797890	
8	797960	798029	798098	798167	798236	798305	798374	798443	798513	798582	
9	798651	798720	798789	798858	798927	798996	799065	799134	799203	799272	69
630	**799341**	**799409**	**799478**	**799547**	**799616**	**799685**	**799754**	**799823**	**799892**	**799961**	
1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	
2	800717	800786	800854	800923	800992	801061	801129	801198	801266	801335	
3	801404	801472	801541	801609	801678	801747	801815	801884	801952	802021	
4	802089	802158	802226	802295	802363	802432	802500	802568	802637	802705	
5	802774	802842	802910	802979	803047	803116	803184	803252	803321	803389	
6	803457	803525	803594	803662	803730	803798	803867	803935	804003	804071	
7	804139	804208	804276	804344	804412	804480	804548	804616	804685	804753	
8	804821	804889	804957	805025	805093	805161	805229	805297	805365	805433	68
9	805501	805569	805637	805705	805773	805841	805908	805976	806044	806112	
640	**806180**	**806248**	**806316**	**806384**	**806451**	**806519**	**806587**	**806655**	**806723**	**806790**	
1	806858	806926	806994	807061	807129	807197	807264	807332	807400	807467	
2	807535	807603	807670	807738	807806	807873	807941	808008	808076	808143	
3	808211	808279	808346	808414	808481	808549	808616	808684	808751	808818	
4	808886	808953	809021	809088	809156	809223	809290	809358	809425	809492	
5	809560	809627	809694	809762	809829	809896	809964	810031	810098	810165	
6	810233	810300	810367	810434	810501	810569	810636	810703	810770	810837	
7	810904	810971	811039	811106	811173	811240	811307	811374	811441	811508	67
8	811575	811642	811709	811776	811843	811910	811977	812044	812111	812178	
9	812245	812312	812379	812445	812512	812579	812646	812713	812780	812847	
650	**812913**	**812980**	**813047**	**813114**	**813181**	**813247**	**813314**	**813381**	**813448**	**813514**	
1	813581	813648	813714	813781	813848	813914	813981	814048	814114	814181	
2	814248	814314	814381	814447	814514	814581	814647	814714	814780	814847	
3	814913	814980	815046	815113	815179	815246	815312	815378	815445	815511	
4	815578	815644	815711	815777	815843	815910	815976	816042	816109	816175	
5	816241	816308	816374	816440	816506	816573	816639	816705	816771	816838	
6	816904	816970	817036	817102	817169	817235	817301	817367	817433	817499	
7	817565	817631	817698	817764	817830	817896	817962	818028	818094	818160	
8	818226	818292	818358	818424	818490	818556	818622	818688	818754	818820	66
9	818885	818951	819017	819083	819149	819215	819281	819346	819412	819478	
660	**819544**	**819610**	**819676**	**819741**	**819807**	**819873**	**819939**	**820004**	**820070**	**820136**	
1	820201	820267	820333	820399	820464	820530	820595	820661	820727	820792	
2	820858	820924	820989	821055	821120	821186	821251	821317	821382	821448	
3	821514	821579	821645	821710	821775	821841	821906	821972	822037	822103	
4	822168	822233	822299	822364	822430	822495	822560	822626	822691	822756	
5	822822	822887	822952	823018	823083	823148	823213	823279	823344	823409	
6	823474	823539	823605	823670	823735	823800	823865	823930	823996	824061	
7	824126	824191	824256	824321	824386	824451	824516	824581	824646	824711	
8	824776	824841	824906	824971	825036	825101	825166	825231	825296	825361	65
9	825426	825491	825556	825621	825686	825751	825815	825880	825945	826010	
670	**826075**	**826140**	**826204**	**826269**	**826334**	**826399**	**826464**	**826528**	**826593**	**826658**	
1	826723	826787	826852	826917	826981	827046	827111	827175	827240	827305	
2	827369	827434	827499	827563	827628	827692	827757	827821	827886	827951	
3	828015	828080	828144	828209	828273	828338	828402	828467	828531	828595	
4	828660	828724	828789	828853	828918	828982	829046	829111	829175	829239	
5	829304	829368	829432	829497	829561	829625	829690	829754	829818	829882	
6	829947	830011	830075	830139	830204	830268	830332	830396	830460	830525	
7	830589	830653	830717	830781	830845	830909	830973	831037	831102	831166	
8	831230	831294	831358	831422	831486	831550	831614	831678	831742	831806	64
9	831870	831934	831998	832062	832126	832189	832253	832317	832381	832445	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
66	6.6	13.2	19.8	26.4	33.0	39.6	46 2	52.8	59.4
64	6.4	12 8	19.2	25.6	32.0	38 4	44.8	51.2	57.6
62	6 2	12 4	18 6	24.8	31 0	37 2	43 4	49 6	55.8

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
680	**832509**	**832573**	**832637**	**832700**	**832764**	**832828**	**832892**	**832956**	**833020**	**833083**	
1	833147	833211	833275	833338	833402	833466	833530	833593	833657	833721	
2	833784	833848	833912	833975	834039	834103	834166	834230	834294	834357	
3	834421	834484	834548	834611	834675	834739	834802	834866	834929	834993	
4	835056	835120	835183	835247	835310	835373	835437	835500	835564	835627	
5	835691	835754	835817	835881	835944	836007	836071	836134	836197	836261	
6	836324	836387	836451	836514	836577	836641	836704	836767	836830	836894	
7	836957	837020	837083	837146	837210	837273	837336	837399	837462	837525	
8	837588	837652	837715	837778	837841	837904	837967	838030	838093	838156	63
9	838219	838282	838345	838408	838471	838534	838597	838660	838723	838786	
690	**838849**	**838912**	**838975**	**839038**	**839101**	**839164**	**839227**	**839289**	**839352**	**839415**	
1	839478	839541	839604	839667	839729	839792	839855	839918	839981	840043	
2	840106	840169	840232	840294	840357	840420	840482	840545	840608	840671	
3	840733	840796	840859	840921	840984	841046	841109	841172	841234	841297	
4	841359	841422	841485	841547	841610	841672	841735	841797	841860	841922	
5	841985	842047	842110	842172	842235	842297	842360	842422	842484	842547	
6	842609	842672	842734	842796	842859	842921	842983	843046	843108	843170	
7	843233	843295	843357	843420	843482	843544	843606	843669	843731	843793	
8	843855	843918	843980	844042	844104	844166	844229	844291	844353	844415	
9	844477	844539	844601	844664	844726	844788	844850	844912	844974	845036	
700	**845098**	**845160**	**845222**	**845284**	**845346**	**845408**	**845470**	**845532**	**845594**	**845656**	62
1	845718	845780	845842	845904	845966	846028	846090	846151	846213	846275	
2	846337	846399	846461	846523	846585	846646	846708	846770	846832	846894	
3	846955	847017	847079	847141	847202	847264	847326	847388	847449	847511	
4	847573	847634	847696	847758	847819	847881	847943	848004	848066	848128	
5	848189	848251	848312	848374	848435	848497	848559	848620	848682	848743	
6	848805	848866	848928	848989	849051	849112	849174	849235	849297	849358	
7	849419	849481	849542	849604	849665	849726	849788	849849	849911	849972	
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	
9	850646	850707	850769	850830	850891	850952	851014	851075	851136	851197	
710	**851258**	**851320**	**851381**	**851442**	**851503**	**851564**	**851625**	**851686**	**851747**	**851809**	
1	851870	851931	851992	852053	852114	852175	852236	852297	852358	852419	61
2	852480	852541	852602	852663	852724	852785	852846	852907	852968	853029	
3	853090	853150	853211	853272	853333	853394	853455	853516	853577	853637	
4	853698	853759	853820	853881	853941	854002	854063	854124	854185	854245	
5	854306	854367	854428	854488	854549	854610	854670	854731	854792	854852	
6	854913	854974	855034	855095	855156	855216	855277	855337	855398	855459	
7	855519	855580	855640	855701	855761	855822	855882	855943	856003	856064	
8	856124	856185	856245	856306	856366	856427	856487	856548	856608	856668	
9	856729	856789	856850	856910	856970	857031	857091	857152	857212	857272	
720	**857332**	**857393**	**857453**	**857513**	**857574**	**857634**	**857694**	**857755**	**857815**	**857875**	
1	857935	857995	858056	858116	858176	858236	858297	858357	858417	858477	
2	858537	858597	858657	858718	858778	858838	858898	858958	859018	859078	
3	859138	859198	859258	859318	859379	859439	859499	859559	859619	859679	60
4	859739	859799	859859	859918	859978	860038	860098	860158	860218	860278	
5	860338	860398	860458	860518	860578	860637	860697	860757	860817	860877	
6	860937	860996	861056	861116	861176	861236	861295	861355	861415	861475	
7	861534	861594	861654	861714	861773	861833	861893	861952	862012	862072	
8	862131	862191	862251	862310	862370	862430	862489	862549	862608	862668	
9	862728	862787	862847	862906	862966	863025	863085	863144	863204	863263	
730	**863323**	**863382**	**863442**	**863501**	**863561**	**863620**	**863680**	**863739**	**863799**	**863858**	
1	863917	863977	864036	864096	864155	864214	864274	864333	864392	864452	
2	864511	864570	864630	864689	864748	864808	864867	864926	864985	865045	
3	865104	865163	865222	865282	865341	865400	865459	865519	865578	865637	
4	865696	865755	865814	865874	865933	865992	866051	866110	866169	866228	
5	866287	866346	866405	866465	866524	866583	866642	866701	866760	866819	
6	866878	866937	866996	867055	867114	867173	867232	867291	867350	867409	59
7	867467	867526	867585	867644	867703	867762	867821	867880	867939	867998	
8	868056	868115	868174	868233	868292	868350	868409	868468	868527	868586	
9	868644	868703	868762	868821	868879	868938	868997	869056	869114	869173	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
740	**869232**	**869290**	**869349**	**869408**	**869466**	**869525**	**869584**	**869642**	**869701**	**869760**	
1	869818	869877	869935	869994	870053	870111	870170	870228	870287	870345	
2	870404	870462	870521	870579	870638	870696	870755	870813	870872	870930	
3	870989	871047	871106	871164	871223	871281	871339	871398	871456	871515	
4	871573	871631	871690	871748	871806	871865	871923	871981	872040	872098	
5	872156	872215	872273	872331	872389	872448	872506	872564	872622	872681	
6	872739	872797	872855	872913	872972	873030	873088	873146	873204	873262	
7	873321	873379	873437	873495	873553	873611	873669	873727	873785	873844	
8	873902	873960	874018	874076	874134	874192	874250	874308	874366	874424	58
9	874482	874540	874598	874656	874714	874772	874830	874888	874945	875003	
750	**875061**	**875119**	**875177**	**875235**	**875293**	**875351**	**875409**	**875466**	**875524**	**875582**	
1	875640	875698	875756	875813	875871	875929	875987	876045	876102	876160	
2	876218	876276	876333	876391	876449	876507	876564	876622	876680	876737	
3	876795	876853	876910	876968	877026	877083	877141	877199	877256	877314	
4	877371	877429	877487	877544	877602	877659	877717	877774	877832	877889	
5	877947	878004	878062	878119	878177	878234	878292	878349	878407	878464	
6	878522	878579	878637	878694	878752	878809	878866	878924	878981	879039	
7	879096	879153	879211	879268	879325	879383	879440	879497	879555	879612	
8	879669	879726	879784	879841	879898	879956	880013	880070	880127	880185	
9	880242	880299	880356	880413	880471	880528	880585	880642	880699	880756	
760	**880814**	**880871**	**880928**	**880985**	**881042**	**881099**	**881156**	**881213**	**881271**	**881328**	
1	881385	881442	881499	881556	881613	881670	881727	881784	881841	881898	
2	881955	882012	882069	882126	882183	882240	882297	882354	882411	882468	57
3	882525	882581	882638	882695	882752	882809	882866	882923	882980	883037	
4	883093	883150	883207	883264	883321	883377	883434	883491	883548	883605	
5	883661	883718	883775	883832	883888	883945	884002	884059	884115	884172	
6	884229	884285	884342	884399	884455	884512	884569	884625	884682	884739	
7	884795	884852	884909	884965	885022	885078	885135	885192	885248	885305	
8	885361	885418	885474	885531	885587	885644	885700	885757	885813	885870	
9	885926	885983	886039	886096	886152	886209	886265	886321	886378	886434	
770	**886491**	**886547**	**886604**	**886660**	**886716**	**886773**	**886829**	**886885**	**886942**	**886998**	
1	887054	887111	887167	887223	887280	887336	887392	887449	887505	887561	
2	887617	887674	887730	887786	887842	887898	887955	888011	888067	888123	
3	888179	888236	888292	888348	888404	888460	888516	888573	888629	888685	
4	888741	888797	888853	888909	888965	889021	889077	889134	889190	889246	
5	889302	889358	889414	889470	889526	889582	889638	889694	889750	889806	56
6	889862	889918	889974	890030	890086	890141	890197	890253	890309	890365	
7	890421	890477	890533	890589	890645	890700	890756	890812	890868	890924	
8	890980	891035	891091	891147	891203	891259	891314	891370	891426	891482	
9	891537	891593	891649	891705	891760	891816	891872	891928	891983	892039	
780	**892095**	**892150**	**892206**	**892262**	**892317**	**892373**	**892429**	**892484**	**892540**	**892595**	
1	892651	892707	892762	892818	892873	892929	892985	893040	893096	893151	
2	893207	893262	893318	893373	893429	893484	893540	893595	893651	893706	
3	893762	893817	893873	893928	893984	894039	894094	894150	894205	894261	
4	894316	894371	894427	894482	894538	894593	894648	894704	894759	894814	
5	894870	894925	894980	895036	895091	895146	895201	895257	895312	895367	
6	895423	895478	895533	895588	895644	895699	895754	895809	895864	895920	
7	895975	896030	896085	896140	896195	896251	896306	896361	896416	896471	
8	896526	896581	896636	896692	896747	896802	896857	896912	896967	897022	
9	897077	897132	897187	897242	897297	897352	897407	897462	897517	897572	
790	**897627**	**897682**	**897737**	**897792**	**897847**	**897902**	**897957**	**898012**	**898067**	**898122**	55
1	898176	898231	898286	898341	898396	898451	898506	898561	898615	898670	
2	898725	898780	898835	898890	898944	898999	899054	899109	899164	899218	
3	899273	899328	899383	899437	899492	899547	899602	899656	899711	899766	
4	899821	899875	899930	899985	900039	900094	900149	900203	900258	900312	
5	900367	900422	900476	900531	900586	900640	900695	900749	900804	900859	
6	900913	900968	901022	901077	901131	901186	901240	901295	901349	901404	
7	901458	901513	901567	901622	901676	901731	901785	901840	901894	901948	
8	902003	902057	902112	902166	902221	902275	902329	902384	902438	902492	
9	902547	902601	902655	902710	902764	902818	902873	902927	902981	903036	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
800	**903090**	**903144**	**903199**	**903253**	**903307**	**903361**	**903416**	**903470**	**903524**	**903578**	
1	903633	903687	903741	903795	903849	903904	903958	904012	904066	904120	
2	904174	904229	904283	904337	904391	904445	904499	904553	904607	904661	
3	904716	904770	904824	904878	904932	904986	905040	905094	905148	905202	
4	905256	905310	905364	905418	905472	905526	905580	905634	905688	905742	54
5	905796	905850	905904	905958	906012	906066	906119	906173	906227	906281	
6	906335	906389	906443	906497	906551	906604	906658	906712	906766	906820	
7	906874	906927	906981	907035	907089	907143	907196	907250	907304	907358	
8	907411	907465	907519	907573	907626	907680	907734	907787	907841	907895	
9	907949	908002	908056	908110	908163	908217	908270	908324	908378	908431	
810	**908485**	**908539**	**908592**	**908646**	**908699**	**908753**	**908807**	**908860**	**908914**	**908967**	
1	909021	909074	909128	909181	909235	909289	909342	909396	909449	909503	
2	909556	909610	909663	909716	909770	909823	909877	909930	909984	910037	
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	910571	
4	910624	910678	910731	910784	910838	910891	910944	910998	911051	911104	
5	911158	911211	911264	911317	911371	911424	911477	911530	911584	911637	
6	911690	911743	911797	911850	911903	911956	912009	912063	912116	912169	
7	912222	912275	912328	912381	912435	912488	912541	912594	912647	912700	
8	912753	912806	912859	912913	912966	913019	913072	913125	913178	913231	
9	913284	913337	913390	913443	913496	913549	913602	913655	913708	913761	53
820	**913814**	**913867**	**913920**	**913973**	**914026**	**914079**	**914132**	**914184**	**914237**	**914290**	
1	914343	914396	914449	914502	914555	914608	914660	914713	914766	914819	
2	914872	914925	914977	915030	915083	915136	915189	915241	915294	915347	
3	915400	915453	915505	915558	915611	915664	915716	915769	915822	915875	
4	915927	915980	916033	916085	916138	916191	916243	916296	916349	916401	
5	916454	916507	916559	916612	916664	916717	916770	916822	916875	916927	
6	916980	917033	917085	917138	917190	917243	917295	917348	917400	917453	
7	917506	917558	917611	917663	917716	917768	917820	917873	917925	917978	
8	918030	918083	918135	918188	918240	918293	918345	918397	918450	918502	
9	918555	918607	918659	918712	918764	918816	918869	918921	918973	919026	
830	**919078**	**919130**	**919183**	**919235**	**919287**	**919340**	**919392**	**919444**	**919496**	**919549**	
1	919601	919653	919706	919758	919810	919862	919914	919967	920019	920071	
2	920123	920176	920228	920280	920332	920384	920436	920489	920541	920593	
3	920645	920697	920749	920801	920853	920906	920958	921010	921062	921114	52
4	921166	921218	921270	921322	921374	921426	921478	921530	921582	921634	
5	921686	921738	921790	921842	921894	921946	921998	922050	922102	922154	
6	922206	922258	922310	922362	922414	922466	922518	922570	922622	922674	
7	922725	922777	922829	922881	922933	922985	923037	923089	923140	923192	
8	923244	923296	923348	923399	923451	923503	923555	923607	923658	923710	
9	923762	923814	923865	923917	923969	924021	924072	924124	924176	924228	
840	**924279**	**924331**	**924383**	**924434**	**924486**	**924538**	**924589**	**924641**	**924693**	**924744**	
1	924796	924848	924899	924951	925003	925054	925106	925157	925209	925261	
2	925312	925364	925415	925467	925518	925570	925621	925673	925725	925776	
3	925828	925879	925931	925982	926034	926085	926137	926188	926240	926291	
4	926342	926394	926445	926497	926548	926600	926651	926702	926754	926805	
5	926857	926908	926959	927011	927062	927114	927165	927216	927268	927319	
6	927370	927422	927473	927524	927576	927627	927678	927730	927781	927832	
7	927883	927935	927986	928037	928088	928140	928191	928242	928293	928345	
8	928396	928447	928498	928549	928601	928652	928703	928754	928805	928857	
9	928908	928959	929010	929061	929112	929163	929215	929266	929317	929368	
850	**929419**	**929470**	**929521**	**929572**	**929623**	**929674**	**929725**	**929776**	**929827**	**929879**	
1	929930	929981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	930542	930592	930643	930694	930745	930796	930847	930898	
3	930949	931000	931051	931102	931153	931204	931254	931305	931356	931407	
4	931458	931509	931560	931610	931661	931712	931763	931814	931865	931915	
5	931966	932017	932068	932118	932169	932220	932271	932322	932372	932423	
6	932474	932524	932575	932626	932677	932727	932778	932829	932879	932930	
7	932981	933031	933082	933133	933183	933234	933285	933335	933386	933437	
8	933487	933538	933589	933639	933690	933740	933791	933841	933892	933943	
9	933993	934044	934094	934145	934195	934246	934296	934347	934397	934448	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
860	**934498**	**934549**	**934599**	**934650**	**934700**	**934751**	**934801**	**934852**	**934902**	**934953**	
1	935003	935054	935104	935154	935205	935255	935306	935356	935406	935457	
2	935507	935558	935608	935658	935709	935759	935809	935860	935910	935960	
3	936011	936061	936111	936162	936212	936262	936313	936363	936413	936463	
4	936514	936564	936614	936665	936715	936765	936815	936865	936916	936966	
5	937016	937066	937116	937167	937217	937267	937317	937367	937418	937468	
6	937518	937568	937618	937668	937718	937769	937819	937869	937919	937969	
7	938019	938069	938119	938169	938219	938269	938320	938370	938420	938470	50
8	938520	938570	938620	938670	938720	938770	938820	938870	938920	938970	
9	939020	939070	939120	939170	939220	939270	939320	939369	939419	939469	
870	**939519**	**939569**	**939619**	**939669**	**939719**	**939769**	**939819**	**939869**	**939918**	**939968**	
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	
2	940516	940566	940616	940666	940716	940765	940815	940865	940915	940964	
3	941014	941064	941114	941163	941213	941263	941313	941362	941412	941462	
4	941511	941561	941611	941660	941710	941760	941809	941859	941909	941958	
5	942008	942058	942107	942157	942207	942256	942306	942355	942405	942455	
6	942504	942554	942603	942653	942702	942752	942801	942851	942901	942950	
7	943000	943049	943099	943148	943198	943247	943297	943346	943396	943445	
8	943495	943544	943593	943643	943692	943742	943791	943841	943890	943939	
9	943989	944038	944088	944137	944186	944236	944285	944335	944384	944433	
880	**944483**	**944532**	**944581**	**944631**	**944680**	**944729**	**944779**	**944828**	**944877**	**944927**	
1	944976	945025	945074	945124	945173	945222	945272	945321	945370	945419	
2	945469	945518	945567	945616	945665	945715	945764	945813	945862	945912	
3	945961	946010	946059	946108	946157	946207	946256	946305	946354	946403	
4	946452	946501	946551	946600	946649	946698	946747	946796	946845	946894	
5	946943	946992	947041	947090	947140	947189	947238	947287	947336	947385	
6	947434	947483	947532	947581	947630	947679	947728	947777	947826	947875	49
7	947924	947973	948022	948070	948119	948168	948217	948266	948315	948364	
8	948413	948462	948511	948560	948608	948657	948706	948755	948804	948853	
9	948902	948951	948999	949048	949097	949146	949195	949244	949292	949341	
890	**949390**	**949439**	**949488**	**949536**	**949585**	**949634**	**949683**	**949731**	**949780**	**949829**	
1	949878	949926	949975	950024	950073	950121	950170	950219	950267	950316	
2	950365	950414	950462	950511	950560	950608	950657	950706	950754	950803	
3	950851	950900	950949	950997	951046	951095	951143	951192	951240	951289	
4	951338	951386	951435	951483	951532	951580	951629	951677	951726	951775	
5	951823	951872	951920	951969	952017	952066	952114	952163	952211	952260	
6	952308	952356	952405	952453	952502	952550	952599	952647	952696	952744	
7	952792	952841	952889	952938	952986	953034	953083	953131	953180	953228	
8	953276	953325	953373	953421	953470	953518	953566	953615	953663	953711	
9	953760	953808	953856	953905	953953	954001	954049	954098	954146	954194	
900	**954243**	**954291**	**954339**	**954387**	**954435**	**954484**	**954532**	**954580**	**954628**	**954677**	
1	954725	954773	954821	954869	954918	954966	955014	955062	955110	955158	
2	955207	955255	955303	955351	955399	955447	955495	955543	955592	955640	
3	955688	955736	955784	955832	955880	955928	955976	956024	956072	956120	
4	956168	956216	956265	956313	956361	956409	956457	956505	956553	956601	
5	956649	956697	956745	956793	956840	956888	956936	956984	957032	957080	48
6	957128	957176	957224	957272	957320	957368	957416	957464	957512	957559	
7	957607	957655	957703	957751	957799	957847	957894	957942	957990	958038	
8	958086	958134	958181	958229	958277	958325	958373	958421	958468	958516	
9	958564	958612	958659	958707	958755	958803	958850	958898	958946	958994	
910	**959041**	**959089**	**959137**	**959185**	**959232**	**959280**	**959328**	**959375**	**959423**	**959471**	
1	959518	959566	959614	959661	959709	959757	959804	959852	959900	959947	
2	959995	960042	960090	960138	960185	960233	960280	960328	960376	960423	
3	960471	960518	960566	960613	960661	960709	960756	960804	960851	960899	
4	960946	960994	961041	961089	961136	961184	961231	961279	961326	961374	
5	961421	961469	961516	961563	961611	961658	961706	961753	961801	961848	
6	961895	961943	961990	962038	962085	962132	962180	962227	962275	962322	
7	962369	962417	962464	962511	962559	962606	962653	962701	962748	962795	
8	962843	962890	962937	962985	963032	963079	963126	963174	963221	963268	
9	963316	963363	963410	963457	963504	963552	963599	963646	963693	963741	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
920	**963788**	**963835**	**963882**	**963929**	**963977**	**964024**	**964071**	**964118**	**964165**	**964212**	
1	964260	964307	964354	964401	964448	964495	964542	964590	964637	964684	
2	964731	964778	964825	964872	964919	964966	965013	965061	965108	965155	
3	965202	965249	965296	965343	965390	965437	965484	965531	965578	965625	
4	965672	965719	965766	965813	965860	965907	965954	966001	966048	966095	47
5	966142	966189	966236	966283	966329	966376	966423	966470	966517	966564	
6	966611	966658	966705	966752	966799	966845	966892	966939	966986	967033	
7	967080	967127	967173	967220	967267	967314	967361	967408	967454	967501	
8	967548	967595	967642	967688	967735	967782	967829	967875	967922	967969	
9	968016	968062	968109	968156	968203	968249	968296	968343	968390	968436	
930	**968483**	**968530**	**968576**	**968623**	**968670**	**968716**	**968763**	**968810**	**968856**	**968903**	
1	968950	968996	969043	969090	969136	969183	969229	969276	969323	969369	
2	969416	969463	969509	969556	969602	969649	969695	969742	969789	969835	
3	969882	969928	969975	970021	970068	970114	970161	970207	970254	970300	
4	970347	970393	970440	970486	970533	970579	970626	970672	970719	970765	
5	970812	970858	970904	970951	970997	971044	971090	971137	971183	971229	
6	971276	971322	971369	971415	971461	971508	971554	971601	971647	971693	
7	971740	971786	971832	971879	971925	971971	972018	972064	972110	972157	
8	972203	972249	972295	972342	972388	972434	972481	972527	972573	972619	
9	972666	972712	972758	972804	972851	972897	972943	972989	973035	973082	
940	**973128**	**973174**	**973220**	**973266**	**973313**	**973359**	**973405**	**973451**	**973497**	**973543**	
1	973590	973636	973682	973728	973774	973820	973866	973913	973959	974005	
2	974051	974097	974143	974189	974235	974281	974327	974374	974420	974466	
3	974512	974558	974604	974650	974696	974742	974788	974834	974880	974926	
4	974972	975018	975064	975110	975156	975202	975248	975294	975340	975386	46
5	975432	975478	975524	975570	975616	975662	975707	975753	975799	975845	
6	975891	975937	975983	976029	976075	976121	976167	976212	976258	976304	
7	976350	976396	976442	976488	976533	976579	976625	976671	976717	976763	
8	976808	976854	976900	976946	976992	977037	977083	977129	977175	977220	
9	977266	977312	977358	977403	977449	977495	977541	977586	977632	977678	
950	**977724**	**977769**	**977815**	**977861**	**977906**	**977952**	**977998**	**978043**	**978089**	**978135**	
1	978181	978226	978272	978317	978363	978409	978454	978500	978546	978591	
2	978637	978683	978728	978774	978819	978865	978911	978956	979002	979047	
3	979093	979138	979184	979230	979275	979321	979366	979412	979457	979503	
4	979548	979594	979639	979685	979730	979776	979821	979867	979912	979958	
5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	
6	980458	980503	980549	980594	980640	980685	980730	980776	980821	980867	
7	980912	980957	981003	981048	981093	981139	981184	981229	981275	981320	
8	981366	981411	981456	981501	981547	981592	981637	981683	981728	981773	
9	981819	981864	981909	981954	982000	982045	982090	982135	982181	982226	
960	**982271**	**982316**	**982362**	**982407**	**982452**	**982497**	**982543**	**982588**	**982633**	**982678**	
1	982723	982769	982814	982859	982904	982949	982994	983040	983085	983130	
2	983175	983220	983265	983310	983356	983401	983446	983491	983536	983581	
3	983626	983671	983716	983762	983807	983852	983897	983942	983987	984032	
4	984077	984122	984167	984212	984257	984302	984347	984392	984437	984482	
5	984527	984572	984617	984662	984707	984752	984797	984842	984887	984932	45
6	984977	985022	985067	985112	985157	985202	985247	985292	985337	985382	
7	985426	985471	985516	985561	985606	985651	985696	985741	985786	985830	
8	985875	985920	985965	986010	986055	986100	986144	986189	986234	986279	
9	986324	986369	986413	986458	986503	986548	986593	986637	986682	986727	
970	**986772**	**986817**	**986861**	**986906**	**986951**	**986996**	**987040**	**987085**	**987130**	**987175**	
1	987219	987264	987309	987353	987398	987443	987488	987532	987577	987622	
2	987666	987711	987756	987800	987845	987890	987934	987979	988024	988068	
3	988113	988157	988202	988247	988291	988336	988381	988425	988470	988514	
4	988559	988604	988648	988693	988737	988782	988826	988871	988916	988960	
5	989005	989049	989094	989138	989183	989227	989272	989316	989361	989405	
6	989450	989494	989539	989583	989628	989672	989717	989761	989806	989850	
7	989895	989939	989983	990028	990072	990117	990161	990206	990250	990294	
8	990339	990383	990428	990472	990516	990561	990605	990650	990694	990738	
9	990783	990827	990871	990916	990960	991004	991049	991093	991137	991182	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
42	4.2	8.4	12.6	16.8	21.0	25.2	29.4	33.6	37.8

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
980	**991226**	**991270**	**991315**	**991359**	**991403**	**991448**	**991492**	**991536**	**991580**	**991625**	
1	991669	991713	991758	991802	991846	991890	991935	991979	992023	992067	
2	992111	992156	992200	992244	992288	992333	992377	992421	992465	992509	
3	992554	992598	992642	992686	992730	992774	992819	992863	992907	992951	
4	992995	993039	993083	993127	993172	993216	993260	993304	993348	993392	
5	993436	993480	993524	993568	993613	993657	993701	993745	993789	993833	
6	993877	993921	993965	994009	994053	994097	994141	994185	994229	994273	
7	994317	994361	994405	994449	994493	994537	994581	994625	994669	994713	44
8	994757	994801	994845	994889	994933	994977	995021	995065	995108	995152	
9	995196	995240	995284	995328	995372	995416	995460	995504	995547	995591	
990	**995635**	**995679**	**995723**	**995767**	**995811**	**995854**	**995898**	**995942**	**995986**	**996030**	
1	996074	996117	996161	996205	996249	996293	996337	996380	996424	996468	
2	996512	996555	996599	996643	996687	996731	996774	996818	996862	996906	
3	996949	996993	997037	997080	997124	997168	997212	997255	997299	997343	
4	997386	997430	997474	997517	997561	997605	997648	997692	997736	997779	
5	997823	997867	997910	997954	997998	998041	998085	998129	998172	998216	
6	998259	998303	998347	998390	998434	998477	998521	998564	998608	998652	
7	998695	998739	998782	998826	998869	998913	998956	999000	999043	999087	
8	999131	999174	999218	999261	999305	999348	999392	999435	999479	999522	
9	999565	999609	999652	999696	999739	999783	999826	999870	999913	999957	
1000	**000000**	**000043**	**000087**	**000130**	**000174**	**000217**	**000250**	**000304**	**000347**	**000391**	43

Table 3. Natural (Napierian or Hyperbolic) Logarithms of Numbers

Table gives natural logarithms of numbers from 1.0 to 9.99 directly. To find logarithms of numbers outside that range add or subtract natural logarithm of the powers of 10 as follows:

$\log_e 10 = 2.302585$ $\log_e 10^4 = 9.210340$ $\log_e 10^7 = 16.118096$
$\log_e 10^2 = 4.605170$ $\log_e 10^5 = 11.512925$ $\log_e 10^8 = 18.420681$
$\log_e 10^3 = 6.907755$ $\log_e 10^6 = 13.815511$ $\log_e 10^9 = 20.703266$

EXAMPLE.—$\log_e 679. = \log_e 6.79 + \log_e 10^2 = 1.9155 + 4.6052 = 5.5207$
$\log_e .0679 = \log_e 6.79 - \log_e 10^2 = 1.9155 - 4.6052 = -2.6897$

The common logarithm is the natural logarithm multiplied by the modulus of $\log_{10}$; $\log_{10} N = 0.434294 \log_e N$.

N	0	1	2	3	4	5	6	7	8	9
1.0	**0.0000**	**0.0100**	**0.0198**	**0.0296**	**0.0392**	**0.0488**	**0.0583**	**0.0677**	**0.0770**	**0.0862**
1.1	0.0953	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1740
1.2	0.1823	0.1906	0.1989	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4383	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366
1.9	0.6419	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881
2.0	**0.6931**	**0.6981**	**0.7031**	**0.7080**	**0.7129**	**0.7178**	**0.7227**	**0.7275**	**0.7324**	**0.7372**
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7885	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	**1.0986**	**1.1019**	**1.1053**	**1.1086**	**1.1119**	**1.1151**	**1.1184**	**1.1217**	**1.1249**	**1.1282**
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838

Table 3. **Natural (Napierian or Hyperbolic) Logarithms of Numbers**—*Continued*

N	0	1	2	3	4	5	6	7	8	9
4.0	**1.3863**	**1.3888**	**1.3913**	**1.3938**	**1.3962**	**1.3987**	**1.4012**	**1.4036**	**1.4061**	**1.4085**
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	**1.6094**	**1.6114**	**1.6134**	**1.6154**	**1.6174**	**1.6194**	**1.6214**	**1.6233**	**1.6253**	**1.6273**
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029
5.5	1.7047	1.7066	1.7084	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901
6.0	**1.7918**	**1.7934**	**1.7951**	**1.7967**	**1.7984**	**1.8001**	**1.8017**	**1.8034**	**1.8050**	**1.8066**
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445
7.0	**1.9459**	**1.9473**	**1.9488**	**1.9502**	**1.9516**	**1.9530**	**1.9544**	**1.9559**	**1.9573**	**1.9587**
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656
7.9	2.0669	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782
8.0	**2.0794**	**2.0807**	**2.0819**	**2.0832**	**2.0844**	**2.0857**	**2.0869**	**2.0882**	**2.0894**	**2.0906**
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961
9.0	**2.1972**	**2.1983**	**2.1994**	**2.2006**	**2.2017**	**2.2028**	**2.2039**	**2.2050**	**2.2061**	**2.3072**
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016

Table 4. Values and Logarithms of Exponentials and Hyperbolic Functions

The following tables give values of e^x, e^{-x}, sinh x, cosh x and tanh x for values of x from 0.00 to 6.00 in intervals of 0.01.

To facilitate computations involving multiplication, the common logarithms of e^x, sinh x, cosh x, and tanh x are also given.

For values of x greater than 6: e^x may be computed from the relationship $e^x = \log^{-1}(x \log_{10} e) = \log^{-1} 0.43429x$; e^{-x} approaches zero; sinh x and cosh x are approximately equal and become $0.5\, e^x$; and tanh x and coth x have values approximately equal to unity.

Where more accurate values of the exponentials and functions are required they may be computed from the following relationships.

$$e = 2.71828\ 18285 \qquad \frac{1}{e} = 0.36787\ 94412$$

$$M = \log_{10} e = 0.43429\ 44819 \qquad \frac{1}{M} = \log_e 10 = 2.30258\ 50930$$

$$e^x = \log^{-1} Mx, \qquad e^{-x} = \log^{-1} -Mx$$

$$\text{Sinh } x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\text{csch } x = \frac{1}{\sinh x} \qquad \text{sech } x = \frac{1}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

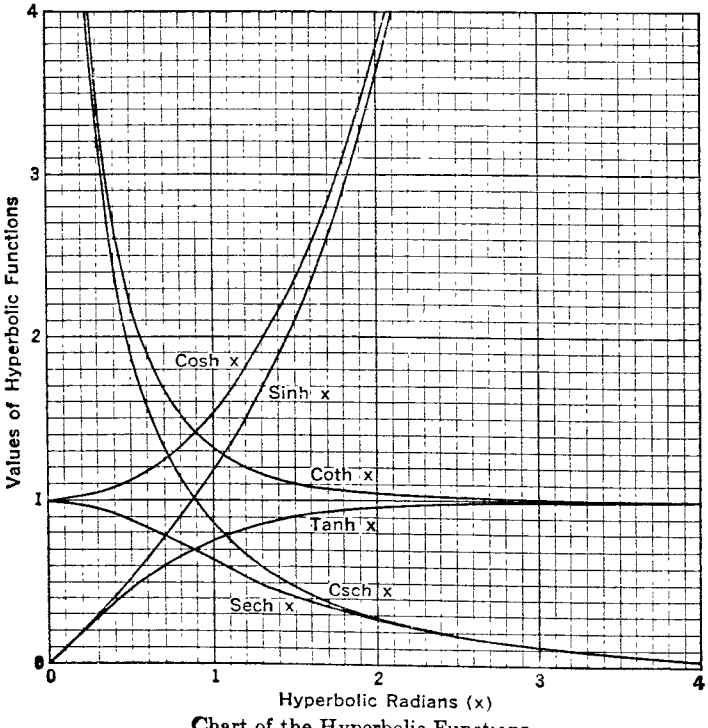

Chart of the Hyperbolic Functions.

Table 4. Values and Logarithms of Exponentials and Hyperbolic Functions—*Continued*

x	Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
0.00	1.0000	1.0000	0.0000	1.0000	.00000	0.00000	$-\infty$	0.00000	$-\infty$
0.10	1.1052	.90484	0.1002	1.0050	.09967	0.04343	$\bar{1}.00072$	0.00217	$\bar{2}.99856$
0.20	1.2214	.81873	0.2013	1.0201	.19738	0.08686	$\bar{1}.30392$	0.00863	$\bar{1}.29529$
0.30	1.3499	.74082	0.3045	1.0453	.29131	0.13029	1.48362	0.10926	$\bar{1}.46436$
0.40	1.4918	.67032	0.4108	1.0811	.37995	0.17372	$\bar{1}.61358$	0.03385	$\bar{1}.57973$
0.50	1.6487	.60653	0.5211	1.1276	.46212	0.21715	1.71692	0.05217	1.66475
0.60	1.8221	.54881	0.6367	1.1855	.53705	0.26058	$\bar{1}.80390$	0.07389	$\bar{1}.73001$
0.70	2.0138	.49659	0.7586	1.2552	.60437	0.30401	$\bar{1}.88000$	0.09870	$\bar{1}.78130$
0.80	2.2255	.44933	0.8881	1.3374	.66404	0.34744	$\bar{1}.94846$	0.12627	1.82219
0.90	2.4596	.40657	1.0265	1.4331	.71630	0.39087	0.01137	0.15627	$\bar{1}.85509$
1.00	2.7183	.36788	1.1752	1.5431	.76159	0.43439	0.07011	0.18839	$\bar{1}.88172$
1.10	3.0042	.33287	1.3356	1.6685	.80050	0.47772	0.12569	0.22233	$\bar{1}.90336$
1.20	3.3201	.30119	1.5095	1.8107	.83365	0.52115	0.17882	0.25784	$\bar{1}.92099$
1.30	3.6693	.27253	1.6984	1.9709	.86172	0.56458	0.23004	0.29467	$\bar{1}.93537$
1.40	4.0552	.24660	1.9043	2.1509	.88535	0.60801	0.27974	0.33262	$\bar{1}.94712$
1.50	4.4817	.22313	2.1293	2.3524	.90515	0.65144	0.32823	0.37151	$\bar{1}.95672$
1.60	4.9530	.20190	2.3756	2.5775	.92167	0.69487	0.37577	0.41119	$\bar{1}.96457$
1.70	5.4739	.18268	2.6456	2.8283	.93541	0.73830	0.42253	0.45153	$\bar{1}.97100$
1.80	6.0496	.16530	2.9422	3.1075	.94681	0.78173	0.46867	0.49241	$\bar{1}.97626$
1.90	6.6859	.14957	3.2682	3.4177	.95624	0.82516	0.51430	0.53374	$\bar{1}.98057$
2.00	7.3891	.13534	3.6269	3.7622	.96403	0.86859	0.55953	0.57544	$\bar{1}.98409$
2.10	8.1662	.12246	4.0219	4.1443	.97045	0.91202	0.60443	0.61745	$\bar{1}.98697$
2.20	9.0250	.11080	4.4571	4.5679	.97574	0.95545	0.64905	0.65972	$\bar{1}.98934$
2.30	9.9742	.10026	4.9370	5.0372	.98010	0.99888	0.69346	0.70219	$\bar{1}.99127$
2.40	11.023	.09072	5.4662	5.5569	.98367	1.04231	0.73769	0.74484	$\bar{1}.99285$
2.50	12.182	.08208	6.0502	6.1323	.98661	1.08574	0.78177	0.78762	$\bar{1}.99415$
2.60	13.464	.07427	6.6947	6.7690	.98903	1.12917	0.82573	0.83052	$\bar{1}.99521$
2.70	14.880	.06721	7.4063	7.4735	.99101	1.17260	0.86960	0.87352	$\bar{1}.99608$
2.80	16.445	.06081	8.1919	8.2527	.99263	1.21602	0.91339	0.91660	$\bar{1}.99679$
2.90	18.174	.05502	9.0596	9.1146	.99396	1.25945	0.95711	0.95974	$\bar{1}.99737$
3.00	20.086	.04979	10.018	10.068	.99505	1.30288	1.00078	1.00293	$\bar{1}.99785$
3.10	22.198	.04505	11.077	11.122	.99595	1.34631	1.04440	1.04616	$\bar{1}.99824$
3.20	24.533	.04076	12.246	12.287	.99668	1.38974	1.08799	1.08943	$\bar{1}.99856$
3.30	27.113	.03688	13.538	13.575	.99728	1.43317	1.13155	1.13273	$\bar{1}.99882$
3.40	29.964	.03337	14.965	14.999	.99777	1.47660	1.17509	1.17605	$\bar{1}.99903$
3.50	33.115	.03020	16.543	16.573	.99818	1.52003	1.21860	1.21940	$\bar{1}.99921$
3.60	36.598	.02732	18.285	18.313	.99851	1.56346	1.26211	1.26275	$\bar{1}.99935$
3.70	40.447	.02472	20.211	20.236	.99878	1.60689	1.30559	1.30612	$\bar{1}.99947$
3.80	44.701	.02237	22.339	22.362	.99900	1.65032	1.34907	1.34951	$\bar{1}.99957$
3.90	49.402	.02024	24.691	24.711	.99918	1.69375	1.39254	1.39290	$\bar{1}.99964$
4.00	54.598	.01832	27.290	27.308	.99933	1.73718	1.43600	1.43629	$\bar{1}.99971$
4.10	60.340	.01657	30.162	30.178	.99945	1.78061	1.47946	1.47970	$\bar{1}.99976$
4.20	66.686	.01500	33.336	33.351	.99955	1.82404	1.52291	1.52310	$\bar{1}.99980$
4.30	73.700	.01357	36.843	36.857	.99963	1.86747	1.56636	1.56652	$\bar{1}.99984$
4.40	81.451	.01228	40.719	40.732	.99970	1.91090	1.60980	1.60993	$\bar{1}.99987$
4.50	90.017	.01111	45.003	45.014	.99975	1.95433	1.65324	1.65335	$\bar{1}.99989$
4.60	99.484	.01005	49.737	49.747	.99980	1.99775	1.69668	1.69677	$\bar{1}.99991$
4.70	109.95	.00910	54.969	54.978	.99983	2.04118	1.74012	1.74019	$\bar{1}.99993$
4.80	121.51	.00823	60.751	60.759	.99986	2.08461	1.78355	1.78361	$\bar{1}.99994$
4.90	134.29	.00745	67.141	67.149	.99989	2.12804	1.82699	1.82704	$\bar{1}.99995$
5.00	148.41	.00674	74.203	74.210	.99991	2.17147	1.87042	1.87046	$\bar{1}.99996$
5.10	164.02	.00610	82.008	82.014	.99993	2.21490	1.91386	1.91389	$\bar{1}.99997$
5.20	181.27	.00552	90.633	90.639	.99994	2.25833	1.95729	1.95731	$\bar{1}.99997$
5.30	200.34	.00499	100.17	100.17	.99995	2.30176	2.00072	2.00074	$\bar{1}.99998$
5.40	221.41	.00452	110.70	110.71	.99996	2.34519	2.04415	2.04417	$\bar{1}.99998$
5.50	244.69	.00409	122.34	122.35	.99997	2.38862	2.08758	2.08760	$\bar{1}.99999$
5.60	270.43	.00370	135.21	135.22	.99997	2.43205	2.13101	2.13103	$\bar{1}.99999$
5.70	298.87	.00335	149.43	149.44	.99998	2.47548	2.17444	2.17445	$\bar{1}.99999$
5.80	330.30	.00303	165.15	165.15	.99998	2.51891	2.21787	2.21788	$\bar{1}.99999$
5.90	365.04	.00274	182.52	182.52	.99998	2.56234	2.26130	2.26131	$\bar{1}.99999$
6.00	403.43	.00248	201.71	201.72	.99999	2.60577	2.30473	2.30474	$\bar{1}.99999$

2. PROPERTIES OF NUMBERS

Table 5. Certain Constants Containing e and π

$e = 2.7182818285$ $\quad M = \log_{10} e = 0.4342944819$

$\pi = 3.1415926536$ $\quad M^{-1} = \log_e 10 = 2.3025850930$

Powers of e			Multiples of π			Fractions of π		
e^n	Value	Logarithm	$n\pi$	Value	Logarithm	π/n	Value	Logarithm
e	2.718282	0.434294	π	3.141593	0.497150	$\pi/2$	1.570780	0.196120
e^{-1}	0.367879	$\bar{1}.565706$	2π	6.283185	0.798180	$\pi/3$	1.047198	0.020029
e^2	7.389057	0.868589	3π	9.424778	0.974271	$\pi/4$	0.785398	$\bar{1}.895090$
e^{-2}	0.135335	$\bar{1}.131411$	4π	12.566371	1.099210	$\pi/180$	0.017453*	$\bar{2}.241877$
$e^{1/2}$	1.648721	0.217147	5π	15.707963	1.196120			

Reciprocals of π			Powers of π			Roots of π		
n/π	Value	Logarithm	$\pi^{\pm n}$	Value	Logarithm	$\pi^{\pm 1/n}$	Value	Logarithm
$1/\pi$	0.318310	$\bar{1}.502850$	π^2	9.869604	0.994300	$\sqrt{\pi}$	1.772454	0.248575
$2/\pi$	0.636620	$\bar{1}.803880$	$1/\pi^2$	0.101321	$\bar{1}.005700$	$1/\sqrt{\pi}$	0.564190	$\bar{1}.751425$
$3/\pi$	0.954930	$\bar{1}.979971$	π^3	31.006277	1.491450	$\sqrt[3]{\pi}$	1.464592	0.165717
$180/\pi$	57.295780†	1.758123	$1/\pi^3$	0.032252	$\bar{2}.508550$	$1/\sqrt[3]{\pi}$	0.681784	$\bar{1}.834283$

* Number of radians per degree. † Number of degrees per radian.

Table 6. Factorials

n	$n! = 1\cdot 2\cdot 3 \ldots n$	$1/n!$	n	$n! = 1\cdot 2\cdot 3 \ldots n$	$1/n!$
1	1	1.	11	$399{,}168 \times 10^2$	0.250521×10^{-7}
2	2	0.5	12	$479{,}002 \times 10^3$	$.208768 \times 10^{-8}$
3	6	.166667	13	$622{,}702 \times 10^4$	$.160590 \times 10^{-9}$
4	24	$.416667 \times 10^{-1}$	14	$871{,}783 \times 10^5$	$.114707 \times 10^{-10}$
5	120	$.833333 \times 10^{-2}$	15	$130{,}767 \times 10^7$	$.764716 \times 10^{-12}$
6	720	$.138889 \times 10^{-2}$	16	$209{,}228 \times 10^8$	$.477948 \times 10^{-13}$
7	5,040	$.198413 \times 10^{-3}$	17	$355{,}687 \times 10^9$	$.281146 \times 10^{-14}$
8	40,320	$.248016 \times 10^{-4}$	18	$640{,}237 \times 10^{10}$	$.156192 \times 10^{-15}$
9	362,880	$.275573 \times 10^{-5}$	19	$121{,}645 \times 10^{12}$	$.822064 \times 10^{-17}$
10	3,628,800	$.275573 \times 10^{-6}$	20	$243{,}290 \times 10^{13}$	$.411032 \times 10^{-19}$

Table 7. Properties of Numbers

Decimal Equivalents, Squares, Cubes, Square Roots, Cube Roots, Three-Halves Powers, Fifth Roots, Reciprocals, Circumference and Area of Circles

Number, N		N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N=Diam.)	
Fraction	Decimal								Circum.	Area
1/64	.015625	0.000244	$.381\times10^{-5}$	.1250	.2500	00195	.4353	64 0	0.04909	00019
1/32	.03125	.000977	$.305\times10^{-4}$	.1768	.3150	00552	.5000	32.0	.09818	.00077
3/64	.046875	.002197	103×10^{-3}	.2165	.3606	01015	.5422	21.3333	.14726	00173
1/16	**.0625**	**.003906**	**244×10^{-3}**	**.2500**	**.3969**	**01563**	**.5744**	**16.0**	**.19635**	**.00307**
5/64	078125	.006104	477×10^{-3}	2795	.4275	02184	6006	12.80	.24544	.00479
3/32	.09375	.008789	824×10^{-3}	.3062	.4543	02871	.6229	10.6667	.29452	.00690
	.10	.010	00100	.3162	.4642	.03162	.6310	10.0	.31416	.00785
7/64	.109375	.01196	001308	.3307	4782	.03617	6424	9.1429	.34361	.00939
1/8	**.125**	**.01563**	**001953**	**.3536**	**.5000**	**04419**	**6598**	**8.0**	**.39270**	**.01227**
9/64	.140625	.01978	002782	.3750	.5200	05273	6755	7.1111	.44179	01554
5/32	.15625	.02441	.003814	.3953	.5386	06176	.6899	6.40	.49087	01917
1/64	.171875	.02954	005077	4146	5560	07126	.7031	5.8182	.53996	.02320
3/16	**.1875**	**.03516**	**.006592**	**4330**	**.5724**	**.08119**	**.7155**	**5.3333**	**.58905**	**02761**
	.20	.040	.0080	.4472	.5848	.08944	.7248	5.0	.62832	.03142
13/64	.203125	.04126	.008381	.4507	5878	09155	.7270	4.9231	.63814	.03241
7/32	.21875	.04785	.01047	.4677	6025	.10231	7379	4.5714	.68722	03758
15/64	.234375	.05493	.01287	.4841	6166	.11347	.7481	4.2667	.73631	.04314
1/4	**.250**	**.0625**	**01563**	**5000**	**6300**	**12500**	**7579**	**4.0**	**.78540**	**.04909**
17/64	.265625	.07056	01874	5154	.6428	13690	.7671	3.7647	.83448	05542
9/32	.28125	.07910	.02225	5303	.6552	.14916	7759	3.5556	.88357	.06213
19/64	.296875	.08813	.02616	.5449	.6671	.16176	.7844	3.3684	.93266	.06922
	.30	.090	.0270	.5477	.6694	.16432	.7860	3.3333	.94248	.07069
5/16	**.3125**	**.09766**	**.03052**	**5590**	**6786**	**17469**	**7925**	**3.2000**	**.98175**	**07670**
21/64	328125	.10767	.03533	5728	6897	.18796	.8002	3.0476	1.0308	08456
11/32	34375	.11816	.04062	5863	.7005	.20154	8077	2.9091	1.0799	09281
23/64	359375	.12915	04641	5995	.7110	21544	.8149	2.7826	1.1290	.10143
3/8	**375**	**.14063**	**.05273**	**6124**	**7211**	**22964**	**.8219**	**2.6667**	**1.1781**	**.11045**
25/64	.390625	.15259	.05961	.6250	.7310	.24414	.8286	2.5600	1.2272	.11984
	.40	.16	.0640	.6325	.7368	25298	8326	2.50	1.2566	.12566
13/32	.40625	.16504	06705	.6374	.7406	25894	8351	2.4615	1.2763	.12962
27/64	421875	.17798	07508	.6495	.7500	27402	.8415	2.3704	1.3254	.13979
7/16	**4375**	**.19141**	**.08374**	**.6614**	**.7592**	**28938**	**.8476**	**2.2857**	**1.3744**	**.15033**
29/64	453125	.20532	09304	.6732	.7681	30502	8536	2.2069	1.4235	16126
15/32	46875	.21973	10300	.6847	7768	32093	.8594	2.1333	1.4726	.17257
31/64	.484375	.23462	.11364	.6960	.7854	.33711	.8650	2.0645	1.5217	.18427
1/2	**50**	**.2500**	**.12500**	**.7071**	**.7937**	**.35355**	**.8706**	**2.0**	**1.5708**	**.19635**
33/64	.515625	.26587	.13709	.7181	8019	.37025	8759	1.9394	1.6199	20881
17/32	53125	.28223	14993	.7289	8099	.38721	.8812	1.8824	1.6690	.22166
35/64	546875	.29907	.16355	.7395	.8178	.40442	.8863	1.8286	1.7181	.23489
9/16	**.5625**	**.31641**	**.17798**	**.7500**	**8255**	**42188**	**8913**	**1.7778**	**1.7671**	**.24850**
37/64	578125	.33423	.19323	7604	.8331	43957	8962	1.7297	1.8162	26250
19/32	.59375	.35254	.20932	.7706	.8405	.45751	.9010	1.6842	1.8653	.27688
	.60	.3600	.21600	.7746	.8434	.46476	.9029	1.6667	1.8850	.28274
39/64	609375	.37134	22628	.7806	.8478	47569	.9057	1.6410	1.9144	.29165
5/8	**625**	**.39063**	**.24414**	**7906**	**.8550**	**.49410**	**.9103**	**1.6000**	**1.9635**	**.30680**
41/64	640625	.41040	26291	8004	8621	.51275	.9148	1.5610	2.0126	.32233
21/32	.65625	.43066	.28262	.8101	8690	.53162	9192	1.5238	2.0617	.33824
43/64	671875	.45142	30330	8197	8759	.55072	.9235	1.4884	2.1108	.35454
11/16	**.6875**	**.47266**	**.32495**	**8297**	**.8826**	**.57005**	**9278**	**1.4545**	**2.1598**	**.37122**
	.70	.4900	.34300	8367	.8879	58566	9312	1.4286	2.1991	.38485
45/64	703125	.49438	34761	8385	.8892	58959	9320	1.4222	2.2089	38829
23/32	71875	.51660	.37131	.8478	.8958	60935	9361	1.3913	2 2580	.40574
47/64	734375	.53931	.39605	8570	9022	62933	.9401	1.3617	2 3071	.42357
3/4	**750**	**.56250**	**42188**	**.8660**	**.9086**	**.64952**	**.9441**	**1.3333**	**2 3562**	**.44179**
49/64	765625	.58618	44879	8750	9148	66992	9480	1.3061	2 4053	.46038
25/32	.78125	.61035	.47684	.8839	9210	.69053	9518	1.2800	2.4544	.47937
51/64	.796875	.63501	.50602	.8927	9271	.71135	.9556	1.2549	2.5035	.49874
	80	.6400	51200	.8944	.9283	.71554	9564	1.2500	2.5133	.50265
13/16	**8125**	**.66016**	**.53638**	**.9014**	**.9331**	**73238**	**9593**	**1.2308**	**2.5525**	**.51849**
53/64	828125	.68579	56792	9100	.9391	.75361	9630	1 2075	2.6016	.53862
27/32	84375	.71191	60067	.9186	.9449	.77503	9666	1 1852	2.6507	.55914
55/64	859375	.73853	63467	9270	9507	.79666	9702	1.1636	2 6998	.58004
7/8	**875**	**.76563**	**66992**	**9354**	**9565**	**81849**	**9737**	**1.1429**	**2.7489**	**.60132**
57/64	.890625	.79321	.70645	.9437	.9621	.84051	.9771	1.1228	2.7980	.62299
	90	.81000	.72900	.9487	.9655	.85435	9792	1.1111	2.8274	.63617
29/32	90625	.82129	.74429	.9520	.9677	.86272	.9805	1.1034	2.8471	.64504
59/64	921875	.84985	.78346	9601	.9733	88513	9839	1.0847	2 8962	.66747
15/16	**9375**	**.87891**	**.82398**	**9683**	**9787**	**90773**	**9872**	**1.0667**	**2.9452**	**.69029**
61/64	953125	.90845	.86587	.9763	9841	93053	9905	1.0492	2.9943	.71349
31/32	96875	.93848	.90915	9843	.9895	95349	9937	1.0323	3.0434	.73708
63/64	.984375	.96899	.95385	.9922	9948	97666	.9969	1.0159	3.0925	.76104

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
1.	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000000**	**3.1416**	**0.7854**
1.125	1.2656	1.4238	1.0606	1.0400	1.1932	1.0238	.8888888	3.5343	.9940
1.25	1.5625	1.9531	1.1180	1.0772	1.3975	1.0456	.80000000	3.9270	1.2272
1.375	1.8906	2.5996	1.1726	1.1120	1.6123	1.0658	.72727272	4.3197	1.4849
1.5	2.25	3.3750	1.2247	1.1447	1.8371	1.0845	.66666666	4.7124	1.7671
1.625	2.6406	4.2910	1.2748	1.1757	2.0715	1.1020	.61538462	5.1051	2.0739
1.75	3.0625	5.3594	1.3229	1.2051	2.3150	1.1186	.57142857	5.4978	2.4053
1.875	3.5156	6.5918	1.3693	1.2331	2.5675	1.1340	.53333333	5.8905	2.7612
2.	**4.0000**	**8.0000**	**1.4142**	**1.2599**	**2.8284**	**1.1487**	**.50000000**	**6.2832**	**3.1416**
2.125	4.5156	9.5957	1.4577	1.2856	3.0977	1.1627	.47058823	6.6759	3.5466
2.25	5.0625	11.3906	1.5000	1.3104	3.3750	1.1761	.44444444	7.0686	3.9761
2.375	5.6406	13.3965	1.5411	1.3342	3.6601	1.1889	.42105263	7.4613	4.4301
2.5	6.2500	15.6250	1.5811	1.3572	3.9529	1.2011	.40000000	7.8540	4.9087
2.625	6.8906	18.0879	1.6202	1.3795	4.2530	1.2129	.38095231	8.2467	5.4119
2.75	7.5625	20.7969	1.6583	1.4011	4.5604	1.2242	.36363636	8.6394	5.9396
2.875	8.2656	23.7637	1.6956	1.4219	4.8748	1.2352	.34782609	9.0321	6.4918
3.	**9.0000**	**27.0000**	**1.7321**	**1.4422**	**5.1962**	**1.2457**	**.33333333**	**9.4248**	**7.0686**
3.125	9.7656	30.5176	1.7678	1.4620	5.5243	1.2559	.32000000	9.8175	7.6699
3.25	10.5625	34.3281	1.8028	1.4813	5.8590	1.2658	.30769231	10.2102	8.2958
3.375	11.3906	38.4434	1.8371	1.5000	6.2003	1.2754	.29629629	10.6029	8.9462
3.5	12.2500	42.8750	1.8708	1.5183	6.5479	1.2847	.28571429	10.9956	9.6211
3.625	13.1406	47.6348	1.9039	1.5362	6.9018	1.2938	.27586207	11.3883	10.3206
3.75	14.0625	52.7344	1.9365	1.5536	7.2619	1.3026	.26666666	11.7810	11.0447
3.875	15.0156	58.1856	1.9685	1.5707	7.6279	1.3112	.25806452	12.1737	11.7932
4.	**16.0000**	**64.0000**	**2.0000**	**1.5874**	**8.0000**	**1.3195**	**.25000000**	**12.5664**	**12.5664**
4.125	17.0156	70.1895	2.0310	1.6038	8.3779	1.3277	.24242424	12.9591	13.3640
4.25	18.0625	76.7656	2.0616	1.6198	8.7616	1.3356	.23529412	13.3518	14.1863
4.375	19.1406	83.7402	2.0916	1.6355	9.1510	1.3434	.22857143	13.7445	15.0330
4.5	20.2500	91.1250	2.1213	1.6510	9.5460	1.3510	.22222222	14.1372	15.9043
4.625	21.3906	98.9317	2.1506	1.6661	9.9465	1.3584	.21621622	14.5299	16.8001
4.75	22.5625	107.1719	2.1795	1.6810	10.3524	1.3656	.21052632	14.9226	17.7205
4.875	23.7656	115.8574	2.2079	1.6956	10.7637	1.3728	.20512821	15.3153	18.6655
5	**25.0000**	**125.0000**	**2.2361**	**1.7100**	**11.1803**	**1.3799**	**.20000000**	**15.7080**	**19.6350**
5.125	26.2656	134.6113	2.2638	1.7241	11.6022	1.3866	.19512195	16.1006	20.6289
5.25	27.5625	144.7031	2.2913	1.7380	12.0293	1.3933	.19047619	16.4933	21.6475
5.375	28.8906	155.2871	2.3184	1.7517	12.4614	1.3998	.18604651	16.8860	22.6906
5.5	30.2500	166.3750	2.3452	1.7652	12.8987	1.4063	.18181818	17.2787	23.7583
5.625	31.6406	177.9785	2.3727	1.7784	13.3409	1.4126	.17777777	17.6714	24.8505
5.75	33.0625	190.1094	2.3979	1.7915	13.7880	1.4188	.17391304	18.0641	25.9672
5.875	34.5156	202.7793	2.4238	1.8044	14.2400	1.4250	.17021277	18.4568	27.1085
6.	**36.0000**	**216.0000**	**2.4495**	**1.8171**	**14.6969**	**1.4310**	**.16666666**	**18.8495**	**28.2743**
6.125	37.5156	229.7832	2.4749	1.8297	15.1586	1.4369	.16326531	19.2422	29.4647
6.25	39.0625	244.1406	2.5000	1.8420	15.6250	1.4427	.16000000	19.6349	30.6796
6.375	40.6406	259.0840	2.5249	1.8542	16.0961	1.4484	.15686275	20.0276	31.9190
6.5	42.2500	274.6250	2.5495	1.8663	16.5718	1.4542	.15384615	20.4203	33.1831
6.625	43.8906	290.7754	2.5739	1.8781	17.0522	1.4596	.15094339	20.8130	34.4716
6.75	45.5625	307.5469	2.5981	1.8899	17.5370	1.4651	.14814815	21.2057	35.7847
6.875	47.2656	324.9512	2.6220	1.9015	18.0264	1.4705	.14545454	21.5984	37.1223
7.	**49.0000**	**343.0000**	**2.6458**	**1.9129**	**18.5203**	**1.4758**	**.14285714**	**21.9911**	**38.4845**
7.125	50.7656	361.7051	2.6693	1.9243	19.0186	1.4810	.14035088	22.3838	39.8712
7.25	52.5625	381.0781	2.6926	1.9354	19.5212	1.4862	.13793103	22.7765	41.2825
7.375	54.3906	401.1309	2.7157	1.9465	20.0283	1.4913	.13559322	23.1692	42.7183
7.5	56.2500	421.8750	2.7386	1.9574	20.5396	1.4963	.13333333	23.5619	44.1786
7.625	58.1406	443.3223	2.7613	1.9683	21.0552	1.5012	.13114754	23.9546	45.6635
7.75	60.0625	465.4844	2.7839	1.9789	21.5751	1.5061	.12903226	24.3473	47.1730
7.875	62.0156	488.3731	2.8063	1.9895	22.0992	1.5110	.12698413	24.7400	48.7069
8.	**64.0000**	**512.0000**	**2.8284**	**2.0000**	**22.6274**	**1.5157**	**.12500000**	**25.1327**	**50.2655**
8.125	66.0156	536.3770	2.8504	2.0104	23.1598	1.5204	.12307692	25.5254	51.8485
8.25	68.0625	561.5156	2.8723	2.0206	23.6963	1.5251	.12121212	25.9181	53.4562
8.375	70.1406	587.4278	2.8940	2.0308	24.2369	1.5297	.11940298	26.3108	55.0883
8.5	72.2500	614.1250	2.9155	2.0408	24.7816	1.5342	.11764706	26.7035	56.7450
8.625	74.3906	641.6192	2.9368	2.0508	25.3301	1.5387	.11594203	27.0962	58.4262
8.75	76.5625	669.9219	2.9580	2.0606	25.8828	1.5431	.11428571	27.4889	60.1320
8.875	78.7656	699.0450	2.9791	2.0704	26.4394	1.5475	.11267605	27.8816	61.8623
9.	**81.0000**	**729.0000**	**3.0000**	**2.0801**	**27.0000**	**1.5518**	**.11111111**	**28.2743**	**63.6172**
9.125	83.2656	759.7989	3.0207	2.0897	27.5645	1.5561	.10958904	28.6670	65.3966
9.25	85.5625	791.4531	3.0414	2.0992	28.1328	1.5604	.10810811	29.0597	67.2006
9.375	87.8906	823.9746	3.0619	2.1086	28.7050	1.5646	.10666666	29.4524	69.0291
9.5	90.2500	857.3750	3.0822	2.1179	29.2810	1.5687	.10526316	29.8451	70.8822
9.625	92.6406	891.6660	3.1024	2.1272	29.8608	1.5728	.10389610	30.2378	72.7597
9.75	95.0625	926.8594	3.1225	2.1363	30.4444	1.5769	.10256410	30.6305	74.6619
9.875	97.5156	962.9668	3.1425	2.1454	31.0317	1.5809	.10126582	31.0232	76.5886

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam) Circum	Area
10	**100**	**1000**	**3.1623**	**2 1544**	**31 623**	**1 5849**	**10000000**	**31.4159**	**78.5398**
11	121	1331	3.3166	2 2240	36 483	1 6154	.09090909	34 5575	95.0332
12	144	1728	3 4641	2 2894	41 569	1 6438	08333333	37 6991	113 0973
13	169	2197	3 6056	2 3513	46.873	1 6703	07692308	40 8407	132.7323
14	196	2744	3 7417	2 4101	52 384	1 6953	07142857	43.9823	153 9380
15	225	3375	3.8730	2 4662	58 095	1 7188	06666667	47.1239	176 7146
16	256	4096	4 0000	2.5198	64 000	1 7411	06250000	50.2654	201.0619
17	289	4913	4 1231	2.5713	70 093	1.7623	05882353	53 4070	226 9801
18	324	5832	4 2426	2 6207	76 367	1.7826	05555556	56.5486	254.4690
19	361	6859	4.3589	2 6684	82 819	1.8020	05263158	59.6902	283.5287
20	**400**	**8000**	**4.4721**	**2.7144**	**89 442**	**1 8206**	**05000000**	**62.8318**	**314.1593**
21	441	9261	4 5826	2.7589	96 235	1.8384	04761905	65 9734	346 3606
22	484	10648	4.6904	2.8020	103 19	1.8556	04545455	69 1150	380.1327
23	529	12167	4.7958	2.8439	110 30	1.8722	04347826	72 2566	415.4756
24	576	13824	4 8990	2 8845	117 58	1 8882	04166667	75 3982	452 3893
25	625	15625	5 0000	2 9240	125 00	1 9037	04000000	78 5398	490 8739
26	676	17576	5.0990	2.9625	132 57	1.9186	03846154	81.6813	530.9292
27	729	19683	5.1962	3.0000	140 30	1 9332	03703704	84 8229	572 5553
28	784	21952	5.2915	3.0366	148 16	1 9473	03571429	87 9645	615 7522
29	841	24389	5.3852	3.0723	156.17	1.9610	03448276	91.1061	660.5198
30	**900**	**27000**	**5.4772**	**3 1072**	**164.32**	**1.9744**	**03333333**	**94 2477**	**706.8583**
31	961	29791	5.5678	3.1414	172.60	1.9873	03225806	97 3893	754 7676
32	1024	32768	5.6569	3 1748	181.02	2.0000	03125000	100 5309	804 2477
33	1089	35937	5.7446	3 2075	189.57	2 0123	03030303	103.6725	855 2986
34	1156	39304	5.8310	3.2396	198.25	2 0244	02941176	106.8141	907 9203
35	1225	42875	5 9161	3.2711	207 06	2 0362	02857143	109 9557	962 1127
36	1296	46656	6.0000	3.3019	216.00	2 0477	.02777778	113.0972	1017.8760
37	1369	50653	6.0828	3 3322	225 06	2 0589	02702703	116.2388	1075 2101
38	1444	54872	6.1644	3 3620	234 25	2 0699	02631579	119 3804	1134 1149
39	1521	59319	6.2450	3.3912	243.56	2 0807	02564103	122 5220	1194.5906
40	**1600**	**64000**	**6.3246**	**3 4200**	**252 98**	**2.0913**	**02500000**	**125.6636**	**1256 6371**
41	1681	68921	6.4031	3.4482	262.53	2 1016	02439024	128 8052	1320 2543
42	1764	74088	6.4807	3.4760	272.19	2 1118	02380952	131 9468	1385 4424
43	1849	79507	6.5574	3.5034	281.97	2.1218	02325581	135.0884	1452.2012
44	1936	85184	6.6332	3 5303	291 86	2 1315	02272727	138.2300	1520 5308
45	2025	91125	6 7082	3.5569	301 87	2 1411	02222222	141 3716	1590 4313
46	2116	97336	6.7823	3.5830	311.99	2.1506	02173913	144.5131	1661.9025
47	2209	103823	6.8557	3 6088	322 22	2 1598	02127660	147 6547	1734 9445
48	2304	110592	6.9282	3.6342	332 55	2 1689	.02083333	150.7963	1809 5574
49	2401	117649	7.0000	3.6593	343 00	2.1779	02040816	153.9379	1885 7410
50	**2500**	**125000**	**7.0711**	**3.6840**	**353.55**	**2 1867**	**02000000**	**157.0795**	**1963.500**
51	2601	132651	7.1414	3.7084	364 21	2.1954	01960784	160.2211	2042.820
52	2704	140608	7.2111	3.7325	374 98	2 2039	01923077	163 3627	2123 716
53	2809	148877	7.2801	3.7563	385.85	2.2124	01886792	166.5043	2206.183
54	2916	157464	7 3485	3 7798	396 82	2 2206	01851852	169 6459	2290 221
55	3025	166375	7.4162	3.8030	407.89	2 2288	01818182	172.7875	2375 829
56	3136	175616	7.4833	3.8259	419.07	2 2369	01785714	175.9290	2463 008
57	3249	185193	7 5498	3.8485	430 35	2.2448	01754386	179 0706	2551.758
58	3364	195112	7 6158	3.8709	441 72	2 2526	01724138	182.2122	2642 079
59	3481	205379	7.6811	3.8930	453.19	2.2603	01694915	185.3538	2733.970
60	**3600**	**216000**	**7.7460**	**3.9149**	**464.76**	**2 2679**	**.01666667**	**188.4954**	**2827.433**
61	3721	226981	7 8102	3 9365	476 43	2 2755	01639344	191 6370	2922.466
62	3844	238328	7 8740	3 9579	488 19	2 2829	.01612903	194.7786	3019.070
63	3969	250047	7 9373	3.9791	500.05	2.2902	01587302	197 9202	3117.245
64	4096	262144	8 0000	4 0000	512 00	2 2974	01562500	201 0618	3216.990
65	4225	274625	8 0623	4.0207	524 05	2 3045	01538462	204 2034	3318.307
66	4356	287496	8.1240	4.0412	536 19	2.3116	01515152	207 3449	3421.194
67	4489	300763	8.1854	4 0615	548 42	2 3186	01492537	210 4865	3525 652
68	4624	314432	8 2462	4 0817	560 74	2 3254	01470588	213 6281	3631 680
69	4761	328509	8.3066	4.1016	573.16	2.3322	01449275	216 7697	3739.280
70	**4900**	**343000**	**8.3666**	**4.1213**	**585.66**	**2 3389**	**01428571**	**219 9113**	**3848.450**
71	5041	357911	8 4261	4 1408	598 26	2 3456	01408451	223 0529	3959.191
72	5184	373248	8 4853	4.1602	610 94	2 3522	.01388889	226 1945	4071.503
73	5329	389017	8.5440	4.1793	623.71	2.3587	.01369863	229.3361	4185.386
74	5476	405224	8 6023	4.1983	636 57	2 3651	01351351	232 4777	4300.839
75	5625	421875	8 6603	4.2172	649 52	2 3714	01333333	235 6193	4417 864
76	5776	438976	8 7178	4.2358	662.55	2.3777	01315789	238 7608	4536.459
77	5929	456533	8 7750	4 2543	675 68	2 3840	01298701	241.9024	4656 625
78	6084	474552	8 8318	4.2727	688 88	2 3901	.01282051	245 0440	4778.361
79	6241	493039	8.8882	4.2908	702.17	2.3962	.01265823	248 1856	4901.669

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Circle (N = Diam.) Area
80	**6400**	**512000**	**8.9443**	**4.3089**	**715 54**	**2.4022**	**.01250000**	**251.327**	**5026.547**
81	6561	531441	9.0000	4 3267	729.00	2.4082	.01234568	254.469	5152.998
82	6724	551368	9.0554	4 3445	742 54	2 4141	.01219512	257 610	5281.016
83	6889	571787	9.1104	4 3621	756 17	2.4200	.01204819	260.752	5410.607
84	7056	592704	9.1652	4.3795	769 88	2.4258	.01190476	263 894	5541.770
85	7225	614125	9 2195	4 3968	783 66	2 4315	01176471	267 035	5674 501
86	7396	636056	9.2736	4.4140	797.53	2.4372	.01162791	270.177	5808.805
87	7569	658503	9.3274	4.4310	811 49	2 4429	01149425	273.318	5944.679
88	7744	681472	9.3808	4.4480	825.52	2.4485	.01136364	276 460	6082.124
89	7921	704969	9.4340	4.4647	839.63	2.4540	01123596	279.602	6221.138
90	**8100**	**729000**	**9.4868**	**4.4814**	**853 82**	**2.4595**	**.01111111**	**282.743**	**6361.725**
91	8281	753571	9.5394	4 4979	868.09	2.4650	.01098901	285.885	6503.882
92	8464	778688	9.5917	4 5144	882.44	2.4705	.01086957	289 026	6647.610
93	8649	804357	9.6437	4.5307	896.86	2.4758	.01075269	292.168	6792.909
94	8836	830584	9.6954	4.5468	911 36	2.4810	.01063830	295.309	6939.778
95	9025	857375	9 7468	4.5629	925.95	2.4863	.01052632	298.451	7088.219
96	9216	884736	9.7980	4.5789	940.61	2.4915	.01041667	301.593	7238.230
97	9409	912673	9.8489	4 5947	955.34	2 4966	.01030928	304.734	7389.812
98	9604	941192	9.8995	4.6104	970.15	2.5018	.01020408	307.876	7542.962
99	9801	970299	9.9499	4.6261	985.04	2.5069	.01010101	311.017	7697.688
100	**10000**	**1000000**	**10.0000**	**4.6416**	**1000.0**	**2.5119**	**.01000000**	**314.159**	**7853.982**
101	10201	1030301	10 0499	4.6570	1015 0	2 5169	00990099	317.301	8011.85
102	10404	1061208	10.0995	4.6723	1030 1	2 5219	00980392	320.442	8171.28
103	10609	1092727	10.1489	4.6875	1045.3	2.5268	.00970874	323.584	8332.29
104	10816	1124864	10.1980	4.7027	1060.6	2.5317	00961538	326.725	8494.87
105	11025	1157625	10.2470	4 7177	1075 9	2.5365	.00952381	329.867	8659.01
106	11236	1191016	10.2956	4.7326	1091.3	2.5413	00943396	333.009	8824.73
107	11449	1225043	10.3441	4.7475	1106 8	2.5461	.00934579	336.150	8992.02
108	11664	1259712	10.3923	4.7622	1122.4	2.5509	.00925926	339.292	9160.88
109	11881	1295029	10.4403	4.7769	1138.0	2.5556	.00917431	342.433	9331.32
110	**12100**	**1331000**	**10.4881**	**4.7914**	**1153.7**	**2.5602**	**.00909091**	**345 575**	**9503.32**
111	12321	1367631	10.5357	4 8059	1169.5	2.5649	.00900901	348.716	9676.89
112	12544	1404928	10.5830	4.8203	1185.3	2.5695	.00892857	351.858	9852.03
113	12769	1442897	10.6301	4.8346	1201.2	2.5740	00884956	355.000	10028.75
114	12996	1481544	10 6771	4.8488	1217.2	2.5786	.00877193	358.141	10207.03
115	13225	1520875	10 7238	4.8629	1233 2	2 5831	.00869565	361.283	10386 89
116	13456	1560896	10.7703	4.8770	1249.4	2.5876	.00862069	364.424	10568.32
117	13689	1601613	10 8167	4.8910	1265 5	2 5920	00854701	367.566	10751.31
118	13924	1643032	10.8628	4 9049	1281.8	2.5964	.00847458	370.708	10935.88
119	14161	1685159	10.9087	4.9187	1298 1	2.6008	.00840336	373 849	11122.02
120	**14400**	**1728000**	**10.9545**	**4.9324**	**1314.5**	**2.6052**	**.00833333**	**376.991**	**11309.73**
121	14641	1771561	11.0000	4.9461	1331.0	2.6095	.00826446	380.132	11499.01
122	14884	1815848	11.0454	4.9597	1347 5	2.6138	.00819672	383.274	11689.86
123	15129	1860867	11.0905	4.9732	1364.1	2.6181	.00813008	386.416	11882.29
124	15376	1906624	11.1355	4.9866	1380.8	2.6223	.00806452	389.557	12076.28
125	15625	1953125	11.1803	5 0000	1397.5	2.6265	.00800000	392 699	12271.84
126	15876	2000376	11.2250	5.0133	1414.4	2.6307	.00793651	395.840	12468.98
127	16129	2048383	11.2694	5 0265	1431.2	2.6349	.00787402	398.982	12667.68
128	16384	2097152	11.3137	5.0397	1448 2	2.6390	.00781250	402.124	12867.96
129	16641	2146689	11.3578	5 0528	1465 2	2.6431	.00775194	405.265	13069.81
130	**16900**	**2197000**	**11.4018**	**5.0658**	**1482.2**	**2.6472**	**.00769231**	**408.407**	**13273.23**
131	17161	2248091	11.4455	5 0788	1499.4	2.6513	.00763359	411.548	13478.22
132	17424	2299968	11.4891	5.0916	1516 6	2.6553	.00757576	414.690	13684.77
133	17689	2352637	11.5326	5.1045	1533.8	2.6593	.00751880	417.831	13892.91
134	17956	2406104	11.5758	5.1172	1551.2	2.6633	.00746269	420 973	14102.61
135	18225	2460375	11 6190	5.1299	1568 6	2 6673	.00740741	424 115	14313.88
136	18496	2515456	11.6619	5.1426	1586 0	2 6712	.00735294	427 256	14526.72
137	18769	2571353	11.7047	5.1551	1603 6	2.6751	.00729927	430.398	14741.14
138	19044	2628072	11.7473	5.1676	1621 1	2 6790	.00724638	433 539	14957.12
139	19321	2685619	11.7898	5.1801	1638.8	2.6829	.00719424	436 681	15174.67
140	**19600**	**2744000**	**11.8322**	**5.1925**	**1656.5**	**2.6867**	**.00714286**	**439.823**	**15393.80**
141	19881	2803221	11.8743	5.2048	1674.3	2 6906	.00709220	442.964	15614.50
142	20164	2863288	11.9164	5.2171	1692 1	2.6944	.00704225	446.106	15836.77
143	20449	2924207	11.9583	5.2293	1710.0	2.6981	.00699301	449.247	16060.60
144	20736	2985984	12.0000	5.2415	1728 0	2.7019	.00694444	452.389	16286.01
145	21025	3048625	12.0416	5.2536	1746 0	2.7057	.00689655	455.531	16512.99
146	21316	3112136	12.0830	5.2656	1764.1	2.7094	.00684932	458.672	16741.54
147	21609	3176523	12.1244	5.2776	1782.2	2.7131	.00680272	461.814	16971.67
148	21904	3241792	12.1655	5.2896	1800 5	2 7168	.00675676	464.955	17203.36
149	22201	3307949	12.2066	5.3015	1818.8	2.7204	.00671141	468.097	17436.62

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam) Circum	Area
150	**22500**	**3375000**	**12 2474**	**5.3133**	**1837.1**	**2 7241**	**00666667**	**471.239**	**17671 46**
151	22801	3442951	12 2882	5 3251	1855 5	2 7277	.00662252	474 380	17907.86
152	23104	3511808	12.3288	5 3368	1874 0	2 7314	.00657895	477.522	18145 84
153	23409	3581577	12.3693	5.3485	1892 5	2 7349	00653595	480 663	18385.38
154	23716	3652264	12.4097	5.3601	1911.1	2 7385	.00649351	483 805	18626.50
155	24025	3723875	12 4499	5 3717	1929 7	2 7420	00645161	486 946	18869.19
156	24336	3796416	12 4900	5.3832	1948.4	2.7455	.00641026	490 088	19113.45
157	24649	3869893	12 5300	5.3947	1967.2	2.7490	00636943	493 230	19359 28
158	24964	3944312	12 5698	5 4061	1986.0	2 7525	00632911	496 371	19606 68
159	25281	4019679	12.6095	5.4175	2004.9	2.7560	00628931	499 513	19855.65
160	**25600**	**4096000**	**12.6491**	**5 4288**	**2023 9**	**2.7595**	**00625000**	**502.654**	**20106.19**
161	25921	4173281	12 6886	5.4401	2042 9	2 7629	00621118	505.796	20358.30
162	26244	4251528	12 7279	5 4514	2061 9	2 7663	00617284	508 938	20611 99
163	26569	4330747	12 7671	5.4626	2081.0	2 7697	00613497	512.079	20867.24
164	26896	4410944	12 8062	5 4737	2100 2	2 7731	00609756	515.221	21124 06
165	27225	4492125	12 8452	5.4848	2119 5	2 7765	00606061	518.362	21382.46
166	27556	4574296	12.8841	5 4959	2138.8	2.7799	.00602410	521 504	21642.43
167	27889	4657463	12 9228	5 5069	2158 1	2 7832	00598802	524 646	21903 96
168	28224	4741632	12 9615	5 5178	2177 5	2 7865	00595238	527 787	22167.07
169	28561	4826809	13.0000	5 5288	2197 0	2 7898	00591716	530 929	22431.75
170	**28900**	**4913000**	**13 0384**	**5 5397**	**2216.6**	**2 7931**	**00588235**	**534.070**	**22698.00**
171	29241	5000211	13.0767	5.5505	2236.1	2 7964	00584795	537 212	22965.82
172	29584	5088448	13 1149	5 5613	2255.8	2 7997	00581395	540 353	23235.21
173	29929	5177717	13.1529	5.5721	2275.5	2 8029	00578035	543 495	23506.18
174	30276	5268024	13 1909	5 5828	2295 2	2 8061	00574713	546 637	23778.71
175	30625	5359375	13 2288	5 5934	2315 0	2 8094	.00571429	549 778	24052.81
176	30976	5451776	13 2665	5 6041	2334 9	2 8126	00568182	552 920	24328.49
177	31329	5545233	13 3041	5.6147	2354 8	2 8158	00564972	556 061	24605.73
178	31684	5639752	13 3417	5 6252	2374.8	2 8189	00561798	559 203	24884 55
179	32041	5735339	13.3791	5 6357	2394.9	2 8221	00558659	562 345	25164 94
180	**32400**	**5832000**	**13.4164**	**5 6462**	**2415.0**	**2 8252**	**00555556**	**565 486**	**25446.90**
181	32761	5929741	13 4536	5 6567	2435 1	2 8284	00552486	568 628	25730 42
182	33124	6028568	13.4907	5 6671	2455 3	2 8315	00549451	571 769	26015.52
183	33489	6128487	13.5277	5.6774	2475 6	2 8346	00546448	574 911	26302.19
184	33856	6229504	13.5647	5 6877	2495.9	2 8377	00543478	578 053	26590 43
185	34225	6331625	13 6015	5 6980	2516 3	2 8408	00540541	581 194	26880 25
186	34596	6434856	13 6382	5 7083	2536 7	2 8438	00537634	584 336	27171.63
187	34969	6539203	13 6748	5 7185	2557 2	2 8469	00534759	587.477	27464 58
188	35344	6644672	13 7113	5 7287	2577 7	2 8499	00531915	590 619	27759 11
189	35721	6751269	13.7477	5 7388	2598.3	2.8529	00529101	593 761	28055.20
190	**36100**	**6859000**	**13.7840**	**5 7489**	**2619 0**	**2 8560**	**00526316**	**596 902**	**28352 87**
191	36481	6967871	13 8203	5 7590	2639 7	2 8590	00523560	600 044	28652.10
192	36864	7077888	13 8564	5 7690	2660 4	2 8619	00520833	603 185	28952 91
193	37249	7189057	13 8924	5 7790	2681 2	2 8649	00518135	606 327	29255.29
194	37636	7301384	13 9284	5 7890	2702 1	2.8679	00515464	609 468	29559.24
195	38025	7414875	13 9642	5 7989	2723 0	2 8708	00512821	612 610	29864.76
196	38416	7529536	14 0000	5 8088	2744 0	2 8738	.00510204	615.752	30171.85
197	38809	7645373	14.0357	5 8186	2765 0	2 8767	00507614	618.893	30480.51
198	39204	7762392	14 0712	5 8285	2786 1	2 8796	00505051	622 035	30790 74
199	39601	7880599	14.1067	5 8383	2807.2	2 8825	00502513	625 176	31102.55
200	**40000**	**8000000**	**14.1421**	**5 8480**	**2828 4**	**2 8854**	**00500000**	**628.318**	**31415 93**
201	40401	8120601	14.1774	5 8578	2849 7	2 8883	00497512	631 460	31730.87
202	40804	8242408	14 2127	5 8675	2871 0	2 8911	00495050	634 601	32047 39
203	41209	8365427	14 2478	5.8771	2892 3	2 8940	00492611	637 743	32365 47
204	41616	8489664	14 2829	5 8868	2913 7	2 8968	00490196	640 884	32685 13
205	42025	8615125	14 3178	5 8964	2935 2	2 8997	00487805	644 026	33006 36
206	42436	8741816	14.3527	5 9059	2956.7	2.9025	00485437	647 168	33329 16
207	42849	8869743	14 3875	5 9155	2978 2	2 9053	00483092	650 309	33653.53
208	43264	8998912	14.4222	5 9250	2999 8	2 9081	00480769	653 451	33979 47
209	43681	9129329	14 4568	5 9345	3021 5	2 9109	00478469	656 592	34306.98
210	**44100**	**9261000**	**14 4914**	**5 9439**	**3043 2**	**2 9137**	**00475190**	**659 734**	**34636 06**
211	44521	9393931	14 5258	5 9533	3065 0	2 9165	00473934	662 875	34966.71
212	44944	9528128	14 5602	5 9627	3086 8	2 9192	00471698	666 017	35298 94
213	45369	9663597	14 5945	5 9721	3108.7	2 9220	00469484	669 159	35632 73
214	45796	9800344	14 6287	5 9814	3130 6	2 9247	00467290	672 300	35968 09
215	46225	9938375	14 6629	5 9907	3152 5	2 9274	00465116	675 442	36305.03
216	46656	10077696	14.6969	6 0000	3174 5	2 9302	00462963	678 583	36643.54
217	47089	10218313	14 7309	6 0092	3196 6	2 9329	00460829	681 725	36983.61
218	47524	10360232	14 7648	6 0185	3218 7	2 9356	00458716	684 867	37325 26
219	47961	10503459	14 7986	6.0277	3240.9	2 9383	00456621	688 008	37668.48

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
220	**48400**	**10648000**	**14.8324**	**6.0368**	**3263.1**	**2.9409**	**.00454545**	**691.150**	**38013.27**
221	48841	10793861	14.8661	6.0459	3285.4	2.9436	.00452489	694.291	38359.63
222	49284	10941048	14.8997	6.0550	3307.7	2.9463	.00450450	697.433	38707.56
223	49729	11089567	14.9332	6.0641	3330.1	2.9489	.00448430	700.575	39057.07
224	50176	11239424	14.9666	6.0732	3352.5	2.9516	.00446429	703.716	39408.14
225	50625	11390625	15.0000	6.0822	3375.0	2.9542	.00444444	706.858	39760.78
226	51076	11543176	15.0333	6.0912	3397.5	2.9568	.00442478	709.999	40115.00
227	51529	11697083	15.0665	6.1002	3420.1	2.9594	.00440529	713.141	40470.78
228	51984	11852352	15.0997	6.1091	3442.7	2.9620	.00438596	716.283	40828.14
229	52441	12008989	15.1327	6.1180	3465.4	2.9646	.00436681	719.424	41187.07
230	**52900**	**12167000**	**15.1658**	**6.1269**	**3488.1**	**2.9672**	**.00434783**	**722.566**	**41547.56**
231	53361	12326391	15.1987	6.1358	3510.9	2.9698	.00432900	725.707	41909.63
232	53824	12487168	15.2315	6.1446	3533.7	2.9723	.00431034	728.849	42273.27
233	54289	12649337	15.2643	6.1534	3556.6	2.9749	.00429185	731.990	42638.48
234	54756	12812904	15.2971	6.1622	3579.5	2.9774	.00427350	735.132	43005.26
235	55225	12977875	15.3297	6.1710	3602.5	2.9800	.00425532	738.274	43373.61
236	55696	13144256	15.3623	6.1797	3625.5	2.9825	.00423729	741.415	43743.54
237	56169	13312053	15.3948	6.1885	3648.6	2.9850	.00421941	744.557	44115.03
238	56644	13481272	15.4272	6.1972	3671.7	2.9875	.00420168	747.698	44488.09
239	57121	13651919	15.4596	6.2058	3694.8	2.9900	.00418410	750.840	44862.73
240	**57600**	**13824000**	**15.4919**	**6.2145**	**3718.0**	**2.9925**	**.00416667**	**753.982**	**45238.93**
241	58081	13997521	15.5242	6.2231	3741.3	2.9950	.00414938	757.123	45616.71
242	58564	14172488	15.5563	6.2317	3764.6	2.9975	.00413223	760.265	45996.06
243	59049	14348907	15.5885	6.2403	3788.0	3.0000	.00411523	763.406	46376.98
244	59536	14526784	15.6205	6.2488	3811.4	3.0025	.00409836	766.548	46759.47
245	60025	14706125	15.6525	6.2573	3834.9	3.0049	.00408163	769.690	47143.52
246	60516	14886936	15.6844	6.2658	3858.4	3.0074	.00406504	772.831	47529.16
247	61009	15069223	15.7162	6.2743	3881.9	3.0098	.00404858	775.973	47916.36
248	61504	15252992	15.7480	6.2828	3905.5	3.0122	.00403226	779.114	48305.13
249	62001	15438249	15.7797	6.2912	3929.2	3.0147	.00401606	782.256	48695.47
250	**62500**	**15625000**	**15.8114**	**6.2996**	**3952.9**	**3.0171**	**.00400000**	**785.398**	**49087.39**
251	63001	15813251	15.8430	6.3080	3976.6	3.0195	.00398406	788.539	49480.87
252	63504	16003008	15.8745	6.3164	4000.4	3.0219	.00396825	791.681	49875.92
253	64009	16194277	15.9060	6.3247	4024.2	3.0243	.00395257	794.822	50272.55
254	64516	16387064	15.9374	6.3330	4048.1	3.0267	.00393701	797.964	50670.75
255	65025	16581375	15.9687	6.3413	4072.0	3.0291	.00392157	801.105	51070.52
256	65536	16777216	16.0000	6.3496	4096.0	3.0314	.00390625	804.247	51471.85
257	66049	16974593	16.0312	6.3579	4120.0	3.0338	.00389105	807.389	51874.76
258	66564	17173512	16.0624	6.3661	4144.1	3.0362	.00387597	810.530	52279.24
259	67081	17373979	16.0935	6.3743	4168.2	3.0385	.00386100	813.672	52685.29
260	**67600**	**17576000**	**16.1245**	**6.3825**	**4192.4**	**3.0418**	**.00384615**	**816.813**	**53092.92**
261	68121	17779581	16.1555	6.3907	4216.6	3.0432	.00383142	819.955	53502.11
262	68644	17984728	16.1864	6.3988	4240.8	3.0455	.00381679	823.097	53912.87
263	69169	18191447	16.2173	6.4070	4265.1	3.0478	.00380228	826.238	54325.21
264	69696	18399744	16.2481	6.4151	4289.5	3.0501	.00378788	829.380	54739.11
265	70225	18609625	16.2788	6.4232	4313.9	3.0524	.00377358	832.521	55154.59
266	70756	18821096	16.3095	6.4312	4338.3	3.0547	.00375940	835.663	55571.63
267	71289	19034163	16.3401	6.4393	4362.8	3.0570	.00374532	838.805	55990.25
268	71824	19248832	16.3707	6.4473	4387.3	3.0593	.00373134	841.946	56410.44
269	72361	19465109	16.4012	6.4553	4411.9	3.0616	.00371747	845.088	56832.20
270	**72900**	**19683000**	**16.4317**	**6.4633**	**4436.5**	**3.0639**	**.00370370**	**848.229**	**57255.53**
271	73441	19902511	16.4621	6.4713	4461.2	3.0662	.00369004	851.371	57680.43
272	73984	20123648	16.4924	6.4792	4485.9	3.0684	.00367647	854.512	58106.90
273	74529	20346417	16.5227	6.4872	4510.7	3.0707	.00366300	857.654	58534.94
274	75076	20570824	16.5529	6.4951	4535.5	3.0729	.00364964	860.796	58964.55
275	75625	20796875	16.5831	6.5030	4560.4	3.0752	.00363636	863.937	59395.74
276	76176	21024576	16.6132	6.5108	4585.3	3.0774	.00362319	867.079	59828.49
277	76729	21253933	16.6433	6.5187	4610.2	3.0796	.00361011	870.220	60262.82
278	77284	21484952	16.6733	6.5265	4635.2	3.0818	.00359712	873.362	60698.71
279	77841	21717639	16.7033	6.5343	4660.2	3.0840	.00358423	876.504	61136.18
280	**78400**	**21952000**	**16.7332**	**6.5421**	**4685.3**	**3.0863**	**.00357143**	**879.645**	**61575.22**
281	78961	22188041	16.7631	6.5499	4710.4	3.0885	.00355872	882.787	62015.82
282	79524	22425768	16.7929	6.5577	4735.6	3.0907	.00354610	885.928	62458.00
283	80089	22665187	16.8226	6.5654	4760.8	3.0928	.00353357	889.070	62901.75
284	80656	22906304	16.8523	6.5731	4786.0	3.0950	.00352113	892.212	63347.07
285	81225	23149125	16.8819	6.5808	4811.3	3.0972	.00350877	895.353	63793.97
286	81796	23393656	16.9115	6.5885	4836.7	3.0994	.00349650	898.495	64242.43
287	82369	23639903	16.9411	6.5962	4862.1	3.1015	.00348432	901.636	64692.46
288	82944	23887872	16.9706	6.6039	4887.5	3.1037	.00347222	904.778	65144.07
289	83521	24137569	17.0000	6.6115	4913.0	3.1058	.00346021	907.920	65597.24

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.)	
								Circum.	Area
290	**84100**	**24389000**	**17.0294**	**6.6191**	**4938.5**	**3.1080**	**.00344828**	**911.061**	**66051.99**
291	84681	24642171	17.0587	6.6267	4964.1	3.1101	.00343643	914.203	66508.30
292	85264	24897088	17.0880	6.6343	4989.7	3.1123	.00342466	917.344	66966.19
293	85849	25153757	17.1172	6.6419	5015.4	3.1144	.00341297	920.486	67425.65
294	86436	25412184	17.1464	6.6494	5041.1	3.1165	.00340136	923.627	67886.68
295	87025	25672375	17.1756	6.6569	5066.8	3.1186	.00338983	926.769	68349.28
296	87616	25934336	17.2047	6.6644	5092.6	3.1207	.00337838	929.911	68813.45
297	88209	26198073	17.2337	6.6719	5118.4	3.1228	.00336700	933.052	69279.19
298	88804	26463592	17.2627	6.6794	5144.3	3.1249	.00335570	936.194	69746.50
299	89401	26730899	17.2916	6.6869	5170.2	3.1270	.00334448	939.335	70215.38
300	**90000**	**27000000**	**17.3205**	**6.6943**	**5196.2**	**3.1291**	**.00333333**	**942.477**	**70685.83**
301	90601	27270901	17.3494	6.7018	5222.2	3.1312	.00332226	945.619	71157.86
302	91204	27543608	17.3781	6.7092	5248.2	3.1333	.00331126	948.760	71631.45
303	91809	27818127	17.4069	6.7166	5274.3	3.1354	.00330033	951.902	72106.62
304	92416	28094464	17.4356	6.7240	5300.4	3.1374	.00328947	955.043	72583.36
305	93025	28372625	17.4642	6.7313	5326.6	3.1395	.00327869	958.185	73061.66
306	93636	28652616	17.4929	6.7387	5352.8	3.1416	.00326797	961.327	73541.54
307	94249	28934443	17.5214	6.7460	5379.1	3.1436	.00325733	964.468	74022.99
308	94864	29218112	17.5499	6.7533	5405.4	3.1456	.00324675	967.610	74506.01
309	95481	29503629	17.5784	6.7606	5431.7	3.1477	.00323625	970.751	74990.60
310	**96100**	**29791000**	**17.6068**	**6.7679**	**5458.1**	**3.1497**	**.00322581**	**973.893**	**75476.76**
311	96721	30080231	17.6352	6.7752	5484.5	3.1518	.00321543	977.034	75964.50
312	97344	30371328	17.6635	6.7824	5511.0	3.1538	.00320513	980.176	76453.80
313	97969	30664297	17.6918	6.7897	5537.5	3.1558	.00319489	983.318	76944.67
314	98596	30959144	17.7200	6.7969	5564.1	3.1578	.00318471	986.459	77437.12
315	99225	31255875	17.7482	6.8041	5590.7	3.1598	.00317460	989.601	77931.13
316	99856	31554496	17.7764	6.8113	5617.3	3.1618	.00316456	992.742	78426.72
317	100489	31855013	17.8045	6.8185	5644.0	3.1638	.00315457	995.884	78923.88
318	101124	32157432	17.8326	6.8256	5670.7	3.1658	.00314465	999.026	79422.60
319	101761	32461759	17.8606	6.8328	5697.5	3.1678	.00313480	1002.167	79922.90
320	**102400**	**32768000**	**17.8885**	**6.8399**	**5724.3**	**3.1698**	**.00312500**	**1005.309**	**80424.77**
321	103041	33076161	17.9165	6.8470	5751.2	3.1718	.00311526	1008.450	80928.21
322	103684	33386248	17.9444	6.8541	5778.1	3.1737	.00310559	1011.592	81433.22
323	104329	33698267	17.9722	6.8612	5805.0	3.1757	.00309598	1014.734	81939.80
324	104976	34012224	18.0000	6.8683	5832.0	3.1777	.00308642	1017.875	82447.96
325	105625	34328125	18.0278	6.8753	5859.0	3.1796	.00307692	1021.017	82957.68
326	106276	34645976	18.0555	6.8824	5886.1	3.1816	.00306748	1024.158	83468.97
327	106929	34965783	18.0831	6.8894	5913.2	3.1835	.00305810	1027.300	83981.84
328	107584	35287552	18.1108	6.8964	5940.3	3.1855	.00304878	1030.442	84496.28
329	108241	35611289	18.1384	6.9034	5967.5	3.1874	.00303951	1033.583	85012.28
330	**108900**	**35937000**	**18.1659**	**6.9104**	**5994.7**	**3.1894**	**.00303030**	**1036.725**	**85529.86**
331	109561	36264691	18.1934	6.9174	6022.0	3.1913	.00302115	1039.866	86049.01
332	110224	36594368	18.2209	6.9244	6049.3	3.1932	.00301205	1043.008	86569.73
333	110889	36926037	18.2483	6.9313	6076.7	3.1951	.00300300	1046.149	87092.02
334	111556	37259704	18.2757	6.9382	6104.1	3.1970	.00299401	1049.291	87615.88
335	112225	37595375	18.3030	6.9451	6131.5	3.1989	.00298507	1052.433	88141.31
336	112896	37933056	18.3303	6.9521	6159.0	3.2009	.00297619	1055.574	88668.31
337	113569	38272753	18.3576	6.9589	6186.5	3.2028	.00296736	1058.716	89196.88
338	114244	38614472	18.3848	6.9658	6214.1	3.2047	.00295858	1061.857	89727.03
339	114921	38958219	18.4120	6.9727	6241.7	3.2066	.00294985	1064.999	90258.74
340	**115600**	**39304000**	**18.4391**	**6.9795**	**6269.3**	**3.2085**	**.00294118**	**1068.141**	**90792.03**
341	116281	39651821	18.4662	6.9864	6297.0	3.2103	.00293255	1071.282	91326.88
342	116964	40001688	18.4932	6.9932	6324.7	3.2122	.00292398	1074.424	91863.31
343	117649	40353607	18.5203	7.0000	6352.4	3.2141	.00291545	1077.565	92401.31
344	118336	40707584	18.5472	7.0068	6380.2	3.2160	.00290698	1080.707	92940.88
345	119025	41063625	18.5742	7.0136	6408.1	3.2178	.00289855	1083.849	93482.02
346	119716	41421736	18.6011	7.0203	6436.0	3.2197	.00289017	1086.990	94024.73
347	120409	41781923	18.6279	7.0271	6463.9	3.2216	.00288184	1090.132	94569.01
348	121104	42144192	18.6548	7.0338	6491.9	3.2234	.00287356	1093.273	95114.86
349	121801	42508549	18.6815	7.0406	6519.9	3.2253	.00286533	1096.415	95662.28
350	**122500**	**42875000**	**18.7083**	**7.0473**	**6547.9**	**3.2271**	**.00285714**	**1099.557**	**96211.28**
351	123201	43243551	18.7350	7.0540	6576.0	3.2289	.00284900	1102.698	96761.84
352	123904	43614208	18.7617	7.0607	6604.1	3.2308	.00284091	1105.840	97313.97
353	124609	43986977	18.7883	7.0674	6632.3	3.2326	.00283286	1108.981	97867.68
354	125316	44361864	18.8149	7.0740	6660.5	3.2345	.00282486	1112.123	98422.96
355	126025	44738875	18.8414	7.0807	6688.7	3.2363	.00281690	1115.264	98979.80
356	126736	45118016	18.8680	7.0873	6717.0	3.2381	.00280899	1118.406	99538.22
357	127449	45499293	18.8944	7.0940	6745.3	3.2399	.00280112	1121.548	100098.21
358	128164	45882712	18.9209	7.1006	6773.7	3.2417	.00279330	1124.689	100659.77
359	128881	46268279	18.9473	7.1072	6802.1	3.2435	.00278552	1127.831	101222.90

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	(Circle N = Diam.) Circum.	Area
360	**129600**	**46656000**	**18.9737**	**7.1138**	**6830.5**	**3.2453**	**.00277778**	**1130.972**	**101787.60**
361	130321	47045881	19.0000	7.1204	6859.0	3.2471	.00277008	1134.114	102353.87
362	131044	47437928	19.0263	7.1269	6887.5	3.2489	.00276243	1137.256	102921.72
363	131769	47832147	19.0526	7.1335	6916.1	3.2507	.00275482	1140.397	103491.13
364	132496	48228544	19.0788	7.1400	6944.7	3.2525	.00274725	1143.539	104062.12
365	133225	48627125	19.1050	7.1466	6973.3	3.2543	.00273973	1146.680	104634.67
366	133956	49027896	19.1311	7.1531	7002.0	3.2561	.00273224	1149.822	105208.80
367	134689	49430863	19.1572	7.1596	7030.7	3.2579	.00272480	1152.964	105784.49
368	135424	49836032	19.1833	7.1661	7059.5	3.2597	.00271739	1156.105	106361.76
369	136161	50243409	19.2094	7.1726	7088.3	3.2614	.00271003	1159.247	106940.60
370	**136900**	**50653000**	**19.2354**	**7.1791**	**7117.1**	**3.2632**	**.00270270**	**1162.388**	**107521.01**
371	137641	51064811	19.2614	7.1855	7146.0	3.2650	.00269542	1165.530	108102.99
372	138384	51478848	19.2873	7.1920	7174.9	3.2668	.00268817	1168.671	108686.54
373	139129	51895117	19.3132	7.1984	7203.9	3.2685	.00268097	1171.813	109271.66
374	139876	52313624	19.3391	7.2048	7232.8	3.2702	.00267380	1174.955	109858.35
375	140625	52734375	19.3649	7.2112	7261.8	3.2719	.00266667	1178.096	110446.62
376	141376	53157376	19.3907	7.2177	7290.9	3.2737	.00265957	1181.238	111036.45
377	142129	53582633	19.4165	7.2240	7320.0	3.2754	.00265252	1184.379	111627.86
378	142884	54010152	19.4422	7.2304	7349.2	3.2772	.00264550	1187.521	112220.83
379	143641	54439939	19.4679	7.2368	7378.4	3.2789	.00263852	1190.663	112815.38
380	**144400**	**54872000**	**19.4936**	**7.2432**	**7407.6**	**3.2807**	**.00263158**	**1193.804**	**113411.49**
381	145161	55306341	19.5192	7.2495	7436.8	3.2824	.00262467	1196.946	114009.18
382	145924	55742968	19.5448	7.2558	7466.1	3.2841	.00261780	1200.087	114608.44
383	146689	56181887	19.5704	7.2622	7495.4	3.2858	.00261097	1203.229	115209.27
384	147456	56623104	19.5959	7.2685	7524.8	3.2875	.00260417	1206.371	115811.67
385	148225	57066625	19.6214	7.2748	7554.2	3.2892	.00259740	1209.512	116415.64
386	148996	57512456	19.6469	7.2811	7583.7	3.2909	.00259067	1212.654	117021.18
387	149769	57960603	19.6723	7.2874	7613.2	3.2926	.00258398	1215.795	117628.30
388	150544	58411072	19.6977	7.2936	7642.7	3.2943	.00257732	1218.937	118236.96
389	151321	58863869	19.7231	7.2999	7672.3	3.2960	.00257069	1222.079	118847.24
390	**152100**	**59319000**	**19.7484**	**7.3061**	**7701.9**	**3.2977**	**.00256410**	**1225.220**	**119459.06**
391	152881	59776471	19.7737	7.3124	7731.5	3.2994	.00255754	1228.362	120072.46
392	153664	60236288	19.7990	7.3186	7761.2	3.3011	.00255102	1231.503	120687.42
393	154449	60698457	19.8242	7.3248	7790.9	3.3028	.00254453	1234.645	121303.95
394	155236	61162984	19.8494	7.3310	7820.7	3.3045	.00253807	1237.786	121922.07
395	156025	61629875	19.8746	7.3372	7850.5	3.3061	.00253165	1240.928	122541.75
396	156816	62099136	19.8997	7.3434	7880.3	3.3078	.00252525	1244.070	123163.00
397	157609	62570773	19.9249	7.3496	7910.2	3.3095	.00251889	1247.211	123785.82
398	158404	63044792	19.9499	7.3558	7940.1	3.3111	.00251256	1250.353	124410.21
399	159201	63521199	19.9750	7.3619	7970.0	3.3128	.00250627	1253.494	125036.17
400	**160000**	**64000000**	**20.0000**	**7.3681**	**8000.0**	**3.3145**	**.00250000**	**1256.636**	**125663.71**
401	160801	64481201	20.0250	7.3742	8030.0	3.3161	.00249377	1259.778	126292.81
402	161604	64964808	20.0499	7.3803	8061.1	3.3178	.00248756	1262.919	126923.48
403	162409	65450827	20.0749	7.3864	8090.2	3.3194	.00248139	1266.061	127555.73
404	163216	65939264	20.0998	7.3925	8120.3	3.3211	.00247525	1269.202	128189.55
405	164025	66430125	20.1246	7.3986	8150.5	3.3227	.00246914	1272.344	128824.93
406	164836	66923416	20.1494	7.4047	8180.7	3.3243	.00246305	1275.486	129461.89
407	165649	67419143	20.1742	7.4108	8210.9	3.3260	.00245700	1278.627	130100.42
408	166464	67917312	20.1990	7.4169	8241.2	3.3276	.00245098	1281.769	130740.52
409	167281	68417929	20.2237	7.4229	8271.5	3.3292	.00244499	1284.910	131382.19
410	**168100**	**68921000**	**20.2485**	**7.4290**	**8301.9**	**3.3308**	**.00243902**	**1288.052**	**132025.43**
411	168921	69426531	20.2731	7.4350	8332.3	3.3325	.00243309	1291.193	132670.24
412	169744	69934528	20.2978	7.4410	8362.7	3.3341	.00242718	1294.335	133316.63
413	170569	70444997	20.3224	7.4470	8393.2	3.3357	.00242131	1297.477	133964.58
414	171396	70957944	20.3470	7.4530	8423.7	3.3373	.00241546	1300.618	134614.10
415	172225	71473375	20.3715	7.4590	8454.2	3.3390	.00240964	1303.760	135265.20
416	173056	71991296	20.3961	7.4650	8484.8	3.3406	.00240385	1306.901	135917.86
417	173889	72511713	20.4206	7.4710	8515.4	3.3422	.00239808	1310.043	136572.10
418	174724	73034632	20.4450	7.4770	8546.0	3.3438	.00239234	1313.185	137227.91
419	175561	73560059	20.4695	7.4829	8576.7	3.3454	.00238663	1316.326	137885.29
420	**176400**	**74088000**	**20.4939**	**7.4889**	**8607.4**	**3.3470**	**.00238095**	**1319.468**	**138544.24**
421	177241	74618461	20.5183	7.4948	8638.2	3.3485	.00237530	1322.609	139204.76
422	178084	75151448	20.5426	7.5007	8669.0	3.3501	.00236967	1325.751	139866.85
423	178929	75686967	20.5670	7.5067	8699.8	3.3517	.00236407	1328.893	140530.51
424	179776	76225024	20.5913	7.5126	8730.7	3.3533	.00235849	1332.034	141195.74
425	180625	76765625	20.6155	7.5185	8761.6	3.3559	.00235294	1335.176	141862.54
426	181476	77308776	20.6398	7.5244	8792.5	3.3564	.00234742	1338.317	142530.92
427	182329	77854483	20.6640	7.5302	8823.5	3.3580	.00234192	1341.459	143200.86
428	183184	78402752	20.6882	7.5361	8854.5	3.3596	.00233645	1344.601	143872.38
429	184041	78953589	20.7123	7.5420	8885.6	3.3612	.00233100	1347.742	144545.46

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
430	**184900**	**79507000**	**20.7364**	**7.5478**	**8915.7**	**3.3627**	**.00232558**	**1350 884**	**145220.12**
431	185761	80062991	20 7605	7.5537	8947.8	3.3643	.00232019	1354.025	145896.35
432	186624	80621568	20 7846	7 5595	8979.0	3 3659	00231481	1357.167	146574 15
433	187489	81182737	20.8087	7.5654	9010.1	3.3674	.00230947	1360.308	147253.52
434	188356	81746504	20 8327	7 5712	9041 4	3 3690	00230415	1363 450	147934.46
435	189225	82312875	20 8567	7.5770	9072 7	3 3705	.00229885	1366 592	148616 97
436	190096	82881856	20.8806	7.5828	9104.0	3.3720	00229358	1369.733	149301.05
437	190969	83453453	20 9045	7 5886	9135.3	3 3736	00228833	1372 875	149986 70
438	191844	84027672	20 9284	7.5944	9166.7	3.3752	00228311	1376.016	150673.92
439	192721	84604519	20.9523	7.6001	9198.1	3.3767	.00227790	1379.158	151362.72
440	**193600**	**85184000**	**20.9762**	**7.6059**	**9229.5**	**3.3783**	**00227273**	**1382.300**	**152053 08**
441	194481	85766121	21.0000	7 6117	9261.0	3.3798	00226757	1385.441	152745.02
442	195364	86350888	21.0238	7 6174	9292.5	3 3813	00226244	1388 583	153438 53
443	196249	86938307	21.0476	7.6232	9324.1	3.3828	.00225734	1391.724	154133.60
444	197136	87528384	21 0713	7.6289	9355 7	3 3844	00225225	1394 866	154830 25
445	198025	88121125	21.0950	7.6346	9387.3	3.3859	.00224719	1398 008	155528.47
446	198916	88716536	21.1187	7.6403	9419.0	3.3874	00224215	1401.149	156228.26
447	199809	89314623	21.1424	7 6460	9450.7	3.3889	00223714	1404.291	156929 62
448	200704	89915392	21.1660	7 6517	9482 4	3 3904	.00223214	1407.432	157632 55
449	201601	90518849	21.1896	7.6574	9514.2	3.3919	.00222717	1410.574	158337.05
450	**202500**	**91125000**	**21.2132**	**7.6631**	**9546.0**	**3.3935**	**00222222**	**1413.716**	**159043.13**
451	203401	91733851	21.2368	7.6688	9577.8	3 3950	.00221729	1416.857	159750.77
452	204304	92345408	21.2603	7 6744	9609.6	3 3965	.00221239	1419.999	160459.99
453	205209	92959677	21.2838	7.6801	9641.5	3.3980	.00220751	1423.140	161170.77
454	206116	93576664	21 3073	7.6857	9673.5	3 3995	00220264	1426.282	161883 13
455	207025	94196375	21.3307	7 6914	9705.5	3 4010	.00219780	1429.423	162597.05
456	207936	94818816	21.3542	7.6970	9737.5	3.4025	00219298	1432.565	163312.55
457	208849	95443993	21 3776	7.7026	9769.5	3.4039	00218818	1435.707	164029.62
458	209764	96071912	21 4009	7 7082	9801 6	3.4054	.00218341	1438.848	164748.26
459	210681	96702579	21.4243	7.7138	9833.8	3.4069	.00217865	1441.990	165468.47
460	**211600**	**97336000**	**21.4476**	**7.7194**	**9865.9**	**3.4084**	**.00217391**	**1445.131**	**166190.25**
461	212521	97972181	21.4709	7.7250	9898 1	3.4199	00216920	1448.273	166913 60
462	213444	98611128	21.4942	7 7306	9930.3	3.4113	00216450	1451.415	167638.52
463	214369	99252847	21.5174	7.7362	9962.6	3.4128	00215983	1454.556	168365 02
464	215296	99897344	21.5407	7.7418	9994.8	3.4143	00215517	1457.698	169093.08
465	216225	100544625	21.5639	7.7473	10027.	3.4158	00215054	1460 839	169822.72
466	217156	101194696	21.5870	7.7529	10060.	3.4173	00214592	1463.981	170553 92
467	218089	101847563	21.6102	7.7584	10092.	3.4187	.00214133	1467.123	171286.70
468	219024	102503232	21.6333	7 7639	10124.	3 4202	.00213675	1470 264	172021.05
469	219961	103161709	21.6564	7.7695	10157.	3.4217	.00213220	1473.406	172756.96
470	**220900**	**103823000**	**21.6795**	**7.7750**	**10189.**	**3.4231**	**.00212766**	**1476.547**	**173494.45**
471	221841	104487111	21.7025	7.7805	10222.	3.4246	00212314	1479.689	174233.51
472	222784	105154048	21.7256	7 7860	10255.	3.4260	.00211864	1482.830	174974.14
473	223729	105823817	21.7486	7.7915	10287.	3.4275	.00211416	1485.972	175716.34
474	224676	106496424	21.7715	7.7970	10320.	3.4289	.00210970	1489.114	176460.12
475	225625	107171875	21 7945	7 8025	10352.	3.4304	00210526	1492.255	177205 46
476	226576	107850176	21.8174	7 8079	10385.	3.4318	.00210084	1495.397	177952.37
477	227529	108531333	21.8403	7.8134	10418.	3.4332	.00209644	1498.538	178700 86
478	228484	109215352	21 8632	7 8188	10450	3.4347	00209205	1501.680	179450 91
479	229441	109902239	21.8861	7 8243	10483.	3.4361	.00208768	1504.822	180202.54
480	**230400**	**110592000**	**21 9089**	**7.8297**	**10516.**	**3.4375**	**00208333**	**1507.963**	**180955.74**
481	231361	111284641	21.9317	7 8352	10549.	3.4390	00207900	1511.105	181710.50
482	232324	111980168	21.9545	7 8406	10582.	3.4404	00207469	1514.246	182466.84
483	233289	112678587	21.9773	7.8460	10615.	3.4418	.00207039	1517.388	183224.75
484	234256	113379904	22.0000	7.8514	10648.	3.4433	00206612	1520.530	183984 23
485	235225	114084125	22.0227	7 8568	10681.	3.4447	00206186	1523 671	184745 28
486	236196	114791256	22.0454	7.8622	10714.	3.4461	.00205761	1526.813	185507.90
487	237169	115501303	22.0681	7 8675	10747.	3.4475	00205339	1529.954	186272.10
488	238144	116214272	22 0907	7 8730	10780.	3.4489	00204918	1533.096	187037.86
489	239121	116930169	22.1133	7.8784	10813.	3.4504	.00204499	1536.238	187805.19
490	**240100**	**117649000**	**22.1359**	**7.8837**	**10847.**	**3.4518**	**00204082**	**1539.379**	**188574.10**
491	241081	118370771	22 1585	7 8891	10880.	3 4532	00203666	1542.521	189344.57
492	242064	119095488	22.1811	7 8944	10913.	3.4546	.00203252	1545 662	190116 62
493	243049	119823157	22.2036	7 8998	10946.	3.4560	.00202840	1548.804	190890.24
494	244036	120553784	22 2261	7 9051	10980.	3.4574	00202429	1551.945	191665 43
495	245025	121287375	22.2486	7 9105	11013.	3 4588	.00202020	1555.087	192442.18
496	246016	122023936	22.2711	7.9158	11046.	3.4602	00201613	1558.229	193220.51
497	**247009**	**122763473**	22.2935	7 9211	11080.	3.4616	.00201207	1561.370	194000.41
498	**248004**	**123505992**	22 3159	7.9264	11113.	3.4630	.00200803	1564.512	194781.89
499	**249001**	**124251499**	22.3383	7.9317	11147.	3.4643	.00200401	1567 653	195564.93

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam) Circum	Circle (N = Diam) Area
500	**250000**	**125000000**	**22.3607**	**7.9370**	**11180**	**3.4657**	**.00200000**	**1570.795**	**196349.54**
501	251001	125751501	22.3830	7.9423	11214	3.4671	.00199601	1573.937	197135.72
502	252004	126506008	22.4054	7.9476	11247	3.4685	.00199203	1577.078	197923.48
503	253009	127263527	22.4277	7.9528	11281	3.4699	.00198807	1580.220	198712.80
504	254016	128024064	22.4499	7.9581	11315	3.4713	.00198413	1583.361	199503.70
505	255025	128787625	22.4722	7.9634	11348	3.4726	.00198020	1586.503	200296.17
506	256036	129554216	22.4944	7.9686	11382	3.4740	.00197628	1589.645	201090.20
507	257049	130323843	22.5167	7.9739	11416	3.4754	.00197239	1592.786	201885.81
508	258064	131096512	22.5389	7.9791	11450	3.4768	.00196850	1595.928	202682.99
509	259081	131872229	22.5610	7.9843	11484	3.4781	.00196464	1599.069	203481.74
510	**260100**	**132651000**	**22.5832**	**7.9896**	**11517**	**3.4795**	**.00196078**	**1602.211**	**204282.06**
511	261121	133432831	22.6053	7.9948	11551	3.4808	.00195695	1605.352	205083.95
512	262144	134217728	22.6274	8.0000	11585	3.4822	.00195313	1608.494	205887.42
513	263169	135005697	22.6495	8.0052	11619	3.4836	.00194932	1611.636	206692.45
514	264196	135796744	22.6716	8.0104	11653	3.4849	.00194553	1614.777	207499.05
515	265225	136590875	22.6936	8.0156	11687	3.4863	.00194175	1617.919	208307.23
516	266256	137388096	22.7156	8.0208	11721	3.4876	.00193798	1621.060	209116.97
517	267289	138188413	22.7376	8.0260	11755	3.4890	.00193424	1624.202	209928.29
518	268324	138991832	22.7596	8.0311	11789	3.4904	.00193050	1627.344	210741.18
519	269361	139798359	22.7816	8.0363	11824	3.4917	.00192678	1630.485	211555.63
520	**270400**	**140608000**	**22.8035**	**8.0415**	**11858**	**3.4930**	**.00192308**	**1633.627**	**212371.66**
521	271441	141420761	22.8254	8.0466	11892	3.4944	.00191939	1636.768	213189.26
522	272484	142236648	22.8473	8.0517	11926	3.4957	.00191571	1639.910	214008.43
523	273529	143055667	22.8692	8.0569	11960	3.4970	.00191205	1643.052	214829.17
524	274576	143877824	22.8910	8.0620	11995	3.4984	.00190840	1646.193	215651.49
525	275625	144703125	22.9129	8.0671	12029	3.4997	.00190476	1649.335	216475.37
526	276676	145531576	22.9347	8.0723	12064	3.5010	.00190114	1652.476	217300.82
527	277729	146363183	22.9565	8.0774	12098	3.5024	.00189753	1655.618	218127.85
528	278784	147197952	22.9783	8.0825	12133	3.5037	.00189394	1658.760	218956.44
529	279841	148035889	23.0000	8.0876	12167	3.5050	.00189036	1661.901	219786.61
530	**280900**	**148877000**	**23.0217**	**8.0927**	**12202**	**3.5064**	**.00188679**	**1665.043**	**220618.34**
531	281961	149721291	23.0434	8.0978	12236	3.5077	.00188324	1668.184	221451.65
532	283024	150568768	23.0651	8.1028	12271	3.5090	.00187970	1671.326	222286.53
533	284089	151419437	23.0868	8.1079	12305	3.5103	.00187617	1674.467	223122.98
534	285156	152273304	23.1084	8.1130	12340	3.5116	.00187266	1677.609	223961.00
535	286225	153130375	23.1301	8.1180	12375	3.5130	.00186916	1680.751	224800.59
536	287296	153990656	23.1517	8.1231	12410	3.5143	.00186567	1683.892	225641.75
537	288369	154854153	23.1733	8.1281	12444	3.5156	.00186220	1687.034	226484.48
538	289444	155720872	23.1948	8.1332	12479	3.5169	.00185874	1690.175	227328.79
539	290521	156590819	23.2164	8.1382	12514	3.5182	.00185529	1693.317	228174.66
540	**291600**	**157464000**	**23.2379**	**8.1433**	**12549**	**3.5195**	**.00185185**	**1696.459**	**229022.10**
541	292681	158340421	23.2594	8.1483	12583	3.5208	.00184843	1699.600	229871.12
542	293764	159220088	23.2809	8.1533	12618	3.5221	.00184502	1702.742	230721.71
543	294849	160103007	23.3024	8.1583	12653	3.5234	.00184162	1705.883	231573.86
544	295936	160989184	23.3238	8.1633	12688	3.5247	.00183824	1709.025	232427.59
545	297025	161878625	23.3452	8.1683	12723	3.5260	.00183486	1712.167	233282.89
546	298116	162771336	23.3666	8.1733	12758	3.5273	.00183150	1715.308	234139.76
547	299209	163667323	23.3880	8.1783	12793	3.5286	.00182815	1718.450	234998.20
548	300304	164566592	23.4094	8.1833	12828	3.5299	.00182482	1721.591	235858.21
549	301401	165469149	23.4307	8.1882	12863	3.5311	.00182149	1724.733	236719.79
550	**302500**	**166375000**	**23.4521**	**8.1932**	**12899**	**3.5324**	**.00181818**	**1727.875**	**237582.94**
551	303601	167284151	23.4734	8.1982	12934	3.5337	.00181488	1731.016	238447.67
552	304704	168196608	23.4947	8.2031	12969	3.5350	.00181159	1734.158	239313.96
553	305809	169112377	23.5160	8.2081	13004	3.5363	.00180832	1737.299	240181.83
554	306916	170031464	23.5372	8.2130	13040	3.5376	.00180505	1740.441	241051.26
555	308025	170953875	23.5584	8.2180	13075	3.5388	.00180180	1743.582	241922.27
556	309136	171879616	23.5797	8.2229	13110	3.5401	.00179856	1746.724	242794.85
557	310249	172808693	23.6008	8.2278	13146	3.5414	.00179533	1749.866	243668.99
558	311364	173741112	23.6220	8.2327	13181	3.5426	.00179211	1753.007	244544.71
559	312481	174676879	23.6432	8.2377	13217	3.5439	.00178891	1756.149	245422.00
560	**313600**	**175616000**	**23.6643**	**8.2426**	**13252**	**3.5451**	**.00178571**	**1759.290**	**246300.86**
561	314721	176558481	23.6854	8.2475	13288	3.5464	.00178253	1762.432	247181.30
562	315844	177504328	23.7065	8.2524	13323	3.5477	.00177936	1765.574	248063.30
563	316969	178453547	23.7276	8.2573	13359	3.5490	.00177620	1768.715	248946.87
564	318096	179406144	23.7487	8.2621	13394	3.5502	.00177305	1771.857	249832.01
565	319225	180362125	23.7697	8.2670	13430	3.5515	.00176991	1774.998	250718.73
566	320356	181321496	23.7908	8.2719	13466	3.5527	.00176678	1778.140	251607.01
567	321489	182284263	23.8118	8.2768	13501	3.5540	.00176367	1781.282	252496.87
568	322624	183250432	23.8328	8.2816	13537	3.5553	.00176056	1784.423	253388.30
569	323761	184220009	23.8537	8.2865	13573	3.5565	.00175747	1787.565	254281.29

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
570	**324900**	**185193000**	**23 8747**	**8 2913**	**13609**	**3.5577**	**00175439**	**1790.706**	**255175.86**
571	326041	186169411	23 8956	8 2962	13644	3 5590	00175131	1793 848	256072 00
572	327184	187149248	23 9165	8 3010	13680	3 5602	00174825	1796 989	256969 71
573	328329	188132517	23 9374	8 3059	13716	3.5615	00174520	1800.131	257868 99
574	329476	189119224	23 9583	8 3107	13752	3 5627	00174216	1803 273	258769 85
575	330625	190109375	23 9792	8 3155	13788	3.5640	00173913	1806 414	259672 27
576	331776	191102976	24 0000	8 3203	13824	3.5652	00173611	1809 556	260576.26
577	332929	192100033	24.0208	8 3251	13860	3 5664	00173310	1812 697	261481.83
578	334084	193100552	24 0416	8 3300	13896	3 5677	00173010	1815 839	262388 96
579	335241	194104539	24.0624	8 3348	13932	3.5689	00172712	1818.981	263297.67
580	**336400**	**195112000**	**24.0832**	**8 3396**	**13968**	**3.5702**	**00172414**	**1822 122**	**264207.94**
581	337561	196122941	24 1039	8 3443	14004	3.5714	00172117	1825 264	265119 79
582	338724	197137368	24 1247	8 3491	14040	3.5726	00171821	1828 405	266033.21
583	339889	198155287	24 1454	8.3539	14077	3.5738	00171527	1831.547	266948.20
584	341056	199176704	24 1661	8.3587	14113	3.5751	00171233	1834.689	267864.76
585	342225	200201625	24 1868	8 3634	14149	3.5763	00170940	1837 830	268782.89
586	343396	201230056	24.2074	8.3682	14186	3.5775	00170648	1840 972	269702.59
587	344569	202262003	24 2281	8 3730	14222	3.5787	00170358	1844 113	270623 86
588	345744	203297472	24 2487	8 3777	14258	3.5799	00170068	1847 255	271546 70
589	346921	204336469	24 2693	8.3825	14295	3.5812	00169779	1850 397	272471.12
590	**348100**	**205379000**	**24 2899**	**8 3872**	**14331**	**3.5824**	**00169492**	**1853 538**	**273397.10**
591	349281	206425071	24 3105	8 3919	14368	3.5886	00169205	1856 680	274324 66
592	350464	207474688	24 3311	8 3967	14404	3 5848	00168919	1859 821	275253 78
593	351649	208527857	24.3516	8 4014	14440	3.5860	00168634	1862.963	276184.48
594	352836	209584584	24.3721	8.4061	14477	3 5872	00168350	1866 104	277116 75
595	354025	210644875	24 3926	8 4108	14514	3 5884	00168067	1869 246	278050 58
596	355216	211708736	24.4131	8.4155	14550	3.5896	00167785	1872.388	278985.99
597	356409	212776173	24 4336	8 4202	14587	3.5908	00167504	1875 529	279922 97
598	357604	213847192	24 4540	8.4249	14624	3.5920	00167224	1878 671	280861.52
599	358801	214921799	24.4745	8 4296	14660	3.5932	00166945	1881 812	281801.65
600	**360000**	**216000000**	**24.4949**	**8.4343**	**14697**	**3.5944**	**.00166667**	**1884 954**	**282743 34**
601	361201	217081801	24 5153	8 4390	14734	3.5956	00166389	1888 096	283686 60
602	362404	218167208	24 5357	8 4437	14770	3.5958	00166113	1891 237	284631 44
603	363609	219256227	24.5561	8.4484	14807	3.5980	.00165837	1894.379	285577 84
604	364816	220348864	24.5764	8.4530	14844	3 5992	00165563	1897 520	286525 82
605	366025	221445125	24 5967	8.4577	14881	3.6004	00165289	1900 662	287475 36
606	367236	222545016	24.6171	8.4623	14918	3.6016	00165017	1903 804	288426.48
607	368449	223648543	24 6374	8.4670	14955	3 6028	00164745	1906 945	289379.17
608	369664	224755712	24.6577	8.4716	14992	3 6040	00164474	1910 087	290333.43
609	370881	225866529	24 6779	8.4763	15029	3 6052	00164204	1913 228	291289.26
610	**372100**	**226981000**	**24.6982**	**8.4809**	**15066**	**3.6063**	**00163934**	**1916 370**	**292246.66**
611	373321	228099131	24 7184	8.4856	15103	3 6075	00163666	1919 511	293205.63
612	374544	229220928	24 7386	8.4902	15140	3 6087	00163399	1922 653	294166.17
613	375769	230346397	24.7588	8.4948	15177	3.6099	00163132	1925.795	295128.28
614	376996	231475544	24 7790	8 4994	15214	3 6111	00162866	1928 936	296091.97
615	378225	232608375	24 7992	8 5040	15252	3 6122	00162602	1932 078	297057 22
616	379456	233744896	24.8193	8.5086	15289	3 6134	00162338	1935.219	298024 05
617	380689	234885113	24.8395	8 5132	15326	3 6146	00162075	1938 361	298992.44
618	381924	236029032	24 8596	8 5178	15363	3 6158	00161812	1941.503	299962 41
619	383161	237176659	24.8797	8 5224	15400	3.6169	00161551	1944 644	300933.95
620	**384400**	**238328000**	**24 8998**	**8 5270**	**15437**	**3.6181**	**00161290**	**1947 786**	**301907 05**
621	385641	239483061	24 9199	8 5316	15475	3 6192	00161031	1950 927	302881 73
622	386884	240641848	24 9399	8 5362	15513	3 6204	00160772	1954 069	303857.98
623	388129	241804367	24 9600	8 5408	15550	3.6216	00160514	1957 211	304835.80
624	389376	242970624	24 9800	8 5453	15588	3 6227	00160256	1960 352	305815.20
625	390625	244140625	25 0000	8 5499	15625	3 6239	00160000	1963 494	306796.16
626	391876	245314376	25 0200	8.5544	15663	3 6250	00159744	1966 635	307778.69
627	393129	246491883	25 0400	8 5590	15700	3 6262	00159490	1969 777	308762 79
628	394384	247673152	25 0599	8 5635	15738	3 6274	00159236	1972 919	309748 47
629	395641	248858189	25 0799	8.5681	15775	3 6285	00158983	1976 060	310735.71
630	**396900**	**250047000**	**25 0998**	**8 5726**	**15813**	**3 6297**	**00158730**	**1979 202**	**311724.53**
631	398161	251239591	25 1197	8 5772	15850	3 6309	00158479	1982.343	312714 92
632	399424	252435968	25 1396	8 5817	15888	3 6320	00158228	1985 485	313706 88
633	400689	253636137	25 1595	8 5862	15926	3.6331	00157978	1988 626	314700.40
634	401956	254840104	25.1794	8 5907	15964	3.6343	00157729	1991 768	315695.50
635	403225	256047875	25 1992	8 5952	16002	3 6354	00157480	1994 910	316692 17
636	404496	257259456	25.2190	8 5997	16040	3 6366	00157233	1998 051	317690.42
637	405769	258474853	25 2389	8 6043	16077	3 6377	00156986	2001.193	318690.23
638	407044	259694072	25 2587	8 6088	16115	3 6389	00156740	2004 334	319691.61
639	408321	260917119	25 2784	8.6132	16153	3 6400	00156495	2007 476	320694.56

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Area
640	**409600**	**262144000**	**25.2982**	**8.6177**	**16191**	**3.6411**	**.00156250**	**2010.618**	**321699.09**
641	410881	263374721	25.3180	8.6222	16229	3.6423	.00156006	2013.759	322705.18
642	412164	264609288	25.3377	8.6267	16267	3.6435	.00155763	2016.901	323712.85
643	413449	265847707	25.3574	8.6312	16305	3.6446	.00155521	2020.042	324722.09
644	414736	267089984	25.3772	8.6357	16343	3.6457	.00155280	2023.184	325732.89
645	416025	268336125	25.3969	8.6401	16381	3.6468	.00155039	2026.326	326745.27
646	417316	269586136	25.4165	8.6446	16419	3.6479	.00154799	2029.467	327759.22
647	418609	270840023	25.4362	8.6490	16457	3.6489	.00154560	2032.609	328774.74
648	419904	272097792	25.4558	8.6535	16495	3.6502	.00154321	2035.750	329791.83
649	421201	273359449	25.4755	8.6579	16534	3.6513	.00154083	2038.892	330810.49
650	**422500**	**274625000**	**25.4951**	**8.6624**	**16572**	**3.6524**	**.00153846**	**2042.034**	**331830.72**
651	423801	275894451	25.5147	8.6668	16610	3.6536	.00153610	2045.175	332852.53
652	425104	277167808	25.5343	8.6713	16648	3.6547	.00153374	2048.317	333875.90
653	426409	278445077	25.5539	8.6757	16687	3.6558	.00153139	2051.458	334900.85
654	427716	279726264	25.5734	8.6801	16725	3.6569	.00152905	2054.600	335927.36
655	429025	281011375	25.5930	8.6845	16764	3.6580	.00152672	2057.741	336955.45
656	430336	282300416	25.6125	8.6890	16802	3.6592	.00152439	2060.883	337985.10
657	431649	283593393	25.6320	8.6934	16840	3.6603	.00152207	2064.025	339016.33
658	432964	284890312	25.6515	8.6978	16879	3.6614	.00151976	2067.166	340049.13
659	434281	286191179	25.6710	8.7022	16917	3.6625	.00151745	2070.308	341083.50
660	**435600**	**287496000**	**25.6905**	**8.7066**	**16956**	**3.6636**	**.00151515**	**2073.449**	**342119.44**
661	436921	288804781	25.7099	8.7110	16994	3.6647	.00151286	2076.591	343156.95
662	438244	290117528	25.7294	8.7154	17033	3.6658	.00151057	2079.733	344196.03
663	439569	291434247	25.7488	8.7198	17071	3.6669	.00150830	2082.874	345236.69
664	440896	292754944	25.7682	8.7241	17110	3.6680	.00150602	2086.016	346278.91
665	442225	294079625	25.7876	8.7285	17149	3.6691	.00150376	2089.157	347322.70
666	443556	295408296	25.8070	8.7329	17187	3.6702	.00150150	2092.299	348368.07
667	444889	296740963	25.8263	8.7373	17226	3.6713	.00149925	2095.441	349415.00
668	446224	298077632	25.8457	8.7416	17265	3.6724	.00149701	2098.582	350463.51
669	447561	299418309	25.8650	8.7460	17304	3.6735	.00149477	2101.724	351513.59
670	**448900**	**300763000**	**25.8844**	**8.7503**	**17343**	**3.6746**	**.00149254**	**2104.865**	**352565.24**
671	450241	302111711	25.9037	8.7547	17381	3.6757	.00149031	2108.007	353618.45
672	451584	303464448	25.9230	8.7590	17420	3.6768	.00148810	2111.148	354673.24
673	452929	304821217	25.9422	8.7634	17459	3.6779	.00148588	2114.290	355729.60
674	454276	306182024	25.9615	8.7677	17498	3.6790	.00148368	2117.432	356787.54
675	455625	307546875	25.9808	8.7721	17537	3.6801	.00148148	2120.573	357847.04
676	456976	308915776	26.0000	8.7764	17576	3.6812	.00147929	2123.715	358908.11
677	458329	310288733	26.0192	8.7807	17615	3.6823	.00147710	2126.856	359970.75
678	459684	311665752	26.0384	8.7850	17654	3.6834	.00147493	2129.998	361034.97
679	461041	313046839	26.0576	8.7893	17693	3.6845	.00147275	2133.140	362100.75
680	**462400**	**314432000**	**26.0768**	**8.7937**	**17732**	**3.6856**	**.00147059**	**2136.281**	**363168.11**
681	463761	315821241	26.0960	8.7980	17771	3.6866	.00146843	2139.423	364237.04
682	465124	317214568	26.1151	8.8023	17810	3.6877	.00146628	2142.564	365307.54
683	466489	318611987	26.1343	8.8066	17850	3.6888	.00146413	2145.706	366379.60
684	467856	320013504	26.1534	8.8109	17889	3.6899	.00146199	2148.848	367453.24
685	469225	321419125	26.1725	8.8152	17928	3.6909	.00145985	2151.989	368528.45
686	470596	322828856	26.1916	8.8194	17967	3.6920	.00145773	2155.131	369605.23
687	471969	324242703	26.2107	8.8237	18007	3.6931	.00145560	2158.272	370683.59
688	473344	325660672	26.2298	8.8280	18046	3.6942	.00145349	2161.414	371763.51
689	474721	327082769	26.2488	8.8323	18085	3.6953	.00145138	2164.556	372845.00
690	**476100**	**328509000**	**26.2679**	**8.8366**	**18125**	**3.6963**	**.00144928**	**2167.697**	**373928.07**
691	477481	329939371	26.2869	8.8408	18164	3.6974	.00144718	2170.839	375012.70
692	478864	331373888	26.3059	8.8451	18204	3.6985	.00144509	2173.980	376098.91
693	480249	332812557	26.3249	8.8493	18243	3.6995	.00144300	2177.122	377186.68
694	481636	334255384	26.3439	8.8536	18283	3.7006	.00144092	2180.263	378276.03
695	483025	335702375	26.3629	8.8578	18322	3.7016	.00143885	2183.405	379366.95
696	484416	337153536	26.3818	8.8621	18362	3.7027	.00143678	2186.547	380459.44
697	485809	338608873	26.4008	8.8663	18401	3.7038	.00143472	2189.688	381553.50
698	487204	340068392	26.4197	8.8706	18441	3.7049	.00143266	2192.830	382649.13
699	488601	341532099	26.4386	8.8748	18480	3.7059	.00143062	2195.971	383746.33
700	**490000**	**343000000**	**26.4575**	**8.8790**	**18520**	**3.7070**	**.00142857**	**2199.113**	**384845.10**
701	491401	344472101	26.4764	8.8833	18560	3.7080	.00142653	2202.255	385945.44
702	492804	345948408	26.4953	8.8875	18600	3.7091	.00142450	2205.396	387047.36
703	494209	347428927	26.5141	8.8917	18640	3.7101	.00142248	2208.538	388150.84
704	495616	348913664	26.5330	8.8959	18679	3.7112	.00142045	2211.679	389255.90
705	497025	350402625	26.5518	8.9001	18719	3.7123	.00141844	2214.821	390362.52
706	498436	351895816	26.5707	8.9043	18759	3.7133	.00141643	2217.963	391470.72
707	499849	353393243	26.5895	8.9085	18799	3.7144	.00141443	2221.104	392580.49
708	501264	354894912	26.6083	8.9127	18839	3.7154	.00141243	2224.246	393691.82
709	502681	356400829	26.6271	8.9169	18879	3.7165	.00141044	2227.387	394804.73

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
710	**504100**	**357911000**	**26.6458**	**8.9211**	**18919**	**3.7175**	**.00140845**	**2230.529**	**395919.21**
711	505521	359425431	26 6646	8 9253	18959	3.7185	.00140647	2233.670	397035.26
712	506944	360944128	26 6833	8 9295	18999	3.7196	00140449	2236.812	398152.89
713	508369	362467097	26.7021	8.9337	19039	3.7206	.00140252	2239.954	399272.08
714	509796	363994344	26.7208	8 9378	19079	3.7217	.00140056	2243.095	400392.84
715	511225	365525875	26 7395	8 9420	19119	3.7227	00139860	2246 237	401515.18
716	512656	367061696	26.7582	8.9462	19159	3.7238	.00139665	2249.378	402639.08
717	514089	368601813	26.7769	8 9503	19199	3.7248	.00139470	2252.520	403764.56
718	515524	370146232	26 7955	8.9545	19239	3.7258	.00139276	2255.662	404891.60
719	516961	371694959	26.8142	8 9587	19280	3.7269	.00139082	2258.803	406020.22
720	**518400**	**373248000**	**26.8328**	**8 9628**	**19320**	**3 7279**	**.00138889**	**2261.945**	**407150.41**
721	519841	374805361	26.8514	8 9670	19360	3.7290	.00138696	2265.086	408282.17
722	521284	376367048	26 8701	8 9711	19400	3 7300	.00138504	2268.228	409415.50
723	522729	377933067	26.8887	8.9752	19440	3.7310	.00138313	2271.370	410550.40
724	524176	379503424	26.9072	8.9794	19481	3.7321	.00138122	2274.511	411686.87
725	525625	381078125	26 9258	8 9835	19521	3 7331	.00137931	2277.653	412824.91
726	527076	382657176	26.9444	8.9876	19562	3.7341	.00137741	2280.794	413964.52
727	528529	384240583	26 9629	8 9918	19602	3.7351	00137552	2283 936	415105.71
728	529984	385828352	26.9815	8.9959	19643	3 7362	00137363	2287.078	416248.46
729	531441	387420489	27.0000	9.0000	19683	3 7372	.00137174	2290.219	417392.79
730	**532900**	**389017000**	**27.0185**	**9.0041**	**19724**	**3.7382**	**.00136986**	**2293.361**	**418538.68**
731	534361	390617891	27.0370	9.0082	19764	3.7392	00136799	2296 502	419686.15
732	535824	392223168	27.0555	9.0123	19805	3 7403	.00136612	2299 644	420835.19
733	537289	393832837	27.0740	9.0164	19845	3.7413	.00136426	2302.785	421985.79
734	538756	395446904	27.0924	9.0205	19886	3.7423	.00136240	2305.927	423137.97
735	540225	397065375	27.1109	9.0246	19927	3.7433	.00136054	2309 069	424291.72
736	541696	398688256	27.1293	9.0287	19967	3.7443	.00135870	2312.210	425447.04
737	543169	400315553	27 1477	9 0328	20008	3.7454	.00135685	2315.352	426603.94
738	544644	401947272	27.1662	9 0369	20049	3.7464	00135501	2318 493	427762.40
739	546121	403583419	27.1846	9.0410	20090	3.7474	.00135318	2321.635	428922.43
740	**547600**	**405224000**	**27.2029**	**9.0450**	**20130**	**3.7484**	**.00135135**	**2324.777**	**430084.03**
741	549081	406869021	27.2213	9.0491	20171	3 7494	00134953	2327.918	431247.21
742	550564	408518488	27.2397	9 0532	20212	3 7504	00134771	2331 060	432411.95
743	552049	410172407	27.2580	9.0572	20253	3.7514	.00134590	2334.201	433578.27
744	553536	411830784	27.2764	9.0613	20294	3.7524	.00134409	2337.343	434746.16
745	555025	413493625	27.2947	9 0654	20335	3 7534	00134228	2340.485	435915.62
746	556516	415160936	27.3130	9 0694	20376	3.7545	.00134048	2343.626	437086.64
747	558009	416832723	27.3313	9.0735	20417	3.7555	.00133869	2346.768	438259.24
748	559504	418508992	27 3496	9 0775	20458	3.7565	.00133690	2349.909	439433.41
749	561001	420189749	27.3679	9.0816	20499	3.7575	.00133511	2353.051	440609.16
750	**562500**	**421875000**	**27.3861**	**9.0856**	**20540**	**3.7585**	**.00133333**	**2356.193**	**441786.47**
751	564001	423564751	27.4044	9.0896	20581	3.7595	.00133156	2359.334	442965.35
752	565504	425259008	27.4226	9 0937	20622	3 7605	.00132979	2362.476	444145.80
753	567009	426957777	27.4408	9.0977	20663	3.7615	.00132802	2365.617	445327.83
754	568516	428661064	27.4591	9 1017	20704	3.7625	.00132626	2368.759	446511.42
755	570025	430368875	27.4773	9.1057	20745	3.7635	.00132450	2371.900	447696.59
756	571536	432081216	27.4955	9.1098	20787	3.7645	.00132275	2375.042	448883.32
757	573049	433798093	27.5136	9.1138	20828	3.7655	.00132100	2378.184	450071.63
758	574564	435519512	27 5318	9.1178	20869	3 7665	.00131926	2381.325	451261.51
759	576081	437245479	27.5500	9.1218	20910	3.7675	.00131752	2384.467	452452.96
760	**577600**	**438976000**	**27.5681**	**9 1258**	**20952**	**3.7685**	**.00131579**	**2387.608**	**453645.98**
761	579121	440711081	27.5862	9.1298	20993	3.7694	.00131406	2390 750	454840.57
762	580644	442450728	27.6043	9.1338	21035	3 7704	.00131234	2393 892	456036.73
763	582169	444194947	27.6225	9.1378	21076	3.7714	.00131062	2397 033	457234.46
764	583696	445943744	27.6405	9.1418	21117	3.7724	.00130890	2400.175	458433.77
765	585225	447697125	27.6586	9.1458	21159	3.7734	00130719	2403.316	459634.64
766	586756	449455096	27.6767	9.1498	21200	3.7744	.00130548	2406.458	460837.08
767	588289	451217663	27.6948	9 1537	21242	3.7754	.00130378	2409.600	462041.10
768	589824	452984832	27 7128	9.1577	21283	3.7764	.00130208	2412.741	463246.69
769	591361	454756609	27.7308	9.1617	21325	3.7774	00130039	2415.883	464453.84
770	**592900**	**456533000**	**27.7489**	**9.1657**	**21367**	**3.7784**	**.00129870**	**2419.024**	**465662.57**
771	594441	458314011	27.7669	9 1696	21408	3 7793	.00129702	2422.166	466872.87
772	595984	460099648	27 7849	9.1736	21450	3 7803	.00129534	2425.307	468084.74
773	597529	461889917	27.8029	9.1775	21492	3.7813	.00129366	2428.449	469298.19
774	599076	463684824	27.8209	9.1815	21533	3.7822	00129199	2431.591	470513.19
775	600625	465484375	27.8388	9.1855	21575	3.7832	.00129032	2434.732	471729.77
776	602176	467288576	27.8568	9.1894	21617	3.7842	.00128866	2437.874	472947.92
777	603729	469097433	27 8747	9 1933	21658	3 7852	.00128700	2441.015	474167.65
778	605284	470910952	27.8927	9.1973	21700	3.7861	00128535	2444.157	475388.94
779	606841	472729139	27.9106	9.2012	21742	3.7871	.00128370	2447.299	476611.81

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam) Circum	Area
780	**608400**	**474552000**	**27 9285**	**9.2052**	**21784**	**3 7881**	**.00128205**	**2450.440**	**477836.24**
781	609961	476379541	27.9464	9 2091	21826	3 7890	.00128041	2453.582	479062.25
782	611524	478211768	27.9643	9.2130	21868	3 7900	.00127877	2456.723	480289 83
783	613089	480048687	27.9821	9.2170	21910	3.7910	.00127714	2459.865	481518.97
784	614656	481890304	28.0000	9.2209	21952	3 7920	00127551	2463.007	482749.69
785	616225	483736625	28.0179	9 2248	21994	3 7929	.00127389	2466.148	483981.98
786	617796	485587656	28.0357	9.2287	22036	3 7939	00127226	2469.290	485215.84
787	619369	487443403	28 0535	9 2326	22078	3 7949	00127065	2472.431	486451 28
788	620944	489303872	28.0713	9 2365	22120	3 7959	.00126904	2475 573	487688.28
789	622521	491169069	28.0891	9.2404	22162	3 7969	.00126743	2478.715	488926.85
790	**624100**	**493039000**	**28.1069**	**9.2443**	**22205**	**3.7978**	**.00126582**	**2481.856**	**490166.99**
791	625681	494913671	28.1247	9.2482	22247	3 7987	00126422	2484.998	491408.71
792	627264	496793088	28.1425	9.2521	22289	3 7997	00126263	2488.139	492651.99
793	628849	498677257	28.1603	9 2560	22331	3.8006	00126103	2491.281	493896.85
794	630436	500566184	28.1780	9 2599	22373	3 8016	00125945	2494.422	495143.28
795	632025	502459875	28.1957	9 2638	22416	3 8025	00125786	2497.564	496391.27
796	633616	504358336	28 2135	9.2677	22458	3.8035	00125628	2500.706	497640.84
797	635209	506261573	28 2312	9 2716	22500	3 8044	00125471	2503 847	498891.98
798	636804	508169592	28 2489	9 2754	22543	3 8054	00125313	2506 989	500144.69
799	638401	510082399	28.2666	9.2793	22585	3.8064	.00125156	2510.130	501398.97
800	**640000**	**512000000**	**28.2843**	**9.2832**	**22627**	**3.8073**	**00125000**	**2513.272**	**502654.82**
801	641601	513922401	28 3019	9 2870	22670	3 8083	00124844	2516 414	503912.25
802	643204	515849608	28.3196	9 2909	22712	3 8092	00124688	2519 555	505171.24
803	644809	517781627	28.3373	9.2948	22755	3.8102	00124533	2522.697	506431.80
804	646416	519718464	28 3549	9 2986	22797	3 8111	00124378	2525.838	507693.94
805	648025	521660125	28 3725	9 3025	22840	3 8121	00124224	2528.980	508957.64
806	649636	523606616	28.3901	9.3063	22883	3 8130	.00124069	2532.122	510222.92
807	651249	525557943	28 4077	9.3102	22925	3 8139	00123916	2535.263	511489.77
808	652864	527514112	28 4253	9 3140	22968	3 8149	00123762	2538.405	512758.19
809	654481	529475129	28.4429	9.3179	23010	3 8158	.00123609	2541.546	514028.18
810	**656100**	**531441000**	**28 4605**	**9 3217**	**23053**	**3 8168**	**00123457**	**2544 688**	**515299.74**
811	657721	533411731	28.4781	9 3255	23096	3 8177	00123305	2547.829	516572.87
812	659344	535387328	28 4956	9 3294	23138	3 8186	00123153	2550 971	517847.57
813	660969	537367797	28.5132	9.3332	23181	3.8196	.00123001	2554.113	519123.84
814	662596	539353144	28 5307	9 3370	23224	3.8205	00122850	2557.254	520401.68
815	664225	541343375	28.5482	9.3408	23267	3 8215	00122699	2560 396	521681.10
816	665856	543338496	28.5657	9.3447	23310	3.8224	00122549	2563.537	522962.08
817	667489	545338513	28.5832	9 3485	23352	3 8234	.00122399	2566 679	524244.63
818	669124	547343432	28.6007	9.3523	23395	3 8243	.00122249	2569.821	525528.76
819	670761	549353259	28.6182	9.3561	23438	3 8252	00122100	2572.962	526814.46
820	**672400**	**551368000**	**28.6356**	**9.3599**	**23481**	**3 8262**	**00121951**	**2576.104**	**528101.73**
821	674041	553387661	28 6531	9 3637	23524	3 8271	.00121803	2579.245	529390.56
822	675684	555412248	28 6705	9 3675	23567	3 8280	.00121655	2582.387	530680.97
823	677329	557441767	28.6880	9.3713	23610	3.8290	.00121507	2585.529	531972.95
824	678976	559476224	28 7054	9 3751	23653	3.8299	00121359	2588.670	533266.50
825	680625	561515625	28.7228	9 3789	23696	3 8308	00121212	2591 812	534561.62
826	682276	563559976	28.7402	9.3827	23740	3 8317	00121065	2594.953	535858.32
827	683929	565609283	28 7576	9 3865	23783	3 8327	00120919	2598 095	537156.58
828	685584	567663552	28 7750	9 3902	23826	3 8336	00120773	2601.237	538456.41
829	687241	569722789	28.7924	9.3940	23869	3 8345	.00120627	2604.378	539757.82
830	**688900**	**571787000**	**28.8097**	**9.3978**	**23912**	**3.8355**	**00120482**	**2607.520**	**541060.79**
831	690561	573856191	28 8271	9 4016	23955	3 8364	00120337	2610.661	542365.34
832	692224	575930368	28 8444	9 4053	23999	3 8373	00120192	2613.803	543671.46
833	693889	578009537	28 8617	9.4091	24042	3 8382	00120048	2616.944	544979.15
834	695556	580093704	28 8791	9 4129	24085	3 8391	00119904	2620.086	546288 40
835	697225	582182875	28 8964	9.4166	24128	3 8401	00119760	2623.228	547599.23
836	698896	584277056	28.9137	9.4204	24172	3 8410	00119617	2626.369	548911.63
837	700569	586376253	28 9310	9.4241	24215	3 8419	00119474	2629.511	550225.61
838	702244	588480472	28 9482	9 4279	24259	3 8428	00119332	2632.652	551541.15
839	703921	590589719	28.9655	9.4316	24302	3 8437	00119190	2635.794	552858 26
840	**705600**	**592704000**	**28.9828**	**9.4354**	**24346**	**3 8446**	**.00119048**	**2638.936**	**554176.94**
841	707281	594823321	29 0000	9 4391	24389	3.8456	00118906	2642 077	555497.20
842	708964	596947688	29.0172	9.4429	24432	3.8465	00118765	2645.219	556819.02
843	710649	599077107	29.0345	9.4466	24476	3.8474	.00118624	2648.360	558142.42
844	712336	601211584	29 0517	9.4503	24520	3.8483	00118483	2651.502	559467 39
845	714025	603351125	29 0689	9.4541	24563	3.8492	00118343	2654.644	560793.92
846	715716	605495736	29.0861	9.4578	24607	3 8501	.00118203	2657.785	562122 03
847	**717409**	**607645423**	**29.1033**	**9.4615**	**24650**	**3 8510**	**00118064**	**2660.927**	**563451 71**
848	**719104**	**609800192**	**29 1204**	**9.4652**	**24694**	**3 8519**	**00117925**	**2664 068**	**564782 96**
849	**720801**	**611960049**	**29 1376**	**9.4690**	**24738**	**3.8528**	**00117786**	**2667.210**	**566115.78**

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Area
850	**722500**	**614125000**	**29.1548**	**9.4727**	**24782**	**3.8538**	**.00117647**	**2670.352**	**567450.17**
851	724201	616295051	29.1719	9.4764	24825	3.8547	.00117509	2673.493	568786.14
852	725904	618470208	29.1890	9.4801	24869	3.8556	.00117371	2676.635	570123.67
853	727609	620650477	29.2062	9.4838	24913	3.8565	.00117233	2679.776	571462.77
854	729316	622835864	29.2233	9.4875	24957	3.8574	.00117096	2682.918	572803.45
855	731025	625026375	29.2404	9.4912	25000	3.8582	.00116959	2686.059	574145.69
856	732736	627222016	29.2575	9.4949	25044	3.8592	.00116822	2689.201	575489.51
857	734449	629422793	29.2746	9.4986	25088	3.8601	.00116686	2692.343	576834.90
858	736164	631628712	29.2916	9.5023	25132	3.8610	.00116550	2695.484	578181.85
859	737881	633839779	29.3087	9.5060	25176	3.8619	.00116414	2698.626	579530.38
860	**739600**	**636056000**	**29.3258**	**9.5097**	**25220**	**3.8628**	**.00116279**	**2701.767**	**580880.48**
861	741321	638277381	29.3428	9.5134	25264	3.8637	.00116144	2704.909	582232.15
862	743044	640503928	29.3598	9.5171	25308	3.8646	.00116009	2708.051	583585.39
863	744769	642735647	29.3769	9.5207	25352	3.8655	.00115875	2711.192	584940.20
864	746496	644972544	29.3939	9.5244	25396	3.8664	.00115741	2714.334	586296.59
865	748225	647214625	29.4109	9.5281	25440	3.8673	.00115607	2717.475	587654.54
866	749956	649461896	29.4279	9.5317	25485	3.8682	.00115473	2720.617	589014.07
867	751689	651714363	29.4449	9.5354	25529	3.8691	.00115340	2723.759	590375.16
868	753424	653972032	29.4618	9.5391	25573	3.8700	.00115207	2726.900	591737.83
869	755161	656234909	29.4788	9.5427	25617	3.8708	.00115075	2730.042	593102.06
870	**756900**	**658503000**	**29.4958**	**9.5464**	**25661**	**3.8717**	**.00114943**	**2733.183**	**594467.87**
871	758641	660776311	29.5127	9.5501	25706	3.8726	.00114811	2736.325	595835.25
872	760384	663054848	29.5296	9.5537	25750	3.8735	.00114679	2739.466	597204.20
873	762129	665338617	29.5466	9.5574	25794	3.8744	.00114548	2742.608	598574.72
874	763876	667627624	29.5635	9.5610	25839	3.8753	.00114416	2745.750	599946.81
875	765625	669921875	29.5804	9.5647	25883	3.8762	.00114286	2748.891	601320.47
876	767376	672221376	29.5973	9.5683	25927	3.8771	.00114155	2752.033	602695.70
877	769129	674526133	29.6142	9.5719	25972	3.8780	.00114025	2755.174	604072.50
878	770884	676836152	29.6311	9.5756	26016	3.8789	.00113895	2758.316	605450.88
879	772641	679151439	29.6479	9.5792	26061	3.8797	.00113766	2761.458	606830.82
880	**774400**	**681472000**	**29.6648**	**9.5828**	**26105**	**3.8806**	**.00113636**	**2764.599**	**608212.34**
881	776161	683797841	29.6816	9.5865	26150	3.8815	.00113507	2767.741	609595.42
882	777924	686128968	29.6985	9.5901	26194	3.8823	.00113379	2770.882	610980.08
883	779689	688465387	29.7153	9.5937	26239	3.8832	.00113250	2774.024	612366.31
884	781456	690807104	29.7321	9.5973	26283	3.8841	.00113122	2777.166	613754.11
885	783225	693154125	29.7489	9.6010	26328	3.8850	.00112994	2780.307	615143.48
886	784996	695506456	29.7658	9.6046	26373	3.8859	.00112867	2783.449	616534.42
887	786769	697864103	29.7825	9.6082	26417	3.8868	.00112740	2786.590	617926.93
888	788544	700227072	29.7993	9.6118	26462	3.8877	.00112613	2789.732	619321.01
889	790321	702595369	29.8161	9.6154	26507	3.8885	.00112486	2792.874	620716.65
890	**792100**	**704969000**	**29.8329**	**9.6190**	**26551**	**3.8894**	**.00112360**	**2796.015**	**622113.89**
891	793881	707347971	29.8496	9.6226	26596	3.8902	.00112233	2799.157	623512.68
892	795664	709732288	29.8664	9.6262	26641	3.8911	.00112108	2802.298	624913.04
893	797449	712121957	29.8831	9.6298	26686	3.8920	.00111982	2805.440	626314.96
894	799236	714516984	29.8998	9.6334	26730	3.8929	.00111857	2808.581	627718.49
895	801025	716917375	29.9166	9.6370	26775	3.8937	.00111732	2811.723	629123.56
896	802816	719323136	29.9333	9.6406	26820	3.8946	.00111607	2814.865	630530.21
897	804609	721734273	29.9500	9.6442	26865	3.8955	.00111483	2818.006	631938.43
898	806404	724150792	29.9666	9.6477	26910	3.8963	.00111359	2821.148	633348.22
899	808201	726572699	29.9833	9.6513	26955	3.8972	.00111235	2824.289	634759.58
900	**810000**	**729000000**	**30.0000**	**9.6549**	**27000**	**3.8981**	**.00111111**	**2827.431**	**636172.51**
901	811801	731432701	30.0167	9.6585	27045	3.8989	.00110988	2830.573	637587.01
902	813604	733870808	30.0333	9.6620	27090	3.8998	.00110865	2833.714	639003.09
903	815409	736314327	30.0500	9.6656	27135	3.9007	.00110742	2836.856	640420.73
904	817216	738763264	30.0666	9.6692	27180	3.9015	.00110619	2839.997	641839.95
905	819025	741217625	30.0832	9.6727	27225	3.9024	.00110497	2843.139	643260.73
906	820836	743677416	30.0998	9.6763	27270	3.9032	.00110375	2846.281	644683.09
907	822649	746142643	30.1164	9.6799	27316	3.9041	.00110254	2849.422	646107.01
908	824464	748613312	30.1330	9.6834	27361	3.9050	.00110132	2852.564	647532.51
909	826281	751089429	30.1496	9.6870	27406	3.9059	.00110011	2855.705	648959.53
910	**828100**	**753571000**	**30.1662**	**9.6905**	**27451**	**3.9067**	**.00109890**	**2858.847**	**650288.22**
911	829921	756058031	30.1828	9.6941	27497	3.9076	.00109769	2861.988	651818.43
912	831744	758550528	30.1993	9.6976	27542	3.9084	.00109649	2865.130	653250.21
913	833569	761048497	30.2159	9.7012	27587	3.9093	.00109529	2868.272	654683.56
914	835396	763551944	30.2324	9.7047	27632	3.9101	.00109409	2871.413	656118.48
915	837225	766060875	30.2490	9.7082	27678	3.9110	.00109290	2874.555	657554.98
916	839056	768575296	30.2655	9.7118	27723	3.9118	.00109170	2877.696	658993.04
917	840889	771095213	30.2820	9.7153	27769	3.9127	.00109051	2880.838	660432.68
918	842724	773620632	30.2985	9.7188	27814	3.9135	.00108932	2883.980	661873.88
919	844561	776151559	30.3150	9.7224	27859	3.9144	.00108814	2887.121	663316.66

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Area
920	**846400**	**778688000**	**30.3315**	**9.7259**	**27905**	**3.9153**	**.00108696**	**2890.263**	**664761.01**
921	848241	781229961	30.3480	9.7294	27950	3.9161	.00108578	2893.404	666206.92
922	850084	783777448	30.3645	9.7329	27996	3.9169	.00108460	2896.546	667654.41
923	851929	786330467	30.3809	9.7364	28042	3.9178	.00108342	2899.688	669103.47
924	853776	788889024	30.3974	9.7400	28087	3.9186	.00108225	2902.829	670554.10
925	855625	791453125	30.4138	9.7435	28133	3.9194	.00108108	2905.971	672006.30
926	857476	794022776	30.4302	9.7470	28179	3.9203	.00107991	2909.112	673460.08
927	859329	796597983	30.4467	9.7505	28224	3.9212	.00107875	2912.254	674915.42
928	861184	799178752	30.4631	9.7540	28270	3.9220	.00107759	2915.396	676372.33
929	863041	801765089	30.4795	9.7575	28315	3.9229	.00107643	2918.537	677830.82
930	**864900**	**804357000**	**30.4959**	**9.7610**	**28361**	**3.9237**	**.00107527**	**2921.679**	**679290.87**
931	866761	806954491	30.5123	9.7645	28407	3.9246	.00107411	2924.820	680752.50
932	868624	809557568	30.5287	9.7680	28453	3.9254	.00107296	2927.962	682215.69
933	870489	812166237	30.5450	9.7715	28499	3.9262	.00107181	2931.103	683680.46
934	872356	814780504	30.5614	9.7750	28544	3.9271	[illegible]	[illegible]	[illegible] 80
935	874225	817400375	30.5778	9.7785	28590	3.9279	[illegible]	[illegible]	[illegible] 71
936	876096	820025856	30.5941	9.7819	28636	3.9288	[illegible]	[illegible]	[illegible] 19
937	877969	822656953	30.6105	9.7854	28682	3.9296	.00106724	2943.670	689555.24
938	879844	825293672	30.6268	9.7889	28728	3.9304	.00106610	2946.811	691027.86
939	881721	827936019	30.6431	9.7924	28774	3.9313	.00106496	2949.953	692502.05
940	**883600**	**830584000**	**30.6594**	**9.7959**	**28820**	**3.9321**	**.00106383**	**2953.095**	**693977.82**
941	885481	833237621	30.6757	9.7993	28866	3.9329	.00106270	2956.236	695455.15
942	887364	835896888	30.6920	9.8028	28912	3.9338	.00106157	2959.378	696934.06
943	889249	838561807	30.7083	9.8063	28958	3.9346	.00106045	2962.519	698414.53
944	891136	841232384	30.7246	9.8097	29004	3.9354	.00105932	2965.661	699896.58
945	893025	843908625	30.7409	9.8132	29050	3.9363	.00105820	2968.803	701380.19
946	894916	846590536	30.7571	9.8167	29096	3.9371	.00105708	2971.944	702865.38
947	896809	849278123	30.7734	9.8201	29142	3.9379	[illegible]	[illegible]	[illegible] 14
948	898704	851971392	30.7896	9.8236	29189	3.9388	[illegible]	[illegible]	[illegible] 17
949	900601	854670349	30.8058	9.8270	29235	3.9396	[illegible]	[illegible]	[illegible] 27
950	**902500**	**857375000**	**30.8221**	**9.8305**	**29281**	**3.9404**	**.00105263**	**2984.511**	**708821.84**
951	904401	860085351	30.8383	9.8339	29327	3.9413	.00105152	2987.652	710314.88
952	906304	862801408	30.8545	9.8374	29374	3.9421	.00105042	2990.794	711809.50
953	908209	865523177	30.8707	9.8408	29420	3.9429	.00104932	2993.935	713305.68
954	910116	868250664	30.8869	9.8443	29466	3.9438	.00104822	2997.077	714803.43
955	912025	870983875	30.9031	9.8477	29513	3.9446	.00104712	3000.218	716302.76
956	913936	873722816	30.9192	9.8511	29559	3.9454	.00104603	3003.360	717803.66
957	915849	876467493	30.9354	9.8546	29605	3.9462	[illegible]	[illegible]	[illegible] 12
958	917764	879217912	30.9516	9.8580	29652	3.9471	[illegible]	[illegible]	[illegible] 19
959	919681	881974079	30.9677	9.8614	29698	3.9479	[illegible]	[illegible]	[illegible] 77
960	**921600**	**884736000**	**30.9839**	**9.8648**	**29745**	**3.9487**	**.00104167**	**3015.926**	**723822.95**
961	923521	887503681	31.0000	9.8683	29791	3.9495	.00104058	3019.068	725331.70
962	925444	890277128	31.0161	9.8717	29838	3.9503	.00103950	3022.210	726842.02
963	927369	893056347	31.0322	9.8751	29884	3.9512	.00103842	3025.351	728353.91
964	929296	895841344	31.0483	9.8785	29931	3.9520	.00103734	3028.493	729867.37
965	931225	898632125	31.0644	9.8819	29977	3.9528	.00103627	3031.634	731382.40
966	933156	901428696	31.0805	9.8854	30024	3.9536	.00103520	3034.776	732899.01
967	935089	904231063	31.0966	9.8888	30070	3.9544	.00103413	3037.918	734417.18
968	937024	907039232	31.1127	9.8922	30117	3.9553	.00103306	3041.059	735936.93
969	938961	909853209	31.1288	9.8956	30164	3.9561	.00103199	3044.201	737458.24
970	**940900**	**912673000**	**31.1448**	**9.8990**	**30210**	**3.9569**	**.00103093**	**3047.342**	**738981.13**
971	942841	915498611	31.1609	9.9024	30257	3.9577	.00102987	3050.484	740505.59
972	944784	918330048	31.1769	9.9058	30304	3.9585	.00102881	3053.625	742031.62
973	946729	921167317	31.1929	9.9092	30351	3.9593	.00102775	3056.767	743559.22
974	948676	924010424	31.2090	9.9126	30398	3.9602	[illegible]	[illegible]	[illegible] 39
975	950625	926859375	31.2250	9.9160	30444	3.9610	[illegible]	[illegible]	[illegible] 13
976	952576	929714176	31.2410	9.9194	30491	3.9618	[illegible]	[illegible]	[illegible] 44
977	954529	932574833	31.2570	9.9227	30538	3.9626	.00102354	3069.333	749685.32
978	956484	935441352	31.2730	9.9261	30585	3.9634	.00102249	3072.475	751220.78
979	958441	938313739	31.2890	9.9295	30632	3.9642	.00102145	3075.617	752757.80
980	**960400**	**941192000**	**31.3050**	**9.9329**	**30679**	**3.9650**	**.00102041**	**3078.758**	**754296.40**
981	962361	944076141	31.3209	9.9363	30726	3.9658	.00101937	3081.900	755836.59
982	964324	946966168	31.3369	9.9396	30773	3.9666	.00101833	3085.041	757378.30
983	966289	949862087	31.3528	9.9430	30820	3.9674	.00101729	3088.183	758921.61
984	968256	952763904	31.3688	9.9464	30867	3.9682	.00101626	3091.325	760466.48
985	970225	955671625	31.3847	9.9497	30914	3.9691	.00101523	3094.466	762012.93
986	972196	958585256	31.4006	9.9531	30961	3.9699	.00101420	3097.608	763560.95
987	974169	961504803	31.4166	9.9565	31008	3.9707	[illegible]	[illegible]	[illegible] 54
988	976144	964430272	31.4325	9.9598	31055	3.9715	[illegible]	[illegible]	[illegible] 70
989	978121	967361669	31.4484	9.9632	31102	3.9723	[illegible]	[illegible]	[illegible] 44

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
								Circum	Area
990	**980100**	**970299000**	**31.4643**	**9.9666**	**31150**	**3.9731**	**00101010**	**3110.174**	**769768.74**
991	982081	973242271	31.4802	9.9699	31197	3.9739	00100908	3113.316	771324.61
992	984064	976191488	31.4960	9.9733	31244	3.9747	00100806	3116.457	772882.06
993	986049	979146657	31.5119	9.9766	31291	3.9755	00100705	3119.599	774441.07
994	988036	982107784	31.5278	9.9800	31339	3.9763	00100604	3122.740	776001.66
995	990025	985074875	31.5436	9.9833	31386	3.9771	00100503	3125.882	777563.82
996	992016	988047936	31.5595	9.9866	31433	3.9779	00100402	3129.024	779127.54
997	994009	991026973	31.5753	9.9900	31480	3.9787	00100301	3132.165	780692.84
998	996004	994011992	31.5911	9.9933	31528	3.9795	00100200	3135.307	782259.71
999	998001	997002999	31.6070	9.9967	31575	3.9803	00100100	3138.448	783828.15
1000	**1000000**	**1000000000**	**31.6228**	**10.0000**	**31623**	**3.9811**	**00100000**	**3141.593**	**785398.16**

3. MEASURES, WEIGHTS, AND UNITS

Flow and capacity equivalents, Tables 8 and 9, are based on the following values.

1 cubic foot = 7.480519 U. S. gallons
1 U. S. gallon = 231 cu in = 0.13368056 cu ft
1 lb of water at 62° F = 0.01603489 cu ft

The British Imperial gallon = 277.418 cu in (U.S.) = 277.420 cu in (British) = 1.20094 U. S. gallons.

The flow equivalents of Table 8 are based on water at 62° F. These flow equivalents for a given weight of water can be converted to flow equivalents for any other fluid by multiplying by the ratio of the density of that fluid to the density of water. For example the flow per minute of 20 lb of oil of specific gravity 0.90 (water = 1.0) is

$$(1/0.90) \times 20 \times 0.1200 = 2.67 \text{ gal. per min.}$$

Table 8. Flow Equivalents

	Cu. ft per sec.	Cu. ft per min	U. S. Gallons per min.	U. S. Gals per hr.	U. S. Gallons per 24 hr	Lb H_2O per min.	Lb. H_2O per hr
1 cu. ft. per sec =	1	60	448 83	26,929 9	646,316 8	3741 00	224,460
1 cu. ft per min. =	1/60	1	7 4805	448 83	10 771 2	62 35	3741 00
1 gal per min. =	0 002228	0 1337	1	60	1440	8 3350	500 1
1 gal per hr =	3713×10^{-5}	0 002280	1/60	1	24	0 1389	8 3350
1 gal per 24 hr. =	1547×10^{-6}	9283×10^{-5}	6944×10^{-4}	1/24	1	0 005788	0 3473
1 lb H_2O per min. =	2673×10^{-3}	0 01604	0 1200	7 1986	172 7658	1	60
1 lb H_2O per hr =	4455×10^{-5}	2673×10^{-3}	0 0020	0 1200	2 8794	1 60	1

Table 9. Gallon and Cubic Feet Equivalents

Gallons or cu ft	Gallons equivalent to cu ft	Cu ft. equivalent to gallons	Gallons or cu. ft.	Gallons equivalent to cu ft.	Cu. ft. equivalent to gallons	Gallons or cu. ft.	Gallons equivalent to cu. ft.	Cu. ft. equivalent to gallons
0 1	0 75	0 0134	100	748 0	13 368	100,000	748,051 9	13,368 1
.2	1 50	.0267	200	1,496.1	26 736	200,000	1,496,103 8	26,736 1
.3	2.24	.0401	300	2,244 2	40 104	300,000	2,244,155 7	40,104.2
.4	2 99	.0535	400	2,992 2	53 472	400,000	2,992,207 6	53,472 2
.5	3.74	.0668	500	3,740 3	66 840	500,000	3,740,259 5	66,840 3
.6	4.49	.0802	600	4,488 3	80.208	600,000	4,488,311.4	80,208 3
.7	5.24	.0936	700	5,236 4	93.576	700,000	5,236,363 3	93,576 4
.8	5.98	.1069	800	5,984 4	106 944	800,000	5,984,415 2	106,944 4
.9	6.73	.1203	900	6,732 5	120.312	900,000	6,732,467 1	120,312 5
1 0	7 48	.134	1,000	7,480 5	133 681	1,000,000	7,480,519	133,680 6
2	14 96	.267	2,000	14,961 0	267 361	2,000,000	14,961,038	267,361 1
3	22.44	.401	3,000	22,441.6	401 042	3,000,000	22,441,557	401,041.7
4	29 92	.535	4,000	29,922 1	534 722	4,000,000	29,922,076	534,722.2
5	37.40	.668	5,000	37,402 6	668 403	5,000,000	37,402,595	668,402.8
6	44.88	.802	6,000	44,883 1	802.083	6,000,000	44,883,114	802,083.3
7	52 36	.936	7,000	52,363 6	935 764	7,000,000	52,363,633	935,763.9
8	59 84	1.069	8,000	59,844 2	1,069 444	8,000,000	59,844,152	1,069,444
9	67.32	1.203	9,000	67,324.7	1,203.125	9,000,000	67,324,671	1,203,125
10	74 80	1.337	10,000	74,805 2	1,336 806	10,000,000	74,805,190	1,336,806
20	149 6	2 674	20,000	149,610 4	2,673 61	20,000,000	149,610,380	2,673,611
30	224.4	4.010	30,000	224,415.6	4,010.42	30,000,000	224,415,570	4,010,417
40	299 2	5.347	40,000	299,220 8	5,347 22	40,000,000	299,220,760	5,347,222
50	374.0	6.684	50,000	374,025 9	6,684 03	50,000,000	374,025,950	6,684,028
60	448.8	8 021	60,000	448,831.1	8,020 83	60,000,000	448,831,140	8,020,833
70	523 6	9 358	70,000	523,636 3	9,357 64	70,000,000	523,636,330	9,357,639
80	598 4	10 694	80,000	598,441.5	10,694 44	80,000,000	598,441,520	10,694,444
90	673.2	12 031	90,000	673,246 7	12,031 25	90,000,000	673,246,710	12,031,250

MEASURES AND WEIGHTS COMMONLY USED IN THE UNITED STATES AND GREAT BRITAIN

Abbreviations

bbl.	= barrel	dwt.	= pennyweight	hr.	= hour	quin.	= quintal
bu.	= bushel	fath.	= fathom	Imp.	= Imperial (bu., gal.)	qr.	= quarter
chn.	= chain	ft.	= foot	in.	= inch	qt.	= quart
circm.	= circumference	fl. drm.	= fluid drachm	lea	= league	scr.	= scruple
cir. in	= circular inch	fl. oz.	= fluid ounce	m. pace	= military pace	sec.	= second
cir. mil.	= circular mil	furl.	= furlong	min	= minute	sq.	= square (in., ft., etc.)
cwt	= hundred weight	gal	= gallon (U.S.)	naut. mi	= nautical mile	stat. mi.	= statute mile
deg.	= degree	grn.	= grain	pnch	= puncheon	trce.	= tierce
drm.	= drachm (dram)	hghd.	= hogshead	quad.	= quadrant	yd.	= yard

Table 10. Measures of Length

Long Measure

12 in. = 1 ft.
3 ft = 1 yd
5 1/2 yd. = 16 1/2 ft. = 1 rod, pole or perch
40 poles = 220 yd = 1 furl
8 furl. = 1760 yd. = 5280 ft. = 1 mile
4 in. = 1 hand
9 in = 1 span
2 1/2 ft. = 1 m pace.

Land, Surveyors, or Gunter's Measure

7.92 in. = 1 link
100 links = 66 ft = 4 rods = 1 chn.
10 chn = 220 yd = 1 furl
8 furl. = 80 chn. = 1 mile.

Nautical Measure

6080 26 ft. = 1 15156 stat. mi. = 1 naut mi
3 naut. mi. = 1 lea.
60 naut. mi. = 1 deg (at equator)
6 ft. = 1 fath
120 fath. = 1 cable
1 naut. mi. per hr. = 1 knot.

Table 11. Measures of Area

Square Measure

144 sq in. = 183 35 cir. in. = 1 sq ft.
9 sq. ft = 1 sq yd
30 1/4 sq. yd = 272 1/4 sq ft = 1 sq rod
160 sq rods = 10 chn } = 1 acre
4840 sq yd. = 43560 sq ft. }
640 acres = 27,878,400 sq. ft = 1 sq mile
1 acre = (208 71 ft)^2
1 cir. in. = { area of circle 1 in diam. = 0 7854 sq in
1 cir mil = { area of circle 0 001 in. diam. = 0 0 7854 sq in
1,000,000 cir mil = 1 cir. in.

Board Measure

144 cu in. = 12 × 12 × 1 in. = 1 board ft.

The above definition of a board foot applies to rough green lumber American practice uses a yard standard of 29/32 in. thick and an industrial standard of 7/8 in thick for rough dry lumber; for dressed lumber the corresponding thicknesses are 25/32 in and 13/16 in. For example, a rough dry piece 12 × 12 × 29/32 in contains 1 board ft Board ft in round timber = $(1/4\, c \times d \times l)$ 144 c = mean circm , d = diam , l = length, all in ft

Table 12. Measures of Weight

(The grain is the same in all systems)

Avoirdupois Weight

16 drm. = 437 5 grn = 1 oz.
16 oz = 7000 grn = 1 lb.
14 lb. = 1 stone
28 lb = 2 stone = 1 qr.
100 lb = 1 quin.
4 qr = 112 lb = 1 cwt
20 cwt. = 2240 lb. = 1 long ton
2000 lb. = 1 short ton.

Troy Weight (for gold and silver)

24 grn = 1 dwt
20 dwt. = 480 grn = 1 oz
12 oz = 5760 grn. = 1 lb
1 Assay ton = 29,167 milligrams
= { Troy oz per 2000 lb. ton avoirdupois

Carat Weight (for precious stones)

1 carat = 3.086 grn = 0 200 gram

Apothecaries' Weight

20 grn. = 1 scr ℈
3 scr = 60 grn = 1 drm ʒ
8 drm = 480 grn = 1 oz ℥
12 oz. = 5760 grn = 1 lb

Table 13. Measures of Volume

Cubic Measure

1728 cu in = 1 cu ft
27 cu ft. = 1 cu yd
1 cord of wood = { 4 × 4 × 8 ft = 128 cu ft
1 perch masonry = { 16 1/2 × 1 1/2 × 1 ft. = 23 3/4 cu. ft.

Liquid Measure

4 gills = 1 pint
2 pints = 1 qt.
4 qt = 231 cu in. = 8 3256 lb H_2O
= 1 U S gal
= 0 8327 Imp. gal.
1 Imp gal. = 277 420 cu in.
= 10 lb H_2O
1 20094 U S gal. = 1 Imp gal
7 4805 U S gal = 1 cu ft

Old Liquid Measure

31 1/2 gal = 1 bbl
42 gal = 1 tree
2 bbl = 63 gal = 1 hghd.
84 gal = 2 tree = 1 pipe
2 hghd = 126 gal = 1 pipe or butt
2 pipes = 3 pchn = 1 tun

Apothecaries' Fluid Measure

60 minims = 1 fl drm
8 drm. = 1 fl oz.
1 fl. oz. = 1,128 U S gal = 1 8047 cu in = 456 3 grn H_2O at 39° F
16 fl. oz. = 1 pint
1 fl oz. (British) = 1 732 cu in = 437.5 grn. H_2O at 62° F.

Dry Measure

2 pints = 1 qt.
8 qts = 1 peck
4 pecks = 2150 42 cu in.
= 1 bu (struck)
1 1/4 bu. (heaped) = 1 bu (heaped)
105 qts (struck) = 7056 cu in = 1 bbl.
8 Imp gal. = 2218 192 cu in.
= 1 Imp bu.
8 Imp bu. = 1 qr

Shipping Measure

100 cu ft = 1 Register ton *
40 cu ft. = 31 143 U S bu
= 1 shipping ton (U. S.) †
42 cu. ft. = 32 719 Imp bu.
= 1 shipping ton (British) †

* For measurement of entire internal capacity † For measurement of cargo

Miner's Inch

A variable unit, defined by statute in different states as the quantity of water that will flow through an orifice of 1 sq in area, under a head ranging from 4 1/2 to 6 in U S Reclamation Service defines the miner's inch as 1 cu in. per sec The law governing the locality in which the measurement is to be made should be consulted

Table 14. Circular Measure

60 sec. = 1 min
60 min = 1 deg
90 deg. = 1 quad.
360 deg. = circm.
57.2957795 deg. = 1 radian
= 57° 17′ 44.806″
1 radian = angle of arc equal to radius.

Table 15. Time

60 sec. = 1 min. 7 days = 1 week
60 min. = 1 hr. 52 weeks = 1 year
24 hr = 1 day
1 year (exact) = 365 days, 5 hr., 48 min., 45.6 sec.

A year divisible by 4 is a leap year and contains 366 days, except centesimal years, of which only those divisible by 400 are leap years.

365.242° mean solar days
= 366.2422 sidereal days
24 hr. sidereal time
= 23 hr. 56 min. 04.091 sec. of mean solar time
1 mean solar day
= 24 hr. 3 min. 56.555 sec. of sidereal time

THE METRIC SYSTEM

The fundamental standard of length in the metric system is the International Standard Meter, derived from the Metre des Archives, Paris. Its length is defined as the distance between two lines at 0° C. on a platinum-iridium bar deposited at the International Bureau of Weights and Measures near Paris. 1 meter = 39.37 in.

The fundamental standard of mass is the International Standard Kilogram, which is a mass of platinum-iridium, deposited at the International Bureau, whose weight *in vacuo* is the same as that of the Kilogram des Archives.

The fundamental unit of volume is the **liter**, which is defined as the volume of a kilogram of water at the temperature of maximum density, 4° C., under a pressure of 76 cm. of mercury. The liter originally was defined as a cubic decimeter, but is slightly greater. The generally accepted relation between the cubic decimeter and the liter is 1 liter = 1.000027 cu. decimeters.

The unit of force is the **dyne**, which is the force which acting for 1 second on a mass of one gram will produce a velocity of 1 centimeter per second. 1 dyne = (1/980.665) gram. 1,000,000 dynes = 1.020 kilograms force = 2.248 lb. force. 1 lb. force = 0.4536 kilogram force = 448,800 dynes.

The unit of heat is the **gram-calorie**, which is the quantity of heat required to raise 1 gram of water 1° C.

Table 16. Units of Metric Measures *

Length			Area		
Unit	Symbol	Value in Meters	Unit	Symbol	Value in Sq. Meters
Micron	μ	0.000 001			
Millimeter	mm.	0.001	Sq. millimeter	sq. mm.	0.000 001
Centimeter	cm.	0.01	Sq. centimeter	sq. cm.	0.000 1
Decimeter	dm.	0.1	Sq. decimeter	sq. dm.	0.01
Meter (unit)	m.	1.0	Sq. meter (centiare)	sq. m.	1.0
Dekameter	dkm.	10.0	Sq. dekameter (are)	sq. dkm.	100.0
Hectometer	hm.	100.0	Hectare	ha.	10,000.0
Kilometer	km.	1,000.0	Sq. kilometer	sq. km.	1,000,000.0
Myriameter	Mm.	10,000.0			
Megameter		1,000,000.0			

Cubic Measure			Volume		
Unit	Symbol	Value in Cubic Meters	Unit	Symbol	Value in Liters
Cubic kilometer	cu. km.	1,000,000,000	Milliliter	ml.	0.001
Cubic hectometer	cu. hm.	1,000,000	Centiliter	cl.	0.01
Cubic dekameter	cu. dkm.	1,000	Deciliter	dl.	0.1
Cubic meter	cu. m.	1	Liter (unit)	l.	1.0
Cubic decimeter	cu. dm.	0.001	Dekaliter	dkl.	10.0
Cubic centimeter	cu. cm.	0.000 001	Hectoliter	hl.	100.0
Cubic millimeter	cu. mm.	0.000 000 001	Kiloliter	kl.	1,000.0
Cubic micron	μ^3	$0.0_{12}1$			

Weight					
Unit	Symbol	Value in Grams	Unit	Symbol	Value in Grams
Microgram	γ	0.000 001	Gram (unit)	g.	1.0
Milligram	mg.	0.001	Dekagram	dkg.	10.0
Centigram	cg.	0.01	Hectogram	hg.	100.0
Decigram	dg.	0.1	Kilogram	kg.	1,000.0
Gram (unit)	g.	1.0	Myriagram	Mg.	10,000.0
			Quintal	q.	100,000.0
			Ton	t.	1,000,000.0

* A subscript after a figure indicates the number of times it is repeated. Thus $0.0_38 = 0.0008$

Table 17. Metric Equivalents of Length *

English to Metric			Log	Metric to English			Log.
1 inch =	25 4	mm.	1.404834	1 mm. =	0.03937	in.	$\bar{2}$ 595165
=	2.54	cm.	0 404834	=	0.003281	ft	$\bar{3}$ 515984
=	0.0254	m.	$\bar{2}$.404834	=	0.001094	yd.	$\bar{3}$.038863
1 foot =	304 800	mm.	2.484015	1 cm =	0 3937	in.	$\bar{1}$.595165
=	30 480	cm.	1 484015	=	0 03281	ft.	$\bar{2}$.515984
=	0.3048	m.	$\bar{1}$.484015	=	0 01094	yd.	$\bar{2}$.038863
1 yard =	91 4402	cm.	1.961137	1 meter =	39 37	in.	1.595162
=	0 9144	m	$\bar{1}$ 961137	=	3 2808	ft	0 515984
=	0 0914	km.	$\bar{4}$.961137	=	1.0936	yd.	0 038863
1 rod =	502 9211	cm.	2 701500	=	0.1988	rd.	$\bar{1}$ 298500
=	5.0292	m	0.701500	=	0.04971	chain	$\bar{2}$ 696440
=	0.005029	km.	$\bar{3}$.701500	=	$0.0_3 6214$	mi.	$\bar{4}$.793350
1 chain =	2011.68	cm.	3 303559	1 kilometer =	3280 833	ft.	3.515954
=	20.1168	m	1 303559	=	1093.611	yd.	3.038863
=	0.02012	km.	$\bar{2}$ 303559	=	198 838	rods	2.298500
1 mile =	1609.344	m.	3 206650	=	49 7095	chains	1.696440
=	1 6093	km.	0 206650	=	0 6214	mi	$\bar{1}$ 793350

Table 18. Metric Equivalents of Area *

English to Metric			Log.	Metric to English			Log.
1 cir. mil =	$0\ 0_3 5067$	sq. mm.	$\bar{4}$.704751	1 sq. mm. =	1,973 55	cir. mils	3.295249
1 sq. in. =	645 $1_6 3$	sq. mm.	2.809669	=	0 001550	sq. in.	$\bar{3}$.190331
=	6.4516	sq. cm.	0.809669	=	$0\ 0_4 10764$	sq ft.	$\bar{5}$.031968
=	$0.0_3 6452$	sq. m.	$\bar{4}$.809669	=	$0\ 0_5 1196$	sq. yd.	$\bar{6}$.077726
1 sq. ft. =	92,903.41	sq mm.	4.968052	1 sq. cm. =	0 1550	sq in.	$\bar{1}$.190331
=	929 0341	sq. cm.	2.968032	=	0.001076	sq. ft.	$\bar{3}$.031968
=	0 0929	sq. m.	$\bar{2}$.968032	=	$0\ 0_3 1196$	sq. yd.	$\bar{4}$ 077726
=	$0\ 0_5 929$	hectare	$\bar{6}$.968032	1 sq. m. =	1,549 9969	sq. in.	3.190331
=	$0\ 0_7 929$	sq. km.	$\bar{8}$ 968032	=	10 7639	sq ft.	1.031968
1 sq. yd. =	836,130 74	sq mm.	5 922274	=	1.1960	sq yd.	0.077726
=	8,361 307	sq. cm.	3 922274	=	0 002471	sq. chain	$\bar{3}$ 392882
=	0 83613	sq. m.	$\bar{1}$.922274	=	$0\ 0_3 2471$	acre	$\bar{4}$.392882
=	$0\ 0_4 836$	hectare	$\bar{5}$ 922274	=	$0\ 0_8 3861$	sq. mi.	$\bar{7}$.586700
=	$0\ 0_6 836$	sq. km.	$\bar{7}$ 922274	1 hectare =	107,638 7	sq ft.	5.031968
1 sq. chain =	404 686	sq. m.	2 607118	=	11,959 85	sq. yd.	4.077726
=	0 04047	hectare	$\bar{2}$ 607118	=	24.710	sq. chain	1.392882
=	$0\ 0_3 4047$	sq. km.	$\bar{4}$ 607118	=	2 4710	acres	0.392882
1 acre =	4,046 86	sq. m.	3.607118	=	0 003861	sq mi.	3.586700
=	0 4047	hectare	$\bar{1}$.607118	1 sq. km. =	10,763,867.36	sq. ft.	7.031968
=	0 004047	sq. km.	$\bar{3}$ 607118	=	1,195,985 26	sq. yd.	6.077726
1 sq. mile =	2,589,998	sq. m.	6 413300	=	2,471 050	sq. chains	3 392882
=	259 0	hectare	2.413300	=	247 1045	acres	2 392882
=	2 590	sq km.	0 413300	=	0 3861	sq. mi.	$\bar{1}$ 586700

Table 19. Metric Equivalents of Weight or Mass *

English to Metric			Log	Metric to English			Log.
1 grain =	64 797	mg.	1.811568	1 milligram =	0 01543	grain	$\bar{2}$ 188433
=	0 0648	gm.	$\bar{2}$.811568	=	$0\ 0_4 3215$	oz. Troy	$\bar{5}$ 507131
1 oz. Troy or				=	$0\ 0_4 3527$	oz. avoir.	$\bar{5}$.547454
apothecary =	31,103 5	mg.	4.492809	1 gram =	15 4324	grains	1.188433
=	31 103	gm.	1.492809	=	0 03215	oz Troy	$\bar{2}$ 507131
=	0 03110	kg.	$\bar{2}$.492809	=	0 03527	oz avoir.	$\bar{2}$ 547454
1 oz. avoirdupois =	28,349 5	mg.	4.452546	=	$0\ 0_2 2679$	lb. Troy	$\bar{3}$.428310
=	28 3495	gm.	1 452546	=	$0\ 0_2 2205$	lb. avoir.	$\bar{3}$.343334
=	0 02835	kg.	$\bar{2}$ 452546	1 kilogram =	32 1508	oz. Troy	1.507131
1 pound Troy or				=	35 2740	oz. avoir.	1.547454
apothecary =	373 2417	gm.	2 571900	=	2 6792	lb. Troy	0.428310
=	0 3732	kg.	$\bar{1}$.571900	=	2 2046	lb. avoir.	0.343334
1 pound avoirdupois =	453 5925	gm.	2 656663	=	$0\ 0_2 1102$	short ton	$\bar{3}$.043304
=	0 4536	kg.	$\bar{1}$ 656663	=	$0\ 0_3 9842$	long ton	$\bar{4}$ 993086
=	$0\ 0_3 4536$	metric ton	$\bar{4}$.656663	1 metric ton =	2204 62	lb. avoir.	3.343334
1 short ton =	907 2	kg.	2 957606	=	1 1023	short tons	0.043304
=	0 9072	metric ton	$\bar{1}$ 957606	=	0 9842	long ton	$\bar{1}$.993086
1 long ton =	1,016 06	kg.	3 006914				
=	1 0161	metric ton	0 006914				

* A subscript after a figure indicates the number of times it is repeated. Thus $0.0_3 8 = 0.0008$

Table 20. Metric Equivalents of Capacity or Volume *

English to Metric			Log.	Metric to English			Log.
1 cu. in. =	16,387 17	cu. mm.	4.214504	1 cu. mm. =	0 $0_4$6102	cu. in.	$\bar{5}$.785496
=	16 3872	cu. cm.	1 214504	=	0 $0_3$2705	fluid dr.	$\bar{4}$ 432185
=	0 016387	l.	$\bar{2}$ 214504	=	0 $0_4$3381	fluid oz.	$\bar{5}$ 529094
=	0 $0_3$16387	hl.	$\bar{4}$.214504	1 cu. cm. =	0 06102	cu. in.	$\bar{2}$ 785496
=	0 $0_4$16387	cu. m.	$\bar{5}$.214504	=	0 $0_4$3531	cu. ft.	$\bar{5}$.547951
1 cu. ft. =	28,317 08	cu. cm.	4 452049	=	0 $0_5$1308	cu. yd.	$\bar{6}$ 116589
=	28 3169	l.	1 452049	=	0 $0_4$2838	bushel	$\bar{5}$ 452972
=	0 2832	hl.	$\bar{1}$.452049	=	0 2705	fluid dr.	$\bar{1}$.432185
=	0 02832	cu.m.	$\bar{2}$ 452049	=	0 03381	fluid oz.	$\bar{2}$.529094
1 cu. yd. =	764,559 5	cu. cm.	5.883411	=	0 001057	quart	$\bar{3}$ 023944
=	764 5595	l.	2.883411	=	0 $0_3$2642	gallon	$\bar{4}$.421884
=	7 6456	hl.	0 883411	1 liter =	61 02398	cu in.	1.785496
=	0 7646	cu. m.	$\bar{1}$.883411	=	0 035313	cu. ft.	$\bar{2}$ 547951
1 bushel =	35,239 3	cu. cm.	4 547027	=	0 0013079	cu. yd.	$\bar{3}$ 116589
=	35 2393	l.	1 547027	=	0 028377	bushel	$\bar{2}$.452972
=	0 3524	hl.	$\bar{1}$.547027	=	1 0567	quart	0.023944
=	0 03524	cu. m.	$\bar{2}$.547027	=	0 2642	gallon	$\bar{1}$.421884
1 fluid drachm =	3,696 7	cu. mm.	3.567815	1 hectoliter =	6,102 398	cu. in.	3.785496
=	3 6967	cu. cm.	0.567815	=	3 5313	cu. ft.	0 547951
1 fluid oz. =	29,573 7	cu. mm.	4.470906	=	0 13079	cu. yd.	$\bar{1}$ 116589
=	29 5737	cu. cm.	1.470906	=	2 8377	bushels	0.452972
1 quart =	946 359	cu. cm.	2 976056	1 cu. meter =	61,023 38	cu. in.	4 785496
=	0 9464	l.	$\bar{1}$ 976056	=	35 3133	cu. ft.	1.547951
=	0 $0_3$946	cu. m.	$\bar{4}$ 976056	=	1 3079	cu. yd.	0.116589
1 U. S. gallon =	3,785 43	cu. cm.	3.578116	=	28 3773	bushels	1.452972
=	3 7854	l.	0 578116	=	1,056 682	quarts	3 023944
=	0 003785	cu. m.	$\bar{3}$.578116	=	264 170	gallons	2.421884

Table 21. Metric Equivalents of Density *

English to Metric	Log.	Metric to English	Log.
1 lb. per cu. in. = 27.680 gm per cu. cm	1 442162	1 gm. per cu. cm. = 0.03613 lb. per cu. in.	$\bar{2}$ 557838
= 27,679.7 kg. per cu. m	4 442162	= 62.430 lb. per cu. ft.	1.795381
1 lb. per cu. ft. = 0 01602 gm. per cu. cm	$\bar{2}$ 204619	= 8 3454 lb. per U. S. gal	0 921450
= 16 0183 kg. per cu. m.	1.204619	1 kg. per cu. m. = 0.$0_4$3613 lb per cu. in.	$\bar{5}$.557838
= 0 01602 metric ton per cu. m.	$\bar{2}$ 204619	= 0 062430 lb. per cu. ft	$\bar{2}$.795381
1 lb. per cu. yd. = 0 5933 kg. per cu. m.	$\bar{1}$ 773254	= 1.6856 lb. per cu yd.	0.226746
= 0 $0_3$5933 metric tons per cu. m.	$\bar{4}$ 773254	= 0.$0_2$8345 lb. per U. S. gal.	$\bar{3}$.921450
1 lb. per U. S. gal. = 0 1198 gm. per cu. cm	$\bar{1}$.078550	1 metric ton per cu. m = 62.4286 lb. per cu. ft.	1 795381
= 119.826 kg. per cu. m.	2.078550	= 1685 487 lb. per cu. yd.	3.226746
1 short ton per cu. yd. = 1.1865 metric tons per cu m.	0 074285	= 0.8428 short ton per cu. yd.	$\bar{1}$.925715
1 long ton per cu. yd. = 1.3289 metric tons per cu m	0 123502	= 0.7525 long ton per cu. yd.	$\bar{1}$ 876498

Table 22. Metric Equivalents of Velocity *

English to Metric	Log.	Metric to English	Log.
1 in. per sec., = 2.54001 cm. per sec.	0.404834	1 cm. per sec. = 0.3937 in. per sec.	$\bar{1}$ 595165
= 0.0254 m. per sec.	$\bar{2}$.404834	= 0.03281 ft. per sec.	$\bar{2}$.515984
= 1.5240 m. per min.	0.182985	= 1.9685 ft. per min.	0.294136
1 ft. per sec. = 30 4801 cm. per sec.	1.484015	= 0.02237 mi. per hr.	$\bar{2}$ 349653
= 0 3048 m. per sec.	$\bar{1}$.484015	= 0.01943 knot	$\bar{2}$.288367
= 18.2880 m. per min.	1 262166	1 m. per sec. = 39 37 in. per sec.	1 595165
= 1.0973 km. per hr.	0.040318	= 3.2808 ft per sec.	0 515984
1 ft. per min. = 0 5080 cm per sec.	$\bar{1}$ 705864	= 196 8500 ft per min.	2 294136
= 0 00508 m. per sec.	$\bar{3}$ 705864	= 2 2369 mi. per hr.	0 349653
= 0 3048 m. per min.	$\bar{1}$ 484015	= 1 9426 knots	0.288367
= 0 018288 km. per hr.	$\bar{2}$ 262166	1 m. per min. = 0 6562 in. per sec.	$\bar{1}$ 817014
1 mile per hr. = 44.704 cm. per sec.	1 650348	= 0.05468 ft. per sec.	$\bar{2}$ 737833
= 0 4470 m. per sec.	$\bar{1}$ 650348	= 3.2808 ft. per min.	0 515984
= 26 8222 m. per min.	1.428495	= 0 03728 mi per hr.	$\bar{2}$ 571503
= 1 6093 km. per hr.	0.206650	= 0 03238 knot	$\bar{2}$ 510218
1 knot = 51 4791 cm. per sec.	1 711631	1 km. per hr. = 0.9113 ft per sec.	$\bar{1}$ 959681
= 0.5148 m. per sec.	$\bar{1}$ 711631	= 54 6806 ft. per min.	1.737833
= 30 8875 m. per min.	1 489782	= 0 62138 mi per hr.	$\bar{1}$ 793357
= 1.8532 km. per hr.	0.267933	= 0 5396 knot	$\bar{1}$ 732072

* A subscript after a figure indicates the number of times it is repeated. Thus 0 $0_3$8 = 0.0008

Table 23. Metric Equivalents of Pressure *

English to Metric	Log.	Metric to English	Log.
1 lb. per sq. in. = 0.7031 gm. per sq. mm.	$\bar{1}.846996$	1 gm. per sq. mm. = 1.4223 lb. per sq. in.	0.153004
= 70.3066 gm. per sq. cm.	2.846996	= 204.8170 lb. per sq. ft.	2.311366
= 0.07031 kg. per sq. cm.	$\bar{2}.846996$	1 gm. per sq. cm. = 0.01422 lb. per sq. in.	$\bar{2}.153004$
= 0.7031 metric ton per sq. m.	$\bar{1}.846996$	= 2.0482 lb. per sq. ft.	0.311366
= 0.07031 metric atmosphere†	$\bar{2}.846996$	1 kg. per sq. cm. = 14.2234 lb. per sq. in.	1.153004
1 lb. per sq. ft. = 0.004882 gm. per sq. mm.	$\bar{3}.688598$	= 2048.1696 lb. per sq. ft.	3.311366
= 0.4882 gm. per sq. cm.	$\bar{1}.688598$	1 metric ton per sq. m. = 1.4223 lb. per sq. in.	0.153004
= $0.0_2 4882$ kg. per sq. cm.	$\bar{4}.688598$	= 204.8170 lb. per sq. ft.	2.311366
= 0.004882 metric ton per sq. m.	$\bar{3}.688598$	= 0.1024 ton per sq. ft. §	$\bar{1}.010342$
= $0.0_2 4882$ metric atmosphere†	$\bar{4}.688598$	1 metric atmosphere† = 14.2234 lb. per sq. in.	1.153004
1 ton per sq. ft.† = 9.7648 metric tons per sq. m.	0.989663	= 2048.1696 lb. per sq. ft.	3.311366
= 0.9765 metric atmosphere†	$\bar{1}.989663$	= 1.0241 tons per sq. ft.	0.010342
1 atmosphere ‡ = 1.0335 metric atmospheres †	0.014310	= 0.9676 atmosphere ‡	$\bar{1}.985696$

† 1 metric atmosphere = 1 kg. per sq. cm. ‡ 1 atmosphere = 14.7 lb. per sq. in. § 1 ton = 2000 lb.

Table 24. Metric Equivalents of Force*

English to Metric	Log.	Metric to English	Log.
1 lb. = 444,806 dynes	5.648165	1 dyne = $0.0_5 2248$ lb.	$\bar{6}.351796$
= 453.6 grams	2.656673	= $0.0_4 7233$ poundal	$\bar{5}.859318$
= 0.04448 joule per cm.	$\bar{2}.648165$	= $0.0_2 1020$ gram	$\bar{3}.008600$
= 0.4536 kg.	$\bar{1}.656673$	= $0.0_5 1020$ kg.	$\bar{6}.008600$
= 32.17 poundals	1.507451	= $0.0_6 1$ joule per cm.	$\bar{7}.000000$
1 poundal = 13,826 dynes	4.140696	1 gram = $0.0_2 2205$ lb.	$\bar{3}.343409$
= 14.10 grams	1.149219	= 0.07093 poundal	$\bar{2}.850830$
= $0.0_2 13826$ joule per cm.	$\bar{3}.140696$	= 980.7 dynes	2.991536
= 0.01410 kg.	$\bar{2}.149219$	= 0.001 kg.	$\bar{3}.000000$
= 0.03108 lb.	$\bar{2}.492481$	= $0.0_4 9807$ joule per cm.	$\bar{5}.991536$
		1 joule per cm. = 22.48 lb.	1.351796
		= 723.3 poundals	2.859318
		= 10,000,000 dynes	7.000000
		= 10,200 grams	4.008600
		= 10.20 kg.	1.008600
		1 kg. = 2.205 lb.	0.343409
		= 70.93 poundals	1.850830
		= 980,700 dynes	5.991536
		= 1000 grams	3.000000
		= 0.09807 joule per cm.	2.991536

Table 25. Metric Equivalents of Power, Work, and Energy*

English to Metric	Log	Metric to English	Log
1 ft-lb = 0.138255 kg-m	$\bar{1}.140680$	1 kg-m = 7.2330 ft-lb	0.859318
= $0.0_8 37650$ kwhr	$\bar{7}.575765$	= $0.0_5 365304$ hp-hr	$\bar{6}.562655$
= 0.32379 IT cal	$\bar{1}.510263$	= $0.0_2 92938$ Btu	$\bar{3}.968193$
1 hp = 76.042 kg-m per sec	1.881053	1 kwhr = 2.656000 ft-lb	6.424228
= 0.76042 poncelet	$\bar{1}.881053$	= 1.3414 hp-hr	0.127558
= 1.01389 cheval vapeur	0.005994	= 3412.75 Btu	3.533103
1 hp-hr = 273,750 kg-m	5.437355	1 cheval vapeur = 0.9863 hp	$\bar{1}.994009$
= 0.74548 kwhr	$\bar{1}.872435$	1 poncelet = 1.3151 hp	0.118959
= 641,100 IT cal	5.806926	1 IT cal † = 3.0884 ft-lb	0.489736
1 Poncelet = 100 kg-m per sec	2.000000	= $0.0_5 155980$ hp-hr	$\bar{6}.193070$
1 cheval-vapeur = 75 kg-m per sec	1.875061	= $0.0_3 39683$ Btu	$\bar{3}.598605$
1 Btu = 251.996 IT cal	2.401400	= 1.00037 mean calories	0.000160
= 107.599 kg-m	2.031808		
= $0.0_3 29302$	$\bar{4}.466898$		

* A subscript after a figure indicates the number of times it is repeated. Thus $0.0_3 8 = 0.0008$.

† The IT cal is defined as $1/860$ of the international watthour, and is nearly equal to the mean calorie; 1 IT cal = 1.00037 mean calories, or 1 Btu = 251.996 IT cal. (See *Mechanical Engineering*, Nov. 1935, p. 710.)

GEOMETRY

4. CIRCULAR ARCS, CHORDS, SEGMENTS, AND SECTORS

Notation.—A = central angle, degrees; C = chord of arc; c = chord of $1/2$ arc; D = diameter; $G = \dfrac{\text{area of segment}}{R^2}$; H = rise of arc or height of segment; L = length of arc, R = radius S = area of sector.

L (exact) $= 2\pi RA/360 = 0.017453RA$

L (approx.) $= (8c - C)/3 = \{(2c \times 10H)/(60D - 27A)\} + 2c$

$= \{(\sqrt{C^2 + 4H^2} \times 10H^2)/(15C^2 + 33H^2)\} + 2c$

$C = 2\sqrt{c^2 - H^2} = \sqrt{D^2 - (D - 2H)^2} = (8c - 3L) = 2\sqrt{R^2 - (R - H)^2} = 2\sqrt{(D - H) \times H}$

$C/2 = \sqrt{H \times (D - H)}$; $c = 1/2\sqrt{C^2 - 4H^2} = \sqrt{D \times H} = (3L + C)/8$

$D = c^2/H = (1/4\,C^2 + H^2)/H$

$H = c^2/D = 1/2\,(D - \sqrt{D^2 - C^2})$ when $H < R$; $= 1/2\,(D + \sqrt{D^2 - C^2}) = \sqrt{c^2 - 1/4\,C^2}$ when $H > R$

$G = S - 1/2\,C\sqrt{R^2 - 1/4\,C^2}$ when $G <$ semi-circle; $= S + 1/2\,C\sqrt{R^2 - 1/4\,C^2}$ when $S >$ semi-circle

$= S - (C/2)\,(R - H) = 1/2\,\{(L - C)\,(D/2) + CH\}$

$S = 1/2\,LR = \pi AR^2/360 = (\pi R^2/360) \times 2[\sin^{-1}\{(C/2)/R\}]$; $R = (C^2 + 4H^2)/8H$

Table 26. Circular Arcs, Chords, and Segments. Radius = 1

Central Angle in Degrees	Arc Length L	Arc Rise H	Chord, Length C	$\frac{H}{C}$	Segment Area $\frac{}{R^2} = G^*$	Central Angle in Degrees	Arc Length L	Arc Rise H	Chord, Length C	$\frac{H}{C}$	Segment Area $\frac{}{R^2} = G^*$
1	0.0175	0.0000	0.0175	0.0022	0.00000	31	0.5411	0.0364	0.5345	0.0680	0.01301
2	.0349	.0002	.0349	.0044	.00000	32	.5585	0387	.5513	.0703	.01429
3	.0524	.0003	.0524	.0066	.00001	33	.5760	.0412	.5680	.0725	.01566
4	.0698	.0006	.0698	.0087	.00003	34	.5934	.0437	.5847	.0747	.01711
5	.0873	0010	.0872	.0109	.00006	35	.6109	.0463	.6014	.0770	.01864
6	.1047	.0014	.1047	.0131	.00010	36	.6283	.0489	.6180	.0792	.02027
7	.1222	.0019	.1221	.0153	.00015	37	.6458	.0517	.6346	.0814	.02198
8	.1396	0024	.1395	.0175	.00023	38	.6632	.0545	.6511	.0837	.02378
9	.1571	.0031	.1569	.0196	.00032	39	.6807	.0574	.6676	.0859	.02568
10	**.1745**	**.0038**	**.1743**	**.0218**	**.00044**	**40**	**.6981**	**.0603**	**.6840**	**.0882**	**.02767**
11	.1920	.0046	.1917	.0240	.00059	41	.7156	.0633	.7004	.0904	.02976
12	.2094	.0055	.2091	.0262	.00076	42	.7330	.0664	.7167	.0927	.03195
13	.2269	.0064	.2264	.0284	.00097	43	.7505	.0696	.7330	.0949	.03425
14	.2443	.0075	.2437	.0306	.00121	44	.7679	.0728	.7492	.0972	.03664
15	.2618	.0086	.2611	.0328	.00149	45	.7854	.0761	.7654	.0995	.03915
16	.2793	.0097	.2783	.0350	.00181	46	.8029	.0795	.7815	.1017	.04176
17	.2967	.0110	.2956	.0372	.00217	47	.8203	.0829	.7975	.1040	.04448
18	.3142	.0123	.3129	.0394	.00257	48	.8378	.0865	.8135	.1063	.04731
19	.3316	.0137	.3301	.0415	.00302	49	.8552	.0900	.8294	.1086	.05025
20	**.3491**	**.0152**	**.3473**	**.0437**	**.00352**	**50**	**8727**	**.0937**	**.8452**	**.1108**	**.05331**
21	.3665	.0167	.3645	.0459	.00408	51	.8901	.0974	.8610	.1131	.05649
22	.3840	.0184	.3816	.0481	.00468	52	.9076	.1012	.8767	.1154	.05978
23	.4014	.0201	.3987	.0503	.00535	53	.9250	.1051	.8924	.1177	.06319
24	.4189	.0219	.4158	.0526	.00607	54	.9425	.1090	.9080	.1200	.06673
25	.4363	.0237	.4329	.0548	.00686	55	.9599	.1130	.9235	.1223	.07039
26	.4538	.0256	.4499	.0570	.00771	56	.9774	.1171	.9389	.1247	.07417
27	.4712	.0276	.4669	.0592	.00862	57	.9848	.1212	.9543	.1270	.07808
28	.4887	.0297	.4838	.0614	.00961	58	1.0123	.1254	.9696	.1293	.08212
29	.5061	.0319	.5008	.0636	.01067	59	1.0297	.1296	.9848	.1316	.08629
30	**.5236**	**.0341**	**.5176**	**.0658**	**.01180**	**60**	**1 0472**	**.1340**	**1.0000**	**.1340**	**.09059**

*Area of segment of any radius = factor $G \times$ square of radius

Table 26. Circular Arcs, Chords, and Segments—*Continued*

Central Angle in Degrees	Arc Length L	Arc Rise H	Chord, Length C	$\frac{H}{C}$	Segment Area $\frac{R^2}{}$ $= G$ *	Central Angle in Degrees	Arc Length L	Arc Rise H	Chord Length C	$\frac{H}{C}$	Segment Area $\frac{R^2}{}$ $= G$ *
61	1.0647	0.1384	1.015	0.1363	0.09502	121	2.1118	0.5076	1.741	0.2916	0.62734
62	1.0821	.1428	1.030	.1387	.09958	122	2.1293	.5152	1.749	.2945	.64063
63	1.0996	.1474	1.045	.1410	.10428	123	2.1468	.5228	1.758	.2975	.65404
64	1.1170	.1520	1.060	.1434	.10911	124	2.1642	.5305	1.766	.3004	.66759
65	1.1345	.1566	1.075	.1457	.11408	125	2.1817	.5383	1.774	.3034	.68125
66	1.1519	.1613	1.089	.1481	.11919	126	2.1991	.5460	1.782	.3064	.69505
67	1.1694	.1661	1.104	.1505	.12443	127	2.2166	.5538	1.790	.3094	.70897
68	1.1868	.1710	1.118	.1529	.12982	128	2.2340	.5616	1.798	.3124	.72301
69	1.2043	.1759	1.133	.1553	.13535	129	2.2515	.5695	1.805	.3155	.73716
70	**1.2217**	**.1808**	**1.147**	**.1576**	**.14102**	**130**	**2.2689**	**.5774**	**1.813**	**.3185**	**.75144**
71	1.2392	.1859	1.161	.1601	.14683	131	2.2864	.5853	1.820	.3216	.76584
72	1.2566	.1910	1.176	.1625	.15279	132	2.3038	.5933	1.827	.3247	.78034
73	1.2741	.1961	1.190	.1649	.15889	133	2.3213	.6013	1.834	.3278	.79497
74	1.2915	.2014	1.204	.1673	.16514	134	2.3387	.6093	1.841	.3309	.80970
75	1.3090	.2066	1.218	.1697	.17154	135	2.3562	.6173	1.848	.3341	.82454
76	1.3265	.2120	1.231	.1722	.17808	136	2.3736	.6254	1.854	.3373	.83949
77	1.3439	.2174	1.245	.1746	.18477	137	2.3911	.6335	1.861	.3404	.85455
78	1.3614	.2229	1.259	.1771	.19160	138	2.4086	.6416	1.867	.3436	.86971
79	1.3788	.2284	1.272	.1795	.19859	139	2.4260	.6498	1.873	.3469	.88497
80	**1.3963**	**.2340**	**1.286**	**.1820**	**.20573**	**140**	**2.4435**	**.6580**	**1.879**	**.3501**	**.90034**
81	1.4137	.2396	1.299	.1845	.21301	141	2.4609	.6662	1.885	.3534	.91580
82	1.4312	.2453	1.312	.1869	.22045	142	2.4784	.6744	1.891	.3566	.93135
83	1.4486	.2510	1.325	.1894	.22804	143	2.4958	.6827	1.897	.3599	.94700
84	1.4661	.2569	1.338	.1919	.23578	144	2.5133	.6910	1.902	.3633	.96274
85	1.4835	.2627	1.351	.1944	.24367	145	2.5307	.6993	1.907	.3666	.97858
86	1.5010	.2686	1.364	.1970	.25171	146	2.5482	.7076	1.913	.3700	.99449
87	1.5184	.2746	1.377	.1995	.25990	147	2.5656	.7160	1.918	.3734	1.0105
88	1.5359	.2807	1.389	.2020	.26825	148	2.5831	.7244	1.923	.3768	1.0266
89	1.5533	.2867	1.402	.2046	.27675	149	2.6005	.7328	1.927	.3802	1.0428
90	**1.5708**	**.2929**	**1.414**	**.2071**	**.28540**	**150**	**2.6180**	**.7412**	**1.932**	**.3837**	**1.0590**
91	1.5882	.2991	1.427	.2097	.29420	151	2.6354	.7496	1.936	.3871	1.0753
92	1.6057	.3053	1.439	.2122	.30316	152	2.6529	.7581	1.941	.3906	1.0917
93	1.6232	.3116	1.451	.2148	.31226	153	2.6704	.7666	1.945	.3942	1.1082
94	1.6406	.3180	1.463	.2174	.32152	154	2.6878	.7750	1.949	.3977	1.1247
95	1.6581	.3244	1.475	.2200	.33093	155	2.7053	.7836	1.953	.4013	1.1413
96	1.6755	.3309	1.486	.2226	.34050	156	2.7227	.7921	1.956	.4049	1.1580
97	1.6930	.3374	1.498	.2252	.35021	157	2.7402	.8006	1.960	.4085	1.1747
98	1.7104	.3439	1.509	.2279	.36008	158	2.7576	.8092	1.963	.4122	1.1915
99	1.7279	.3506	1.521	.2305	.37009	159	2.7751	.8178	1.967	.4158	1.2084
100	**1.7453**	**.3572**	**1.532**	**.2332**	**.38026**	**160**	**2.7925**	**.8264**	**1.970**	**.4195**	**1.2253**
101	1.7628	.3639	1.543	.2358	.39058	161	2.8100	.8350	1.973	.4233	1.2422
102	1.7802	.3707	1.554	.2385	.40104	162	2.8274	.8436	1.975	.4270	1.2592
103	1.7977	.3775	1.565	.2412	.41166	163	2.8449	.8522	1.978	.4308	1.2763
104	1.8151	.3843	1.576	.2439	.42242	164	2.8623	.8608	1.981	.4346	1.2934
105	1.8326	.3912	1.587	.2466	.43333	165	2.8798	.8695	1.983	.4385	1.3105
106	1.8500	.3982	1.597	.2493	.44439	166	2.8972	.8781	1.985	.4424	1.3277
107	1.8675	.4052	1.608	.2520	.45560	167	2.9147	.8868	1.987	.4463	1.3449
108	1.8850	.4122	1.618	.2548	.46695	168	2.9322	.8955	1.989	.4502	1.3621
109	1.9024	.4193	1.628	.2575	.47844	169	2.9496	.9042	1.991	.4542	1.3794
110	**1.9199**	**.4264**	**1.638**	**.2603**	**.49008**	**170**	**2.9671**	**.9128**	**1.992**	**.4582**	**1.3967**
111	1.9373	.4336	1.648	.2631	.50187	171	2.9845	.9215	1.994	.4622	1.4140
112	1.9548	.4408	1.658	.2659	.51379	172	3.0020	.9302	1.995	.4663	1.4314
113	1.9722	.4481	1.668	.2687	.52586	173	3.0194	.9396	1.996	.4704	1.4488
114	1.9897	.4554	1.677	.2715	.53806	174	3.0369	.9477	1.997	.4745	1.4662
115	2.0071	.4627	1.687	.2743	.55041	175	3.0543	.9564	1.998	.4786	1.4836
116	2.0246	.4701	1.696	.2772	.56289	176	3.0718	.9651	1.9988	.4828	1.5010
117	2.0420	.4775	1.705	.2800	.57551	177	3.0892	.9738	1.9993	.4871	1.5184
118	2.0595	.4850	1.714	.2829	.58827	178	3.1067	.9825	1.9997	.4913	1.5359
119	2.0769	.4925	1.723	.2858	.60116	179	3.1241	.9913	1.9999	.4957	1.5533
120	**2.0944**	**.5000**	**1.732**	**.2887**	**.61419**	**180**	**3.1416**	**1.0000**	**2.000**	**.5000**	**1.5708**

* Area of segment of any radius = factor $G \times$ square of radius

Table 27. Areas of Segments of a Circle

Area = $F \times D^2$ where segment < semicircle; = $1/4\pi D^2$ − (area of segment of rise h). H = rise or height of segment; D = diam. of circle; F = factor corresponding to H/D. $h = (D - H)$

H/D	F	H/D	F	H/D	F	H/D	F	H/D	F	H/D	F
0.000	0.00000	0.07	0.02417	0.14	0.06683	0.21	0.11990	0.28	0.18002	0.35	0.24498
.001	.00004	.071	.02468	.141	.06753	.211	.12071	.281	.18092	.351	.24593
.002	.00012	.072	.02520	.142	.06822	.212	.12153	.282	.18182	.352	.24689
.003	.00022	.073	.02571	.143	.06892	.213	.12235	.283	.18272	.353	.24784
.004	.00034	.074	.02624	.144	.06963	.214	.12317	.284	.18362	.354	.24880
.005	.00047	.075	.02676	.145	.07033	.215	.12399	.285	.18452	.355	.24976
.006	.00062	.076	.02729	.146	.07103	.216	.12481	.286	.18542	.356	.25071
.007	.00078	.077	.02782	.147	.07174	.217	.12563	.287	.18633	.357	.25167
.008	.00095	.078	.02836	.148	.07245	.218	.12646	.288	.18723	.358	.25263
.009	.00113	.079	.02889	.149	.07316	.219	.12729	.289	.18814	.359	.25359
.01	.00133	.08	.02943	.15	.07387	.22	.12811	.29	.18905	.36	.25455
.011	.00153	.081	.02998	.151	.07459	.221	.12894	.291	.18996	.361	.25551
.012	.00175	.082	.03053	.152	.07531	.222	.12977	.292	.19086	.362	.25647
.013	.00197	.083	.03108	.153	.07603	.223	.13060	.293	.19177	.363	.25743
.014	.00220	.084	.03163	.154	.07675	.224	.13144	.294	.19268	.364	.25839
.015	.00244	.085	.03219	.155	.07747	.225	.13227	.295	.19360	.365	.25936
.016	.00268	.086	.03275	.156	.07819	.226	.13311	.296	.19451	.366	.26032
.017	.00294	.087	.03331	.157	.07892	.227	.13395	.297	.19542	.367	.26128
.018	.00320	.088	.03387	.158	.07965	.228	.13478	.298	.19634	.368	.26225
.019	.00347	.089	.03444	.159	.08038	.229	.13562	.299	.19725	.369	.26321
.02	.00375	.09	.03501	.16	.08111	.23	.13646	.3	.19817	.37	.26418
.021	.00403	.091	.03559	.161	.08185	.231	.13731	.301	.19908	.371	.26514
.022	.00432	.092	.03616	.162	.08258	.232	.13815	.302	.20000	.372	.26611
.023	.00462	.093	.03674	.163	.08332	.233	.13900	.303	.20092	.373	.26708
.024	.00492	.094	.03732	.164	.08406	.234	.13984	.304	.20184	.374	.26805
.025	.00523	.095	.03791	.165	.08480	.235	.14069	.305	.20276	.375	.26901
.026	.00555	.096	.03850	.166	.08554	.236	.14154	.306	.20368	.376	.26998
.027	.00587	.097	.03909	.167	.08629	.237	.14239	.307	.20460	.377	.27095
.028	.00619	.098	.03968	.168	.08704	.238	.14324	.308	.20553	.378	.27192
.029	.00653	.099	.04028	.169	.08779	.239	.14409	.309	.20645	.379	.27289
.03	.00687	.1	.04087	.17	.08854	.24	.14494	.31	.20738	.38	.27386
.031	.00721	.101	.04148	.171	.08929	.241	.14580	.311	.20830	.381	.27483
.032	.00756	.102	.04208	.172	.09004	.242	.14666	.312	.20923	.382	.27580
.033	.00791	.103	.04269	.173	.09080	.243	.14751	.313	.21015	.383	.27678
.034	.00827	.104	.04330	.174	.09155	.244	.14837	.314	.21108	.384	.27775
.035	.00864	.105	.04391	.175	.09231	.245	.14923	.315	.21201	.385	.27872
.036	.00901	.106	.04452	.176	.09307	.246	.15009	.316	.21294	.386	.27969
.037	.00938	.107	.04514	.177	.09384	.247	.15095	.317	.21387	.387	.28067
.038	.00976	.108	.04576	.178	.09460	.248	.15182	.318	.21480	.388	.28164
.039	.01015	.109	.04638	.179	.09537	.249	.15268	.319	.21573	.389	.28262
.04	.01054	.11	.04701	.18	.09613	.25	.15355	.32	.21667	.39	.28359
.041	.01093	.111	.04763	.181	.09690	.251	.15441	.321	.21760	.391	.28457
.042	.01133	.112	.04826	.182	.09767	.252	.15528	.322	.21853	.392	.28554
.043	.01173	.113	.04889	.183	.09845	.253	.15615	.323	.21947	.393	.28652
.044	.01214	.114	.04953	.184	.09922	.254	.15702	.324	.22040	.394	.28750
.045	.01255	.115	.05016	.185	.10000	.255	.15789	.325	.22134	.395	.28848
.046	.01297	.116	.05080	.186	.10077	.256	.15876	.326	.22228	.396	.28945
.047	.01339	.117	.05145	.187	.10155	.257	.15964	.327	.22322	.397	.29043
.048	.01382	.118	.05209	.188	.10233	.258	.16051	.328	.22415	.398	.29141
.049	.01425	.119	.05274	.189	.10312	.259	.16139	.329	.22509	.399	.29239
.05	.01468	.12	.05338	.19	.10390	.26	.16226	.33	.22603	.4	.29337
.051	.01512	.121	.05404	.191	.10469	.261	.16314	.331	.22697	.401	.29435
.052	.01556	.122	.05469	.192	.10547	.262	.16402	.332	.22792	.402	.29533
.053	.01601	.123	.05535	.193	.10626	.263	.16490	.333	.22886	.403	.29631
.054	.01646	.124	.05600	.194	.10705	.264	.16578	.334	.22980	.404	.29729
.055	.01691	.125	.05666	.195	.10784	.265	.16666	.335	.23074	.405	.29827
.056	.01737	.126	.05733	.196	.10864	.266	.16755	.336	.23169	.406	.29926
.057	.01783	.127	.05799	.197	.10943	.267	.16843	.337	.23263	.407	.30024
.058	.01830	.128	.05866	.198	.11023	.268	.16932	.338	.23358	.408	.30122
.059	.01877	.129	.05933	.199	.11102	.269	.17020	.339	.23453	.409	.30220
.06	.01924	.13	.06000	.2	.11182	.27	.17109	.34	.23547	.41	.30319
.061	.01972	.131	.06067	.201	.11262	.271	.17198	.341	.23642	.411	.30417
.062	.02020	.132	.06135	.202	.11343	.272	.17287	.342	.23737	.412	.30516
.063	.02068	.133	.06203	.203	.11423	.273	.17376	.343	.23832	.413	.30614
.064	.02117	.134	.06271	.204	.11504	.274	.17465	.344	.23927	.414	.30712
.065	.02166	.135	.06339	.205	.11584	.275	.17554	.345	.24022	.415	.30811
.066	.02215	.136	.06407	.206	.11665	.276	.17644	.346	.24117	.416	.30910
.067	.02265	.137	.06476	.207	.11746	.277	.17733	.347	.24212	.417	.31008
.068	.02315	.138	.06545	.208	.11827	.278	.17823	.348	.24307	.418	.31107
.069	.02366	.139	.06614	.209	.11908	.279	.17912	.349	.24403	.419	.31205

Table 27. Area of Segments of a Circle—*Continued*

H/D	F	H/D	F	H/D	F	H/D	F	H/D	F	H/D	F
0.42	0.31304	0.435	0.32788	0.45	0.34278	0.465	0.35773	0.48	0.37270	0.495	0.38770
.421	31403	436	.32887	.451	.34378	.466	35873	481	.37370	.496	.38870
.422	.31502	.437	.32987	.452	.34477	.467	.35972	.482	.37470	.497	.38970
.423	31600	.438	.33086	.453	34577	.468	.36072	.483	.37570	.498	.39070
.424	.31699	.439	.33185	.454	.34676	.469	.36172	.484	.37670	.499	.39170
.425	.31798	.44	.33284	.455	.34776	.47	.36272	.485	.37770	.500	.39270
.426	31897	441	.33384	.456	.34876	.471	.36372	.486	37870		
.427	31996	.442	.33483	.457	34975	.472	.36471	487	37970		
.428	32095	.443	.33582	.458	35075	.473	.36571	488	38070		
.429	.32194	.444	.33682	.459	.35175	.474	.36671	.489	.38170		
.43	.32293	.445	.33781	.46	.35274	.475	.36771	.49	.38270		
.431	32392	446	.33880	.461	.35374	.476	.36871	.491	38370		
.432	.32491	447	.33980	.462	.35474	.477	.36971	492	38470		
.433	32590	.448	.34079	.463	.35573	.478	.37071	.493	.38570		
.434	32689	.449	.34179	.464	.35673	.479	.37171	.494	38670		

Table 28. Values of Degrees and Minutes in Radians

Deg	Radians	Deg	Radians	Deg	Radians	Deg	Radians	Deg	Radians	Deg	Radians	Min	Radians	Min	Radians
1	0.01745	31	0.54105	61	1.06465	91	1.58825	121	2.11185	151	2.63545	1	0.00029	31	0.00902
2	03491	32	.55851	62	1.08210	92	1.60570	122	2.12930	152	2.65290	2	00058	32	00931
3	05236	33	.57596	63	1.09956	93	1.62316	123	2.14676	153	2.67035	3	00087	33	00960
4	06981	34	59341	64	1.11701	94	1.64061	124	2.16421	154	2.68781	4	00116	34	00989
5	08727	35	61087	65	1.13446	95	1.65806	125	2.18166	155	2.70526	5	00145	35	01018
6	10472	36	62832	66	1.15192	96	1.67552	126	2.19912	156	2.72271	6	00175	36	01047
7	12217	37	64577	67	1.16937	97	1.69297	127	2.21657	157	2.74017	7	00204	37	.01076
8	13963	38	66323	68	1.18682	98	1.71042	128	2.23402	158	2.75762	8	00233	38	01105
9	15708	39	68068	69	1.20428	99	1.72788	129	2.25148	159	2.77507	9	00262	39	01134
10	.17453	40	69813	70	1.22173	100	1.74533	130	2.26893	160	2.79253	10	00291	40	01164
11	19199	41	71559	71	1.23918	101	1.76278	131	2.28638	161	2.80998	11	00320	41	01193
12	20944	42	73304	72	1.25664	102	1.78024	132	2.30384	162	2.82743	12	00349	42	.01222
13	22689	43	75049	73	1.27409	103	1.79769	133	2.32129	163	2.84489	13	00378	43	01251
14	24435	44	76795	74	1.29154	104	1.81514	134	2.33874	164	2.86234	14	00407	44	01280
15	26180	45	78540	75	1.30900	105	1.83260	135	2.35620	165	2.87979	15	00436	45	01309
16	.27925	46	80285	76	1.32645	106	1.85005	136	2.37365	166	2.89725	16	00465	46	01338
17	29671	47	82031	77	1.34390	107	1.86750	137	2.39110	167	2.91470	17	00495	47	01367
18	31416	48	83776	78	1.36136	108	1.88496	138	2.40856	168	2.93215	18	00524	48	.01396
19	33161	49	85521	79	1.37881	109	1.90241	139	2.42601	169	2.94961	19	00553	49	01425
20	34907	50	87267	80	1.39626	110	1.91986	140	2.44346	170	2.96706	20	00582	50	01454
21	36652	51	89012	81	1.41372	111	1.93732	141	2.46092	171	2.98451	21	00611	51	01484
22	38397	52	90757	82	1.43117	112	1.95477	142	2.47837	172	3.00197	22	00640	52	01513
23	40143	53	92502	83	1.44862	113	1.97222	143	2.49582	173	3.01942	23	00669	53	01542
24	.41888	54	94248	84	1.46608	114	1.98968	144	2.51328	174	3.03687	24	00698	54	01571
25	.43633	55	95993	85	1.48353	115	2.00713	145	2.53073	175	3.05433	25	00727	55	01600
26	.45379	56	97738	86	1.50098	116	2.02458	146	2.54818	176	3.07178	26	00756	56	01629
27	.47124	57	99484	87	1.51844	117	2.04204	147	2.56564	177	3.08923	27	00785	57	01658
28	48869	58	1.01299	88	1.53589	118	2.05949	148	2.58309	178	3.10669	28	00814	58	.01687
29	50615	59	1.02974	89	1.55334	119	2.07694	149	2.60054	179	3.12414	29	00844	59	01716
30	52360	60	1.04720	90	1.57080	120	2.09440	150	2.61800	180	3.14159	30	00873	60	01745

Table 29. Values of Radians in Degrees

Rad.	00	01	02	03	04	05	06	07	08	09
	Deg	Deg	Deg.	Deg	Deg	Deg.	Deg.	Deg	Deg	Deg
0.0	0.0000	0.5730	1.1459	1.7189	2.2918	2.8648	3.4377	4.0107	4.5837	5.1566
.1	5.7296	6.3025	6.8755	7.4485	8.0214	8.5944	9.1673	9.7403	10.3132	10.8862
.2	11.4591	12.0321	12.6051	13.1780	13.7510	14.3239	14.8969	15.4699	16.0428	16.6158
.3	17.1887	17.7617	18.3346	18.9076	19.4806	20.0535	20.6265	21.1994	21.7724	22.3454
.4	22.9183	23.4913	24.0642	24.6372	25.2101	25.7831	26.3561	26.9290	27.5020	28.0749
.5	28.6479	29.2208	29.7938	30.3668	30.9397	31.1527	32.0856	32.6586	33.2316	33.8045
.6	34.3775	34.9504	35.5234	36.0963	36.6693	37.2423	37.8152	38.3882	38.9611	39.5341
.7	40.1070	40.6800	41.2530	41.8259	42.3989	42.9718	43.5448	44.1178	44.6907	45.2637
.8	45.8366	46.4096	46.9825	47.5555	48.1285	48.7014	49.2744	49.8473	50.4203	50.9932
.9	51.5662	52.1392	52.7121	53.2851	53.8580	54.4310	55.0039	55.5769	56.1499	56.7228

1 Radian = 57.29578 deg. | 2 Radians = 114.59156 deg. | 3 Radians = 171.88734 deg.

Table 30. Decimals of a Degree in Minutes and Seconds

Decimal	00		01		02		03		04		05		06		07		08		09	
	Min.	Sec.	Min.	Sec	Min	Sec	Min	Sec	Min	Sec	Min.	Sec	Min	Sec	Min	Sec	Min	Sec	Min	Sec.
0.0	0	0	0	36	1	12	1	48	2	24	3	0	3	36	4	12	4	48	5	24
.1	6	0	6	36	7	12	7	48	8	24	9	0	9	36	10	12	10	48	11	24
.2	12	0	12	36	13	12	13	48	14	24	15	0	15	36	16	12	16	48	17	24
.3	18	0	18	36	19	12	19	48	20	24	21	0	21	36	22	12	22	48	23	24
.4	24	0	24	36	25	12	25	48	26	24	27	0	27	36	28	12	28	48	29	24
.5	30	0	30	36	31	12	31	48	32	24	33	0	33	36	34	12	24	48	35	24
.6	36	0	36	36	37	12	37	48	38	24	39	0	39	36	40	12	40	48	41	24
7	42	0	42	36	43	12	43	48	44	24	45	0	45	36	46	12	46	48	47	24
8	48	0	48	36	49	12	49	48	50	24	51	0	51	36	52	12	52	48	53	24
9	54	0	54	36	55	12	55	48	56	24	57	0	57	36	58	12	58	48	59	24

Table 31. Minutes in Decimals of a Degree

Minutes	0	1	2	3	4	5	6	7	8	9
	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees	Degrees
0	0.00000	0.01667	0.03333	0.05000	0 06667	0 08333	0.10000	0 11667	0.13333	0.15000
10	.16667	.18333	.20000	.21667	.23333	.25000	.26667	.28333	.30000	.31667
20	.33333	.35000	.36667	.38333	.40000	.41667	.43333	.45000	.46667	.48333
30	.50000	.51667	.53333	.55000	.56667	.58333	60000	.61667	.63333	.65000
40	.66667	68333	.70000	71667	.73333	75000	76667	.78333	.80000	.81667
50	83333	.85000	86667	.88333	90000	91667	93333	.95000	.96667	.98333

Table 32. Seconds in Decimals of a Degree

Seconds	0	1	2	3	4
	Degrees	Degrees	Degrees	Degrees	Degrees
0	0	0.0002778	0.0005555	0.0008333	0 0011111
10	0 0027778	.0030555	.0033333	0036111	.0038888
20	.0055555	.0058333	.0061111	.0063888	.0066667
30	.0083333	.0086111	.0088888	.0091667	.0094444
40	.0111111	.0113888	.0116667	.0119444	.0122222
50	.0138888	.0141667	.0144444	.0147222	.0150000

Seconds	5	6	7	8	9
	Degrees	Degrees	Degrees	Degrees	Degrees
0	0 0013888	0.0016667	0.0019444	0.0022222	0.0024999
10	.0041667	.0044444	.0047222	.0050000	.0052778
20	.0069444	.0072222	.0075000	.0077778	.0080555
30	.0097222	.0010000	.0102778	.0105555	.0108333
40	.0125000	.0127778	.0130555	.0133333	.0136111
50	0152778	.0155555	.0158333	.0161111	.0163888

Table 33. Circumferences and Areas of Circles—Diameters in Feet and Inches

Circumferences in feet: areas in square feet

Diam., feet	Diameter, inches											
	0	1	2	3	4	5	6	7	8	9	10	11
	Circumference											
0	0 0000	0 2618	0 5236	0 7854	1 0472	1 3090	1 5708	1 8326	2 0944	2 3562	2 6180	2 8798
1	3 1416	3 4034	3 6653	3 9270	4 1877	4 4507	4 7124	4 9741	5 2361	5 4978	5 7595	6 0215
2	6 2832	6 5450	6 8069	7 0686	7 3293	7 5923	7 8540	8 1157	8 3777	8 6394	8 9011	9 1631
3	9 4248	9 6866	9 9485	10 210	10 471	10 734	10 996	11 257	11 519	11 781	12 043	12 305
4	12 566	12 828	13 090	13 352	13 613	13 875	14 137	14 399	14 661	14 923	15 184	15 446
5	15 708	15 970	16 232	16 493	16 754	17 017	17 279	17 540	17 802	18 064	18 326	18 588
6	18 850	19 111	19 373	19 635	19 896	20 159	20 420	20 682	20 944	21 206	21 467	21 729
7	21 991	22 253	22 515	22 777	23 037	23 300	23 562	23 824	24 086	24 347	24 609	24 871
8	25 133	25 395	25 656	25 918	26 179	26 442	26 704	26 965	27 227	27 489	27 751	28 013
9	28 274	28 536	28 798	29 060	29 320	29 583	29 845	30 107	30 369	30 631	30 892	31 154
10	31 416	31 678	31 940	32 201	32 462	32 725	32 987	33 248	33 510	33 772	34 034	34 296
11	34 557	34 819	35 081	35 343	35 604	35 867	36 128	36 390	36 652	36 914	37 175	37 437
12	37 699	37 961	38 223	38 484	38 745	39 008	39 270	39 532	39 794	40 055	40 317	40 579
13	40 841	41 102	41 364	41 626	41 887	42 150	42 411	42 673	42 935	43 197	43 459	43 721
14	43 982	44 244	44 506	44 768	45 028	45 291	45 553	45 815	46 077	46 338	46 600	46 862
15	47 124	47 386	47 648	47 909	48 170	48 433	48 695	48 956	49 218	49 480	49 742	50 004
16	50 265	50 527	50 789	51 051	51 312	51 575	51 836	52 098	52 360	52 622	52 883	53 145
17	53 407	53 669	53 931	54 192	54 453	54 716	54 978	55 240	55 502	55 763	56 025	56 287
18	56 549	56 810	57 072	57 334	57 595	57 858	58 119	58 381	58 643	58 905	59 167	59 429
19	59 690	59 952	60 214	60 476	60 736	61 000	61 261	61 523	61 785	62 046	62 308	62 570
20	62 832	63 094	63 356	63 617	63 878	64 141	64 403	64 664	64 926	65 188	65 450	65 712
21	65 973	66 235	66 497	66 759	67 020	67 282	67 544	67 806	68 068	68 330	68 591	68 853
22	69 115	69 377	69 639	69 900	70 161	70 424	70 686	70 947	71 209	71 471	71 733	71 995
23	72 257	72 518	72 780	73 042	73 303	73 566	73 827	74 089	74 351	74 613	74 874	75 136
24	75 398	75 660	75 922	76 184	76 444	76 707	76 969	77 231	77 493	77 754	78 016	78 278
25	78 540	78 802	79 063	79 325	79 586	79 849	80 111	80 372	80 634	80 896	81 158	81 420
26	81 681	81 943	82 205	82 467	82 727	82 990	83 252	83 514	83 776	84 038	84 299	84 561
27	84 823	85 085	85 347	85 608	85 869	86 132	86 394	86 655	86 917	87 179	87 441	87 703
28	87 965	88 226	88 488	88 749	89 011	89 274	89 535	89 797	90 059	90 321	90 582	90 844
29	91 106	91 367	91 630	91 892	92 152	92 415	92 677	92 939	93 201	93 462	93 724	93 986
30	94 248	94 509	94 771	95 033	95 294	95 557	95 818	96 080	96 342	96 604	96 866	97 128
	Area											
0	0 0000	0 0055	0 0218	0 0491	0 0873	0 1364	0 1963	0 2673	0 3491	0 4418	0 5454	0 6600
1	0 7854	0 9218	1 069	1 227	1 396	1 576	1 767	1 969	2 182	2 405	2 640	2 885
2	3 142	3 409	3 687	3 976	4 276	4 587	4 909	5 241	5 585	5 940	6 305	6 681
3	7 069	7 467	7 876	8 296	8 727	9 168	9 621	10 08	10 56	11 04	11 54	12 05
4	12 57	13 10	13 64	14 19	14 75	15 32	15 90	16 50	17 10	17 72	18 35	18 99
5	19 63	20 29	20 97	21 65	22 34	23 04	23 76	24 48	25 22	25 97	26 73	27 49
6	28 27	29 07	29 87	30 68	31 50	32 34	33 18	34 04	34 91	35 78	36 67	37 57
7	38 48	39 41	40 34	41 28	42 24	43 20	44 18	45 17	46 16	47 17	48 19	49 22
8	50 27	51 32	52 38	53 46	54 54	55 64	56 75	57 86	58 99	60 13	61 28	62 44
9	63 62	64 80	66 00	67 20	68 42	69 64	70 88	72 13	73 39	74 66	75 94	77 24
10	78 54	79 85	81 18	82 52	83 86	85 22	86 59	87 97	89 36	90 76	92 18	93 60
11	95 03	96.48	97 93	99 40	100 9	102 4	103 9	105 4	106 9	108 4	110 0	111 5
12	113 1	114 7	116 3	117 9	119 5	121 1	122 7	124 4	126 0	127 7	129 4	131 0
13	132 7	134 4	136 2	137 9	139 6	141 4	143 1	144 9	146 7	148 5	150 3	152 1
14	153 9	155 8	157 6	159 5	161 4	163 2	165 1	167 0	168 9	170 9	172 8	174 8
15	176 7	178 7	180 7	182 7	184 7	186 7	188 7	190 7	192 8	194 8	196 9	199 0
16	201 1	203 2	205 3	207 4	209 5	211 7	213 8	216 0	218 2	220 4	222 5	224 8
17	227 0	229 2	231 4	233 7	236 0	238 2	240 5	242 8	245 1	247 5	249 8	252 1
18	254 5	256 8	259 2	261 6	264 0	266 4	268 8	271 2	273 7	276 1	278 6	281 0
19	283 5	286 0	288 5	291 0	293 6	296.1	298 7	301 2	303 8	306 4	308 9	311 5
20	314 2	316 8	319 4	322 1	324 7	327 4	330 1	332 7	335 4	338 2	340 9	343 6
21	346 4	349 1	351 9	354 7	357 4	360 2	363 1	365 9	368 7	371 5	374 4	377 2
22	380 1	383 0	386 0	388 8	391 7	394 7	397 6	400 5	403 5	406 5	409 5	412 5
23	415 5	418 5	421 5	424 6	427 6	430 7	433 7	436 8	439 9	443 0	446 1	449 2
24	452 4	455 5	458 7	461 9	465 0	468 2	471 4	474 6	477 9	481 1	484 3	487 6
25	490 9	494 1	497 4	500 7	504 0	507 4	510 7	514 0	517 4	520 8	524 1	527 5
26	530 9	534 3	537 7	541 2	544 6	548 1	551 6	555 0	558 5	562 0	565 5	569 0
27	572 6	576 1	579 6	583 2	586 8	590 3	594 0	597 5	601 2	604 8	608 4	612 1
28	615.8	619 4	623 1	626 8	630 5	634 2	637 9	641 7	645 4	649 2	652 9	656 7
29	660 5	664 3	668 1	672 0	675 8	679 6	683 5	687 3	691 2	695 1	699 0	702 9
30	706 9	710 8	714 7	718 7	722 6	726 6	730 6	734 6	738 6	742 6	746 7	750 7

5. MENSURATION

Table 34. Plane Rectilinear Figures

Notation: Lines, $a, b, c, \ldots$; angles, $\alpha, \beta, \gamma, \ldots$; altitude (perpendicular height), h; side, s; diagonals, $d, d_1, \ldots$; perimeter, p; radius of inscribed circle, r; radius of circumscribed circle, R; area, A

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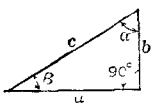
1. Right Triangle



(One angle 90°)

$p = a + b + c$; $c^2 = a^2 + b^2$;

$$A = \frac{ab}{2} = \frac{a^2}{2}\tan\beta = \frac{c^2}{4}\sin 2\beta = \frac{c^2}{4}\sin 2\alpha.$$

For additional formulas, see *General Triangle* below, and also trigonometry.

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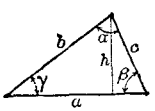
2. General Triangle (and Equilateral Triangle)



For General Triangle:

$p = a + b + c$. Let $s = \frac{1}{2}(a + b + c)$.

$$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s};\ R = \frac{a}{2\sin\alpha} = \frac{abc}{4rs};$$

$$A = \frac{ah}{2} = \frac{ab}{2}\sin\gamma = \frac{b^2\sin\gamma\sin\alpha}{2\sin\beta} = rs = \frac{abc}{4R}.$$

Length of median to side $c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$.

Length of bisector of angle $\gamma = \dfrac{\sqrt{ab[(a+b)^2 - c^2]}}{a + b}$.

For Equilateral Triangle ($a = b = c = s$ and $\alpha = \beta = \gamma = 60°$):

(Equal sides and equal angles)

$$p = 3s;\ r = \frac{s}{2\sqrt{3}};\ R = \frac{s}{\sqrt{3}} = 2r;$$

$$h = \frac{s\sqrt{3}}{2};\ s = \frac{2h}{\sqrt{3}};\ A = \frac{s^2\sqrt{3}}{4}.$$

For additional formulas, see trigonometry.

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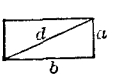
3. Rectangle (and Square)



For Rectangle:

$p = 2(a + b)$; $d = \sqrt{a^2 + b^2}$; $A = ab$.

For Square ($a = b = s$):

$$p = 4s;\ d = s\sqrt{2};\ s = \frac{d}{\sqrt{2}};\ A = s^2 = \frac{d^2}{2}.$$

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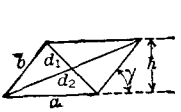
4. General Parallelogram (Rhomboid) (and Rhombus)



For General Parallelogram (Rhomboid):

(Opposite sides parallel)

$p = 2(a + b)$; $d_1 = \sqrt{a^2 + b^2 - 2ab\cos\gamma}$;

$d_2 = \sqrt{a^2 + b^2 + 2ab\cos\gamma}$; $d_1^2 + d_2^2 = 2(a^2 + b^2)$;

$A = ah = ab\sin\gamma$.

For Rhombus ($a = b = s$):

(Opposite sides parallel and all sides equal)

$$p = 4s;\ d_1 = 2s\sin\frac{\gamma}{2};\ d_2 = 2s\cos\frac{\gamma}{2};\ d_1^2 + d_2^2 = 4s^2;$$

$$d_1 d_2 = 2s^2\sin\gamma;\ A = sh = s^2\sin\gamma = \frac{d_1 d_2}{2}.$$

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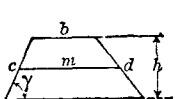
5. General Trapezoid (and Isosceles Trapezoid)



Let mid-line bisecting non-parallel sides $= m$. Then $m = \dfrac{a + b}{2}$.

For General Trapezoid:

(Only one pair of opposite sides parallel)

$$p = a + b + c + d;\ A = \frac{(a + b)h}{2} = mh.$$

For Isosceles Trapezoid ($d = c$):

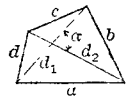
(Non-parallel sides equal)

$$A = \frac{(a + b)h}{2} = mh = \frac{(a + b)c\sin\gamma}{2}$$

$$= (a - c\cos\gamma)c\sin\gamma = (b + c\cos\gamma)c\sin\gamma.$$

Table 34. Plane Rectilinear Figures—*Continued*

6. General Quadrilateral (Trapezium)

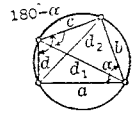


(No sides parallel)

$p = a + b + c + d$

$A = \frac{1}{2} d_1 d_2 \sin \alpha$ = sum of areas of the two triangles formed by either diagonal and the four sides

7. Quadrilateral Inscribed in Circle



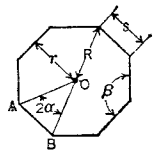
(Sum of opposite angles = 180°)

$ac + bd = d_1 d_2$

Let $s = \frac{1}{2}(a + b + c + d) = \frac{p}{2}$ and α = angle between sides a and b.

$A = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \frac{1}{2}(ab + cd) \sin \alpha$

8. Regular Polygon (and General Polygon)



For Regular Polygon

(Equal sides and equal angles)

Let n = number of sides.

Central angle $= 2\alpha = \frac{2\pi}{n}$ radians;

Vertex angle $= \beta = \frac{(n-2)}{n}\pi$ radians.

$p = ns$; $s = 2r \tan \alpha = 2R \sin \alpha$;

$r = \frac{s}{2} \cot \alpha$; $R = \frac{s}{2} \csc \alpha$,

$A = \frac{nsr}{2} = nr^2 \tan \alpha = \frac{nR^2}{2} \sin 2\alpha = \frac{ns^2}{4} \cot \alpha$ = sum of areas of the n equal triangles such as OAB.

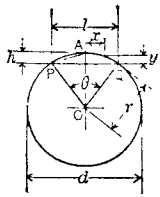
For General Polygon.

A = sum of areas of constituent triangles into which it can be divided

Table 35. Plane Curvilinear Figures

Notation. Lines, a, b, . . . ; radius, r; diameter, d, perimeter, p; circumference, c; central angle in radians, θ, arc, s; chord of arc (s), l; chord of half arc $(s/2)$, l'; rise, h; area, A.

9. Circle (and Circular Arc)



For Circle:

$d = 2r$; $c = 2\pi r = \pi d$; $A = \pi r^2 = \frac{\pi d^2}{4} = \frac{c^2}{4\pi}$.

For Circular Arc:

Let arc $PAQ = s$; and chord $PA = \left(\text{chord of } \frac{s}{2}\right) = l'$. Then,

$s = r\theta = \frac{d\theta}{2}$; $s = \frac{8l' - l}{3}$. (The latter equation is Huyghen's approximate formula. For θ small, error is very small, for $\theta = 120°$, error equals about 1 part in 400; for $\theta = 180°$, error is less than 1.25%.)

$l = 2r \sin \frac{\theta}{2}$; $l = 2\sqrt{2hr - h^2}$ (approximate formula)

$r = \frac{s}{\theta} = \frac{l}{2 \sin \frac{\theta}{2}}$; $r = \frac{4h^2 + l^2}{8h}$ (approximate formula)

$h = r \mp \sqrt{r^2 - \frac{l^2}{4}}$ (− if $\theta \leq 180°$; + if $\theta \geq 180°$) $= r\left(1 - \cos \frac{\theta}{2}\right)$

$= r \operatorname{versin} \frac{\theta}{2} = 2r \sin^2 \frac{\theta}{4} = \frac{l}{2} \tan \frac{\theta}{4} = r + y - \sqrt{r^2 - x^2}$.

Side ordinate $y = h - r + \sqrt{r^2 - x^2}$.

(*Table continued on p. 11-58*)

Table 35. Plane Curvilinear Figures—*Continued*

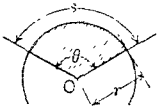
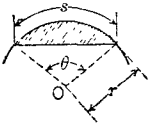
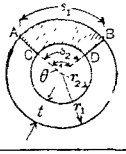
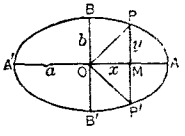
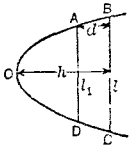
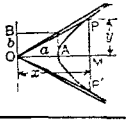
10. Circular Sector (and Semicircle) 	*For Circular Sector*: $A = \frac{\theta r^2}{2} = \frac{sr}{2}$. *For Semicircle*: $A = \frac{\pi r^2}{2}$.
11. Circular Segment 	$A = \frac{r^2}{2}(\theta - \sin\theta)$ $= 1/2\,[sr \mp l\,(r - h)]$ ($-$ if $h \leqq r$; $+$ if $h \geqq r$) $A = \frac{2lh}{3}$ or $\frac{h}{15}(3l' + 6l)$. (Approximate formulas For h small compared with r, error is very small; for $h = \frac{r}{4}$, first formula errs about 3.5% and second less than 1.0%.)
12. Annulus 	(Surface between two concentric circles) $A = \pi(r_1^2 - r_2^2) = \pi(r_1 + r_2)(r_1 - r_2)$; A of sector $ABCD = \frac{\theta}{2}(r_1^2 - r_2^2) = \frac{\theta}{2}(r_1 + r_2)(r_1 - r_2)$ $= \frac{t}{2}(s_1 + s_2)$
13. Ellipse 	$p = \pi(a + b)\left(1 + \frac{R^2}{4} + \frac{R^4}{64} + \frac{R^6}{256} + \ldots\right)$ where $R = \frac{a - b}{a + b}$. $p = \pi(a + b)\frac{64 - 3R^4}{64 - 16R^2}$ (approximate formula) $A = \pi ab$; A of quadrant $AOB = \frac{\pi ab}{4}$; A of sector $AOP = \frac{ab}{2}\cos^{-1}\frac{x}{a}$; A of sector $POB = \frac{ab}{2}\sin^{-1}\frac{x}{a}$; A of section $BPP'B' = xy + ab\sin^{-1}\frac{x}{a}$; A of segment $PAP'P = -xy + ab\cos^{-1}\frac{x}{a}$. For additional formulas, see analytic geometry
14. Parabola 	Arc $BOC = s = 1/2\sqrt{l^2 + 16h^2} + \frac{l^2}{8h}\log_e\frac{4h + \sqrt{l^2 + 16h^2}}{l}$. Let $R = \frac{h}{l}$. Then, $s = l\left(1 + \frac{8R^2}{3} - \frac{32R^4}{5} + \ldots\right)$ (approximate formula). $d = \frac{h}{l^2}(l^2 - l_1^2)$; $l_1 = l\sqrt{\frac{h - d}{h}}$; $h = \frac{dl^2}{l^2 - l_1^2}$; A of segment $BOC = \frac{2hl}{3}$; A of section $ABCD = \frac{2}{3}d\left(\frac{l^3 - l_1^3}{l^2 - l_1^2}\right)$. For additional formulas, see analytic geometry.
15. Hyperbola 	A of figure $OPAP'O = ab\log_e\left(\frac{x}{a} + \frac{y}{b}\right) = ab\cosh^{-1}\frac{x}{a}$; A of segment $PAP' = xy - ab\log_e\left(\frac{x}{a} + \frac{y}{b}\right) = xy - ab\cosh^{-1}\frac{x}{a}$. For additional formulas, see analytic geometry.

Table 35. Plane Curvilinear Figures—*Continued*

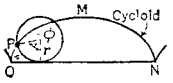
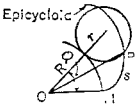
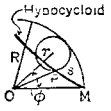
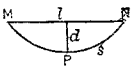
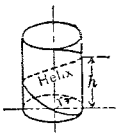
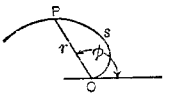
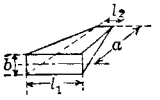
16. Cycloid 	Arc $OP = s = 4r\left(1 - \cos\frac{\phi}{2}\right)$; Arc $OMN = 8r$; A under curve $OMN = 3\pi r^2$. For additional formulas, see analytic geometry
17. Epicycloid 	Arc $MP = s = \frac{4r}{R}(R + r)\left(1 - \cos\frac{R\phi}{2r}\right)$; Area $MOP = A = \frac{r}{2R}(R + r)(R + 2r)\left(\frac{R\phi}{r} - \sin\frac{R\phi}{r}\right)$. For additional formulas, see analytic geometry.
18. Hypocycloid 	Arc $MP = s = \frac{4r}{R}(R - r)\left(1 - \cos\frac{R\phi}{2r}\right)$; Area $MOP = A = \frac{r}{2R}(R - r)(R - 2r)\left(\frac{R\phi}{r} - \sin\frac{R\phi}{r}\right)$. For additional formulas, see analytic geometry.
19. Catenary 	If d is small compared with l Arc $MPN = s = l\left[1 + \frac{2}{3}\left(\frac{2d}{l}\right)^2\right]$ (approximately). For additional formulas, see analytic geometry.
20. Helix 	Let length of helix $= s$, radius of coil (r = radius of cylinder in figure) $= r$, distance advanced in one revolution = pitch $= h$; and number of revolutions $= n$. Then, $s = n\sqrt{(2\pi r)^2 + h^2}$.
21. Spiral of Archimedes 	Let $a = \frac{r}{\phi}$. Then, Arc $OP = s = \frac{a}{2}\left[\phi\sqrt{1 + \phi^2} + \log_e\left(\phi + \sqrt{1 + \phi^2}\right)\right]$. For additional formulas, see analytic geometry
22. Irregular Figure	Divide the figure into an even number, n, of strips by means of $(n + 1)$ ordinates, y_n, spaced equal distances, w. The area can then be determined approximately by any of the following formulas, which are presented in the order of usual increasing approach to accuracy. In any of the first three cases, the greater the number of strips used, the more nearly accurate will be the result. (Approximate Formulas)
Trapezoidal Rule	$A = w\left[\frac{y_0 + y_n}{2} + y_1 + y_2 + \ldots + y_{n-1}\right]$.
Durand's Rule	$A = w[0.4(y_0 + y_n) + 1.1(y_1 + y_{n-1}) + y_2 + y_3 + \ldots + y_{n-2}]$;
Simpson's Rule (*n must* be even)	$A = \frac{w}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})]$.
Weddle's Rule (for 6 strips only)	$A = \frac{3w}{10}[5(y_1 + y_5) + 6y_3 + y_0 + y_2 + y_4 + y_6]$.
	Areas of irregular surfaces can often be determined more quickly by such methods as plotting on squared paper and counting the squares; graphical coordinate representation (see analytic geometry); or use of a planimeter

Table 36. Solids Having Plane Surfaces

Notation. Lines, a, b, c, . . . ; altitude (perpendicular height), h; slant height, s; perimeter of base, p_b or p_B; perimeter of a right section, p_r; area of base, A_b or A_B; area of a right section, A_r; total area of lateral surfaces, A_l, total area of all surfaces, A_t; volume, V.

23. Wedge (and Right Triangular Prism)



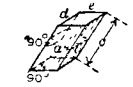
For Wedge:

(Narrow-side rectangular); $V = \frac{ab}{6}(2l_1 + l_2)$.

For Right Triangular Prism (or wedge having parallel triangular bases perpendicular to sides) $l_2 = l_1 = l$:

$V = \frac{abl}{2}$.

24. Rectangular Prism (or Rectangular Parallelepiped) (and Cube)



For Rectangular Prism or Rectangular Parallelepiped:

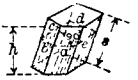
$A_l = 2c(a + b)$; $A_t = 2(de + ac + bc)$;

$V = A_r c = abc$.

For Cube (letting $b = c = a$):

$A_t = 6a^2$; $V = a^3$; Diagonal $= a\sqrt{3}$.

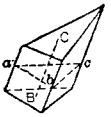
25. General Prism



$A_l = hp_b = sp_r = s(a + b + \ldots + n)$;

$V = hA_b = sA_r$

26. General Truncated Prism (and Truncated Triangular Prism)



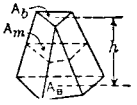
For General Truncated Prism:

$V = A_r \cdot$(length of line BC joining centers of gravity of bases).

For Truncated Triangular Prism:

$V = \frac{A_r}{3}(a + b + c)$.

27. Prismatoid



Let area of mid-section $= A_m$.

$V = \frac{h}{6}(A_B + A_b + 4A_m)$.

28. Right Regular Pyramid (and Frustum of Right Regular Pyramid)

For Right Regular Pyramid.

$A_l = \frac{sp_B}{2}$; $V = \frac{hA_B}{3}$.

For Frustum of Right Regular Pyramid:

$A_l = \frac{s}{2}(p_B + p_b)$; $V = \frac{h}{3}(A_B + A_b + \sqrt{A_B A_b})$.

29. General Pyramid (and Frustum of Pyramid)

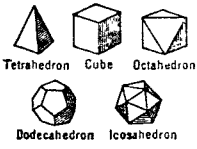
For General Pyramid.

$V = \frac{hA_B}{3}$.

For Frustum of General Pyramid:

$V = \frac{h}{3}(A_B + A_b + \sqrt{A_B A_b})$.

30. Regular Polyhedrons



Let edge $= a$, and radius of inscribed sphere $= r$. Then,

$r = \frac{3V}{A_t}$, and.

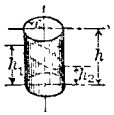
Number of Faces	Form of Faces	Total Area A_t	Volume V
4	Equilateral triangle	$1.7321\,a^2$	$0.1179\,a^3$
6	Square	$6.0000\,a^2$	$1.0000\,a^3$
8	Equilateral triangle	$3.4641\,a^2$	$0.4714\,a^3$
12	Regular pentagon	$20.6457\,a^2$	$7.6631\,a^3$
20	Equilateral triangle	$8.6603\,a^2$	$2.1817\,a^3$

(Factors shown only to four decimal places.)

Table 37. Solids Having Curved Surfaces

Notation. Lines, a, b, c, . . . ; altitude (perpendicular height), h, h_1, . . . ; slant height, s; radius, r; perimeter of base, p_b; perimeter of a right section, p_r; angle in radians, ϕ; arc, s; chord of segment, l; rise, h; area of base, A_b or A_B; area of a right section, A_r; total area of convex surface, A_l; total area of all surfaces, A_t; volume, V.

31. Right Circular Cylinder (and Truncated Right Circular Cylinder)



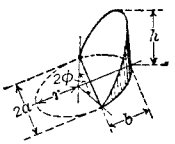
For Right Circular Cylinder:

$$A_l = 2\pi rh;\ A_t = 2\pi r\,(r + h);$$
$$V = \pi r^2 h.$$

For Truncated Right Circular Cylinder:

$$A_l = \pi r\,(h_1 + h_2);\ A_t = \pi r\left[h_1 + h_2 + r + \sqrt{r^2 + \left(\frac{h_1 - h_2}{2}\right)^2}\right];$$
$$V = \frac{\pi r^2}{2}\,(h_1 + h_2).$$

32. Ungula (Wedge) of Right Circular Cylinder

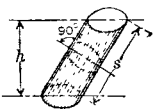


$$A_l = \frac{2\,rh}{b}\,[a + (b - r)\phi];$$
$$V = \frac{h}{3b}\,[a\,(3r^2 - a^2) + 3\,r^2\,(b - r)\phi]$$
$$= \frac{hr^3}{b}\left[\sin\phi - \frac{\sin^3\phi}{3} - \phi\cos\phi\right].$$

For Semicircular Base (letting $a = b = r$):

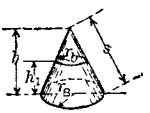
$$A_l = 2\,rh;\ V = \frac{2\,r^2h}{3}.$$

33. General Cylinder



$$A_l = p_b h = p_r s;$$
$$V = A_b h = A_r s.$$

34. Right Circular Cone (and Frustum of Right Circular Cone)



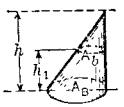
For Right Circular Cone:

$$A_l = \pi r_B s = \pi r_B\sqrt{r_B^2 + h^2};\ A_t = \pi r_B\,(r_B + s);$$
$$V = \frac{\pi r_B^2 h}{3}.$$

For Frustum of Right Circular Cone:

$$s = \sqrt{h_1^2 + (r_B - r_b)^2},\ A_l = \pi s\,(r_B + r_b);$$
$$V = \frac{\pi h_1}{3}\,(r_B^2 + r_b^2 + r_B r_b).$$

35. General Cone (and Frustum of General Cone)



For General Cone.

$$V = \frac{A_B h}{3}$$

For Frustum of General Cone:

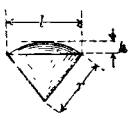
$$V = \frac{h_1}{3}\,(A_B + A_b + \sqrt{A_B A_b})$$

36. Sphere

Let diameter $= d$.

$$A_t = 4\pi r^2 = \pi d^2,$$
$$V = \frac{4\pi r^3}{3} = \frac{\pi d^3}{6}.$$

37. Spherical Sector (and Hemisphere)



For Spherical Sector.

$$A_t = \frac{\pi r}{2}\,(4h + l);\ V = \frac{2\pi r^2 h}{3}.$$

For Hemisphere (letting $h = \frac{l}{2} = r$):

$$A_t = 3\pi r^2;\ V = \frac{2\pi r^3}{3}.$$

(Table continued on p 11-62)

Table 37. Solids Having Curved Surfaces—*Continued*

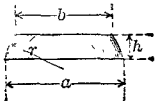
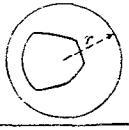
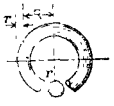
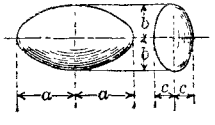
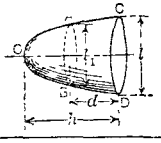
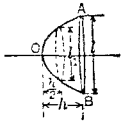
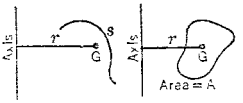
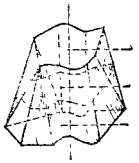
38. Spherical Zone (and Spherical Segment) 	*For Spherical Zone Bounded by Two Planes.* $A_l = 2\pi rh;\ A_t = \frac{\pi}{4}(8rh + a^2 + b^2).$ *For Spherical Zone Bounded by One Plane* ($b = 0$): $A_l = 2\pi rh = \frac{\pi}{2}(4h^2 + a^2);$ $A_t = \frac{\pi}{4}(8rh + a^2) = \frac{\pi}{2}(2h^2 + a^2).$ *For Spherical Segment with Two Bases:* $V = \frac{\pi h}{24}(3a^2 + 3b^2 + 4h^2).$ *For Spherical Segment with One Base* ($b = 0$): $V = \frac{\pi h}{24}(3a^2 + 4h^2) = \pi h^2\left(r - \frac{h}{3}\right).$
39. Spherical Polygon (and Spherical Triangle) 	*For Spherical Polygon.* Let sum of angles in radians $= \theta$ and number of sides $= n$. $A = [\theta - (n - 2)\pi]r^2$ (The quantity $[\theta - (n - 2)\pi]$ is called "spherical excess.") *For Spherical Triangle* ($n = 3$): $A = (\theta - \pi)r^2$ For additional formulas, see trigonometry
40. Torus 	$A_l = 4\pi^2 Rr;$ $V = 2\pi^2 Rr^2.$
41. Ellipsoid (and Spheroids) 	*For Ellipsoid.* $V = \frac{4}{3}\pi abc.$ *For Prolate Spheroid:* Let $c = b$ and $\frac{\sqrt{a^2 - b^2}}{a} = e.$ $A_t = 2\pi b^2 + 2\pi ab\frac{\sin^{-1} e}{e};\ V = \frac{4}{3}\pi ab^2.$ *For Oblate Spheroid:* Let $c = a$ and $\frac{\sqrt{a^2 - b^2}}{a} = e.$ $A_t = 2\pi a^2 + \frac{\pi a^2}{e}\ln\left(\frac{1+e}{1-e}\right);\ V = \frac{4}{3}\pi a^2 b.$
42. Paraboloid of Revolution 	A_l of segment $DOC = \frac{2\pi l}{3h^2}\left[\left(\frac{l^2}{16} + h^2\right)^{3/2} - \left(\frac{l}{4}\right)^3\right].$ *For Paraboloidal Segment with Two Bases:* V of $ABCD = \frac{\pi d}{8}(l^2 + l_1^2)$ *For Paraboloidal Segment with One Base* ($l_1 = 0$ and $d = h$): V of $DOC = \frac{\pi h l^2}{8}$
43. Hyperboloid of Revolution 	V of segment $AOB = \frac{\pi h}{24}(l^2 + 4l_1^2).$

Table 37. Solids Having Curved Surfaces—*Continued*

44. Surface and Solid of Revolution 	Let perpendicular distance from axis to center of gravity (G) of curve (or surface) $= r$. Curve (or surface) must not cross axis. Then, *Area of Surface* generated by curve revolving about axis: $A_l = 2\pi rs$. *Volume of Solid* generated by surface revolving about axis: $V = 2\pi rA$.
45. Irregular Solid 	One of the following methods can often be employed to determine the volume of an irregular solid with a reasonable approach to accuracy: (*a*) Divide the solid into prisms, cylinders, etc., and sum their individual volumes. (*b*) Divide one surface into triangles, after replacing curved lines by straight ones and curved surfaces by plane ones. Then multiply the area of each triangle by the mean depth of the section beneath it (which generally approximates the average of the depths at its corners) Sum the volumes thus obtained. (*c*) If two surfaces are parallel, replace any curved lateral surfaces by plane surfaces best suited to the contour and then employ the prismatoidal formula

6. ANALYTIC GEOMETRY

See Eshbach's *Handbook of Engineering Fundamentals*, 2nd Ed., pages 2-80 to 2-85 (Point and Line; Transformation of Coordinates; Conic Sections) and 2-93 to 2-96 (Point, Line, and Plane; Transformation of Coordinates, Quadric Surfaces).

TRIGONOMETRY

7. CIRCULAR FUNCTIONS OF PLANE ANGLES

Definitions and Values

Trigonometric Functions. The angle α in Fig. 1 is measured in degrees or radians. The ratio of any two of the quantities x, y, or r determines the extent of the opening between the lines OP and OX. Since these ratios are functions of the angle they may be used to measure or construct it. The definitions and terms used to designate the functions are as follows:

$$\text{Sine } \alpha = \frac{y}{r} = \sin \alpha$$

$$\text{Cosine } \alpha = \frac{x}{r} = \cos \alpha$$

$$\text{Tangent } \alpha = \frac{y}{x} = \tan \alpha$$

$$\text{Cotangent } \alpha = \frac{x}{y} = \cot \alpha$$

$$\text{Secant } \alpha = \frac{r}{x} = \sec \alpha$$

$$\text{Cosecant } \alpha = \frac{r}{y} = \csc \alpha$$

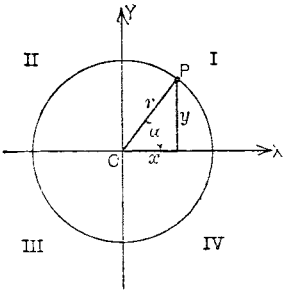


FIG. 1

$$\text{Versine } \alpha = \frac{r - x}{r} = \text{vers } \alpha = 1 - \cos \alpha$$

$$\text{Coversine } \alpha = \frac{r - y}{r} = \text{covers } \alpha = 1 - \sin \alpha$$

$$\text{Haversine } \alpha = \frac{r - x}{2r} = \text{hav } \alpha = \frac{1}{2} \text{vers } \alpha$$

Positive and Negative Values. An angle α (Fig. 1), if measured in a *counterclockwise* direction, is said to be *positive;* if measured *clockwise, negative.* Following the convention that x is positive if measured along OX to the right of the OY axis and negative if measured to the left, and similarly, y is positive if measured along OY above the OX axis and negative if measured below, the signs of the trigonometric functions are different for angles in the quadrants I, II, III, and IV.

Table 38. Signs of Trigonometric Functions

Quadrant	sin	cos	tan	cot	sec	csc
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−

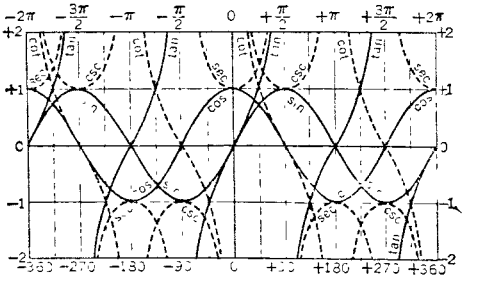


FIG 2

Values of Trigonometric Functions are periodic, the period of the sin, cos, sec, csc being 2π radians, and that of the tan and cot, π radians. For example, in Fig. 2 (n an integer)

$$\sin (\alpha + 2\pi n) = \sin \alpha$$

$$\tan (\alpha + \pi n) = \tan \alpha$$

Table 39. Functions of Angles in Any Quadrant in Terms of Angles in the First Quadrant

	$-\alpha$	$90° \pm \alpha$	$180° \pm \alpha$	$270° \pm \alpha$	$360° \pm \alpha$
sin	$-\sin\alpha$	$+\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$
cos	$+\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$	$+\cos\alpha$
tan	$-\tan\alpha$	$\mp\cot\alpha$	$\pm\tan\alpha$	$\mp\cot\alpha$	$\pm\tan\alpha$
cot	$-\cot\alpha$	$\mp\tan\alpha$	$\pm\cot\alpha$	$\mp\tan\alpha$	$\pm\cot\alpha$
sec	$+\sec\alpha$	$\mp\csc\alpha$	$-\sec\alpha$	$\pm\csc\alpha$	$+\sec\alpha$
csc	$-\csc\alpha$	$+\sec\alpha$	$\mp\csc\alpha$	$-\sec\alpha$	$\pm\csc\alpha$

Table 40. Functions of Certain Angles

	0°	30°	45°	60°	90°	180°	270°	360°
sin	0	$1/2$	$1/2\sqrt{2}$	$1/2\sqrt{3}$	1	0	-1	0
cos	1	$1/2\sqrt{3}$	$1/2\sqrt{2}$	$1/2$	0	-1	0	1
tan	0	$1/3\sqrt{3}$	1	$\sqrt{3}$	∞	0	∞	0
cot	∞	$\sqrt{3}$	1	$1/3\sqrt{3}$	0	∞	0	∞
sec	1	$2/3\sqrt{3}$	$\sqrt{2}$	2	∞	-1	∞	1
csc	∞	2	$\sqrt{2}$	$2/3\sqrt{3}$	1	∞	-1	∞

Inverse, or Anti-functions. The symbol $\sin^{-1} x$ means the angle whose sine is x, and is read inverse sine of x, anti-sine of x, or arc sine x. Similarly for $\cos^{-1} x$, $\tan^{-1} x$, $\cot^{-1} x$, $\sec^{-1} x$, $\csc^{-1} x$, $\text{vers}^{-1} x$, the last meaning an angle α such that $(1 - \cos\alpha) = x$. While the direct functions (sine, etc.) are single valued, the indirect are many valued, thus $\sin 30° = 0.5$, but $\sin^{-1} 0.5 = 30°, 150°, \cdots$.

Functional Relations Identities

Table 41. Functions of an Angle in Terms of Each of the Others

	$\sin\alpha = a$	$\cos\alpha = a$	$\tan\alpha = a$	$\cot\alpha = a$	$\sec\alpha = a$	$\csc\alpha = a$
sin	a	$\sqrt{1-a^2}$	$\frac{a}{\sqrt{1+a^2}}$	$\frac{1}{\sqrt{1+a^2}}$	$\frac{\sqrt{a^2-1}}{a}$	$\frac{1}{a}$
cos	$\sqrt{1-a^2}$	a	$\frac{1}{\sqrt{1+a^2}}$	$\frac{a}{\sqrt{1+a^2}}$	$\frac{1}{a}$	$\frac{\sqrt{a^2-1}}{a}$
tan	$\frac{a}{\sqrt{1-a^2}}$	$\frac{\sqrt{1-a^2}}{a}$	a	$\frac{1}{a}$	$\sqrt{a^2-1}$	$\frac{1}{\sqrt{a^2-1}}$
cot	$\frac{\sqrt{1-a^2}}{a}$	$\frac{a}{\sqrt{1-a^2}}$	$\frac{1}{a}$	a	$\frac{1}{\sqrt{a^2-1}}$	$\sqrt{a^2-1}$
sec	$\frac{1}{\sqrt{1-a^2}}$	$\frac{1}{a}$	$\sqrt{1+a^2}$	$\frac{\sqrt{1+a^2}}{a}$	a	$\frac{a}{\sqrt{a^2-1}}$
csc	$\frac{1}{a}$	$\frac{1}{\sqrt{1-a^2}}$	$\frac{\sqrt{1+a^2}}{a}$	$\sqrt{1+a^2}$	$\frac{a}{\sqrt{a^2-1}}$	a

Note: The sign of the radical is to be determined by the quadrant.

Functions of the Sum and Difference of Two Angles.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = (\tan\alpha \pm \tan\beta)/(1 \mp \tan\alpha\tan\beta)$$

$$\cot(\alpha \pm \beta) = (\cot\beta\cot\alpha \mp 1)/(\cot\beta \pm \cot\alpha)$$

If x is small, say 3° or 4°, then the following are close approximations, in which the quantity x is to be expressed in radians (1° = 0.01745 radian).

$$\sin\alpha \approx \alpha, \quad \cos\alpha \approx 1, \quad \tan\alpha \approx \alpha$$

$$\sin(\alpha \pm x) \approx \sin\alpha \pm x\cos\alpha, \quad \cos(\alpha \pm x) \approx \cos\alpha \mp x\sin\alpha$$

Functions of Half Angles.

$$\sin \tfrac{1}{2}\alpha = \sqrt{\tfrac{1}{2}(1 - \cos\alpha)} = \tfrac{1}{2}\sqrt{1 + \sin\alpha} - \tfrac{1}{2}\sqrt{1 - \sin\alpha}$$

$$\cos \tfrac{1}{2}\alpha = \sqrt{\tfrac{1}{2}(1 + \cos\alpha)} = \tfrac{1}{2}\sqrt{1 + \sin\alpha} + \tfrac{1}{2}\sqrt{1 - \sin\alpha}$$

$$\tan \tfrac{1}{2}\alpha = \sqrt{(1 - \cos\alpha)/(1 + \cos\alpha)} = (1 - \cos\alpha)/\sin\alpha = \sin\alpha/(1 + \cos\alpha)$$

$$\cot \tfrac{1}{2}\alpha = \sqrt{(1 + \cos\alpha)/(1 - \cos\alpha)} = (1 + \cos\alpha)/\sin\alpha = \sin\alpha/(1 - \cos\alpha)$$

Functions of Multiples of Angles.

$$\sin 2\alpha = 2 \sin\alpha \cos\alpha$$

$$\tan 2\alpha = 2 \tan\alpha/(1 - \tan^2\alpha)$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\cot 2\alpha = (\cot^2\alpha - 1)/2\cot\alpha$$

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\sin 4\alpha = 8\cos^3\alpha \sin\alpha - 4\cos\alpha \sin\alpha$$

$$\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$$

$$\sin n\alpha = 2\sin(n-1)\alpha \cos\alpha - \sin(n-2)\alpha$$

$$= n \sin\alpha \cos^{n-1}\alpha - {}_nC_3 \sin^3\alpha \cos^{n-3}\alpha + {}_nC_5 \sin^5\alpha \cos^{n-5}\alpha - \cdots$$

$$\cos n\alpha = 2\cos(n-1)\alpha \cos\alpha - \cos(n-2)\alpha$$

$$= \cos^n\alpha - {}_nC_2 \sin^2\alpha \cos^{n-2}\alpha + {}_nC_4 \sin^4\alpha \cos^{n-4}\alpha - \cdots$$

Products and Powers of Functions.

$$\sin\alpha \sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\cos\alpha \cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$

$$\sin\alpha \cos\beta = \tfrac{1}{2}\sin(\alpha - \beta) + \tfrac{1}{2}\sin(\alpha + \beta)$$

$$\tan\alpha \cot\alpha = \sin\alpha \csc\alpha = \cos\alpha \sec\alpha = 1$$

$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha); \quad \cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha)$$

$$\sin^3\alpha = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha); \quad \cos^3\alpha = \tfrac{1}{4}(3\cos\alpha + \cos 3\alpha)$$

$$\sin^4\alpha = \tfrac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha), \quad \cos^4\alpha = \tfrac{1}{8}(3 + 4\cos 2\alpha + \cos 4\alpha)$$

$$\sin^5\alpha = \tfrac{1}{16}(10\sin\alpha - 5\sin 3\alpha + \sin 5\alpha)$$

$$\sin^6\alpha = \tfrac{1}{32}(10 - 15\cos 2\alpha + 6\cos 4\alpha - \cos 6\alpha)$$

$$\cos^5\alpha = \tfrac{1}{16}(10\cos\alpha + 5\cos 3\alpha + \cos 5\alpha)$$

$$\cos^6\alpha = \tfrac{1}{32}(10 + 15\cos 2\alpha + 6\cos 4\alpha + \cos 6\alpha)$$

Sums and Differences of Functions.

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$

$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$

$$\tan\alpha + \tan\beta = \frac{\sin(\alpha + \beta)}{\cos\alpha \cos\beta}; \quad \cot\alpha + \cot\beta = \frac{\sin(\alpha + \beta)}{\sin\alpha \sin\beta}$$

$$\tan\alpha - \tan\beta = \frac{\sin(\alpha - \beta)}{\cos\alpha \cos\beta}; \quad \cot\alpha - \cot\beta = -\frac{\sin(\alpha - \beta)}{\sin\alpha \sin\beta}$$

$$\sin^2\alpha - \sin^2\beta = \sin(\alpha + \beta)\sin(\alpha - \beta)$$

$$\cos^2\alpha - \cos^2\beta = -\sin(\alpha + \beta)\sin(\alpha - \beta)$$

$$\cos^2\alpha - \sin^2\beta = \cos(\alpha + \beta)\cos(\alpha - \beta)$$

Anti-Trigonometric or Inverse Functional Relations. In the following formulas the periodic constant is omitted.

$$\sin^{-1} x = -\sin^{-1}(-x) = \frac{\pi}{2} - \cos^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$= \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \csc^{-1}\frac{1}{x} = \sec^{-1}\frac{1}{\sqrt{1-x^2}}$$

$$\cos^{-1} x = \pi - \cos^{-1}(-x) = \frac{\pi}{2} - \sin^{-1} x = \tfrac{1}{2}\cos^{-1}(2x^2-1) = \sin^{-1}\sqrt{1-x^2}$$

$$= \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sec^{-1}\frac{1}{x} = \csc^{-1}\frac{1}{\sqrt{1-x^2}}$$

$$\tan^{-1} x = -\tan^{-1}(-x) = \frac{\pi}{2} - \cot^{-1} x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \cot^{-1}\frac{1}{x}$$

$$= \sec^{-1}\sqrt{1+x^2} = \csc^{-1}\frac{\sqrt{1+x^2}}{x}$$

$$\cot^{-1} x = \tan^{-1}\frac{1}{x}; \quad \sec^{-1} x = \cos^{-1}\frac{1}{x}, \quad \csc^{-1} x = \sin^{-1}\frac{1}{x}$$

$$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\}$$

$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\{xy \mp \sqrt{(1-x^2)(1-y^2)}\}$$

$$\sin^{-1} x \pm \cos^{-1} y = \sin^{-1}\{xy \pm \sqrt{(1-x^2)(1-y^2)}\} = \cos^{-1}\{y\sqrt{1-x^2} \mp x\sqrt{1-y^2}\}$$

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\frac{x \pm y}{1 \mp xy}$$

$$\tan^{-1} x \pm \cot^{-1} y = \tan^{-1}\frac{xy \pm 1}{y \mp x} = \cot^{-1}\frac{y \mp x}{xy \pm 1}$$

8. SOLUTION OF TRIANGLES

Relations between Angles and Sides of Plane Triangles. Let a, b, c = sides of triangle; α, β, γ = angles opposite, a, b, c, respectively; A = area of triangle; $s = \frac{1}{2}(a+b+c)$; r = radius of inscribed circle (Fig. 3).

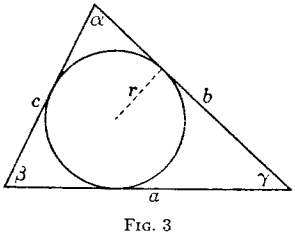


FIG. 3

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \quad \text{(Law of Sines)}$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \text{(Law of Cosines)}$$

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)} \quad \text{(Law of Tangents)}$$

$$\alpha + \beta + \gamma = 180°$$

$$a = b\cos\gamma + c\cos\beta; \quad b = c\cos\alpha + a\cos\gamma; \quad c = a\cos\beta + b\cos\alpha$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\sin\alpha = \frac{2}{bc}A; \quad \sin\beta = \frac{2}{ca}A; \quad \sin\gamma = \frac{2}{ab}A$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}; \quad \sin\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}; \quad \sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}; \quad \cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}}; \quad \cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}; \quad \tan\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}; \quad \tan\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

Solution of Plane Oblique Triangles.

Given a, b, c. (If logarithms are to be used, use 1.)

1. $r = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$; $A = \sqrt{s(s-a)(s-b)(s-c)} = rs$;

$$\tan\frac{\alpha}{2} = \frac{r}{s-a}\ ;\ \tan\frac{\beta}{2} = \frac{r}{s-b}\ ;\ \tan\frac{\gamma}{2} = \frac{r}{s-c}.$$

2. $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$; $\cos\beta = \dfrac{a^2+c^2-b^2}{2ac}$;

$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab},\ \text{or}\ \gamma = 180^\circ - (\alpha+\beta).$$

Given a, b, α.

$\sin\beta = \dfrac{b\sin\alpha}{a}$ (if $a > b$, $\beta < \dfrac{\pi}{2}$ and has only one value, if $b > a$, β has two values,

β_1 and $\beta_2 = 180^\circ - \beta_1$), $\gamma = 180^\circ - (\alpha+\beta)$, $c = \dfrac{a\sin\gamma}{\sin\alpha}$, $A = {}^1/_2\, ab\sin\gamma$.

Given a, α, β

$b = \dfrac{a\sin\beta}{\sin\alpha}$; $\gamma = 180^\circ - (\alpha+\beta)$; $c = \dfrac{a\sin\gamma}{\sin\alpha}$; $A = {}^1/_2\, ab\sin\gamma$.

Given a, b, γ. (If logarithms are to be used use 1.)

1. $\tan {}^1/_2(\alpha-\beta) = \dfrac{a-b}{a+b}\cot {}^1/_2\gamma$; ${}^1/_2(\alpha+\beta) = 90^\circ - {}^1/_2\gamma$; $c = \dfrac{a\sin\gamma}{\sin\alpha}$;

$A = {}^1/_2\, ab\sin\gamma$.

2. $c = \sqrt{a^2+b^2-2ab\cos\gamma}$, $\sin\alpha = \dfrac{a\sin\gamma}{c}$; $\beta = 180^\circ - (\alpha+\gamma)$.

3. $\tan\alpha = \dfrac{a\sin\gamma}{b - a\cos\gamma}$, $\beta = 180^\circ - (\alpha+\gamma)$, $c = \dfrac{a\sin\gamma}{\sin\alpha}$.

Mollweide's Check Formulas.

1. $\dfrac{a-b}{c} = \dfrac{\sin {}^1/_2(\alpha-\beta)}{\cos {}^1/_2\gamma}$.

2. $\dfrac{a+b}{c} = \dfrac{\cos {}^1/_2(\alpha-\beta)}{\sin {}^1/_2\gamma}$.

Solution of Plane Right Triangles. Let $\gamma = 90^\circ$ and c be the hypotenuse Given any two sides or one side and an acute angle α

$$a = \sqrt{c^2-b^2} = \sqrt{(c+b)(c-b)} = b\tan\alpha = c\sin\alpha$$

$$b = \sqrt{c^2-a^2} = \sqrt{(c-a)(c-a)} = \frac{a}{\tan\alpha} = c\cos\alpha$$

$$c = \sqrt{a^2+b^2} = \frac{a}{\sin\alpha} = \frac{b}{\cos\alpha}$$

$$\alpha = \sin^{-1}\frac{a}{c} = \cos^{-1}\frac{b}{c} = \tan^{-1}\frac{a}{b},\ \beta = 90^\circ - \alpha$$

$$A = \frac{ab}{2} = \frac{a^2}{2\tan\alpha} = \frac{b^2\tan\alpha}{2} = \frac{c^2\sin 2\alpha}{4}$$

9. TRIGONOMETRIC FUNCTIONS

Table 42 gives the values $\sin x$, $\cos x$, $\tan x$, and $\cot x$ for x from 0 to 90°. By making use of the periodicity of these functions, values can be determined from this table for all values of x. If the angle is given in radians: Number of radians $\times \dfrac{180}{\pi}$ (57 295) = Number of degrees.

Table 42. Values and Logarithms of Trigonometric Functions

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.00000	1.00000	.00000	+∞	−∞	10.000000	−∞	+∞	60	1.00
	1	.00029	1.00000	.00029	3437.75	6.463726	.000000	6.463726	13.536274	59	
	2	.00058	1.00000	.00058	1718.87	.764756	.000000	.764756	.235244	58	
.05	3	.00087	1.00000	.00087	1145.92	.939847	.000000	.940847	.059153	57	.95
	4	.00116	1.00000	.00116	859.436	7.065786	.000000	7.065786	12.934214	56	
	5	.00145	1.00000	.00145	687.549	.162696	.000000	.162696	.837304	55	
.10	6	.00175	1.00000	.00175	572.957	.241877	9.999999	.241878	.758122	54	.90
	7	.00204	1.00000	.00204	491.106	.308824	.999999	.308825	.691175	53	
	8	.00233	1.00000	.00233	429.718	.366816	.999999	.366817	.633183	52	
.15	9	.00262	1.00000	.00262	381.971	.417968	.999999	.417970	.582030	51	.85
	10	.00291	1.00000	.00291	343.774	.463726	.999998	.463727	.536273	50	
	11	.00320	.99999	.00320	312.521	.505118	.999998	.505120	.494880	49	
.20	12	.00349	.99999	.00349	286.478	.542906	.999997	.542909	.457091	48	.80
	13	.00378	.99999	.00378	264.441	.577668	.999997	.577672	.422328	47	
	14	.00407	.99999	.00407	245.552	.609853	.999996	.609857	.390143	46	
.25	15	.00436	.99999	.00436	229.182	.639816	.999996	.639820	.360180	45	.75
	16	.00465	.99999	.00465	214.858	.667845	.999995	.667849	.332151	44	
	17	.00495	.99999	.00495	202.219	.694173	.999995	.694179	.305821	43	
.30	18	.00524	.99999	.00524	190.984	.718997	.999994	.719003	.280997	42	.70
	19	.00553	.99998	.00553	180.932	.742478	.999993	.742484	.257516	41	
	20	.00582	.99998	.00582	171.885	.764754	.999993	.764761	.235239	40	
.35	21	.00611	.99998	.00611	163.700	.785943	.999992	.785951	.214049	39	.65
	22	.00640	.99998	.00640	156.259	.806146	.999991	.806155	.193845	38	
	23	.00669	.99998	.00669	149.465	.825451	.999990	.825460	.174540	37	
.40	24	.00698	.99998	.00698	143.237	.843934	.999989	.843944	.156056	36	.60
	25	.00727	.99997	.00727	137.507	.861662	.999989	.861674	.138326	35	
	26	.00756	.99997	.00756	132.219	.878695	.999988	.878708	.121292	34	
.45	27	.00785	.99997	.00785	127.321	.895085	.999987	.895099	.104901	33	.55
	28	.00814	.99997	.00815	122.774	.910879	.999986	.910894	.089106	32	
	29	.00844	.99996	.00844	118.540	.926119	.999985	.926134	.073866	31	
.50	30	.00873	.99996	.00873	114.589	.940842	.999983	.940858	.059142	30	.50
	31	.00902	.99996	.00902	110.892	.955082	.999982	.955100	.044900	29	
	32	.00931	.99996	.00931	107.426	.968870	.999981	.968889	.031111	28	
.55	33	.00960	.99995	.00960	104.171	.982233	.999980	.982253	.017747	27	.45
	34	.00989	.99995	.00989	101.107	.995198	.999979	.995219	.004781	26	
	35	.01018	.99995	.01018	98.2179	8.007787	.999977	8.007809	11.992191	25	
.60	36	.01047	.99995	.01047	95.4895	.020021	.999976	.020044	.979956	24	.40
	37	.01076	.99994	.01076	92.9085	.031919	.999975	.031945	.968055	23	
	38	.01105	.99994	.01105	90.4633	.043501	.999973	.043527	.956473	22	
.65	39	.01134	.99994	.01135	88.1436	.054781	.999972	.054809	.945191	21	.35
	40	.01164	.99993	.01164	85.9398	.065776	.999971	.065806	.934194	20	
	41	.01193	.99993	.01193	83.8435	.076500	.999969	.076531	.923469	19	
.70	42	.01222	.99993	.01222	81.8470	.086965	.999968	.086997	.913003	18	.30
	43	.01251	.99992	.01251	79.9434	.097183	.999966	.097217	.902783	17	
	44	.01280	.99992	.01280	78.1263	.107167	.999964	.107203	.892797	16	
.75	45	.01309	.99991	.01309	76.3900	.116926	.999963	.116963	.883037	15	.25
	46	.01338	.99991	.01338	74.7292	.126471	.999961	.126510	.873490	14	
	47	.01367	.99991	.01367	73.1390	.135810	.999959	.135851	.864149	13	
.80	48	.01396	.99990	.01396	71.6151	.144953	.999958	.144996	.855004	12	.20
	49	.01425	.99990	.01425	70.1533	.153907	.999956	.153952	.846048	11	
	50	.01454	.99989	.01455	68.7501	.162681	.999954	.162727	.837273	10	
.85	51	.01483	.99989	.01484	67.4019	.171280	.999952	.171328	.828672	9	.15
	52	.01513	.99989	.01513	66.1055	.179713	.999950	.179763	.820237	8	
	53	.01542	.99988	.01542	64.8580	.187985	.999948	.188036	.811964	7	
.90	54	.01571	.99988	.01571	63.6567	.196102	.999946	.196156	.803844	6	.10
	55	.01600	.99987	.01600	62.4992	.204070	.999944	.204126	.795874	5	
	56	.01629	.99987	.01629	61.3829	.211895	.999942	.211953	.788047	4	
.95	57	.01658	.99986	.01658	60.3058	.219581	.999940	.219641	.780359	3	.05
	58	.01687	.99986	.01687	59.2659	.227134	.999938	.227195	.772805	2	
	59	.01716	.99985	.01716	58.2612	.234557	.999936	.234621	.765379	1	
1.00	60	.01745	.99985	.01746	57.2900	8.241855	9.999934	8.241921	11.758079	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00		.01745	.99985	.01746	57.2900	8.241855	9.999934	8.241921	11.758079	60	1.00
						.249033	.999932	.249102	.750898	59	
	2	.01803	.99984	.01804	55.4415	.256094	.999929	.256165	.743835	58	
.05	3	.01832	.99983	.01833	54.5613	.263042	.999927	.263115	.736885	57	.95
	4	.01862	.99983	.01862	53.7086	.269881	.999925	.269956	.730044	56	
	5	.01891	.99982	.01891	52.8821	.276614	.999922	.276691	.723309	55	
.10	6	.01920	.99982	.01920	52.0807	.283243	.999920	.283323	.716677	54	.90
	7	.01949	.99981	.01949	51.3032	.289773	.999918	.289856	.710144	53	
	8	.01978	.99980	.01978	50.5485	.296207	.999915	.296292	.703708	52	
.15	9	.02007	.99980	.02007	49.8157	.302546	.999913	.302634	.697366	51	.85
	10	.02036	.99979	.02036	49.1039	.308794	.999910	.308884	.691116	50	
	11	.02065	.99979	.02066	48.4121	.314954	.999907	.315046	.684954	49	
.20	12	.02094	.99978	.02095	47.7395	.321027	.999905	.321122	.678878	48	.80
	13	.02123	.99977	.02124	47.0853	.327016	.999902	.327114	.672886	47	
	14	.02152	.99977	.02153	46.4489	.332924	.999899	.333025	.666975	46	
.25	15	.02181	.99976	.02182	45.8294	.338753	.999897	.338856	.661144	45	.75
	16	.02211	.99976	.02211	45.2261	.344504	.999894	.344610	.655390	44	
	17	.02240	.99975	.02240	44.6386	.350181	.999891	.350289	.649711	43	
.30	18	.02269	.99974	.02269	44.0661	.355783	.999888	.355895	.644105	42	.70
	19	.02298	.99974	.02298	43.5081	.361315	.999885	.361430	.638570	41	
	20	.02327	.99973	.02328	42.9641	.366777	.999882	.366895	.633105	40	
.35	21	.02356	.99972	.02357	42.4335	.372171	.999879	.372292	.627708	39	.65
	22	.02385	.99972	.02386	41.9158	.377499	.999876	.377622	.622378	38	
	23	.02414	.99971	.02415	41.4106	.382762	.999873	.382889	.617111	37	
.40	24	.02443	.99970	.02444	40.9174	.387962	.999870	.388092	.611908	36	.60
	25	.02472	.99969	.02473	40.4358	.393101	.999867	.393234	.606766	35	
	26	.02501	.99969	.02502	39.9655	.398179	.999864	.398315	.601685	34	
.45	27	.02530	.99968	.02531	39.5059	.403199	.999861	.403338	.596662	33	.55
	28	.02560	.99967	.02560	39.0568	.408161	.999858	.408304	.591696	32	
	29	.02589	.99966	.02589	38.6177	.413068	.999854	.413213	.586787	31	
.50	30	.02618	.99966	.02619	38.1885	.417919	.999851	.418068	.581932	30	.50
	31	.02647	.99965	.02648	37.7686	.422717	.999848	.422869	.577131	29	
	32	.02676	.99964	.02677	37.3579	.427462	.999844	.427618	.572382	28	
.55	33	.02705	.99963	.02706	36.9560	.432156	.999841	.432315	.567685	27	.45
	34	.02734	.99963	.02735	36.5627	.436800	.999838	.436962	.563038	26	
	35	.02763	.99962	.02764	36.1776	.441394	.999834	.441560	.558440	25	
.60	36	.02792	.99961	.02793	35.8005	.445941	.999831	.446110	.553890	24	.40
	37	.02821	.99960	.02822	35.4313	.450440	.999827	.450613	.549387	23	
	38	.02850	.99959	.02851	35.0695	.454893	.999824	.455070	.544930	22	
.65	39	.02879	.99959	.02881	34.7151	.459301	.999820	.459481	.540519	21	.35
	40	.02908	.99958	.02910	34.3678	.463665	.999816	.463849	.536151	20	
	41	.02938	.99957	.02939	34.0273	.467985	.999813	.468172	.531828	19	
.70	42	.02967	.99956	.02968	33.6935	.472263	.999809	.472454	.527546	18	.30
	43	.02996	.99955	.02997	33.3662	.476498	.999805	.476693	.523307	17	
	44	.03025	.99954	.03026	33.0452	.480693	.999801	.480892	.519108	16	
.75	45	.03054	.99953	.03055	32.7303	.484843	.999797	.485050	.514950	15	.25
	46	.03083	.99952	.03084	32.4213	.488963	.999794	.489170	.510830	14	
	47	.03112	.99952	.03114	32.1181	.493040	.999790	.493250	.506750	13	
.80	48	.03141	.99951	.03143	31.8205	.497078	.999786	.497293	.502707	12	.20
	46	.03170	.99950	.03172	31.5284	.501080	.999782	.501298	.498702	11	
	50	.03199	.99949	.03201	31.2416	.505045	.999778	.505267	.494733	10	
.85	51	.03228	.99948	.03230	30.9599	.508974	.999774	.509200	.490800	9	.15
	52	.03257	.99947	.03259	30.6833	.512867	.999769	.513098	.486902	8	
	53	.03286	.99946	.03288	30.4116	.516726	.999765	.516961	.483039	7	
.90	54	.03316	.99945	.03317	30.1446	.520551	.999761	.520790	.479210	6	.10
	55	.03345	.99944	.03346	29.8823	.524343	.999757	.524586	.475414	5	
	56	.03374	.99943	.03376	29.6245	.528102	.999753	.528349	.471651	4	
.95	57	.03463	.99942	.03405	29.3711	.531828	.999748	.532030	.467920	3	.05
	58	.03432	.99941	.03434	29.1220	.535523	.999744	.535779	.464221	2	
	59	.03461	.99940	.03463	28.8771	.539186	.999740	.539447	.460553	1	
1.00	60	.03490	.99939	.03492	28.6363	8.542813	9.999735	8.543084	11.456916	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.03490	.99939	.03492	28.6363	8.542319	9.999735	8.543084	11.456916	60	1.00
	1	.03519	.99938	.03521	28.3994	.546422	.999731	.546691	.453309	59	
	2	.03548	.99937	.03550	28.1664	.549995	.999726	.550268	.449732	58	
.05	3	.03577	.99936	.03579	27.9372	.553539	.999722	.553817	.446183	57	.95
	4	.03606	.99935	.03609	27.7117	.557054	.999717	.557336	.442664	56	
	5	.03635	.99934	.03638	27.4899	.560540	.999713	.560828	.439172	55	
.10	6	.03664	.99933	.03667	27.2715	.563999	.999708	.564291	.435709	54	.90
	7	.03693	.99932	.03696	27.0566	.567431	.999704	.567727	.432273	53	
	8	.03723	.99931	.03725	26.8450	.570836	.999699	.571137	.428863	52	
.15	9	.03752	.99930	.03754	26.6367	.574214	.999694	.574520	.425480	51	.85
	10	.03781	.99929	.03783	26.4316	.577566	.999689	.577877	.422123	50	
	11	.03810	.99927	.03812	26.2296	.580892	.999685	.581208	.418792	49	
.20	12	.03839	.99926	.03842	26.0307	.584193	.999680	.584514	.415486	48	.80
	13	.03868	.99925	.03871	25.8348	.587469	.999675	.587795	.412205	47	
	14	.03897	.99924	.03900	25.6418	.590721	.999670	.591051	.408949	46	
.25	15	.03926	.99923	.03929	25.4517	.593948	.999665	.594283	.405717	45	.75
	16	.03955	.99922	.03958	25.2644	.597152	.999660	.597492	.402508	44	
	17	.03984	.99921	.03987	25.0798	.600332	.999655	.600677	.399323	43	
.30	18	.04013	.99919	.04016	24.8978	.603489	.999650	.603839	.396161	42	.70
	19	.04042	.99918	.04046	24.7185	.606623	.999645	.606978	.393022	41	
	20	.04071	.99917	.04075	24.5418	.609734	.999640	.610094	.389906	40	
.35	21	.04100	.99916	.04104	24.3675	.612823	.999635	.613189	.386811	39	.65
	22	.04129	.99915	.04133	24.1957	.615891	.999629	.616252	.383748	38	
	23	.04159	.99913	.04162	24.0263	.618937	.999624	.619313	.380687	37	
.40	24	.04188	.99912	.04191	23.8593	.621962	.999619	.622343	.377657	36	.60
	25	.04217	.99911	.04220	23.6945	.624965	.999614	.625352	.374648	35	
	26	.04246	.99910	.04250	23.5321	.627946	.999608	.628340	.371660	34	
.45	27	.04275	.99909	.04279	23.3718	.630911	.999603	.631308	.368692	33	.55
	28	.04304	.99907	.04308	23.2137	.633854	.999597	.634256	.365744	32	
	29	.04333	.99906	.04337	23.0577	.636776	.999592	.637184	.362816	31	
.50	30	.04362	.99905	.04366	22.9038	.639680	.999586	.640093	.359907	30	.50
	31	.04391	.99904	.04395	22.7519	.642563	.999581	.642982	.357018	29	
	32	.04420	.99902	.04424	22.6020	.645428	.999575	.645853	.354147	28	
.55	33	.04449	.99901	.04454	22.4541	.648274	.999570	.648704	.351296	27	.45
	34	.04478	.99900	.04483	22.3081	.651102	.999564	.651537	.348463	26	
	35	.04507	.99898	.04512	22.1640	.653911	.999558	.654352	.345648	25	
.60	36	.04536	.99897	.04541	22.0217	.656702	.999553	.657149	.342851	24	.40
	37	.04565	.99896	.04570	21.8813	.659475	.999547	.659928	.340072	23	
	38	.04594	.99894	.04599	21.7426	.662230	.999541	.662689	.337311	22	
.65	39	.04623	.99893	.04628	21.6056	.664968	.999535	.665433	.334567	21	.35
	40	.04653	.99892	.04658	21.4704	.667689	.999529	.668160	.331840	20	
	41	.04682	.99890	.04687	21.3369	.670393	.999524	.670870	.329130	19	
.70	42	.04711	.99889	.04716	21.2049	.673080	.999518	.673563	.326437	18	.30
	43	.04740	.99888	.04745	21.0747	.675751	.999512	.676239	.323761	17	
	44	.04769	.99886	.04774	20.9460	.678405	.999506	.678900	.321100	16	
.75	45	.04798	.99885	.04803	20.8188	.681043	.999500	.681544	.318456	15	.25
	46	.04827	.99883	.04833	20.6932	.683665	.999493	.684172	.315828	14	
	47	.04856	.99882	.04862	20.5691	.686272	.999487	.686784	.313216	13	
.80	48	.04885	.99881	.04891	20.4465	.688863	.999481	.689381	.310619	12	.20
	49	.04914	.99879	.04920	20.3253	.691438	.999475	.691963	.308037	11	
	50	.04943	.99878	.04949	20.2056	.693998	.999469	.694529	.305471	10	
.85	51	.04972	.99876	.04978	20.0872	.696543	.999463	.697081	.302919	9	.15
	52	.05001	.99875	.05007	19.9702	.699073	.999456	.699617	.300383	8	
	53	.05030	.99873	.05037	19.8546	.701589	.999450	.702139	.297861	7	
.90	54	.05059	.99872	.05066	19.7403	.704090	.999443	.704646	.295354	6	.10
	55	.05088	.99870	.05095	19.6273	.706577	.999437	.707140	.292860	5	
	56	.05117	.99869	.05124	19.5156	.709049	.999431	.709618	.290382	4	
.95	57	.05146	.99867	.05153	19.4051	.711507	.999424	.712083	.287917	3	.05
	58	.05175	.99866	.05182	19.2959	.713952	.999418	.714534	.285466	2	
	59	.05205	.99864	.05212	19.1879	.716383	.999411	.716972	.283028	1	
1.00	60	.05234	.99863	.05241	19.0811	8.718800	9.999404	8.719396	11.280604	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.05234	.99863	.05241	19.0811	8.718800	9.999404	8.719396	11.280604	60	1.00
	1	.05263	.99861	.05270	18.9755	.721204	.999398	.721806	.278194	59	
	2	.05292	.99860	.05299	18.8711	.723595	.999391	.724204	.275796	58	
.05	3	.05321	.99858	.05328	18.7678	.725972	.999384	.726588	.273412	57	.95
	4	.05350	.99857	.05357	18.6656	.728337	.999378	.728959	.271041	56	
	5	.05379	.99855	.05387	18.5645	.730688	.999371	.731317	.268683	55	
.10	6	.05408	.99854	.05416	18.4645	.733027	.999364	.733663	.266337	54	.90
	7	.05437	.99852	.05445	18.3655	.735354	.999357	.735996	.264004	53	
	8	.05466	.99851	.05474	18.2677	.737667	.999350	.738317	.261683	52	
.15	9	.05495	.99849	.05503	18.1708	.739969	.999343	.740626	.259374	51	.85
	10	.05524	.99847	.05533	18.0750	.742259	.999336	.742922	.257078	50	
	11	.05553	.99846	.05562	17.9802	.744536	.999329	.745207	.254793	49	
.20	12	.05582	.99844	.05591	17.8863	.746802	.999322	.747479	.252521	48	.80
	13	.05611	.99842	.05620	17.7934	.749055	.999315	.749740	.250260	47	
	14	.05640	.99841	.05649	17.7015	.751297	.999308	.751989	.248011	46	
.25	15	.05669	.99839	.05678	17.6106	.753528	.999301	.754227	.245773	45	.75
	16	.05698	.99838	.05708	17.5205	.755747	.999294	.756453	.243547	44	
	17	.05727	.99836	.05737	17.4314	.757955	.999287	.758668	.241332	43	
.30	18	.05756	.99834	.05766	17.3432	.760151	.999279	.760872	.239128	42	.70
	19	.05785	.99833	.05795	17.2558	.762337	.999272	.763065	.236935	41	
	20	.05814	.99831	.05824	17.1693	.764511	.999265	.765246	.234754	40	
.35	21	.05844	.99829	.05854	17.0837	.766675	.999257	.767417	.232583	39	.65
	22	.05873	.99827	.05883	16.9990	.768828	.999250	.769578	.230422	38	
	23	.05902	.99826	.05912	16.9150	.770970	.999242	.771727	.228273	37	
.40	24	.05931	.99824	.05941	16.8319	.773101	.999235	.773866	.226134	36	.60
	25	.05960	.99822	.05970	16.7496	.775223	.999227	.775995	.224005	35	
	26	.05989	.99821	.05999	16.6681	.777333	.999220	.778114	.221886	34	
.45	27	.06018	.99819	.06029	16.5874	.779434	.999212	.780222	.219778	33	.55
	28	.06047	.99817	.06058	16.5075	.781524	.999205	.782320	.217680	32	
	29	.06076	.99815	.06087	16.4283	.783605	.999197	.784408	.215592	31	
.50	30	.06105	.99813	.06116	16.3499	.785675	.999189	.786486	.213514	30	.50
	31	.06134	.99812	.06145	16.2722	.787736	.999181	.788554	.211446	29	
	32	.06163	.99810	.06175	16.1952	.789787	.999174	.790613	.209387	28	
.55	33	.06192	.99808	.06204	16.1190	.791828	.999166	.792662	.207338	27	.45
	34	.06221	.99806	.06233	16.0435	.793859	.999158	.794701	.205299	26	
	35	.06250	.99804	.06262	15.9687	.795881	.999150	.796731	.203269	25	
.60	36	.06279	.99803	.06291	15.8945	.797894	.999142	.798752	.201248	24	.40
	37	.06308	.99801	.06321	15.8211	.799897	.999134	.800763	.199237	23	
	38	.06337	.99799	.06350	15.7483	.801892	.999126	.802765	.197235	22	
.65	39	.06366	.99797	.06379	15.6762	.803876	.999118	.804758	.195242	21	.35
	40	.06395	.99795	.06408	15.6048	.805852	.999110	.806742	.193258	20	
	41	.06424	.99793	.06437	15.5340	.807819	.999102	.808717	.191283	19	
.70	42	.06453	.99792	.06467	15.4638	.809777	.999094	.810683	.189317	18	.30
	43	.06482	.99790	.06496	15.3943	.811726	.999086	.812641	.187359	17	
	44	.06511	.99788	.06525	15.3254	.813667	.999077	.814589	.185411	16	
.75	45	.06540	.99786	.06554	15.2571	.815599	.999069	.816529	.183471	15	.25
	46	.06569	.99784	.06584	15.1893	.817522	.999061	.818461	.181539	14	
	47	.06598	.99782	.06613	15.1222	.819436	.999053	.820384	.179616	13	
.80	48	.06627	.99780	.06642	15.0557	.821343	.999044	.822298	.177702	12	.20
	49	.06656	.99778	.06671	14.9898	.823240	.999036	.824205	.175795	11	
	50	.06685	.99776	.06700	14.9244	.825130	.999027	.826103	.173897	10	
.85	51	.06714	.99774	.06730	14.8596	.827011	.999019	.827992	.172008	9	.15
	52	.06743	.99772	.06759	14.7954	.828884	.999010	.829874	.170126	8	
	53	.06773	.99770	.06788	14.7317	.830749	.999002	.831748	.168252	7	
.90	54	.06802	.99768	.06817	14.6685	.832607	.998993	.833613	.166387	6	.10
	55	.06831	.99766	.06847	14.6059	.834456	.998984	.835471	.164529	5	
	56	.06860	.99764	.06876	14.5438	.836297	.998976	.837321	.162679	4	
.95	57	.06889	.99762	.06905	14.4823	.838130	.998967	.839163	.160837	3	.05
	58	.06918	.99760	.06934	14.4212	.839956	.998958	.840998	.159002	2	
	59	.06947	.99758	.06963	14.3607	.841774	.998950	.842825	.157175	1	
1.00	60	.06976	.99756	.06993	14.3007	8.843585	9.998941	8.844644	11.155356	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.06976	.99756	.06993	14.3007	8.843535	9.998941	8.844644	11.155356	60	1.00
	1	.07005	.99754	.07022	14.2411	.845387	.998932	.846455	.153545	59	
	2	.07034	.99752	.07051	14.1821	.847183	.998923	.848260	.151740	58	
.05	3	.07063	.99750	.07080	14.1235	.848971	.998914	.850057	.149943	57	.95
	4	.07092	.99748	.07110	14.0655	.850751	.998905	.851846	.148154	56	
	5	.07121	.99746	.07139	14.0079	.852525	.998896	.853628	.146372	55	
.10	6	.07150	.99744	.07168	13.9507	.854291	.998887	.855403	.144597	54	.90
	7	.07179	.99742	.07197	13.8940	.856049	.998878	.857171	.142829	53	
	8	.07208	.99740	.07227	13.8378	.857801	.998869	.858932	.141068	52	
.15	9	.07237	.99738	.07256	13.7821	.859546	.998860	.860686	.139314	51	.85
	10	.07266	.99736	.07285	13.7267	.861283	.998851	.862433	.137567	50	
	11	.07295	.99734	.07314	13.6719	.863014	.998841	.864173	.135827	49	
.20	12	.07324	.99731	.07344	13.6174	.864738	.998832	.865906	.134094	48	.80
	13	.07353	.99729	.07373	13.5634	.866455	.998823	.867632	.132368	47	
	14	.07382	.99727	.07402	13.5098	.868165	.998813	.869351	.130649	46	
.25	15	.07411	.99725	.07431	13.4566	.869868	.998804	.871064	.128936	45	.75
	16	.07440	.99723	.07461	13.4039	.871565	.998795	.872770	.127230	44	
	17	.07469	.99721	.07490	13.3515	.873255	.998785	.874469	.125531	43	
.30	18	.07498	.99719	.07519	13.2996	.874938	.998776	.876162	.123838	42	.70
	19	.07527	.99716	.07548	13.2480	.876615	.998766	.877849	.122151	41	
	20	.07556	.99714	.07578	13.1969	.878285	.998757	.879529	.120471	40	
.35	21	.07585	.99712	.07607	13.1461	.879949	.998747	.881202	.118798	39	.65
	22	.07614	.99710	.07636	13.0958	.881607	.998738	.882869	.117131	38	
	23	.07643	.99708	.07665	13.0458	.883258	.998728	.884530	.115470	37	
.40	24	.07672	.99705	.07695	12.9962	.884903	.998718	.886185	.113815	36	.60
	25	.07701	.99703	.07724	12.9469	.886542	.998708	.887833	.112167	35	
	26	.07730	.99701	.07753	12.8981	.888174	.998699	.889476	.110524	34	
.45	27	.07759	.99699	.07782	12.8496	.889801	.998689	.891112	.108888	33	.55
	28	.07788	.99696	.07812	12.8014	.891421	.998679	.892742	.107258	32	
	29	.07817	.99694	.07841	12.7536	.893035	.998669	.894366	.105634	31	
.50	30	.07846	.99692	.07870	12.7062	.894643	.998659	.895984	.104016	30	.50
	31	.07875	.99689	.07899	12.6591	.896246	.998649	.897596	.102404	29	
	32	.07904	.99687	.07929	12.6124	.897842	.998639	.899203	.100797	28	
.55	33	.07933	.99685	.07958	12.5660	.899432	.998629	.900803	.099197	27	.45
	34	.07962	.99683	.07987	12.5199	.901017	.998619	.902398	.097602	26	
	35	.07991	.99680	.08017	12.4742	.902596	.998609	.903987	.096013	25	
.60	36	.08020	.99678	.08046	12.4288	.904169	.998599	.905570	.094430	24	.40
	37	.08049	.99676	.08075	12.3838	.905736	.998589	.907147	.092853	23	
	38	.08078	.99673	.08104	12.3390	.907297	.998578	.908719	.091281	22	
.65	39	.08107	.99671	.08134	12.2946	.908853	.998568	.910285	.089715	21	.35
	40	.08136	.99668	.08163	12.2505	.910404	.998558	.911846	.088154	20	
	41	.08165	.99666	.08192	12.2067	.911949	.998548	.913401	.086599	19	
.70	42	.08194	.99664	.08221	12.1632	.913488	.998537	.914951	.085049	18	.30
	43	.08223	.99661	.08251	12.1201	.915022	.998527	.916495	.083505	17	
	44	.08252	.99659	.08280	12.0772	.916550	.998516	.918034	.081966	16	
.75	45	.08281	.99657	.08309	12.0346	.918073	.998506	.919568	.080432	15	.25
	46	.08310	.99654	.08339	11.9923	.919591	.998495	.921096	.078904	14	
	47	.08339	.99652	.08368	11.9504	.921103	.998485	.922619	.077381	13	
.80	48	.08368	.99649	.08397	11.9087	.922610	.998474	.924136	.075864	12	.20
	49	.08397	.99647	.08427	11.8673	.924112	.998464	.925649	.074351	11	
	50	.08426	.99644	.08456	11.8262	.925609	.998453	.927156	.072844	10	
.85	51	.08455	.99642	.08485	11.7853	.927100	.998442	.928658	.071342	9	.15
	52	.08484	.99639	.08514	11.7448	.928587	.998431	.930155	.069845	8	
	53	.08513	.99637	.08544	11.7045	.930068	.998421	.931647	.068353	7	
.90	54	.08542	.99635	.08573	11.6645	.931544	.998410	.933134	.066866	6	.10
	55	.08571	.99632	.08602	11.6248	.933015	.998399	.934616	.065384	5	
	56	.08600	.99630	.08632	11.5853	.934481	.998388	.936093	.063907	4	
.95	57	.08629	.99627	.08661	11.5461	.935942	.998377	.937565	.062435	3	.05
	58	.08658	.99625	.08690	11.5072	.937398	.998366	.939032	.060968	2	
	59	.08687	.99622	.08720	11.4685	.938850	.998355	.940494	.059506	1	
1.00	60	.08716	.99619	.08749	11.4301	8.940296	9.998344	8.941952	11.058048	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.08716	.99619	.08749	11.4301	8.940296	9.998344	8.941952	11.058048	60	1.00
	1	.08745	.99617	.08778	11.3919	.941738	.998333	.943404	.056596	59	
	2	.08774	.99614	.08807	11.3540	.943174	.998322	.944852	.055148	58	
.05	3	.08803	.99612	.08837	11.3163	.944606	.998311	.946295	.053705	57	.95
	4	.08831	.99609	.08866	11.2789	.946034	.998300	.947734	.052266	56	
	5	.08860	.99607	.08895	11.2417	.947456	.998289	.949168	.050832	55	
.10	6	.08889	.99604	.08925	11.2048	.948874	.998277	.950597	.049403	54	.90
	7	.08918	.99602	.08954	11.1681	.950287	.998266	.952021	.047979	53	
	8	.08947	.99599	.08983	11.1316	.951696	.998255	.953441	.046559	52	
.15	9	.08976	.99596	.09013	11.0954	.953100	.998243	.954856	.045144	51	.85
	10	.09005	.99594	.09042	11.0594	.954499	.998232	.956267	.043733	50	
	11	.09034	.99591	.09071	11.0237	.955894	.998220	.957674	.042326	49	
.20	12	.09063	.99588	.09101	10.9882	.957284	.998209	.959075	.040925	48	.80
	13	.09092	.99586	.09130	10.9529	.958670	.998197	.960473	.039527	47	
	14	.09121	.99583	.09159	10.9178	.960052	.998186	.961866	.038134	46	
.25	15	.09150	.99580	.09189	10.8829	.961429	.998174	.963255	.036745	45	.75
	16	.09179	.99578	.09218	10.8483	.962801	.998163	.964639	.035361	44	
	17	.09208	.99575	.09247	10.8139	.964170	.998151	.966019	.033981	43	
.30	18	.09237	.99572	.09277	10.7797	.965534	.998139	.967394	.032606	42	.70
	19	.09266	.99570	.09306	10.7457	.966893	.998128	.968766	.031234	41	
	20	.09295	.99567	.09335	10.7119	.968249	.998116	.970133	.029867	40	
.35	21	.09324	.99564	.09365	10.6783	.969600	.998104	.971496	.028504	39	.65
	22	.09353	.99562	.09394	10.6450	.970947	.998092	.972855	.027145	38	
	23	.09382	.99559	.09423	10.6118	.972289	.998080	.974209	.025791	37	
.40	24	.09411	.99556	.09453	10.5789	.973628	.998068	.975560	.024440	36	.60
	25	.09440	.99553	.09482	10.5462	.974962	.998056	.976906	.023094	35	
	26	.09469	.99551	.09511	10.5136	.976293	.998044	.978248	.021752	34	
.45	27	.09498	.99548	.09541	10.4813	.977619	.998032	.979586	.020414	33	.55
	28	.09527	.99545	.09570	10.4491	.978941	.998020	.980921	.019079	32	
	29	.09556	.99542	.09600	10.4172	.980259	.998008	.982251	.017749	31	
.50	30	.09585	.99540	.09629	10.3854	.981573	.997996	.983577	.016423	30	.50
	31	.09614	.99537	.09658	10.3538	.982883	.997984	.984899	.015101	29	
	32	.09642	.99534	.09688	10.3224	.984189	.997972	.986217	.013783	28	
.55	33	.09671	.99531	.09717	10.2913	.985491	.997959	.987532	.012468	27	.45
	34	.09700	.99528	.09746	10.2602	.986789	.997947	.988842	.011158	26	
	35	.09729	.99526	.09776	10.2294	.988083	.997935	.990149	.009851	25	
.60	36	.09758	.99523	.09805	10.1988	.989374	.997922	.991451	.008549	24	.40
	37	.09787	.99520	.09834	10.1683	.990660	.997910	.992750	.007250	23	
	38	.09816	.99517	.09864	10.1381	.991943	.997897	.994045	.005955	22	
.65	39	.09845	.99514	.09893	10.1080	.993222	.997885	.995337	.004663	21	.35
	40	.09874	.99511	.09923	10.0780	.994497	.997872	.996624	.003376	20	
	41	.09903	.99508	.09952	10.0483	.995768	.997860	.997908	.002092	19	
.70	42	.09932	.99506	.09981	10.0187	.997036	.997847	.999188	.000812	18	.30
	43	.09961	.99503	.10011	9.9893	.998299	.997835	9.000465	10.999535	17	
	44	.09990	.99500	.10040	9.9600	.999560	.997822	.001738	.998262	16	
.75	45	.10019	.99497	.10069	9.9310	9.000816	.997809	.003007	.996993	15	.25
	46	.10048	.99494	.10099	9.9021	.002069	.997797	.004272	.995728	14	
	47	.10077	.99491	.10128	9.8733	.003318	.997784	.005534	.994466	13	
.80	48	.10106	.99488	.10158	9.8448	.004563	.997771	.006792	.993208	12	.20
	49	.10135	.99485	.10187	9.8164	.005805	.997758	.008047	.991953	11	
	50	.10164	.99482	.10216	9.7881	.007044	.997745	.009298	.990702	10	
.85	51	.10192	.99479	.10246	9.7600	.008278	.997732	.010546	.989454	9	.15
	52	.10221	.99476	.10275	9.7321	.009510	.997719	.011790	.988210	8	
	53	.10250	.99473	.10305	9.7044	.010737	.997706	.013031	.986969	7	
.90	54	.10279	.99470	.10334	9.6768	.011962	.997693	.014268	.985732	6	.10
	55	.10308	.99467	.10363	9.6493	.013182	.997680	.015502	.984498	5	
	56	.10337	.99464	.10393	9.6220	.014400	.997667	.016732	.983268	4	
.95	57	.10366	.99461	.10422	9.5949	.015613	.997654	.017959	.982041	3	.05
	58	.10395	.99458	.10452	9.5679	.016824	.997641	.019183	.980817	2	
	59	.10424	.99455	.10481	9.5410	.018031	.997628	.020403	.979597	1	
1.00	60	.10453	.99452	.10510	9.5143	9.019235	9.997614	9.021620	10.978380	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.10453	.99452	.10510	9.51436	9.019235	9.997614	9.021620	10.978380	60	1.00
	1	.10482	.99449	.10540	9.48781	.020435	.997601	.022834	.977166	59	
	2	.10511	.99446	.10569	9.46141	.021632	.997588	.024044	.975956	58	
.05	3	.10540	.99443	.10599	9.43515	.022825	.997574	.025251	.974749	57	.95
	4	.10569	.99440	.10628	9.40904	.024016	.997561	.026455	.973545	56	
	5	.10597	.99437	.10657	9.38307	.025203	.997547	.027655	.972345	55	
.10	6	.10626	.99434	.10687	9.35724	.026386	.997534	.028852	.971148	54	.90
	7	.10655	.99431	.10716	9.33155	.027567	.997520	.030046	.969954	53	
	8	.10684	.99428	.10746	9.30599	.028744	.997507	.031237	.968763	52	
.15	9	.10713	.99424	.10775	9.28058	.029918	.997493	.032425	.967575	51	.85
	10	.10742	.99421	.10805	9.25530	.031089	.997480	.033609	.966391	50	
	11	.10771	.99418	.10834	9.23016	.032257	.997466	.034791	.965209	49	
.20	12	.10800	.99415	.10863	9.20516	.033421	.997452	.035969	.964031	48	.80
	13	.10829	.99412	.10893	9.18028	.034582	.997439	.037144	.962856	47	
	14	.10858	.99409	.10922	9.15554	.035741	.997425	.038316	.961684	46	
.25	15	.10887	.99406	.10952	9.13093	.036896	.997411	.039485	.960515	45	.75
	16	.10916	.99402	.10981	9.10646	.038048	.997397	.040651	.959349	44	
	17	.10945	.99399	.11011	9.08211	.039197	.997383	.041813	.958187	43	
.30	18	.10973	.99396	.11040	9.05789	.040342	.997369	.042973	.957027	42	.70
	19	.11002	.99393	.11070	9.03379	.041485	.997355	.044130	.955870	41	
	20	.11031	.99390	.11099	9.00983	.042625	.997341	.045284	.954716	40	
.35	21	.11060	.99386	.11128	8.98598	.043762	.997327	.046434	.953566	39	.65
	22	.11089	.99383	.11158	8.96227	.044895	.997313	.047582	.952418	38	
	23	.11118	.99380	.11187	8.93867	.046026	.997299	.048727	.951273	37	
.40	24	.11147	.99377	.11217	8.91520	.047154	.997285	.049869	.950131	36	.60
	25	.11176	.99374	.11246	8.89185	.048279	.997271	.051008	.948992	35	
	26	.11205	.99370	.11276	8.86862	.049400	.997257	.052144	.947856	34	
.45	27	.11234	.99367	.11305	8.84551	.050519	.997242	.053277	.946723	33	.55
	28	.11263	.99364	.11335	8.82252	.051635	.997228	.054407	.945593	32	
	29	.11291	.99360	.11364	8.79964	.052749	.997214	.055535	.944465	31	
.50	30	.11320	.99357	.11394	8.77689	.053859	.997199	.056659	.943341	30	.50
	31	.11349	.99354	.11423	8.75425	.054966	.997185	.057781	.942219	29	
	32	.11378	.99351	.11452	8.73172	.056071	.997170	.058900	.941100	28	
.55	33	.11407	.99347	.11482	8.70931	.057172	.997156	.060016	.939984	27	.45
	34	.11436	.99344	.11511	8.68701	.058271	.997141	.061130	.938870	26	
	35	.11465	.99341	.11541	8.66482	.059367	.997127	.062240	.937760	25	
.60	36	.11494	.99337	.11570	8.64275	.060460	.997112	.063348	.936652	24	.40
	37	.11523	.99334	.11600	8.62078	.061551	.997098	.064453	.935547	23	
	38	.11552	.99331	.11629	8.59893	.062639	.997083	.065556	.934444	22	
.65	39	.11580	.99327	.11659	8.57718	.063724	.997068	.066655	.933345	21	.35
	40	.11609	.99324	.11688	8.55555	.064806	.997053	.067752	.932248	20	
	41	.11638	.99320	.11718	8.53402	.065885	.997039	.068846	.931154	19	
.70	42	.11667	.99317	.11747	8.51259	.066962	.997024	.069938	.930062	18	.30
	43	.11696	.99314	.11777	8.49128	.068036	.997009	.071027	.928973	17	
	44	.11725	.99310	.11806	8.47007	.069107	.996994	.072113	.927887	16	
.75	45	.11754	.99307	.11836	8.44896	.070176	.996979	.073197	.926803	15	.25
	46	.11783	.99303	.11865	8.42795	.071242	.996964	.074278	.925722	14	
	47	.11812	.99300	.11895	8.40705	.072306	.996949	.075356	.924644	13	
.80	48	.11840	.99297	.11924	8.38625	.073366	.996934	.076432	.923568	12	.20
	49	.11869	.99293	.11954	8.36555	.074424	.996919	.077505	.922495	11	
	50	.11898	.99290	.11983	8.34496	.075480	.996904	.078576	.921424	10	
.85	51	.11927	.99286	.12013	8.32446	.076533	.996889	.079644	.920356	9	.15
	52	.11956	.99283	.12042	8.30406	.077583	.996874	.080710	.919290	8	
	53	.11985	.99279	.12072	8.28376	.078631	.996858	.081773	.918227	7	
.90	54	.12014	.99276	.12101	8.26355	.079676	.996843	.082833	.917167	6	.10
	55	.12043	.99272	.12131	8.24345	.080719	.996828	.083891	.916109	5	
	56	.12071	.99269	.12160	8.22344	.081759	.996812	.084947	.915053	4	
.95	57	.12100	.99265	.12190	8.20352	.082797	.996797	.086000	.914000	3	.05
	58	.12129	.99262	.12219	8.18370	.083832	.996782	.087050	.912950	2	
	59	.12158	.99258	.12249	8.16398	.084864	.996766	.088098	.911902	1	
1.00	60	.12187	.99255	.12278	8.14435	9.085894	9.996751	9.089144	10.910856	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.12187	.99255	.12278	8.14435	9.085894	9.996751	9.089144	10.910856	60	1.00
	1	.12216	.99251	.12308	8.12481	.086922	.996735	.090187	.909813	59	
	2	.12245	.99248	.12338	8.10536	.087947	.996720	.091228	.908772	58	
.05	3	.12274	.99244	.12367	8 08600	.088970	.996704	.092266	.907734	57	.95
	4	.12302	.99240	.12397	8.06674	.089990	.996688	.093302	.906698	56	
	5	.12331	.99237	.12426	8.04756	.091008	.996673	.094336	.905664	55	
.10	6	.12360	.99233	.12456	8.02848	.092024	.996657	.095367	.904633	54	.90
	7	.12389	.99230	.12485	8.00948	.093037	.996641	.096395	.903605	53	
	8	.12418	.99226	.12515	7.99058	.094047	.996625	.097422	.902578	52	
.15	9	.12447	.99222	.12544	7 97176	.095056	.996610	.098446	.901554	51	.85
	10	.12476	.99219	.12574	7.95302	.096062	.996594	.099468	.900532	50	
	11	.12504	.99215	.12603	7.93438	.097065	.996578	.100487	.899513	49	
.20	12	.12533	.99211	.12633	7.91582	.098066	.996562	.101504	.898496	43	.80
	13	.12562	.99208	.12662	7.89734	.099065	.996546	.102519	.897481	47	
	14	.12591	.99204	.12692	7.87895	.100062	.996530	.103532	.896468	46	
.25	15	.12620	.99200	.12722	7.86064	.101056	.996514	.104542	.895458	45	.75
	16	.12649	.99197	.12751	7.84242	.102048	.996498	.105550	.894450	44	
	17	.12678	.99193	.12781	7.82428	.103037	.996482	.106556	.893444	43	
.30	18	.12706	.99189	.12810	7.80622	.104025	.996465	.107559	.892441	42	.70
	19	.12735	.99186	.12840	7.78825	.105010	.996449	.108560	.891440	41	
	20	.12764	.99182	.12869	7.77035	.105992	.996433	.109559	.890441	40	
.35	21	.12793	.99178	.12899	7.75254	.106973	.996417	.110556	.889444	39	.65
	22	.12822	.99175	.12929	7.73480	.107951	.996400	.111551	.888449	38	
	23	.12851	.99171	.12958	7.71715	.108927	.996384	.112543	.887457	37	
.40	24	.12880	.99167	.12988	7.69957	.109901	.996368	.113533	.886467	36	.60
	25	.12908	.99163	.13017	7.68208	.110873	.996351	.114521	.885479	35	
	26	.12937	.99160	.13047	7.66466	.111842	.996335	.115507	.884493	34	
.45	27	.12966	.99156	.13076	7.64732	.112809	.996318	.116491	.883509	33	.55
	28	.12995	.99152	.13106	7 63005	.113774	.996302	.117472	.882528	32	
	29	.13024	.99148	.13136	7.61287	.114737	.996285	.118452	.881548	31	
.50	30	.13053	.99144	.13165	7.59575	.115698	.996269	.119429	.880571	30	.50
	31	.13081	.99141	.13195	7.57872	.116656	.996252	.120404	.879596	29	
	32	.13110	.99137	.13224	7.56176	.117613	.996235	.121377	.878623	28	
.55	33	.13139	.99133	.13254	7.54487	.118567	.996219	.122348	.877652	27	.45
	34	.13168	.99129	.13284	7 52806	.119519	.996202	.123317	.876683	26	
	35	.13197	.99125	.13313	7.51132	.120469	.996185	.124284	.875716	25	
.60	36	.13226	.99122	.13343	7.49466	.121417	.996168	.125249	.874751	24	.40
	37	.13254	.99118	.13372	7.47806	.122362	.996151	.126211	.873789	23	
	38	.13283	.99114	.13402	7.46154	.123306	.996134	.127172	.872828	22	
.65	39	.13312	.99110	.13432	7.44509	.124248	.996117	.128130	.871870	21	.35
	40	.13341	.99106	.13461	7.42871	.125187	.996100	.129087	.870913	20	
	41	.13370	.99102	.13491	7.41240	.126125	.996083	.130041	.869959	19	
.70	42	.13399	.99098	.13521	7.39616	.127060	.996066	.130994	.869006	18	.30
	43	.13427	.99094	.13550	7.37999	.127993	.996049	.131944	.868056	17	
	44	.13456	.99091	.13580	7.36389	.128925	.996032	.132893	.867107	16	
.75	45	.13485	.99087	.13609	7.34786	.129854	.996015	.133839	.866161	15	.25
	46	.13514	.99083	.13639	7.33190	.130781	.995998	.134784	.865216	14	
	47	.13543	.99079	.13669	7.31600	.131706	.995980	.135726	.864274	13	
.80	48	.13572	.99075	.13698	7.30018	.132630	.995963	.136667	.863333	12	.20
	49	.13600	.99071	.13728	7.28442	.133551	.995946	.137605	.862395	11	
	50	.13629	.99067	.13758	7.26873	.134470	.995928	.138542	.861458	10	
.85	51	.13658	.99063	.13787	7.25310	.135387	.995911	.139476	.860524	9	.15
	52	.13687	.99059	.13817	7.23754	.136303	.995894	.140409	.859591	8	
	53	.13716	.99055	.13846	7.22204	.137216	.995876	.141340	.858660	7	
.90	54	.13744	.99051	.13876	7.20661	.138128	.995859	.142269	.857731	6	.10
	55	.13773	.99047	.13906	7.19125	.139037	.995841	.143196	.856804	5	
	56	.13802	.99043	.13935	7.17594	.139944	.995823	.144121	.855879	4	
.95	57	.13831	99039	.13965	7.16071	.140850	.995806	.145044	.854956	3	.05
	58	.13860	.99035	.13995	7.14553	.141754	.995788	.145966	.854034	2	
	59	.13889	.99031	14024	7.13042	.142655	.995771	.146885	.853115	1	
1.00	60	.13917	.99027	14054	7 11537	9.143555	9.995753	9.147803	10.852197	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

VALUES AND LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.13917	.99027	.14054	7.11537	9.143555	9.995753	9.147803	10.852197	60	1.00
	1	.13946	.99023	.14084	7.10038	.144453	.995735	.148718	.851282	59	
	2	.13975	.99019	.14113	7.08546	.145349	.995717	.149632	.850368	58	
.05	3	.14004	.99015	.14143	7.07059	.146243	.995699	.150544	.849456	57	.95
	4	.14033	.99011	.14173	7.05579	.147136	.995681	.151454	.848546	56	
	5	.14061	.99006	.14202	7.04105	.148026	.995664	.152363	.847637	55	
.10	6	.14090	.99002	.14232	7.02637	.148915	.995646	.153269	.846731	54	.90
	7	.14119	.98998	.14262	7.01174	.149802	.995628	.154174	.845826	53	
	8	.14148	.98994	.14291	6.99718	.150686	.995610	.155077	.844923	52	
.15	9	.14177	.98990	.14321	6.98268	.151569	.995591	.155978	.844022	51	.85
	10	.14205	.98986	.14351	6.96823	.152451	.995573	.156877	.843123	50	
	11	.14234	.98982	.14381	6.95385	.153330	.995555	.157775	.842225	49	
.20	12	.14263	.98978	.14410	6.93952	.154208	.995537	.158671	.841329	48	.80
	13	.14292	.98973	.14440	6.92525	.155083	.995519	.159565	.840435	47	
	14	.14320	.98969	.14470	6.91104	.155957	.995501	.160457	.839543	46	
.25	15	.14349	.98965	.14499	6.89688	.156830	.995482	.161347	.838653	45	.75
	16	.14378	.98961	.14529	6.88278	.157700	.995464	.162236	.837764	44	
	17	.14407	.98957	.14559	6.86874	.158569	.995446	.163123	.836877	43	
.30	18	.14436	.98953	.14588	6.85475	.159435	.995427	.164008	.835992	42	.70
	19	.14464	.98948	.14618	6.84082	.160301	.995409	.164892	.835108	41	
	20	.14493	.98944	.14648	6.82694	.161164	.995390	.165774	.834226	40	
.35	21	.14522	.98940	.14678	6.81312	.162025	.995372	.166654	.833346	39	.65
	22	.14551	.98936	.14707	6.79936	.162885	.995353	.167532	.832468	38	
	23	.14580	.98931	.14737	6.78564	.163743	.995334	.168409	.831591	37	
.40	24	.14608	.98927	.14767	6.77199	.164600	.995316	.169284	.830716	36	.60
	25	.14637	.98923	.14796	6.75838	.165454	.995297	.170157	.829845	35	
	26	.14666	.98919	.14826	6.74483	.166307	.995278	.171029	.828971	34	
.45	27	.14695	.98914	.14856	6.73133	.167159	.995260	.171899	.828101	33	.55
	28	.14723	.98910	.14886	6.71789	.168008	.995241	.172767	.827233	32	
	29	.14752	.98906	.14915	6.70450	.168856	.995222	.173634	.826366	31	
.50	30	.14781	.98902	.14945	6.69116	.169702	.995203	.174499	.825501	30	.50
	31	.14810	.98897	.14975	6.67787	.170547	.995184	.175362	.824638	29	
	32	.14838	.98893	.15005	6.66463	.171389	.995165	.176224	.823776	28	
.55	33	.14867	.98889	.15034	6.65144	.172230	.995146	.177084	.822916	27	.45
	34	.14896	.98884	.15064	6.63831	.173070	.995127	.177942	.822058	26	
	35	.14925	.98880	.15094	6.62523	.173908	.995108	.178799	.821201	25	
.60	36	.14954	.98876	.15124	6.61219	.174744	.995089	.179655	.820345	24	.40
	37	.14982	.98871	.15153	6.59921	.175578	.995070	.180508	.819492	23	
	38	.15011	.98867	.15183	6.58627	.176411	.995051	.181360	.818640	22	
.65	39	.15040	.98863	.15213	6.57339	.177242	.995032	.182211	.817789	21	.35
	40	.15069	.98858	.15243	6.56055	.178072	.995013	.183059	.816941	20	
	41	.15097	.98854	.15272	6.54777	.178900	.994993	.183907	.816093	19	
.70	42	.15126	.98849	.15302	6.53503	.179726	.994974	.184752	.815248	18	.30
	43	.15155	.98845	.15332	6.52234	.180551	.994955	.185597	.814403	17	
	44	.15184	.98841	.15362	6.50970	.181374	.994935	.186439	.813561	16	
.75	45	.15212	.98836	.15391	6.49710	.182196	.994916	.187280	.812720	15	.25
	46	.15241	.98832	.15421	6.48456	.183016	.994896	.188120	.811880	14	
	47	.15270	.98827	.15451	6.47206	.183834	.994877	.188958	.811042	13	
.80	48	.15299	.98823	.15481	6.45961	.184651	.994857	.189794	.810206	12	.20
	49	.15327	.98818	.15511	6.44720	.185466	.994838	.190629	.809371	11	
	50	.15356	.98814	.15540	6.43484	.186280	.994818	.191462	.808538	10	
.85	51	.15385	.98809	.15570	6.42253	.187092	.994798	.192294	.807706	9	.15
	52	.15414	.98805	.15600	6.41026	.187903	.994779	.193124	.806876	8	
	53	.15442	.98800	.15630	6.39804	.188712	.994759	.193953	.806047	7	
.90	54	.15471	.98796	.15660	6.38587	.189519	.994739	.194780	.805220	6	.10
	55	.15500	.98791	.15689	6.37374	.190325	.994720	.195606	.804394	5	
	56	.15529	.98787	.15719	6.36165	.191130	.994700	.196430	.803570	4	
.95	57	.15557	.98782	.15749	6.34961	.191933	.994680	.197253	.802747	3	.05
	58	.15586	.98778	.15779	6.33761	.192734	.994660	.198074	.801926	2	
	59	.15615	.98773	.15809	6.32566	.193534	.994640	.198894	.801106	1	
1.00	60	.15643	.98769	.15838	6.31375	9.194332	9.994620	9.199713	10.800287	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.15643	.98769	.15838	6.31375	9.194332	9.994620	9.199713	10.800287	60	1.00
	1	.15672	.98764	.15868	6.30189	.195129	.994600	.200529	.799471	59	
	2	.15701	.98760	.15898	6.29007	.195925	.994580	.201345	.798655	58	
.05	3	.15730	.98755	.15928	6.27829	.196719	.994560	.202159	.797841	57	.95
	4	.15759	.98751	.15958	6.26655	.197511	.994540	.202971	.797029	56	
	5	.15787	.98746	.15988	6.25486	.198302	.994519	.203782	.796218	55	
.10	6	.15816	.98741	.16017	6.24321	.199091	.994499	.204592	.795408	54	.90
	7	.15845	.98737	.16047	6.23160	.199879	.994479	.205400	.794600	53	
	8	.15873	.98732	.16077	6.22003	.200666	.994459	.206207	.793793	52	
.15	9	.15902	.98728	.16107	6.20851	.201451	.994438	.207013	.792987	51	.85
	10	.15931	.98723	.16137	6.19703	.202234	.994418	.207817	.792183	50	
	11	.15959	.98718	.16167	6.18559	.203017	.994398	.208619	.791381	49	
.20	12	.15988	.98714	.16196	6.17419	.203797	.994377	.209420	.790580	48	.80
	13	.16017	.98709	.16226	6.16283	.204577	.994357	.210220	.789780	47	
	14	.16046	.98704	.16256	6.15151	.205354	.994336	.211018	.788982	46	
.25	15	.16074	.98700	.16286	6.14023	.206131	.994316	.211815	.788185	45	.75
	16	.16103	.98695	.16316	6.12899	.206906	.994295	.212611	.787389	44	
	17	.16132	.98690	.16346	6.11779	.207679	.994274	.213405	.786595	43	
.30	18	.16160	.98686	.16376	6.10664	.208452	.994254	.214198	.785802	42	.70
	19	.16189	.98681	.16405	6.09552	.209222	.994233	.214989	.785011	41	
	20	.16218	.98676	.16435	6.08444	.209992	.994212	.215780	.784220	40	
.35	21	.16246	.98671	.16465	6.07340	.210760	.994191	.216568	.783432	39	.65
	22	.16275	.98667	.16495	6.06240	.211526	.994171	.217355	.782644	38	
	23	.16304	.98662	.16525	6.05143	.212291	.994150	.218142	.781858	37	
.40	24	.16333	.98657	.16555	6.04051	.213055	.994129	.218926	.781074	36	.60
	25	.16361	.98652	.16585	6.02962	.213818	.994108	.219710	.780290	35	
	26	.16390	.98648	.16615	6.01878	.214579	.994087	.220492	.779508	34	
.45	27	.16419	.98643	.16645	6.00797	.215338	.994066	.221272	.778728	33	.55
	28	.16447	.98638	.16674	5.99720	.216097	.994045	.222052	.777948	32	
	29	.16476	.98633	.16704	5.98646	.216854	.994024	.222830	.777170	31	
.50	30	.16505	.98629	.16734	5.97576	.217609	.994003	.223607	.776393	30	.50
	31	.16533	.98624	.16764	5.96510	.218363	.993982	.224382	.775618	29	
	32	.16562	.98619	.16794	5.95448	.219116	.993960	.225156	.774844	28	
.55	33	.16591	.98614	.16824	5.94390	.219868	.993939	.225928	.774071	27	.45
	34	.16620	.98609	.16854	5.93335	.220618	.993918	.226700	.773300	26	
	35	.16648	.98604	.16884	5.92285	.221367	.993897	.227471	.772529	25	
.60	36	.16677	.98600	.16914	5.91236	.222115	.993875	.228239	.771761	24	.40
	37	.16706	.98595	.16944	5.90191	.222861	.993854	.229007	.770993	23	
	38	.16734	.98590	.16974	5.89151	.223606	.993832	.229773	.770227	22	
.65	39	.16763	.98585	.17004	5.88114	.224349	.993811	.230539	.769461	21	.35
	40	.16792	.98580	.17033	5.87080	.225092	.993789	.231302	.768698	20	
	41	.16820	.98575	.17063	5.86051	.225833	.993768	.232065	.767935	19	
.70	42	.16849	.98570	.17093	5.85024	.226573	.993746	.232826	.767174	18	.30
	43	.16878	.98565	.17123	5.84001	.227311	.993725	.233586	.766414	17	
	44	.16906	.98561	.17153	5.82982	.228048	.993703	.234345	.765655	16	
.75	45	.16935	.98556	.17183	5.81966	.228784	.993681	.235103	.764897	15	.25
	46	.16964	.98551	.17213	5.80953	.229518	.993660	.235859	.764141	14	
	47	.16992	.98546	.17243	5.79944	.230252	.993638	.236614	.763386	13	
.80	48	.17021	.98541	.17273	5.78938	.230984	.993616	.237368	.762632	12	.20
	49	.17050	.98536	.17303	5.77936	.231715	.993594	.238120	.761880	11	
	50	.17078	.98531	.17333	5.76937	.232444	.993572	.238872	.761128	10	
.85	51	.17107	.98526	.17363	5.75941	.233172	.993550	.239622	.760378	9	.15
	52	.17136	.98521	.17393	5.74949	.233899	.993528	.240371	.759629	8	
	53	.17164	.98516	.17423	5.73960	.234625	.993506	.241118	.758882	7	
.90	54	.17193	.98511	.17453	5.72974	.235349	.993484	.241865	.758135	6	.10
	55	.17222	.98506	.17483	5.71992	.236073	.993462	.242610	.757390	5	
	56	.17250	.98501	.17513	5.71013	.236795	.993440	.243354	.756646	4	
.95	57	.17279	.98496	.17543	5.70037	.237515	.993418	.244097	.755903	3	.05
	58	.17308	.98491	.17573	5.69064	.238235	.993396	.244839	.755161	2	
	59	.17336	.98486	.17603	5.68094	.238953	.993374	.245579	.754421	1	
1.00	60	.17365	.98481	.17633	5.67128	9.239670	9.993351	9.246319	10.753681	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

VALUES AND LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.17365	.98481	.17633	5.67128	9.239670	9.993351	9.246319	10.753681	60	1.00
	1	.17393	.98476	.17663	5.66165	.240386	.993329	.247057	.752943	59	
	2	.17422	.98471	.17693	5.65205	.241101	.993307	.247794	.752206	58	
.05	3	.17451	.98466	.17723	5.64248	.241814	.993284	.248530	.751470	57	.95
	4	.17479	.98461	.17753	5.63295	.242526	.993262	.249264	.750736	56	
	5	.17508	.98455	.17783	5.62344	.243237	.993240	.249998	.750002	55	
.10	6	.17537	.98450	.17813	5.61397	.243947	.993217	.250730	.749270	54	.90
	7	.17565	.98445	.17843	5.60452	.244656	.993195	.251461	.748539	53	
	8	.17594	.98440	.17873	5.59511	.245363	.993172	.252191	.747809	52	
.15	9	.17623	.98435	.17903	5.58573	.246069	.993149	.252920	.747080	51	.85
	10	.17651	.98430	.17933	5.57638	.246775	.993127	.253648	.746352	50	
	11	.17680	.98425	.17963	5.56706	.247478	.993104	.254374	.745626	49	
.20	12	.17708	.98420	.17993	5.55777	.248181	.993081	.255100	.744900	48	.80
	13	.17737	.98414	.18023	5.54851	.248883	.993059	.255824	.744176	47	
	14	.17766	.98409	.18053	5.53927	.249583	.993036	.256547	.743453	46	
.25	15	.17794	.98404	.18083	5.53007	.250282	.993013	.257269	.742731	45	.75
	16	.17823	.98399	.18113	5.52090	.250980	.992990	.257990	.742010	44	
	17	.17852	.98394	.18143	5.51176	.251677	.992967	.258710	.741290	43	
.30	18	.17880	.98389	.18173	5.50264	.252373	.992944	.259429	.740571	42	.70
	19	.17909	.98383	.18203	5.49356	.253067	.992921	.260146	.739854	41	
	20	.17937	.98378	.18233	5.48451	.253761	.992898	.260863	.739137	40	
.35	21	.17966	.98373	.18263	5.47548	.254453	.992875	.261578	.738422	39	.65
	22	.17995	.98368	.18293	5.46648	.255144	.992852	.262292	.737708	38	
	23	.18023	.98362	.18323	5.45751	.255834	.992829	.263005	.736995	37	
.40	24	.18052	.98357	.18353	5.44857	.256523	.992806	.263717	.736283	36	.60
	25	.18081	.98352	.18384	5.43966	.257211	.992783	.264428	.735572	35	
	26	.18109	.98347	.18414	5.43077	.257898	.992759	.265138	.734862	34	
.45	27	.18138	.98341	.18444	5.42192	.258583	.992736	.265847	.734153	33	.55
	28	.18166	.98336	.18474	5.41309	.259268	.992713	.266555	.733445	32	
	29	.18195	.98331	.18504	5.40429	.259951	.992690	.267261	.732739	31	
.50	30	.18224	.98325	.18534	5.39552	.260633	.992666	.267967	.732033	30	.50
	31	.18252	.98320	.18564	5.38677	.261314	.992643	.268671	.731329	29	
	32	.18281	.98315	.18594	5.37805	.261994	.992619	.269375	.730625	28	
.55	33	.18309	.98310	.18624	5.36936	.262673	.992596	.270077	.729923	27	.45
	34	.18338	.98304	.18654	5.36070	.263351	.992572	.270779	.729221	26	
	35	.18367	.98299	.18684	5.35206	.264027	.992549	.271479	.728521	25	
.60	36	.18395	.98294	.18714	5.34345	.264703	.992525	.272178	.727822	24	.40
	37	.18424	.98288	.18745	5.33487	.265377	.992501	.272876	.727124	23	
	38	.18452	.98283	.18775	5.32631	.266051	.992478	.273573	.726427	22	
.65	39	.18481	.98277	.18805	5.31778	.266723	.992454	.274269	.725731	21	.35
	40	.18509	.98272	.18835	5.30928	.267395	.992430	.274964	.725036	20	
	41	.18538	.98267	.18865	5.30080	.268065	.992406	.275658	.724342	19	
.70	42	.18567	.98261	.18895	5.29235	.268734	.992382	.276351	.723649	18	.30
	43	.18595	.98256	.18925	5.28393	.269402	.992359	.277043	.722957	17	
	44	.18624	.98250	.18955	5.27553	.270069	.992335	.277734	.722266	16	
.75	45	.18652	.98245	.18986	5.26715	.270735	.992311	.278424	.721576	15	.25
	46	.18681	.98240	.19016	5.25880	.271400	.992287	.279113	.720887	14	
	47	.18710	.98234	.19046	5.25048	.272064	.992263	.279801	.720199	13	
.80	48	.18738	.98229	.19076	5.24218	.272726	.992239	.280488	.719512	12	.20
	49	.18767	.98223	.19106	5.23391	.273388	.992214	.281174	.718826	11	
	50	.18795	.98218	.19136	5.22566	.274049	.992190	.281858	.718142	10	
.85	51	.18824	.98212	.19166	5.21744	.274708	.992166	.282542	.717458	9	.15
	52	.18852	.98207	.19197	5.20925	.275367	.992142	.283225	.716775	8	
	53	.18881	.98201	.19227	5.20107	.276025	.992118	.283907	.716093	7	
.90	54	.18910	.98196	.19257	5.19293	.276681	.992093	.284588	.715412	6	.10
	55	.18938	.98190	.19287	5.18480	.277337	.992069	.285268	.714732	5	
	56	.18967	.98185	.19317	5.17671	.277991	.992044	.285947	.714053	4	
.95	57	.18995	.98179	.19347	5.16863	.278645	.992020	.286624	.713376	3	.05
	58	.19024	.98174	.19378	5.16058	.279297	.991996	.287301	.712699	2	
	59	.19052	.98168	.19408	5.15256	.279948	.991971	.287977	.712023	1	
1.00	60	.19081	.98163	.19438	5.14455	9.280599	9.991947	9.288652	10.711348	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.19081	.98163	.19438	5.14455	9.280599	9.991947	9.288652	10.711348	60	1.00
	1	.19109	.98157	.19468	5.13658	.281248	.991922	.289326	.710674	59	
	2	.19138	.98152	.19498	5.12862	.281897	.991897	.289999	.710001	58	
.05	3	.19167	.98146	.19529	5.12069	.282544	.991873	.290671	.709329	57	.95
	4	.19195	.98140	.19559	5.11279	.283190	.991848	.291342	.708658	56	
	5	.19224	.98135	.19589	5.10490	.283836	.991823	.292013	.707987	55	
.10	6	.19252	.98129	.19619	5.09704	.284480	.991799	.292682	.707318	54	.90
	7	.19281	.98124	.19649	5.08921	.285124	.991774	.293350	.706650	53	
	8	.19309	.98118	.19680	5.08139	.285766	.991749	.294017	.705983	52	
.15	9	.19338	.98112	.19710	5.07360	.286408	.991724	.294684	.705316	51	.85
	10	.19366	.98107	.19740	5.06584	.287048	.991699	.295349	.704651	50	
	11	.19395	.98101	.19770	5.05809	.287688	.991674	.296013	.703987	49	
.20	12	.19423	.98096	.19801	5.05037	.288326	.991649	.296677	.703323	48	.80
	13	.19452	.98090	.19831	5.04267	.288964	.991624	.297339	.702661	47	
	14	.19481	.98084	.19861	5.03499	.289600	.991599	.298001	.701999	46	
.25	15	.19509	.98079	.19891	5.02734	.290236	.991574	.298662	.701338	45	.75
	16	.19538	.98073	.19921	5.01971	.290870	.991549	.299322	.700678	44	
	17	.19566	.98067	.19952	5.01210	.291504	.991524	.299980	.700020	43	
.30	18	.19595	.98061	.19982	5.00451	.292137	.991498	.300638	.699362	42	.70
	19	.19623	.98056	.20012	4.99695	.292768	.991473	.301295	.698705	41	
	20	.19652	.98050	.20042	4.98940	.293399	.991448	.301951	.698049	40	
.35	21	.19680	.98044	.20073	4.98188	.294029	.991422	.302607	.697393	39	.65
	22	.19709	.98039	.20103	4.97438	.294658	.991397	.303261	.696739	38	
	23	.19737	.98033	.20133	4.96690	.295286	.991372	.303914	.696086	37	
.40	24	.19766	.98027	.20164	4.95945	.295913	.991346	.304567	.695433	36	.60
	25	.19794	.98021	.20194	4.95201	.296539	.991321	.305218	.694782	35	
	26	.19823	.98016	.20224	4.94460	.297164	.991295	.305869	.694131	34	
.45	27	.19851	.98010	.20254	4.93721	.297788	.991270	.306519	.693481	33	.55
	28	.19880	.98004	.20285	4.92984	.298412	.991244	.307168	.692832	32	
	29	.19908	.97998	.20315	4.92249	.299034	.991218	.307816	.692184	31	
.50	30	.19937	.97992	.20345	4.91516	.299655	.991193	.308463	.691537	30	.50
	31	.19965	.97987	.20376	4.90785	.300276	.991167	.309109	.690891	29	
	32	.19994	.97981	.20406	4.90056	.300895	.991141	.309754	.690246	28	
.55	33	.20022	.97975	.20436	4.89330	.301514	.991115	.310399	.689601	27	.45
	34	.20051	.97969	.20466	4.88605	.302132	.991090	.311042	.688958	26	
	35	.20079	.97963	.20497	4.87882	.302748	.991064	.311685	.688315	25	
.60	36	.20108	.97958	.20527	4.87162	.303364	.991038	.312327	.687673	24	.40
	37	.20136	.97952	.20557	4.86444	.303979	.991012	.312968	.687032	23	
	38	.20165	.97946	.20588	4.85727	.304593	.990986	.313608	.686392	22	
.65	39	.20193	.97940	.20618	4.85013	.305207	.990960	.314247	.685753	21	.35
	40	.20222	.97934	.20648	4.84300	.305819	.990934	.314885	.685115	20	
	41	.20250	.97928	.20679	4.83590	.306430	.990908	.315523	.684477	19	
.70	42	.20279	.97922	.20709	4.82882	.307041	.990882	.316159	.683841	18	.30
	43	.20307	.97916	.20739	4.82175	.307650	.990855	.316795	.683205	17	
	44	.20336	.97910	.20770	4.81471	.308259	.990829	.317430	.682570	16	
.75	45	.20364	.97905	.20800	4.80769	.308867	.990803	.318064	.681936	15	.25
	46	.20393	.97899	.20830	4.80068	.309474	.990777	.318697	.681303	14	
	47	.20421	.97893	.20861	4.79370	.310080	.990750	.319330	.680670	13	
.80	48	.20450	.97887	.20891	4.78673	.310685	.990724	.319961	.680039	12	.20
	49	.20478	.97881	.20921	4.77978	.311289	.990697	.320592	.679408	11	
	50	.20507	.97875	.20952	4.77286	.311893	.990671	.321222	.678778	10	
.85	51	.20535	.97869	.20982	4.76595	.312495	.990645	.321851	.678149	9	.15
	52	.20563	.97863	.21013	4.75906	.313097	.990618	.322479	.677521	8	
	53	.20592	.97857	.21043	4.75219	.313698	.990591	.323106	.676894	7	
.90	54	.20620	.97851	.21073	4.74534	.314297	.990565	.323733	.676267	6	.10
	55	.20649	.97845	.21104	4.73851	.314897	.990538	.324358	.675642	5	
	56	.20677	.97839	.21134	4.73170	.315495	.990511	.324983	.675017	4	
.95	57	.20706	.97833	.21164	4.72490	.316092	.990485	.325607	.674393	3	.05
	58	.20734	.97827	.21195	4.71813	.316689	.990458	.326231	.673769	2	
	59	.20763	.97821	.21225	4.71137	.317284	.990431	.326853	.673147	1	
1.00	60	.20791	.97815	.21256	4.70463	9.317879	9.990404	9.327475	10.672525	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.20791	.97815	.21256	4.70463	9.317879	9.990404	9.327475	10.672525	60	1.00
	1	.20820	.97809	.21286	4.69791	.318473	.990378	.328095	.671905	59	
	2	.20848	.97803	.21316	4.69121	.319066	.990351	.328715	.671285	58	
.05	3	.20877	.97797	.21347	4.68452	.319658	.990324	.329334	.670666	57	.95
	4	.20905	.97791	.21377	4.67786	.320249	.990297	.329953	.670047	56	
	5	.20933	.97784	.21408	4.67121	.320840	.990270	.330570	.669430	55	
.10	6	.20962	.97778	.21438	4.66458	.321430	.990243	.331187	.668813	54	.90
	7	.20990	.97772	.21469	4.65797	.322019	.990215	.331803	.668197	53	
	8	.21019	.97766	.21499	4.65138	.322607	.990188	.332418	.667582	52	
.15	9	.21047	.97760	.21529	4.64480	.323194	.990161	.333033	.666967	51	.85
	10	.21076	.97754	.21560	4.63825	.323780	.990134	.333646	.666354	50	
	11	.21104	.97748	.21590	4.63171	.324366	.990107	.334259	.665741	49	
.20	12	.21132	.97742	.21621	4.62518	.324950	.990079	.334871	.665129	48	.80
	13	.21161	.97735	.21651	4.61868	.325534	.990052	.335482	.664518	47	
	14	.21189	.97729	.21682	4.61219	.326117	.990025	.336093	.663907	46	
.25	15	.21218	.97723	.21712	4.60572	.326700	.989997	.336702	.663298	45	.75
	16	.21246	.97717	.21743	4.59927	.327281	.989970	.337311	.662689	44	
	17	.21275	.97711	.21773	4.59283	.327862	.989942	.337919	.662081	43	
.30	18	.21303	.97705	.21804	4.58641	.328442	.989915	.338527	.661473	42	.70
	19	.21331	.97698	.21834	4.58001	.329021	.989887	.339133	.660867	41	
	20	.21360	.97692	.21864	4.57363	.329599	.989860	.339739	.660261	40	
.35	21	.21388	.97686	.21895	4.56726	.330176	.989832	.340344	.659656	39	.65
	22	.21417	.97680	.21925	4.56091	.330753	.989804	.340948	.659052	38	
	23	.21445	.97673	.21956	4.55458	.331329	.989777	.341552	.658448	37	
.40	24	.21474	.97667	.21986	4.54826	.331903	.989749	.342155	.657845	36	.60
	25	.21502	.97661	.22017	4.54196	.332478	.989721	.342757	.657243	35	
	26	.21530	.97655	.22047	4.53568	.333051	.989693	.343358	.656642	34	
.45	27	.21559	.97648	.22078	4.52941	.333624	.989665	.343958	.656042	33	.55
	28	.21587	.97642	.22108	4.52316	.334195	.989637	.344558	.655442	32	
	29	.21616	.97636	.22139	4.51693	.334767	.989610	.345157	.654843	31	
.50	30	.21644	.97630	.22169	4.51071	.335337	.989582	.345755	.654245	30	.50
	31	.21672	.97623	.22200	4.50451	.335906	.989553	.346353	.653647	29	
	32	.21701	.97617	.22231	4.49832	.336475	.989525	.346949	.653051	28	
.55	33	.21729	.97611	.22261	4.49215	.337043	.989497	.347545	.652455	27	.45
	34	.21758	.97604	.22292	4.48600	.337610	.989469	.348141	.651859	26	
	35	.21786	.97598	.22322	4.47986	.338176	.989441	.348735	.651265	25	
.60	36	.21814	.97592	.22353	4.47374	.338742	.989413	.349329	.650671	24	.40
	37	.21843	.97585	.22383	4.46764	.339307	.989385	.349922	.650078	23	
	38	.21871	.97579	.22414	4.46155	.339871	.989356	.350514	.649486	22	
.65	39	.21899	.97573	.22444	4.45548	.340434	.989328	.351106	.648894	21	.35
	40	.21928	.97566	.22475	4.44942	.340996	.989300	.351697	.648303	20	
	41	.21956	.97560	.22505	4.44338	.341558	.989271	.352287	.647713	19	
.70	42	.21985	.97553	.22536	4.43735	.342119	.989243	.352876	.647124	18	.30
	43	.22013	.97547	.22567	4.43134	.342679	.989214	.353465	.646535	17	
	44	.22041	.97541	.22597	4.42534	.343239	.989186	.354053	.645947	16	
.75	45	.22070	.97534	.22628	4.41936	.343797	.989157	.354640	.645360	15	.25
	46	.22098	.97528	.22658	4.41340	.344355	.989128	.355227	.644773	14	
	47	.22126	.97521	.22689	4.40745	.344912	.989100	.355813	.644187	13	
.80	48	.22155	.97515	.22719	4.40152	.345469	.989071	.356398	.643602	12	.20
	49	.22183	.97508	.22750	4.39560	.346024	.989042	.356982	.643018	11	
	50	.22212	.97502	.22781	4.38969	.346579	.989014	.357566	.642434	10	
.85	51	.22240	.97496	.22811	4.38381	.347134	.988985	.358149	.641851	9	.15
	52	.22268	.97489	.22842	4.37793	.347687	.988956	.358731	.641269	8	
	53	.22297	.97483	.22872	4.37207	.348240	.988927	.359313	.640687	7	
.90	54	.22325	.97476	.22903	4.36623	.348792	.988898	.359893	.640107	6	.10
	55	.22353	.97470	.22934	4.36040	.349343	.988869	.360474	.639526	5	
	56	.22382	.97463	.22964	4.35459	.349893	.988840	.361053	.638947	4	
.95	57	.22410	.97457	.22995	4.24879	.350443	.988811	.361632	.638368	3	.05
	58	.22438	.97450	.23026	4.34300	.350992	.988782	.362210	.637790	2	
	59	.22467	.97444	.23056	4.33723	.351540	.988753	.362787	.637213	1	
1.00	60	.22495	.97437	.23087	4.33148	9.352088	9.988724	9.363364	10.635636	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.22495	.97437	.23087	4.33148	9.352088	9.988724	9.363361	10.636635	60	1.00
	1	.22523	.97430	.23117	4.32573	.352635	.988695	.363940	.636060	59	
	2	.22552	.97424	.23148	4.32001	.353181	.988666	.364515	.635485	58	
.05	3	.22580	.97417	.23179	4.31430	.353726	.988636	.365090	.634910	57	.95
	4	.22608	.97411	.23209	4.30860	.354271	.988607	.365664	.634336	56	
	5	.22637	.97404	.23240	4.30291	.354815	.988578	.366237	.633763	55	
.10	6	.22665	.97398	.23271	4.29724	.355358	.988548	.366810	.633190	54	.90
	7	.22693	.97391	.23301	4.29159	.355901	.988519	.367382	.632618	53	
	8	.22722	.97384	.23332	4.28595	.356443	.988489	.367953	.632047	52	
.15	9	.22750	.97378	.23363	4.28032	.356984	.988460	.368524	.631476	51	.85
	10	.22778	.97371	.23393	4.27471	.357524	.988430	.369094	.630906	50	
	11	.22807	.97365	.23424	4.26911	.358064	.988401	.369663	.630337	49	
.20	12	.22835	.97358	.23455	4.26352	.358603	.988371	.370232	.629768	48	.80
	13	.22863	.97351	.23485	4.25795	.359141	.988342	.370799	.629201	47	
	14	.22892	.97345	.23516	4.25239	.359678	.988312	.371367	.628633	46	
.25	15	.22920	.97338	.23547	4.24685	.360215	.988282	.371933	.628067	45	.75
	16	.22948	.97331	.23578	4.24132	.360752	.988252	.372499	.627501	44	
	17	.22977	.97325	.23608	4.23580	.361287	.988223	.373064	.626936	43	
.30	18	.23005	.97318	.23639	4.23030	.361822	.988193	.373629	.626371	42	.70
	19	.23033	.97311	.23670	4.22481	.362356	.988163	.374193	.625807	41	
	20	.23062	.97304	.23700	4.21933	.362889	.988133	.374756	.625244	40	
.35	21	.23090	.97298	.23731	4.21387	.363422	.988103	.375319	.624681	39	.65
	22	.23118	.97291	.23762	4.20842	.363954	.988073	.375881	.624119	38	
	23	.23146	.97284	.23793	4.20298	.364485	.988043	.376442	.623558	37	
.40	24	.23175	.97278	.23823	4.19756	.365016	.988013	.377003	.622997	36	.60
	25	.23203	.97271	.23854	4.19215	.365546	.987983	.377563	.622437	35	
	26	.23231	.97264	.23885	4.18675	.366075	.987953	.378122	.621878	34	
.45	27	.23260	.97257	.23916	4.18137	.366604	.987922	.378681	.621319	33	.55
	28	.23288	.97251	.23946	4.17600	.367131	.987892	.379239	.620761	32	
	29	.23316	.97244	.23977	4.17064	.367659	.987862	.379797	.620203	31	
.50	30	.23345	.97237	.24008	4.16530	.368185	.987832	.380354	.619646	30	.50
	31	.23373	.97230	.24039	4.15997	.368711	.987801	.380910	.619090	29	
	32	.23401	.97223	.24069	4.15465	.369236	.987771	.381466	.618534	28	
.55	33	.23429	.97217	.24100	4.14934	.369761	.987740	.382020	.617980	27	.45
	34	.23458	.97210	.24131	4.14405	.370285	.987710	.382575	.617425	26	
	35	.23486	.97203	.24162	4.13877	.370808	.987679	.383129	.616871	25	
.60	36	.23514	.97196	.24193	4.13350	.371330	.987649	.383682	.616318	24	.40
	37	.23542	.97189	.24223	4.12825	.371852	.987618	.384234	.615766	23	
	38	.23571	.97182	.24254	4.12301	.372373	.987588	.384786	.615214	22	
.65	39	.23599	.97176	.24285	4.11778	.372894	.987557	.385337	.614663	21	.35
	40	.23627	.97169	.24316	4.11256	.373414	.987526	.385888	.614112	20	
	41	.23656	.97162	.24347	4.10736	.373933	.987496	.386438	.613562	19	
.70	42	.23684	.97155	.24377	4.10216	.374452	.987465	.386987	.613013	18	.30
	43	.23712	.97148	.24408	4.09699	.374970	.987434	.387536	.612464	17	
	44	.23740	.97141	.24439	4.09182	.375487	.987403	.388084	.611916	16	
.75	45	.23769	.97134	.24470	4.08666	.376003	.987372	.388631	.611369	15	.25
	46	.23797	.97127	.24501	4.08152	.376519	.987341	.389178	.610822	14	
	47	.23825	.97120	.24532	4.07639	.377035	.987310	.389724	.610276	13	
.80	48	.23853	.97113	.24562	4.07127	.377549	.987279	.390270	.609730	12	.20
	49	.23882	.97106	.24593	4.06616	.378063	.987248	.390815	.609185	11	
	50	.23910	.97100	.24624	4.06107	.378577	.987217	.391360	.608640	10	
.85	51	.23938	.97093	.24655	4.05599	.379089	.987186	.391903	.608097	9	.15
	52	.23966	.97086	.24686	4.05092	.379601	.987155	.392447	.607553	8	
	53	.23995	.97079	.24717	4.04586	.380113	.987124	.392989	.607011	7	
.90	54	.24023	.97072	.24747	4.04081	.380624	.987092	.393531	.606469	6	.10
	55	.24051	.97065	.24778	4.03578	.381134	.987061	.394073	.605927	5	
	56	.24079	.97058	.24809	4.03076	.381643	.987030	.394614	.605386	4	
.95	57	.24108	.97051	.24840	4.02574	.382152	.986998	.395154	.604846	3	.05
	58	.24136	.97044	.24871	4.02074	.382661	.986967	.395694	.604306	2	
	59	.24164	.97037	.24902	4.01576	.383168	.986936	.396233	.603767	1	
1.00	60	.24192	.97030	.24933	4.01078	9.383675	9.986904	9.396771	10.603229	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.24192	.97030	.24933	4.01078	9.383675	9.986904	9.396771	10.603229	60	1.00
	1	.24220	.97023	.24964	4.00582	.384182	.986873	.397309	.602691	59	
	2	.24249	.97015	.24995	4.00086	.384687	.986841	.397846	.602154	58	
.05	3	.24277	.97008	.25026	3.99592	.385192	.986809	.398383	.601617	57	.95
	4	.24305	.97001	.25056	3.99099	.385697	.986778	.398919	.601081	56	
	5	.24333	.96994	.25087	3.98607	.386201	.986746	.399455	.600545	55	
.10	6	.24362	.96987	.25118	3.98117	.386704	.986714	.399990	.600010	54	.90
	7	.24390	.96980	.25149	3.97627	.387207	.986683	.400524	.599476	53	
	8	.24418	.96973	.25180	3.97139	.387709	.986651	.401058	.598942	52	
.15	9	.24446	.96966	.25211	3.96651	.388210	.986619	.401591	.598409	51	.85
	10	.24474	.96959	.25242	3.96165	.388711	.986587	.402124	.597876	50	
	11	.24503	.96952	.25273	3.95680	.389211	.986555	.402656	.597344	49	
.20	12	.24531	.96945	.25304	3.95196	.389711	.986523	.403187	.596813	48	.80
	13	.24559	.96937	.25335	3.94713	.390210	.986491	.403718	.596282	47	
	14	.24587	.96930	.25366	3.94232	.390708	.986459	.404249	.595751	46	
.25	15	.24615	.96923	.25397	3.93751	.391206	.986427	.404778	.595222	45	.75
	16	.24644	.96916	.25428	3.93271	.391703	.986395	.405308	.594692	44	
	17	.24672	.96909	.25459	3.92793	.392199	.986363	.405836	.594164	43	
.30	18	.24700	.96902	.25490	3.92316	.392695	.986331	.406364	.593636	42	.70
	19	.24728	.96894	.25521	3.91839	.393191	.986299	.406892	.593108	41	
	20	.24756	.96887	.25552	3.91364	.393685	.986266	.407419	.592581	40	
.35	21	.24784	.96880	.25583	3.90890	.394179	.986234	.407945	.592055	39	.65
	22	.24813	.96873	.25614	3.90417	.394673	.986202	.408471	.591529	38	
	23	.24841	.96866	.25645	3.89945	.395166	.986169	.408996	.591004	37	
.40	24	.24869	.96858	.25676	3.89474	.395658	.986137	.409521	.590479	36	.60
	25	.24897	.96851	.25707	3.89004	.396150	.986104	.410045	.589955	35	
	26	.24925	.96844	.25738	3.88536	.396641	.986072	.410569	.589431	34	
.45	27	.24954	.96837	.25769	3.88068	.397132	.986039	.411092	.588908	33	.55
	28	.24982	.96829	.25800	3.87601	.397621	.986007	.411615	.588385	32	
	29	.25010	.96822	.25831	3.87136	.398111	.985974	.412137	.587863	31	
.50	30	.25038	.96815	.25862	3.86671	.398600	.985942	.412658	.587342	30	.50
	31	.25066	.96807	.25893	3.86208	.399088	.985909	.413179	.586821	29	
	32	.25094	.96800	.25924	3.85745	.399575	.985876	.413699	.586301	28	
.55	33	.25122	.96793	.25955	3.85284	.400062	.985843	.414219	.585781	27	.45
	34	.25151	.96786	.25986	3.84824	.400549	.985811	.414738	.585262	26	
	35	.25179	.96778	.26017	3.84364	.401035	.985778	.415257	.584743	25	
.60	36	.25207	.96771	.26048	3.83906	.401520	.985745	.415775	.584225	24	.40
	37	.25235	.96764	.26079	3.83449	.402005	.985712	.416293	.583707	23	
	38	.25263	.96756	.26110	3.82992	.402489	.985679	.416810	.583190	22	
.65	39	.25291	.96749	.26141	3.82537	.402972	.985646	.417326	.582674	21	.35
	40	.25320	.96742	.26172	3.82083	.403455	.985613	.417842	.582158	20	
	41	.25348	.96734	.26203	3.81630	.403938	.985580	.418358	.581642	19	
.70	42	.25376	.96727	.26235	3.81177	.404420	.985547	.418873	.581127	18	.30
	43	.25404	.96719	.26266	3.80726	.404901	.985514	.419387	.580613	17	
	44	.25432	.96712	.26297	3.80276	.405382	.985480	.419901	.580099	16	
.75	45	.25460	.96705	.26328	3.79827	.405862	.985447	.420415	.579585	15	.25
	46	.25488	.96697	.26359	3.79378	.406341	.985414	.420927	.579073	14	
	47	.25516	.96690	.26390	3.78931	.406820	.985381	.421440	.578560	13	
.80	48	.25545	.96682	.26421	3.78485	.407299	.985347	.421952	.578048	12	.20
	49	.25573	.96675	.26452	3.78040	.407777	.985314	.422463	.577537	11	
	50	.25601	.96667	.26483	3.77595	.408254	.985280	.422974	.577026	10	
.85	51	.25629	.96660	.26515	3.77152	.408731	.985247	.423484	.576516	9	.15
	52	.25657	.96653	.26546	3.76709	.409207	.985213	.423993	.576007	8	
	53	.25685	.96645	.26577	3.76268	.409682	.985180	.424503	.575497	7	
.90	54	.25713	.96638	.26608	3.75828	.410157	.985146	.425011	.574989	6	.10
	55	.25741	.96630	.26639	3.75388	.410632	.985113	.425519	.574481	5	
	56	.25769	.96623	.26670	3.74950	.411106	.985079	.426027	.573973	4	
.95	57	.25798	.96615	.26701	3.74512	.411579	.985045	.426534	.573466	3	.05
	58	.25826	.96608	.26733	3.74075	.412052	.985011	.427041	.572959	2	
	59	.25854	.96600	.26764	3.73640	.412524	.984978	.427547	.572453	1	
1.00	60	.25882	.96593	.26795	3.73205	9.412996	9.984944	9.428052	10.571948	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.25882	.96593	.26795	3.73205	9.412996	9.984944	9.428052	10.571948	60	1.00
	1	.25910	.96585	.26826	3.72771	.413467	.984910	.428558	.571442	59	
	2	.25938	.96578	.26857	3.72338	.413938	.984876	.429062	.570938	58	
.05	3	.25966	.96570	.26888	3.71907	.414408	.984842	.429566	.570434	57	.95
	4	.25994	.96562	.26920	3.71476	.414878	.984808	.430070	.569930	56	
	5	.26022	.96555	.26951	3.71046	.415347	.984774	.430573	.569427	55	
.10	6	.26050	.96547	.26982	3.70616	.415815	.984740	.431075	.568925	54	.90
	7	.26079	.96540	.27013	3.70188	.416283	.984706	.431577	.568423	53	
	8	.26107	.96532	.27044	3.69761	.416751	.984672	.432079	.567921	52	
.15	9	.26135	.96524	.27076	3.69335	.417217	.984638	.432580	.567420	51	.85
	10	.26163	.96517	.27107	3.68909	.417684	.984603	.433080	.566920	50	
	11	.26191	.96509	.27138	3.68485	.418150	.984569	.433580	.566420	49	
.20	12	.26219	.96502	.27169	3.68061	.418615	.984535	.434080	.565920	48	.80
	13	.26247	.96494	.27201	3.67638	.419079	.984500	.434579	.565421	47	
	14	.26275	.96486	.27232	3.67217	.419544	.984466	.435078	.564922	46	
.25	15	.26303	.96479	.27263	3.66796	.420007	.984432	.435576	.564424	45	.75
	16	.26331	.96471	.27294	3.66376	.420470	.984397	.436073	.563927	44	
	17	.26359	.96463	.27326	3.65957	.420933	.984363	.436570	.563430	43	
.30	18	.26387	.96456	.27357	3.65538	.421395	.984328	.437067	.562933	42	.70
	19	.26415	.96448	.27388	3.65121	.421857	.984294	.437563	.562437	41	
	20	.26443	.96440	.27419	3.64705	.422318	.984259	.438059	.561941	40	
.35	21	.26471	.96433	.27451	3.64289	.422778	.984224	.438554	.561446	39	.65
	22	.26500	.96425	.27482	3.63874	.423238	.984190	.439048	.560952	38	
	23	.26528	.96417	.27513	3.63461	.423697	.984155	.439543	.560457	37	
.40	24	.26556	.96410	.27545	3.63048	.424156	.984120	.440036	.559964	36	.60
	25	.26584	.96402	.27576	3.62636	.424615	.984085	.440529	.559471	35	
	26	.26612	.96394	.27607	3.62224	.425073	.984050	.441022	.558978	34	
.45	27	.26640	.96386	.27638	3.61814	.425530	.984015	.441514	.558486	33	.55
	28	.26668	.96379	.27670	3.61405	.425987	.983981	.442006	.557994	32	
	29	.26696	.96371	.27701	3.60996	.426443	.983946	.442497	.557503	31	
.50	30	.26724	.96363	.27732	3.60588	.426899	.983911	.442988	.557012	30	.50
	31	.26752	.96355	.27764	3.60181	.427354	.983875	.443479	.556521	29	
	32	.26780	.96347	.27795	3.59775	.427809	.983840	.443968	.556032	28	
.55	33	.26808	.96340	.27826	3.59370	.428263	.983805	.444458	.555542	27	.45
	34	.26836	.96332	.27858	3.58966	.428717	.983770	.444947	.555053	26	
	35	.26864	.96324	.27889	3.58562	.429170	.983735	.445435	.554565	25	
.60	36	.26892	.96316	.27921	3.58160	.429623	.983700	.445923	.554077	24	.40
	37	.26920	.96308	.27952	3.57758	.430075	.983664	.446411	.553589	23	
	38	.26948	.96301	.27983	3.57357	.430527	.983629	.446898	.553102	22	
.65	39	.26976	.96293	.28015	3.56957	.430978	.983594	.447384	.552616	21	.35
	40	.27004	.96285	.28046	3.56557	.431429	.983558	.447870	.552130	20	
	41	.27032	.96277	.28077	3.56159	.431879	.983523	.448356	.551644	19	
.70	42	.27060	.96269	.28109	3.55761	.432329	.983487	.448841	.551159	18	.30
	43	.27088	.96261	.28140	3.55364	.432778	.983452	.449326	.550674	17	
	44	.27116	.96253	.28172	3.54968	.433226	.983416	.449810	.550190	16	
.75	45	.27144	.96246	.28203	3.54573	.433675	.983381	.450294	.549706	15	.25
	46	.27172	.96238	.28234	3.54179	.434122	.983345	.450777	.549223	14	
	47	.27200	.96230	.28266	3.53785	.434569	.983309	.451260	.548740	13	
.80	48	.27228	.96222	.28297	3.53393	.435016	.983273	.451743	.548257	12	.20
	49	.27256	.96214	.28329	3.53001	.435462	.983238	.452225	.547775	11	
	50	.27284	.96206	.28360	3.52609	.435908	.983202	.452706	.547294	10	
.85	51	.27312	.96198	.28391	3.52219	.436353	.983166	.453187	.546813	9	.15
	52	.27340	.96190	.28423	3.51829	.436798	.983130	.453668	.546332	8	
	53	.27368	.96182	.28454	3.51441	.437242	.983094	.454148	.545852	7	
.90	54	.27396	.96174	.28486	3.51053	.437686	.983058	.454628	.545372	6	.10
	55	.27424	.96166	.28517	3.50666	.438129	.983022	.455107	.544893	5	
	56	.27452	.96158	.28549	3.50279	.438572	.982986	.455586	.544414	4	
.95	57	.27480	.96150	.28580	3.49894	.439014	.982950	.456064	.543936	3	.05
	58	.27508	.96142	.28612	3.49509	.439456	.982914	.456542	.543458	2	
	59	.27536	.96134	.28643	3.49125	.439897	.982878	.457019	.542981	1	
1.00	60	.27564	.96126	.28675	3.48741	9.440338	9.982842	9.457496	10.542504	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.27564	.96126	.28675	3.48741	9.440338	9.982842	9.457496	10.542504	60	1.00
	1	.27592	.96118	.28706	3.48359	.440778	.982805	.457973	.542027	59	
	2	.27620	.96110	.28738	3.47977	.441218	.982769	.458449	.541551	58	
.05	3	.27648	.96102	.28769	3.47596	.441658	.982733	.458925	.541075	57	.95
	4	.27676	.96094	.28800	3.47216	.442096	.982696	.459400	.540600	56	
	5	.27704	.96086	.28832	3.46837	.442535	.982660	.459875	.540125	55	
.10	6	.27731	.96078	.28864	3.46458	.442973	.982624	.460349	.539651	54	.90
	7	.27759	.96070	.28895	3.46080	.443410	.982587	.460823	.539177	53	
	8	.27787	.96062	.28927	3.45703	.443847	.982551	.461297	.538703	52	
.15	9	.27815	.96054	.28958	3.45327	.444284	.982514	.461770	.538230	51	.85
	10	.27843	.96046	.28990	3.44951	.444720	.982477	.462242	.537758	50	
	11	.27871	.96037	.29021	3.44576	.445155	.982441	.462715	.537285	49	
.20	12	.27899	.96029	.29053	3.44202	.445590	.982404	.463186	.536814	48	.80
	13	.27927	.96021	.29084	3.43829	.446025	.982367	.463658	.536342	47	
	14	.27955	.96013	.29116	3.43456	.446459	.982331	.464128	.535872	46	
.25	15	.27983	.96005	.29147	3.43084	.446893	.982294	.464599	.535401	45	.75
	16	.28011	.95997	.29179	3.42713	.447326	.982257	.465069	.534931	44	
	17	.28039	.95989	.29210	3.42343	.447759	.982220	.465539	.534461	43	
.30	18	.28067	.95981	.29242	3.41973	.448191	.982183	.466008	.533992	42	.70
	19	.28095	.95972	.29274	3.41604	.448623	.982146	.466477	.533523	41	
	20	.28123	.95964	.29305	3.41236	.449054	.982109	.466945	.533055	40	
.35	21	.28150	.95956	.29337	3.40869	.449485	.982072	.467413	.532587	39	.65
	22	.28178	.95948	.29368	3.40502	.449915	.982035	.467880	.532120	38	
	23	.28206	.95940	.29400	3.40136	.450345	.981998	.468347	.531653	37	
.40	24	.28234	.95931	.29432	3.39771	.450775	.981961	.468814	.531186	36	.60
	25	.28262	.95923	.29463	3.39406	.451204	.981924	.469280	.530720	35	
	26	.28290	.95915	.29495	3.39042	.451632	.981886	.469746	.530254	34	
.45	27	.28318	.95907	.29526	3.38679	.452060	.981849	.470211	.529789	33	.55
	28	.28346	.95898	.29558	3.38317	.452488	.981812	.470676	.529324	32	
	29	.28374	.95890	.29590	3.37955	.452915	.981774	.471141	.528859	31	
.50	30	.28402	.95882	.29621	3.37594	.453342	.981737	.471605	.528395	30	.50
	31	.28429	.95874	.29653	3.37234	.453768	.981700	.472069	.527931	29	
	32	.28457	.95865	.29685	3.36875	.454194	.981662	.472532	.527468	28	
.55	33	.28485	.95857	.29716	3.36516	.454619	.981625	.472995	.527005	27	.45
	34	.28513	.95849	.29748	3.36158	.455044	.981587	.473457	.526543	26	
	35	.28541	.95841	.29780	3.35800	.455469	.981549	.473919	.526081	25	
.60	36	.28569	.95832	.29811	3.35443	.455893	.981512	.474381	.525619	24	.40
	37	.28597	.95824	.29843	3.35087	.456316	.981474	.474842	.525158	23	
	38	.28625	.95816	.29875	3.34732	.456739	.981436	.475303	.524697	22	
.65	39	.28652	.95807	.29906	3.34377	.457162	.981399	.475763	.524237	21	.35
	40	.28680	.95799	.29938	3.34023	.457584	.981361	.476223	.523777	20	
	41	.28708	.95791	.29970	3.33670	.458006	.981323	.476683	.523317	19	
.70	42	.28736	.95782	.30001	3.33317	.458427	.981285	.477142	.522858	18	.30
	43	.28764	.95774	.30033	3.32965	.458848	.981247	.477601	.522399	17	
	44	.28792	.95766	.30065	3.32614	.459268	.981209	.478059	.521941	16	
.75	45	.28820	.95757	.30097	3.32264	.459688	.981171	.478517	.521483	15	.25
	46	.28847	.95749	.30128	3.31914	.460108	.981133	.478975	.521025	14	
	47	.28875	.95740	.30160	3.31565	.460527	.981095	.479432	.520568	13	
.80	48	.28903	.95732	.30192	3.31216	.460946	.981057	.479889	.520111	12	.20
	49	.28931	.95724	.30224	3.30868	.461364	.981019	.480345	.519655	11	
	50	.28959	.95715	.30255	3.30521	.461782	.980981	.480801	.519199	10	
.85	51	.28987	.95707	.30287	3.30174	.462199	.980942	.481257	.518743	9	.15
	52	.29015	.95698	.30319	3.29829	.462616	.980904	.481712	.518288	8	
	53	.29042	.95690	.30351	3.29483	.463032	.980866	.482167	.517833	7	
.90	54	.29070	.95681	.30382	3.29139	.463448	.980827	.482621	.517379	6	.10
	55	.29098	.95673	.30414	3.28795	.463864	.980789	.483075	.516925	5	
	56	.29126	.95664	.30446	3.28452	.464279	.980750	.483529	.516471	4	
.95	57	.29154	.95656	.30478	3.28109	.464694	.980712	.483982	.516018	3	.05
	58	.29182	.95647	.30509	3.27767	.465108	.980673	.484435	.515565	2	
	59	.29209	.95639	.30541	3.27426	.465522	.980635	.484887	.515113	1	
1.00	60	.29237	.95630	.30573	3.27085	9.465935	9.980596	9.485339	10.514661	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.29237	.95630	.30573	3.27085	9.465935	9.980596	9.485339	10.514661	60	1.00
	1	.29265	.95622	.30605	3.26745	.466348	.980558	.485791	.514209	59	
	2	.29293	.95613	.30637	3.26406	.466761	.980519	.486242	.513758	58	
.05	3	.29321	.95605	.30669	3.26067	.467173	.980480	.486693	.513307	57	.95
	4	.29348	.95596	.30700	3.25729	.467585	.980442	.487143	.512857	56	
	5	.29376	.95588	.30732	3.25392	.467996	.980403	.487593	.512407	55	
.10	6	.29404	.95579	.30764	3.25055	.468407	.980364	.488043	.511957	54	.90
	7	.29432	.95571	.30796	3.24719	.468817	.980325	.488492	.511508	53	
	8	.29460	.95562	.30828	3.24383	.469227	.980286	.488941	.511059	52	
.15	9	.29487	.95554	.30860	3.24049	.469637	.980247	.489390	.510610	51	.85
	10	.29515	.95545	.30891	3.23714	.470046	.980208	.489838	.510162	50	
	11	.29543	.95536	.30923	3.23381	.470455	.980169	.490286	.509714	49	
.20	12	.29571	.95528	.30955	3.23048	.470863	.980130	.490733	.509267	48	.80
	13	.29599	.95519	.30987	3.22715	.471271	.980091	.491180	.508820	47	
	14	.29626	.95511	.31019	3.22384	.471679	.980052	.491627	.508373	46	
.25	15	.29654	.95502	.31051	3.22053	.472086	.980012	.492073	.507927	45	.75
	16	.29682	.95493	.31083	3.21722	.472492	.979973	.492519	.507481	44	
	17	.29710	.95485	.31115	3.21392	.472898	.979934	.492965	.507035	43	
.30	18	.29737	.95476	.31147	3.21063	.473304	.979895	.493410	.506590	42	.70
	19	.29765	.95467	.31178	3.20734	.473710	.979855	.493854	.506146	41	
	20	.29793	.95459	.31210	3.20406	.474115	.979816	.494299	.505701	40	
.35	21	.29821	.95450	.31242	3.20079	.474519	.979776	.494743	.505257	39	.65
	22	.29849	.95441	.31274	3.19752	.474923	.979737	.495186	.504814	38	
	23	.29876	.95433	.31306	3.19426	.475327	.979697	.495630	.504370	37	
.40	24	.29904	.95424	.31338	3.19100	.475730	.979658	.496073	.503927	36	.60
	25	.29932	.95415	.31370	3.18775	.476133	.979618	.496515	.503485	35	
	26	.29960	.95407	.31402	3.18451	.476536	.979579	.496957	.503043	34	
.45	27	.29987	.95398	.31434	3.18127	.476938	.979539	.497399	.502601	33	.55
	28	.30015	.95389	.31466	3.17804	.477340	.979499	.497841	.502159	32	
	29	.30043	.95380	.31498	3.17481	.477741	.979459	.498282	.501718	31	
.50	30	.30071	.95372	.31530	3.17159	.478142	.979420	.498722	.501278	30	.50
	31	.30098	.95363	.31562	3.16838	.478542	.979380	.499163	.500837	29	
	32	.30126	.95354	.31594	3.16517	.478942	.979340	.499603	.500397	28	
.55	33	.30154	.95345	.31626	3.16197	.479342	.979300	.500042	.499958	27	.45
	34	.30182	.95337	.31658	3.15877	.479741	.979260	.500481	.499519	26	
	35	.30209	.95328	.31690	3.15558	.480140	.979220	.500920	.499080	25	
.60	36	.30237	.95319	.31722	3.15240	.480539	.979180	.501359	.498641	24	.40
	37	.30265	.95310	.31754	3.14922	.480937	.979140	.501797	.498203	23	
	38	.30292	.95301	.31786	3.14605	.481334	.979100	.502235	.497765	22	
.65	39	.30320	.95293	.31818	3.14288	.481731	.979059	.502672	.497328	21	.35
	40	.30348	.95284	.31850	3.13972	.482128	.979019	.503109	.496891	20	
	41	.30376	.95275	.31882	3.13656	.482525	.978979	.503546	.496454	19	
.70	42	.30403	.95266	.31914	3.13341	.482921	.978939	.503982	.496018	18	.30
	43	.30431	.95257	.31946	3.13027	.483316	.978898	.504418	.495582	17	
	44	.30459	.95248	.31978	3.12713	.483712	.978858	.504854	.495146	16	
.75	45	.30486	.95240	.32010	3.12400	.484107	.978817	.505289	.494711	15	.25
	46	.30514	.95231	.32042	3.12087	.484501	.978777	.505724	.494276	14	
	47	.30542	.95222	.32074	3.11775	.484895	.978737	.506159	.493841	13	
.80	48	.30570	.95213	.32106	3.11464	.485289	.978696	.506593	.493407	12	.20
	49	.30597	.95204	.32139	3.11153	.485682	.978655	.507027	.492973	11	
	50	.30625	.95195	.32171	3.10842	.486075	.978615	.507460	.492540	10	
.85	51	.30653	.95186	.32203	3.10532	.486467	.978574	.507893	.492107	9	.15
	52	.30680	.95177	.32235	3.10223	.486860	.978533	.508326	.491674	8	
	53	.30708	.95168	.32267	3.09914	.487251	.978493	.508759	.491241	7	
.90	54	.30736	.95159	.32299	3.09606	.487643	.978452	.509191	.490809	6	.10
	55	.30763	.95150	.32331	3.09298	.488034	.978411	.509622	.490378	5	
	56	.30791	.95142	.32363	3.08991	.488424	.978370	.510054	.489946	4	
.95	57	.30819	.95133	.32396	3.08685	.488814	.978329	.510485	.489515	3	.05
	58	.30846	.95124	.32428	3.08379	.489204	.978288	.510916	.489084	2	
	59	.30874	.95115	.32460	3.08073	.489593	.978247	.511346	.488654	1	
1.00	60	.30902	.95106	.32492	3.07768	9.489982	9.978206	9.511776	10.488224	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

VALUES AND LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.30902	.95106	.32492	3.07768	9.489982	9.978206	9.511776	10.488224	60	1.00
	1	.30929	.95097	.32524	3.07464	.490371	.978165	.512206	.487794	59	
	2	.30957	.95088	.32556	3.07160	.490759	.978124	.512635	.487365	58	
.05	3	.30985	.95079	.32588	3.06857	.491147	.978083	.513064	.486936	57	.95
	4	.31012	.95070	.32621	3.06554	.491535	.978042	.513493	.486507	56	
	5	.31040	.95061	.32653	3.06252	.491922	.978001	.513921	.486079	55	
.10	6	.31068	.95052	.32685	3.05950	.492308	.977959	.514349	.485651	54	.90
	7	.31095	.95043	.32717	3.05649	.492695	.977918	.514777	.485223	53	
	8	.31123	.95033	.32749	3.05349	.493081	.977877	.515204	.484796	52	
.15	9	.31151	.95024	.32782	3.05049	.493466	.977835	.515631	.484369	51	.85
	10	.31178	.95015	.32814	3.04749	.493851	.977794	.516057	.483943	50	
	11	.31206	.95006	.32846	3.04450	.494236	.977752	.516484	.483516	49	
.20	12	.31233	.94997	.32878	3.04152	.494621	.977711	.516910	.483090	48	.80
	13	.31261	.94988	.32911	3.03854	.495005	.977669	.517335	.482665	47	
	14	.31289	.94979	.32943	3.03556	.495388	.977628	.517761	.482239	46	
.25	15	.31316	.94970	.32975	3.03260	.495772	.977586	.518186	.481814	45	.75
	16	.31344	.94961	.33007	3.02963	.496154	.977544	.518610	.481390	44	
	17	.31372	.94952	.33040	3.02667	.496537	.977503	.519034	.480966	43	
.30	18	.31399	.94943	.33072	3.02372	.496919	.977461	.519458	.480542	42	.70
	19	.31427	.94933	.33104	3.02077	.497301	.977419	.519882	.480118	41	
	20	.31454	.94924	.33136	3.01783	.497682	.977377	.520305	.479695	40	
.35	21	.31482	.94915	.33169	3.01489	.498064	.977335	.520728	.479272	39	.65
	22	.31510	.94906	.33201	3.01196	.498444	.977293	.521151	.478849	38	
	23	.31537	.94897	.33233	3.00903	.498825	.977251	.521573	.478427	37	
.40	24	.31565	.94888	.33266	3.00611	.499204	.977209	.521995	.478005	36	.60
	25	.31593	.94878	.33298	3.00319	.499584	.977167	.522417	.477583	35	
	26	.31620	.94869	.33330	3.00028	.499963	.977125	.522838	.477162	34	
.45	27	.31648	.94860	.33363	2.99738	.500342	.977083	.523259	.476741	33	.55
	28	.31675	.94851	.33395	2.99447	.500721	.977041	.523680	.476320	32	
	29	.31703	.94842	.33427	2.99158	.501099	.976999	.524100	.475900	31	
.50	30	.31730	.94832	.33460	2.98868	.501476	.976957	.524520	.475480	30	.50
	31	.31758	.94823	.33492	2.98580	.501854	.976914	.524940	.475060	29	
	32	.31786	.94814	.33524	2.98292	.502231	.976872	.525359	.474641	28	
.55	33	.31813	.94805	.33557	2.98004	.502607	.976830	.525778	.474222	27	.45
	34	.31841	.94795	.33589	2.97717	.502984	.976787	.526197	.473803	26	
	35	.31868	.94786	.33621	2.97430	.503360	.976745	.526615	.473385	25	
.60	36	.31896	.94777	.33654	2.97144	.503735	.976702	.527033	.472967	24	.40
	37	.31923	.94768	.33686	2.96858	.504110	.976660	.527451	.472549	23	
	38	.31951	.94758	.33718	2.96573	.504485	.976617	.527868	.472132	22	
.65	39	.31979	.94749	.33751	2.96288	.504860	.976574	.528285	.471715	21	.35
	40	.32006	.94740	.33783	2.96004	.505234	.976532	.528702	.471298	20	
	41	.32034	.94730	.33816	2.95721	.505608	.976489	.529119	.470881	19	
.70	42	.32061	.94721	.33848	2.95437	.505981	.976446	.529535	.470465	18	.30
	43	.32089	.94712	.33881	2.95155	.506354	.976404	.529951	.470049	17	
	44	.32116	.94702	.33913	2.94872	.506727	.976361	.530366	.469634	16	
.75	45	.32144	.94693	.33945	2.94591	.507099	.976318	.530781	.469219	15	.25
	46	.32171	.94684	.33978	2.94309	.507471	.976275	.531196	.468804	14	
	47	.32199	.94674	.34010	2.94028	.507843	.976232	.531611	.468389	13	
.80	48	.32227	.94665	.34043	2.93748	.508214	.976189	.532025	.467975	12	.20
	49	.32254	.94656	.34075	2.93468	.508585	.976146	.532439	.467561	11	
	50	.32282	.94646	.34108	2.93189	.508956	.976103	.532853	.467147	10	
.85	51	.32309	.94637	.34140	2.92910	.509326	.976060	.533266	.466734	9	.15
	52	.32337	.94627	.34173	2.92632	.509696	.976017	.533679	.466321	8	
	53	.32364	.94618	.34205	2.92354	.510065	.975974	.534092	.465908	7	
.90	54	.32392	.94609	.34238	2.92076	.510434	.975930	.534504	.465496	6	.10
	55	.32419	.94599	.34270	2.91799	.510803	.975887	.534916	.465084	5	
	56	.32447	.94590	.34303	2.91523	.511172	.975844	.535328	.464672	4	
.95	57	.32474	.94580	.34335	2.91246	.511540	.975800	.535739	.464261	3	.05
	58	.32502	.94571	.34368	2.90971	.511907	.975757	.536150	.463850	2	
	59	.32529	.94561	.34400	2.90696	.512275	.975714	.536561	.463439	1	
1.00	60	.32557	.94552	.34433	2.90421	9.512642	9.975670	9.536972	10.463028	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values Sin	Cos	Tan	Cot	Common Logarithms Sin	Cos	Tan	Cot	Minutes	Decimals
.00	0	.32557	.94552	.34433	2.90421	9.512642	9.975670	9.536972	10.463028	60	1.00
	1	.32584	.94542	.34465	2.90147	.513009	.975627	.537382	.462618	59	
	2	.32612	.94533	.34498	2.89873	.513375	.975583	.537792	.462208	58	
.05	3	.32639	.94523	.34530	2.89600	.513741	.975539	.538202	.461798	57	.95
	4	.32667	.94514	.34563	2.89327	.514107	.975496	.538611	.461389	56	
	5	.32694	.94504	.34596	2.89055	.514472	.975452	.539020	.460980	55	
.10	6	.32722	.94495	.34628	2.88783	.514837	.975408	.539429	.460571	54	.90
	7	.32749	.94485	.34661	2.88511	.515202	.975365	.539837	.460163	53	
	8	.32777	.94476	.34693	2.88240	.515566	.975321	.540245	.459755	52	
.15	9	.32804	.94466	.34726	2.87970	.515930	.975277	.540653	.459347	51	.85
	10	.32832	.94457	.34758	2.87700	.516294	.975233	.541061	.458939	50	
	11	.32859	.94447	.34791	2.87430	.516657	.975189	.541468	.458532	49	
.20	12	.32887	.94438	.34824	2.87161	.517020	.975145	.541875	.458125	48	.80
	13	.32914	.94428	.34856	2.86892	.517382	.975101	.542281	.457719	47	
	14	.32942	.94418	.34889	2.86624	.517745	.975057	.542688	.457312	46	
.25	15	.32969	.94409	.34922	2.86356	.518107	.975013	.543094	.456906	45	.75
	16	.32997	.94399	.34954	2.86089	.518468	.974969	.543499	.456501	44	
	17	.33024	.94390	.34987	2.85822	.518829	.974925	.543905	.456095	43	
.30	18	.33051	.94380	.35020	2.85555	.519190	.974880	.544310	.455690	42	.70
	19	.33079	.94370	.35052	2.85289	.519551	.974836	.544715	.455285	41	
	20	.33106	.94361	.35085	2.85023	.519911	.974792	.545119	.454881	40	
.35	21	.33134	.94351	.35118	2.84758	.520271	.974748	.545524	.454476	39	.65
	22	.33161	.94342	.35150	2.84494	.520631	.974703	.545928	.454072	38	
	23	.33189	.94332	.35183	2.84229	.520990	.974659	.546331	.453669	37	
.40	24	.33216	.94322	.35216	2.83965	.521349	.974614	.546735	.453265	36	.60
	25	.33244	.94313	.35248	2.83702	.521707	.974570	.547138	.452862	35	
	26	.33271	.94303	.35281	2.83439	.522066	.974525	.547540	.452460	34	
.45	27	.33298	.94293	.35314	2.83176	.522424	.974481	.547943	.452057	33	.55
	28	.33326	.94284	.35346	2.82914	.522781	.974436	.548345	.451655	32	
	29	.33353	.94274	.35379	2.82653	.523138	.974391	.548747	.451253	31	
.50	30	.33381	.94264	.35412	2.82391	.523495	.974347	.549149	.450851	30	.50
	31	.33408	.94254	.35445	2.82130	.523852	.974302	.549550	.450450	29	
	32	.33436	.94245	.35477	2.81870	.524208	.974257	.549951	.450049	28	
.55	33	.33463	.94235	.35510	2.81610	.524564	.974212	.550352	.449648	27	.45
	34	.33490	.94225	.35543	2.81350	.524920	.974167	.550752	.449248	26	
	35	.33518	.94215	.35576	2.81091	.525275	.974122	.551153	.448847	25	
.60	36	.33545	.94206	.35608	2.80833	.525630	.974077	.551552	.448448	24	.40
	37	.33573	.94196	.35641	2.80574	.525984	.974032	.551952	.448048	23	
	38	.33600	.94186	.35674	2.80316	.526339	.973987	.552351	.447649	22	
.65	39	.33627	.94176	.35707	2.80059	.526693	.973942	.552750	.447250	21	.35
	40	.33655	.94167	.35740	2.79802	.527046	.973897	.553149	.446851	20	
	41	.33682	.94157	.35772	2.79545	.527400	.973852	.553548	.446452	19	
.70	42	.33710	.94147	.35805	2.79289	.527753	.973807	.553946	.446054	18	.30
	43	.33737	.94137	.35838	2.79033	.528105	.973761	.554344	.445656	17	
	44	.33764	.94127	.35871	2.78778	.528458	.973716	.554741	.445259	16	
.75	45	.33792	.94118	.35904	2.78523	.528810	.973671	.555139	.444861	15	.25
	46	.33819	.94108	.35937	2.78269	.529161	.973625	.555536	.444464	14	
	47	.33846	.94098	.35969	2.78014	.529513	.973580	.555933	.444067	13	
.80	48	.33874	.94088	.36002	2.77761	.529864	.973535	.556329	.443671	12	.20
	49	.33901	.94078	.36035	2.77507	.530215	.973489	.556725	.443275	11	
	50	.33929	.94068	.36068	2.77254	.530565	.973444	.557121	.442879	10	
.85	51	.33956	.94058	.36101	2.77002	.530915	.973398	.557517	.442483	9	.15
	52	.33983	.94049	.36134	2.76750	.531265	.973352	.557913	.442087	8	
	53	.34011	.94039	.36167	2.76498	.531614	.973307	.558308	.441692	7	
.90	54	.34038	.94029	.36199	2.76247	.531963	.973261	.558703	.441297	6	.10
	55	.34065	.94019	.36232	2.75996	.532312	.973215	.559097	.440903	5	
	56	.34093	.94009	.36265	2.75746	.532661	.973169	.559491	.440509	4	
.95	57	.34120	.93999	.36298	2.75496	.533009	.973124	.559885	.440115	3	.05
	58	.34147	.93989	.36331	2.75246	.533357	.973078	.560279	.439721	2	
	59	.34175	.93979	.36364	2.74997	.533704	.973032	.560673	.439327	1	
1.00	60	.34202	.93969	.36397	2.74748	9.534052	9.972986	9.561066	10.438934	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

VALUES AND LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.34202	.93969	.36397	2.74748	9.534052	9.972986	9.561066	10.438934	60	1.00
	1	.34229	.93959	.36430	2.74499	.534399	.972940	.561459	.438541	59	
	2	.34257	.93949	.36463	2.74251	.534745	.972894	.561851	.438149	58	
.05	3	.34284	.93939	.36496	2.74004	.535092	.972848	.562244	.437756	57	.95
	4	.34311	.93929	.36529	2.73756	.535438	.972802	.562636	.437364	56	
	5	.34339	.93919	.36562	2.73509	.535783	.972755	.563028	.436972	55	
.10	6	.34366	.93909	.36595	2.73263	.536129	.972709	.563419	.436581	54	.90
	7	.34393	.93899	.36628	2.73017	.536474	.972663	.563811	.436189	53	
	8	.34421	.93889	.36661	2.72771	.536818	.972617	.564202	.435798	52	
.15	9	.34448	.93879	.36694	2.72526	.537163	.972570	.564593	.435407	51	.85
	10	.34475	.93869	.36727	2.72281	.537507	.972524	.564983	.435017	50	
	11	.34503	.93859	.36760	2.72036	.537851	.972478	.565373	.434627	49	
.20	12	.34530	.93849	.36793	2.71792	.538194	.972431	.565763	.434237	48	.80
	13	.34557	.93839	.36826	2.71548	.538538	.972385	.566153	.433847	47	
	14	.34584	.93829	.36859	2.71305	.538880	.972338	.566542	.433458	46	
.25	15	.34612	.93819	.36892	2.71062	.539223	.972291	.566932	.433068	45	.75
	16	.34639	.93809	.36925	2.70819	.539565	.972245	.567320	.432680	44	
	17	.34666	.93799	.36958	2.70577	.539907	.972198	.567709	.432291	43	
.30	18	.34694	.93789	.36991	2.70335	.540249	.972151	.568098	.431902	42	.70
	19	.34721	.93779	.37024	2.70094	.540590	.972105	.568486	.431514	41	
	20	.34748	.93769	.37057	2.69853	.540931	.972058	.568873	.431127	40	
.35	21	.34775	.93759	.37090	2.69612	.541272	.972011	.569261	.430739	39	.65
	22	.34803	.93748	.37123	2.69371	.541613	.971964	.569648	.430352	38	
	23	.34830	.93738	.37157	2.69131	.541953	.971917	.570035	.429965	37	
.40	24	.34857	.93728	.37190	2.68892	.542293	.971870	.570422	.429578	36	.60
	25	.34884	.93718	.37223	2.68653	.542632	.971823	.570809	.429191	35	
	26	.34912	.93708	.37256	2.68414	.542971	.971776	.571195	.428805	34	
.45	27	.34939	.93698	.37289	2.68175	.543310	.971729	.571581	.428419	33	.55
	28	.34966	.93688	.37322	2.67937	.543649	.971682	.571967	.428033	32	
	29	.34993	.93677	.37355	2.67700	.543987	.971635	.572352	.427648	31	
.50	30	.35021	.93667	.37388	2.67462	.544325	.971588	.572738	.427262	30	.50
	31	.35048	.93657	.37422	2.67225	.544663	.971540	.573123	.426877	29	
	32	.35075	.93647	.37455	2.66989	.545000	.971493	.573507	.426493	28	
.55	33	.35102	.93637	.37488	2.66752	.545338	.971446	.573892	.426108	27	.45
	34	.35130	.93626	.37521	2.66516	.545674	.971398	.574276	.425724	26	
	35	.35157	.93616	.37554	2.66281	.546011	.971351	.574660	.425340	25	
.60	36	.35184	.93606	.37588	2.66046	.546347	.971303	.575044	.424956	24	.40
	37	.35211	.93596	.37621	2.65811	.546683	.971256	.575427	.424573	23	
	38	.35239	.93585	.37654	2.65576	.547019	.971208	.575810	.424190	22	
.65	39	.35266	.93575	.37687	2.65342	.547354	.971161	.576193	.423807	21	.35
	40	.35293	.93565	.37720	2.65109	.547689	.971113	.576576	.423424	20	
	41	.35320	.93555	.37754	2.64875	.548024	.971066	.576959	.423041	19	
.70	42	.35347	.93544	.37787	2.64642	.548359	.971018	.577341	.422659	18	.30
	43	.35375	.93534	.37820	2.64410	.548693	.970970	.577723	.422277	17	
	44	.35402	.93524	.37853	2.64177	.549027	.970922	.578104	.421896	16	
.75	45	.35429	.93514	.37887	2.63945	.549360	.970874	.578486	.421514	15	.25
	46	.35456	.93503	.37920	2.63714	.549693	.970827	.578867	.421133	14	
	47	.35484	.93493	.37953	2.63483	.550026	.970779	.579248	.420752	13	
.80	48	.35511	.93483	.37986	2.63252	.550359	.970731	.579629	.420371	12	.20
	49	.35538	.93472	.38020	2.63021	.550692	.970683	.580009	.419991	11	
	50	.35565	.93462	.38053	2.62791	.551024	.970635	.580389	.419611	10	
.85	51	.35592	.93452	.38086	2.62561	.551356	.970586	.580769	.419231	9	.15
	52	.35619	.93441	.38120	2.62332	.551687	.970538	.581149	.418851	8	
	53	.35647	.93431	.38153	2.62103	.552018	.970490	.581528	.418472	7	
.90	54	.35674	.93420	.38186	2.61874	.552349	.970442	.581907	.418093	6	.10
	55	.35701	.93410	.38220	2.61646	.552680	.970394	.582286	.417714	5	
	56	.35728	.93400	.38253	2.61418	.553010	.970345	.582665	.417335	4	
.95	57	.35755	.93389	.38286	2.61190	.553341	.970297	.583044	.416956	3	.05
	58	.35782	.93379	.38320	2.60963	.553670	.970249	.583422	.416578	2	
	59	.35810	.93368	.38353	2.60736	.554000	.970200	.583800	.416200	1	
1.00	60	.35837	.93358	.38386	2.60509	9.554329	9.970152	9.584177	10.415823	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.35837	.93358	.38386	2.60509	9.554329	9.970152	9.584177	10.415823	60	1.00
	1	.35864	.93348	.38420	2.60283	.554658	.970103	.584555	.415445	59	
	2	.35891	.93337	.38453	2.60057	.554987	.970055	.584932	.415068	58	
.05	3	.35918	.93327	.38487	2.59831	.555315	.970006	.585309	.414691	57	.95
	4	.35945	.93316	.38520	2.59606	.555643	.969957	.585686	.414314	56	
	5	.35973	.93306	.38553	2.59381	.555971	.969909	.586062	.413938	55	
.10	6	.36000	.93295	.38587	2.59156	.556299	.969860	.586439	.413561	54	.90
	7	.36027	.93285	.38620	2.58932	.556626	.969811	.586815	.413185	53	
	8	.36054	.93274	.38654	2.58708	.556953	.969762	.587190	.412810	52	
.15	9	.36081	.93264	.38687	2.58484	.557280	.969714	.587566	.412434	51	.85
	10	.36108	.93253	.38721	2.58261	.557606	.969665	.587941	.412059	50	
	11	.36135	.93243	.38754	2.58038	.557932	.969616	.588316	.411684	49	
.20	12	.36162	.93232	.38787	2.57815	.558258	.969567	.588691	.411309	48	.80
	13	.36190	.93222	.38821	2.57593	.558583	.969518	.589066	.410934	47	
	14	.36217	.93211	.38854	2.57371	.558909	.969469	.589440	.410560	46	
.25	15	.36244	.93201	.38888	2.57150	.559234	.969420	.589814	.410186	45	.75
	16	.36271	.93190	.38921	2.56928	.559558	.969370	.590188	.409812	44	
	17	.36298	.93180	.38955	2.56707	.559883	.969321	.590562	.409438	43	
.30	18	.36325	.93169	.38988	2.56487	.560207	.969272	.590935	.409065	42	.70
	19	.36352	.93159	.39022	2.56266	.560531	.969223	.591308	.408692	41	
	20	.36379	.93148	.39055	2.56046	.560855	.969173	.591681	.408319	40	
.35	21	.36406	.93137	.39089	2.55827	.561178	.969124	.592054	.407946	39	.65
	22	.36434	.93127	.39122	2.55608	.561501	.969075	.592426	.407574	38	
	23	.36461	.93116	.39156	2.55389	.561824	.969025	.592799	.407201	37	
.40	24	.36488	.93106	.39190	2.55170	.562146	.968976	.593171	.406829	36	.60
	25	.36515	.93095	.39223	2.54952	.562468	.968926	.593542	.406458	35	
	26	.36542	.93084	.39257	2.54734	.562790	.968877	.593914	.406086	34	
.45	27	.36569	.93074	.39290	2.54516	.563112	.968827	.594285	.405715	33	.55
	28	.36596	.93063	.39324	2.54299	.563433	.968777	.594656	.405344	32	
	29	.36623	.93052	.39357	2.54082	.563755	.968728	.595027	.404973	31	
.50	30	.36650	.93042	.39391	2.53865	.564075	.968678	.595398	.404602	30	.50
	31	.36677	.93031	.39425	2.53648	.564396	.968628	.595768	.404232	29	
	32	.36704	.93020	.39458	2.53432	.564716	.968578	.596138	.403862	28	
.55	33	.36731	.93010	.39492	2.53217	.565036	.968528	.596508	.403492	27	.45
	34	.36758	.92999	.39526	2.53001	.565356	.968479	.596878	.403122	26	
	35	.36785	.92988	.39559	2.52786	.565676	.968429	.597247	.402753	25	
.60	36	.36812	.92978	.39593	2.52571	.565995	.968379	.597616	.402384	24	.40
	37	.36839	.92967	.39626	2.52357	.566314	.968329	.597985	.402015	23	
	38	.36867	.92956	.39660	2.52142	.566632	.968278	.598354	.401646	22	
.65	39	.36894	.92945	.39694	2.51929	.566951	.968228	.598722	.401278	21	.35
	40	.36921	.92935	.39727	2.51715	.567269	.968178	.599091	.400909	20	
	41	.36948	.92924	.39761	2.51502	.567587	.968128	.599459	.400541	19	
.70	42	.36975	.92913	.39795	2.51289	.567904	.968078	.599827	.400173	18	.30
	43	.37002	.92902	.39829	2.51076	.568222	.968027	.600194	.399806	17	
	44	.37029	.92892	.39862	2.50864	.568539	.967977	.600562	.399438	16	
.75	45	.37056	.92881	.39896	2.50652	.568856	.967927	.600929	.399071	15	.25
	46	.37083	.92870	.39930	2.50440	.569172	.967876	.601296	.398704	14	
	47	.37110	.92859	.39963	2.50229	.569488	.967826	.601663	.398337	13	
.80	48	.37137	.92849	.39997	2.50018	.569804	.967775	.602029	.397971	12	.20
	49	.37164	.92838	.40031	2.49807	.570120	.967725	.602395	.397605	11	
	50	.37191	.92827	.40065	2.49597	.570435	.967674	.602761	.397239	10	
.85	51	.37218	.92816	.40098	2.49386	.570751	.967624	.603127	.396873	9	.15
	52	.37245	.92805	.40132	2.49177	.571066	.967573	.603493	.396507	8	
	53	.37272	.92794	.40166	2.48967	.571380	.967522	.603858	.396142	7	
.90	54	.37299	.92784	.40200	2.48758	.571695	.967471	.604223	.395777	6	.10
	55	.37326	.92773	.40234	2.48549	.572009	.967421	.604588	.395412	5	
	56	.37353	.92762	.40267	2.48340	.572323	.967370	.604953	.395047	4	
.95	57	.37380	.92751	.40301	2.48132	.572636	.967319	.605317	.394683	3	.05
	58	.37407	.92740	.40335	2.47924	.572950	.967268	.605682	.394318	2	
	59	.37434	.92729	.40369	2.47716	.573263	.967217	.606046	.393954	1	
1.00	60	.37461	.92718	.40403	2.47509	9.573575	9.967166	9.606410	10.393590	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.37461	.92718	.40403	2.47509	9.573575	9.967166	9.606410	10.393590	60	1.00
	1	.37488	.92707	.40436	2.47302	.573888	.967115	.606773	.393227	59	
	2	.37515	.92697	.40470	2.47095	.574200	.967064	.607137	.392863	58	
.05	3	.37542	.92686	.40504	2.46888	.574512	.967013	.607500	.392500	57	.95
	4	.37569	.92675	.40538	2.46682	.574824	.966961	.607863	.392137	56	
	5	.37595	.92664	.40572	2.46476	.575136	.966910	.608225	.391775	55	
.10	6	.37622	.92653	.40606	2.46270	.575447	.966859	.608588	.391412	54	.90
	7	.37649	.92642	.40640	2.46065	.575758	.966808	.608950	.391050	53	
	8	.37676	.92631	.40674	2.45860	.576069	.966756	.609312	.390688	52	
.15	9	.37703	.92620	.40707	2.45655	.576379	.966705	.609674	.390326	51	.85
	10	.37730	.92609	.40741	2.45451	.576689	.966653	.610036	.389964	50	
	11	.37757	.92598	.40775	2.45246	.576999	.966602	.610397	.389603	49	
.20	12	.37784	.92587	.40809	2.45043	.577309	.966550	.610759	.389241	48	.80
	13	.37811	.92576	.40843	2.44839	.577618	.966499	.611120	.388880	47	
	14	.37838	.92565	.40877	2.44636	.577927	.966447	.611480	.388520	46	
.25	15	.37865	.92554	.40911	2.44433	.578236	.966395	.611841	.388159	45	.75
	16	.37892	.92543	.40945	2.44230	.578545	.966344	.612201	.387799	44	
	17	.37919	.92532	.40979	2.44027	.578853	.966292	.612561	.387439	43	
.30	18	.37946	.92521	.41013	2.43825	.579162	.966240	.612921	.387079	42	.70
	19	.37973	.92510	.41047	2.43623	.579470	.966188	.613281	.386719	41	
	20	.37999	.92499	.41081	2.43422	.579777	.966136	.613641	.386359	40	
.35	21	.38026	.92488	.41115	2.43220	.580085	.966085	.614000	.386000	39	.65
	22	.38053	.92477	.41149	2.43019	.580392	.966033	.614359	.385641	38	
	23	.38080	.92466	.41183	2.42819	.580699	.965981	.614718	.385282	37	
.40	24	.38107	.92455	.41217	2.42618	.581005	.965929	.615077	.384923	36	.60
	25	.38134	.92444	.41251	2.42418	.581312	.965876	.615435	.384565	35	
	26	.38161	.92432	.41285	2.42218	.581618	.965824	.615793	.384207	34	
.45	27	.38188	.92421	.41319	2.42019	.581924	.965772	.616151	.383849	33	.55
	28	.38215	.92410	.41353	2.41819	.582229	.965720	.616509	.383491	32	
	29	.38241	.92399	.41387	2.41620	.582535	.965668	.616867	.383133	31	
.50	30	.38268	.92388	.41421	2.41421	.582840	.965615	.617224	.382776	30	.50
	31	.38295	.92377	.41455	2.41223	.583145	.965563	.617582	.382418	29	
	32	.38322	.92366	.41490	2.41025	.583449	.965511	.617939	.382061	28	
.55	33	.38349	.92355	.41524	2.40827	.583754	.965458	.618295	.381705	27	.45
	34	.38376	.92343	.41558	2.40629	.584058	.965406	.618652	.381348	26	
	35	.38403	.92332	.41592	2.40432	.584361	.965353	.619008	.380992	25	
.60	36	.38430	.92321	.41626	2.40235	.584665	.965301	.619364	.380636	24	.40
	37	.38456	.92310	.41660	2.40038	.584968	.965248	.619720	.380280	23	
	38	.38483	.92299	.41694	2.39841	.585272	.965195	.620076	.379924	22	
.65	39	.38510	.92287	.41728	2.39645	.585574	.965143	.620432	.379568	21	.35
	40	.38537	.92276	.41763	2.39449	.585877	.965090	.620787	.379213	20	
	41	.38564	.92265	.41797	2.39253	.586179	.965037	.621142	.378858	19	
.70	42	.38591	.92254	.41831	2.39058	.586482	.964984	.621497	.378503	18	.30
	43	.38617	.92243	.41865	2.38863	.586783	.964931	.621852	.378148	17	
	44	.38644	.92231	.41899	2.38668	.587085	.964879	.622207	.377793	16	
.75	45	.38671	.92220	.41933	2.38473	.587386	.964826	.622561	.377439	15	.25
	46	.38698	.92209	.41968	2.38279	.587688	.964773	.622915	.377085	14	
	47	.38725	.92198	.42002	2.38084	.587989	.964720	.623269	.376731	13	
.80	48	.38752	.92186	.42036	2.37891	.588289	.964666	.623623	.376377	12	.20
	49	.38778	.92175	.42070	2.37697	.588590	.964613	.623976	.376024	11	
	50	.38805	.92164	.42105	2.37504	.588890	.964560	.624330	.375670	10	
.85	51	.38832	.92152	.42139	2.37311	.589190	.964507	.624683	.375317	9	.15
	52	.38859	.92141	.42173	2.37118	.589489	.964454	.625036	.374964	8	
	53	.38886	.92130	.42207	2.36925	.589789	.964400	.625388	.374612	7	
.90	54	.38912	.92119	.42242	2.36733	.590088	.964347	.625741	.374259	6	.10
	55	.38939	.92107	.42276	2.36541	.590387	.964294	.626093	.373907	5	
	56	.38966	.92096	.42310	2.36349	.590686	.964240	.626445	.373555	4	
.95	57	.38993	.92085	.42345	2.36158	.590984	.964187	.626797	.373203	3	.05
	58	.39020	.92073	.42379	2.35967	.591282	.964133	.627149	.372851	2	
	59	.39046	.92062	.42413	2.35776	.591580	.964080	.627501	.372499	1	
1.00	60	.39073	.92050	.42447	2.35585	9.591878	9.964026	9.627852	10.372148	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.39073	.92050	.42447	2.35585	9.591878	9.964026	9.627852	10.372148	60	1.00
	1	.39100	.92039	.42482	2.35395	.592176	.963972	.628203	.371797	59	
	2	.39127	.92028	.42516	2.35205	.592473	.963919	.628554	.371446	58	
.05	3	.39153	.92016	.42551	2.35015	.592770	.963865	.628905	.371095	57	.95
	4	.39180	.92005	.42585	2.34825	.593067	.963811	.629255	.370745	56	
	5	.39207	.91994	.42619	2.34636	.593363	.963757	.629606	.370394	55	
.10	6	.39234	.91982	.42654	2.34447	.593659	.963704	.629956	.370044	54	.90
	7	.39260	.91971	.42688	2.34258	.593955	.963650	.630306	.369694	53	
	8	.39287	.91959	.42722	2.34069	.594251	.963596	.630656	.369344	52	
.15	9	.39314	.91948	.42757	2.33881	.594547	.963542	.631005	.368995	51	.85
	10	.39341	.91936	.42791	2.33693	.594842	.963488	.631355	.368645	50	
	11	.39367	.91925	.42826	2.33505	.595137	.963434	.631704	.368296	49	
.20	12	.39394	.91914	.42860	2.33317	.595432	.963379	.632053	.367947	48	.80
	13	.39421	.91902	.42894	2.33130	.595727	.963325	.632402	.367598	47	
	14	.39448	.91891	.42929	2.32943	.596021	.963271	.632750	.367250	46	
.25	15	.39474	.91879	.42963	2.32756	.596315	.963217	.633099	.366901	45	.75
	16	.39501	.91868	.42998	2.32570	.596609	.963163	.633447	.366553	44	
	17	.39528	.91856	.43032	2.32383	.596903	.963108	.633795	.366205	43	
.30	18	.39555	.91845	.43067	2.32197	.597196	.963054	.634143	.365857	42	.70
	19	.39581	.91833	.43101	2.32012	.597490	.962999	.634490	.365510	41	
	20	.39608	.91822	.43136	2.31826	.597783	.962945	.634838	.365162	40	
.35	21	.39635	.91810	.43170	2.31641	.598075	.962890	.635185	.364815	39	.65
	22	.39661	.91799	.43205	2.31456	.598368	.962836	.635532	.364468	38	
	23	.39688	.91787	.43239	2.31271	.598660	.962781	.635879	.364121	37	
.40	24	.39715	.91775	.43274	2.31086	.598952	.962727	.636226	.363774	36	.60
	25	.39741	.91764	.43308	2.30902	.599244	.962672	.636572	.363428	35	
	26	.39768	.91752	.43343	2.30718	.599536	.962617	.636919	.363081	34	
.45	27	.39795	.91741	.43378	2.30534	.599827	.962562	.637265	.362735	33	.55
	28	.39822	.91729	.43412	2.30351	.600118	.962508	.637611	.362389	32	
	29	.39848	.91718	.43447	2.30167	.600409	.962453	.637956	.362044	31	
.50	30	.39875	.91706	.43481	2.29984	.600700	.962398	.638302	.361698	30	.50
	31	.39902	.91694	.43516	2.29801	.600990	.962343	.638647	.361353	29	
	32	.39928	.91683	.43550	2.29619	.601280	.962288	.638992	.361008	28	
.55	33	.39955	.91671	.43585	2.29437	.601570	.962233	.639337	.360663	27	.45
	34	.39982	.91660	.43620	2.29254	.601860	.962178	.639682	.360318	26	
	35	.40008	.91648	.43654	2.29073	.602150	.962123	.640027	.359973	25	
.60	36	.40035	.91636	.43689	2.28891	.602439	.962067	.640371	.359629	24	.40
	37	.40062	.91625	.43724	2.28710	.602728	.962012	.640716	.359284	23	
	38	.40088	.91613	.43758	2.28528	.603017	.961957	.641060	.358940	22	
.65	39	.40115	.91601	.43793	2.28348	.603305	.961902	.641404	.358596	21	.35
	40	.40141	.91590	.43828	2.28167	.603594	.961846	.641747	.358253	20	
	41	.40168	.91578	.43862	2.27987	.603882	.961791	.642091	.357909	19	
.70	42	.40195	.91566	.43897	2.27806	.604170	.961735	.642434	.357566	18	.30
	43	.40221	.91555	.43932	2.27626	.604457	.961680	.642777	.357223	17	
	44	.40248	.91543	.43966	2.27447	.604745	.961624	.643120	.356880	16	
.75	45	.40275	.91531	.44001	2.27267	.605032	.961569	.643463	.356537	15	.25
	46	.40301	.91519	.44036	2.27088	.605319	.961513	.643806	.356194	14	
	47	.40328	.91508	.44071	2.26909	.605606	.961458	.644148	.355852	13	
.80	48	.40355	.91496	.44105	2.26730	.605892	.961402	.644490	.355510	12	.20
	49	.40381	.91484	.44140	2.26552	.606179	.961346	.644832	.355168	11	
	50	.40408	.91472	.44175	2.26374	.606465	.961290	.645174	.354826	10	
.85	51	.40434	.91461	.44210	2.26196	.606751	.961235	.645516	.354484	9	.15
	52	.40461	.91449	.44244	2.26018	.607036	.961179	.645857	.354143	8	
	53	.40488	.91437	.44279	2.25840	.607322	.961123	.646199	.353801	7	
.90	54	.40514	.91425	.44314	2.25663	.607607	.961067	.646540	.353460	6	.10
	55	.40541	.91414	.44349	2.25486	.607892	.961011	.646881	.353119	5	
	56	.40567	.91402	.44384	2.25309	.608177	.960955	.647222	.352778	4	
.95	57	.40594	.91390	.44418	2.25132	.608461	.960899	.647562	.352438	3	.05
	58	.40621	.91378	.44453	2.24956	.608745	.960843	.647903	.352097	2	
	59	.40647	.91366	.44488	2.24780	.609029	.960786	.648243	.351757	1	
1.00	60	.40674	.91355	.44523	2.24604	9.609313	9.960730	9.648583	10.351417	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values Sin	Cos	Tan	Cot	Common Logarithms Sin	Cos	Tan	Cot	Minutes	Decimals
.00	0	.40674	.91355	.44523	2.24604	9.609313	9.960730	9.648583	10.351417	60	1.00
	1	.40700	.91343	.44558	2.24428	.609597	.960674	.648923	.351077	59	
	2	.40727	.91331	.44593	2.24252	.609880	.960618	.649263	.350737	58	
.05	3	.40753	.91319	.44627	2.24077	.610164	.960561	.649602	.350398	57	.95
	4	.40780	.91307	.44662	2.23902	.610447	.960505	.649942	.350058	56	
	5	.40806	.91295	.44697	2.23727	.610729	.960448	.650281	.349719	55	
.10	6	.40833	.91283	.44732	2.23553	.611012	.960392	.650620	.349380	54	.90
	7	.40860	.91272	.44767	2.23378	.611294	.960335	.650959	.349041	53	
	8	.40886	.91260	.44802	2.23204	.611576	.960279	.651297	.348703	52	
.15	9	.40913	.91248	.44837	2.23030	.611858	.960222	.651636	.348364	51	.85
	10	.40939	.91236	.44872	2.22857	.612140	.960165	.651974	.348026	50	
	11	.40966	.91224	.44907	2.22683	.612421	.960109	.652312	.347688	49	
.20	12	.40992	.91212	.44942	2.22510	.612702	.960052	.652650	.347350	48	.80
	13	.41019	.91200	.44977	2.22337	.612983	.959995	.652988	.347012	47	
	14	.41045	.91188	.45012	2.22164	.613264	.959938	.653326	.346674	46	
.25	15	.41072	.91176	.45047	2.21992	.613545	.959882	.653663	.346337	45	.75
	16	.41098	.91164	.45082	2.21819	.613825	.959825	.654000	.346000	44	
	17	.41125	.91152	.45117	2.21647	.614105	.959768	.654337	.345663	43	
.30	18	.41151	.91140	.45152	2.21475	.614385	.959711	.654674	.345326	42	.70
	19	.41178	.91128	.45187	2.21304	.614665	.959654	.655011	.344989	41	
	20	.41204	.91116	.45222	2.21132	.614944	.959596	.655348	.344652	40	
.35	21	.41231	.91104	.45257	2.20961	.615223	.959539	.655684	.344316	39	.65
	22	.41257	.91092	.45292	2.20790	.615502	.959482	.656020	.343980	38	
	23	.41284	.91080	.45327	2.20619	.615781	.959425	.656356	.343644	37	
.40	24	.41310	.91068	.45362	2.20449	.616060	.959368	.656692	.343308	36	.60
	25	.41337	.91056	.45397	2.20278	.616338	.959310	.657028	.342972	35	
	26	.41363	.91044	.45432	2.20108	.616616	.959253	.657364	.342636	34	
.45	27	.41390	.91032	.45467	2.19938	.616894	.959195	.657699	.342301	33	.55
	28	.41416	.91020	.45502	2.19769	.617172	.959138	.658034	.341966	32	
	29	.41443	.91008	.45538	2.19599	.617450	.959080	.658369	.341631	31	
.50	30	.41469	.90996	.45573	2.19430	.617727	.959023	.658704	.341296	30	.50
	31	.41496	.90984	.45608	2.19261	.618004	.958965	.659039	.340961	29	
	32	.41522	.90972	.45643	2.19092	.618281	.958908	.659373	.340627	28	
.55	33	.41549	.90960	.45678	2.18923	.618558	.958850	.659708	.340292	27	.45
	34	.41575	.90948	.45713	2.18755	.618834	.958792	.660042	.339958	26	
	35	.41602	.90936	.45748	2.18587	.619110	.958734	.660376	.339624	25	
.60	36	.41628	.90924	.45784	2.18419	.619386	.958677	.660710	.339290	24	.40
	37	.41655	.90911	.45819	2.18251	.619662	.958619	.661043	.338957	23	
	38	.41681	.90899	.45854	2.18084	.619938	.958561	.661377	.338623	22	
.65	39	.41707	.90887	.45889	2.17916	.620213	.958503	.661710	.338290	21	.35
	40	.41734	.90875	.45924	2.17749	.620488	.958445	.662043	.337957	20	
	41	.41760	.90863	.45960	2.17582	.620763	.958387	.662376	.337624	19	
.70	42	.41787	.90851	.45995	2.17416	.621038	.958329	.662709	.337291	18	.30
	43	.41813	.90839	.46030	2.17249	.621313	.958271	.663042	.336958	17	
	44	.41840	.90826	.46065	2.17083	.621587	.958213	.663375	.336625	16	
.75	45	.41866	.90814	.46101	2.16917	.621861	.958154	.663707	.336293	15	.25
	46	.41892	.90802	.46136	2.16751	.622135	.958096	.664039	.335961	14	
	47	.41919	.90790	.46171	2.16585	.622409	.958038	.664371	.335629	13	
.80	48	.41945	.90778	.46206	2.16420	.622682	.957979	.664703	.335297	12	.20
	49	.41972	.90766	.46242	2.16255	.622956	.957921	.665035	.334965	11	
	50	.41998	.90753	.46277	2.16090	.623229	.957863	.665366	.334634	10	
.85	51	.42024	.90741	.46312	2.15925	.623502	.957804	.665698	.334302	9	.15
	52	.42051	.90729	.46348	2.15760	.623774	.957746	.666029	.333971	8	
	53	.42077	.90717	.46383	2.15596	.624047	.957687	.666360	.333640	7	
.90	54	.42104	.90704	.46418	2.15432	.624319	.957628	.666691	.333309	6	.10
	55	.42130	.90692	.46454	2.15268	.624591	.957570	.667021	.332979	5	
	56	.42156	.90680	.46489	2.15104	.624863	.957511	.667352	.332648	4	
.95	57	.42183	.90668	.46525	2.14940	.625135	.957452	.667682	.332318	3	.05
	58	.42209	.90655	.46560	2.14777	.625406	.957393	.668013	.331987	2	
	59	.42235	.90643	.46595	2.14614	.625677	.957335	.668343	.331657	1	
1.00	60	.42262	.90631	.46631	2.14451	9.625948	9.957276	9.668673	10.331327	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.42262	.90631	.46631	2.14451	9.625948	9.957276	9.668673	10.331327	60	1.00
	1	.42288	.90618	.46666	2.14288	.626219	.957217	.669002	.330998	59	
	2	.42315	.90606	.46702	2.14125	.626490	.957158	.669332	.330668	58	
.05	3	.42341	.90594	.46737	2.13963	.626760	.957099	.669661	.330339	57	.95
	4	.42367	.90582	.46772	2.13801	.627030	.957040	.669991	.330009	56	
	5	.42394	.90569	.46808	2.13639	.627300	.956981	.670320	.329680	55	
.10	6	.42420	.90557	.46843	2.13477	.627570	.956921	.670649	.329351	54	.90
	7	.42446	.90545	.46879	2.13316	.627840	.956862	.670977	.329023	53	
	8	.42473	.90532	.46914	2.13154	.628109	.956803	.671306	.328694	52	
.15	9	.42499	.90520	.46950	2.12993	.628378	.956744	.671635	.328365	51	.85
	10	.42525	.90507	.46985	2.12832	.628647	.956684	.671963	.328037	50	
	11	.42552	.90495	.47021	2.12671	.628916	.956625	.672291	.327709	49	
.20	12	.42578	.90483	.47056	2.12511	.629185	.956566	.672619	.327381	48	.80
	13	.42604	.90470	.47092	2.12350	.629453	.956506	.672947	.327053	47	
	14	.42631	.90458	.47128	2.12190	.629721	.956447	.673274	.326726	46	
.25	15	.42657	.90446	.47163	2.12030	.629989	.956387	.673602	.326398	45	.75
	16	.42683	.90433	.47199	2.11871	.630257	.956327	.673929	.326071	44	
	17	.42709	.90421	.47234	2.11711	.630524	.956268	.674257	.325743	43	
.30	18	.42736	.90408	.47270	2.11552	.630792	.956208	.674584	.325416	42	.70
	19	.42762	.90396	.47305	2.11392	.631059	.956148	.674911	.325089	41	
	20	.42788	.90383	.47341	2.11233	.631326	.956089	.675237	.324763	40	
.35	21	.42815	.90371	.47377	2.11075	.631593	.956029	.675564	.324436	39	.65
	22	.42841	.90358	.47412	2.10916	.631859	.955969	.675890	.324110	38	
	23	.42867	.90346	.47448	2.10758	.632125	.955909	.676217	.323783	37	
.40	24	.42894	.90334	.47483	2.10600	.632392	.955849	.676543	.323457	36	.60
	25	.42920	.90321	.47519	2.10442	.632658	.955789	.676869	.323131	35	
	26	.42946	.90309	.47555	2.10284	.632923	.955729	.677194	.322806	34	
.45	27	.42972	.90296	.47590	2.10126	.633189	.955669	.677520	.322480	33	.55
	28	.42999	.90284	.47626	2.09969	.633454	.955609	.677846	.322154	32	
	29	.43025	.90271	.47662	2.09811	.633719	.955548	.678171	.321829	31	
.50	30	.43051	.90259	.47698	2.09654	.633984	.955488	.678496	.321504	30	.50
	31	.43077	.90246	.47733	2.09498	.634249	.955428	.678821	.321179	29	
	32	.43104	.90233	.47769	2.09341	.634514	.955368	.679146	.320854	28	
.55	33	.43130	.90221	.47805	2.09184	.634778	.955307	.679471	.320529	27	.45
	34	.43156	.90208	.47840	2.09028	.635042	.955247	.679795	.320205	26	
	35	.43182	.90196	.47876	2.08872	.635306	.955186	.680120	.319880	25	
.60	36	.43209	.90183	.47912	2.08716	.635570	.955126	.680444	.319556	24	.40
	37	.43235	.90171	.47948	2.08560	.635834	.955065	.680768	.319232	23	
	38	.43261	.90158	.47984	2.08405	.636097	.955005	.681092	.318908	22	
.65	39	.43287	.90146	.48019	2.08250	.636360	.954944	.681416	.318584	21	.35
	40	.43313	.90133	.48055	2.08094	.636623	.954883	.681740	.318260	20	
	41	.43340	.90120	.48091	2.07939	.636886	.954823	.682063	.317937	19	
.70	42	.43366	.90108	.48127	2.07785	.637148	.954762	.682387	.317613	18	.30
	43	.43392	.90095	.48163	2.07630	.637411	.954701	.682710	.317290	17	
	44	.43418	.90082	.48198	2.07476	.637673	.954640	.683033	.316967	16	
.75	45	.43445	.90070	.48234	2.07321	.637935	.954579	.683356	.316644	15	.25
	46	.43471	.90057	.48270	2.07167	.638197	.954518	.683679	.316321	14	
	47	.43497	.90045	.48306	2.07014	.638458	.954457	.684001	.315999	13	
.80	48	.43523	.90032	.48342	2.06860	.638720	.954396	.684324	.315676	12	.20
	49	.43549	.90019	.48378	2.06706	.638981	.954335	.684646	.315354	11	
	50	.43575	.90007	.48414	2.06553	.639242	.954274	.684968	.315032	10	
.85	51	.43602	.89994	.48450	2.06400	.639503	.954213	.685290	.314710	9	.15
	52	.43628	.89981	.48486	2.06247	.639764	.954152	.685612	.314388	8	
	53	.43654	.89968	.48521	2.06094	.640024	.954090	.685934	.314066	7	
.90	54	.43680	.89956	.48557	2.05942	.640284	.954029	.686255	.313745	6	.10
	55	.43706	.89943	.48593	2.05790	.640544	.953968	.686577	.313423	5	
	56	.43733	.89930	.48629	2.05637	.640804	.953906	.686898	.313102	4	
.95	57	.43759	.89918	.48665	2.05485	.641064	.953845	.687219	.312781	3	.05
	58	.43785	.89905	.48701	2.05333	.641324	.953783	.687540	.312460	2	
	59	.43811	.89892	.48737	2.05182	.641583	.953722	.687861	.312139	1	
1.00	60	.43837	.89879	.48773	2.05030	9.641842	9.953660	9.688182	10.311818	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.43837	.89879	.48773	2.05030	9.641842	9.953650	9.688182	10.311818	60	1.00
	1	.43863	.89867	.48809	2.04879	.642101	.953599	.688502	.311498	59	
	2	.43889	.89854	.48845	2.04728	.642360	.953537	.688823	.311177	58	
.05	3	.43916	.89841	.48881	2.04577	.642618	.953475	.689143	.310857	57	.95
	4	.43942	.89828	.48917	2.04426	.642877	.953413	.689463	.310537	56	
	5	.43968	.89816	.48953	2.04276	.643135	.953352	.689783	.310217	55	
.10	6	.43994	.89803	.48989	2.04125	.643393	.953290	.690103	.309897	54	.90
	7	.44020	.89790	.49026	2.03975	.643650	.953228	.690423	.309577	53	
	8	.44046	.89777	.49062	2.03825	.643908	.953166	.690742	.309258	52	
.15	9	.44072	.89764	.49098	2.03675	.644165	.953104	.691062	.308938	51	.85
	10	.44098	.89752	.49134	2.03526	.644423	.953042	.691381	.308619	50	
	11	.44124	.89739	.49170	2.03376	.644680	.952980	.691700	.308300	49	
.20	12	.44151	.89726	.49206	2.03227	.644936	.952918	.692019	.307981	48	.80
	13	.44177	.89713	.49242	2.03078	.645193	.952855	.692338	.307662	47	
	14	.44203	.89700	.49278	2.02929	.645450	.952793	.692656	.307344	46	
.25	15	.44229	.89687	.49315	2.02780	.645706	.952731	.692975	.307025	45	.75
	16	.44255	.89674	.49351	2.02631	.645962	.952669	.693293	.306707	44	
	17	.44281	.89662	.49387	2.02483	.646218	.952606	.693612	.306388	43	
.30	18	.44307	.89649	.49423	2.02335	.646474	.952544	.693930	.306070	42	.70
	19	.44333	.89636	.49459	2.02187	.646729	.952481	.694248	.305752	41	
	20	.44359	.89623	.49495	2.02039	.646984	.952419	.694566	.305434	40	
.35	21	.44385	.89610	.49532	2.01891	.647240	.952356	.694883	.305117	39	.65
	22	.44411	.89597	.49568	2.01743	.647494	.952294	.695201	.304799	38	
	23	.44437	.89584	.49604	2.01596	.647749	.952231	.695518	.304482	37	
.40	24	.44464	.89571	.49640	2.01449	.648004	.952168	.695836	.304164	36	.60
	25	.44490	.89558	.49677	2.01302	.648258	.952106	.696153	.303847	35	
	26	.44516	.89545	.49713	2.01155	.648512	.952043	.696470	.303530	34	
.45	27	.44542	.89532	.49749	2.01008	.648766	.951980	.696787	.303213	33	.55
	28	.44568	.89519	.49786	2.00862	.649020	.951917	.697103	.302897	32	
	29	.44594	.89506	.49822	2.00715	.649274	.951854	.697420	.302580	31	
.50	30	.44620	.89493	.49858	2.00569	.649527	.951791	.697736	.302264	30	.50
	31	.44646	.89480	.49894	2.00423	.649781	.951728	.698053	.301947	29	
	32	.44672	.89467	.49931	2.00277	.650034	.951665	.698369	.301631	28	
.55	33	.44698	.89454	.49967	2.00131	.650287	.951602	.698685	.301315	27	.45
	34	.44724	.89441	.50004	1.99986	.650539	.951539	.699001	.300999	26	
	35	.44750	.89428	.50040	1.99841	.650792	.951476	.699316	.300684	25	
.60	36	.44776	.89415	.50076	1.99695	.651044	.951412	.699632	.300368	24	.40
	37	.44802	.89402	.50113	1.99550	.651297	.951349	.699947	.300053	23	
	38	.44828	.89389	.50149	1.99406	.651549	.951286	.700263	.299737	22	
.65	39	.44854	.89376	.50185	1.99261	.651800	.951222	.700578	.299422	21	.35
	40	.44880	.89363	.50222	1.99116	.652052	.951159	.700893	.299107	20	
	41	.44906	.89350	.50258	1.98972	.652304	.951096	.701208	.298792	19	
.70	42	.44932	.89337	.50295	1.98828	.652555	.951032	.701523	.298477	18	.30
	43	.44958	.89324	.50331	1.98684	.652806	.950968	.701837	.298163	17	
	44	.44984	.89311	.50368	1.98540	.653057	.950905	.702152	.297848	16	
.75	45	.45010	.89298	.50404	1.98396	.653308	.950841	.702466	.297534	15	.25
	46	.45036	.89285	.50441	1.98253	.653558	.950778	.702781	.297219	14	
	47	.45062	.89272	.50477	1.98110	.653808	.950714	.703095	.296905	13	
.80	48	.45088	.89259	.50514	1.97966	.654059	.950650	.703409	.296591	12	.20
	49	.45114	.89245	.50550	1.97823	.654309	.950586	.703722	.296278	11	
	50	.45140	.89232	.50587	1.97681	.654558	.950522	.704036	.295964	10	
.85	51	.45166	.89219	.50623	1.97538	.654808	.950458	.704350	.295650	9	.15
	52	.45192	.89206	.50660	1.97395	.655058	.950394	.704663	.295337	8	
	53	.45218	.89193	.50696	1.97253	.655307	.950330	.704976	.295024	7	
.90	54	.45243	.89180	.50733	1.97111	.655556	.950266	.705290	.294710	6	.10
	55	.45269	.89167	.50769	1.96969	.655805	.950202	.705603	.294397	5	
	56	.45295	.89153	.50806	1.96827	.656054	.950138	.705916	.294084	4	
.95	57	.45321	.89140	.50843	1.96685	.656302	.950074	.706228	.293772	3	.05
	57	.45347	.89127	.50879	1.96544	.656551	.950010	.706541	.293459	2	
	59	.45373	.89114	.50916	1.96402	.656799	.949945	.706854	.293146	1	
1.00	60	.45399	.89101	.50953	1.96261	9.657047	9.949881	9.707166	10.292834	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.45399	.89101	.50953	1.96261	9.657047	9.949881	9.707166	10.292834	60	1.00
	1	.45425	.89087	.50989	1.96120	.657295	.949816	.707478	.292522	59	
	2	.45451	.89074	.51026	1.95979	.657542	.949752	.707790	.292210	58	
.05	3	.45477	.89061	.51063	1.95838	.657790	.949688	.708102	.291898	57	.95
	4	.45503	.89048	.51099	1.95698	.658037	.949623	.708414	.291586	56	
	5	.45529	.89035	.51136	1.95557	.658284	.949558	.708726	.291274	55	
.10	6	.45554	.89021	.51173	1.95417	.658531	.949494	.709037	.290963	54	.90
	7	.45580	.89008	.51209	1.95277	.658778	.949429	.709349	.290651	53	
	8	.45606	.88995	.51246	1.95137	.659025	.949364	.709660	.290340	52	
.15	9	.45632	.88981	.51283	1.94997	.659271	.949300	.709971	.290029	51	.85
	10	.45658	.88968	.51319	1.94858	.659517	.949235	.710282	.289718	50	
	11	.45684	.88955	.51356	1.94718	.659763	.949170	.710593	.289407	49	
.20	12	.45710	.88942	.51393	1.94579	.660009	.949105	.710904	.289096	48	.80
	13	.45736	.88928	.51430	1.94440	.660255	.949040	.711215	.288785	47	
	14	.45762	.88915	.51467	1.94301	.660501	.948975	.711525	.288475	46	
.25	15	.45787	.88902	.51503	1.94162	.660746	.948910	.711836	.288164	45	.75
	16	.45813	.88888	.51540	1.94023	.660991	.948845	.712146	.287854	44	
	17	.45839	.88875	.51577	1.93885	.661236	.948780	.712456	.287544	43	
.30	18	.45865	.88862	.51614	1.93746	.661481	.948715	.712766	.287234	42	.70
	19	.45891	.88848	.51651	1.93608	.661726	.948650	.713076	.286924	41	
	20	.45917	.88835	.51688	1.93470	.661970	.948584	.713386	.286614	40	
.35	21	.45942	.88822	.51724	1.93332	.662214	.948519	.713696	.286304	39	.65
	22	.45968	.88808	.51761	1.93195	.662459	.948454	.714005	.285995	38	
	23	.45994	.88795	.51798	1.93057	.662703	.948388	.714314	.285686	37	
.40	24	.46020	.88782	.51835	1.92920	.662946	.948323	.714624	.285376	36	.60
	25	.46046	.88768	.51872	1.92782	.663190	.948257	.714933	.285067	35	
	26	.46072	.88755	.51909	1.92645	.663433	.948192	.715242	.284758	34	
.45	27	.46097	.88741	.51946	1.92508	.663677	.948126	.715551	.284449	33	.55
	28	.46123	.88728	.51983	1.92371	.663920	.948060	.715860	.284140	32	
	29	.46149	.88715	.52020	1.92235	.664163	.947995	.716168	.283832	31	
.50	30	.46175	.88701	.52057	1.92098	.664406	.947929	.716477	.283523	30	.50
	31	.46201	.88688	.52094	1.91962	.664648	.947863	.716785	.283215	29	
	32	.46226	.88674	.52131	1.91826	.664891	.947797	.717093	.282907	28	
.55	33	.46252	.88661	.52168	1.91690	.665133	.947731	.717401	.282599	27	.45
	34	.46278	.88647	.52205	1.91554	.665375	.947665	.717709	.282291	26	
	35	.46304	.88634	.52242	1.91418	.665617	.947600	.718017	.281983	25	
.60	36	.46330	.88620	.52279	1.91282	.665859	.947533	.718325	.281675	24	.40
	37	.46355	.88607	.52316	1.91147	.666100	.947467	.718633	.281367	23	
	38	.46381	.88593	.52353	1.91012	.666342	.947401	.718940	.281060	22	
.65	39	.46407	.88580	.52390	1.90876	.666583	.947335	.719248	.280752	21	.35
	40	.46433	.88566	.52427	1.90741	.666824	.947269	.719555	.280445	20	
	41	.46458	.88553	.52464	1.90607	.667065	.947203	.719862	.280138	19	
.70	42	.46484	.88539	.52501	1.90472	.667305	.947136	.720169	.279831	18	.30
	43	.46510	.88526	.52538	1.90337	.667546	.947070	.720476	.279524	17	
	44	.46536	.88512	.52575	1.90203	.667786	.947004	.720783	.279217	16	
.75	45	.46561	.88499	.52613	1.90068	.668027	.946937	.721089	.278911	15	.25
	46	.46587	.88485	.52650	1.89935	.668267	.946871	.721396	.278604	14	
	47	.46613	.88472	.52687	1.89801	.668506	.946804	.721702	.278298	13	
.80	48	.46639	.88458	.52724	1.89667	.668746	.946738	.722009	.277991	12	.20
	49	.46664	.88445	.52761	1.89533	.668986	.946671	.722315	.277685	11	
	50	.46690	.88431	.52798	1.89400	.669225	.946604	.722621	.277379	10	
.85	51	.46716	.88417	.52836	1.89266	.669464	.946538	.722927	.277073	9	.15
	52	.46742	.88404	.52873	1.89133	.669703	.946471	.723232	.276768	8	
	53	.46767	.88390	.52910	1.89000	.669942	.946404	.723538	.276462	7	
.90	54	.46793	.88377	.52947	1.88867	.670181	.946337	.723844	.276156	6	.10
	55	.46819	.88363	.52985	1.88734	.670419	.946270	.724149	.275851	5	
	56	.46844	.88349	.53022	1.88602	.670658	.946203	.724454	.275546	4	
.95	57	.46870	.88336	.53059	1.88469	.670896	.946136	.724760	.275240	3	.05
	58	.46896	.88322	.53096	1.88337	.671134	.946069	.725065	.274935	2	
	59	.46921	.88308	.53134	1.88205	.671372	.946002	.725370	.274630	1	
1.00	60	.46947	.88295	.53171	1.88073	9.671609	9.945935	9.725674	10.274326	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

VALUES AND LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.46947	.88295	.53171	1.88073	9.671609	9.945935	9.725674	10.274326	60	1.00
	1	.46973	.88281	.53208	1.87941	.671847	.945868	.725979	.274021	59	
	2	.46999	.88267	.53246	1.87809	.672084	.945800	.726284	.273716	58	
.05	3	.47024	.88254	.53283	1.87677	.672321	.945733	.726588	.273412	57	.95
	4	.47050	.88240	.53320	1.87546	.672558	.945666	.726892	.273108	56	
	5	.47076	.88226	.53358	1.87415	.672795	.945598	.727197	.272803	55	
.10	6	.47101	.88213	.53395	1.87283	.673032	.945531	.727501	.272499	54	.90
	7	.47127	.88199	.53432	1.87152	.673268	.945464	.727805	.272195	53	
	8	.47153	.88185	.53470	1.87021	.673505	.945396	.728109	.271891	52	
.15	9	.47178	.88172	.53507	1.86891	.673741	.945328	.728412	.271588	51	.85
	10	.47204	.88158	.53545	1.86760	.673977	.945261	.728716	.271284	50	
	11	.47229	.88144	.53582	1.86630	.674213	.945193	.729020	.270980	49	
.20	12	.47255	.88130	.53620	1.86499	.674448	.945125	.729323	.270677	48	.80
	13	.47281	.88117	.53657	1.86369	.674684	.945058	.729626	.270374	47	
	14	.47306	.88103	.53694	1.86239	.674919	.944990	.729929	.270071	46	
.25	15	.47332	.88089	.53732	1.86109	.675155	.944922	.730233	.269767	45	.75
	16	.47358	.88075	.53769	1.85979	.675390	.944854	.730535	.269465	44	
	17	.47383	.88062	.53807	1.85850	.675624	.944786	.730838	.269162	43	
.30	18	.47409	.88048	.53844	1.85720	.675859	.944718	.731141	.268859	42	.70
	19	.47434	.88034	.53882	1.85591	.676094	.944650	.731444	.268556	41	
	20	.47460	.88020	.53920	1.85462	.676328	.944582	.731746	.268254	40	
.35	21	.47486	.88006	.53957	1.85333	.676562	.944514	.732048	.267952	39	.65
	22	.47511	.87993	.53995	1.85204	.676796	.944446	.732351	.267649	38	
	23	.47537	.87979	.54032	1.85075	.677030	.944377	.732653	.267347	37	
.40	24	.47562	.87965	.54070	1.84946	.677264	.944309	.732955	.267045	36	.60
	25	.47588	.87951	.54107	1.84818	.677498	.944241	.733257	.266743	35	
	26	.47614	.87937	.54145	1.84689	.677731	.944172	.733558	.266442	34	
.45	27	.47639	.87923	.54183	1.84561	.677964	.944104	.733860	.266140	33	.55
	28	.47665	.87909	.54220	1.84433	.678197	.944036	.734162	.265838	32	
	29	.47690	.87896	.54258	1.84305	.678430	.943967	.734463	.265537	31	
.50	30	.47716	.87882	.54296	1.84177	.678663	.943899	.734764	.265236	30	.50
	31	.47741	.87868	.54333	1.84049	.678895	.943830	.735066	.264934	29	
	32	.47767	.87854	.54371	1.83922	.679128	.943761	.735367	.264633	28	
.55	33	.47793	.87840	.54409	1.83794	.679360	.943693	.735668	.264332	27	.45
	34	.47818	.87826	.54446	1.83667	.679592	.943624	.735969	.264031	26	
	35	.47844	.87812	.54484	1.83540	.679824	.943555	.736269	.263731	25	
.60	36	.47869	.87798	.54522	1.83413	.680056	.943486	.736570	.263430	24	.40
	37	.47895	.87784	.54560	1.83286	.680288	.943417	.736870	.263130	23	
	38	.47920	.87770	.54597	1.83159	.680519	.943348	.737171	.262829	22	
.65	39	.47946	.87756	.54635	1.83033	.680750	.943279	.737471	.262529	21	.35
	40	.47971	.87743	.54673	1.82906	.680982	.943210	.737771	.262229	20	
	41	.47997	.87729	.54711	1.82780	.681213	.943141	.738071	.261929	19	
.70	42	.48022	.87715	.54748	1.82654	.681443	.943072	.738371	.261629	18	.30
	43	.48048	.87701	.54786	1.82528	.681674	.943003	.738671	.261329	17	
	44	.48073	.87687	.54824	1.82402	.681905	.942934	.738971	.261029	16	
.75	45	.48099	.87673	.54862	1.82276	.682135	.942864	.739271	.260729	15	.25
	46	.48124	.87659	.54900	1.82150	.682365	.942795	.739570	.260430	14	
	47	.48150	.87645	.54938	1.82025	.682595	.942726	.739870	.260130	13	
.80	48	.48175	.87631	.54975	1.81899	.682825	.942656	.740169	.259831	12	.20
	49	.48201	.87617	.55013	1.81774	.683055	.942587	.740468	.259532	11	
	50	.48226	.87603	.55051	1.81649	.683284	.942517	.740767	.259233	10	
.85	51	.48252	.87589	.55089	1.81524	.683514	.942448	.741066	.258934	9	.15
	52	.48277	.87575	.55127	1.81399	.683743	.942378	.741365	.258635	8	
	53	.48303	.87561	.55165	1.81274	.683972	.942308	.741664	.258336	7	
.90	54	.48328	.87546	.55203	1.81150	.684201	.942239	.741962	.258038	6	.10
	55	.48354	.87532	.55241	1.81025	.684430	.942169	.742261	.257739	5	
	56	.48379	.87518	.55279	1.80901	.684658	.942099	.742559	.257441	4	
.95	57	.48405	.87504	.55317	1.80777	.684887	.942029	.742858	.257142	3	.05
	58	.48430	.87490	.55355	1.80653	.685115	.941959	.743156	.256844	2	
	59	.48456	.87476	.55393	1.80529	.685343	.941889	.743454	.256546	1	
1.00	60	.48481	.87462	.55431	1.80405	9.685571	9.941819	9.743752	10.256248	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.48481	.87462	.55431	1.80405	9.685571	9.941819	9.743752	10.256248	60	1.00
	1	.48506	.87448	.55469	1.80281	.685799	.941749	.744050	.255950	59	
	2	.48532	.87434	.55507	1.80158	.686027	.941679	.744348	.255652	58	
.05	3	.48557	.87420	.55545	1.80034	.686254	.941609	.744645	.255355	57	.95
	4	.48583	.87406	.55583	1.79911	.686482	.941539	.744943	.255057	56	
	5	.48608	.87391	.55621	1.79788	.686709	.941469	.745240	.254760	55	
.10	6	.48634	.87377	.55659	1.79665	.686936	.941398	.745538	.254462	54	.90
	7	.48659	.87363	.55697	1.79542	.687163	.941328	.745835	.254165	53	
	8	.48684	.87349	.55736	1.79419	.687389	.941258	.746132	.253868	52	
.15	9	.48710	.87335	.55774	1.79296	.687616	.941187	.746429	.253571	51	.85
	10	.48735	.87321	.55812	1.79174	.687843	.941117	.746726	.253274	50	
	11	.48761	.87306	.55850	1.79051	.688069	.941046	.747023	.252977	49	
.20	12	.48786	.87292	.55888	1.78929	.688295	.940975	.747319	.252681	48	.80
	13	.48811	.87278	.55926	1.78807	.688521	.940905	.747616	.252384	47	
	14	.48837	.87264	.55964	1.78685	.688747	.940834	.747913	.252087	46	
.25	15	.48862	.87250	.56003	1.78563	.688972	.940763	.748209	.251791	45	.75
	16	.48888	.87235	.56041	1.78441	.689198	.940693	.748505	.251495	44	
	17	.48913	.87221	.56079	1.78319	.689423	.940622	.748801	.251199	43	
.30	18	.48938	.87207	.56117	1.78198	.689648	.940551	.749097	.250903	42	.70
	19	.48964	.87193	.56156	1.78077	.689873	.940480	.749393	.250607	41	
	20	.48989	.87178	.56194	1.77955	.690098	.940409	.749689	.250311	40	
.35	21	.49014	.87164	.56232	1.77834	.690323	.940338	.749985	.250015	39	.65
	22	.49040	.87150	.56270	1.77713	.690548	.940267	.750281	.249719	38	
	23	.49065	.87136	.56309	1.77592	.690772	.940196	.750576	.249424	37	
.40	24	.49090	.87121	.56347	1.77471	.690996	.940125	.750872	.249128	36	.60
	25	.49116	.87107	.56385	1.77351	.691220	.940054	.751167	.248833	35	
	26	.49141	.87093	.56424	1.77230	.691444	.939982	.751462	.248538	34	
.45	27	.49166	.87079	.56462	1.77110	.691668	.939911	.751757	.248243	33	.55
	28	.49192	.87064	.56501	1.76990	.691892	.939840	.752052	.247948	32	
	29	.49217	.87050	.56539	1.76869	.692115	.939768	.752347	.247653	31	
.50	30	.49242	.87036	.56577	1.76749	.692339	.939697	.752642	.247358	30	.50
	31	.49268	.87021	.56616	1.76629	.692562	.939625	.752937	.247063	29	
	32	.49293	.87007	.56654	1.76510	.692785	.939554	.753231	.246769	28	
.55	33	.49318	.86993	.56693	1.76390	.693008	.939482	.753526	.246474	27	.45
	34	.49344	.86978	.56731	1.76271	.693231	.939410	.753820	.246180	26	
	35	.49369	.86964	.56769	1.76151	.693453	.939339	.754115	.245885	25	
.60	36	.49394	.86949	.56808	1.76032	.693676	.939267	.754409	.245591	24	.40
	37	.49419	.86935	.56846	1.75913	.693898	.939195	.754703	.245297	23	
	38	.49445	.86921	.56885	1.75794	.694120	.939123	.754997	.245003	22	
.65	39	.49470	.86906	.56923	1.75675	.694342	.939052	.755291	.244709	21	.35
	40	.49495	.86892	.56962	1.75556	.694564	.938980	.755585	.244415	20	
	41	.49521	.86878	.57000	1.75437	.694786	.938908	.755878	.244122	19	
.70	42	.49546	.86863	.57039	1.75319	.695007	.938836	.756172	.243828	18	.30
	43	.49571	.86849	.57078	1.75200	.695229	.938763	.756465	.243535	17	
	44	.49596	.86834	.57116	1.75082	.695450	.938691	.756759	.243241	16	
.75	45	.49622	.86820	.57155	1.74964	.695671	.938619	.757052	.242948	15	.25
	46	.49647	.86805	.57193	1.74846	.695892	.938547	.757345	.242655	14	
	47	.49672	.86791	.57232	1.74728	.696113	.938475	.757638	.242362	13	
.80	48	.49697	.86777	.57271	1.74610	.696334	.938402	.757931	.242069	12	.20
	49	.49723	.86762	.57309	1.74492	.696554	.938330	.758224	.241776	11	
	50	.49748	.86748	.57348	1.74375	.696775	.938258	.758517	.241483	10	
.85	51	.49773	.86733	.57386	1.74257	.696995	.938185	.758810	.241190	9	.15
	52	.49798	.86719	.57425	1.74140	.697215	.938113	.759102	.240898	8	
	53	.49824	.86704	.57464	1.74022	.697435	.938040	.759395	.240605	7	
.90	54	.49849	.86690	.57503	1.73905	.697654	.937967	.759687	.240313	6	.10
	55	.49874	.86675	.57541	1.73788	.697874	.937895	.759979	.240021	5	
	56	.49899	.86661	.57580	1.73671	.698094	.937822	.760272	.239728	4	
.95	57	.49924	.86646	.57619	1.73555	.698313	.937749	.760564	.239436	3	.05
	58	.49950	.86632	.57657	1.73438	.698532	.937676	.760856	.239144	2	
	59	.49975	.86617	.57696	1.73321	.698751	.937604	.761148	.238852	1	
1.00	60	.50000	.86603	.57735	1.73205	9.698970	9.937531	9.761439	10.238561	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.50000	.86603	.57735	1.73205	9.698970	9.937531	9.761439	10.238561	60	1.00
	1	.50025	.86588	.57774	1.73089	.699189	.937458	.761731	.238269	59	
	2	.50050	.86573	.57813	1.72973	.699407	.937385	.762023	.237977	58	
.05	3	.50076	.86559	.57851	1.72857	.699626	.937312	.762314	.237686	57	.95
	4	.50101	.86544	.57890	1.72741	.699844	.937238	.762606	.237394	56	
	5	.50126	.86530	.57929	1.72625	.700062	.937165	.762897	.237103	55	
.10	6	.50151	.86515	.57968	1.72509	.700280	.937092	.763188	.236812	54	.90
	7	.50176	.86501	.58007	1.72393	.700498	.937019	.763479	.236521	53	
	8	.50201	.86486	.58046	1.72278	.700716	.936946	.763770	.236230	52	
.15	9	.50227	.86471	.58085	1.72163	.700933	.936872	.764061	.235939	51	.85
	10	.50252	.86457	.58124	1.72047	.701151	.936799	.764352	.235648	50	
	11	.50277	.86442	.58162	1.71932	.701368	.936725	.764643	.235357	49	
.20	12	.50302	.86427	.58201	1.71817	.701585	.936652	.764933	.235067	48	.80
	13	.50327	.86413	.58240	1.71702	.701802	.936578	.765224	.234776	47	
	14	.50352	.86398	.58279	1.71588	.702019	.936505	.765514	.234486	46	
.25	15	.50377	.86384	.58318	1.71473	.702236	.936431	.765805	.234195	45	.75
	16	.50403	.86369	.58357	1.71358	.702452	.936357	.766095	.233905	44	
	17	.50428	.86354	.58396	1.71244	.702669	.936284	.766385	.233615	43	
.30	18	.50453	.86340	.58435	1.71129	.702885	.936210	.766675	.233325	42	.70
	19	.50478	.86325	.58474	1.71015	.703101	.936136	.766965	.233035	41	
	20	.50503	.86310	.58513	1.70901	.703317	.936062	.767255	.232745	40	
.35	21	.50528	.86295	.58552	1.70787	.703533	.935988	.767545	.232455	39	.65
	22	.50553	.86281	.58591	1.70673	.703749	.935914	.767834	.232166	38	
	23	.50578	.86266	.58631	1.70560	.703964	.935840	.768124	.231876	37	
.40	24	.50603	.86251	.58670	1.70446	.704179	.935766	.768414	.231586	36	.60
	25	.50628	.86237	.58709	1.70332	.704395	.935692	.768703	.231297	35	
	26	.50654	.86222	.58748	1.70219	.704610	.935618	.768992	.231008	34	
.45	27	.50679	.86207	.58787	1.70106	.704825	.935543	.769281	.230719	33	.55
	28	.50704	.86192	.58826	1.69992	.705040	.935469	.769571	.230429	32	
	29	.50729	.86178	.58865	1.69879	.705254	.935395	.769860	.230140	31	
.50	30	.50754	.86163	.58905	1.69766	.705469	.935320	.770148	.229852	30	.50
	31	.50779	.86148	.58944	1.69653	.705683	.935246	.770437	.229563	29	
	32	.50804	.86133	.58983	1.69541	.705898	.935171	.770726	.229274	28	
.55	33	.50829	.86119	.59022	1.69428	.706112	.935097	.771015	.228985	27	.45
	34	.50854	.86104	.59061	1.69316	.706326	.935022	.771303	.228697	26	
	35	.50879	.86089	.59101	1.69203	.706539	.934948	.771592	.228408	25	
.60	36	.50904	.86074	.59140	1.69091	.706753	.934873	.771880	.228120	24	.40
	37	.50929	.86059	.59179	1.68979	.706967	.934798	.772168	.227832	23	
	38	.50954	.86045	.59218	1.68866	.707180	.934723	.772457	.227543	22	
.65	39	.50979	.86030	.59258	1.68754	.707393	.934649	.772745	.227255	21	.35
	40	.51004	.86015	.59297	1.68643	.707606	.934574	.773033	.226967	20	
	41	.51029	.86000	.59336	1.68531	.707819	.934499	.773321	.226679	19	
.70	42	.51054	.85985	.59376	1.68419	.708032	.934424	.773608	.225392	18	.30
	43	.51079	.85970	.59415	1.68308	.708245	.934349	.773896	.226104	17	
	44	.51104	.85956	.59454	1.68196	.708458	.934274	.774184	.225816	16	
.75	45	.51129	.85941	.59494	1.68085	.708670	.934199	.774471	.225529	15	.25
	46	.51154	.85926	.59533	1.67974	.708882	.934123	.774759	.225241	14	
	47	.51179	.85911	.59573	1.67863	.709094	.934048	.775046	.224954	13	
.80	48	.51204	.85896	.59612	1.67752	.709306	.933973	.775333	.224667	12	.20
	49	.51229	.85881	.59651	1.67641	.709518	.933898	.775621	.224379	11	
	50	.51254	.85866	.59691	1.67530	.709730	.933822	.775908	.224092	10	
.85	51	.51279	.85851	.59730	1.67419	.709941	.933747	.776195	.223805	9	.15
	52	.51304	.85836	.59770	1.67309	.710153	.933671	.776482	.223518	8	
	53	.51329	.85821	.59809	1.67198	.710364	.933596	.776768	.223232	7	
.90	54	.51354	.85806	.59849	1.67088	.710575	.933520	.777055	.222945	6	.10
	55	.51379	.85792	.59888	1.66978	.710786	.933445	.777342	.222658	5	
	56	.51404	.85777	.59928	1.66867	.710997	.933369	.777628	.222372	4	
.95	57	.51429	.85762	.59967	1.66757	.711208	.933293	.777915	.222085	3	.05
	58	.51454	.85747	.60007	1.66647	.711419	.933217	.778201	.221799	2	
	59	.51479	.85732	.60046	1.66538	.711629	.933141	.778488	.221512	1	
1.00	60	.51504	.85717	.60086	1.66428	9.711839	9.933066	9.778774	10.221226	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.51504	.85717	.60086	1.66428	9.711839	9.933066	9.778774	10.221226	60	1.00
	1	.51529	.85702	.60126	1.66318	.712050	.932990	.779060	.220940	59	
	2	.51554	.85687	.60165	1.66209	.712260	.932914	.779346	.220654	58	
.05	3	.51579	.85672	.60205	1.66099	.712469	.932838	.779632	.220368	57	.95
	4	.51604	.85657	.60245	1.65990	.712679	.932762	.779918	.220082	56	
	5	.51628	.85642	.60284	1.65881	.712889	.932685	.780203	.219797	55	
.10	6	.51653	.85627	.60324	1.65772	.713098	.932609	.780489	.219511	54	.90
	7	.51678	.85612	.60364	1.65663	.713308	.932533	.780775	.219225	53	
	8	.51703	.85597	.60403	1.65554	.713517	.932457	.781060	.218940	52	
.15	9	.51728	.85582	.60443	1.65445	.713726	.932380	.781346	.218654	51	.85
	10	.51753	.85567	.60483	1.65337	.713935	.932304	.781631	.218369	50	
	11	.51778	.85551	.60522	1.65228	.714144	.932228	.781916	.218084	49	
.20	12	.51803	.85536	.60562	1.65120	.714352	.932151	.782201	.217799	48	.80
	13	.51828	.85521	.60602	1.65011	.714561	.932075	.782486	.217514	47	
	14	.51852	.85506	.60642	1.64903	.714769	.931998	.782771	.217229	46	
.25	15	.51877	.85491	.60681	1.64795	.714978	.931921	.783056	.216944	45	.75
	16	.51902	.85476	.60721	1.64687	.715186	.931845	.783341	.216659	44	
	17	.51927	.85461	.60761	1.64579	.715394	.931768	.783626	.216374	43	
.30	18	.51952	.85446	.60801	1.64471	.715602	.931691	.783910	.216090	42	.70
	19	.51977	.85431	.60841	1.64363	.715809	.931614	.784195	.215805	41	
	20	.52002	.85416	.60881	1.64256	.716017	.931537	.784479	.215521	40	
.35	21	.52026	.85401	.60921	1.64148	.716224	.931460	.784764	.215236	39	.65
	22	.52051	.85385	.60960	1.64041	.716432	.931383	.785048	.214952	38	
	23	.52076	.85370	.61000	1.63934	.716639	.931306	.785332	.214668	37	
.40	24	.52101	.85355	.61040	1.63826	.716846	.931229	.785616	.214384	36	.60
	25	.52126	.85340	.61080	1.63719	.717053	.931152	.785900	.214100	35	
	26	.52151	.85325	.61120	1.63612	.717259	.931075	.786184	.213816	34	
.45	27	.52175	.85310	.61160	1.63505	.717466	.930998	.786468	.213532	33	.55
	28	.52200	.85294	.61200	1.63398	.717673	.930921	.736752	.213248	32	
	29	.52225	.85279	.61240	1.63292	.717879	.930843	.787036	.212964	31	
.50	30	.52250	.85264	.61280	1.63185	.718085	.930766	.787319	.212681	30	.50
	31	.52275	.85249	.61320	1.63079	.718291	.930688	.787603	.212397	29	
	32	.52299	.85234	.61360	1.62972	.718497	.930611	.787886	.212114	28	
.55	33	.52324	.85218	.61400	1.62866	.718703	.930533	.788170	.211830	27	.45
	34	.52349	.85203	.61440	1.62760	.718909	.930456	.788453	.211547	26	
	35	.52374	.85188	.61480	1.62654	.719114	.930378	.788736	.211264	25	
.60	36	.52399	.85173	.61520	1.62548	.719320	.930300	.789019	.210981	24	.40
	37	.52423	.85157	.61561	1.62442	.719525	.930223	.789302	.210698	23	
	38	.52448	.85142	.61601	1.62336	.719730	.930145	.789585	.210415	22	
.65	39	.52473	.85127	.61641	1.62230	.719935	.930067	.789868	.210132	21	.35
	40	.52498	.85112	.61681	1.62125	.720140	.929989	.790151	.209849	20	
	41	.52522	.85096	.61721	1.62019	.720345	.929911	.790434	.209566	19	
.70	42	.52547	.85081	.61761	1.61914	.720549	.929833	.790716	.209284	18	.30
	43	.52572	.85066	.61801	1.61808	.720754	.929755	.790999	.209001	17	
	44	.52597	.85051	.61842	1.61703	.720958	.929677	.791281	.208719	16	
.75	45	.52621	.85035	.61882	1.61598	.721162	.929599	.791563	.208437	15	.25
	46	.52646	.85020	.61922	1.61493	.721366	.929521	.791846	.208154	14	
	47	.52671	.85005	.61962	1.61388	.721570	.929442	.792128	.207872	13	
.80	48	.52696	.84989	.62003	1.61283	.721774	.929364	.792410	.207590	12	.20
	49	.52720	.84974	.62043	1.61179	.721978	.929286	.792692	.207308	11	
	50	.52745	.84959	.62083	1.61074	.722181	.929207	.792974	.207026	10	
.85	51	.52770	.84943	.62124	1.60970	.722385	.929129	.793256	.206744	9	.15
	52	.52794	.84928	.62164	1.60865	.722588	.929050	.793538	.206462	8	
	53	.52819	.84913	.62204	1.60761	.722791	.928972	.793819	.206181	7	
.90	54	.52844	.84897	.62245	1.60657	.722994	.928893	.794101	.205899	6	.10
	55	.52869	.84882	.62285	1.60553	.723197	.928815	.794383	.205617	5	
	56	.52893	.84866	.62325	1.60449	.723400	.928736	.794664	.205336	4	
.95	57	.52918	.84851	.62366	1.60345	.723603	.928657	.794946	.205054	3	.05
	58	.52943	.84836	.62406	1.60241	.723805	.928578	.795227	.204773	2	
	59	.52967	.84820	.62446	1.60137	.724007	.928499	.795508	.204492	1	
1.00	60	.52992	.84805	.62487	1.60033	9.724210	9.928420	9.795789	10.204211	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.52992	.84805	.62487	1.60033	9.724210	9.928420	9.795789	10.204211	60	1.00
	1	.53017	.84789	.62527	1.59930	.724412	.928342	.796070	.203930	59	
	2	.53041	.84774	.62568	1.59826	.724614	.928263	.796351	.203649	58	
.05	3	.53066	.84759	.62608	1.59723	.724816	.928183	.796632	.203368	57	.95
	4	.53091	.84743	.62649	1.59620	.725017	.928104	.796913	.203087	56	
	5	.53115	.84728	.62689	1.59517	.725219	.928025	.797194	.202806	55	
.10	6	.53140	.84712	.62730	1.59414	.725420	.927946	.797474	.202526	54	.90
	7	.53164	.84697	.62770	1.59311	.725622	.927867	.797755	.202245	53	
	8	.53189	.84681	.62811	1.59208	.725823	.927787	.798036	.201964	52	
.15	9	.53214	.84666	.62852	1.59105	.726024	.927708	.798316	.201684	51	.85
	10	.53238	.84650	.62892	1.59002	.726225	.927629	.798596	.201404	50	
	11	.53263	.84635	.62933	1.58900	.726426	.927549	.798877	.201123	49	
.20	12	.53288	.84619	.62973	1.58797	.726626	.927470	.799157	.200843	48	.80
	13	.53312	.84604	.63014	1.58695	.726827	.927390	.799437	.200563	47	
	14	.53337	.84588	.63055	1.58593	.727027	.927310	.799717	.200283	46	
.25	15	.53361	.84573	.63095	1.58490	.727228	.927231	.799997	.200003	45	.75
	16	.53386	.84557	.63136	1.58388	.727428	.927151	.800277	.199723	44	
	17	.53411	.84542	.63177	1.58286	.727628	.927071	.800557	.199443	43	
.30	18	.53435	.84526	.63217	1.58184	.727828	.926991	.800836	.199164	42	.70
	19	.53460	.84511	.63258	1.58083	.728027	.926911	.801116	.198884	41	
	20	.53484	.84495	.63299	1.57981	.728227	.926831	.801396	.198604	40	
.35	21	.53509	.84480	.63340	1.57879	.728427	.926751	.801675	.198325	39	.65
	22	.53534	.84464	.63380	1.57778	.728626	.926671	.801955	.198045	38	
	23	.53558	.84448	.63421	1.57676	.728825	.926591	.802234	.197766	37	
.40	24	.53583	.84433	.63462	1.57575	.729024	.926511	.802513	.197487	36	.60
	25	.53607	.84417	.63503	1.57474	.729223	.926431	.802792	.197208	35	
	26	.53632	.84402	.63544	1.57372	.729422	.926351	.803072	.196928	34	
.45	27	.53656	.84386	.63584	1.57271	.729621	.926270	.803351	.196649	33	.55
	28	.53681	.84370	.63625	1.57170	.729820	.926190	.803630	.196370	32	
	29	.53705	.84355	.63666	1.57069	.730018	.926110	.803909	.196091	31	
.50	30	.53730	.84339	.63707	1.56969	.730217	.926029	.804187	.195813	30	.50
	31	.53754	.84324	.63748	1.56868	.730415	.925949	.804466	.195534	29	
	32	.53779	.84308	.63789	1.56767	.730613	.925868	.804745	.195255	28	
.55	33	.53804	.84292	.63830	1.56667	.730811	.925788	.805023	.194977	27	.45
	34	.53828	.84277	.63871	1.56566	.731009	.925707	.805302	.194698	26	
	35	.53853	.84261	.63912	1.56466	.731206	.925626	.805580	.194420	25	
.60	36	.53877	.84245	.63953	1.56366	.731404	.925545	.805859	.194141	24	.40
	37	.53902	.84230	.63994	1.56265	.731602	.925465	.806137	.193863	23	
	38	.53926	.84214	.64035	1.56165	.731799	.925384	.806415	.193585	22	
.65	39	.53951	.84198	.64076	1.56065	.731996	.925303	.806693	.193307	21	.35
	40	.53975	.84182	.64117	1.55966	.732193	.925222	.806971	.193029	20	
	41	.54000	.84167	.64158	1.55866	.732390	.925141	.807249	.192751	19	
.70	42	.54024	.84151	.64199	1.55766	.732587	.925060	.807527	.192473	18	.30
	43	.54049	.84135	.64240	1.55666	.732784	.924979	.807805	.192195	17	
	44	.54073	.84120	.64281	1.55567	.732980	.924897	.808083	.191917	16	
.75	45	.54097	.84104	.64322	1.55467	.733177	.924816	.808361	.191639	15	.25
	46	.54122	.84088	.64363	1.55368	.733373	.924735	.808638	.191362	14	
	47	.54146	.84072	.64404	1.55269	.733569	.924654	.808916	.191084	13	
.80	48	.54171	.84057	.64446	1.55170	.733765	.924572	.809193	.190807	12	.20
	49	.54195	.84041	.64487	1.55071	.733961	.924491	.809471	.190529	11	
	50	.54220	.84025	.64528	1.54972	.734157	.924409	.809748	.190252	10	
.85	51	.54244	.84009	.64569	1.54873	.734353	.924328	.810025	.189975	9	.15
	52	.54269	.83994	.64610	1.54774	.734549	.924246	.810302	.189698	8	
	53	.54293	.83978	.64652	1.54675	.734744	.924164	.810580	.189420	7	
.90	54	.54317	.83962	.64693	1.54576	.734939	.924083	.810857	.189143	6	.10
	55	.54342	.83946	.64734	1.54478	.735135	.924001	.811134	.188866	5	
	56	.54366	.83930	.64775	1.54379	.735330	.923919	.811410	.188590	4	
.95	57	.54391	.83915	.64817	1.54281	.735525	.923837	.811687	.188313	3	.05
	58	.54415	.83899	.64858	1.54183	.735719	.923755	.811964	.188036	2	
	59	.54440	.83883	.64899	1.54085	.735914	.923673	.812241	.187759	1	
1.00	60	.54464	.83867	.64941	1.53986	9.736109	9.923591	9.812517	10.187483	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.54464	.83867	.64941	1.53986	9.736109	9.923591	9.812517	10.187483	60	1.00
	1	.54488	.83851	.64982	1.53888	.736303	.923509	.812794	.187206	59	
	2	.54513	.83835	.65024	1.53791	.736498	.923427	.813070	.186930	58	
.05	3	.54537	.83819	.65065	1.53693	.736692	.923345	.813347	.186653	57	.95
	4	.54561	.83804	.65106	1.53595	.736886	.923263	.813623	.186377	56	
	5	.54586	.83788	.65148	1.53497	.737080	.923181	.813899	.186101	55	
.10	6	.54610	.83772	.65189	1.53400	.737274	.923098	.814176	.185824	54	.90
	7	.54635	.83756	.65231	1.53302	.737467	.923016	.814452	.185548	53	
	8	.54659	.83740	.65272	1.53205	.737661	.922933	.814728	.185272	52	
.15	9	.54683	.83724	.65314	1.53107	.737855	.922851	.815004	.184996	51	.85
	10	.54708	.83708	.65355	1.53010	.738048	.922768	.815280	.184720	50	
	11	.54732	.83692	.65397	1.52913	.738241	.922686	.815555	.184445	49	
.20	12	.54756	.83676	.65438	1.52816	.738434	.922603	.815831	.184169	48	.80
	13	.54781	.83660	.65480	1.52719	.738627	.922520	.816107	.183893	47	
	14	.54805	.83645	.65521	1.52622	.738820	.922438	.816382	.183618	46	
.25	15	.54829	.83629	.65563	1.52525	.739013	.922355	.816658	.183342	45	.75
	16	.54854	.83613	.65604	1.52429	.739206	.922272	.816933	.183067	44	
	17	.54878	.83597	.65646	1.52332	.739398	.922189	.817209	.182791	43	
.30	18	.54902	.83581	.65688	1.52235	.739590	.922106	.817484	.182516	42	.70
	19	.54927	.83565	.65729	1.52139	.739783	.922023	.817759	.182241	41	
	20	.54951	.83549	.65771	1.52043	.739975	.921940	.818035	.181965	40	
.35	21	.54975	.83533	.65813	1.51946	.740167	.921857	.818310	.181690	39	.65
	22	.54999	.83517	.65854	1.51850	.740359	.921774	.818585	.181415	38	
	23	.55024	.83501	.65896	1.51754	.740550	.921691	.818860	.181140	37	
.40	24	.55048	.83485	.65938	1.51658	.740742	.921607	.819135	.180865	36	.60
	25	.55072	.83469	.65980	1.51562	.740934	.921524	.819410	.180590	35	
	26	.55097	.83453	.66021	1.51466	.741125	.921441	.819684	.180316	34	
.45	27	.55121	.83437	.66063	1.51370	.741316	.921357	.819959	.180041	33	.55
	28	.55145	.83421	.66105	1.51275	.741508	.921274	.820234	.179766	32	
	29	.55169	.83405	.66147	1.51179	.741699	.921190	.820508	.179492	31	
.50	30	.55194	.83389	.66189	1.51084	.741889	.921107	.820783	.179217	30	.50
	31	.55218	.83373	.66230	1.50988	.742080	.921023	.821057	.178943	29	
	32	.55242	.83356	.66272	1.50893	.742271	.920939	.821332	.178668	28	
.55	33	.55266	.83340	.63314	1.50797	.742462	.920856	.821606	.178394	27	.45
	34	.55291	.83324	.66356	1.50702	.742652	.920772	.821880	.178120	26	
	35	.55315	.83308	.66398	1.50607	.742842	.920688	.822154	.177846	25	
.60	36	.55339	.83292	.66440	1.50512	.743033	.920604	.822429	.177571	24	.40
	37	.55363	.83276	.66482	1.50417	.743223	.920520	.822703	.177297	23	
	38	.55388	.83260	.66524	1.50322	.743413	.920436	.822977	.177023	22	
.65	39	.55412	.83244	.66566	1.50228	.743602	.920352	.823251	.176749	21	.35
	40	.55436	.83228	.66608	1.50133	.743792	.920268	.823524	.176476	20	
	41	.55460	.83212	.66650	1.50038	.743982	.920184	.823798	.176202	19	
.70	42	.55484	.83195	.66692	1.49944	.744171	.920099	.824072	.175923	18	.30
	43	.55509	.83179	.66734	1.49849	.744361	.920015	.824345	.175655	17	
	44	.55533	.83163	.66776	1.49755	.744550	.919931	.824619	.175381	16	
.75	45	.55557	.83147	.66818	1.49661	.744739	.919846	.824893	.175107	15	.25
	46	.55581	.83131	.66860	1.49566	.744928	.919762	.825166	.174834	14	
	47	.55605	.83115	.66902	1.49472	.745117	.919677	.825439	.174561	13	
.80	48	.55630	.83098	.66944	1.49378	.745306	.919593	.825713	.174287	12	.20
	49	.55654	.83082	.66986	1.49284	.745494	.919508	.825986	.174014	11	
	50	.55678	.83066	.67028	1.49190	.745683	.919424	.826259	.173741	10	
.85	51	.55702	.83050	.67071	1.49097	.745871	.919339	.826532	.173468	9	.15
	52	.55726	.83034	.67113	1.49003	.746060	.919254	.826805	.173195	8	
	53	.55750	.83017	.67155	1.48909	.746248	.919169	.827078	.172922	7	
.90	54	.55775	.83001	.67197	1.48816	.746436	.919085	.827350	.172650	6	.10
	55	.55799	.82985	.67239	1.48722	.746624	.919000	.827624	.172376	5	
	56	.55823	.82969	.67282	1.48629	.746812	.918915	.827897	.172103	4	
.95	57	.55847	.82953	.67324	1.48536	.746999	.918830	.828170	.171830	3	.05
	58	.55871	.82936	.67366	1.48442	.747187	.918745	.828442	.171558	2	
	59	.55895	.82920	.67409	1.48349	.747374	.918659	.828715	.171285	1	
1.00	60	.55919	.82904	.67451	1.48256	9.747562	9.918574	9.828987	10.171013	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.55919	.82904	.67451	1.48256	9.747562	9.918574	9.828987	10.171013	60	1.00
	1	.55943	.82887	.67493	1.48163	.747749	.918489	.829260	.170740	59	
	2	.55968	.82871	.67536	1.48070	.747936	.918404	.829532	.170468	58	
.05	3	.55992	.82855	.67578	1.47977	.748123	.918318	.829805	.170195	57	.95
	4	.56016	.82839	.67620	1.47885	.748310	.918233	.830077	.169923	56	
	5	.56040	.82822	.67663	1.47792	.748497	.918147	.830349	.169651	55	
.10	6	.56064	.82806	.67705	1.47699	.748683	.918062	.830621	.169379	54	.90
	7	.56088	.82790	.67748	1.47607	.748870	.917976	.830893	.169107	53	
	8	.56112	.82773	.67790	1.47514	.749056	.917891	.831165	.168835	52	
.15	9	.56136	.82757	.67832	1.47422	.749243	.917805	.831437	.168563	51	.85
	10	.56160	.82741	.67875	1.47330	.749429	.917719	.831709	.168291	50	
	11	.56184	.82724	.67917	1.47238	.749615	.917634	.831981	.168019	49	
.20	12	.56208	.82708	.67960	1.47146	.749801	.917548	.832253	.167747	48	.80
	13	.56232	.82692	.68002	1.47053	.749987	.917462	.832525	.167475	47	
	14	.56256	.82675	.68045	1.46962	.750172	.917376	.832796	.167204	46	
.25	15	.56280	.82659	.68088	1.46870	.750358	.917290	.833068	.166932	45	.75
	16	.56305	.82643	.68130	1.46778	.750543	.917204	.833339	.166661	44	
	17	.56329	.82626	.68173	1.46686	.750729	.917118	.833611	.166389	43	
.30	18	.56353	.82610	.68215	1.46595	.750914	.917032	.833882	.166118	42	.70
	19	.56377	.82593	.68258	1.46503	.751099	.916946	.834154	.165846	41	
	20	.56401	.82577	.68301	1.46411	.751284	.916859	.834425	.165575	40	
.35	21	.56425	.82561	.68343	1.46320	.751469	.916773	.834696	.165304	39	.65
	22	.56449	.82544	.68386	1.46229	.751654	.916687	.834967	.165033	38	
	23	.56473	.82528	.68429	1.46137	.751839	.916600	.835238	.164762	37	
.40	24	.56497	.82511	.68471	1.46046	.752023	.916514	.835509	.164491	36	.60
	25	.56521	.82495	.68514	1.45955	.752208	.916427	.835780	.164220	35	
	26	.56545	.82478	.68557	1.45864	.752392	.916341	.836051	.163949	34	
.45	27	.56569	.82462	.68600	1.45773	.752576	.916254	.836322	.163678	33	.55
	28	.56593	.82446	.68642	1.45682	.752760	.916167	.836593	.163407	32	
	29	.56617	.82429	.68685	1.45592	.752944	.916081	.836864	.163136	31	
.50	30	.56641	.82413	.68728	1.45501	.753128	.915994	.837134	.162866	30	.50
	31	.56665	.82396	.68771	1.45410	.753312	.915907	.837405	.162595	29	
	32	.56689	.82380	.68814	1.45320	.753495	.915820	.837675	.162325	28	
.55	33	.56713	.82363	.68857	1.45229	.753679	.915733	.837946	.162054	27	.45
	34	.56736	.82347	.68900	1.45139	.753862	.915646	.838216	.161784	26	
	35	.56760	.82330	.68942	1.45049	.754046	.915559	.838487	.161513	25	
.60	36	.56784	.82314	.68985	1.44958	.754229	.915472	.838757	.161243	24	.40
	37	.56808	.82297	.69028	1.44868	.754412	.915385	.839027	.160973	23	
	38	.56832	.82281	.69071	1.44778	.754595	.915297	.839297	.160703	22	
.65	39	.56856	.82264	.69114	1.44688	.754778	.915210	.839568	.160432	21	.35
	40	.56880	.82248	.69157	1.44598	.754960	.915123	.839838	.160162	20	
	41	.56904	.82231	.69200	1.44508	.755143	.915035	.840108	.159892	19	
.70	42	.56928	.82214	.69243	1.44418	.755326	.914948	.840378	.159622	18	.30
	43	.56952	.82198	.69286	1.44329	.755508	.914860	.840648	.159352	17	
	44	.56976	.82181	.69329	1.44239	.755690	.914773	.840917	.159083	16	
.75	45	.57000	.82165	.69372	1.44149	.755872	.914685	.841187	.158813	15	.25
	46	.57024	.82148	.69416	1.44060	.756054	.914598	.841457	.158543	14	
	47	.57047	.82132	.69459	1.43970	.756236	.914510	.841727	.158273	13	
.80	48	.57071	.82115	.69502	1.43881	.756418	.914422	.841996	.158004	12	.20
	49	.57095	.82098	.69545	1.43792	.756600	.914334	.842266	.157734	11	
	50	.57119	.82082	.69588	1.43703	.756782	.914246	.842535	.157465	10	
.85	51	.57143	.82065	.69631	1.43614	.756963	.914158	.842805	.157195	9	.15
	52	.57167	.82048	.69675	1.43525	.757144	.914070	.843074	.156926	8	
	53	.57191	.82032	.69718	1.43436	.757326	.913982	.843343	.156657	7	
.90	54	.57215	.82015	.69761	1.43347	.757507	.913894	.843612	.156388	6	.10
	55	.57238	.81999	.69804	1.43258	.757688	.913806	.843882	.156118	5	
	56	.57262	.81982	.69847	1.43169	.757869	.913718	.844151	.155849	4	
.95	57	.57286	.81965	.69891	1.43080	.758050	.913630	.844420	.155580	3	.05
	58	.57310	.81949	.69934	1.42992	.758230	.913541	.844689	.155311	2	
	59	.57334	.81932	.69977	1.42903	.758411	.913453	.844958	.155042	1	
1.00	60	.57358	.81915	.70021	1.42815	9.758591	9.913365	9.845227	10.154773	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values Sin	Cos	Tan	Cot	Common Logarithms Sin	Cos	Tan	Cot	Minutes	Decimals
.00	0	.57358	.81915	.70021	1.42815	9.758591	9.913365	9.845227	10.154773	60	1.00
	1	.57381	.81899	.70064	1.42726	.758772	.913276	.845496	.154504	59	
	2	.57405	.81882	.70170	1.42638	.758952	.913187	.845764	.154236	58	
.05	3	.57429	.81865	.70151	1.42550	.759132	.913099	.846033	.153967	57	.95
	4	.57453	.81848	.70194	1.42462	.759312	.913010	.846302	.153698	56	
	5	.57477	.81832	.70238	1.42374	.759492	.912922	.846570	.153430	55	
.10	6	.57501	.81815	.70281	1.42286	.759672	.912833	.846839	.153161	54	.90
	7	.57524	.81798	.70325	1.42198	.759852	.912744	.847108	.152892	53	
	8	.57548	.81782	.70368	1.42110	.760031	.912655	.847376	.152624	52	
.15	9	.57572	.81765	.70412	1.42022	.760211	.912566	.847644	.152356	51	.85
	10	.57596	.81748	.70455	1.41934	.760390	.912477	.847913	.152087	50	
	11	.57619	.81731	.70499	1.41847	.760569	.912388	.848181	.151819	49	
.20	12	.57643	.81714	.70542	1.41759	.760748	.912299	.848449	.151551	48	.80
	13	.57667	.81698	.70586	1.41672	.760927	.912210	.848717	.151283	47	
	14	.57691	.81681	.70629	1.41584	.761106	.912121	.848986	.151014	46	
.25	15	.57715	.81664	.70673	1.41497	.761285	.912031	.849254	.150746	45	.75
	16	.57738	.81647	.70717	1.41409	.761464	.911942	.849522	.150478	44	
	17	.57762	.81631	.70760	1.41322	.761642	.911853	.849790	.150210	43	
.30	18	.57786	.81614	.70804	1.41235	.761821	.911763	.850057	.149943	42	.70
	19	.57810	.81597	.70848	1.41148	.761999	.911674	.850325	.149675	41	
	20	.57833	.81580	.70891	1.41061	.762177	.911584	.850593	.149407	40	
.35	21	.57857	.81563	.70935	1.40974	.762356	.911495	.850861	.149139	39	.65
	22	.57881	.81546	.70979	1.40887	.762534	.911405	.851129	.148871	38	
	23	.57904	.81530	.71023	1.40800	.762712	.911315	.851396	.148604	37	
.40	24	.57928	.81513	.71066	1.40714	.762889	.911226	.851664	.148336	36	.60
	25	.57952	.81496	.71110	1.40627	.763067	.911136	.851931	.148069	35	
	26	.57976	.81479	.71154	1.40540	.763245	.911046	.852199	.147801	34	
.45	27	.57999	.81462	.71198	1.40454	.763422	.910956	.852466	.147534	33	.55
	28	.58023	.81445	.71242	1.40367	.763600	.910866	.852733	.147267	32	
	29	.58047	.81428	.71285	1.40281	.763777	.910776	.853001	.146999	31	
.50	30	.58070	.81412	.71329	1.40195	.763954	.910686	.853268	.146732	30	.50
	31	.58094	.81395	.71373	1.40109	.764131	.910596	.853535	.146465	29	
	32	.58118	.81378	.71417	1.40022	.764308	.910506	.853802	.146198	28	
.55	33	.58141	.81361	.71461	1.39936	.764485	.910415	.854069	.145931	27	.45
	34	.58165	.81344	.71505	1.39850	.764662	.910325	.854336	.145664	26	
	35	.58189	.81327	.71549	1.39764	.764838	.910235	.854603	.145397	25	
.60	36	.58212	.81310	.71593	1.39679	.765015	.910144	.854870	.145130	24	.40
	37	.58236	.81293	.71637	1.39593	.765191	.910054	.855137	.144863	23	
	38	.58260	.81276	.71681	1.39507	.765367	.909963	.855404	.144596	22	
.65	39	.58283	.81259	.71725	1.39421	.765544	.909873	.855671	.144329	21	.35
	40	.58307	.81242	.71769	1.39336	.765720	.909782	.855938	.144062	20	
	41	.58330	.81225	.71813	1.39250	.765896	.909691	.856204	.143796	19	
.70	42	.58354	.81208	.71857	1.39165	.766072	.909601	.856471	.143529	18	.30
	43	.58378	.81191	.71901	1.39079	.766247	.909510	.856737	.143263	17	
	44	.58401	.81174	.71946	1.38994	.766423	.909419	.857004	.142996	16	
.75	45	.58425	.81157	.71990	1.38909	.766598	.909328	.857270	.142730	15	.25
	46	.58449	.81140	.72034	1.38824	.766774	.909237	.857537	.142463	14	
	47	.58472	.81123	.72078	1.38738	.766949	.909146	.857803	.142197	13	
.80	48	.58496	.81106	.72122	1.38653	.767124	.909055	.858069	.141931	12	.20
	49	.58519	.81089	.72167	1.38568	.767300	.908964	.858336	.141664	11	
	50	.58543	.81072	.72211	1.38484	.767475	.908873	.858602	.141398	10	
.85	51	.58567	.81055	.72255	1.38399	.767649	.908781	.858868	.141132	9	.15
	52	.58590	.81038	.72299	1.38314	.767824	.908690	.859134	.140866	8	
	53	.58614	.81021	.72344	1.38229	.767999	.908599	.859400	.140600	7	
.90	54	.58637	.81004	.72388	1.38145	.768173	.908507	.859666	.140334	6	.10
	55	.58661	.80987	.72432	1.38060	.768348	.908416	.859932	.140068	5	
	56	.58684	.80970	.72477	1.37976	.768522	.908324	.860198	.139802	4	
.95	57	.58708	.80953	.72521	1.37891	.768697	.908233	.860464	.139536	3	.05
	58	.58731	.80936	.72565	1.37807	.768871	.908141	.860730	.139270	2	
	59	.58755	.80919	.72610	1.37722	.769045	.908049	.860995	.139005	1	
1.00	60	.58779	.80902	.72654	1.37638	9.769219	9.907958	9.861261	10.138739	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.58779	.80902	.72654	1.37638	9.769219	9.907958	9.861261	10.138739	60	1.00
	1	.58802	.80885	.72699	1.37554	.769393	.907866	.861527	.138473	59	
	2	.58826	.80867	.72743	1.37470	.769566	.907774	.861792	.138208	58	
.05	3	.58849	.80850	.72788	1.37386	.769740	.907682	.862058	.137942	57	.95
	4	.58873	.80833	.72832	1.37302	.769913	.907590	.862323	.137677	56	
	5	.58896	.80816	.72877	1.37218	.770087	.907498	.862589	.137411	55	
.10	6	.58920	.80799	.72921	1.37134	.770260	.907406	.862854	.137146	54	.90
	7	.58943	.80782	.72966	1.37050	.770433	.907314	.863119	.136881	53	
	8	.58967	.80765	.73010	1.36967	.770606	.907222	.863385	.136615	52	
.15	9	.58990	.80748	.73055	1.36883	.770779	.907129	.863650	.136350	51	.85
	10	.59014	.80730	.73100	1.36800	.770952	.907037	.863915	.136085	50	
	11	.59037	.80713	.73144	1.36716	.771125	.906945	.864180	.135820	49	
.20	12	.59061	.80696	.73189	1.36633	.771298	.906852	.864445	.135555	48	.80
	13	.59084	.80679	.73234	1.36549	.771470	.906760	.864710	.135290	47	
	14	.59108	.80662	.73278	1.36466	.771643	.906667	.864975	.135025	46	
.25	15	.59131	.80644	.73323	1.36383	.771815	.906575	.865240	.134760	45	.75
	16	.59154	.80627	.73368	1.36300	.771987	.906482	.865505	.134495	44	
	17	.59178	.80610	.73413	1.36217	.772159	.906389	.865770	.134230	43	
.30	18	.59201	.80593	.73457	1.36134	.772331	.906296	.866035	.133965	42	.70
	19	.59225	.80576	.73502	1.36051	.772503	.906204	.866300	.133700	41	
	20	.59248	.80558	.73547	1.35968	.772675	.906111	.866564	.133436	40	
.35	21	.59272	.80541	.73592	1.35885	.772847	.906018	.866829	.133171	39	.65
	22	.59295	.80524	.73637	1.35802	.773018	.905925	.867094	.132906	38	
	23	.59318	.80507	.73681	1.35719	.773190	.905832	.867358	.132642	37	
.40	24	.59342	.80489	.73726	1.35637	.773361	.905739	.867623	.132377	36	.60
	25	.59365	.80472	.73771	1.35554	.773533	.905645	.867887	.132113	35	
	26	.59389	.80455	.73816	1.35472	.773704	.905552	.868152	.131848	34	
.45	27	.59412	.80438	.73861	1.35389	.773875	.905459	.868416	.131584	33	.55
	28	.59436	.80420	.73906	1.35307	.774046	.905366	.868680	.131320	32	
	29	.59459	.80403	.73951	1.35224	.774217	.905272	.868945	.131055	31	
.50	30	.59482	.80386	.73996	1.35142	.774388	.905179	.869209	.130791	30	.50
	31	.59506	.80368	.74041	1.35060	.774558	.905085	.869473	.130527	29	
	32	.59529	.80351	.74086	1.34978	.774729	.904992	.869737	.130263	28	
.55	33	.59552	.80334	.74131	1.34896	.774899	.904898	.870001	.129999	27	.45
	34	.59576	.80316	.74176	1.34814	.775070	.904804	.870265	.129735	26	
	35	.59599	.80299	.74221	1.34732	.775240	.904711	.870529	.129471	25	
.60	36	.59622	.80282	.74267	1.34650	.775410	.904617	.870793	.129207	24	.40
	37	.59646	.80264	.74312	1.34568	.775580	.904523	.871057	.128943	23	
	38	.59669	.80247	.74357	1.34487	.775750	.904429	.871321	.128679	22	
.65	39	.59693	.80230	.74402	1.34405	.775920	.904335	.871585	.128415	21	.35
	40	.59716	.80212	.74447	1.34323	.776090	.904241	.871849	.128151	20	
	41	.59739	.80195	.74492	1.34242	.776259	.904147	.872112	.127888	19	
.70	42	.59763	.80178	.74538	1.34160	.776429	.904053	.872376	.127624	18	.30
	43	.59786	.80160	.74583	1.34079	.776598	.903959	.872640	.127360	17	
	44	.59809	.80143	.74628	1.33998	.776768	.903864	.872903	.127097	16	
.75	45	.59832	.80125	.74674	1.33916	.776937	.903770	.873167	.126833	15	.25
	46	.59856	.80108	.74719	1.33835	.777106	.903676	.873430	.126570	14	
	47	.59879	.80091	.74764	1.33754	.777275	.903581	.873694	.126306	13	
.80	48	.59902	.80073	.74810	1.33673	.777444	.903487	.873957	.126043	12	.20
	49	.59926	.80056	.74855	1.33592	.777613	.903392	.874220	.125780	11	
	50	.59949	.80038	.74900	1.33511	.777781	.903298	.874484	.125516	10	
.85	51	.59972	.80021	.74946	1.33430	.777950	.903203	.874747	.125253	9	.15
	52	.59995	.80003	.74991	1.33349	.778119	.903108	.875010	.124990	8	
	53	.60019	.79986	.75037	1.33268	.778287	.903014	.875273	.124727	7	
.90	54	.60042	.79968	.75082	1.33187	.778455	.902919	.875537	.124463	6	.10
	55	.60065	.79951	.75128	1.33107	.778624	.902824	.875800	.124200	5	
	56	.60089	.79934	.75173	1.33026	.778792	.902729	.876063	.123937	4	
.95	57	.60112	.79916	.75219	1.32946	.778960	.902634	.876326	.123674	3	.05
	58	.60135	.79899	.75264	1.32865	.779128	.902539	.876589	.123411	2	
	59	.60158	.79881	.75310	1.32785	.779295	.902444	.876852	.123148	1	
1.00	60	.60182	.79864	.75355	1.32704	9.779463	9.902349	9.877114	10.122886	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values Sin	Cos	Tan	Cot	Common Logarithms Sin	Cos	Tan	Cot	Minutes	Decimals
.00	0	.60182	.79864	.75355	1.32704	9.779463	9.902349	9.877114	10.122335	60	1.00
	1	.60205	.79846	.75401	1.32624	.779631	.902253	.877377	.122623	59	
	2	.60228	.79829	.75447	1.32544	.779798	.902158	.877640	.122360	58	
.05	3	.60251	.79811	.75492	1.32464	.779966	.902063	.877903	.122097	57	.95
	4	.60274	.79793	.75538	1.32384	.780133	.901967	.878165	.121835	56	
	5	.60298	.79776	.75584	1.32304	.780300	.901872	.878428	.121572	55	
.10	6	.60321	.79758	.75629	1.32224	.780467	.901776	.878691	.121309	54	.90
	7	.60344	.79741	.75675	1.32144	.780634	.901681	.878953	.121047	53	
	8	.60367	.79723	.75721	1.32064	.780801	.901585	.879216	.120784	52	
.15	9	.60390	.79706	.75767	1.31984	.780968	.901490	.879478	.120522	51	.85
	10	.60414	.79688	.75812	1.31904	.781134	.901394	.879741	.120259	50	
	11	.60437	.79671	.75858	1.31825	.781301	.901298	.880003	.119997	49	
.20	12	.60460	.79653	.75904	1.31745	.781468	.901202	.880265	.119735	48	.80
	13	.60483	.79635	.75950	1.31666	.781634	.901106	.880528	.119472	47	
	14	.60506	.79618	.75996	1.31586	.781800	.901010	.880790	.119210	46	
.25	15	.60529	.79600	.76042	1.31507	.781966	.900914	.881052	.118948	45	.75
	16	.60553	.79583	.76088	1.31427	.782132	.900818	.881314	.118686	44	
	17	.60576	.79565	.76134	1.31348	.782298	.900722	.881577	.118423	43	
.30	18	.60599	.79547	.76180	1.31269	.782464	.900626	.881839	.118161	42	.70
	19	.60622	.79530	.76226	1.31190	.782630	.900529	.882101	.117899	41	
	20	.60645	.79512	.76272	1.31110	.782796	.900433	.882363	.117637	40	
.35	21	.60668	.79494	.76318	1.31031	.782961	.900337	.882625	.117375	39	.65
	22	.60691	.79477	.76364	1.30952	.783127	.900240	.882887	.117113	38	
	23	.60714	.79459	.76410	1.30873	.783292	.900144	.883148	.116852	37	
.40	24	.60738	.79441	.76456	1.30795	.783458	.900047	.883410	.116590	36	.60
	25	.60761	.79424	.76502	1.30716	.783623	.899951	.883672	.116328	35	
	26	.60784	.79406	.76548	1.30637	.783788	.899854	.883934	.116066	34	
.45	27	.60807	.79388	.76594	1.30558	.783953	.899757	.884196	.115804	33	.55
	28	.60830	.79371	.76640	1.30480	.784118	.899660	.884457	.115543	32	
	29	.60853	.79353	.76686	1.30401	.784282	.899564	.884719	.115281	31	
.50	30	.60876	.79335	.76733	1.30323	.784447	.899467	.884980	.115020	30	.50
	31	.60899	.79318	.76779	1.30244	.784612	.899370	.885242	.114758	29	
	32	.60922	.79300	.76825	1.30166	.784776	.899273	.885504	.114496	28	
.55	33	.60945	.79282	.76871	1.30087	.784941	.899176	.885765	.114235	27	.45
	34	.60968	.79264	.76918	1.30009	.785105	.899078	.886026	.113974	26	
	35	.60991	.79247	.76964	1.29931	.785269	.898981	.886288	.113712	25	
.60	36	.61015	.79229	.77010	1.29853	.785433	.898884	.886549	.113451	24	.40
	37	.61038	.79211	.77057	1.29775	.785597	.898787	.886811	.113189	23	
	38	.61061	.79193	.77103	1.29696	.785761	.898689	.887072	.112928	22	
.65	39	.61084	.79176	.77149	1.29618	.785925	.898592	.887333	.112667	21	.35
	40	.61107	.79158	.77196	1.29541	.786089	.898494	.887594	.112406	20	
	41	.61130	.79140	.77242	1.29463	.786252	.898397	.887855	.112145	19	
.70	42	.61153	.79122	.77289	1.29385	.786416	.898299	.888116	.111884	18	.30
	43	.61176	.79105	.77335	1.29307	.786579	.898202	.888378	.111622	17	
	44	.61199	.79087	.77382	1.29229	.786742	.898104	.888639	.111361	16	
.75	45	.61222	.79069	.77428	1.29152	.786906	.898006	.888900	.111100	15	.25
	46	.61245	.79051	.77475	1.29074	.787069	.897908	.889161	.110839	14	
	47	.61268	.79033	.77521	1.28997	.787232	.897810	.889421	.110579	13	
.80	48	.61291	.79016	.77568	1.28919	.787395	.897712	.889682	.110318	12	.20
	49	.61314	.78998	.77615	1.28842	.787557	.897614	.889943	.110057	11	
	50	.61337	.78980	.77661	1.28764	.787720	.897516	.890204	.109796	10	
.85	51	.61360	.78962	.77708	1.28687	.787883	.897418	.890465	.109535	9	.15
	52	.61383	.78944	.77754	1.28610	.788045	.897320	.890725	.109275	8	
	53	.61406	.78926	.77801	1.28533	.788208	.897222	.890986	.109014	7	
.90	54	.61429	.78908	.77848	1.28456	.788370	.897123	.891247	.108753	6	.10
	55	.61451	.78891	.77895	1.28379	.788532	.897025	.891507	.108493	5	
	56	.61474	.78873	.77941	1.28302	.788694	.896926	.891768	.108232	4	
.95	57	.61497	.78855	.77988	1.28225	.788856	.896828	.892028	.107972	3	.05
	58	.61520	.78837	.78035	1.28148	.789018	.896729	.892289	.107711	2	
	59	.61543	.78819	.78082	1.28071	.789180	.896631	.892549	.107451	1	
1.00	60	.61566	.78801	.78129	1.27994	9.789342	9.896532	9.892810	10.107190	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values Sin	Cos	Tan	Cot	Common Logarithms Sin	Cos	Tan	Cot	Minutes	Decimals
.00	0	.61566	.78801	.78129	1.27994	9.789313	9.896532	9.892810	10.107190	60	1.00
	1	.61589	.78783	.78175	1.27917	.789504	.896433	.893070	.106930	59	
	2	.61612	.78765	.78222	1.27841	.789665	.896335	.893331	.106669	58	
.05	3	.61635	.78747	.78269	1.27764	.789827	.896236	.893591	.106409	57	.95
	4	.61658	.78729	.78316	1.27688	.789988	.896137	.893851	.106149	56	
	5	.61681	.78711	.78363	1.27611	.790149	.896038	.894111	.105889	55	
.10	6	.61704	.78694	.78410	1.27535	.790310	.895939	.894372	.105628	54	.90
	7	.61726	.78676	.78457	1.27458	.790471	.895840	.894632	.105368	53	
	8	.61749	.78658	.78504	1.27382	.790632	.895741	.894892	.105108	52	
.15	9	.61772	.78640	.78551	1.27305	.790793	.895641	.895152	.104848	51	.85
	10	.61795	.78622	.78598	1.27230	.790954	.895542	.895412	.104588	50	
	11	.61818	.78604	.78645	1.27153	.791115	.895443	.895672	.104328	49	
.20	12	.61841	.78586	.78692	1.27077	.791275	.895343	.895932	.104068	48	.80
	13	.61864	.78568	.78739	1.27001	.791436	.895244	.896192	.103808	47	
	14	.61887	.78550	.78786	1.26925	.791596	.895145	.896452	.103548	46	
.25	15	.61909	.78532	.78834	1.25849	.791757	.895045	.896712	.103288	45	.75
	16	.61932	.78514	.78881	1.26774	.791917	.894945	.896971	.103029	44	
	17	.61955	.78496	.78928	1.26698	.792077	.894846	.897231	.102769	43	
.30	18	.61978	.78478	.78975	1.26622	.792237	.894746	.897491	.102509	42	.70
	19	.62001	.78460	.79022	1.26546	.792397	.894646	.897751	.102249	41	
	20	.62024	.78442	.79070	1.26471	.792557	.894546	.898010	.101990	40	
.35	21	.62046	.78424	.79117	1.26395	.792716	.894446	.898270	.101730	39	.65
	22	.62069	.78405	.79164	1.26319	.792876	.894346	.898530	.101470	38	
	23	.62092	.78387	.79212	1.26244	.793035	.894246	.898789	.101211	37	
.40	24	.62115	.78369	.79259	1.26169	.793195	.894146	.899049	.100951	36	.60
	25	.62138	.78351	.79306	1.26093	.793354	.894046	.899308	.100692	35	
	26	.62160	.78333	.79354	1.26018	.793514	.893946	.899568	.100432	34	
.45	27	.62183	.78315	.79401	1.25943	.793673	.893846	.899827	.100173	33	.55
	28	.62206	.78297	.79449	1.25867	.793832	.893745	.900087	.099913	32	
	29	.62229	.78279	.79496	1.25792	.793991	.893645	.900346	.099654	31	
.50	30	.62251	.78261	.79544	1.25717	.794150	.893544	.900605	.099395	30	.50
	31	.62274	.78243	.79591	1.25642	.794308	.893444	.900864	.099136	29	
	32	.62297	.78225	.79639	1.25567	.794467	.893343	.901124	.098876	28	
.55	33	.62320	.78206	.79686	1.25492	.794626	.893243	.901383	.098617	27	.45
	34	.62342	.78188	.79734	1.25417	.794784	.893142	.901642	.098358	26	
	35	.62365	.78170	.79781	1.25343	.794942	.893041	.901901	.098099	25	
.60	36	.62388	.78152	.79829	1.25268	.795101	.892940	.902160	.097840	24	.40
	37	.62411	.78134	.79877	1.25193	.795259	.892839	.902420	.097580	23	
	38	.62433	.78116	.79924	1.25118	.795417	.892739	.902679	.097321	22	
.65	39	.62456	.78098	.79972	1.25044	.795575	.892638	.902938	.097062	21	.35
	40	.62479	.78079	.80020	1.24969	.795733	.892536	.903197	.096803	20	
	41	.62502	.78061	.80067	1.24895	.795891	.892435	.903456	.096544	19	
.70	42	.62524	.78043	.80115	1.24820	.796049	.892334	.903714	.096286	18	.30
	43	.62547	.78025	.80163	1.24746	.796206	.892233	.903973	.096027	17	
	44	.62570	.78007	.80211	1.24672	.796364	.892132	.904232	.095768	16	
.75	45	.62592	.77988	.80258	1.24597	.796521	.892030	.904491	.095509	15	.25
	46	.62615	.77970	.80306	1.24523	.796679	.891929	.904750	.095250	14	
	47	.62638	.77952	.80354	1.24449	.796836	.891827	.905008	.094992	13	
.80	48	.62660	.77934	.80402	1.24375	.796993	.891726	.905267	.094733	12	.20
	49	.62683	.77916	.80450	1.24301	.797150	.891624	.905526	.094474	11	
	50	.62706	.77897	.80498	1.24227	.797307	.891523	.905785	.094215	10	
.85	51	.62728	.77879	.80546	1.24153	.797464	.891421	.906043	.093957	9	.15
	52	.62751	.77861	.80594	1.24079	.797621	.891319	.906302	.093698	8	
	53	.62774	.77843	.80642	1.24005	.797777	.891217	.906560	.093440	7	
.90	54	.62796	.77824	.80690	1.23931	.797934	.891115	.906819	.093181	6	.10
	55	.62819	.77806	.80738	1.23858	.798091	.891013	.907077	.092923	5	
	56	.62842	.77788	.80786	1.23784	.798247	.890911	.907336	.092664	4	
.95	57	.62864	.77769	.80834	1.23710	.798403	.890809	.907594	.092406	3	.05
	58	.62887	.77751	.80882	1.23637	.798560	.890707	.907853	.092147	2	
	59	.62909	.77733	.80930	1.23563	.798716	.890605	.908111	.091889	1	
1.00	60	.62932	.77715	.80978	1.23490	9.798872	9.890503	9.908369	10.091631	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.62932	.77715	.80973	1.23490	9.798872	9.890503	9.908369	10.091631	60	1.00
	1	.62955	.77696	.81027	1.23416	.799028	.890400	.908623	.091372	59	
	2	.62977	.77678	.81075	1.23343	.799184	.890298	.908886	.091114	58	
.05	3	.63000	.77660	.81123	1.23270	.799339	.890195	.909144	.090856	57	.95
	4	.63022	.77641	.81171	1.23196	.799495	.890093	.909402	.090598	56	
	5	.63045	.77623	.81220	1.23123	.799651	.889990	.909660	.090340	55	
.10	6	.63068	.77605	.81268	1.23050	.799806	.889888	.909918	.090082	54	.90
	7	.63090	.77586	.81316	1.22977	.799962	.889785	.910177	.089823	53	
	8	.63113	.77568	.81364	1.22904	.800117	.889682	.910435	.089565	52	
.15	9	.63135	.77550	.81413	1.22831	.800272	.889579	.910693	.089307	51	.85
	10	.63158	.77531	.81461	1.22758	.800427	.889477	.910951	.089049	50	
	11	.63180	.77513	.81510	1.22685	.800582	.889374	.911209	.088791	49	
.20	12	.63203	.77494	.81558	1.22612	.800737	.889271	.911467	.088533	48	.80
	13	.63225	.77476	.81606	1.22539	.800892	.889168	.911725	.088275	47	
	14	.63248	.77458	.81655	1.22467	.801047	.889064	.911982	.088018	46	
.25	15	.63271	.77439	.81703	1.22394	.801201	.888961	.912240	.087760	45	.75
	16	.63293	.77421	.81752	1.22321	.801356	.888858	.912498	.087502	44	
	17	.63316	.77402	.81800	1.22249	.801511	.888755	.912756	.087244	43	
.30	18	.63338	.77384	.81849	1.22176	.801665	.888651	.913014	.086986	42	.70
	19	.63361	.77366	.81898	1.22104	.801819	.888548	.913271	.086729	41	
	20	.63383	.77347	.81946	1.22031	.801973	.888444	.913529	.086471	40	
.35	21	.63406	.77329	.81995	1.21959	.802128	.888341	.913787	.086213	39	.65
	22	.63428	.77310	.82044	1.21886	.802282	.888237	.914044	.085956	38	
	23	.63451	.77292	.82092	1.21814	.802436	.888134	.914302	.085698	37	
.40	24	.63473	.77273	.82141	1.21742	.802589	.888030	.914560	.085440	36	.60
	25	.63496	.77255	.82190	1.21670	.802743	.887926	.914817	.085183	35	
	26	.63518	.77236	.82238	1.21598	.802897	.887822	.915075	.084925	34	
.45	27	.63540	.77218	.82287	1.21526	.803050	.887718	.915332	.084668	33	.55
	28	.63563	.77199	.82336	1.21454	.803204	.887614	.915590	.084410	32	
	29	.63585	.77181	.82385	1.21382	.803357	.887510	.915847	.084153	31	
.50	30	.63608	.77162	.82434	1.21310	.803511	.887406	.916104	.083896	30	.50
	31	.63630	.77144	.82483	1.21238	.803664	.887302	.916362	.083638	29	
	32	.63653	.77125	.82531	1.21166	.803817	.887198	.916619	.083381	28	
.55	33	.63675	.77107	.82580	1.21094	.803970	.887093	.916877	.083123	27	.45
	34	.63698	.77088	.82629	1.21023	.804123	.886989	.917134	.082866	26	
	35	.63720	.77070	.82678	1.20951	.804276	.886885	.917391	.082609	25	
.60	36	.63742	.77051	.82727	1.20879	.804428	.886780	.917648	.082352	24	.40
	37	.63765	.77033	.82776	1.20808	.804581	.886676	.917906	.082094	23	
	38	.63787	.77014	.82825	1.20736	.804734	.886571	.918163	.081837	22	
.65	39	.63810	.76996	.82874	1.20665	.804886	.886466	.918420	.081580	21	.35
	40	.63832	.76977	.82923	1.20593	.805039	.886362	.918677	.081323	20	
	41	.63854	.76959	.82972	1.20522	.805191	.886257	.918934	.081066	19	
.70	42	.63877	.76940	.83022	1.20451	.805343	.886152	.919191	.080809	18	.30
	43	.63899	.76921	.83071	1.20379	.805495	.886047	.919448	.080552	17	
	44	.63922	.76903	.83120	1.20308	.805647	.885942	.919705	.080295	16	
.75	45	.63944	.76884	.83169	1.20237	.805799	.885837	.919962	.080038	15	.25
	46	.63966	.76866	.83218	1.20166	.805951	.885732	.920219	.079781	14	
	47	.63989	.76847	.83268	1.20095	.806103	.885627	.920476	.079524	13	
.80	48	.64011	.76828	.83317	1.20024	.806254	.885522	.920733	.079267	12	.20
	49	.64033	.76810	.83366	1.19953	.806406	.885416	.920990	.079010	11	
	50	.64056	.76791	.83415	1.19882	.806557	.885311	.921247	.078753	10	
.85	51	.64078	.76772	.83465	1.19811	.806709	.885205	.921503	.078497	9	.15
	52	.64100	.76754	.83514	1.19740	.806860	.885100	.921760	.078240	8	
	53	.64123	.76735	.83564	1.19669	.807011	.884994	.922017	.077983	7	
.90	54	.64145	.76717	.83613	1.19599	.807163	.884889	.922274	.077726	6	.10
	55	.64167	.76698	.83662	1.19528	.807314	.884783	.922530	.077470	5	
	56	.64190	.76679	.83712	1.19457	.807465	.884677	.922787	.077213	4	
.95	57	.64212	.76661	.83761	1.19387	.807615	.884572	.923044	.076956	3	.05
	58	.64234	.76642	.83811	1.19316	.807766	.884466	.923300	.076700	2	
	59	.64256	.76623	.83860	1.19246	.807917	.884360	.923557	.076443	1	
1.00	60	.64279	.76604	.83910	1.19175	9.808067	9.884254	9.923814	10.076186	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

VALUES AND LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	**0**	**.64279**	**.76604**	**.83910**	**1.19175**	**9.808067**	**9.884254**	**9.923814**	**10.076186**	**60**	**1.00**
	1	.64301	.76586	.83960	1.19105	.808218	.884148	.924070	.075930	59	
	2	.64323	.76567	.84009	1.19035	.808368	.884042	.924327	.075673	58	
.05	**3**	**.64346**	**.76548**	**.84059**	**1.18964**	**.808519**	**.883936**	**.924583**	**.075417**	**57**	**.95**
	4	.64368	.76530	.84108	1.18894	.808669	.883829	.924840	.075160	56	
	5	.64390	.76511	.84158	1.18824	.808819	.883723	.925096	.074904	55	
.10	**6**	**.64412**	**.76492**	**.84208**	**1.18754**	**.808969**	**.883617**	**.925352**	**.074648**	**54**	**.90**
	7	.64435	.76473	.84258	1.18684	.809119	.883510	.925609	.074391	53	
	8	.64457	.76455	.84307	1.18614	.809269	.883404	.925865	.074135	52	
.15	**9**	**.64479**	**.76436**	**.84357**	**1.18544**	**.809419**	**.883297**	**.926122**	**.073878**	**51**	**.85**
	10	.64501	.76417	.84407	1.18474	.809569	.883191	.926378	.073622	50	
	11	.64524	.76398	.84457	1.18404	.809718	.883084	.926634	.073366	49	
.20	**12**	**.64546**	**.76380**	**.84507**	**1.18334**	**.809868**	**.882977**	**.926890**	**.073110**	**48**	**.80**
	13	.64568	.76361	.84556	1.18264	.810017	.882871	.927147	.072853	47	
	14	.64590	.76342	.84606	1.18194	.810167	.882764	.927403	.072597	46	
.25	**15**	**.64612**	**.76323**	**.84656**	**1.18125**	**.810316**	**.882657**	**.927659**	**.072341**	**45**	**.75**
	16	.64635	.76304	.84706	1.18055	.810465	.882550	.927915	.072085	44	
	17	.64657	.76286	.84756	1.17986	.810614	.882443	.928171	.071829	43	
.30	**18**	**.64679**	**.76267**	**.84806**	**1.17916**	**.810763**	**.882336**	**.928427**	**.071573**	**42**	**.70**
	19	.64701	.76248	.84856	1.17846	.810912	.882229	.928684	.071316	41	
	20	.64723	.76229	.84906	1.17777	.811061	.882121	.928940	.071060	40	
.35	**21**	**.64746**	**.76210**	**.84956**	**1.17708**	**.811210**	**.882014**	**.929196**	**.070804**	**39**	**.65**
	22	.64768	.76192	.85006	1.17638	.811358	.881907	.929452	.070548	38	
	23	.64790	.76173	.85057	1.17569	.811507	.881799	.929708	.070292	37	
.40	**24**	**.64812**	**.76154**	**.85107**	**1.17500**	**.811655**	**.881692**	**.929964**	**.070036**	**36**	**.60**
	25	.64834	.76135	.85157	1.17430	.811804	.881584	.930220	.069780	35	
	26	.64856	.76116	.85207	1.17361	.811952	.881477	.930475	.069525	34	
.45	**27**	**.64878**	**.76097**	**.85257**	**1.17292**	**.812100**	**.881369**	**.930731**	**.069269**	**33**	**.55**
	28	.64901	.76078	.85308	1.17223	.812248	.881261	.930987	.069013	32	
	29	.64923	.76059	.85358	1.17154	.812396	.881153	.931243	.068757	31	
.50	**30**	**.64945**	**.76041**	**.85408**	**1.17085**	**.812544**	**.881046**	**.931499**	**.068501**	**30**	**.50**
	31	.64967	.76022	.85458	1.17016	.812692	.880938	.931755	.068245	29	
	32	.64989	.76003	.85509	1.16947	.812840	.880830	.932010	.067990	28	
.55	**33**	**.65011**	**.75984**	**.85559**	**1.16878**	**.812988**	**.880722**	**.932266**	**.067734**	**27**	**.45**
	34	.65033	.75965	.85609	1.16809	.813135	.880613	.932522	.067478	26	
	35	.65055	.75946	.85660	1.16741	.813283	.880505	.932778	.067222	25	
.60	**36**	**.65077**	**.75927**	**.85710**	**1.16672**	**.813430**	**.880397**	**.933033**	**.066967**	**24**	**.40**
	37	.65100	.75908	.85761	1.16603	.813578	.880289	.933289	.066711	23	
	38	.65122	.75889	.85811	1.16535	.813725	.880180	.933545	.066455	22	
.65	**39**	**.65144**	**.75870**	**.85862**	**1.16466**	**.813872**	**.880072**	**.933800**	**.066200**	**21**	**.35**
	40	.65166	.75851	.85912	1.16398	.814019	.879963	.934056	.065944	20	
	41	.65188	.75832	.85963	1.16329	.814166	.879855	.934311	.065689	19	
.70	**42**	**.65210**	**.75813**	**.86014**	**1.16261**	**.814313**	**.879746**	**.934567**	**.065433**	**18**	**.30**
	43	.65232	.75794	.86064	1.16192	.814460	.879637	.934822	.065178	17	
	44	.65254	.75775	.86115	1.16124	.814607	.879529	.935078	.064922	16	
.75	**45**	**.65276**	**.75756**	**.86166**	**1.16056**	**.814753**	**.879420**	**.935333**	**.064667**	**15**	**.25**
	46	.65298	.75738	.86216	1.15987	.814900	.879311	.935589	.064411	14	
	47	.65320	.75719	.86267	1.15919	.815046	.879202	.935844	.064156	13	
.80	**48**	**.65342**	**.75700**	**.86318**	**1.15851**	**.815193**	**.879093**	**.936100**	**.063900**	**12**	**.20**
	49	.65364	.75680	.86368	1.15783	.815339	.878984	.936355	.063645	11	
	50	.65386	.75661	.86419	1.15715	.815485	.878875	.936611	.063389	10	
.85	**51**	**.65408**	**.75642**	**.86470**	**1.15647**	**.815632**	**.878766**	**.936866**	**.063134**	**9**	**.15**
	52	.65430	.75623	.86521	1.15579	.815778	.878656	.937121	.062879	8	
	53	.65452	.75604	.86572	1.15511	.815924	.878547	.937377	.062623	7	
.90	**54**	**.65474**	**.75585**	**.86623**	**1.15443**	**.816069**	**.878438**	**.937632**	**.062368**	**6**	**.10**
	55	.65496	.75566	.86674	1.15375	.816215	.878328	.937887	.062113	5	
	56	.65518	.75547	.86725	1.15308	.816361	.878219	.938142	.061858	4	
.95	**57**	**.65540**	**.75528**	**.86776**	**1.15240**	**.816507**	**.878109**	**.938398**	**.061602**	**3**	**.05**
	58	.65562	.75509	.86827	1.15172	.816652	.877999	.938653	.061347	2	
	59	.65584	.75490	.86878	1.15104	.816798	.877890	.938908	.061092	1	
1.00	**60**	**.65606**	**.75471**	**.86929**	**1.15037**	**9.816943**	**9.877780**	**9.939163**	**10.060837**	**0**	**.00**
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values Sin	Cos	Tan	Cot	Common Logarithms Sin	Cos	Tan	Cot	Minutes	Decimals
.00	0	.65603	.75471	.86929	1.15037	9.816943	9.877780	9.939163	10.060837	60	1.00
	1	.65628	.75452	.86980	1.14969	.817008	.877670	.939418	.060582	59	
	2	.65650	.75433	.87031	1.14902	.817233	.877560	.939673	.060327	58	
.05	3	.65672	.75414	.87082	1.14834	.817379	.877450	.939928	.060072	57	.95
	4	.65694	.75395	.87133	1.14767	.817524	.877340	.940183	.059817	56	
	5	.65716	.75375	.87184	1.14699	.817668	.877230	.940439	.059561	55	
.10	6	.65738	.75356	.87236	1.14632	.817813	.877120	.940694	.059306	54	.90
	7	.65759	.75337	.87287	1.14565	.817958	.877010	.940949	.059051	53	
	8	.65781	.75318	.87338	1.14498	.818103	.876899	.941204	.058796	52	
.15	9	.65803	.75299	.87389	1.14430	.818247	.876789	.941459	.058541	51	.85
	10	.65825	.75280	.87441	1.14363	.818392	.876678	.941713	.058287	50	
	11	.65847	.75261	.87492	1.14296	.818536	.876568	.941968	.058032	49	
.20	12	.65869	.75241	.87543	1.14229	.818681	.876457	.942223	.057777	48	.80
	13	.65891	.75222	.87595	1.14162	.818825	.876347	.942478	.057522	47	
	14	.65913	.75203	.87646	1.14095	.818969	.876236	.942733	.057267	46	
.25	15	.65935	.75184	.87698	1.14028	.819113	.876125	.942988	.057012	45	.75
	16	.65956	.75165	.87749	1.13961	.819257	.876014	.943243	.056757	44	
	17	.65978	.75146	.87801	1.13894	.819401	.875904	.943498	.056502	43	
.30	18	.66000	.75126	.87852	1.13828	.819545	.875793	.943752	.056248	42	.70
	19	.66022	.75107	.87904	1.13761	.819689	.875682	.944007	.055993	41	
	20	.66044	.75088	.87955	1.13694	.819832	.875571	.944262	.055738	40	
.35	21	.66065	.75069	.88007	1.13627	.819976	.875459	.944517	.055483	39	.65
	22	.66088	.75050	.88059	1.13561	.820120	.875348	.944771	.055229	38	
	23	.66109	.75030	.88110	1.13494	.820263	.875237	.945026	.054974	37	
.40	24	.66131	.75011	.88162	1.13428	.820406	.875125	.945281	.054719	36	.60
	25	.66153	.74992	.88214	1.13361	.820550	.875014	.945535	.054465	35	
	26	.66175	.74973	.88265	1.13295	.820693	.874903	.945790	.054210	34	
.45	27	.66197	.74953	.88317	1.13228	.820836	.874791	.946045	.053955	33	.55
	28	.66218	.74934	.88369	1.13162	.820979	.874680	.946299	.053701	32	
	29	.66240	.74915	.88421	1.13096	.821122	.874568	.946554	.053446	31	
.50	30	.66262	.74896	.88473	1.13029	.821265	.874456	.946808	.053192	30	.50
	31	.66284	.74876	.88524	1.12963	.821407	.874344	.947063	.052937	29	
	32	.66306	.74857	.88576	1.12897	.821550	.874232	.947318	.052682	28	
.55	33	.66327	.74838	.88628	1.12831	.821693	.874121	.947572	.052428	27	.45
	34	.66349	.74818	.88680	1.12765	.821835	.874009	.947827	.052173	26	
	35	.66371	.74799	.88732	1.12699	.821977	.873896	.948081	.051919	25	
.60	36	.66393	.74780	.88784	1.12633	.822120	.873784	.948335	.051665	24	.40
	37	.66414	.74760	.88836	1.12567	.822262	.873672	.948590	.051410	23	
	38	.66436	.74741	.88888	1.12501	.822404	.873560	.948844	.051156	22	
.65	39	.66458	.74722	.88940	1.12435	.822546	.873448	.949099	.050901	21	.35
	40	.66480	.74703	.88992	1.12369	.822688	.873335	.949353	.050647	20	
	41	.66501	.74683	.89045	1.12303	.822830	.873223	.949608	.050392	19	
.70	42	.66523	.74664	.89097	1.12238	.822972	.873110	.949862	.050138	18	.30
	43	.66545	.74644	.89149	1.12172	.823114	.872998	.950116	.049884	17	
	44	.66566	.74625	.89201	1.12106	.823255	.872885	.950371	.049629	16	
.75	45	.66588	.74606	.89253	1.12041	.823397	.872772	.950625	.049375	15	.25
	46	.66610	.74586	.89306	1.11975	.823539	.872659	.950879	.049121	14	
	47	.66632	.74567	.89358	1.11909	.823680	.872547	.951133	.048867	13	
.80	48	.66653	.74548	.89410	1.11844	.823821	.872434	.951388	.048612	12	.20
	49	.66675	.74528	.89463	1.11778	.823963	.872321	.951642	.048358	11	
	50	.66697	.74509	.89515	1.11713	.824104	.872208	.951896	.048104	10	
.85	51	.66718	.74489	.89567	1.11648	.824245	.872095	.952150	.047850	9	.15
	52	.66740	.74470	.89620	1.11582	.824386	.871981	.952405	.047595	8	
	53	.66762	.74451	.89672	1.11517	.824527	.871868	.952659	.047341	7	
.90	54	.66783	.74431	.89725	1.11452	.824668	.871755	.952913	.047087	6	.10
	55	.66805	.74412	.89777	1.11387	.824808	.871641	.953167	.046833	5	
	56	.66827	.74392	.89830	1.11321	.824949	.871528	.953421	.046579	4	
.95	57	.66848	.74373	.89883	1.11256	.825090	.871414	.953675	.046325	3	.05
	58	.66870	.74353	.89935	1.11191	.825230	.871301	.953929	.046071	2	
	59	.66891	.74334	.89988	1.11126	.825371	.871187	.954183	.045817	1	
1.00	60	.66913	.74314	.90040	1.11061	9.825511	9.871073	9.954437	10.045563	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.66913	.74314	.90040	1.11061	9.825511	9.871073	9.954437	10.045563	60	1.00
	1	.66935	.74295	.90093	1.10996	.825651	.870960	.954691	.045309	59	
	2	.66956	.74276	.90146	1.10931	.825791	.870846	.954946	.045054	58	
.05	3	.66978	.74256	.90199	1.10867	.825931	.870732	.955200	.044800	57	.95
	4	.66999	.74237	.90251	1.10802	.826071	.870618	.955454	.044546	56	
	5	.67021	.74217	.90304	1.10737	.826211	.870504	.955708	.044292	55	
.10	6	.67043	.74198	.90357	1.10672	.826351	.870390	.955961	.044039	54	.90
	7	.67064	.74178	.90410	1.10607	.826491	.870276	.956215	.043785	53	
	8	.67086	.74159	.90463	1.10543	.826631	.870161	.956469	.043531	52	
.15	9	.67107	.74139	.90516	1.10478	.826770	.870047	.956723	.043277	51	.85
	10	.67129	.74120	.90569	1.10414	.826910	.869933	.956977	.043023	50	
	11	.67151	.74100	.90621	1.10349	.827049	.869818	.957231	.042769	49	
.20	12	.67172	.74080	.90674	1.10285	.827189	.869704	.957485	.042515	48	.80
	13	.67194	.74061	.90727	1.10220	.827328	.869589	.957739	.042261	47	
	14	.67215	.74041	.90781	1.10156	.827467	.869474	.957993	.042007	46	
.25	15	.67237	.74022	.90834	1.10091	.827606	.869360	.958247	.041753	45	.75
	16	.67258	.74002	.90887	1.10027	.827745	.869245	.958500	.041500	44	
	17	.67280	.73983	.90940	1.09963	.827884	.869130	.958754	.041246	43	
.30	18	.67301	.73963	.90993	1.09899	.828023	.869015	.959008	.040992	42	.70
	19	.67323	.73944	.91046	1.09834	.828162	.868900	.959262	.040738	41	
	20	.67344	.73924	.91099	1.09770	.828301	.868785	.959516	.040484	40	
.35	21	.67366	.73904	.91153	1.09706	.828439	.868670	.959769	.040231	39	.65
	22	.67387	.73885	.91206	1.09642	.828578	.868555	.960023	.039977	38	
	23	.67409	.73865	.91259	1.09578	.828716	.868440	.960277	.039723	37	
.40	24	.67430	.73846	.91313	1.09514	.828855	.868324	.960530	.039470	36	.60
	25	.67452	.73826	.91366	1.09450	.828993	.868209	.960784	.039216	35	
	26	.67473	.73806	.91419	1.09386	.829131	.868093	.961038	.038962	34	
.45	27	.67495	.73787	.91473	1.09322	.829269	.867978	.961292	.038708	33	.55
	28	.67516	.73767	.91526	1.09258	.829407	.867862	.961545	.038455	32	
	29	.67538	.73747	.91580	1.09195	.829545	.867747	.961799	.038201	31	
.50	30	.67559	.73728	.91633	1.09131	.829683	.867631	.962052	.037948	30	.50
	31	.67580	.73708	.91687	1.09067	.829821	.867515	.962306	.037694	29	
	32	.67602	.73688	.91740	1.09003	.829959	.867399	.962560	.037440	28	
.55	33	.67623	.73669	.91794	1.08940	.830097	.867283	.962813	.037187	27	.45
	34	.67645	.73649	.91847	1.08876	.830234	.867167	.963067	.036933	26	
	35	.67666	.73629	.91901	1.08813	.830372	.867051	.963320	.036680	25	
.60	36	.67688	.73610	.91955	1.08749	.830509	.866935	.963574	.036426	24	.40
	37	.67709	.73590	.92008	1.08686	.830646	.866819	.963828	.036172	23	
	38	.67730	.73570	.92062	1.08622	.830784	.866703	.964081	.035919	22	
.65	39	.67752	.73551	.92116	1.08559	.830921	.866586	.964335	.035665	21	.35
	40	.67773	.73531	.92170	1.08496	.831058	.866470	.964588	.035412	20	
	41	.67795	.73511	.92224	1.08432	.831195	.866353	.964842	.035158	19	
.70	42	.67816	.73491	.92277	1.08369	.831332	.866237	.965095	.034905	18	.30
	43	.67837	.73472	.92331	1.08306	.831469	.866120	.965349	.034651	17	
	44	.67859	.73452	.92385	1.08243	.831606	.866004	.965602	.034398	16	
.75	45	.67880	.73432	.92439	1.08179	.831742	.865887	.965855	.034145	15	.25
	46	.67901	.73413	.92493	1.08116	.831879	.865770	.966109	.033891	14	
	47	.67923	.73393	.92547	1.08053	.832015	.865653	.966362	.033638	13	
.80	48	.67944	.73373	.92601	1.07990	.832152	.865536	.966616	.033384	12	.20
	49	.67965	.73353	.92655	1.07927	.832288	.865419	.966869	.033131	11	
	50	.67987	.73333	.92709	1.07864	.832425	.865302	.967123	.032877	10	
.85	51	.68008	.73314	.92763	1.07801	.832561	.865185	.967376	.032624	9	.15
	52	.68029	.73294	.92817	1.07738	.832697	.865068	.967629	.032371	8	
	53	.68051	.73274	.92872	1.07676	.832833	.864950	.967883	.032117	7	
.90	54	.68072	.73254	.92926	1.07613	.832969	.864833	.968136	.031864	6	.10
	55	.68093	.73234	.92980	1.07550	.833105	.864716	.968389	.031611	5	
	56	.68115	.73215	.93034	1.07487	.833241	.864598	.968643	.031357	4	
.95	57	.68136	.73195	.93088	1.07425	.833377	.864481	.968896	.031104	3	.05
	58	.68157	.73175	.93143	1.07362	.833512	.864363	.969149	.030851	2	
	59	.68179	.73155	.93197	1.07299	.833648	.864245	.969403	.030597	1	
1.00	60	.68200	.73135	.93252	1.07237	9.833783	9.864127	9.969656	10.030344	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.68200	.73135	.93252	1.07237	9.833783	9.864127	9.969656	10.030344	60	1.00
	1	.68221	.73116	.93306	1.07174	.833919	.864010	.969909	.030091	59	
	2	.68242	.73096	.93360	1.07112	.834054	.863892	.970162	.029838	58	
.05	3	.68264	.73076	.93415	1.07049	.834189	.863774	.970416	.029584	57	.95
	4	.68285	.73056	.93469	1.06987	.834325	.863656	.970669	.029331	56	
	5	.68306	.73036	.93524	1.06925	.834460	.863538	.970922	.029078	55	
.10	6	.68327	.73016	.93578	1.06862	.834595	.863419	.971175	.028825	54	.90
	7	.68349	.72996	.93633	1.06800	.834730	.863301	.971429	.028571	53	
	8	.68370	.72976	.93688	1.06738	.834865	.863183	.971682	.028318	52	
.15	9	.68391	.72957	.93742	1.06676	.834999	.863064	.971935	.028065	51	.85
	10	.68412	.72937	.93797	1.06613	.835134	.862946	.972188	.027812	50	
	11	.68434	.72917	.93852	1.06551	.835269	.862827	.972441	.027559	49	
.20	12	.68455	.72897	.93906	1.06489	.835403	.862709	.972695	.027305	48	.80
	13	.68476	.72877	.93961	1.06427	.835538	.862590	.972948	.027052	47	
	14	.68497	.72857	.94016	1.06365	.835672	.862471	.973201	.026799	46	
.25	15	.68518	.72837	.94071	1.06303	.835807	.862353	.973454	.026546	45	.75
	16	.68539	.72817	.94125	1.06241	.835941	.862234	.973707	.026293	44	
	17	.68561	.72797	.94180	1.06179	.836075	.862115	.973960	.026040	43	
.30	18	.68582	.72777	.94235	1.06117	.836209	.861996	.974213	.025787	42	.70
	19	.68603	.72757	.94290	1.06056	.836343	.861877	.974466	.025534	41	
	20	.68624	.72737	.94345	1.05994	.836477	.861758	.974720	.025280	40	
.35	21	.68645	.72717	.94400	1.05932	.836611	.861638	.974973	.025027	39	.65
	22	.68666	.72697	.94455	1.05870	.836745	.861519	.975226	.024774	38	
	23	.68688	.72677	.94510	1.05809	.836878	.861400	.975479	.024521	37	
.40	24	.68709	.72657	.94565	1.05747	.837012	.861280	.975732	.024268	36	.60
	25	.68730	.72637	.94620	1.05685	.837146	.861161	.975985	.024015	35	
	26	.68751	.72617	.94676	1.05624	.837279	.861041	.976238	.023762	34	
.45	27	.68772	.72597	.94731	1.05562	.837412	.860922	.976491	.023509	33	.55
	28	.68793	.72577	.94786	1.05501	.837546	.860802	.976744	.023256	32	
	29	.68814	.72557	.94841	1.05439	.837679	.860682	.976997	.023003	31	
.50	30	.68835	.72537	.94896	1.05378	.837812	.860562	.977250	.022750	30	.50
	31	.68857	.72517	.94952	1.05317	.837945	.860442	.977503	.022497	29	
	32	.68878	.72497	.95007	1.05255	.838078	.860322	.977756	.022244	28	
.55	33	.68899	.72477	.95062	1.05194	.838211	.860202	.978009	.021991	27	.45
	34	.68920	.72457	.95118	1.05133	.838344	.860082	.978262	.021738	26	
	35	.68941	.72437	.95173	1.05072	.838477	.859962	.978515	.021485	25	
.60	36	.68962	.72417	.95229	1.05010	.838610	.859842	.978768	.021232	24	.40
	37	.68983	.72397	.95284	1.04949	.838742	.859721	.979021	.020979	23	
	38	.69004	.72377	.95340	1.04888	.838875	.859601	.979274	.020726	22	
.65	39	.69025	.72357	.95395	1.04827	.839007	.859480	.979527	.020473	21	.35
	40	.69046	.72337	.95451	1.04766	.839140	.859360	.979780	.020220	20	
	41	.69067	.72317	.95506	1.04705	.839272	.859239	.980033	.019967	19	
.70	42	.69088	.72297	.95562	1.04644	.839404	.859119	.980286	.019714	18	.30
	43	.69109	.72277	.95618	1.04583	.839536	.858998	.980538	.019462	17	
	44	.69130	.72257	.95673	1.04522	.839668	.858877	.980791	.019209	16	
.75	45	.69151	.72236	.95729	1.04461	.839800	.858756	.981044	.018956	15	.25
	46	.69172	.72216	.95785	1.04401	.839932	.858635	.981297	.018703	14	
	47	.69193	.72196	.95841	1.04340	.840064	.858514	.981550	.018450	13	
.80	48	.69214	.72176	.95897	1.04279	.840196	.858393	.981803	.018197	12	.20
	49	.69235	.72156	.95952	1.04218	.840328	.858272	.982056	.017944	11	
	50	.69256	.72136	.96008	1.04158	.840459	.858151	.982309	.017691	10	
.85	51	.69277	.72116	.96064	1.04097	.840591	.858029	.982562	.017438	9	.15
	52	.69298	.72095	.96120	1.04036	.840722	.857908	.982814	.017186	8	
	53	.69319	.72075	.96176	1.03976	.840854	.857786	.983067	.016933	7	
.90	54	.69340	.72055	.96232	1.03915	.840985	.857665	.983320	.016680	6	.10
	55	.69361	.72035	.96288	1.03855	.841116	.857543	.983573	.016427	5	
	56	.69382	.72015	.96344	1.03794	.841247	.857422	.983826	.016174	4	
.95	57	.69403	.71995	.96400	1.03734	.841378	.857300	.984079	.015921	3	.05
	58	.69424	.71974	.96457	1.03674	.841509	.857178	.984332	.015668	2	
	59	.69445	.71954	.96513	1.03613	.841640	.857056	.984584	.015416	1	
1.00	60	.69466	.71934	.96569	1.03553	9.841771	9.856934	9.984837	10.015163	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

Decimals	Minutes	Natural Values				Common Logarithms				Minutes	Decimals
		Sin	Cos	Tan	Cot	Sin	Cos	Tan	Cot		
.00	0	.69466	.71934	.96569	1.03553	9.841771	9.856934	9.984837	10.015163	60	1.00
	1	.69487	.71914	.96625	1.03493	.841902	.856812	.985090	.014910	59	
	2	.69508	.71894	.96681	1.03433	.842033	.856690	.985343	.014657	58	
.05	3	.69529	.71873	.96738	1.03372	.842163	.856568	.985596	.014404	57	.95
	4	.69549	.71853	.96794	1.03312	.842294	.856446	.985848	.014152	56	
	5	.69570	.71833	.96850	1.03252	.842424	.856323	.986101	.013899	55	
.10	6	.69591	.71813	.96907	1.03192	.842555	.856201	.986354	.013646	54	.90
	7	.69612	.71792	.96963	1.03132	.842685	.856078	.986607	.013393	53	
	8	.69633	.71772	.97020	1.03072	.842815	.855956	.986860	.013140	52	
.15	9	.69654	.71752	.97076	1.03012	.842946	.855833	.987112	.012888	51	.85
	10	.69675	.71732	.97133	1.02952	.843076	.855711	.987365	.012635	50	
	11	.69696	.71711	.97189	1.02892	.843206	.855588	.987618	.012382	49	
.20	12	.69717	.71691	.97246	1.02832	.843336	.855465	.987871	.012129	48	.80
	13	.69737	.71671	.97302	1.02772	.843466	.855342	.988123	.011877	47	
	14	.69758	.71650	.97359	1.02713	.843595	.855219	.988376	.011624	46	
.25	15	.69779	.71630	.97416	1.02653	.843725	.855096	.988629	.011371	45	.75
	16	.69800	.71610	.97472	1.02593	.843855	.854973	.988882	.011118	44	
	17	.69821	.71590	.97529	1.02533	.843984	.854850	.989134	.010866	43	
.30	18	.69842	.71569	.97586	1.02474	.844114	.854727	.989387	.010613	42	.70
	19	.69862	.71549	.97643	1.02414	.844243	.854603	.989640	.010360	41	
	20	.69883	.71529	.97700	1.02355	.844372	.854480	.989893	.010107	40	
.35	21	.69904	.71508	.97756	1.02295	.844502	.854356	.990145	.009855	39	.65
	22	.69925	.71488	.97813	1.02236	.844631	.854233	.990398	.009602	38	
	23	.69946	.71468	.97870	1.02176	.844760	.854109	.990651	.009349	37	
.40	24	.69966	.71447	.97927	1.02117	.844889	.853986	.990903	.009097	36	.60
	25	.69987	.71427	.97984	1.02057	.845018	.853862	.991156	.008844	35	
	26	.70008	.71407	.98041	1.01998	.845147	.853738	.991409	.008591	34	
.45	27	.70029	.71386	.98098	1.01939	.845276	.853614	.991662	.008338	33	.55
	28	.70049	.71366	.98155	1.01879	.845405	.853490	.991914	.008086	32	
	29	.70070	.71345	.98213	1.01820	.845533	.853366	.992167	.007833	31	
.50	30	.70091	.71325	.98270	1.01761	.845662	.853242	.992420	.007580	30	.50
	31	.70112	.71305	.98327	1.01702	.845790	.853118	.992672	.007328	29	
	32	.70132	.71284	.98384	1.01642	.845919	.852994	.992925	.007075	28	
.55	33	.70153	.71264	.98441	1.01583	.846047	.852869	.993178	.006822	27	.45
	34	.70174	.71243	.98499	1.01524	.846175	.852745	.993431	.006569	26	
	35	.70195	.71223	.98556	1.01465	.846304	.852620	.993683	.006317	25	
.60	36	.70215	.71203	.98613	1.01406	.846432	.852496	.993936	.006064	24	.40
	37	.70236	.71182	.98671	1.01347	.846560	.852371	.994189	.005811	23	
	38	.70257	.71162	.98728	1.01288	.846688	.852247	.994441	.005559	22	
.65	39	.70277	.71141	.98786	1.01229	.846816	.852122	.994694	.005306	21	.35
	40	.70298	.71121	.98843	1.01170	.846944	.851997	.994947	.005053	20	
	41	.70319	.71100	.98901	1.01112	.847071	.851872	.995199	.004801	19	
.70	42	.70339	.71080	.98958	1.01053	.847199	.851747	.995452	.004548	18	.30
	43	.70360	.71059	.99016	1.00994	.847327	.851622	.995705	.004295	17	
	44	.70381	.71039	.99073	1.00935	.847454	.851497	.995957	.004043	16	
.75	45	.70401	.71019	.99131	1.00876	.847582	.851372	.996210	.003790	15	.25
	46	.70422	.70998	.99189	1.00818	.847709	.851246	.996463	.003537	14	
	47	.70443	.70978	.99247	1.00759	.847836	.851121	.996715	.003285	13	
.80	48	.70463	.70957	.99304	1.00701	.847964	.850996	.996968	.003032	12	.20
	49	.70484	.70937	.99362	1.00642	.848091	.850870	.997221	.002779	11	
	50	.70505	.70916	.99420	1.00583	.848218	.850745	.997473	.002527	10	
.85	51	.70525	.70896	.99478	1.00525	.848345	.850619	.997726	.002274	9	.15
	52	.70546	.70875	.99536	1.00467	.848472	.850493	.997979	.002021	8	
	53	.70567	.70855	.99594	1.00408	.848599	.850368	.998231	.001769	7	
.90	54	.70587	.70834	.99652	1.00350	.848726	.850242	.998484	.001516	6	.10
	55	.70608	.70813	.99710	1.00291	.848852	.850116	.998737	.001263	5	
	56	.70628	.70793	.99768	1.00233	.848979	.849990	.998989	.001011	4	
.95	57	.70649	.70772	.99826	1.00175	.849106	.849864	.999242	.000758	3	.05
	58	.70670	.70752	.99884	1.00116	.849232	.849738	.999495	.000505	2	
	59	.70690	.70731	.99942	1.00058	.849359	.849611	.999747	.000253	1	
1.00	60	.70711	.70711	1.00000	1.00000	9.849485	9.849485	10.000000	10.000000	0	.00
Decimals	Minutes	Cos	Sin	Cot	Tan	Cos	Sin	Cot	Tan	Minutes	Decimals
		Natural Values				Common Logarithms					

CALCULUS

10. DIFFERENTIAL CALCULUS

Definition of a Function. A variable y is said to be a function of a variable x if the value of y is determined when the value of x is given. In this definition, x is called the independent variable and y the dependent variable. The symbols $F(x)$, $f(x)$, $\phi(x)$, etc., are used to represent various functions of x, while the symbol $f(a)$ represents the value of $f(x)$ when $x = a$.

Limit, Derivative, Differential, Continuity. The *constant* a is said to be the *limit of a variable* x, if, as the variable changes its value, the numerical difference between the variable and constant becomes and remains less than any small positive constant which may be assigned. The symbol $x \to a$ or $\lim x = a$ is used for this definition. An example of a variable becoming equal to its limit is a swinging pendulum finally coming to rest. An illustration of a variable never reaching its limit is a polygon of n sides inscribed in a circle. No matter how large n is taken, the circumference of area of the polygon never equals that of the circle.

x becomes *infinitely large*, $x \to \infty$, means that the value of x becomes *larger* than any assigned *positive* number. $x \to -\infty$ means that the value of x becomes *smaller* than any assigned *negative* number.

Usually a change in x causes a change in y. A change in x is called an *increment* of x and is denoted by Δx. Similarly a change in y is denoted by Δy. If

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

has a *definite* value, it is called the *derivative* of y with respect to x and is denoted by $\frac{dy}{dx}$ or $f'(x)$.

The geometric interpretation of $f'(x)$ is

$$f'(x) = \frac{dy}{dx} = \tan\theta \tag{1}$$

or $f'(x)$ is equal to the slope of the tangent to the curve $y = f(x)$ at the point of contact $P(x, y)$ (Fig. 4).

$$\lim_{PR \to 0} \frac{RQ}{PR} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{dy}{dx} = f'(x) = \tan\theta \tag{2}$$

The differentials of x and y, respectively, are

$$dx = \Delta x$$

$$dy = f'(x)dx$$

Continuity of a Function in an Interval. A function is called *continuous* at $x = b$ if it has a definite value at b and approaches that value as a limit whenever x approaches b as a limit. The notion of continuity at a point suggests that the graph of the function is a smooth curve in the neighborhood of the point. The analytic conditions that $f(x)$ be continuous at b are that $f(b)$ have a definite value and that

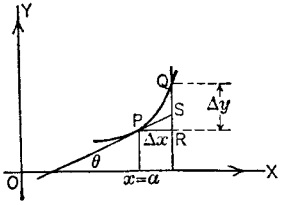


FIG. 4

$$|f(x) - f(b)| < \epsilon \text{ for } |x - b| < \delta(\epsilon) \tag{3}$$

where ϵ is any positive number which can be chosen as small as desired, while $\delta(\epsilon)$ depends on ϵ. The bars outside of $|f(x) - f(b)|$ show that the *absolute value* or value without the algebraic sign is to be taken; thus $|2 - 5| = |5 - 2| = 3$. A function which is continuous at each point of an interval is said to be continuous in that interval. An example of a continuous function is $f(x) = x^2$. The function $\phi(x) = \frac{1}{x - a}$ is continuous for all values of x other than $x = a$, at which point it becomes

infinite. Every differentiable function is continuous, although the reverse is not always true.

If, in the above definition of continuity, the number δ can be chosen the same for all points in the interval, the function is said to be *uniformly continuous* in that interval.

Derivatives of Higher Order. The *derivative of the first derivative of y with respect to x* is called the *second derivative of y with respect to x* and is denoted by

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \quad \text{or} \quad f''(x) \tag{4}$$

$$\frac{d^3y}{dx^3} \quad \text{or} \quad f'''(x) \tag{5}$$

is the third derivative of y with respect to x. If $y = f(x)$, the second differential of $f(x)$ is

$$d^2f = f''(x)dx^2 \tag{6}$$

Indeterminate Forms

If a function $f(x)$ for $x = a$ (where a can also be ∞) has no determined value but appears in one of the meaningless forms

$$\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty, \infty - \infty, 0^0, \infty^0, 0^\infty, 1^\infty$$

then it may happen that the $\lim\limits_{x\to a} f(x)$ has a definite value. For the determination of this limiting value, if it exists, the following rules can be used:

$\frac{0}{0}$. If $f(x) = \frac{\phi(x)}{\psi(x)}$, $\phi(a) = 0$, and $\psi(a) = 0$, then

$$\lim_{x\to a} f(x) = \lim_{x\to a} \frac{\phi'(x)}{\psi'(x)} \qquad \text{(L'Hospital's rule)} \tag{7}$$

If, however, $\phi'(a) = 0$ and $\psi'(a) = 0$, the rule is applied again, with the result

$$\lim_{x\to a} \frac{\phi(x)}{\psi(x)} = \lim_{\xi\to a} \frac{\phi'(\xi)}{\psi'(\xi)} = \frac{\phi''(a)}{\psi''(a)} \tag{8}$$

unless $\phi''(a) = 0$ and $\psi''(a) = 0$. In this case, the rule is applied again.

Example. Find the value of $\frac{\sin x}{x}$ for $x = 0$.

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1.$$

$\frac{\infty}{\infty}$. If $f(x) = \frac{\phi(x)}{\psi(x)}$, $\phi(a) = \infty$, and $\psi(a) = \infty$, then

$$\lim_{x\to a} \frac{\phi(x)}{\psi(x)} = \lim_{x\to a} \frac{\phi'(x)}{\psi'(x)} \tag{9}$$

as before.

$0\cdot\infty$. If $f(x) = \phi(x)\cdot\psi(x)$, $\phi(a) = 0$, and $\psi(a) = \infty$, then place $\frac{1}{\psi(x)} = \omega(x)$ and obtain the previous case $\frac{0}{0}$.

$\infty - \infty$. If $f(x) = \phi(x) - \psi(x)$, $\phi(a) = \infty$, and $\psi(a) = \infty$, then place $\phi(x) = \frac{1}{u(x)}$, $\psi(x) = \frac{1}{v(x)}$ and obtain

$$f(x) = \frac{v(x) - u(x)}{u(x)v(x)} \tag{10}$$

which takes the form $\frac{0}{0}$.

0^0, ∞^0, 0^∞, 1^∞. An expression of the type $[\psi(x)]^{\phi(x)}$ may, for $x = a$, give rise to the forms 0^0, ∞^0, 0^∞, 1^∞.

Such an expression may be reduced to a type $\frac{0}{0}$ or $\frac{\infty}{\infty}$ by the use of logarithms. Thus,

$$\left.\begin{aligned} u &= [\psi(x)]^{\phi(x)} \\ \log_e u &= \phi(x)\cdot\log_e \psi(x) \end{aligned}\right\} \tag{11}$$

If $\lim\limits_{x\to a} \phi(x)\cdot\log_e \psi(x)$ can be found by the previous methods, the limit approached by u can be found.

Example. $u = (1 - x)^{1/x}$ for $x = 0$

$$\log_e u = \frac{\log_e (1 - x)}{x}$$

$$\lim_{x\to 0} \frac{\log_e (1 - x)}{x} = -1$$

Therefore $\lim\limits_{x\to 0} \log_e u = -1$ and $\lim\limits_{x\to 0} u = e^{-1}$

Table 43. Differentiation Formulas

Let $u, v, w, \ldots$ be functions of x; a and n be constants; and e be the base of the natural or Napierian logarithms. Then $e = 2.7183^{-}$.

$$\frac{d}{dx} a = 0$$

$$\frac{d}{dx}(u + v + w + \ldots) = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} + \ldots$$

$$\frac{d}{dx} au = a\frac{du}{dx}$$

$$\frac{d}{dx} uv = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw\ldots) = \left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx} + \ldots\right)(uvw\ldots)$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$\frac{d}{dx} u^n = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx} \log_e u = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx} \log_{10} u = \frac{1}{u}\frac{du}{dx}\log_{10} e = (0.4343)\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx} e^u = e^u\frac{du}{dx}$$

$$\frac{d}{dx} u^v = vu^{v-1}\frac{du}{dx} + u^v\frac{dv}{dx}\log_e u$$

$$\frac{d}{dx} f(u) = \frac{df(u)}{du}\cdot\frac{du}{dx}$$

$$\frac{d^2 f(u)}{dx^2} = \frac{df(u)}{du}\cdot\frac{d^2 u}{dx^2} + \frac{d^2 f(u)}{du^2}\left(\frac{du}{dx}\right)^2$$

$$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u\tan u\frac{du}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u\cot u\frac{du}{dx}$$

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}\left(-\frac{\pi}{2} \leqq \sin^{-1} u \leqq \frac{\pi}{2}\right)$$

$$\frac{d}{dx}\cos^{-1} u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}\ (0 \leqq \cos^{-1} u \leqq \pi)$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\cot^{-1} u = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}\ *$$

$$\frac{d}{dx}\csc^{-1} u = -\frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}\ *$$

$$\frac{d}{dx}\sinh u = \cosh u\frac{du}{dx}$$

$$\frac{d}{dx}\cosh u = \sinh u\frac{du}{dx}$$

$$\frac{d}{dx}\tanh u = \operatorname{sech}^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\coth u = -\operatorname{csch}^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u\tanh u\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u\coth u\frac{du}{dx}$$

$$\frac{d}{dx}\sinh^{-1} u = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$$

$$\frac{d}{dx}\cosh^{-1} u = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}\tanh^{-1} u = \frac{1}{1-u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\coth^{-1} u = \frac{1}{1-u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sech}^{-1} u = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{csch}^{-1} u = -\frac{1}{u\sqrt{u^2+1}}\frac{du}{dx}$$

* For angles in the first and third quadrants. Use the opposite sign in the second and fourth quadrants.

SERIES EXPANSION OF FUNCTIONS. Taylor's Formula. If $f(x)$ and all its derivatives are continuous in the neighborhood of the point $x = a$, then $f(x)$ can be developed into a power series arranged according to ascending powers of $x - a$. The series is.

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n-1)}(a)}{(n-1)!}(x-a)^{n-1} + R_n \qquad (12)$$

where $n! = n(n-1)(n-2)\ldots 2\cdot 1$, and is called factorial n, and

$$R_n = \frac{f^n(\xi)}{n!}(x-a)^n \qquad (13)$$

is the *remainder* after n terms of the series and

$$\xi = a + \theta(x-a),\ 0 < \theta < 1 \qquad (14)$$

Another form:

$$f(x+h) = f(x) + \frac{h}{1!}f'(x) + \frac{h^2}{2!}f''(x) + \ldots + \frac{h^{n-1}}{(n-1)!}f^{(n-1)}(x) + R_n \qquad (15)$$

where $$R_n = \frac{h^n}{n!}f^n(\xi),\ \xi = x + \theta h,\ 0 < \theta < 1 \qquad (16)$$

Maclaurin's Form of Taylor's Formula (for $a = 0$).

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + R_n \qquad (17)$$

where $$R_n = \frac{f^n(\xi)}{n!}x^n,\ \xi = \theta x,\ 0 < \theta < 1 \qquad (18)$$

Care must be exercised in using these formulas that the *series converges*, that is, $\lim_{n\to\infty} R_n = 0$. If Taylor's series converges rapidly, the sum of the first few terms gives a good approximation to $f(x)$ for values of x near $x = a$. If Maclaurin's series converges rapidly, the sum of the first few terms gives a good approximation to $f(x)$ for values of x near $x = 0$. That not all functions can be expanded into Maclaurin's series is shown by the examples; $f(x) = \frac{1}{x}$; $\frac{1}{x^2}$; $\sqrt{x}$, $\frac{1}{\sqrt{x}}$; $\log_e x$; $\cot x$; etc.

Example. Expand e^{nx} into a series of ascending powers of x.

$$\begin{aligned}
f(x) &= e^{nx}, & f(0) &= 1\\
f'(x) &= ne^{nx}, & f'(0) &= n\\
f''(x) &= n^2e^{nx}, & f''(0) &= n^2\\
f'''(x) &= n^3e^{nx}, & f'''(0) &= n^3\\
f^{\mathrm{IV}}(x) &= n^4e^{nx}. & f^{\mathrm{IV}}(0) &= n^4
\end{aligned}$$

$$e^{nx} = 1 + \frac{n}{2!}x + \frac{n^2}{2!}x^2 + \frac{n^3}{3!}x^3 + \ldots$$

If x and n are less than unity, this series converges rapidly.

Table 44. Functions Expanded into Series

($\log = \log_e$)

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}x^3 + \ldots \qquad (x^2 < a^2)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots \qquad (-\infty < x < \infty)$$

$$a^x = 1 + x\log a + \frac{(x\log a)^2}{2!} + \frac{(x\log a)^3}{3!} + \ldots \qquad (-\infty < x < \infty)$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \ldots \qquad (-\infty < x < \infty)$$

$$e^{\sin x} = 1 + x + \frac{x^2}{2!} - \frac{3x^4}{4!} - \frac{8x^5}{5!} - \frac{3x^6}{6!} + \frac{56x^7}{7!} + \ldots \qquad (-\infty < x < \infty)$$

$$e^{\cos x} = e\left(1 - \frac{x^2}{2!} + \frac{4x^4}{4!} - \frac{31x^6}{6!} + \ldots\right) \qquad (-\infty < x < \infty)$$

$$e^{\tan x} = 1 + x + \frac{x^2}{2!} + \frac{3x^3}{3!} + \frac{9x^4}{4!} + \frac{37x^5}{5!} + \ldots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\log x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots \qquad \left(x > \frac{1}{2}\right)$$

$$\log x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right] \qquad (x > 0)$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \qquad (-1 < x < 1)$$

$$\log\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots\right] \qquad (-1 < x < 1)$$

Table 44. Functions Expanded into Series—*Continued*

$$\log\left(\frac{x+1}{x-1}\right) = 2\left[\frac{1}{x} + \frac{1}{3}\left(\frac{1}{x}\right)^3 + \frac{1}{5}\left(\frac{1}{x}\right)^5 + \dots\right] \qquad (-1 < x < 1)$$

$$\log \sin x = \log x - \frac{x^2}{6} - \frac{x^4}{180} - \frac{x^6}{2835} - \dots \qquad (-\pi < x < \pi)$$

$$\log \cos x = -\frac{x^2}{2} - \frac{x^4}{12} - \frac{x^6}{45} - \frac{17x^8}{2520} - \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\log \tan x = \log x + \frac{x^2}{3} + \frac{7x^4}{90} + \frac{62x^6}{2835} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \qquad (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \qquad (-\infty < x < \infty)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \frac{x^7}{4725} - \dots \qquad (-\pi < x < \pi)$$

$$\sec x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\csc x = \frac{1}{x} + \frac{x}{3!} + \frac{7x^3}{3\cdot 5!} + \frac{31x^5}{3\cdot 7!} + \dots \qquad (-\pi < x < \pi)$$

$$\sin^{-1} x = x + \frac{x^3}{2\cdot 3} + \frac{3x^5}{2\cdot 4\cdot 5} + \frac{3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} + \dots \qquad (-1 < x < 1)$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\tan^{-1} x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \dots \qquad (-1 > x > 1)$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$\sec^{-1} x = \frac{\pi}{2} - \frac{1}{x} - \frac{1}{6x^3} - \frac{3}{2\cdot 4\cdot 5x^5} - \frac{3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} - \dots \qquad (-1 > x > 1)$$

$$\csc^{-1} x = \frac{\pi}{2} - \sec^{-1} x$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots \qquad (-\infty < x < \infty)$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \dots \qquad (-\infty < x < \infty)$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} - \frac{x^7}{4725} + \dots \qquad (-\pi < x < \pi)$$

$$\operatorname{sech} x = 1 - \frac{x^2}{2!} + \frac{5x^4}{4!} - \frac{61x^6}{6!} + \frac{1385x^8}{8!} - \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\operatorname{csch} x = \frac{1}{x} - \frac{x}{6} + \frac{7x^3}{360} - \frac{31x^5}{15120} + \dots \qquad (-\pi < x < \pi)$$

$$\sinh^{-1} x = x - \frac{x^3}{2\cdot 3} + \frac{3x^5}{2\cdot 4\cdot 5} - \frac{3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} + \dots \qquad (-1 < x < 1)$$

$$\sinh^{-1} x = \log 2x + \frac{1}{2\cdot 2x^2} - \frac{3}{2\cdot 4\cdot 4x^4} + \frac{3\cdot 5}{2\cdot 4\cdot 6\cdot 6x^6} + \dots \qquad (x > 1)$$

$$\cosh^{-1} x = \pm\left(\log 2x - \frac{1}{2\cdot 2x^2} - \frac{1\cdot 3}{2\cdot 4\cdot 4x^4} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6x^6} - \dots\right) \qquad (x > 1)$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \qquad (-1 < x < 1)$$

$$\coth^{-1} x = \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} + \dots \qquad (-1 > x > 1)$$

$$\operatorname{sech}^{-1} x = \pm\left(\log\frac{2}{x} - \frac{1}{2\cdot 2}x^2 - \frac{1\cdot 3}{2\cdot 4\cdot 4}x^4 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6}x^6 - \dots\right) \qquad (0 < x < 1)$$

$$\operatorname{csch}^{-1} x = \frac{1}{x} - \frac{1}{2\cdot 3x^3} + \frac{3}{2\cdot 4\cdot 5x^5} - \frac{3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \dots \qquad (-1 > x > 1)$$

11. INTEGRAL CALCULUS

Integration is the inverse operation of differentiation. It is indicated by the symbol $\int$ followed by the differential function to be integrated. For example,

$$d(x^3) = 3x^2\,dx \tag{19}$$

and

$$3\int x^2\,dx = x^3 + c \tag{20}$$

or in general

$$\int f'(x)\,dx = f(x) + c \tag{21}$$

where $f(x) + c$ is *indefinite* owing to the necessity of adding the arbitrary constant c whenever the operation of integration is performed.

$$\int_a^b f'(x)\,dx = f(b) - f(a) \tag{22}$$

is called the *definite integral*, where $f'(x)$ is continuous in the interval a to b or has at most a finite number of finite discontinuities in the interval.

Fundamental Forms. Since integration is an anti-differentiation operation, facility in integrating depends upon the ability to recognize the forms of the derivatives of elementary functions and also the knowledge of how to transform a given function into an elementary form. Table 45 is a list of fundamental forms to which many integrals may be reduced by simple transformation.

Integration by Parts is a method frequently employed if it is advantageous to consider the integral of a function as the integral of the product of a function by the differential of another function, then

$$\int u\,dv = uv - \int v\,du \tag{23}$$

since

$$d(uv) = u\,dv + v\,du \tag{24}$$

where u and v are both functions of a variable x.

Integration of Rational Fractions. If the degree of the numerator is not less than that of the denominator in the equation

$$R(x) = \frac{\phi(x)}{f(x)}$$

where $\phi(x)$ and $f(x)$ are rational polynomials, $R(x)$ can be put in the form of a polynomial and a remainder by performing the division indicated. The remainder may then be represented by partial fractions, and both the polynomial and remainder integrated directly by application of formulas 5, 6, 17, and 18 in Table 45.

Integration of Irrational Functions may frequently be accomplished by reducing them to rational integrals by changing the variable. The method is called integration by rationlization or integration by substitution.

For example, integrals containing the following forms may be rationalized by the substitutions indicated:

Form	*Substitution*
$f[(ax + b)^{p/q}]\,dx$	let $ax + b = y^q$
$f[(ax + b)^{p/q}(ax + b)^{r/s}]\,dx$	let $ax + b = y^n$, where n is the L.C.M. of q, s
$f[x,\sqrt{x^2 + ax + b}]\,dx$	let $\sqrt{x^2 + ax + b} = y - x$
$f[x,\sqrt{-x^2 + ax + b}]\,dx$	let $\sqrt{-x^2 + ax + b} = \sqrt{(\alpha - x)(\beta + x)}$ $= (\alpha - x)y$ or $= (\beta + x)y$
$f[\sin x, \cos x]\,dx$	let $\tan \frac{x}{2} = y$
$f[x,\sqrt{a^2 - x^2}]\,dx$	let $x = a \sin y$
$f[x,\sqrt{x^2 - a^2}]\,dx$	let $x = a \sec y$ or $x = a \cosh y$
$f[x,\sqrt{x^2 + a^2}]\,dx$	let $x = a \tan y$ or $x = a \sinh y$

Table 45. Table of Integrals

Elementary Indefinite Integrals

1. $\int a\,dx = ax + c$
2. $\int (u + v + w + \ldots)dx = \int u\,dx + \int v\,dx + \int w\,dx + \ldots$
3. $\int u\,dv = uv - \int v\,du$, integration by parts
4. $\int f(x)\,dx = \int f[\phi(y)]\phi'(y)dy$, $x = \phi(y)$, change of variable
5. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c$, $(n \neq -1)$
6. $\int \dfrac{dx}{x} = \log_e x + c = \log_e c_1 x$, $[\log_e x = \log_e(-x) + (2k+1)\pi i]$
7. $\int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$
8. $\int a^x\,dx = \dfrac{a^x}{\log_e a} + c$
9. $\int a^x \log_e a\,dx = a^x + c$
10. $\int \sin ax\,dx = -\dfrac{1}{a}\cos ax + c$
11. $\int \cos ax\,dx = \dfrac{1}{a}\sin ax + c$
12. $\int \tan ax\,dx = -\dfrac{1}{a}\log_e \cos ax + c = \dfrac{1}{a}\log_e \sec ax + c$
13. $\int \cot ax\,dx = \dfrac{1}{a}\log_e \sin ax + c = -\dfrac{1}{a}\log_e \csc ax + c$
14. $\int \sec ax\,dx = \dfrac{1}{a}\log_e(\sec ax + \tan ax) + c = \dfrac{1}{a}\log_e \tan\left(\dfrac{ax}{2} + \dfrac{\pi}{4}\right) + c$
15. $\int \csc ax\,dx = \dfrac{1}{a}\log_e(\csc ax - \cot ax) + c = \dfrac{1}{a}\log_e \tan\dfrac{ax}{2} + c$
16. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\dfrac{x}{a} + c = -\cos^{-1}\dfrac{x}{a} + c \quad (x^2 < a^2)$
17. $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c = -\dfrac{1}{a}\cot^{-1}\dfrac{x}{a} + c$
18. $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\log\dfrac{x-a}{x+a} = -\dfrac{1}{a}\tanh^{-1}\dfrac{x}{a} = -\dfrac{1}{a}\coth^{-1}\dfrac{x}{a}$
19. $\int \sinh ax\,dx = \dfrac{1}{a}\cosh ax + c$
20. $\int \cosh ax\,dx = \dfrac{1}{a}\sinh ax + c$
21. $\int \tanh ax\,dx = \dfrac{1}{a}\log_e(\cosh ax) + c$
22. $\int \coth ax\,dx = \dfrac{1}{a}\log_e(\sinh ax) + c$
23. $\int \operatorname{sech} ax\,dx = \dfrac{1}{a}\sin^{-1}(\tanh ax) + c$
24. $\int \operatorname{csch} ax\,dx = \dfrac{1}{a}\log_e\left(\tanh\dfrac{ax}{2}\right) + c$
25. $\int \sin^2 ax\,dx = \dfrac{1}{2}x - \dfrac{1}{2a}\sin ax\cos ax + c = \dfrac{1}{2}x - \dfrac{1}{4a}\sin 2ax + c$
26. $\int \cos^2 ax\,dx = \dfrac{1}{2}x + \dfrac{1}{2a}\sin ax\cos ax + c = \dfrac{1}{2}x + \dfrac{1}{4a}\sin 2ax + c$
27. $\int \tan^2 ax\,dx = \dfrac{1}{a}\tan ax - x + c$
28. $\int \cot^2 ax\,dx = -\dfrac{1}{a}\cot ax - x + c$

Elementary Indefinite Integrals—*Continued*

29. $\int \sec^2 ax\, dx = \frac{1}{a} \tan ax + c$

30. $\int \csc^2 ax\, dx = -\frac{1}{a} \cot ax + c$

31. $\int \sin^{-1} ax\, dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1 - a^2 x^2} + c$

32. $\int \cos^{-1} ax\, dx = x \cos^{-1} ax - \frac{1}{a} \sqrt{1 - a^2 x^2} + c$

33. $\int \tan^{-1} ax\, dx = x \tan^{-1} ax - \frac{1}{2a} \log_e (1 + a^2 x^2) + c$

34. $\int \cot^{-1} ax\, dx = x \cot^{-1} ax + \frac{1}{2a} \log_e (1 + a^2 x^2) + c$

35. $\int \sec^{-1} ax\, dx = x \sec^{-1} ax - \frac{1}{a} \log_e (ax + \sqrt{a^2 x^2 - 1}) + c$

36. $\int \csc^{-1} ax\, dx = x \csc^{-1} ax + \frac{1}{a} \log_e (ax + \sqrt{a^2 x^2 - 1}) + c$

Integrals Involving $(ax + b)$

37. $\int (ax + b)^n\, dx = \frac{1}{a(n + 1)} (ax + b)^{n+1} \quad (n \neq -1)$

38. $\int \frac{dx}{ax + b} = \frac{1}{a} \log_e (ax + b)$

39. $\int x(ax + b)^n\, dx = \frac{1}{a^2(n + 2)} (ax + b)^{n+2} - \frac{b}{a^2(n + 1)} (ax + b)^{n+1} \quad (n \neq -1, -2)$

40. $\int \frac{x\, dx}{ax + b} = \frac{x}{a} - \frac{b}{a^2} \log_e (ax + b)$

41. $\int \frac{x\, dx}{(ax + b)^2} = \frac{b}{a^2(ax + b)} + \frac{1}{a^2} \log_e (ax + b)$

42. $\int \frac{x^2\, dx}{ax + b} = \frac{1}{a^3}\left[\frac{1}{2}(ax + b)^2 - 2b(ax + b) + b^2 \log_e (ax + b)\right]$

43. $\int \frac{x^2\, dx}{(ax + b)^2} = \frac{1}{a^3}\left[(ax + b) - 2b \log_e (ax + b) - \frac{b^2}{ax + b}\right]$

44. $\int \frac{x^2\, dx}{(ax + b)^3} = \frac{1}{a^3}\left[\log_e (ax + b) + \frac{2b}{ax + b} - \frac{b^2}{2(ax + b)^2}\right]$

45. $\int \frac{dx}{x(ax + b)} = \frac{1}{b} \log_e \frac{x}{ax + b}$

46. $\int \frac{dx}{x^2 (ax + b)} = -\frac{1}{bx} + \frac{a}{b^2} \log_e \frac{ax + b}{x}$

47. $\int \frac{dx}{x(ax + b)^2} = \frac{1}{b(ax + b)} - \frac{1}{b^2} \log_e \frac{ax + b}{x}$

48. $\int \frac{dx}{x^2 (ax + b)^2} = -\frac{b + 2ax}{b^2 x(ax + b)} + \frac{2a}{b^3} \log_e \frac{ax + b}{x}$

49. $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \log_e \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \quad (b \text{ positive})$

50. $\int \frac{dx}{x\sqrt{ax + b}} = \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{ax + b}{-b}} \quad (b \text{ negative})$

51. $\int \frac{\sqrt{ax + b}}{x}\, dx = 2\sqrt{ax + b} + \sqrt{b} \log_e \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \quad (b \text{ positive})$

52. $\int \frac{\sqrt{ax + b}}{x}\, dx = 2\sqrt{ax + b} - 2\sqrt{-b} \tan^{-1} \sqrt{\frac{ax + b}{-b}} \quad (b \text{ negative})$

(*Table continued on p. 11-122*)

Integrals Involving $(ax + b)$—*Continued*

53. $\displaystyle\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b\sqrt{b}}\log_e \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}$ (b positive)

54. $\displaystyle\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{b\sqrt{-b}}\tan^{-1}\sqrt{\frac{ax+b}{-b}}$ (b negative)

55. $\displaystyle\int \frac{ax+b}{fx+g}\,dx = \frac{ax}{f} + \frac{bf-ag}{f^2}\log_e(fx+g)$

56. $\displaystyle\int \frac{dx}{(ax+b)(fx+g)} = \frac{1}{bf-ag}\log_e\left(\frac{fx+g}{ax+b}\right)$ $(ag \neq bf)$

57. $\displaystyle\int \frac{x\,dx}{(ax+b)(fx+g)} = \frac{1}{bf-ag}\left[\frac{b}{a}\log_e(ax+b) - \frac{g}{f}\log_e(fx+g)\right]$ $(ag \neq bf)$

58. $\displaystyle\int \frac{dx}{(ax+b)^2(fx+g)} = \frac{1}{bf-ag}\left(\frac{1}{ax+b} + \frac{f}{bf-ag}\log_e\frac{fx+g}{ax+b}\right)$ $(ag \neq bf)$

Integrals Involving $(ax^n + b)$

59. $\displaystyle\int (ax^2+b)^n x\,dx = \frac{1}{2a}\,\frac{(ax^2+b)^{n+1}}{n+1}$ $(n \neq -1)$

60. $\displaystyle\int \frac{dx}{ax^2+b} = \frac{1}{\sqrt{ab}}\tan^{-1}\left(x\sqrt{\frac{a}{b}}\right)$ (a and b positive)

61. $\displaystyle\int \frac{dx}{ax^2+b} = \frac{1}{2\sqrt{-ab}}\log_e\frac{x\sqrt{a}-\sqrt{-b}}{x\sqrt{a}+\sqrt{-b}}$ (a positive, b negative)

$\displaystyle= \frac{1}{2\sqrt{-ab}}\log_e\frac{\sqrt{b}+x\sqrt{-a}}{\sqrt{b}-x\sqrt{-a}}$ (a negative, b positive)

62. $\displaystyle\int \frac{dx}{x(ax^2+b)} = \frac{1}{2b}\log_e\frac{x^2}{ax^2+b}$

63. $\displaystyle\int \frac{dx}{(ax^2+b)^n} = \frac{1}{2(n-1)b}\,\frac{x}{(ax^2+b)^{n-1}} + \frac{2n-3}{2(n-1)b}\int\frac{dx}{(ax^2+b)^{n-1}}$ (n integer > 1)

64. $\displaystyle\int \frac{x^2\,dx}{ax^2+b} = \frac{x}{a} - \frac{b}{a}\int\frac{dx}{ax^2+b}$

65. $\displaystyle\int \frac{x^2\,dx}{(ax^2+b)^n} = -\frac{1}{2(n-1)a}\,\frac{x}{(ax^2+b)^{n-1}} + \frac{1}{2(n-1)a}\int\frac{dx}{(ax^2+b)^{n-1}}$ (n integer > 1

66. $\displaystyle\int \frac{dx}{x^2(ax^2+b)^n} = \frac{1}{b}\int\frac{dx}{x^2(ax^2+b)^{n-1}} - \frac{a}{b}\int\frac{dx}{(ax^2+b)^n}$ (n = positive integer)

67. $\displaystyle\int \sqrt{ax^2+b}\,dx = \frac{x}{2}\sqrt{ax^2+b} + \frac{b}{2\sqrt{a}}\log_e(x\sqrt{a}+\sqrt{ax^2+b})$ (a positive)

68. $\displaystyle\int \sqrt{ax^2+b}\,dx = \frac{x}{2}\sqrt{ax^2+b} + \frac{b}{2\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right)$ (a negative)

69. $\displaystyle\int \frac{dx}{\sqrt{ax^2+b}} = \frac{1}{\sqrt{a}}\log_e(x\sqrt{a}+\sqrt{ax^2+b})$ (a positive)

70. $\displaystyle\int \frac{dx}{\sqrt{ax^2+b}} = \frac{1}{\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right)$ (a negative)

71. $\displaystyle\int \frac{x\,dx}{\sqrt{ax^2+b}} = \frac{1}{a}\sqrt{ax^2+b}$

72. $\displaystyle\int \frac{\sqrt{ax^2+b}}{x}\,dx = \sqrt{ax^2+b} + \sqrt{b}\log_e\frac{\sqrt{ax^2+b}-\sqrt{b}}{x}$ (b positive)

73. $\displaystyle\int \frac{\sqrt{ax^2+b}}{x}\,dx = \sqrt{ax^2+b} - \sqrt{-b}\tan^{-1}\frac{\sqrt{ax^2+b}}{\sqrt{-b}}$ (b negative)

74. $\displaystyle\int x\sqrt{ax^2+b}\,dx = \frac{1}{3a}(ax^2+b)^{3/2}$

Integrals Involving $(ax^n + b)$—*Continued*

75. $\int x^2 \sqrt{ax^2+b}\, dx = \frac{x}{4a}(ax^2+b)^{3/2} - \frac{bx}{8a}\sqrt{ax^2+b} - \frac{b^2}{8a\sqrt{a}} \log_e (x\sqrt{a} + \sqrt{ax^2+b})$ (a positive)

76. $\int x^2 \sqrt{ax^2+b}\, dx = \frac{x}{4a}(ax^2+b)^{3/2} - \frac{bx}{8a}\sqrt{ax^2+b} - \frac{b^2}{8a\sqrt{-a}} \sin^{-1}\left(x\sqrt{\frac{-a}{b}}\right)$ (a negative)

77. $\int \frac{dx}{x\sqrt{ax^2+b}} = \frac{1}{\sqrt{b}} \log_e \frac{\sqrt{ax^2+b} - \sqrt{b}}{x}$ (b positive)

78. $\int \frac{dx}{x\sqrt{ax^2+b}} = \frac{1}{\sqrt{-b}} \sec^{-1}\left(x\sqrt{-\frac{a}{b}}\right)$ (b negative)

79. $\int \frac{x^2\, dx}{\sqrt{ax^2+b}} = \frac{x}{2a}\sqrt{ax^2+b} - \frac{b}{2a\sqrt{a}} \log_e (x\sqrt{a} + \sqrt{ax^2+b})$ (a positive)

80. $\int \frac{x^2\, dx}{\sqrt{ax^2+b}} = \frac{x}{2a}\sqrt{ax^2+b} - \frac{b}{2a\sqrt{-a}} \sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right)$ (a negative)

81. $\int \frac{\sqrt{ax^2+b}}{x^2}\, dx = -\frac{\sqrt{ax^2+b}}{x} + \sqrt{a} \log_e (x\sqrt{a} + \sqrt{ax^2+b})$ (a positive)

82. $\int \frac{\sqrt{ax^2+b}}{x^2}\, dx = -\frac{\sqrt{ax^2+b}}{x} - \sqrt{-a} \sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right)$ (a negative)

83. $\int \frac{dx}{x(ax^n+b)} = \frac{1}{bn} \log_e \frac{x^n}{ax^n+b}$

84. $\int \frac{dx}{x\sqrt{ax^n+b}} = \frac{1}{n\sqrt{b}} \log_e \frac{\sqrt{ax^n+b} - \sqrt{b}}{\sqrt{ax^n+b} + \sqrt{b}}$ (b positive)

85. $\int \frac{dx}{x\sqrt{ax^n+b}} = \frac{2}{n\sqrt{-b}} \sec^{-1}\sqrt{-\frac{ax^n}{b}}$ (b negative)

Integrals Involving $(ax^2 + bx + d)$

86. $\int \frac{dx}{ax^2+bx+d} = \frac{1}{\sqrt{b^2-4ad}} \log_e \frac{2ax+b-\sqrt{b^2-4ad}}{2ax+b+\sqrt{b^2-4ad}}$ ($b^2 > 4ad$)

87. $\int \frac{dx}{ax^2+bx+d} = \frac{2}{\sqrt{4ad-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ad-b^2}}$ ($b^2 < 4ad$)

88. $\int \frac{dx}{ax^2+bx+d} = -\frac{2}{2ax+b}$ ($b^2 = 4ad$)

89. $\int \frac{dx}{\sqrt{ax^2+bx+d}} = \frac{1}{\sqrt{a}} \log_e \left(2ax+b+2\sqrt{a(ax^2+bx+d)}\right)$ (a positive)

90. $\int \frac{dx}{\sqrt{ax^2+bx+d}} = \frac{1}{\sqrt{-a}} \sin^{-1} \frac{-2ax-b}{\sqrt{b^2-4ad}}$ (a negative)

91. $\int \frac{x\, dx}{ax^2+bx+d} = \frac{1}{2a} \log_e (ax^2+bx+d) - \frac{b}{2a}\int \frac{dx}{ax^2+bx+d}$

92. $\int \frac{x\, dx}{\sqrt{ax^2+bx+d}} = \frac{\sqrt{ax^2+bx+d}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{ax^2+bx+d}}$

93. $\int \frac{dx}{x\sqrt{ax^2+bx+d}} = -\frac{1}{\sqrt{d}} \log_e \left(\frac{\sqrt{ax^2+bx+d}+\sqrt{d}}{x} + \frac{b}{2\sqrt{d}}\right)$ (d positive)

94. $\int \frac{dx}{x\sqrt{ax^2+bx+d}} = \frac{1}{\sqrt{-d}} \sin^{-1} \frac{bx+2d}{x\sqrt{b^2-4ad}}$ (d negative)

95. $\int \frac{dx}{x\sqrt{ax^2+bx}} = -\frac{2}{bx}\sqrt{ax^2+bx}$

(Table continued on p. 11-124)

Integrals Involving $(ax^2 + bx + d)$—*Continued*

96. $\int \sqrt{ax^2 + bx + d}\, dx = \frac{2ax + b}{4a}\sqrt{ax^2 + bx + d} + \frac{4ad - b^2}{8a}\int \frac{dx}{\sqrt{ax^2 + bx + d}}$

97. $\int x\sqrt{ax^2 + bx + d}\, dx = \frac{(ax^2 + bx + d)^{3/2}}{3a} - \frac{b}{2a}\int \sqrt{ax^2 + bx + d}\, dx$

Integrals Involving $\sin^n ax$

98. $\int \sin^3 ax\, dx = -\frac{1}{a}\cos ax + \frac{1}{3a}\cos^3 ax$

99. $\int \sin^4 ax\, dx = \frac{3}{8}x - \frac{1}{4a}\sin 2ax + \frac{1}{32a}\sin 4ax$

100. $\int \sin^n ax\, dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} ax\, dx$ (n = positive integer)

101. $\int x \sin ax\, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$

102. $\int x^2 \sin ax\, dx = \frac{2x}{a^2}\sin ax - \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\cos ax$

103. $\int x^3 \sin ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right)\sin ax - \left(\frac{x^3}{a} - \frac{6x}{a^3}\right)\cos ax$

104. $\int x^n \sin ax\, dx = -\frac{x^n}{a}\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\, dx$ ($n > 0$)

105. $\int \frac{\sin ax}{x^n}\, dx = -\frac{1}{n-1}\frac{\sin ax}{x^{n-1}} + \frac{a}{n-1}\int \frac{\cos ax}{x^{n-1}}\, dx$

106. $\int \frac{dx}{\sin^n ax} = -\frac{1}{a(n-1)}\frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} ax}$ (n integer > 1)

107. $\int \frac{x\, dx}{\sin^2 ax} = -\frac{x}{a}\cot ax + \frac{1}{a^2}\log_e \sin ax$

108. $\int \frac{dx}{1 + \sin ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

109. $\int \frac{dx}{1 - \sin ax} = \frac{1}{a}\cot\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

110. $\int \frac{x\, dx}{1 + \sin ax} = -\frac{x}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2}\log_e \cos\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

111. $\int \frac{x\, dx}{1 - \sin ax} = \frac{x}{a}\cot\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2}\log_e \sin\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

112. $\int \frac{dx}{b + d\sin ax} = \frac{-2}{a\sqrt{b^2 - d^2}}\tan^{-1}\left[\sqrt{\frac{b-d}{b+d}}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right]$ ($b^2 > d^2$)

113. $\int \frac{dx}{b + d\sin ax} = \frac{-1}{a\sqrt{d^2 - b^2}}\log_e \frac{d + b\sin ax + \sqrt{d^2 - b^2}\cos ax}{b + d\sin ax}$ ($d^2 > b^2$)

114. $\int \sin ax \sin bx\, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}$ ($a^2 \neq b^2$)

Integrals Involving $\cos^n ax$

115. $\int \cos^3 ax\, dx = \frac{1}{a}\sin ax - \frac{1}{3a}\sin^3 ax$

116. $\int \cos^4 ax\, dx = \frac{3}{8}x + \frac{1}{4a}\sin 2ax + \frac{1}{32a}\sin 4ax$

117. $\int \cos^n ax\, dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n}\int \cos^{n-2} ax\, dx$ (n = positive integer)

118. $\int x \cos ax\, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$

Integrals Involving $\cos^n ax$—*Continued*

119. $\int x^2 \cos ax\, dx = \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right) \sin ax$

120. $\int x^3 \cos ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right) \cos ax + \left(\frac{x^3}{a} - \frac{6x}{a^3}\right) \sin ax$

121. $\int x^n \cos ax\, dx = \frac{x^n \sin ax}{a} - \frac{n}{a}\int x^{n-1} \sin ax\, dx \quad (n > 0)$

122. $\int \frac{\cos ax}{x^n}\, dx = -\frac{1}{n-1} \frac{\cos ax}{x^{n-1}} - \frac{a}{n-1}\int \frac{\sin ax}{x^{n-1}}\, dx$

123. $\int \frac{dx}{\cos^n ax} = \frac{1}{a(n-1)} \frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} ax} \quad (n \text{ integer} > 1)$

124. $\int \frac{x\, dx}{\cos^2 ax} = \frac{x}{a} \tan ax + \frac{1}{a^2} \log_e \cos ax$

125. $\int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2}$

126. $\int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot \frac{ax}{2}$

127. $\int \frac{x\, dx}{1 + \cos ax} = \frac{x}{a} \tan \frac{ax}{2} + \frac{2}{a^2} \log_e \cos \frac{ax}{2}$

128. $\int \frac{x\, dx}{1 - \cos ax} = -\frac{x}{a} \cot \frac{ax}{2} + \frac{2}{a^2} \log_e \sin \frac{ax}{2}$

129. $\int \frac{dx}{b + d \cos ax} = \frac{2}{a\sqrt{b^2 - d^2}} \tan^{-1}\left(\sqrt{\frac{b-d}{b+d}} \tan \frac{ax}{2}\right) \quad (b^2 > d^2)$

130. $\int \frac{dx}{b + d \cos ax} = \frac{1}{a\sqrt{d^2 - b^2}} \log_e \frac{d + b \cos ax + \sqrt{d^2 - b^2} \sin ax}{b + d \cos ax} \quad (d^2 > b^2)$

131. $\int \cos ax \cos bx\, dx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$

Integrals Involving $\sin^n ax$, $\cos^n ax$

132. $\int \sin ax \cos bx\, dx = -\frac{1}{2}\left[\frac{\cos (a-b)x}{a-b} + \frac{\cos (a+b)x}{a+b}\right] \quad (a^2 \neq b^2)$

133. $\int \sin^2 ax \cos^2 ax\, dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$

134. $\int \sin^n ax \cos ax\, dx = \frac{1}{a(n+1)} \sin^{n+1} ax \quad (n \neq -1)$

135. $\int \sin ax \cos^n ax\, dx = -\frac{1}{a(n+1)} \cos^{n+1} ax \quad (n \neq -1)$

136. $\int \sin^n ax \cos^m ax\, dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(n+m)} + \frac{n-1}{n+m}\int \sin^{n-2} ax \cos^m ax\, dx \quad (m, n \text{ pos})$

137. $\int \frac{\sin^n ax}{\cos^m ax}\, dx = \frac{\sin^{n+1} ax}{a(m-1) \cos^{m-1} ax} - \frac{n-m+2}{m-1}\int \frac{\sin^n ax}{\cos^{m-2} ax}\, dx \quad (m, n \text{ pos}, m \neq 1)$

138. $\int \frac{\cos^m ax}{\sin^n ax}\, dx = \frac{-\cos^{m+1} ax}{a(n-1) \sin^{n-1} ax} + \frac{n-m-2}{(n-1)}\int \frac{\cos^m ax}{\sin^{n-2} ax}\, dx \quad (m, n \text{ pos}, n \neq 1)$

139. $\int \frac{dx}{\sin ax \cos ax} = \frac{1}{a} \log_e \tan ax$

140. $\int \frac{dx}{b \sin ax + d \cos ax} = \frac{1}{a\sqrt{b^2 + d^2}} \log_e \tan \tfrac{1}{2}\left(ax + \tan^{-1} \frac{d}{b}\right)$

141. $\int \frac{\sin ax}{b + d \cos ax}\, dx = -\frac{1}{ad} \log_e (b + d \cos ax)$

142. $\int \frac{\cos ax}{b + d \sin ax}\, dx = \frac{1}{ad} \log_e (b + d \sin ax)$

(Table continued on p. 11-126)

Integrals Involving $\tan^n ax$, $\cot^n ax$, $\sec^n ax$, $\csc^n ax$

143. $\int \tan^n ax\,dx = \frac{1}{a(n-1)} \tan^{n-1} ax - \int \tan^{n-2} ax\,dx \quad (n \text{ integer} > 1)$

144. $\int \cot^n ax\,dx = -\frac{1}{a(n-1)} \cot^{n-1} ax - \int \cot^{n-2} ax\,dx \quad (n \text{ integer} > 1)$

145. $\int \sec^n ax\,dx = \frac{1}{a(n-1)} \frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1} \int \sec^{n-2} ax\,dx \quad (n \text{ integer} > 1)$

146. $\int \csc^n ax\,dx = -\frac{1}{a(n-1)} \frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1} \int \csc^{n-2} ax\,dx \quad (n \text{ integer} > 1)$

147. $\int \frac{dx}{b + d \tan ax} = \frac{1}{b^2 + d^2} \left[bx + \frac{d}{a} \log_e (b \cos ax + d \sin ax) \right]$

148. $\int \frac{dx}{\sqrt{b + d \tan^2 ax}} = \frac{1}{a\sqrt{b-d}} \sin^{-1} \left[\sqrt{\frac{b-d}{b}} \sin ax \right] \quad (b \text{ pos}, b^2 > d^2)$

149. $\int \tan ax \sec ax\,dx = \frac{1}{a} \sec ax$

150. $\int \tan^n ax \sec^2 ax\,dx = \frac{1}{a(n+1)} \tan^{n+1} ax \quad (n \neq -1)$

151. $\int \frac{\sec^2 ax\,dx}{\tan ax} = \frac{1}{a} \log_e \tan ax$

152. $\int \cot ax \csc ax\,dx = -\frac{1}{a} \csc ax$

153. $\int \cot^n ax \csc^2 ax\,dx = -\frac{1}{a(n+1)} \cot^{n+1} ax \quad (n \neq -1)$

154. $\int \frac{\csc^2 ax}{\cot ax}\,dx = -\frac{1}{a} \log_e \cot ax$

Integrals Involving b^{ax}, e^{ax}, $\sin bx$, $\cos bx$

155. $\int x b^{ax}\,dx = \frac{x b^{ax}}{a \log_e b} - \frac{b^{ax}}{a^2 (\log_e b)^2}$

156. $\int x e^{ax}\,dx = \frac{e^{ax}}{a^2} (ax - 1)$

157. $\int x^n b^{ax}\,dx = \frac{x^n b^{ax}}{a \log_e b} - \frac{n}{a \log_e b} \int x^{n-1} b^{ax}\,dx \quad (n \text{ positive})$

158. $\int x^n e^{ax}\,dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax}\,dx \quad (n \text{ positive})$

159. $\int \frac{dx}{b + de^{ax}} = \frac{1}{ab} \left[ax - \log_e (b + de^{ax}) \right]$

160. $\int \frac{e^{ax}\,dx}{b + de^{ax}} = \frac{1}{ad} \log_e (b + de^{ax})$

161. $\int \frac{dx}{be^{ax} + de^{-ax}} = \frac{1}{a\sqrt{bd}} \tan^{-1} \left(e^{ax} \sqrt{\frac{b}{d}} \right) \quad (b \text{ and } d \text{ positive})$

162. $\int \frac{e^{ax}}{x}\,dx = \log_e x + ax + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^3}{3 \cdot 3!} + \ldots$

163. $\int \frac{e^{ax}}{x^n}\,dx = \frac{1}{n-1} \left(-\frac{e^{ax}}{x^{n-1}} + a \int \frac{e^{ax}}{x^{n-1}}\,dx \right) \quad (n \text{ integer} > 1)$

164. $\int e^{ax} \sin bx\,dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$

165. $\int e^{ax} \cos bx\,dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$

166. $\int x e^{ax} \sin bx\,dx = \frac{x e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$
$$- \frac{e^{ax}}{(a^2 + b^2)^2} [(a^2 - b^2) \sin bx - 2ab \cos bx]$$

167. $\int x e^{ax} \cos bx\,dx = \frac{x e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$
$$- \frac{e^{ax}}{(a^2 + b^2)^2} \left[(a^2 - b^2) \cos bx + 2ab \sin bx \right]$$

Some Definite Integrals

1. $\int_0^a \sqrt{a^2 - x^2}\, dx = \frac{\pi a^2}{4}$

2. $\int_0^a \sqrt{2\,ax - x^2}\, dx = \frac{\pi a^2}{4}$

3. $\int_0^\infty \frac{dx}{a + bx^2} = \frac{\pi}{2\sqrt{ab}}$ (a and b positive)

4. $\int_0^{\sqrt{a/b}} \frac{dx}{a + bx^2}\, dx = \int_{\sqrt{a/b}}^\infty \frac{dx}{a + bx^2} = \frac{\pi}{4\sqrt{ab}}$ (a and b positive)

5. $\int_0^{\sqrt{a/b}} \frac{dx}{\sqrt{a - bx^2}} = \frac{\pi}{2\sqrt{b}}$ (a and b positive)

6. $\int_0^\infty \frac{\sin bx}{x}\, dx = \frac{\pi}{2} \quad (b > 0)$
$= 0 \quad (b = 0)$
$= -\frac{\pi}{2} \quad (b < 0)$

7. $\int_0^\infty \frac{\tan x}{x}\, dx = \frac{\pi}{2}$

8. $\int_0^{\pi/2} \sin^{2n+1} x\, dx = \int_0^{\pi/2} \cos^{2n+1} x\, dx = \frac{2\cdot 4\cdot 6\cdot \ldots \cdot 2n}{3\cdot 5\cdot 7\cdot \ldots \cdot (2n+1)} \quad (n > 0)$

9. $\int_0^{\pi/2} \sin^{2n} x\, dx = \int_0^{\pi/2} \cos^{2n} x\, dx = \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot 6\cdot \ldots \cdot 2n}\cdot\frac{\pi}{2} \quad (n > 0)$

10. $\int_0^\pi \sin ax \sin bx\, dx = \int_0^\pi \cos ax \cos bx\, dx = 0 \quad (a \neq b)$

11. $\int_0^\pi \sin^2 ax\, dx = \int_0^\pi \cos^2 ax\, dx = \frac{\pi}{2}$

12. $\int_0^{\pi/2} \log_e \cos x\, dx = \int_0^{\pi/2} \log_e \sin x\, dx = -\frac{\pi}{2}\log_e 2$

13. $\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

14. $\int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}} \quad (a > 0,\ n = 1, 2, 3, \ldots)$

15. $\int_0^1 \frac{\log_e x}{1 - x}\, dx = -\frac{\pi^2}{6}$

16. $\int_0^1 \frac{\log_e x}{1 + x}\, dx = -\frac{\pi^2}{12}$

17. $\int_0^1 \frac{\log_e x}{1 - x^2}\, dx = \frac{\pi^2}{8}$

ELLIPTIC INTEGRALS

Reference: *Smithsonian Mathematical Formulae and Tables of Elliptic Functions* by E. P. Adams and R. L. Hippisley, Washington, D.C., 1922.

SYMBOLS AND ABBREVIATIONS

Greek Letters

A	α	Alpha	H	η	Eta	N	ν	Nu	T	τ	Tau
B	β	Beta	Θ	ϑ θ	Theta	Ξ	ξ	Xi	Y	υ	Upsilon
Γ	γ	Gamma	I	ι	Iota	O	o	Omicron	Φ	ϕ	Phi
Δ	δ	Delta	K	κ	Kappa	Π	π	Pi	X	χ	Chi
E	ϵ	Epsilon	Λ	λ	Lambda	P	ρ	Rho	Ψ	ψ	Psi
Z	ζ	Zeta	M	μ	Mu	Σ	σ ς	Sigma	Ω	ω	Omega

Mathematical Signs and Abbreviations

$+$ plus (addition).
$+$ positive.
$-$ minus (subtraction).
$-$ negative.
$\pm$ plus or minus.
$\mp$ minus or plus.
$=$ equals.
$\geqq$ equals or greater than.
$\leqq$ equals or is less than.
$\approx$ approximately equals.
$\times$ multiplied by.
ab or $a\,b = a \times b$.
$\div$ divided by.
$/$ divided by.
$\frac{a}{b} = a/b = a \div b$. $15/16 = \frac{15}{16}$.
$0.2 = \frac{2}{10}$, $0.002 = \frac{2}{1000}$.
$\sqrt{\ }$ square root.
$\sqrt[3]{\ }$ cube root.
$\sqrt[4]{\ }$ 4th root.
: is to, : : so is, : to (proportion).
2 : 4 : : 3 : 6, 2 is to 4 as 3 is to 6.
: ratio; divided by.
2 : 4, ratio of 2 to 4 = 2/4.
$>$ greater than.
$<$ less than.
° degrees, arc or thermometer.
′ minutes or feet.
″ seconds or inches.
′ ″ ‴ accents to distinguish letters, as a', a'', a'''.
a_1, a_2, a_3, a_b, a_c, read a sub 1, a sub b, etc.
( ) [ ] { } —— parentheses, brackets, braces, vinculum; denoting that the numbers enclosed are to be taken together; as, $(a + b)c = \overline{4 + 3} \times 5 = 35$.
a^2, a^3, a squared, a cubed.
a^n, a raised to the nth power.
$a^{2/3} = \sqrt[3]{a^2}$, $a^{3/2} = \sqrt{a^3}$.
$a^{-1} = \frac{1}{a}$, $a^{-2} = \frac{1}{a^2}$.
10^9 = 10 to the 9th power = 1,000,000,000.
$\sin a$ = the sine of a.
$\sin^{-1} a$ = the arc whose sine is a.
$\sin a^{-1} = \frac{1}{\sin a}$.
log = logarithm.
$\log_e$ or hyp log = hyperbolic logarithm.
% per cent.
$\angle$ angle.
$\triangle$ triangle.
sin, sine.
cos, cosine.
tan, tangent.
sec, secant
versin, versed sine.
cot, cotangent.
cosec, cosecant.
covers, co-versed sine.

In Algebra, the first letters of the alphabet, a, b, c, d, etc., are generally used to denote known quantities, and the last letters, w, x, y, z, etc., unknown quantities.

Abbreviations and Symbols commonly used.

d, differential (in calculus).
$\int$, integral (in calculus).
$\int_b^a$, integral between limits a and b.
Δ, delta, difference.
Σ, sigma, sign of summation.
π, pi, ratio of circumference of circle to diameter = 3.14159.
g, acceleration due to gravity = 32.16 ft. per second per second.

MATHEMATICS OF FINANCE

12. INTEREST AND DISCOUNT

Interest is *money* paid for the use of *capital* during a period of time. The capital or sum of money borrowed is called the *principal*.

The Rate of Interest is the ratio of the interest earned *in a unit time* to the principal. It is usually expressed as the per cent paid for the use of the principal for *one year*.

Simple Interest, I, is computed by the relation

$$I = Pit \tag{25}$$

where P is the principal, i the rate of interest, and t the time in years.

The *sum*, S, due at the end of the interval of time, t, is

$$S = P + I \tag{26}$$

In Ordinary Simple Interest calculations, a month is regarded as 30 days, and a year as 12 months, or 360 days. Thus, if d denotes the time in days

$$I = Pi\frac{d}{360} \tag{27}$$

Exact Simple Interest. Simple interest computed by taking the exact number of days or 365 days for the year is called *exact simple interest*. Thus

$$I_e = Pi\frac{d}{365} \tag{28}$$

where I_e is the exact simple interest. To facilitate calculation Table 46 shows the number of days between two dates, a year or less apart, and Table 47 the exact simple interest on $1000 for the number of days indicated.

Example. To obtain the number of days between April 13, and December 4, determine the number of days *from* April *to* December, 244, subtract 13, and add 4 Thus, 244 − 13 + 4 = 235 days.

Table 46. Days between the Same Dates of Different Months

(Leap year adds one day in February.)

To → From ↓	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	333	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

Compound Interest is the interest on the principal and its unpaid interest combined at regular intervals. The sum S, or amount to which P will accumulate in n equal conversion periods or time units, is

$$S = P(1 + i)^n \tag{29}$$

If there are m conversion periods per year, n years, and j is the nominal or quoted interest rate per year,

$$S = P\left(1 + \frac{j}{m}\right)^{mn} \tag{30}$$

Table 47. Exact Simple Interest on $1000

$$I_e = 1000i\,\frac{d}{365}$$

d	1 1/2%	2%	2 1/2%	3%	3 1/2%	4%	4 1/2%	5%	5 1/2%	6%
1	0.04110	0.05479	0.06849	0.08219	0.09589	0.10959	0.12329	0.13699	0.15069	0.16438
2	0.08219	0.10959	0.13699	0.16438	0.19178	0.21918	0.24658	0.27397	0.30137	0.32877
3	0.12329	0.16438	0.20548	0.24658	0.28767	0.32877	0.36986	0.41096	0.45206	0.49315
4	0.16438	0.21918	0.27397	0.32877	0.38356	0.43836	0.49315	0.54795	0.60274	0.65753
5	0.20548	0.27397	0.34247	0.41096	0.47945	0.54795	0.61644	0.68493	0.75342	0.82192
6	0.24658	0.32877	0.41096	0.49315	0.57534	0.65754	0.73973	0.82192	0.90411	0.98630
7	0.28767	0.38356	0.47945	0.57534	0.67123	0.76712	0.86301	0.95890	1.05479	1.15068
8	0.32877	0.43836	0.54795	0.65753	0.76712	0.87671	0.98630	1.09589	1.20548	1.31507
9	0.36986	0.49315	0.61644	0.73973	0.86301	0.98630	1.10959	1.23288	1.35616	1.47945
10	0.41096	0.54795	0.68493	0.82192	0.95890	1.09589	1.23288	1.36986	1.50685	1.64384
20	0.82192	1.09589	1.36986	1.64384	1.91781	2.19178	2.46575	2.73973	3.01370	3.28767
30	1.23288	1.64384	2.05480	2.46575	2.87671	3.28767	3.69863	4.10959	4.52055	4.93151
40	1.64384	2.19178	2.73973	3.28767	3.83561	4.38357	4.93151	5.47945	6.02740	6.57534
50	2.05479	2.73973	3.42466	4.10959	4.79452	5.47946	6.16438	6.84932	7.53425	8.21918
60	2.46575	3.28767	4.10959	4.93151	5.75342	6.57535	7.39726	8.21918	9.04110	9.86301
70	2.87671	3.83562	4.79452	5.75342	6.71233	7.67124	8.63017	9.58904	10.5479	11.5068
80	3.28767	4.38356	5.47945	6.57534	7.67123	8.76713	9.86301	10.9589	12.0548	13.1507
90	3.69863	4.93151	6.16438	7.39726	8.63014	9.86302	11.0959	12.3288	13.5616	14.7945
100	4.10959	5.47945	6.84932	8.21918	9.58904	10.9589	12.3288	13.6986	15.0685	16.4384
110	4.52055	6.02740	7.53425	9.04110	10.5479	12.0548	13.5616	15.0685	16.5753	18.0822
120	4.93151	6.57534	8.21918	9.86301	11.5068	13.1507	14.7945	16.4384	18.0822	19.7260
130	5.34247	7.12329	8.90414	10.6849	12.4658	14.2466	16.0274	17.8082	19.5890	21.3699
140	5.75342	7.67123	9.58904	11.5068	13.4246	15.3425	17.2603	19.1781	21.0959	23.0137
150	6.16438	8.21918	10.2740	12.3288	14.3836	16.4384	18.4932	20.5479	22.6027	24.6575
160	6.57534	8.76712	10.9589	13.1507	15.3425	17.5343	19.7260	21.9178	24.1096	26.3014
170	6.98630	9.31507	11.6438	13.9726	16.3014	18.6302	20.9589	23.2877	25.6164	27.9452
180	7.39726	9.86301	12.3288	14.7945	17.2603	19.7260	22.1918	24.6575	27.1233	29.5890
190	7.80822	10.41096	13.0137	15.6164	18.2192	20.8219	23.4247	26.0274	28.6301	31.2329
200	8.21918	10.95890	13.6986	16.4384	19.1781	21.9178	24.6575	27.3973	30.1370	32.8767
210	8.63014	11.50685	14.3836	17.2603	20.1370	23.0137	25.8904	28.7671	31.6438	34.5205
220	9.04110	12.05479	15.0685	18.0822	21.0959	24.1096	27.1233	30.1370	33.1507	36.1644
230	9.45205	12.60274	15.7534	18.9041	22.0548	25.2055	28.3562	31.5068	34.6575	37.8082
240	9.86301	13.15068	16.4383	19.7260	23.0137	26.3014	29.5890	32.8767	36.1644	39.4521
250	10.27397	13.69863	17.1233	20.5479	23.9726	27.3973	30.8219	34.2466	37.6712	41.0959
260	10.68493	14.24658	17.8082	21.3699	24.9315	28.4932	32.0548	35.6164	39.1781	42.7397
270	11.09589	14.79452	18.4931	22.1918	25.8904	29.5891	33.2877	36.9863	40.6849	44.3836
280	11.50685	15.34247	19.1781	23.0137	26.8493	30.6850	34.5205	38.3562	42.1918	46.0274
290	11.91781	15.89041	19.8630	23.8356	27.8082	31.7809	35.7534	39.7260	43.6986	47.6712
300	12.32877	16.43836	20.5479	24.6575	28.7671	32.8767	36.9863	41.0959	45.2055	49.3151
310	12.73973	16.98630	21.2329	25.4795	29.7260	33.9726	38.2192	42.4658	46.7123	50.9589
320	13.15068	17.53425	21.9178	26.3014	30.6849	35.0685	39.4521	43.8356	48.2192	52.6027
330	13.56164	18.08219	22.6027	27.1233	31.6438	36.1644	40.6849	45.2055	49.7260	54.2466
340	13.97260	18.63014	23.2876	27.9452	32.6027	37.2603	41.9178	46.5753	51.2329	55.8904
350	14.38356	19.17808	23.9726	28.7671	33.5616	38.3562	43.1507	47.9452	52.7397	57.5342
360	14.79452	19.72603	24.6575	29.5890	34.5205	39.4521	44.3836	49.3151	54.2466	59.1781

Values of the sum, S, if $P = 1$ (amount of one) may be obtained from Table 48 for values of n from 1 to 50 at different rates, i. The value of S for (29) may also be calculated with the aid of logarithms, thus

$$\log S = \log P + n \log (1 + i) \tag{31}$$

Example. To find the amount of $3900 at 3 1/2 per cent compounded annually for 29 years proceed in this way. (*a*) Enter the table at the column headed 3 1/2 per cent and opposite $n = 29$ read 2.71188 ···, the amount of 1. (*b*) Multiply this tabular value by 3900 to obtain the result. To use the table for a value of time not given use the relation that the amount of 1 for $n_1 + n_2$ years is equal to the amount for n_1 years times the amount for n_2 years. Thus if the table extends to only 50 periods of time, the amount of 1 for 67 periods at the rate 0.04 is $(1.04)^{67} = (1.04)^{50}\,(1.04)^{17}$, and the two factors of the second member can be read from the table.

Table 48. Amount of 1 (Compound Interest)

$s = (1 + i)^n$

n	1 1/2%	2%	2 1/2%	3%	3 1/2%	4%	4 1/2%	5%	5 1/2%	6%
1	1.01500	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000	1.05500	1.06000
2	1.03023	1.04040	1.05062	1.06090	1.07122	1.08160	1.09203	1.10250	1.11303	1.12360
3	1.04568	1.06121	1.07689	1.09273	1.10872	1.12486	1.14117	1.15763	1.17424	1.19102
4	1.06136	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551	1.23882	1.26248
5	1.07728	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628	1.30696	1.33823
6	1.09344	1.12616	1.15969	1.19405	1.22926	1.26532	1.30226	1.34010	1.37884	1.41852
7	1.10984	1.14869	1.18869	1.22987	1.27228	1.31593	1.36086	1.40710	1.45468	1.50363
8	1.12649	1.17166	1.21840	1.26677	1.31681	1.36857	1.42210	1.47746	1.53469	1.59385
9	1.14339	1.19509	1.24886	1.30477	1.36290	1.42331	1.48610	1.55133	1.61909	1.68948
10	1.16054	1.21899	1.28008	1.34392	1.41060	1.48024	1.55297	1.62889	1.70814	1.79085
11	1.17795	1.24337	1.31209	1.38423	1.45997	1.53945	1.62285	1.71034	1.80209	1.89830
12	1.19562	1.26824	1.34489	1.42576	1.51107	1.60103	1.69588	1.79586	1.90121	2.01220
13	1.21355	1.29361	1.37851	1.46853	1.56396	1.66507	1.77220	1.88565	2.00577	2.13293
14	1.23176	1.31948	1.41297	1.51259	1.61869	1.73168	1.85194	1.97993	2.11609	2.26090
15	1.25023	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893	2.23248	2.39656
16	1.26899	1.37279	1.48451	1.60471	1.73399	1.87298	2.02237	2.18287	2.35526	2.54035
17	1.28802	1.40024	1.52162	1.65285	1.79468	1.94790	2.11338	2.29202	2.48480	2.69277
18	1.30734	1.42825	1.55966	1.70243	1.85749	2.02582	2.20848	2.40662	2.62147	2.85434
19	1.32695	1.45681	1.59865	1.75351	1.92250	2.10685	2.30786	2.52695	2.76565	3.02560
20	1.34685	1.48595	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330	2.91776	3.20714
21	1.36706	1.51567	1.67958	1.86029	2.05943	2.27877	2.52024	2.78596	3.07823	3.39956
22	1.38756	1.54598	1.72157	1.91610	2.13151	2.36992	2.63365	2.92526	3.24754	3.60354
23	1.40838	1.57690	1.76461	1.97359	2.20611	2.46472	2.75217	3.07152	3.42615	3.81975
24	1.42950	1.60844	1.80873	2.03279	2.28333	2.56330	2.87601	3.22510	3.61459	4.04893
25	1.45095	1.64061	1.85394	2.09378	2.36324	2.66584	3.00543	3.38635	3.81339	4.29187
26	1.47271	1.67342	1.90029	2.15659	2.44596	2.77247	3.14068	3.55567	4.02313	4.54938
27	1.49480	1.70689	1.94780	2.22129	2.53157	2.88337	3.28201	3.73346	4.24440	4.82235
28	1.51722	1.74102	1.99650	2.28793	2.62017	2.99870	3.42970	3.92013	4.47784	5.11169
29	1.53998	1.77584	2.04640	2.35657	2.71188	3.11865	3.58404	4.11614	4.72412	5.41839
30	1.56308	1.81136	2.09757	2.42726	2.80679	3.24340	3.74532	4.32194	4.98395	5.74349
31	1.58653	1.84759	2.15001	2.50008	2.90503	3.37313	3.91386	4.53804	5.25807	6.08810
32	1.61032	1.88454	2.20376	2.57508	3.00671	3.50806	4.08998	4.76494	5.54726	6.45339
33	1.63448	1.92223	2.25885	2.65234	3.11194	3.64838	4.27403	5.00319	5.85236	6.84059
34	1.65900	1.96068	2.31532	2.73191	3.22086	3.79432	4.46636	5.25335	6.17424	7.25103
35	1.68388	1.99989	2.37321	2.81386	3.33359	3.94609	4.66735	5.51602	6.51383	7.68609
36	1.70914	2.03989	2.43254	2.89828	3.45027	4.10393	4.87738	5.79182	6.87209	8.14725
37	1.73478	2.08069	2.49335	2.98523	3.57103	4.26809	5.09686	6.08141	7.25005	8.63609
38	1.76080	2.12230	2.55568	3.07478	3.69601	4.43881	5.32622	6.38548	7.64880	9.15425
39	1.78721	2.16474	2.61957	3.16703	3.82537	4.61637	5.56590	6.70475	8.06949	9.70351
40	1.81402	2.20804	2.68506	3.26204	3.95926	4.80102	5.81636	7.03999	8.51331	10.2857
41	1.84123	2.25220	2.75219	3.35990	4.09783	4.99306	6.07810	7.39199	8.98154	10.9029
42	1.86885	2.29724	2.82100	3.46070	4.24126	5.19278	6.35162	7.76159	9.47553	11.5570
43	1.89688	2.34319	2.89152	3.56452	4.38970	5.40050	6.63744	8.14967	9.99668	12.2505
44	1.92533	2.39005	2.96381	3.67145	4.54334	5.61652	6.93612	8.55715	10.5465	12.9855
45	1.95241	2.43785	3.03790	3.78160	4.70236	5.84118	7.24825	8.98501	11.1266	13.7646
46	1.98353	2.48661	3.11385	3.89504	4.86694	6.07482	7.57442	9.43426	11.7385	14.5905
47	2.01328	2.53634	3.19170	4.01190	5.03728	6.31782	7.91527	9.90597	12.3841	15.4659
48	2.04348	2.58707	3.27149	4.13225	5.21359	6.57053	8.27146	10.4013	13.0653	16.3939
49	2.07413	2.63881	3.35328	4.25622	5.39606	6.83335	8.64367	10.9213	13.7838	17.3775
50	2.10524	2.69159	3.43711	4.38391	5.58493	7.10668	9.03264	11.4674	14.5420	18.4202

The Present Value of a sum S due in n years is defined as the principal P which drawing a given rate of (compound) interest i will in n years amount to S,

$$P = S(1 + i)^{-n} = Sv^n, \quad \text{if} \quad v = (1 + i)^{-1} \tag{32}$$

Values of P or v^n for $S = 1$, may be obtained from Table 49.

Table 49. Present Value of 1

$$v^n = (1 + i)^{-n}$$

n	1 1/2%	2%	2 1/2%	3%	3 1/2%	4%	4 1/2%	5%	5 1/2%	6%
1	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94787	0.94340
2	0.97066	0.96117	0.95181	0.94260	0.93351	0.92456	0.91573	0.90703	0.89845	0.89000
3	0.95632	0.94232	0.92860	0.91514	0.90194	0.88900	0.87630	0.86384	0.85161	0.83962
4	0.94218	0.92385	0.90595	0.88849	0.87144	0.85480	0.83856	0.82270	0.80722	0.79209
5	0.92826	0.90573	0.88385	0.86261	0.84197	0.82193	0.80245	0.78353	0.76513	0.74726
6	0.91454	0.88797	0.86230	0.83748	0.81350	0.79031	0.76790	0.74622	0.72525	0.70496
7	0.90103	0.87056	0.84127	0.81309	0.78599	0.75992	0.73483	0.71068	0.68744	0.66506
8	0.88771	0.85349	0.82075	0.78941	0.75941	0.73069	0.70319	0.67684	0.65160	0.62741
9	0.87459	0.83676	0.80073	0.76642	0.73373	0.70259	0.67290	0.64461	0.61763	0.59190
10	0.86167	0.82035	0.78120	0.74409	0.70892	0.67556	0.64393	0.61391	0.58543	0.55839
11	0.84893	0.80426	0.76214	0.72242	0.68495	0.64958	0.61620	0.58468	0.55491	0.52679
12	0.83639	0.78849	0.74356	0.70138	0.66178	0.62460	0.58966	0.55684	0.52598	0.49697
13	0.82403	0.77303	0.72542	0.68095	0.63940	0.60057	0.56427	0.53032	0.49856	0.46884
14	0.81185	0.75788	0.70773	0.66112	0.61778	0.57748	0.53997	0.50507	0.47257	0.44230
15	0.79985	0.74301	0.69047	0.64186	0.59689	0.55526	0.51672	0.48102	0.44793	0.41727
16	0.78803	0.72845	0.67362	0.62317	0.57671	0.53391	0.49447	0.45811	0.42458	0.39365
17	0.77639	0.71416	0.65720	0.60502	0.55720	0.51337	0.47318	0.43630	0.40245	0.37136
18	0.76491	0.70016	0.64117	0.58739	0.53836	0.49363	0.45280	0.41552	0.38147	0.35034
19	0.75361	0.68643	0.62553	0.57029	0.52016	0.47464	0.43330	0.39573	0.36158	0.33051
20	0.74247	0.67297	0.61027	0.55368	0.50257	0.45639	0.41464	0.37689	0.34273	0.31180
21	0.73150	0.65978	0.59539	0.53755	0.48557	0.43883	0.39679	0.35894	0.32486	0.29416
22	0.72069	0.64684	0.58086	0.52189	0.46915	0.42196	0.37970	0.34185	0.30793	0.27751
23	0.71004	0.63416	0.56670	0.50669	0.45329	0.40573	0.36335	0.32557	0.29187	0.26180
24	0.69954	0.62172	0.55288	0.49193	0.43796	0.39012	0.34770	0.31007	0.27666	0.24698
25	0.68921	0.60953	0.53939	0.47761	0.42315	0.37512	0.33273	0.29530	0.26223	0.23300
26	0.67902	0.59758	0.52623	0.46369	0.40884	0.36069	0.31840	0.28124	0.24856	0.21981
27	0.66899	0.58586	0.51340	0.45019	0.39501	0.34682	0.30469	0.26785	0.23560	0.20737
28	0.65910	0.57437	0.50088	0.43708	0.38165	0.33348	0.29157	0.25509	0.22332	0.19563
29	0.64936	0.56311	0.48866	0.42435	0.36875	0.32065	0.27902	0.24295	0.21168	0.18456
30	0.63976	0.55207	0.47674	0.41199	0.35628	0.30832	0.26700	0.23138	0.20064	0.17411
31	0.63031	0.54125	0.46511	0.39999	0.34423	0.29646	0.25550	0.22036	0.19018	0.16425
32	0.62099	0.53063	0.45377	0.38834	0.33259	0.28506	0.24450	0.20987	0.18027	0.15496
33	0.61182	0.52023	0.44270	0.37703	0.32134	0.27409	0.23397	0.19987	0.17087	0.14619
34	0.60277	0.51003	0.43191	0.36604	0.31048	0.26355	0.22390	0.19035	0.16196	0.13791
35	0.59387	0.50003	0.42137	0.35538	0.29998	0.25342	0.21425	0.18129	0.15352	0.13011
36	0.58509	0.49022	0.41109	0.34503	0.28983	0.24367	0.20503	0.17266	0.14552	0.12274
37	0.57644	0.48061	0.40107	0.33498	0.28003	0.23430	0.19620	0.16444	0.13793	0.11579
38	0.56792	0.47119	0.39128	0.32523	0.27056	0.22529	0.18775	0.15661	0.13074	0.10924
39	0.55953	0.46195	0.38174	0.31575	0.26141	0.21662	0.17967	0.14915	0.12392	0.10306
40	0.55126	0.45289	0.37243	0.30656	0.25257	0.20829	0.17193	0.14205	0.11746	0.09722
41	0.54312	0.44401	0.36335	0.29763	0.24403	0.20028	0.16453	0.13528	0.11134	0.09172
42	0.53509	0.43530	0.35448	0.28896	0.23578	0.19257	0.15744	0.12884	0.10554	0.08653
43	0.52718	0.42677	0.34584	0.28054	0.22781	0.18517	0.15066	0.12270	0.10003	0.08163
44	0.51939	0.41840	0.33740	0.27237	0.22010	0.17805	0.14417	0.11686	0.09482	0.07701
45	0.51171	0.41020	0.32917	0.26444	0.21266	0.17120	0.13796	0.11130	0.08988	0.07265
46	0.50415	0.40215	0.32115	0.25674	0.20547	0.16461	0.13202	0.10600	0.08519	0.06854
47	0.49670	0.39427	0.31331	0.24926	0.19852	0.15828	0.12634	0.10095	0.08075	0.06466
48	0.48936	0.38654	0.30567	0.24200	0.19181	0.15219	0.12090	0.09614	0.07654	0.06100
49	0.48213	0.37896	0.29822	0.23495	0.18532	0.14634	0.11569	0.09156	0.07255	0.05755
50	0.47500	0.37153	0.29094	0.22811	0.17905	0.14071	0.11071	0.08720	0.06877	0.05429

Example. The present value of $3900 due in 17 years with the (compound) interest rate 3 1/2 per cent is found in the following way: (*a*) Enter the table at the column headed 3 1/2 per cent and opposite $n = 17$ read 0.55720 ···, the present value of 1. (*b*) Multiply this tabular value by 3900 to obtain the result.

Discount is the money paid at the beginning of an interval of time for the use of capital

during that interval. Denote the rate of interest by i, and the *rate of discount* by d. Then

$$d = \frac{i}{1 + i} \tag{33}$$

In place of the type of discount just defined, which can be called *compound discount*, it is usual practice for the banker to give for each unit of the bill an amount $1 - d/m$, in which d is the rate of discount per year and $1/m$ indicates the fractional part of a year. For fractional parts of a year, then, the banker uses what is called *simple discount*.

The Average Due Time for a number of sums, due at different times, is the time at which they may all be paid with due regard to the equities involved. Denote by S_1, S_2, $\cdots$, S_r the sums due at the ends of n_1, n_2, $\cdots$, n_r years, respectively, and the average due time by T. On the assumption that money is worth a (compound) rate of interest i,

$$T = \frac{\log \sum_{k=1}^{r} S_k - \log \sum_{k=1}^{r} \frac{S_k}{(1 + i)^{n_k}}}{\log (1 + i)} \tag{34}$$

If the times are small, the following approximate relation may be employed:

$$T \approx \frac{\sum_{k=1}^{r} n_k S_k}{\sum_{k=1}^{r} S_k} \tag{35}$$

13. ANNUITIES AND SINKING FUNDS

An Annuity Certain is a succession of periodic payments for a definite (certain) term of years. The first payment of an annuity is supposed to be made at the end of the first period. The sum or amount to which the payments accumulate is called the *amount of the annuity*. Ordinarily the nominal rate of interest j and the number m of conversion intervals per year are known, so the effective rate i can be determined. It is given by

$$i = \left(1 + \frac{j}{m}\right)^m - 1 \tag{36}$$

Denote the amount of an annuity of 1 per year payable annually for n years by $s_{\overline{n}|}$. Then

$$s_{\overline{n}|} = \frac{(1 + i)^n - 1}{i} \tag{37}$$

The yearly payment of an annuity is called the *annual rent*. Denote the annual rent of an annuity by R, and the amount for n years by S. Then

$$S = Rs_{\overline{n}|} = \frac{R[(1 + i)^n - 1]}{i} \tag{38}$$

This relation may be solved for n,

$$n = \frac{\log (Si + R) - \log R}{\log (1 + i)} \tag{39}$$

a relation which is exact if n is an integer and approximate if n is not an integer.

The Present Value of an Annuity Certain is the sum of the present values of all the future payments. Denote the present value of an annuity certain of 1 per year for n years by $a_{\overline{n}|}$ and the effective rate of interest by i. Then

$$a_{\overline{n}|} = \frac{1 - (1 + i)^{-n}}{i} \tag{40}$$

Denote the present value of an annuity certain of R per year by A. Then

$$A = Ra_{\overline{n}|} = \frac{R[1 - (1 + i)^{-n}]}{i} \tag{41}$$

As in the preceding paragraph,

$$i = \left(1 + \frac{j}{m}\right)^m - 1 \tag{42}$$

Table 50. Amount of 1 per Annum

$$s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

n	1 1/2%	2%	2 1/2%	3%	3 1/2%	4%	4 1/2%	5%	5 1/2%	6%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.01500	2.02000	2.02500	2.03000	2.03500	2.04000	2.04500	2.05000	2.05500	2.06000
3	3.04522	3.06040	3.07562	3 09090	3.10623	3.12160	3.13702	3 15250	3.16803	3.18360
4	4.09090	4.12161	4.15252	4.18363	4.21494	4.24646	4.27819	4.31013	4.34227	4.37462
5	5.15227	5.20404	5.25633	5.30914	5.36247	5.41632	5.47071	5.52563	5.58109	5.63709
6	6.22955	6.30812	6.38774	6.46841	6.55015	6.63298	6 71689	6.80191	6.88805	6.97532
7	7.32299	7.43428	7.54743	7.66246	7.77941	7.89829	8.01915	8.14201	8.26689	8.39384
8	8.43284	8.58297	8.73612	8.89234	9.05169	9.21423	9.38001	9.54911	9.72157	9.89747
9	9.55933	9 75463	9.95452	10.1591	10.3685	10.5828	10.8021	11.0266	11.2563	11.4913
10	10.70272	10.94972	11.2034	11.4639	11.7314	12.0061	12.2882	12.5779	12.8754	13.1808
11	11.86326	12.16872	12.4835	12.8078	13.1420	13.4864	13.8412	14.2068	14.5835	14.9716
12	13.04121	13.41209	13.7956	14.1920	14.6020	15.0258	15.4640	15.9171	16.3856	16.8699
13	14.23683	14.68033	15.1404	15.6178	16.1130	16.6268	17.1599	17.7130	18.2868	18.8821
14	15.45038	15.97394	16.5190	17 0863	17.6770	18.2919	18.9321	19.5986	20.2926	21.0151
15	16.68214	17.29342	17.9319	18.5989	19.2957	20.0236	20.7841	21.5786	22.4087	23.2760
16	17.93237	18.62929	19.3802	20.1569	20.9710	21.8245	22.7193	23.6575	24.6411	25.6725
17	19.20136	20 01207	20.8647	21.7616	22.7050	23.6975	24.7417	25.8404	26.9964	28.2129
18	20.48938	21.41231	22.3863	23.4144	24.4997	25.6454	26.8551	28.1324	29.4812	30.9057
19	21.79672	22 84056	23.9460	25.1169	26.3572	27.6712	29.0636	30.5390	32.1027	33.7600
20	23.12367	24.29737	25.5447	26.8704	28.2797	29.7781	31.3714	33.0660	34.8683	36.7856
21	24.47052	25.78332	27.1833	28.6765	30.2695	31.9692	33.7831	35.7193	37.7861	39.9927
22	25.83758	27.29899	28.8629	30.5368	32.3289	34.2480	36.3034	38.5052	40.8643	43.3923
23	27.22514	28.84496	30.5844	32.4529	34.4604	36.6179	38.9370	41.4305	44.1118	46.9958
24	28.63352	30.42186	32 3490	34.4265	36 6665	39.0826	41.6892	44.5020	47.5380	50.8156
25	30.06302	32.03030	34.1578	36.4593	38.9499	41.6459	44.5652	47.7271	51.1526	54.8645
26	31.51397	33.67091	36.0117	38.5530	41.3131	44.3117	47.5706	51.1135	54.9660	59.1564
27	32.98668	35.34432	37.9120	40.7096	43.7591	47 0842	50.7113	54.6691	58.9891	63.7058
28	34.48148	37.05121	39.8598	42.9309	46.2906	49.9676	53.9933	58.4026	63.2335	68.5281
29	35.99870	38.79223	41.8563	45.2189	48.9108	52.9663	57.4230	62.3227	67.7114	73.6398
30	37.53868	40.56808	43.9027	47.5754	51.6227	56 0849	61.0071	66.4388	72.4355	79.0582
31	39.10176	42.37944	46 0003	50.0027	54.4295	59.3283	64.7524	70.7608	77.4194	84.8017
32	40.68829	44.22703	48.1503	52.5028	57.3345	62.7015	68.6662	75.2988	82.6775	90.8898
33	42.29861	46.11157	50.3540	55.0778	60 3412	66.2095	72.7562	80.0638	88.2248	97.3432
34	43.93309	48.03380	52.6129	57.7302	63.4532	69.8579	77.0303	85.0670	94.0771	104.184
35	45.59208	49.99448	54.9282	60.4621	66.6740	73.6522	81.4966	90.3203	100.251	111.435
36	47.27597	51.99437	57.3014	63.2759	70.0076	77.5983	86 1640	95.8363	106.765	119.121
37	48.98511	54.03425	59.7339	66.1742	73.4579	81.7022	91.0413	101.628	113.637	127.268
38	50.71989	56.11494	62.2273	69.1594	77.0289	85.9703	96.1382	107.710	120.887	135.904
39	52.48068	58.23724	64.7830	72.2342	80.7249	90.4091	101.464	114.095	128.536	145.058
40	54.26789	60.40198	67.4026	75.4013	84.5503	95.0255	107.030	120.800	136.606	154.762
41	56.08191	62.61002	70.0876	78.6633	88.5095	99 8265	112.847	127.840	145.119	165.048
42	57.92314	64.86222	72.8398	82.0232	92.6074	104.820	118.925	135.232	154.100	175.951
43	59.79199	67.15947	75.6608	85 4839	96.8486	110 012	125.276	142.993	163 576	187.508
44	61.68887	69.50266	78.5523	89.0484	101.238	115.413	131.914	151.143	173.573	199.758
45	63.61420	71.89271	81.5161	92.7199	105.782	121.029	138.850	159.700	184.119	212.744
46	65.56841	74.33056	84.5540	96.5015	110.484	126.871	146.098	168.685	195 246	226.508
47	67.55194	76.81718	87.6679	100.397	115.351	132.945	153.673	178.119	206.984	241.099
48	69.56522	79.35352	90.8596	104.408	120.388	139.263	161.588	188.025	219.368	256.565
49	71.60870	81.94059	94.1311	108 541	125.602	145.834	169.859	198.427	232.434	272.958
50	73.68283	84.57940	97 4843	112.797	130 998	152.667	178.503	209.348	246.217	290.336

Example. The amount of an annuity certain yielding an annual rent of $400 for 10 years with effective interest at 4 per cent may be found in the following way: (*a*) Enter the table in the column headed 4 per cent and opposite $n = 10$ read 12.0061 ···, the amount of 1. (*b*) Multiply this tabular value by 400 to obtain the result.

Table 51. Present Value of 1 per Annum

$$a_{\overline{n}|} = \frac{1 - (1 + i)^{-n}}{i}$$

n	1 1/2%	2%	2 1/2%	3%	3 1/2%	4%	4 1/2%	5%	5 1/2%	6%
1	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94787	0.94340
2	1.95588	1.94156	1.92742	1.91347	1.89969	1.88609	1.87267	1.85941	1.84632	1.83339
3	2.91220	2.88388	2.85602	2.82861	2.80164	2.77509	2.74896	2.72325	2.69793	2.67301
4	3.85438	3.80773	3.76197	3.71710	3.67308	3.62990	3.58753	3.54595	3.50515	3.46511
5	4.78264	4.71346	4.64583	4.57971	4.51505	4.45182	4.38998	4.32948	4.27028	4.21236
6	5.69719	5.60143	5.50813	5.41719	5.32855	5.24214	5.15787	5.07569	4.99553	4.91732
7	6.59821	6.47199	6.34939	6.23028	6.11454	6.00205	5.89270	5.78637	5.68297	5.58238
8	7.48593	7.32548	7.17014	7.01969	6.87396	6.73274	6.59589	6.46321	6.33457	6.20979
9	8.36052	8.16224	7.97087	7.78611	7.60769	7.43533	7.26879	7.10782	6.95220	6.80169
10	9.22218	8.98258	8.75206	8.53020	8.31661	8.11090	7.91272	7.72173	7.53763	7.36009
11	10.07112	9.78685	9.51421	9.25262	9.00155	8.76048	8.52892	8.30641	8.09254	7.88687
12	10.90751	10.57534	10.2578	9.95400	9.66333	9.38507	9.11858	8.86325	8.61852	8.38384
13	11.73153	11.34837	10.9832	10.6350	10.3027	9.98565	9.68285	9.39357	9.11708	8.85268
14	12.54338	12.10625	11.6909	11.2961	10.9205	10.5631	10.2228	9.89864	9.58965	9.29498
15	13.34323	12.84926	12.3814	11.9379	11.5174	11.1184	10.7395	10.3797	10.0376	9.71225
16	14.13126	13.57771	13.0550	12.5611	12.0941	11.6523	11.2340	10.8378	10.4622	10.1059
17	14.90765	14.29187	13.7122	13.1661	12.6513	12.1657	11.7072	11.2741	10.8646	10.4773
18	15.67256	14.99203	14.3534	13.7535	13.1897	12.6593	12.1600	11.6896	11.2461	10.8276
19	16.42617	15.67846	14.9789	14.3238	13.7098	13.1339	12.5933	12.0853	11.6077	11.1581
20	17.16864	16.35143	15.5892	14.8775	14.2124	13.5903	13.0079	12.4622	11.9504	11.4699
21	17.90014	17.01121	16.1845	15.4150	14.6980	14.0292	13.4047	12.8212	12.2752	11.7641
22	18.62082	17.65805	16.7654	15.9369	15.1671	14.4511	13.7844	13.1630	12.5832	12.0416
23	19.33086	18.29220	17.3321	16.4436	15.6204	14.8568	14.1478	13.4886	12.8750	12.3034
24	20.03041	18.91393	17.8850	16.9355	16.0584	15.2470	14.4955	13.7986	13.1517	12.5504
25	20.71961	19.52346	18.4244	17.4131	16.4815	15.6221	14.8282	14.0939	13.4139	12.7834
26	21.39863	20.12104	18.9506	17.8768	16.8904	15.9828	15.1466	14.3752	13.6625	13.0032
27	22.06762	20.70690	19.4640	18.3270	17.2854	16.3296	15.4513	14.6430	13.8981	13.2105
28	22.72672	21.28127	19.9649	18.7641	17.6670	16.6631	15.7429	14.8981	14.1214	13.4062
29	23.37608	21.84438	20.4535	19.1885	18.0358	16.9837	16.0219	15.1411	14.3331	13.5907
30	24.01584	22.39646	20.9303	19.6004	18.3920	17.2920	16.2889	15.3725	14.5337	13.7648
31	24.64615	22.93770	21.3954	20.0004	18.7363	17.5885	16.5444	15.5928	14.7239	13.9291
32	25.26714	23.46833	21.8492	20.3888	19.0689	17.8736	16.7889	15.8027	14.9042	14.0840
33	25.87895	23.98856	22.2919	20.7658	19.3902	18.1476	17.0229	16.0025	15.0751	14.2302
34	26.48173	24.49859	22.7238	21.1318	19.7007	18.4112	17.2468	16.1929	15.2370	14.3681
35	27.07559	24.99862	23.1452	21.4872	20.0007	18.6646	17.4610	16.3742	15.3906	14.4982
36	27.66068	25.48884	23.5563	21.8323	20.2905	18.9083	17.6660	16.5469	15.5361	14.6210
37	28.23713	25.96945	23.9573	22.1672	20.5705	19.1426	17.8622	16.7113	15.6740	14.7368
38	28.80505	26.44064	24.3486	22.4925	20.8411	19.3679	18.0500	16.8679	15.8047	14.8460
39	29.36458	26.90259	24.7303	22.8082	21.1025	19.5845	18.2297	17.0170	15.9287	14.9491
40	29.91585	27.35548	25.1028	23.1148	21.3551	19.7928	18.4016	17.1591	16.0461	15.0463
41	30.45896	27.79949	25.4661	23.4124	21.5991	19.9931	18.5661	17.2944	16.1575	15.1380
42	30.99405	28.23479	25.8206	23.7014	21.8349	20.1856	18.7235	17.4232	16.2630	15.2245
43	31.52123	28.66156	26.1664	23.9819	22.0627	20.3708	18.8742	17.5459	16.3630	15.3062
44	32.04062	29.07996	26.5038	24.2543	22.2828	20.5488	19.0184	17.6628	16.4579	15.3832
45	32.55234	29.49016	26.8330	24.5187	22.4955	20.7200	19.1563	17.7741	16.5477	15.4558
46	33.05649	29.89231	27.1542	24.7754	22.7009	20.8847	19.2884	17.8801	16.6329	15.5244
47	33.55319	30.28658	27.4675	25.0247	22.8994	21.0429	19.4147	17.9810	16.7137	15.5890
48	34.04255	30.67312	27.7732	25.2667	23.0912	21.1951	19.5356	18.0772	16.7902	15.6500
49	34.52468	31.05208	28.0714	25.5017	23.2766	21.3415	19.6513	18.1687	16.8628	15.7076
50	34.99969	31.42361	28.3623	25.7298	23.4556	21.4822	19.7620	18.2559	16.9315	15.7619

Example. The present value of $39 per year for 20 years with an effective interest rate of 5 per cent is found in the following way: (*a*) Enter the table at the column headed 5 per cent and opposite $n = 20$ read 12.4622 ···, the present value of 1 per year. (*b*) Multiply the tabular value by 39 to obtain the result.

The Annuity Which 1 Will Purchase is the annuity whose present value is 1. Denote the annual rent of such an annuity by r. Then

$$r = \frac{1}{a_{\overline{n}|}} \tag{43}$$

If the purchase price is A, the annual rent is

$$R = rA = \frac{A}{a_{\overline{n}|}} = \frac{Ai}{1 - (1 + i)^{-n}} \tag{44}$$

Table 52. Annuity Which 1 Will Purchase

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1 - (1 + i)^{-n}} \quad \text{or} \quad \frac{1}{s_{\overline{n}|}} + i$$

n	1 1/2%	2%	2 1/2%	3%	3 1/2%	4%	4 1/2%	5%	5 1/2%	6%
1	1.015000	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000	1.050000	1.055000	1.060000
2	0.511278	0.515050	0.518827	0.522611	0.526400	0.530196	0.533998	0.537805	0.541618	0.545437
3	0.343383	0.346755	0.350137	0.353530	0.356934	0.360349	0.363773	0.367209	0.370654	0.374110
4	0.259445	0.262624	0.265818	0.269027	0.272251	0.275490	0.278744	0.282012	0.285294	0.288591
5	0.209089	0.212158	0.215247	0.218355	0.221481	0.224627	0.227792	0.230975	0.234176	0.237396
6	0.175525	0.178526	0.181550	0.184598	0.187668	0.190762	0.193878	0.197017	0.200179	0.203363
7	0.151556	0.154512	0.157495	0.160506	0.163544	0.166610	0.169701	0.172820	0.175964	0.179135
8	0.133584	0.136510	0.139467	0.142456	0.145477	0.148528	0.151610	0.154722	0.157864	0.161036
9	0.119610	0.122515	0.125457	0.128434	0.131446	0.134493	0.137574	0.140690	0.143839	0.147022
10	0.108434	0.111327	0.114259	0.117231	0.120241	0.123291	0.126379	0.129505	0.132668	0.135868
11	0.099294	0.102178	0.105106	0.108077	0.111092	0.114149	0.117248	0.120389	0.123571	0.126793
12	0.091680	0.094560	0.097487	0.100462	0.103484	0.106552	0.109666	0.112825	0.116029	0.119277
13	0.085240	0.088118	0.091048	0.094030	0.097062	0.100144	0.103275	0.106456	0.109684	0.112960
14	0.079723	0.082602	0.085537	0.088526	0.091571	0.094669	0.097820	0.101024	0.104279	0.107585
15	0.074944	0.077826	0.080766	0.083767	0.086825	0.089941	0.093114	0.096342	0.099626	0.102963
16	0.070765	0.073650	0.076599	0.079611	0.082685	0.085820	0.089015	0.092270	0.095583	0.098952
17	0.067080	0.069970	0.072928	0.075953	0.079043	0.082199	0.085418	0.088699	0.092042	0.095445
18	0.063806	0.066702	0.069670	0.072709	0.075817	0.078993	0.082237	0.085546	0.088920	0.092357
19	0.060879	0.063782	0.066761	0.069814	0.072940	0.076139	0.079407	0.082745	0.086150	0.089621
20	0.058246	0.061157	0.064147	0.067216	0.070361	0.073582	0.076876	0.080243	0.083679	0.087185
21	0.055866	0.058785	0.061787	0.064872	0.068037	0.071280	0.074601	0.077996	0.081465	0.085005
22	0.053703	0.056631	0.059647	0.062747	0.065932	0.069199	0.072546	0.075971	0.079471	0.083046
23	0.051731	0.054668	0.057696	0.060814	0.064019	0.067309	0.070682	0.074137	0.077670	0.081278
24	0.049924	0.052871	0.055913	0.059047	0.062273	0.065587	0.068987	0.072471	0.076036	0.079679
25	0.048264	0.051220	0.054276	0.057428	0.060674	0.064012	0.067439	0.070952	0.074549	0.078227
26	0.046732	0.049699	0.052769	0.055938	0.059205	0.062567	0.066021	0.069564	0.073193	0.076904
27	0.045315	0.048293	0.051377	0.054564	0.057852	0.061239	0.064719	0.068292	0.071952	0.075697
28	0.044001	0.046990	0.050088	0.053293	0.056603	0.060013	0.063521	0.067123	0.070814	0.074593
29	0.042779	0.045778	0.048891	0.052115	0.055445	0.058880	0.062415	0.066046	0.069769	0.073580
30	0.041639	0.044650	0.047778	0.051019	0.054371	0.057830	0.061392	0.065051	0.068805	0.072649
31	0.040574	0.043596	0.046739	0.049999	0.053372	0.056855	0.060443	0.064132	0.067917	0.071792
32	0.039577	0.042611	0.045768	0.049047	0.052442	0.055949	0.059563	0.063280	0.067095	0.071002
33	0.038641	0.041687	0.044859	0.048156	0.051572	0.055104	0.058745	0.062490	0.066335	0.070273
34	0.037762	0.040819	0.044007	0.047322	0.050760	0.054315	0.057982	0.061755	0.065630	0.069598
35	0.036934	0.040002	0.043206	0.046539	0.049998	0.053577	0.057270	0.061072	0.064975	0.068974
36	0.036152	0.039233	0.042452	0.045804	0.049284	0.052887	0.056606	0.060434	0.064366	0.068395
37	0.035414	0.038507	0.041741	0.045112	0.048613	0.052240	0.055984	0.059840	0.063800	0.067857
38	0.034716	0.037821	0.041070	0.044459	0.047982	0.051632	0.055402	0.059284	0.063272	0.067358
39	0.034055	0.037171	0.040436	0.043844	0.047388	0.051061	0.054856	0.058765	0.062780	0.066894
40	0.033427	0.036556	0.039836	0.043262	0.046827	0.050523	0.054343	0.058278	0.062320	0.066462
41	0.032831	0.035972	0.039268	0.042712	0.046298	0.050017	0.053862	0.057822	0.061891	0.066059
42	0.032264	0.035417	0.038729	0.042192	0.045798	0.049540	0.053409	0.057395	0.061489	0.065683
43	0.031725	0.034890	0.038217	0.041698	0.045325	0.049090	0.052982	0.056993	0.061113	0.065333
44	0.031210	0.034388	0.037730	0.041230	0.044878	0.048665	0.052581	0.056616	0.060761	0.065006
45	0.030720	0.033910	0.037268	0.040785	0.044453	0.048262	0.052202	0.056262	0.060431	0.064701
46	0.030251	0.033453	0.036827	0.040363	0.044051	0.047882	0.051845	0.055928	0.060122	0.064415
47	0.029803	0.033018	0.036407	0.039961	0.043669	0.047522	0.051507	0.055614	0.059831	0.064148
48	0.029375	0.032602	0.036006	0.039578	0.043306	0.047181	0.051189	0.055318	0.059559	0.063898
49	0.028965	0.032204	0.035623	0.039213	0.042962	0.046857	0.050887	0.055040	0.059302	0.063664
50	0.028572	0.031823	0.035258	0.038865	0.042634	0.046550	0.050602	0.054777	0.059061	0.063444

Example. The annual rent for 10 years which $3900 will purchase at an effective interest rate of 5 per cent is found in the following way. (*a*) Enter the table at the column headed 5 per cent and opposite $n = 10$ read 0.129505 ···, the rent purchased by 1. (*b*) Multiply the tabular value by 3900 to obtain the result.

A Sinking or Amortization Fund is a fund set aside to provide a definite sum (usually for canceling a debt) at a certain time; it is usually created by equal and regular contri-

Table 53. Annuity Which Will Amount to 1 (Sinking Fund)

$$\frac{1}{s_{\overline{n}|}} = \frac{i}{(1+i)^n - 1} \quad \text{or} \quad \frac{1}{a_{\overline{n}|}} - i$$

n	1 1/2%	2%	2 1/2%	3%	3 1/2%	4%	4 1/2%	5%	5 1/2%	6%
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	0.496278	0.495049	0.493827	0.492611	0.491400	0.490196	0.488998	0.487805	0.486618	0.485437
3	0.328383	0.326755	0.325137	0.323530	0.321934	0.320349	0.318773	0.317209	0.315654	0.314110
4	0.244445	0.242624	0.240818	0.239027	0.237251	0.235490	0.233744	0.232012	0.230294	0.228591
5	0.194089	0.192158	0.190247	0.188355	0.186481	0.184627	0.182792	0.180975	0.179176	0.177396
6	0.160525	0.158526	0.156550	0.154598	0.152668	0.150762	0.148878	0.147017	0.145179	0.143363
7	0.136556	0.134512	0.132495	0.130506	0.128544	0.126610	0.124701	0.122820	0.120964	0.119135
8	0.118584	0.116510	0.114467	0.112456	0.110477	0.108528	0.106610	0.104722	0.102864	0.101036
9	0.104610	0.102515	0.100457	0.098434	0.096446	0.094493	0.092574	0.090690	0.088839	0.087022
10	0.093434	0.091327	0.089259	0.087231	0.085241	0.083291	0.081379	0.079505	0.077668	0.075868
11	0.084294	0.082178	0.080106	0.078077	0.076092	0.074149	0.072248	0.070389	0.068571	0.066793
12	0.076680	0.074560	0.072487	0.070462	0.068484	0.066552	0.064666	0.062825	0.061029	0.059277
13	0.070240	0.068118	0.066048	0.064030	0.062062	0.060144	0.058275	0.056456	0.054684	0.052960
14	0.064723	0.062602	0.060536	0.058526	0.056571	0.054669	0.052820	0.051024	0.049279	0.047585
15	0.059944	0.057825	0.055766	0.053767	0.051825	0.049941	0.048114	0.046342	0.044626	0.042963
16	0.055765	0.053650	0.051599	0.049611	0.047685	0.045820	0.044015	0.042270	0.040583	0.038952
17	0.052080	0.049970	0.047928	0.045953	0.044043	0.042199	0.040418	0.038699	0.037042	0.035445
18	0.048806	0.046702	0.044670	0.042709	0.040817	0.038993	0.037237	0.035546	0.033920	0.032357
19	0.045878	0.043782	0.041760	0.039814	0.037940	0.036139	0.034407	0.032745	0.031150	0.029621
20	0.043246	0.041157	0.039147	0.037216	0.035361	0.033582	0.031876	0.030243	0.028679	0.027185
21	0.040865	0.038785	0.036787	0.034872	0.033037	0.031280	0.029601	0.027996	0.026465	0.025005
22	0.038703	0.036631	0.034646	0.032747	0.030932	0.029199	0.027546	0.025971	0.024471	0.023046
23	0.036731	0.034668	0.032696	0.030814	0.029019	0.027309	0.025682	0.024137	0.022670	0.021278
24	0.034924	0.032871	0.030913	0.029047	0.027273	0.025587	0.023987	0.022471	0.021036	0.019679
25	0.033263	0.031220	0.029276	0.027428	0.025674	0.024012	0.022439	0.020952	0.019549	0.018227
26	0.031732	0.029699	0.027769	0.025938	0.024205	0.022567	0.021021	0.019564	0.018193	0.016904
27	0.030315	0.028293	0.026377	0.024564	0.022852	0.021239	0.019719	0.018292	0.016952	0.015697
28	0.029001	0.026990	0.025088	0.023293	0.021603	0.020013	0.018521	0.017123	0.015814	0.014593
29	0.027779	0.025778	0.023891	0.022115	0.020445	0.018880	0.017415	0.016046	0.014769	0.013580
30	0.026639	0.024650	0.022778	0.021019	0.019371	0.017830	0.016392	0.015051	0.013805	0.012649
31	0.025574	0.023596	0.021739	0.019999	0.018372	0.016855	0.015443	0.014132	0.012917	0.011792
32	0.024577	0.022611	0.020768	0.019047	0.017442	0.015949	0.014563	0.013280	0.012095	0.011002
33	0.023641	0.021687	0.019859	0.018156	0.016572	0.015104	0.013745	0.012490	0.011335	0.010273
34	0.022762	0.020819	0.019007	0.017322	0.015760	0.014315	0.012982	0.011755	0.010630	0.009598
35	0.021934	0.020002	0.018206	0.016539	0.014998	0.013577	0.012270	0.011072	0.009975	0.008974
36	0.021152	0.019233	0.017452	0.015804	0.014284	0.012887	0.011606	0.010434	0.009366	0.008395
37	0.020414	0.018507	0.016741	0.015112	0.013613	0.012240	0.010984	0.009840	0.008800	0.007857
38	0.019716	0.017821	0.016070	0.014459	0.012982	0.011632	0.010402	0.009284	0.008272	0.007358
39	0.019055	0.017171	0.015436	0.013844	0.012388	0.011061	0.009856	0.008765	0.007780	0.006894
40	0.018427	0.016556	0.014836	0.013262	0.011827	0.010523	0.009343	0.008278	0.007320	0.006462
41	0.017831	0.015972	0.014268	0.012712	0.011298	0.010017	0.008862	0.007822	0.006891	0.006059
42	0.017264	0.015417	0.013728	0.012192	0.010798	0.009540	0.008409	0.007395	0.006489	0.005683
43	0.016725	0.014890	0.013217	0.011698	0.010325	0.009090	0.007982	0.006993	0.006113	0.005333
44	0.016210	0.014388	0.012730	0.011230	0.009878	0.008665	0.007581	0.006616	0.005761	0.005006
45	0.015720	0.013910	0.012268	0.010785	0.009453	0.008262	0.007202	0.006262	0.005431	0.004701
46	0.015251	0.013453	0.011827	0.010363	0.009051	0.007882	0.006845	0.005928	0.005122	0.004415
47	0.014803	0.013018	0.011407	0.009961	0.008669	0.007522	0.006507	0.005614	0.004831	0.004148
48	0.014375	0.012602	0.011006	0.009578	0.008306	0.007181	0.006189	0.005318	0.004559	0.003898
49	0.013965	0.012204	0.010623	0.009213	0.007962	0.006857	0.005887	0.005040	0.004302	0.003664
50	0.013572	0.011823	0.010258	0.008865	0.007634	0.006550	0.005602	0.004777	0.004061	0.003444

butions and their interest earnings, compounded, that is, by an annuity and its interest earnings. Denote the annual instalment or rent required by R, the amount of the sinking fund by S, the number of years for its creation by n, and the effective rate of interest by i. Then

$$R = \frac{S}{s_{\overline{n}|}} = \frac{Si}{(1+i)^n - 1} \tag{45}$$

Example. The annual instalment required to create a fund of $22,000 in 25 years at 4 per cent may be found in the following way: (*a*) Enter the table at the column headed 4 per cent and opposite $n = 25$ read 0.024012, the annual instalment to accumulate 1. (*b*) Multiply this tabular value by 22,000 to obtain the result.

A Life Annuity is one whose payments continue only during the lifetime of the recipient. Its cost and present value depend not only upon the rate of interest but also upon the *probability of living* and differs in this respect from an annuity certain.

Mortality tables (Table 54) are constructed from the experience of agencies such as insurance companies to indicate the probable deaths occurring in each succeeding year from a chosen initial group. Denote the age of the group by x, the number living by l_x,

Table 54. American Experience Table of Mortality

Age x	Number living l_x	Number of deaths d_x	Yearly probability of dying q_x	Yearly probability of living p_x	Age x	Number living l_x	Number of deaths d_x	Yearly probability of dying q_x	Yearly probability of living p_x
10	100,000	749	0.007 490	0.992 510	53	66,797	1,091	0.016 333	0.983 667
11	99,251	746	0.007 516	0.992 484	54	65,706	1,143	0.017 396	0.982 604
12	98,505	743	0.007 543	0.992 457	55	64,563	1,199	0.018 571	0.981 429
13	97,762	740	0.007 569	0.992 431	56	63,364	1,260	0.019 885	0.980 115
14	97,022	737	0.007 596	0.992 404	57	62,104	1,325	0.021 335	0.978 665
15	96,285	735	0.007 634	0.992 366	58	60,779	1,394	0.022 936	0.977 064
16	95,550	732	0.007 661	0.992 339	59	59,385	1,468	0.024 720	0.975 280
17	94,818	729	0.007 688	0.992 312	60	57,917	1,546	0.026 693	0.973 307
18	94,089	727	0.007 727	0.992 273	61	56,371	1,628	0.028 880	0.971 120
19	93,362	725	0.007 765	0.992 235	62	54,743	1,713	0.031 292	0.968 708
20	92,637	723	0.007 805	0.992 195	63	53,030	1,800	0.033 943	0.966 057
21	91,914	722	0.007 855	0.992 145	64	51,230	1,889	0.036 873	0.963 127
22	91,192	721	0.007 906	0.992 094	65	49,341	1,980	0.040 129	0.959 871
23	90,471	720	0.007 958	0.992 042	66	47,361	2,070	0.043 707	0.956 293
24	89,751	719	0.008 011	0.991 989	67	45,291	2,158	0.047 647	0.952 353
25	89,032	718	0.008 065	0.991 935	68	43,133	2,243	0.052 002	0.947 998
26	88,314	718	0.008 130	0.991 870	69	40,890	2,321	0.056 762	0.943 238
27	87,596	718	0.008 197	0.991 803	70	38,569	2,391	0.061 993	0.938 007
28	86,878	718	0.008 264	0.991 736	71	36,178	2,448	0.067 665	0.932 335
29	86,160	719	0.008 345	0.991 655	72	33,730	2,487	0.073 733	0.926 267
30	85,441	720	0.008 427	0.991 573	73	31,243	2,505	0.080 178	0.919 822
31	84,721	721	0.008 510	0.991 490	74	28,738	2,501	0.087 028	0.912 972
32	84,000	723	0.008 607	0.991 393	75	26,237	2,476	0.094 371	0.905 629
33	83,277	726	0.008 718	0.991 282	76	23,761	2,431	0.102 311	0.897 689
34	82,551	729	0.008 831	0.991 169	77	21,330	2,369	0.111 064	0.888 936
35	81,822	732	0.008 946	0.991 054	78	18,961	2,291	0.120 827	0.879 173
36	81,090	737	0.009 089	0.990 911	79	16,670	2,196	0.131 734	0.868 266
37	80,353	742	0.009 234	0.990 766	80	14,474	2,091	0.144 466	0.855 534
38	79,611	749	0.009 406	0.990 592	81	12,383	1,964	0.158 605	0.841 395
39	78,862	756	0.009 586	0.990 414	82	10,419	1,816	0.174 297	0.825 703
40	78,106	765	0.009 794	0.990 206	83	8,603	1,648	0.191 561	0.808 439
41	77,341	774	0.010 008	0.989 992	84	6,955	1,470	0.211 359	0.788 641
42	76,567	785	0.010 252	0.989 748	85	5,485	1,292	0.235 552	0.764 448
43	75,782	797	0.010 517	0.989 483	86	4,193	1,114	0.265 681	0.734 319
44	74,985	812	0.010 829	0.989 171	87	3,079	933	0.303 020	0.696 980
45	74,173	828	0.011 163	0.988 837	88	2,146	744	0.346 692	0.653 308
46	73,345	848	0.011 562	0.988 438	89	1,402	555	0.395 863	0.604 137
47	72,497	870	0.012 000	0.988 000	90	847	385	0.454 545	0.545 455
48	71,627	896	0.012 509	0.987 491	91	462	246	0.532 468	0.467 532
49	70,731	927	0.013 106	0.986 894	92	216	137	0.634 259	0.365 741
50	69,804	962	0.013 781	0.986 219	93	79	58	0.734 177	0.265 823
51	68,842	1,011	0.014 541	0.985 459	94	21	18	0.857 143	0.142 857
52	67,841	1,044	0.015 389	0.984 611	95	3	3	1.000 000	0.000 000

the number dying in the year interval from x to $x + 1$ by d_x, the probability of living during the year x by p_x, and the probability of dying within the year by q_x, then:

$$d_x = l_x - l_{x+1} \tag{46}$$

$$p_x = \frac{l_{x+1}}{l_x} \tag{47}$$

$$q_x = \frac{d_x}{l_x} = 1 - p_x \tag{48}$$

The probability of a person age x living n years is:

$$_np_x = \frac{l_{x+n}}{l_x} \tag{49}$$

and of not living n years is:

$$_nq_x = 1 - {}_np_x \tag{50}$$

Present Value of a Life Annuity. If p is the probability of receiving a certain sum of money S, the value of the *expectation* is pS and the present value of S due in n years is,

$$pS(1 + i)^{-n} = pSv^n \tag{51}$$

The present value of the expectation of a person, x years of age, who is to receive 1 if he lives n more years is called the *n-year pure endowment of 1* denoted by $_nE_x$.

$$_nE_x = {}_np_x \cdot v^n \tag{52}$$

Similarly, the present value of a *life annuity* of 1 beginning at age $x + m$ to a person of age x is,

$$a_{x+m} = \sum_{n=m}^{95-x} {}_nE_x \tag{53}$$

since 95 is the upper bound of age in the American Experience Table of Mortality.

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